Non zero electron EDM from TeV Majorana neutrinos

Saif Rayyan — Beloit College

Tatsu Takeuchi — Virginia Tech

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- Content:

  – CP violation and EDM.
  – Diagrams with Dirac Neutrinos.
  – Diagrams with Majorana neutrinos.
  – Large mixing→ large contribution.
  – THE CALCULATION.
  – Results.

EDM and CP violation

- Interaction between spin and electro magnetic field: Electric and Magnetic dipole moments:

- Electric dipole moment: $d\,\vec{\sigma}\cdot\vec{E}$

- Magnetic dipole moment: $\mu\,\vec{\sigma}\cdot\vec{B}$

- Under C,P,T:

$$\begin{array}{ccccccc}
\vec{\sigma} & \xrightarrow{P} & \vec{\sigma} & \xrightarrow{C} & -\vec{\sigma} & \xrightarrow{T} & +\vec{\sigma} \\
\vec{B} & \xrightarrow{P} & \vec{B} & \xrightarrow{C} & -\vec{B} & \xrightarrow{T} & +\vec{B} \\
\vec{E} & \xrightarrow{P} & -\vec{E} & \xrightarrow{C} & +\vec{E} & \xrightarrow{T} & +\vec{E}
\end{array}$$

- A non zero edm violates CP.

•The most general form of the interaction of leptons with a vector boson (electromagnetic field):

$$\overline{u}(p_2)\,\Gamma^\mu(p_1,p_2)\,u(p_1)$$
$$= \overline{u}(p_2)\left[\{F_1(q^2)+G_1(q^2)\gamma_5\}\,\gamma^\mu + \{F_2(q^2)+G_2(q^2)\gamma_5\}\,\frac{i\sigma^{\mu\nu}q_\nu}{2m} + \{F_3(q^2)+G_3(q^2)\gamma_5\}\,q^\mu\right]u(p_1)\,,$$

•The G_2 term is related to the EDM:

$$\frac{G_2(q^2)}{2m}\gamma_5\,i\sigma^{\mu\nu}q_\nu A_\mu \longrightarrow \frac{G_2(q^2)}{2mi}\left(\vec{\sigma}\cdot\vec{E}\right);$$

•Comparing with: $d\,\vec{\sigma}\cdot\vec{E}$

•The EDM is $d = \frac{G_2(0)}{2mi}$

•The form factors can be written as:

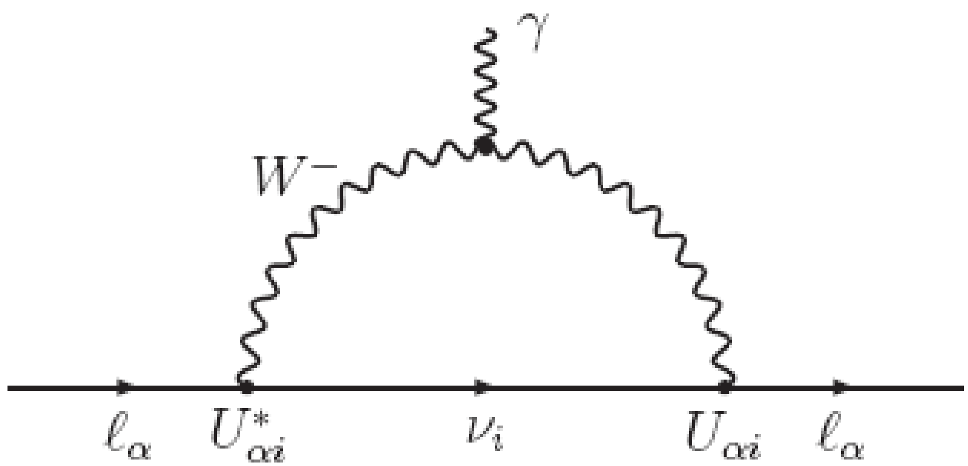
$$\bar{u}_2(p_2)\,\Gamma^\mu(p_1,p_2)\,u_1(p_1)$$
$$= \bar{u}_2(p_2)\left[\{(F_1-F_2)+G_1\gamma_5\}\,\gamma^\mu + \{F_2+G_2\gamma_5\}\,\frac{(p_1+p_2)^\mu}{2m} + \{F_3+G_3\gamma_5\}\,q^\mu\right]u_1(p_1)\,,$$

•So what we need is to pick the terms proportional to: $\gamma_5(p_1+p_2)^\mu$.

•Pick the imaginary part.

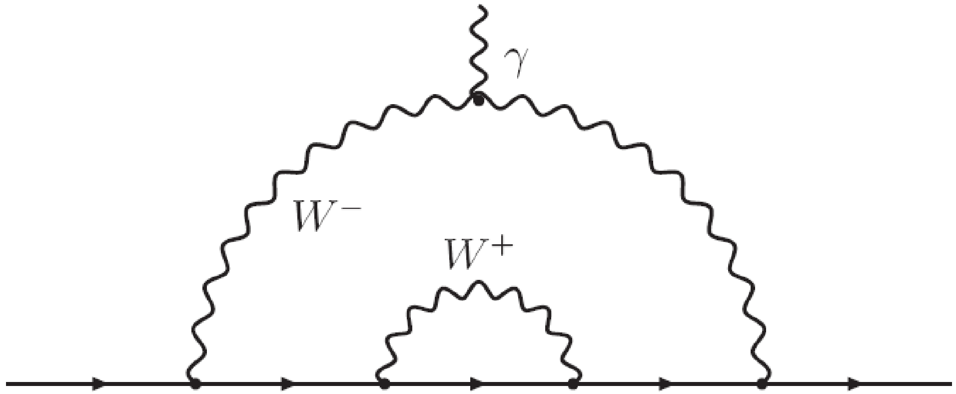
Neutrino mixing → EDM?

One Loop- Dirac case:



- one loop→U* U= real, no EDM.

Two Loops-Dirac case:



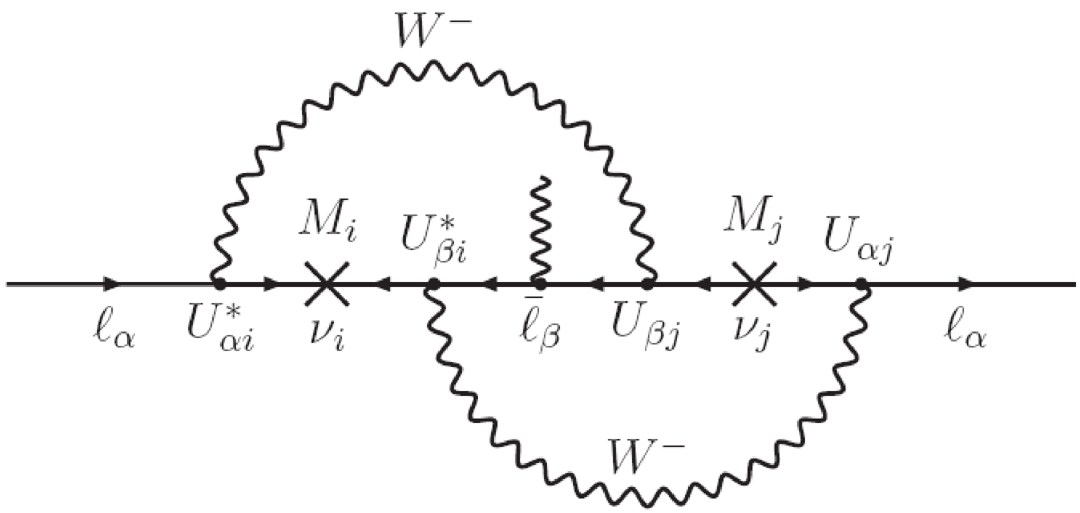
$$\propto (U^*_{\alpha i}U_{\beta i})(U_{\alpha j}U^*_{\beta j})$$

- The diagrams is symmetric under interchange of the masses.
- Sum over i,j and j,i

$$\begin{aligned} & (U^*_{\alpha i}U_{\beta i})(U_{\alpha j}U^*_{\beta j}) + (U^*_{\alpha j}U_{\beta j})(U_{\alpha i}U^*_{\beta i}) \\ = & (U^*_{\alpha i}U_{\beta i})(U_{\alpha j}U^*_{\beta j}) + ((U^*_{\alpha i}U_{\beta i})(U_{\alpha j}U^*_{\beta j}))^* \\ = & 2Re(U^*_{\alpha i}U_{\beta i})(U_{\alpha j}U^*_{\beta j}) \end{aligned}$$

- The amplitude is real, no cp violation

Two Loops- Majorana case:



$$\propto (U^*_{\alpha i} U^*_{\beta i})(U_{\alpha j} U_{\beta j})$$

- Diagrams are anti symmetric in i,j.
- Leads to non zero EDM. Depends on Masses and couplings.

Seesaw mechanism: One generation:

- If we consider a regular one generation seesaw, we only have two parameters to start with: the Dirac mass m and the Heavy neutrinos Majorana Mass M.

$$\begin{bmatrix} \nu & \chi \end{bmatrix} \begin{bmatrix} 0 & m \\ m & M \end{bmatrix} \begin{bmatrix} \nu \\ \chi \end{bmatrix}$$

- After diagonalization we have three quantities:

$$\begin{bmatrix} n & N \end{bmatrix} \begin{bmatrix} \frac{m^2}{M} & 0 \\ 0 & M \end{bmatrix} \begin{bmatrix} n \\ N \end{bmatrix}$$

  - the mass of the light neutrinos
  - the mass of the heavy neutrinos
  - The mixing angle:.

$$\begin{bmatrix} \nu \\ \chi \end{bmatrix} = \begin{bmatrix} icos(\theta)\,n + sin(\theta)\,N \\ -isin(\theta)\,n + cos(\theta)\,N \end{bmatrix}$$

$$\theta = \frac{m}{M} \,;\, m = 100GeV \,;\, M = 10^{16}GeV \,;\, \theta = 10^{-14}$$

Two generation, special case:

$$\left[\, \nu_1\ \nu_2\ \chi_1\ \chi_2 \,\right] \begin{bmatrix} 0 & 0 & m & -m \\ 0 & 0 & m & -m \\ m & m & M & 0 \\ -m & -m & 0 & -M \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \chi_1 \\ \chi_2 \end{bmatrix} .$$

$$O^{\mathrm{T}} \begin{bmatrix} 0 & 0 & m & -m \\ 0 & 0 & m & -m \\ m & m & M & 0 \\ -m & -m & 0 & -M \end{bmatrix} O = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & m/sin(\theta)cos(\theta) & 0 \\ 0 & 0 & 0 & -m/sin(\theta)cos(\theta) \end{bmatrix}$$

• The mass of the light neutrinos is always zero→ we can adjust the masses and the mixings independently.

$$\begin{aligned} m_{\text{light}} &= 0 \\ m_{\text{heavy}} &= \pm\sqrt{M^2 + 4m^2} \approx \pm M \\ \theta^2 &= \frac{m^2}{M^2} \end{aligned}$$

Okamura texture

$$\left[\,\nu_1\ \nu_2\ \nu_3\ \chi_1\ \chi_2\ \chi_3\,\right]\begin{bmatrix} 0 & 0 & 0 & \alpha m & \beta m & \gamma m \\ 0 & 0 & 0 & \alpha m & \beta m & \gamma m \\ 0 & 0 & 0 & \alpha m & \beta m & \gamma m \\ \alpha m & \alpha m & \alpha m & \alpha M & 0 & 0 \\ \beta m & \beta m & \beta m & 0 & \beta M & 0 \\ \gamma m & \gamma m & \gamma m & 0 & 0 & \gamma M \end{bmatrix}\begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \chi_1 \\ \chi_2 \\ \chi_3 \end{bmatrix}$$

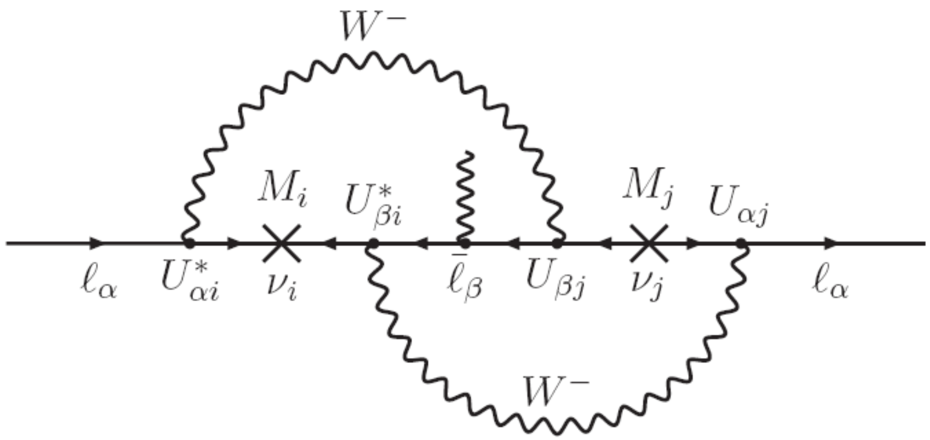
$$\alpha + \beta + \gamma = 0$$

- The generalization to 3 generations can be given by the Okamura texture [hep-ph/0304004]. Also Glashow [hep-ph/0306100]
- large mixing (θ=0.055) ?
- Interesting phenomenology! [hep-ph/0403306]

diagonalize

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_3 \end{bmatrix}$$

Estimates?



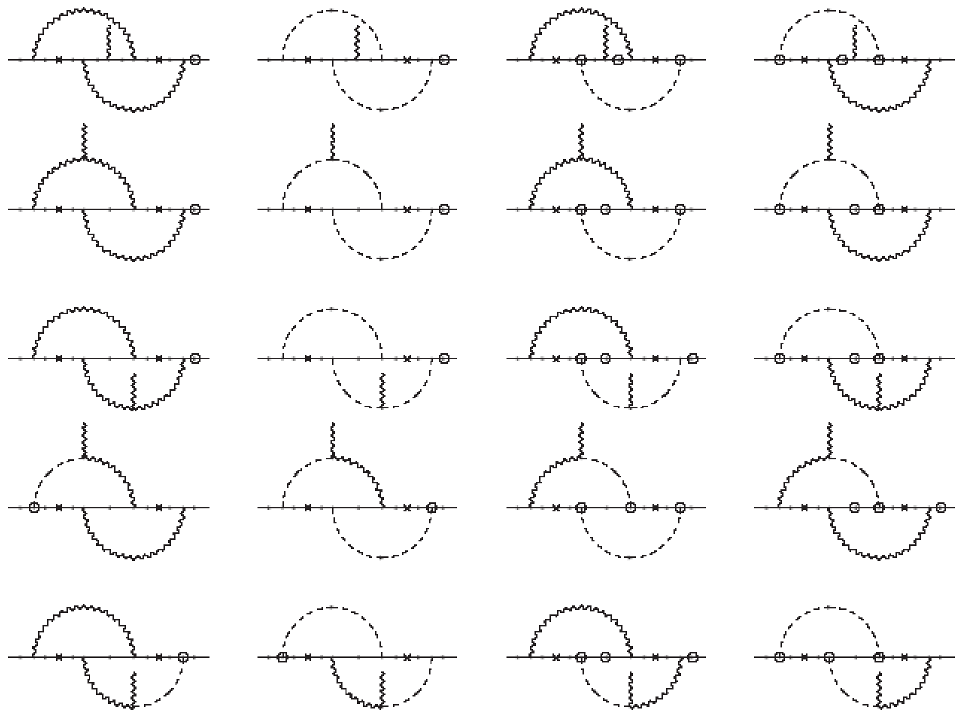
$\propto (U^*_{\alpha i} U^*_{\beta i})(U_{\alpha j} U_{\beta j})$

$$\mathrm{d}_e \approx e m_e G_F d M \theta^4$$

$$d_e \approx e m_e G_F d M \frac{D^4}{M^4}$$

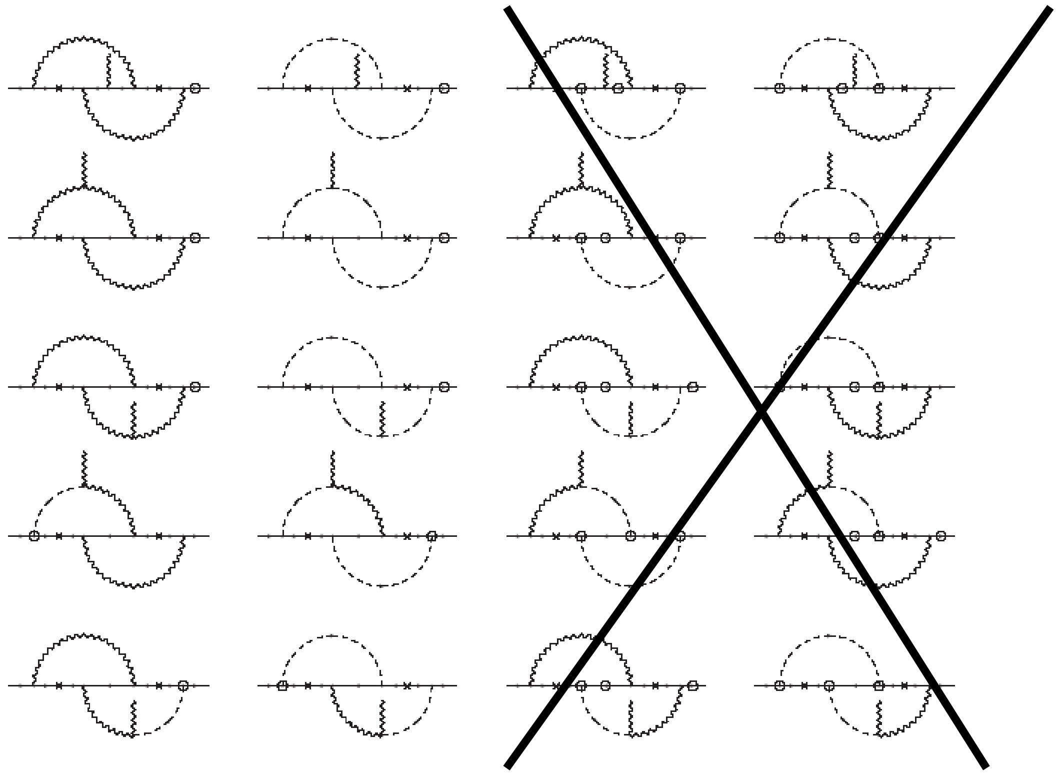
- large mixing→ large EDM

The Calculation

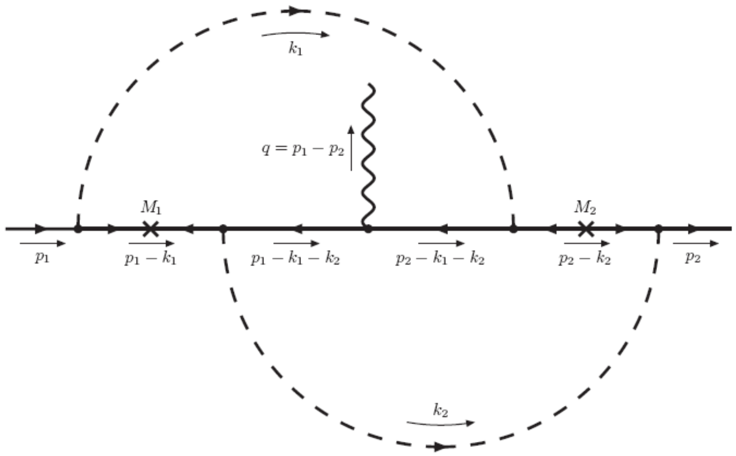


•In the Feynman-t'Hoof gauge we have 20 diagrams.

The Calculation



•10 diagrams will be suppressed.



- Integrate over loop momenta → Passarino Veltman Functions

$$
\begin{aligned}
&A_1\, p_1^{\kappa} + A_2\, p_2^{\kappa} \\
&= \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{K^{\kappa}}{(p_1-K)^2(p_2-K)^2(k_1^2-M_W^2)(k_2^2-M_W^2)[(p_1-k_1)^2-M_1^2][(p_2-k_2)^2-M_2^2]} \\
&= \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{K^{\kappa}}{(K^2-2p_1\cdot K)(K^2-2p_2\cdot K)(k_1^2-M_W^2)(k_2^2-M_W^2)[(k_1^2-M_1^2)-2p_1\cdot k_1]} \\
&\qquad \times \frac{1}{[(k_2^2-M_2^2)-2p_2\cdot k_2]}
\end{aligned}
$$

$$
\begin{aligned}
&(A_1 - A_2) \\
&= (\tilde{A}_1 - \tilde{A}_2) \\
&= \frac{1}{2}\int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)} \left(\frac{K\cdot k_1}{k_1^2-M_1^2} - \frac{K\cdot k_2}{k_2^2-M_2^2}\right) .
\end{aligned}
$$

- After many many many of them → Lots of logs and dilogs.

Okamura- Model:

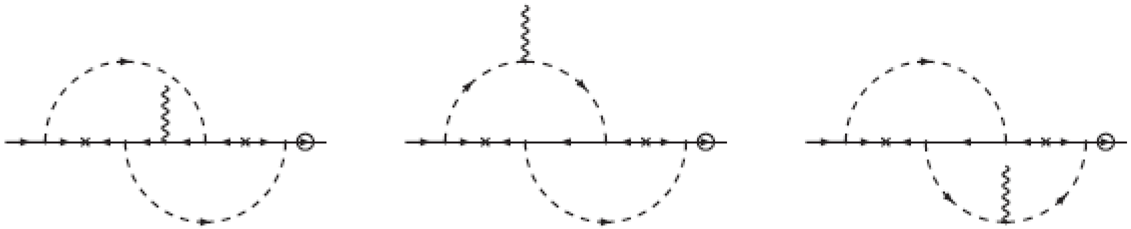


Diagram 1B Diagram 2B Diagram 3B

- The coupling in the diagrams→ $\left(\tilde{\Lambda}_{\alpha 2}\tilde{\Lambda}_{\beta 2}\tilde{\Lambda}^*_{\beta 1}\tilde{\Lambda}^*_{\alpha 1}\right)$
- In the Okamura texture the coupling is independent from the mixing→ largest contribution.

$$d_{1\mathrm{B}} = +2e\,m\sum_{i,j,\beta} M_i M_j \left(i\tilde{\Lambda}^*_{\alpha i}\tilde{\Lambda}^*_{\beta i}\tilde{\Lambda}_{\beta j}\tilde{\Lambda}_{\alpha j}\right)\left[(A_1 - A_2) - (B_1 - B_2)\right]_{(M_i,M_j)}$$

$$d_{2\mathrm{B}} = -e\,m\sum_{i,j,\beta} M_i M_j \left(i\tilde{\Lambda}^*_{\alpha i}\tilde{\Lambda}^*_{\beta i}\tilde{\Lambda}_{\beta j}\tilde{\Lambda}_{\alpha j}\right)\left[\frac{(C_1 - C_2) - (D_{11} - D_{22}) + (D_{12} - D_{21})}{2}\right]_{(M_i,M_j)}$$

$$d_{3\mathrm{B}} = +e\,m\sum_{i,j,\beta} M_i M_j \left(i\tilde{\Lambda}^*_{\alpha i}\tilde{\Lambda}^*_{\beta i}\tilde{\Lambda}_{\beta j}\tilde{\Lambda}_{\alpha j}\right)\left[\frac{(\overline{C}_1 - \overline{C}_2) - (\overline{D}_{11} - \overline{D}_{22}) + (\overline{D}_{12} - \overline{D}_{21})}{2}\right]_{(M_i,M_j)}$$

- The sum will pick the imaginray antisymmetric part:

$$d_{1\mathrm{B}} + d_{2\mathrm{B}} + d_{3\mathrm{B}} = +e\,m\sum_{i,j,\beta} M_i M_j \left(i\tilde{\Lambda}^*_{\alpha i}\tilde{\Lambda}^*_{\beta i}\tilde{\Lambda}_{\beta j}\tilde{\Lambda}_{\alpha j}\right) R_2(M_i, M_j)\ ;$$

$$= +2e\,m\sum_{\beta}\sum_{i>j} M_i M_j\, \Im\left(\tilde{\Lambda}_{\alpha i}\tilde{\Lambda}_{\beta i}\tilde{\Lambda}^*_{\beta j}\tilde{\Lambda}^*_{\alpha j}\right) R_2(M_i, M_j)\ .$$

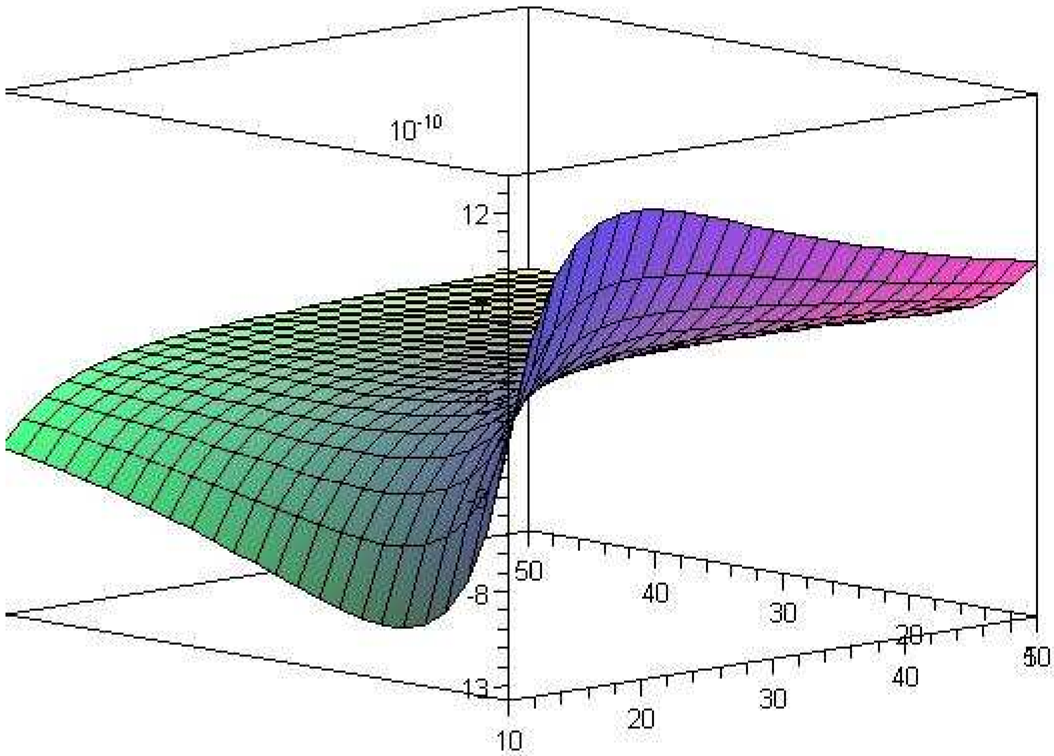
$$
\begin{aligned}
&R_2 \\
&= \frac{1}{12(4\pi)^4 M_W^4 (M_1^2-1)^3(M_2^2-1)^3} \times \\
&\Big[2(M_1^2-1)(M_2^2-1)(M_1^2-M_2^2)\Big\{-9+4(M_1^2+M_2^2)\Big\} \\
&+M_1^2(M_2^2-1)\Big\{17-(15M_1^2+19M_2^2)+(8M_1^4+M_1^2M_2^2+8M_2^4)\Big\}\log M_1^2 \\
&-M_2^2(M_1^2-1)\Big\{17-(15M_2^2+19M_1^2)+(8M_2^4+M_2^2M_1^2+8M_1^4)\Big\}\log M_2^2 \\
&+\Big\{5-(3M_1^2+12M_2^2)+(10M_1^2M_2^2+5M_2^4)-5M_1^2M_2^4\Big\}\mathrm{Li}_2(1-M_1^2) \\
&-\Big\{5-(3M_2^2+12M_1^2)+(10M_2^2M_1^2+5M_1^4)-5M_2^2M_1^4\Big\}\mathrm{Li}_2(1-M_2^2) \\
&+M_1^2\Big\{5-(14M_1^2+M_2^2)+(19M_1^4-4M_1^2M_2^2)-(8M_1^6-3M_1^4M_2^2)\Big\}\left\{\mathrm{Li}_2\left(1-\frac{1}{M_1^2}\right)-\mathrm{Li}_2\left(1-\frac{M_2^2}{M_1^2}\right)\right\} \\
&-M_2^2\Big\{5-(14M_2^2+M_1^2)+(19M_2^4-4M_2^2M_1^2)-(8M_2^6-3M_2^4M_1^2)\Big\}\left\{\mathrm{Li}_2\left(1-\frac{1}{M_2^2}\right)-\mathrm{Li}_2\left(1-\frac{M_1^2}{M_2^2}\right)\right\}\Big]
\end{aligned}
$$

• Since the function is antisymmetric in the masses we can expand in terms of their difference (and sum):

$$
R_2 = +\frac{1}{(4\pi)^4 M_W^4}\left(\frac{688}{3}-\frac{208\pi^2}{9}\right)\frac{d}{M^5}+\cdots
$$

$$
\begin{aligned}
M &= M_1+M_2 \\
d &= M_1-M_2
\end{aligned}
$$

•Plot:



- The total general result:

$$\begin{aligned} d &= +\frac{eg^4}{2}\, m \sum_{\beta} \sum_{i>j} M_i M_j \,\Im\left(\tilde{V}_{\alpha i}\tilde{V}_{\beta i}\tilde{V}^*_{\beta j}\tilde{V}^*_{\alpha j}\right) R_1(M_i, M_j) \\ &\quad +2e\, m \sum_{\beta} \sum_{i>j} M_i M_j \,\Im\left(\tilde{\Lambda}_{\alpha i}\tilde{\Lambda}_{\beta i}\tilde{\Lambda}^*_{\beta j}\tilde{\Lambda}^*_{\alpha j}\right) R_2(M_i, M_j) \\ &\quad +\frac{eg^3}{2\sqrt{2}} M_W \sum_{\beta} \sum_{i,j} M_i M_j \,\Im\left(\tilde{\lambda}_{\alpha i}\tilde{V}_{\beta i}\tilde{V}^*_{\beta j}\tilde{V}^*_{\alpha j}\right) R_3(M_i, M_j) \\ &\quad +\frac{eg}{\sqrt{2}}\, M_W \sum_{\beta} \sum_{i,j} M_i M_j \,\Im\left(\tilde{\Lambda}_{\alpha i}\tilde{\Lambda}_{\beta i}\tilde{V}^*_{\beta j}\tilde{\lambda}^*_{\alpha j}\right) R_4(M_i, M_j)\ . \end{aligned}$$

- Details: etd-05242007-151447. Paper coming out soon!

$$
\begin{aligned}
&R_1 \\
&= \frac{1}{6(4\pi)^4 M_W^4 (M_1^2-1)^3(M_2^2-1)^3}\times \\
&\quad \Big[(M_1^2-1)(M_2^2-1)(M_1^2-M_2^2)\Big\{-17+11(M_1^2+M_2^2)\Big\} \\
&+M_1^2(M_2^2-1)\Big\{24-(11M_1^2+10M_2^2)+(11M_1^4-25M_1^2M_2^2+11M_2^4)\Big\}\log M_1^2 \\
&-M_2^2(M_1^2-1)\Big\{24-(11M_2^2+10M_1^2)+(11M_2^4-25M_1^2M_2^2+11M_1^4)\Big\}\log M_2^2 \\
&+\Big\{12-(17M_1^2+28M_2^2)+(42M_1^2M_2^2+3M_2^4)-3M_1^2M_2^4-9M_1^4M_2^4\Big\}\mathrm{Li}_2(1-M_1^2) \\
&-\Big\{12-(17M_2^2+28M_1^2)+(42M_1^2M_2^2+3M_1^4)-3M_2^2M_1^4-9M_2^4M_1^4\Big\}\mathrm{Li}_2(1-M_2^2) \\
&+M_1^2\Big\{3+9M_2^2+(21M_1^4-48M_1^2M_2^2)-(11M_1^6-17M_1^4M_2^2-9M_1^2M_2^4)\Big\}\left\{\mathrm{Li}_2\left(1-\frac{1}{M_1^2}\right)-\mathrm{Li}_2\left(1-\frac{M_2^2}{M_1^2}\right)\right\} \\
&-M_2^2\Big\{3+9M_1^2+(21M_2^4-48M_2^2M_1^2)-(11M_2^6-17M_2^4M_1^2-9M_2^2M_1^4)\Big\}\left\{\mathrm{Li}_2\left(1-\frac{1}{M_2^2}\right)-\mathrm{Li}_2\left(1-\frac{M_1^2}{M_2^2}\right)\right\}\Big]
\end{aligned}
$$

$$
\begin{aligned}
&R_3\\
&= \frac{1}{2(4\pi)^4 M_W^2 (M_1^2-1)^3 (M_2^2-1)^3}\times\\
&\Big[2(M_1^2-1)(M_2^2-1)\Big\{1-(2M_1^2+3M_2^2)+(3M_1^2M_2^2+M_2^4)\Big\}\\
&+M_1^2(M_2^2-1)^2\Big\{1+(M_1^2-2M_2^2)\Big\}\log M_1^2\\
&+M_2^2(M_1^2-1)\Big\{3+3(M_1^2-M_2^2)-(5M_1^2M_2^2-2M_2^4)\Big\}\log M_2^2\\
&+(M_2^2-1)\Big\{1-(3M_1^2+2M_2^2)+4M_1^2M_2^2\Big\}\,\mathrm{Li}_2(1-M_1^2)\\
&+(M_2^2-1)\Big\{(M_1^2-1)-2M_1^4(M_2^2-1)\Big\}\,\mathrm{Li}_2(1-M_2^2)\\
&+M_1^2(M_2^2-1)\Big\{-2+5M_1^2-(M_1^4+2M_1^2M_2^2)\Big\}\left\{\mathrm{Li}_2\left(1-\frac{1}{M_1^2}\right)-\mathrm{Li}_2\left(1-\frac{M_2^2}{M_1^2}\right)\right\}\\
&+M_2^2(M_2^2-1)\Big\{-2-(2M_1^2-3M_2^2)-(2M_2^4-3M_2^2M_1^2)\Big\}\left\{\mathrm{Li}_2\left(1-\frac{1}{M_2^2}\right)-\mathrm{Li}_2\left(1-\frac{M_1^2}{M_2^2}\right)\right\}\Big]
\end{aligned}
$$

$$
\begin{aligned}
& R_4 \\
&= \frac{1}{8(4\pi)^4 M_W^2 (M_1^2-1)^3(M_2^2-1)^3} \times \\
& \Big[(M_1^2-1)(M_2^2-1)\Big\{1-(3M_1^2+5M_2^2)+(5M_1^2M_2^2+2M_2^4)\Big\} \\
& +2M_1^2(M_2^2-1)^2(M_1^2-2M_2^2)\log M_1^2 \\
& +2M_2^2(M_1^2-1)\Big\{2+(M_1^2-2M_2^2)-(2M_1^2M_2^2-M_2^4)\Big\}\log M_2^2 \\
& +2(M_2^2-1)\Big\{-(M_1^2+M_2^2)+2M_1^2M_2^2\Big\}\mathrm{Li}_2(1-M_1^2) \\
& +2(M_2^2-1)\Big\{(M_1^2-1)-M_1^4(M_2^2-1)\Big\}\mathrm{Li}_2(1-M_2^2) \\
& -2M_1^2(M_2^2-1)\Big\{2-4M_1^2+(M_1^4+M_1^2M_2^2)\Big\}\left\{\mathrm{Li}_2\left(1-\frac{1}{M_1^2}\right)-\mathrm{Li}_2\left(1-\frac{M_2^2}{M_1^2}\right)\right\} \\
& -2M_2^2(M_2^2-1)\Big\{2-2M_2^2+(M_2^4-M_2^2M_1^2)\Big\}\left\{\mathrm{Li}_2\left(1-\frac{1}{M_2^2}\right)-\mathrm{Li}_2\left(1-\frac{M_1^2}{M_2^2}\right)\right\}\Big]
\end{aligned}
$$

Apply to the Okamura texture:

- Limits from experiment:

$$\begin{aligned} d_e &= (6.9 \pm 7.4) \times 10^{-28}\,\mathrm{e \cdot cm} \\ d_\mu &= (3.7 \pm 3.4) \times 10^{-19}\,\mathrm{e \cdot cm} \end{aligned}$$

- Regular seesaw will give a contribution ≈ $O(10^{-43})$e.cm
- Okamura texture (TeV Majorana, large mixing)→ $O(10^{-31})$e.cm
- Quarks→$O(10^{-35})$e.cm

Conclusion

The majorana nature of neutrinos result in new CP violating 2-loop diagrams contributing to the lepton edm.

•A detailed calculation is presented in terms of general parameters.

•The results are within few orders of magnitude from the current limit