RG evolution of neutrino masses and mixing: Type-III seesaw

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Outline

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Introduction

- Neutrinos massless in SM

- Current oscillation data:

	Best fit	3σ range
Δm^2_{21} $[10^{-5}\mathrm{eV}^2]$	7.65	7.05 - 8.34
$\lvert\Delta m^2_{31}\rvert$ $[10^{-3}\mathrm{eV}^2]$	2.40	2.07 - 2.75
$\sin^2\theta_{12}$	0.304	0.25 - 0.37
$\sin^2\theta_{23}$	0.50	0.36 - 0.67
$\sin^2\theta_{13}$	0.01	$\leq$ 0.056

Schwetz, Tortolla, Valle (2008); Fogli, Lisi, Marrone, Palazzo and Rotunno, PRL **101** (2008);

Bandyopadhyay, Choubey, Goswami, Petcov and Roy, (2008)

Introduction

- Oscillation experiments:

$$\Delta m^2_{21} = 7.65^{+0.69}_{-0.60} \times 10^{-5}\ \text{eV}^2$$
$$|\Delta m^2_{31}| = 2.40^{+0.35}_{-0.33} \times 10^{-3}\ \text{eV}^2$$

- Cosmology: $\sum m_i < 1.3$ eV

Fogli, Lisi, Marrone, Melchiorri, Palazzo, Rotunno, Serra, Silk and Slosar, (2008)

- Neutrinos are MASSIVE, but $m_{\nu_i} \ll m_{\text{quark}}, m_{e,\mu,\tau}$

" Seesaw Mechanisms "

Introduction

- Oscillation experiments:

$$\sin^2\theta_{12} = 0.304^{+0.066}_{-0.054}$$
$$\sin^2\theta_{23} = 0.5^{+0.17}_{-0.14}$$
$$\sin^2\theta_{13} \leq 0.056$$

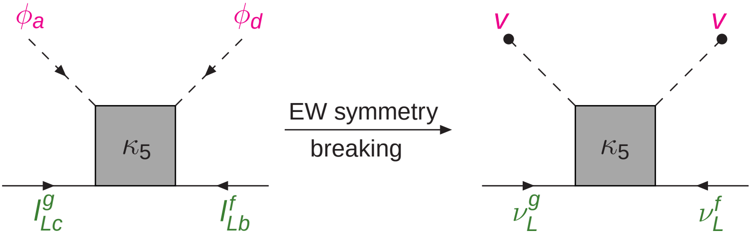
- Two mixing angles θ_{12}, θ_{23} large
- θ_{13} small, $\theta_{13} = 0$ allowed
- Very different from quark sector

" Renormalization Group (RG) Evolution "

The seesaw mechanism

- Heavy field present at high scale Λ
- At low scale, dimension-5 effective operator

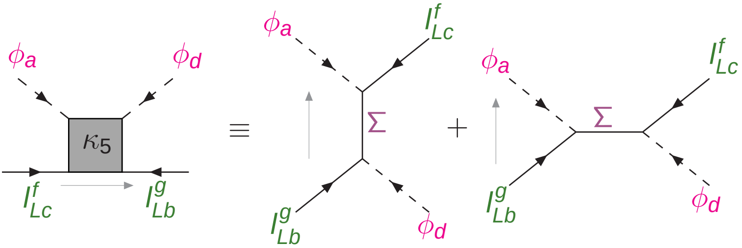
$$\mathcal{L}_5 = \kappa_5 l_L l_L \phi\phi \; ; \quad \kappa_5 = a_5/\Lambda$$



- Majorana mass
- κ_5: Depends on “Heavy particles” & their couplings

Seesaw with fermion triplets: Type-III seesaw

- l_L-ϕ form a fermion triplet
- Fermion triplet (Σ)



Type-III: Foot, Lew, He, Joshi (1989); Ma,Roy (2002); Bajc, Senjanovic, Nemevsek (2006, 2007); Perez *et al.* (2007, 2008); Abada,Biggio,Bonnet,Gavela, Hambye (2008); Mohapatra,Okada,Yu (2008); Fischler, Flauger (2008); Gogoladze, Okada, Shafi (2008); del Aguila, Aguiler-Saavedra, (2008) Hambye, Strumia (2008); Adhikari,Erler,Ma (2008)

Seesaw with fermion triplets: Type-III seesaw

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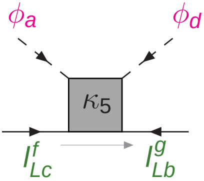
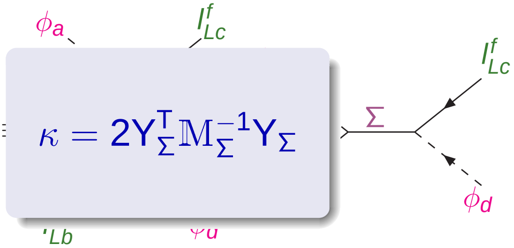
- Fermion triplet (Σ)





$$\kappa = 2 Y_\Sigma^T \mathbb{M}_\Sigma^{-1} Y_\Sigma$$

- $m_\nu = -\frac{v^2}{2} Y_\Sigma^T \mathbb{M}_\Sigma^{-1} Y_\Sigma$
- Neutrino mass ~ 0.1 eV
- $Y_\Sigma \sim 0.1 - 1 \Rightarrow M_\Sigma \sim \mathcal{O}(10^{11} - 10^{15})$ GeV

Seesaw with fermion triplets: Type-III seesaw

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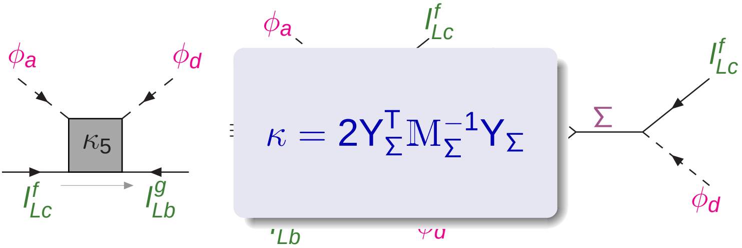
- Fermion triplet (Σ)



$$\kappa = 2Y_\Sigma^T \mathbb{M}_\Sigma^{-1} Y_\Sigma$$

- $m_\nu = -\frac{v^2}{2} Y_\Sigma^T \mathbb{M}_\Sigma^{-1} Y_\Sigma$
- Detectable at LHC $\Rightarrow M_\Sigma \sim$ TeV
- $Y_\Sigma \sim \mathcal{O}(10^{-6}) \Rightarrow$ too small
- Gauge interaction of $\Sigma \Rightarrow$ Detection at LHC

- RG evolution of neutrino masses & mixing parameters in Type-III seesaw
- Compatibility of the low energy data with some high energy scenarios

Based on:

Chakrabortty, Dighe, Goswami and SR, arXiv:0812.2776[hep-ph].

Outline

Type-III Seesaw

- Σ_R: U(1)$_Y$ singlet
- Σ_R: Adjoint rep. of SU(2)$_L$

$$\Sigma_R = \begin{pmatrix} \Sigma_R^0/\sqrt{2} & \Sigma_R^+ \\ \Sigma_R^- & -\Sigma_R^0/\sqrt{2} \end{pmatrix} \equiv \frac{\Sigma_R^i \sigma^i}{\sqrt{2}}$$

- $\Sigma \equiv \Sigma_R + \Sigma_R^C$: Adjoint rep.
- $\Sigma = \Sigma^C$
  - Diagonal entries neutral : Majorana
  - Off-diagonal entries: charged

Type-III Seesaw

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- $\Sigma \equiv \Sigma_R + \Sigma_R^C$: Adjoint rep.
- $\Sigma = \Sigma^C$
- $\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_\Sigma$
- $\mathcal{L}_\Sigma = \mathcal{L}_{\Sigma,kin} + \mathcal{L}_{\Sigma,mass} + \mathcal{L}_{\Sigma,Yuk}$

Type-III Seesaw

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$$\Downarrow$$

$$\mathrm{Tr}[\overline{\Sigma} i \not{D} \Sigma]$$

- $D_\mu \Sigma = \partial_\mu \Sigma + i g_2 [W_\mu, \Sigma]$
- New gauge boson coupling

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$$\Downarrow$$

$$\frac{1}{2}\,\mathrm{Tr}[\overline{\Sigma}\mathbb{M}_\Sigma\Sigma]$$

- The Majorana mass term $|\Delta L| = 2$

Type-III Seesaw

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- Σ_R: Adjoint rep. of $SU(2)_L$
- $\Sigma \equiv \Sigma_R + \Sigma_R^C$: Adjoint rep.
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$$\Downarrow$$

$$-\overline{l_L}\sqrt{2}Y_\Sigma^\dagger \Sigma \widetilde{\phi} + \text{h.c.}$$

- Yukawa coupling : Dirac mass term

Type-III Seesaw

- Σ_R: $U(1)_Y$ singlet
- Σ_R: Adjoint rep. of $SU(2)_L$
- $\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_\Sigma$

- Small seesaw mass for active neutrinos
- New coupling with gauge boson

- $\mathbb{M}_\Sigma = \text{diag}(M_1, M_2, \cdots, M_r)$; $\quad M_1 < M_2 < \cdots < M_r$
- $\mu < M_1 \Rightarrow \mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_5$

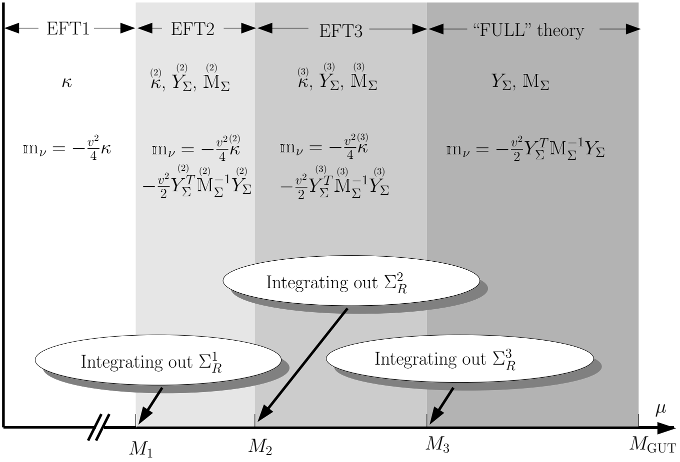
- High energy effects $\Rightarrow$ Renormalization Group evolution
- Non-degenerate triplets $\Rightarrow$ Threshold effects

Outline

Threshold effect

- Energy $\gg M_i \Rightarrow M_i$ coupled to theory
- Energy $\ll M_i \Rightarrow M_i$ decoupled
- Degrees of freedom changes
- Matching condition needed at the energy M_i

Threshold effects: sequential decoupling



- $\mathbb{M}_\Sigma$ evolves $\Rightarrow \mathsf{M_n} = \mathsf{M_n}(\mu)$

Outline

RG evolution

- Dimensional regularization
- Minimal subtraction scheme
- Calculations done in R_ξ gauge
- The β functions are gauge independent

Gauge coupling running

- Σ couples to gauge bosons $\Rightarrow$ Affects g_2 running
- $16\pi^2\beta_{g_2} = b_2 g_2^3$
- $\mathrm{M_{n\text{-}1}} < \mu < \mathrm{M_n} \Rightarrow$ $\boxed{b_2 = -\frac{19}{6} + \frac{4(n-1)}{3}}$
- b_2 changes sign for $n \geq 3$

Gauge coupling running

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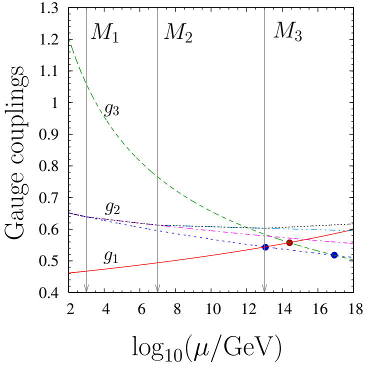

- Unification not realized in SM

Gauge coupling running

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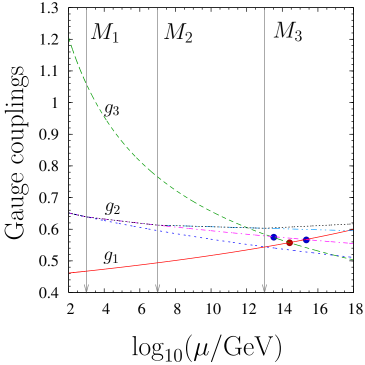

- g_1-g_2 $\Rightarrow$ higher μ
- g_2-g_3 $\Rightarrow$ lower μ
- Strong M_i dependence
- Unification may be achieved!

β functions – Couplings & $\mathbb{M}_\Sigma$

- RG evolution of Y_e and Y_Σ :

- $16\pi^2\beta_{Y_e} = Y_e\left(\frac{3}{2}Y_e^\dagger Y_e + \frac{15}{2}Y_\Sigma^\dagger Y_\Sigma + T - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2\right)$
- $16\pi^2\beta_{Y_\Sigma} = Y_\Sigma\left(\frac{5}{2}Y_e^\dagger Y_e + \frac{5}{2}Y_\Sigma^\dagger Y_\Sigma + T - \frac{9}{20}g_1^2 - \frac{33}{4}g_2^2\right)$
- $T = \mathrm{Tr}\left[Y_e^\dagger Y_e + 3Y_\Sigma^\dagger Y_\Sigma + 3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d\right]$
- $Y_\Sigma \to \overset{(n)}{Y}_\Sigma$

$$\beta_X = \mu dX/d\mu$$

- Y_u, Y_d and Higgs self-coupling running are modified

β functions – Couplings & $\mathbb{M}_\Sigma$

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- $T = \mathrm{Tr} \left[Y_e^\dagger Y_e + 3 Y_\Sigma^\dagger Y_\Sigma + 3 Y_u^\dagger Y_u + 3 Y_d^\dagger Y_d \right]$
- $Y_\Sigma \rightarrow \overset{(n)}{Y}_\Sigma$

- Running of fermion triplet mass:

- $16\pi^2 \beta_{\mathbb{M}_\Sigma} = \left(Y_\Sigma Y_\Sigma^\dagger \right) \mathbb{M}_\Sigma + \mathbb{M}_\Sigma \left(Y_\Sigma Y_\Sigma^\dagger \right)^T - 12 g_2^2 \mathbb{M}_\Sigma$
- Large $\mathbb{M}_\Sigma$ running expected

β function – Neutrino mass matrix m_ν

- In between thresholds $\Rightarrow$ No new contribution to κ
- $\kappa \Rightarrow$ Contribution from decoupled triplets
- $Q \Rightarrow$ Contribution from coupled triplets $(Q \equiv Y_\Sigma^T \, \mathbb{M}_\Sigma^{-1} Y_\Sigma)$

$$16\pi^2 \beta_\kappa = \alpha_\kappa \kappa + P_\kappa^T \kappa + \kappa P_\kappa$$

$$16\pi^2 \beta_Q = \alpha_Q Q + P_Q^T Q + Q P_Q$$

- $16\pi^2 \beta_{\mathrm{m}_\nu} = \alpha_\nu \mathrm{m}_\nu + P^T \mathrm{m}_\nu + \mathrm{m}_\nu P$
- $\mathrm{m}_\nu = -\frac{v^2}{4} (\kappa + 2Q)$
- $\mu > \mathrm{M}_r \Rightarrow P = P_Q$
- $\mu < \mathrm{M}_1 \Rightarrow P = P_\kappa$

Evolution of neutrino mass & mixing

- $U_\nu(\mu)^T \mathrm{m}_\nu(\mu) U_\nu(\mu) = \mathrm{diag}\left(m_1(\mu), m_2(\mu), m_3(\mu)\right)$
- $U_e(\mu)^\dagger Y_e^\dagger Y_e(\mu) U_e(\mu) = \mathrm{diag}\left(y_e^2(\mu), y_\mu^2(\mu), y_\tau^2(\mu)\right)$
- $U_{\mathrm{PMNS}}(\mu) = U_e^\dagger(\mu) U_\nu(\mu)$

- $\mathbb{M}_\Sigma$ and $Y_e^\dagger Y_e$ diagonal at high scale

- $16\pi^2 \beta_{Y_e} = Y_e \left(\frac{3}{2} Y_e^\dagger Y_e + \frac{15}{2} Y_\Sigma^\dagger Y_\Sigma + T - \frac{9}{4} g_1^2 - \frac{9}{4} g_2^2\right)$
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- $\mathbb{M}_\Sigma$ and $Y_e^\dagger Y_e \Rightarrow$ off-diagonal terms with evolution

Evolution of neutrino mass & mixing

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- $\mathbb{M}_\Sigma$ and $Y_e^\dagger Y_e$ diagonal at high scale

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- $\mathbb{M}_\Sigma$ and $Y_e^\dagger Y_e \Rightarrow$ off-diagonal terms with evolution

Basis Rotation

U_e contribution to U_{PMNS} evolution

Evolution of neutrino mass & mixing

- $U_\nu(\mu)^T \mathrm{m}_\nu(\mu) U_\nu(\mu) = \mathrm{diag}\left(m_1(\mu), m_2(\mu), m_3(\mu)\right)$
- $U_e(\mu)^\dagger Y_e^\dagger Y_e(\mu) U_e(\mu) = \mathrm{diag}\left(y_e^2(\mu), y_\mu^2(\mu), y_\tau^2(\mu)\right)$
- $U_{\rm PMNS}(\mu) = U_e^\dagger(\mu) U_\nu(\mu)$

- $U_{\rm PMNS} = \mathrm{diag}(e^{i\delta_e}, e^{i\delta_\mu}, e^{i\delta_\tau}) \,.\, \mathcal{U} \,.\, \mathrm{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1)$
- $\mathcal{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$
- "flavor" phases $\delta_{e,\mu,\tau} \Rightarrow$ No role in ν oscillations

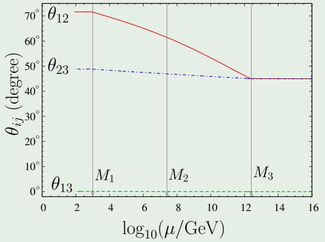
Evolution of neutrino mass & mixing

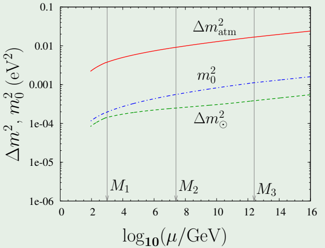
Typical running in Type-III seesaw

- High scale $\mu_0 = 10^{16}$ GeV
- $\mathbb{M}_\Sigma$, $Y_e^\dagger Y_e$ diagonal basis
- $U_{\rm PMNS} = U_\nu$
- Three heavy fermion triplets: $M_1 < M_2 < M_3$
- $M_1 \sim 10^3$ GeV $\Rightarrow$ LHC
- Normal mass ordering

Example I: Bimaximal mixing at μ_0

Example

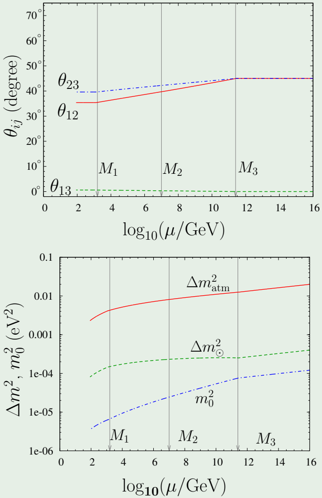




- $\phi_i = 0$
- No CP violation at all energy
- $\theta_{12} \uparrow$ between thresholds
- θ_{12} NOT compatible
- Δm^2 compatible
- m_0 small ~ 0.04 eV
- Running is quite large

Example I: Bimaximal mixing at μ_0

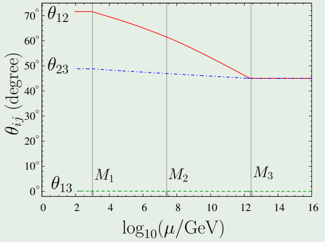
Example



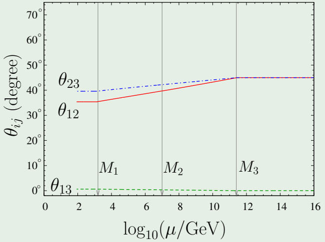
- $\phi_1 \sim 89°$; $\phi_2 \sim 0.4°$
- θ_{12} COMPATIBLE !
- Δm^2 compatible
- m_0 small ~ 0.01 eV
- Running is quite large

Example I: Bimaximal mixing at μ_0

Example



$\phi_1 = \phi_2 = 0$



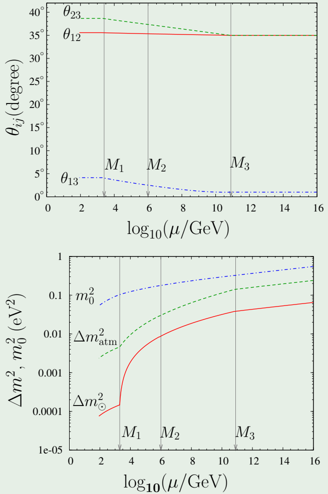
$\phi_1 \sim 89°;\ \phi_2 \sim 0.4°$

- CP violation $\Rightarrow$ crucial
- Threshold effects important

Bimaximal $\Rightarrow$ Compatible at low energy

Example II: Quasi-degenerate ν at μ_0

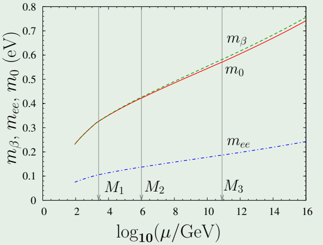
Example

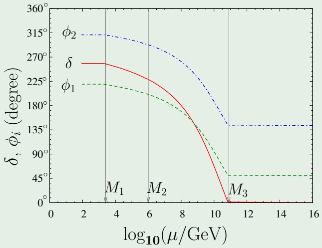


- $m_0 \gtrsim \sqrt{\Delta m^2_{\text{atm}}}$
- Large θ_{23} running
- Phase: $|\phi_1 - \phi_2| \approx \pi/2$
- θ_{12} running suppressed
- Large $\Delta m^2_{\odot}$ running

Example II: Quasi-degenerate ν at μ_0

Example





- $m_\beta \equiv \sqrt{\sum_i |U_{ei}|^2 m_i^2}$
  $\Rightarrow \beta$ decay
- Large m_β running
- $m_{ee} \equiv |\sum_i U_{ei}^2 m_i| \Rightarrow 0\nu\beta\beta$
- $|\phi_1 - \phi_2| \approx \pi/2$
  $\Rightarrow m_{ee} \approx m_0 \cos 2\theta_{12}$
- Large δ generated from zero
- Large ϕ_i running

Outline

Summary

- RG evolution in Type-III seesaw scenario
  - New contribution to evolution of SU(2)$_L$ coupling g_2
  - Running of Higgs self-coupling λ & Yukawa couplings
  - Large M_Σ running
- Evolution of neutrino masses and mixing in Type-III seesaw
- Bimaximal mixing at the high scale
  - Threshold effects important
  - Crucial role of Majorana phases
  - Large running of masses
- Quasi-degenerate neutrino at the high scale
  - Large $\Delta m^2_\odot$ and m_β running
  - Small θ_{12} running
  - Generation of large δ from zero at high scale

THANK YOU

Detection of fermion triplets at LHC

- LHC $\Rightarrow$ CM energy 14 TeV back
- $\mathrm{M_i} \sim$ TeV $\Rightarrow Y \sim 10^{-6}$
- Production:
  - $q\overline{q} \to Z^*/\gamma^* \to \Sigma^+\Sigma^-$
  - $q\overline{q'} \to W^* \to \Sigma^\pm\Sigma^0$
- Detection:

$\Sigma^0 \to l^\mp W\pm$	$\Sigma^\pm \to \overline{\nu} W^\pm$
$\Sigma^0 \to \nu Z$	$\Sigma^\pm \to l^\pm Z$
$\Sigma^0 \to \nu\phi$	$\Sigma^\pm \to l^\pm \phi$

Detection of fermion triplets at LHC

- Signals exclusive to Type-III seesaw: back
  - Six lepton:Branching ratio too small
  - Five lepton or $l^{\pm}l^{\pm}l^{\pm}$: observable at 30 fb^{-1} at 300 GeV
  - $l^{\pm}l^{\pm}l^{\pm}l^{\mp}(Q = \pm 2)$: observable at 15 fb^{-1}
- Signals common to Type-II and Type-III:
  - Five lepton or $l^{\pm}l^{\pm}l^{\mp}l^{\mp}$: E production at 6 fb^{-1} (5σ)
  - $l^{\pm}l^{\pm}l^{\mp}$: observable at $<$ 3 fb^{-1}
  - $l^{\pm}l^{\pm}$ – like-sign dilepton: discovery potential same as $l^{\pm}l^{\pm}l^{\mp}$

B. Bajc and G. Senjanovic, JHEP **0708**, 014 (2007) [arXiv:hep-ph/0612029].

B. Bajc, M. Nemevsek and G. Senjanovic, Phys. Rev. D **76**, 055011 (2007) [arXiv:hep-ph/0703080].