

Process-independent determination of two-loop electroweak logarithms

Bernd Jantzen

RWTH Aachen University

In collaboration with **Ansgar Denner** and **Stefano Pozzorini**

- I Electroweak (EW) corrections at high energies
- II Two-loop next-to-leading logarithmic (NLL) corrections
- III Results for processes involving massless and massive fermions
- IV Summary & outlook

Nucl. Phys. B761 (2007) 1–62 [[arXiv:hep-ph/0608326](#)] (massless fermions)

JHEP 11 (2008) 062 [[arXiv:0809.0800 \[hep-ph\]](#)] (heavy quarks)

I Electroweak corrections at high energies

Precision collider physics

Precision measurements at colliders and theoretical predictions with electroweak (EW) + QCD corrections enable us to test the Standard Model at various energy scales:

- up to now (LEP, Tevatron) at energy scales $\lesssim M_{W,Z}$
- upcoming colliders (LHC, ILC/CLIC) \rightarrow reach TeV regime
 \hookrightarrow new energy domain $\sqrt{s} \gg M_W$ becomes accessible!

EW radiative corrections at high energies $\sqrt{s} \gg M_W$

\Rightarrow enhanced by large Sudakov logarithms

$$\text{per loop: } \ln^2 \left(\frac{s}{M_W^2} \right) \sim 25 \quad \text{at } \sqrt{s} \sim 1 \text{ TeV}$$

\hookrightarrow corrections rise with energy

Origin of large EW logs

- **mass singularities**: real or virtual emission of **soft/collinear gauge bosons** from external particles
- remnants from **UV singularities**

Massless gauge bosons

real emission of soft/collinear photons/gluons **cannot be detected separately**

↪ mass singularities cancel between real & virtual corrections (**KLN theorem**)

Massive gauge bosons

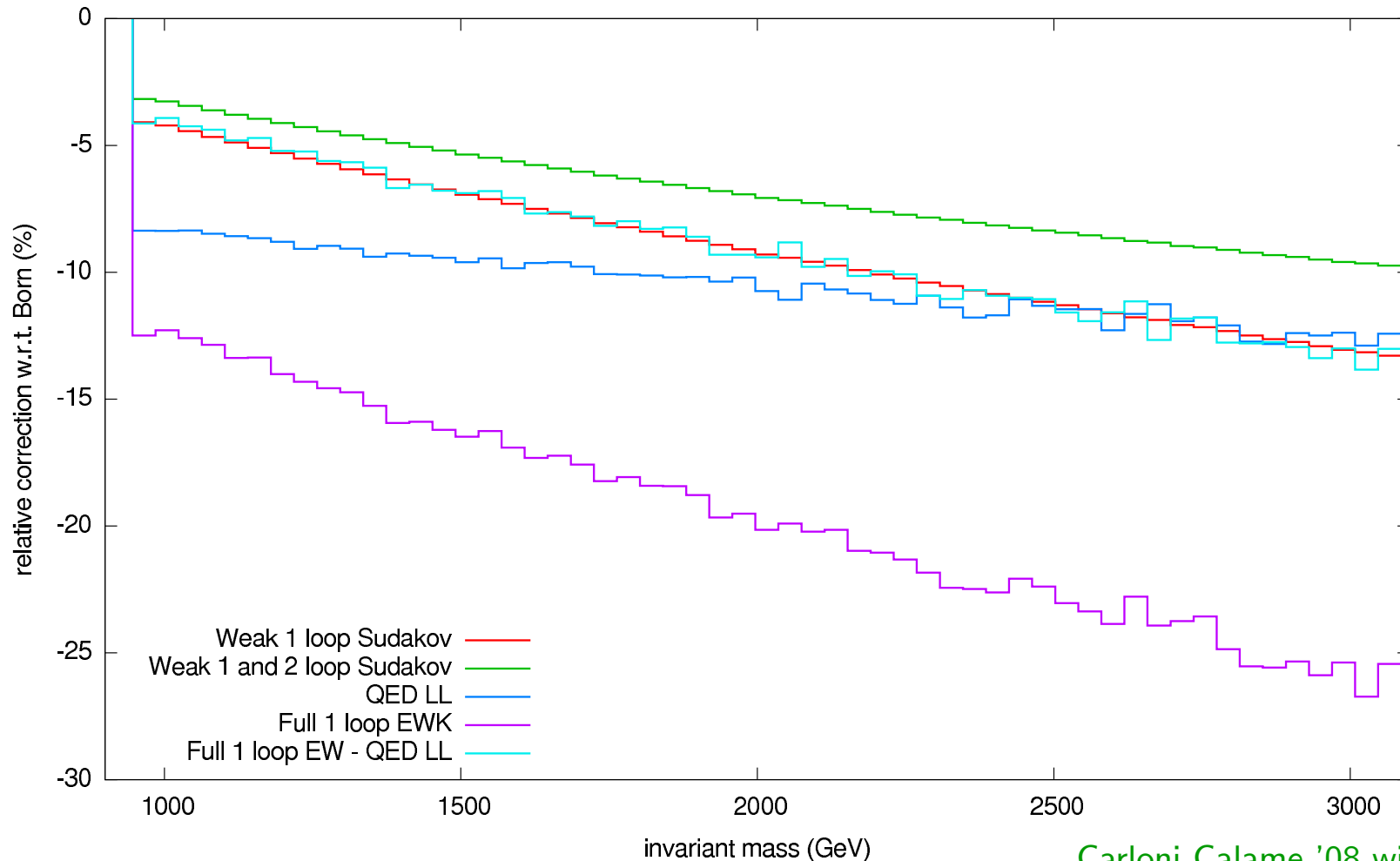
real emission of W 's, Z 's **can** (in principle) **be detected separately**

↪ **exclusive observables** (only **virtual corrections**): large logs remain present,

↪ inclusive observables: **Bloch–Nordsieck violations** \rightsquigarrow only partial cancellation

Virtual EW 1-/2-loop corrections at the LHC

Drell–Yan $pp \rightarrow \mu^+ \mu^-$: (electro)weak 1-loop & 2-loop corrections



Carlone Calame '08 with HORACE
and logarithmic ("Sudakov") results from
B.J., Kühn, Penin, Smirnov '05

⇒ logarithmic approximation very good at high energies

⇒ 2-loop effects $\sim \mathcal{O}(\%)$

cf. Les Houches 2007 report, arXiv:0803.0678 [hep-ph]

[Real emission of invisibly decaying W 's/ Z 's missing → partially cancels virtual corrections!]

Existing virtual EW 2-loop corrections

LL = leading logarithmic (\ln^4 at 2 loops), **NLL** = next-to-leading-logarithmic (\ln^3), ...

Resummation of 1-loop results using **evolution equations**:

$$\alpha^2 \left[\underbrace{C_{\text{LL}} \ln^4 \left(\frac{s}{M_W^2} \right)}_{\substack{\text{Fadin, Lipatov,} \\ \text{Martin, Melles '99}}} + \underbrace{C_{\text{NLL}} \ln^3 \left(\frac{s}{M_W^2} \right)}_{\text{Melles '00, '01}} + \underbrace{C_{\text{N}^2\text{LL}} \ln^2 \left(\frac{s}{M_W^2} \right)}_{\substack{\text{Kühn, Moch, Penin,} \\ \text{Smirnov '99–'01}}} + \underbrace{C_{\text{N}^3\text{LL}} \ln \left(\frac{s}{M_W^2} \right)}_{\substack{\text{B.J., Kühn, Moch, Penin,} \\ \text{Smirnov '03–'05}}} \right]$$

arbitrary processes
massless $f \bar{f} \rightarrow f' \bar{f}'$ processes

+ SCET approach [Chiu, Fuhrer, Golf, Hoang, Kelley, Manohar '07, '08, '09]

+ N²LL for $e^+ e^- \rightarrow W^+ W^-$ [Kühn, Metzler, Penin '07]

Explicit 2-loop calculations based on spontaneously broken $SU(2) \times U(1)$, $M_Z \neq M_W$:

$$\alpha^2 \left[\underbrace{C_{\text{LL}} \ln^4 \left(\frac{s}{M_W^2} \right)}_{\substack{\text{Melles '00;} \\ \text{Hori, Kawamura, Kodaira '00;} \\ \text{Beenakker, Werthenbach '00, '01}}} + \underbrace{C_{\text{NLL}}^{\text{ang}} \ln \left(\frac{t}{s} \right) \ln^3 \left(\frac{s}{M_W^2} \right)}_{\text{Denner, Melles, Pozzorini '03}} + \underbrace{C_{\text{NLL}}^{\text{rem}} \ln^3 \left(\frac{s}{M_W^2} \right)}_{\substack{\text{Pozzorini '04;} \\ \text{Denner, B.J., Pozzorini '06, '08}}} \right]$$

arbitrary processes
 n -fermion processes

II Two-loop next-to-leading logarithmic corrections

Goal: virtual 2-loop EW corrections for arbitrary processes in NLL accuracy

Process-independent: separate loop integrals from Born matrix elements

↪ already completed: processes involving massless & massive external fermions

Parameters:

$[D = 4 - 2\epsilon]$

- different large kinematical invariants $r_{ij} = (p_i + p_j)^2 \sim Q^2 \gg M_W^2$
- different heavy particle masses $M_W^2 \sim M_Z^2 \sim m_t^2 \sim M_{\text{Higgs}}^2$
- massive top quark, other fermions massless

⇒ logs $L = \ln \left(\frac{Q^2}{M_W^2} \right)$ and $\frac{1}{\epsilon}$ poles (from massless photons, counted like logs)

1 loop: LL $\rightarrow \epsilon^{-2}, L\epsilon^{-1}, L^2, L^3\epsilon, L^4\epsilon^2$; NLL $\rightarrow \epsilon^{-1}, L, L^2\epsilon, L^3\epsilon^2$

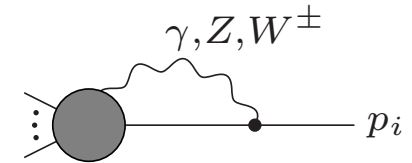
2 loops: LL $\rightarrow \epsilon^{-4}, L\epsilon^{-3}, L^2\epsilon^{-2}, L^3\epsilon^{-1}, L^4$; NLL $\rightarrow \epsilon^{-3}, L\epsilon^{-2}, L^2\epsilon^{-1}, L^3$

⇒ NLL coefficients involve small logs $\ln \left(\frac{-r_{ij}}{Q^2} \right)$ and $\ln \left(\frac{M_Z^2, m_t^2}{M_W^2} \right)$

Extraction of NLL contributions

Logs originate from **mass singularities** when a virtual gauge boson (γ, Z, W^\pm) couples to an **on-shell external leg**

→ **single log** from **collinear region** (+ UV logs)



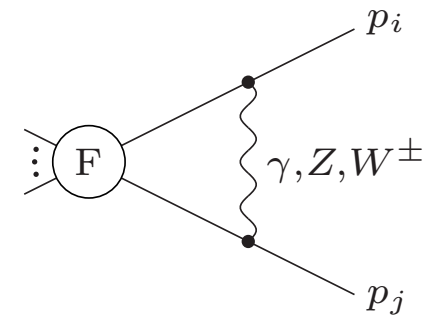
Isolate factorizable contributions:

gauge boson exchanged between external legs;

separate loop integral from Born diagram (F)

via **soft-collinear approximation**

→ **double log** from **soft & collinear region**



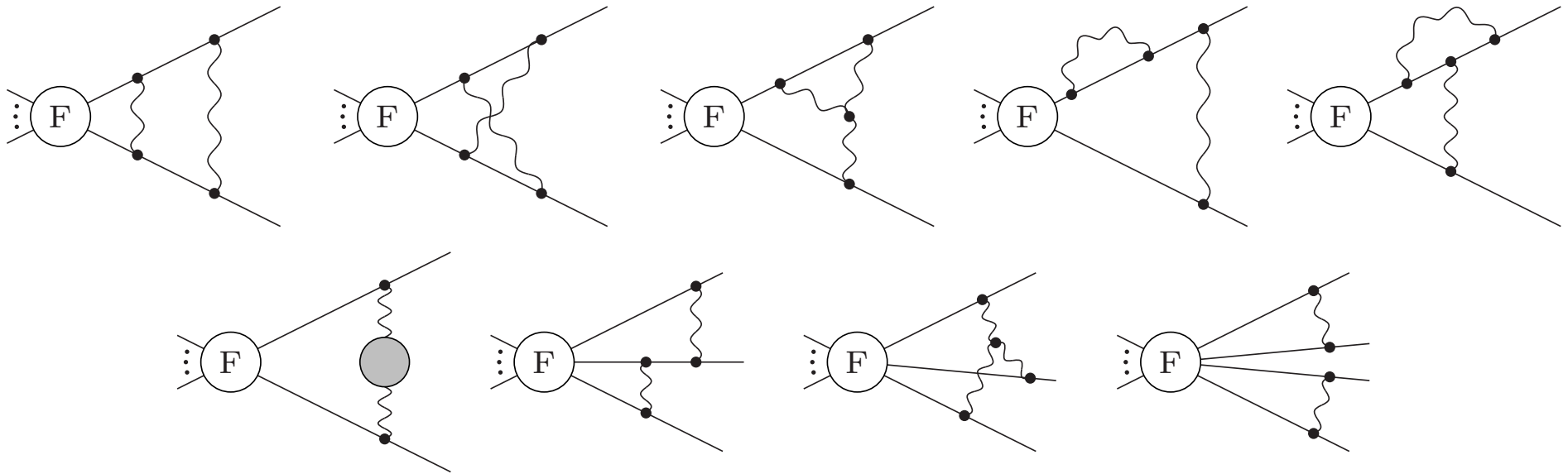
Remaining non-factorizable contributions: **collinear Ward identities**

Denner, Pozzorini '00, '01

An equation showing the cancellation of non-factorizable contributions. On the left, a grey circular blob with a wavy line and an external leg i is subtracted from another similar diagram. This is followed by a minus sign and a summation over $j \neq i$ of a diagram where a circular blob 'F' is connected to a vertex, which is then connected to a wavy line and an external leg i , with another external leg j also connected to the vertex. The result is shown as $\stackrel{\text{NLL}}{=} 0$.

The factorizable contributions contain all soft or collinear NLL mass singularities.

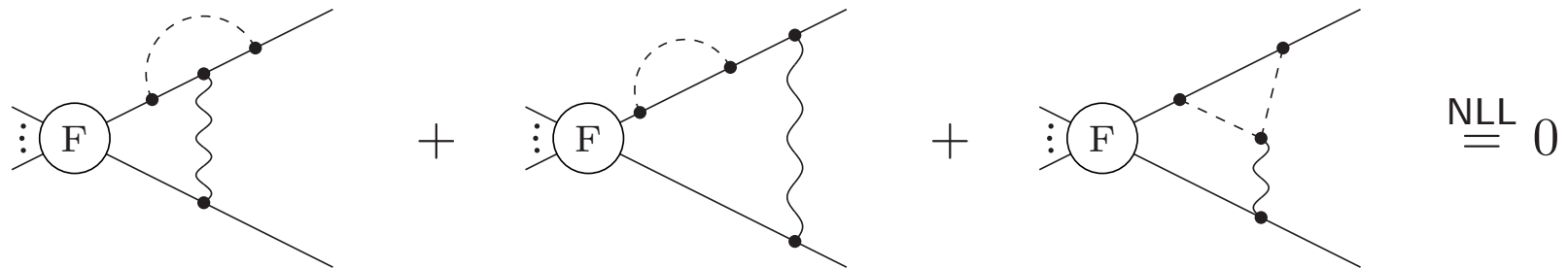
Factorizable contributions at 2 loops:



↪ non-factorizable contributions vanish

Yukawa couplings of massive fermions to Higgs & Goldstone bosons

↪ only three non-suppressed factorizable diagrams:



⇒ Sum vanishes due to **gauge invariance of Yukawa interaction**

↪ Yukawa interaction contributes only to wave-function renormalization

Treatment of ultraviolet (UV) singularities

UV $1/\epsilon$ poles in subdiagrams with scale μ_{loop}^2 & renormalization at scale μ_{R}^2 :

$$\underbrace{\frac{1}{\epsilon} \left(\frac{Q^2}{\mu_{\text{loop}}^2} \right)^\epsilon}_{\text{bare diagrams}} - \underbrace{\frac{1}{\epsilon} \left(\frac{Q^2}{\mu_{\text{R}}^2} \right)^\epsilon}_{\text{counterterms}} = \ln \left(\frac{\mu_{\text{R}}^2}{\mu_{\text{loop}}^2} \right) + \mathcal{O}(\epsilon) \quad \Rightarrow \quad \text{possible NLL contribution}$$

Perform **minimal UV subtraction** in UV-singular (sub)diagrams and counterterms:

$$\underbrace{\frac{1}{\epsilon} \left[\left(\frac{Q^2}{\mu_{\text{loop}}^2} \right)^\epsilon - 1 \right]}_{\text{bare diagrams}} - \underbrace{\frac{1}{\epsilon} \left[\left(\frac{Q^2}{\mu_{\text{R}}^2} \right)^\epsilon - 1 \right]}_{\text{counterterms}}$$

Advantages:

- no UV NLL terms from **hard** subdiagrams ($\mu_{\text{loop}}^2 \sim Q^2$)
 \hookrightarrow no UV contributions from **internal** parts of tree subdiagrams
- can use soft-collinear approximation (not valid in UV regime!)
 also for hard UV-singular subdiagrams

III Results for processes involving massless and massive fermions

Evaluation of factorizable contributions

Loop integrals calculated with two independent methods:

- automatized algorithm based on **sector decomposition** Denner, Pozzorini '04
- combination of **expansion by regions & Mellin–Barnes representations** B.J., Smirnov '06 & refs. therein

All relevant combinations of $\left\{ \begin{array}{l} \text{massless} \\ \text{massive} \end{array} \right\}$ $\left\{ \begin{array}{l} \text{external} \\ \text{internal} \end{array} \right\}$ fermions evaluated explicitly!

Result for amplitude of fermionic processes $f_1 f_2 \rightarrow f_3 \cdots f_n$ in $\mathcal{O}(\alpha^2)$

$$\mathcal{M} \stackrel{\text{NLL}}{=} \underbrace{\exp(\Delta F^{\text{em}})}_{\substack{\text{electromagnetic} \\ M_\gamma = 0}} \times \underbrace{\exp(F^{\text{sew}})}_{\substack{\text{symmetric-electroweak} \\ M_\gamma = M_Z = M_W}} \times \underbrace{(1 + \Delta F^Z)}_{\substack{\text{corrections} \\ \text{from } M_Z \neq M_W}} \times \underbrace{\mathcal{M}_0}_{\text{Born}}$$

- **universal** result: F^{sew} , ΔF^{em} , ΔF^Z depend only on external quantum numbers
- electromagnetic singularities (in ΔF^{em}) factorized \rightarrow separable

$$\mathcal{M} \stackrel{\text{NLL}}{=} \exp(\Delta F^{\text{em}}) \times \exp(F^{\text{sew}}) \times (1 + \Delta F^{\text{Z}}) \times \mathcal{M}_0$$

Symmetric-electroweak terms: independent of fermion masses

$$F^{\text{sew}} = \frac{1}{2} \sum_{i=1}^n \left\{ -\frac{\alpha}{4\pi} \left[\sum_{j \neq i} \sum_{V=\gamma, Z, W^\pm} \overbrace{I_i^{\bar{V}} I_j^V}^{\text{isospin matrices applied to external legs}} I_{ij}(\epsilon, M_W) + \overbrace{\frac{z_i^{\text{Yuk}} m_t^2}{4s_W^2 M_W^2} \left(L + \frac{1}{2} L^2 \epsilon + \frac{1}{6} L^3 \epsilon^2 + \mathcal{O}(\epsilon^3) \right)}^{\text{Yukawa contribution}} \right] \right. \\ \left. + \left(\frac{\alpha}{4\pi} \right)^2 \left[\frac{b_1^{(1)}}{c_W^2} \left(\frac{Y_i}{2} \right)^2 + \frac{b_2^{(1)}}{s_W^2} C_i^W \right] J_{ii}(\epsilon, M_W, \mu_R^2) \right\},$$

$$I_{ij}(\epsilon, M_W) \stackrel{\text{NLL}}{=} -L^2 - \frac{2}{3} L^3 \epsilon - \frac{1}{4} L^4 \epsilon^2 + \left[3 - 2 \ln \left(\frac{-r_{ij}}{Q^2} \right) \right] \left(L + \frac{1}{2} L^2 \epsilon + \frac{1}{6} L^3 \epsilon^2 \right) + \mathcal{O}(\epsilon^3),$$

$$J_{ij}(\epsilon, M_W, \mu^2) = \frac{1}{\epsilon} \left[I_{ij}(2\epsilon, M_W) - \left(\frac{Q^2}{\mu^2} \right)^\epsilon I_{ij}(\epsilon, M_W) \right]$$

Terms from $M_Z \neq M_W$:

$$\Delta F^{\text{Z}} \stackrel{\text{NLL}}{=} \frac{1}{2} \sum_{i=1}^n \frac{\alpha}{4\pi} (I_i^{\text{Z}})^2 \overbrace{\ln \left(\frac{M_Z^2}{M_W^2} \right) (2L + 2L^2 \epsilon + L^3 \epsilon^2)}^{= I_{ii}(\epsilon, M_Z) - I_{ii}(\epsilon, M_W)} + \mathcal{O}(\epsilon^3)$$

$$\mathcal{M} \stackrel{\text{NLL}}{=} \exp(\Delta F^{\text{em}}) \times \exp(F^{\text{sew}}) \times (1 + \Delta F^{\text{Z}}) \times \mathcal{M}_0$$

Electromagnetic terms:

$$[\mu_{\text{R}}^2 = M_{\text{W}}^2]$$

↪ correspond to (QED with $M_\gamma = 0$) minus (QED with $M_\gamma = M_{\text{W}}$)

$$\Delta F^{\text{em}} = \frac{1}{2} \sum_{i=1}^n \left\{ -\frac{\alpha}{4\pi} \sum_{j \neq i} Q_i Q_j \left[I_{ij}(\epsilon, 0) - I_{ij}(\epsilon, M_{\text{W}}) \right] + \left(\frac{\alpha}{4\pi} \right)^2 b_{\text{QED}}^{(1)} Q_i^2 \left[J_{ii}(\epsilon, 0, M_{\text{W}}^2) - J_{ii}(\epsilon, M_{\text{W}}, M_{\text{W}}^2) \right] \right\},$$

dependence on fermion mass m_i

$$I_{ij}(\epsilon, 0) \stackrel{\text{NLL}}{=} - \left[3 - 2 \ln \left(\frac{-r_{ij}}{Q^2} \right) \right] \epsilon^{-1} + \left\{ \overbrace{-\delta_{i,0} \epsilon^{-2} + \delta_{i,t}} \left[L \epsilon^{-1} + \frac{1}{2} L^2 + \frac{1}{6} L^3 \epsilon + \frac{1}{24} L^4 \epsilon^2 \right] + \left(\frac{1}{2} - \ln \left(\frac{m_i^2}{M_{\text{W}}^2} \right) \right) \left(\epsilon^{-1} + L + \frac{1}{2} L^2 \epsilon + \frac{1}{6} L^3 \epsilon^2 \right) \right] + (i \rightarrow j) \right\} + \mathcal{O}(\epsilon^3),$$

$$J_{ij}(\epsilon, 0, \mu^2) = \frac{1}{\epsilon} \left[I_{ij}(2\epsilon, 0) - \left(\frac{Q^2}{\mu^2} \right)^\epsilon I_{ij}(\epsilon, 0) \right]$$

Comparison to existing results

- previous results for **form factor** and **angular-dependent NLLs** reproduced and extended Denner, Melles, Pozzorini '03; Pozzorini '04
- agreement with general **resummation predictions** based on evolution equations Melles '00, '01
- agreement with **SCET results** Chiu, Golf, Kelley, Manohar '07, '08

Application to 4-fermion scattering

- **neutral current** $f\bar{f} \rightarrow f'\bar{f}'$: NLL-agreement with massless-fermion N³LL calculation, additional fermion-mass effects B.J., Kühn, Penin, Smirnov '05
- **charged current** $f_1\bar{f}_2 \rightarrow f_3\bar{f}_4$: new NLL result
- also applicable to processes with fermions and **gluons**, e.g. $g g \rightarrow f\bar{f}$:
gluons = legs with zero EW quantum numbers

IV Summary & outlook

Massless and massive fermionic processes $f_1 f_2 \rightarrow f_3 \cdots f_n$ (+ gluons)

with $(p_i + p_j)^2 \gg M_W^2$ and different masses $M_W^2 \sim M_Z^2 \sim m_t^2 \sim M_{\text{Higgs}}^2$:

- **2-loop EW NLL corrections** in $D = 4 - 2\epsilon$ dimensions
- loop integrals calculated with two independent methods
- Yukawa contributions only in wave-function renormalization
- **universal correction factors**, electromagnetic singularities separable
- **process-independent**: applicable for $e^+ e^- \rightarrow f \bar{f}$, Drell–Yan, $gg \rightarrow f \bar{f}$, ...

Outlook: EW corrections to arbitrary high-energy processes

- process-independent results for **all Standard-Model particles** possible at **1 loop**
Denner, Pozzorini '00, '01
- generalize 2-loop method for **external gauge & Higgs bosons**
- calculate relevant loop integrals \rightsquigarrow many already done for fermions

Extra slides

Virtual + Real W, Z emission: only partial cancellation