The Collinear Factorization Scheme

Rob Putman

F. Maltoni, T. McElmurry, S. Willenbrock
University of Illinois at Urbana-Champaign
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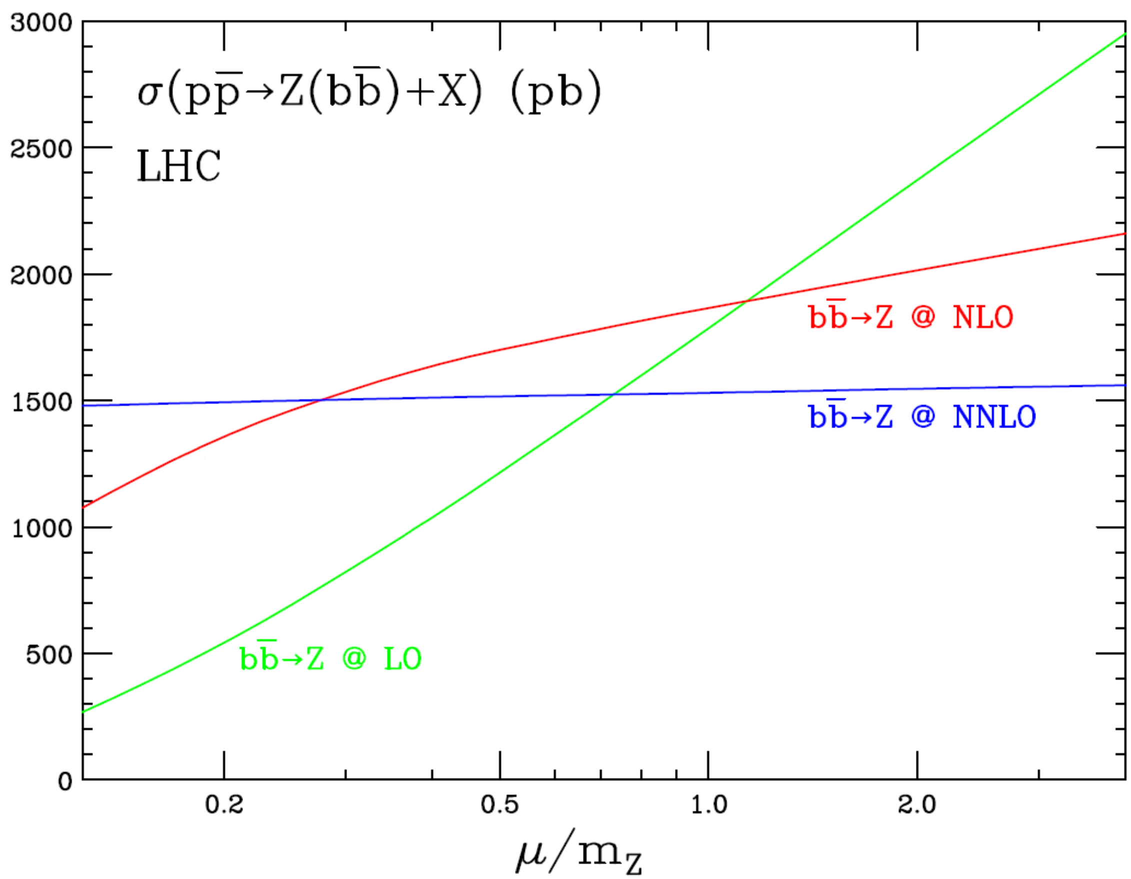
Factorization

- Factorization is the key to being able to calculate in pQCD.
- At NLO we have divergences we have to subtract back into the PDFs. This introduces a factorization scheme & scale (μ).
- The hadronic cross section is a physical observable: it does not depend on μ.

$$\sigma(S) = \int_{Q^2}^{S} \frac{ds}{S} \{f_i \otimes f_j\} \left(\frac{s}{S}, \mu\right) \hat{\sigma}_{ij}(s, \mu)$$

Factorization

- However at any given order in perturbation theory we do have dependence on μ.



Maltoni, McElmurry, Willenbrock hep-ph/0505014

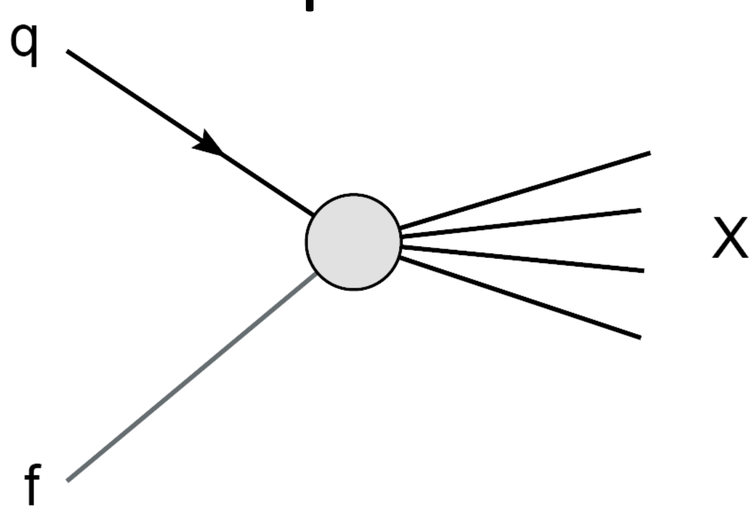
Factorization

- Clearly the best solution is to calculate to higher orders.
- What do we do if that’s not possible and our calculation still depends highly on μ?
- Either we must accept a large uncertainty due to varying the scale, or we must find a way to pin down the scale μ.

The Collinear Scheme

- Consider a process with an incoming quark:

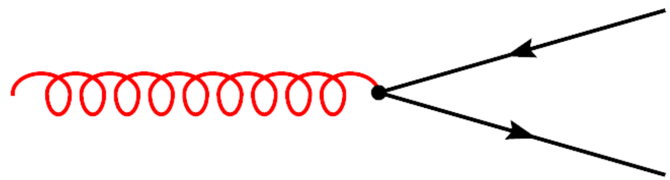
$$qf \rightarrow X$$



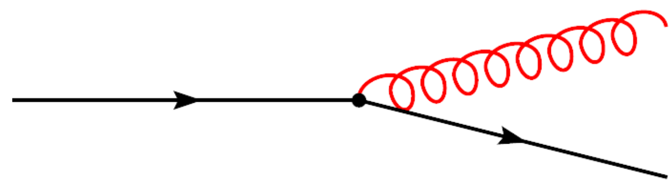
- How does the cross section change when we change the initial state?

The Collinear Scheme

- We can alter the initial state one of two ways:
  - The quark that scatters is radiated by a gluon



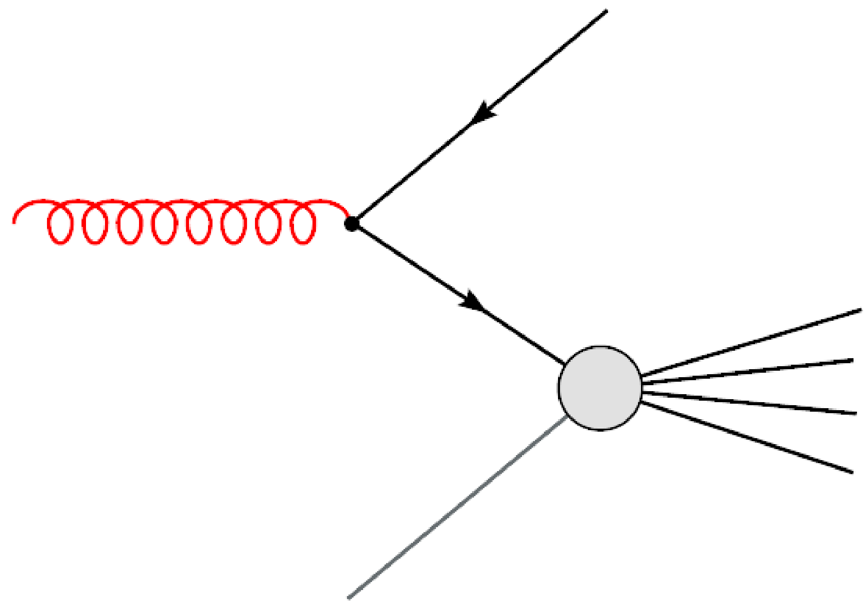
  - The quark that scatters is radiated by a quark



- Let’s consider the case of the quark radiated by a gluon first.

The Collinear Scheme

- In general the matrix element will be very different.



- In the collinear limit it must be a product:

$$\overline{|\mathcal{M}_{gY\to\bar{q}X}|^2} = \frac{2g_s^2\mu_D^{2\epsilon}p_\perp^2}{z(1-z)} P_{gq}^D(z)\left(\frac{1}{q^2}\right)^2 \overline{|\mathcal{M}_{qY\to X}|^2}$$

The Collinear Scheme

- Using the collinear limit as an anchor point we define a counterterm by cutting in $v = \frac{p_\perp^2}{s(1-z)^2}$

$$\sigma^{CT}_{gX\to\bar{q}Y} = \frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu_D^2}{Q^2}\right)^{\epsilon}\frac{1}{\Gamma(1-\epsilon)}\int_{z_0}^{1}\frac{dz}{z}\int_0^{v_{\rm cut}}\frac{dv}{v^{1+\epsilon}}z^{\epsilon}(1-z)^{-2\epsilon}P^D_{gq}(z)\hat{\sigma}_{qY\to X}(z)$$

- This yields a PDF counterterm:

$$\delta q = \frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu_D^2}{Q^2 v_{\rm cut}}\right)^{\epsilon}\frac{1}{\Gamma(1-\epsilon)}\int_x^1\frac{dz}{z}g\left(\frac{x}{z}\right)\left(-\frac{1}{\epsilon}P^D_{gq}(z)+\log\left[\frac{(1-z)^2}{z}\right]P^D_{gq}(z)\right)$$

- To satisfy DGLAP we must have $v_{cut} = \mu^2/Q^2$

The Collinear Scheme

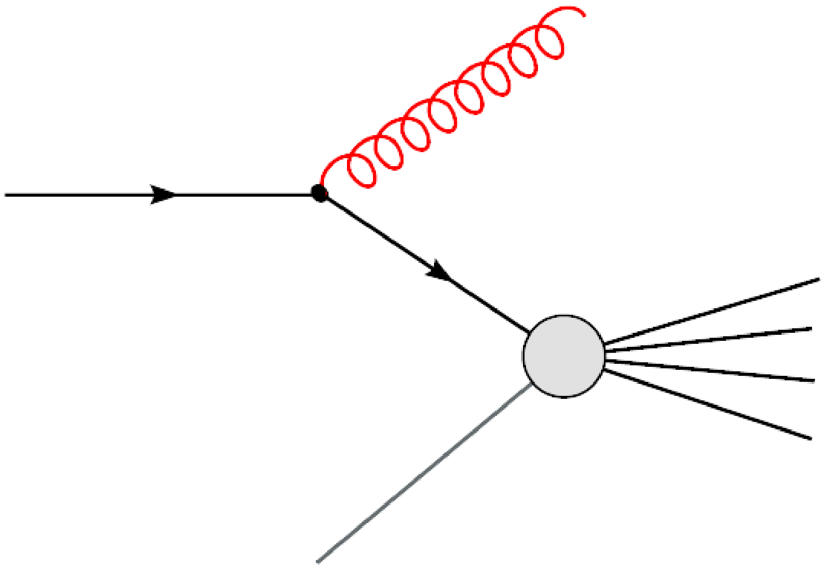
- How is this different from MS bar?

$$\delta q = \frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu_D^2}{\mu^2}\right)^{\epsilon}\frac{1}{\Gamma(1-\epsilon)}\int_x^1 \frac{dz}{z} g\left(\frac{x}{z}\right)\left[-\frac{1}{\epsilon}P_{qg}(z) + \log\left(\frac{(1-z)^2}{z}\right)P_{qg}(z) + z(1-z)\right]$$

- The two terms of order ϵ that hit the collinear divergence:
  - Phase space
  - D dimensional splitting function

The Collinear Scheme

- We can follow a similar procedure for the other correction.
- Virtual corrections complicate things.



$$\delta q_i(x) = \frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu_D^2}{\mu^2}\right)^{\epsilon}\frac{1}{\Gamma(1-\epsilon)}\int_{z_0}^{1}\frac{dz}{z}q_i\left(\frac{x}{z}\right)\left[-\frac{1}{\epsilon}P_{qq}(z)+\frac{4}{3}(1-z)\right.$$
$$\left.+\frac{8}{3}(1+z^2)\left(\frac{\log[1-z]}{1-z}\right)_+ - \frac{4}{3}\frac{1+z^2}{1-z}\log z\right]$$

The Collinear Scheme

- We can follow the same approach to find the corrections to the gluon PDF.

$$\delta g(x) = \frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu_D^2}{\mu^2}\right)^{\epsilon}\frac{1}{\Gamma(1-\epsilon)}\int_{z_0}^{1}\frac{dz}{z}\Bigg[$$
$$2C_A g\left(\frac{x}{z}\right)\left(-\frac{1}{\epsilon}p_{gg}[z] + 2\frac{(1-z+z^2)^2}{z}\left(\frac{\log[1-z]}{1-z}\right)_+ - \frac{(1-z+z^2)^2}{z(1-z)}\log[z]\right) +$$
$$C_F\sum_i q_i\left(\frac{x}{z}\right)\left(-\frac{1}{\epsilon}p_{gq}[z] + \log\left[\frac{(1-z)^2}{z}\right]\frac{1+(1-z)^2}{z} + z\right)\Bigg]$$

- Note again additional terms that don't appear in the MS bar factorization scheme.

Choosing a Factorization Scale

- Now that we have our counterterms, how do we choose μ?
  - We should factorize as much as we can into the PDFs.
  - We should subtract the entire term that arises from the squared matrix element factorizing.
- This corresponds to taking $v_{cut} = 1$, or $\mu = Q$.

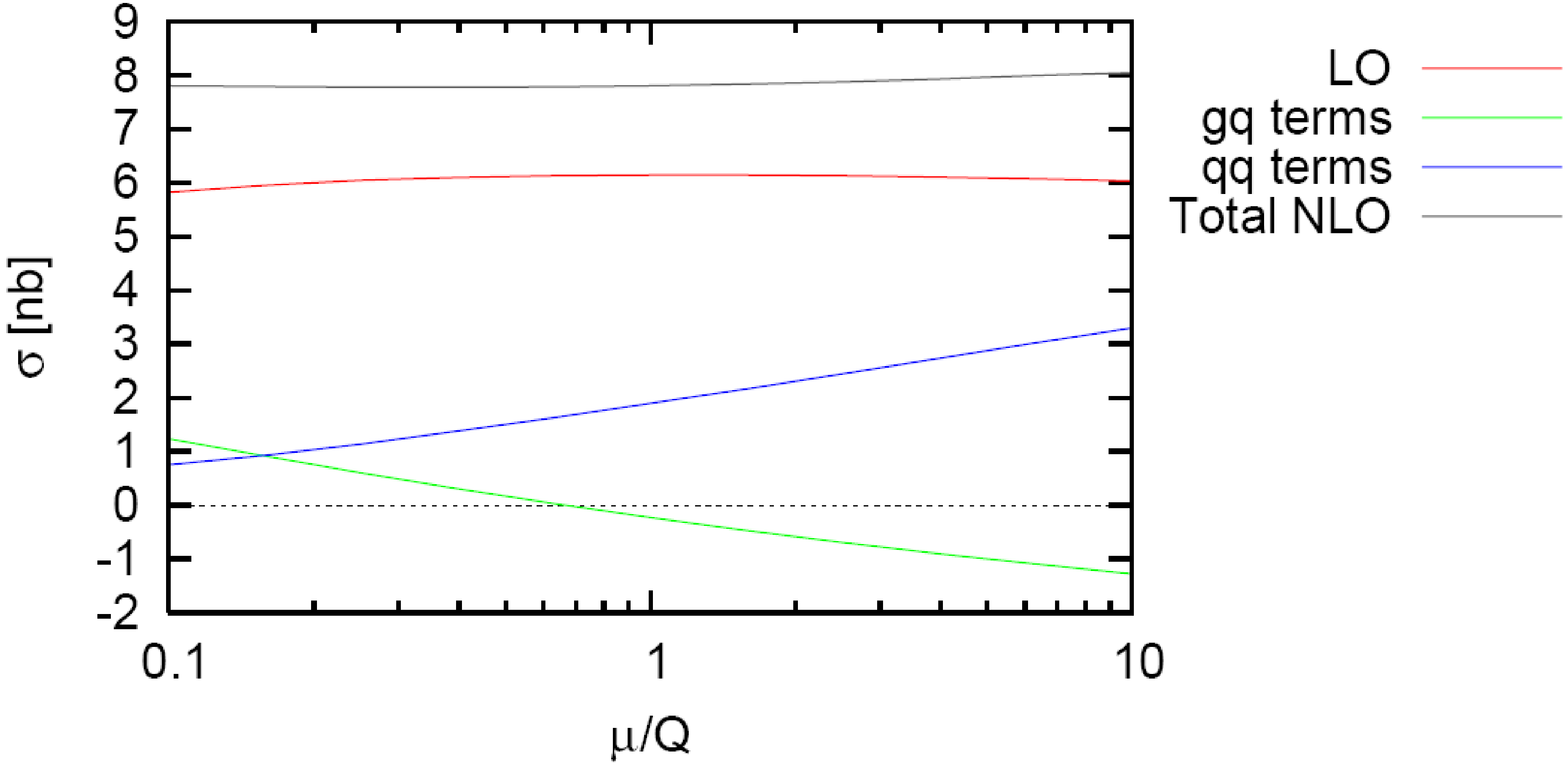
Results

- We have to create a set of collinear scheme PDFs.
- How well does it work?
- We consider two different processes:
  - Drell Yan
  - Higgs Production by Gluon Fusion

Results

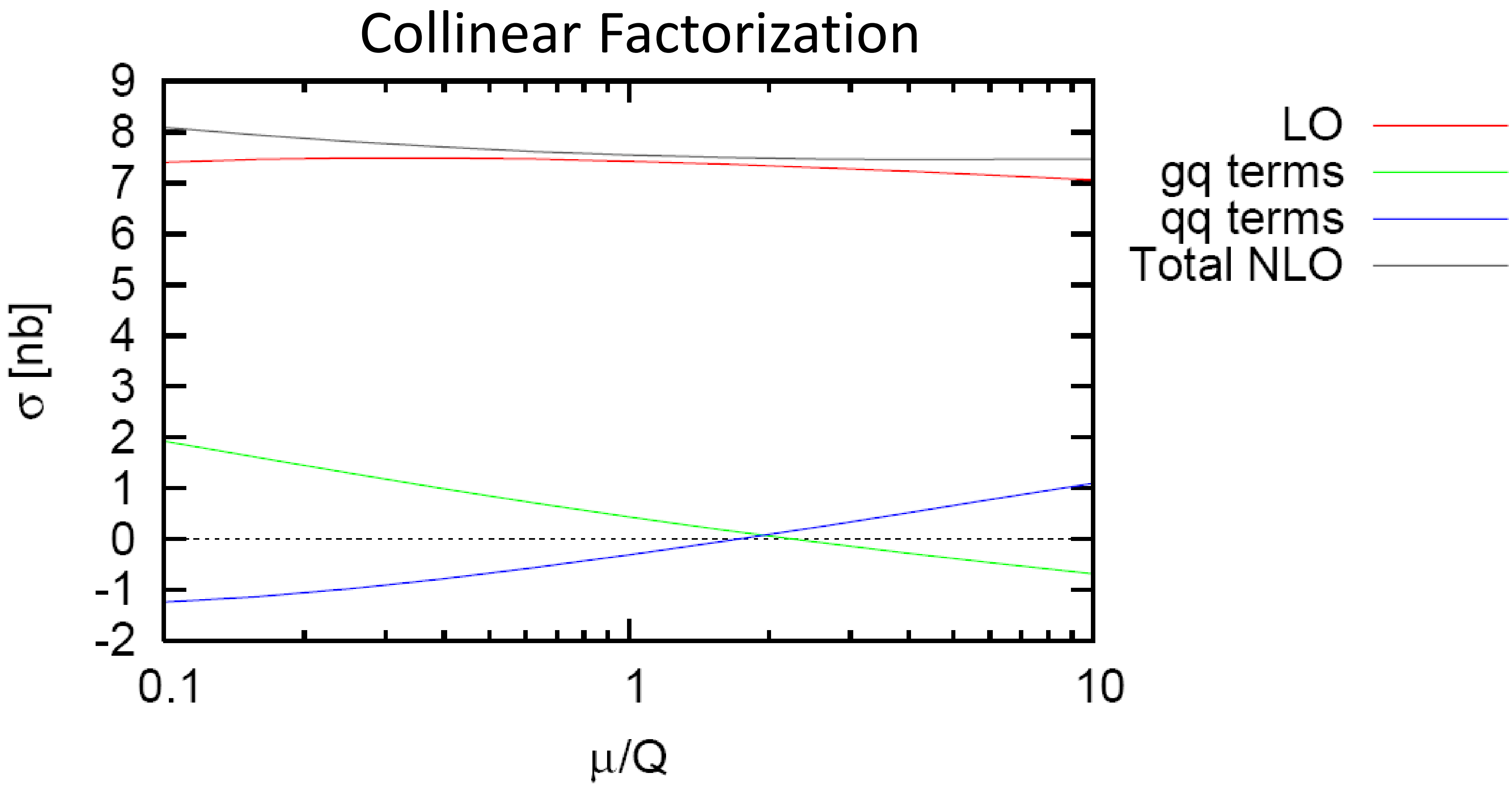
Real Z Production at the Tevatron

MS Bar Factorization



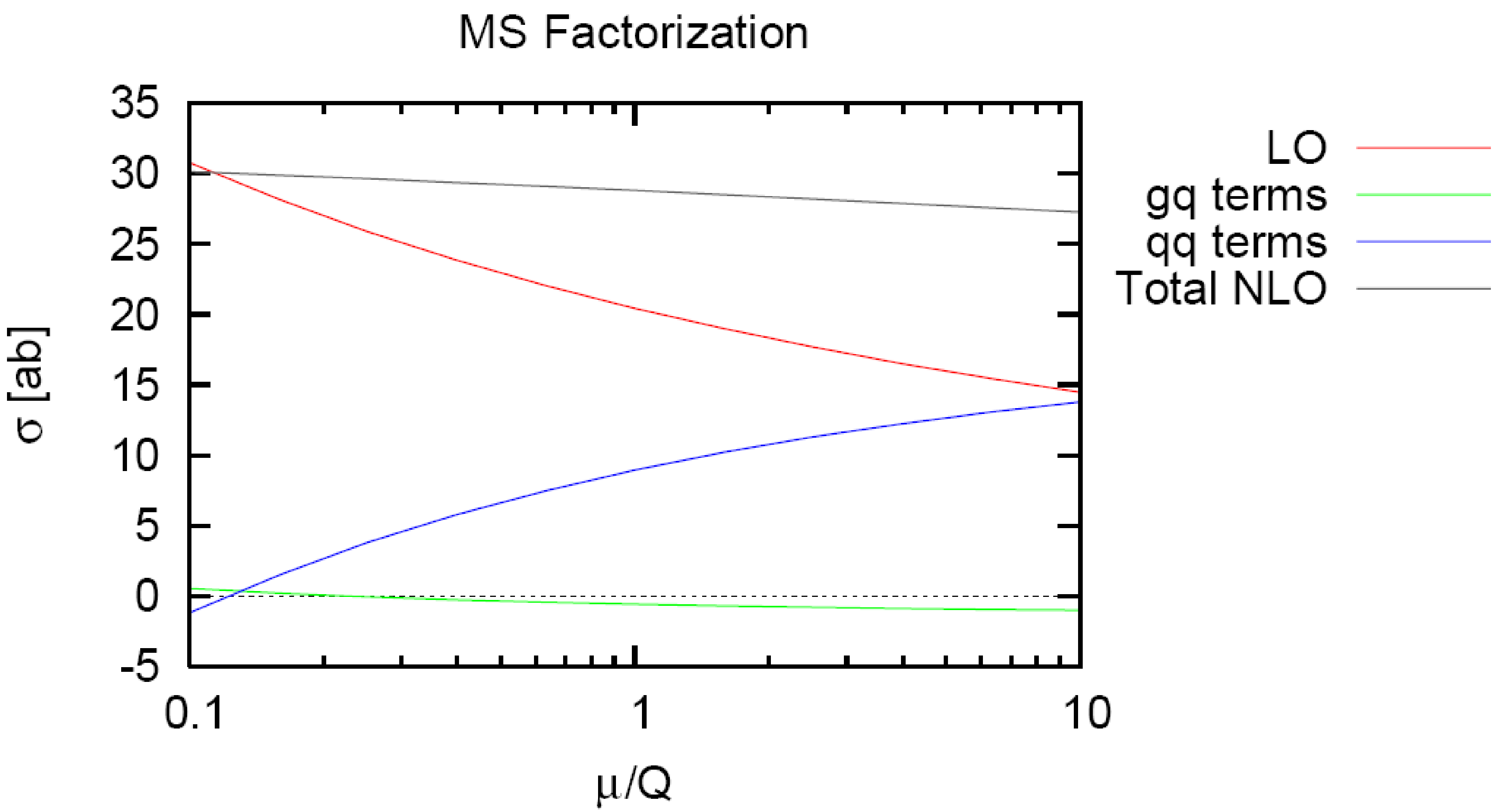
Results

Real Z Production at the Tevatron



Results

Z Production at the LHC Q = 3.5 TeV



Results

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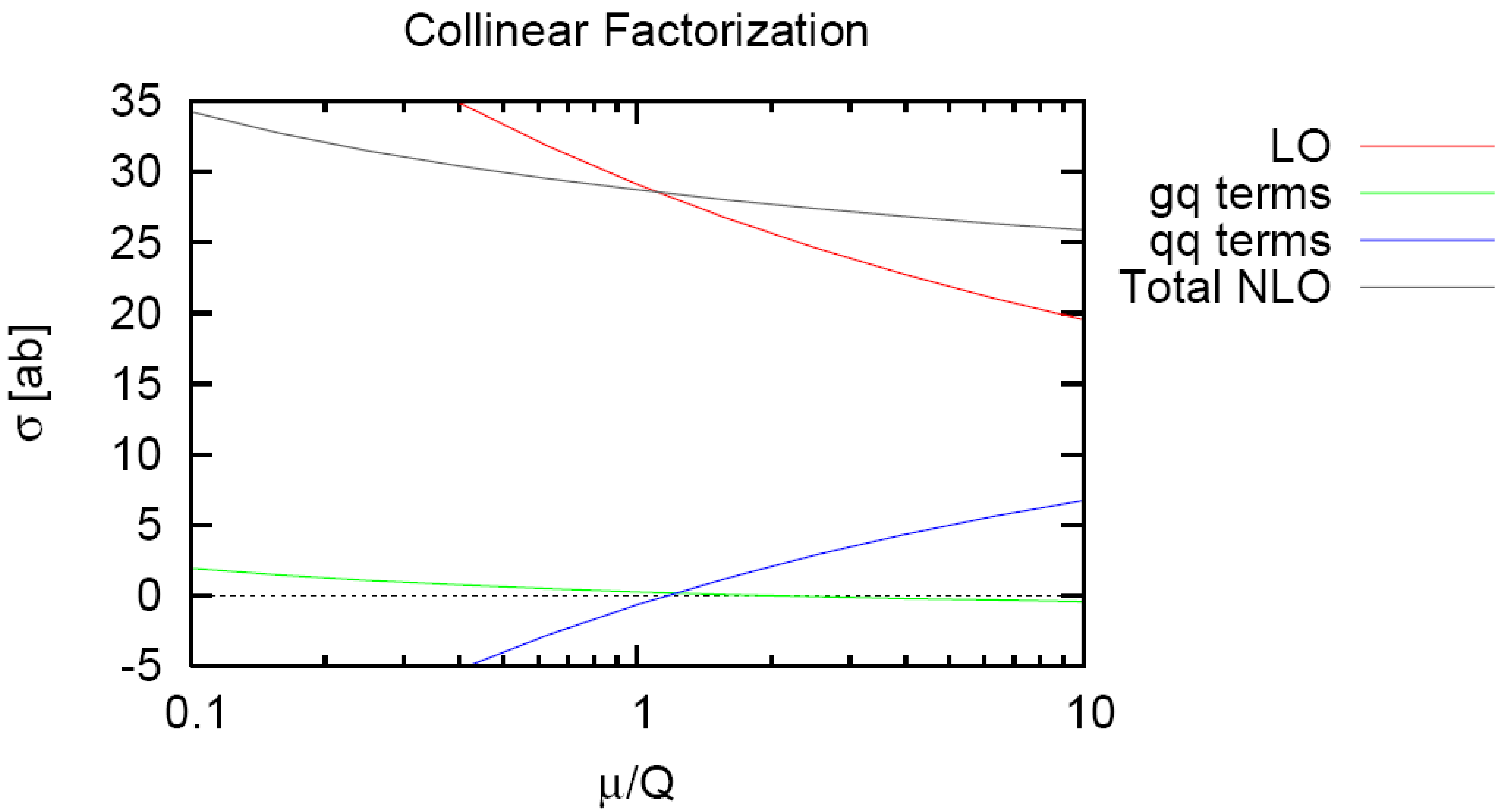
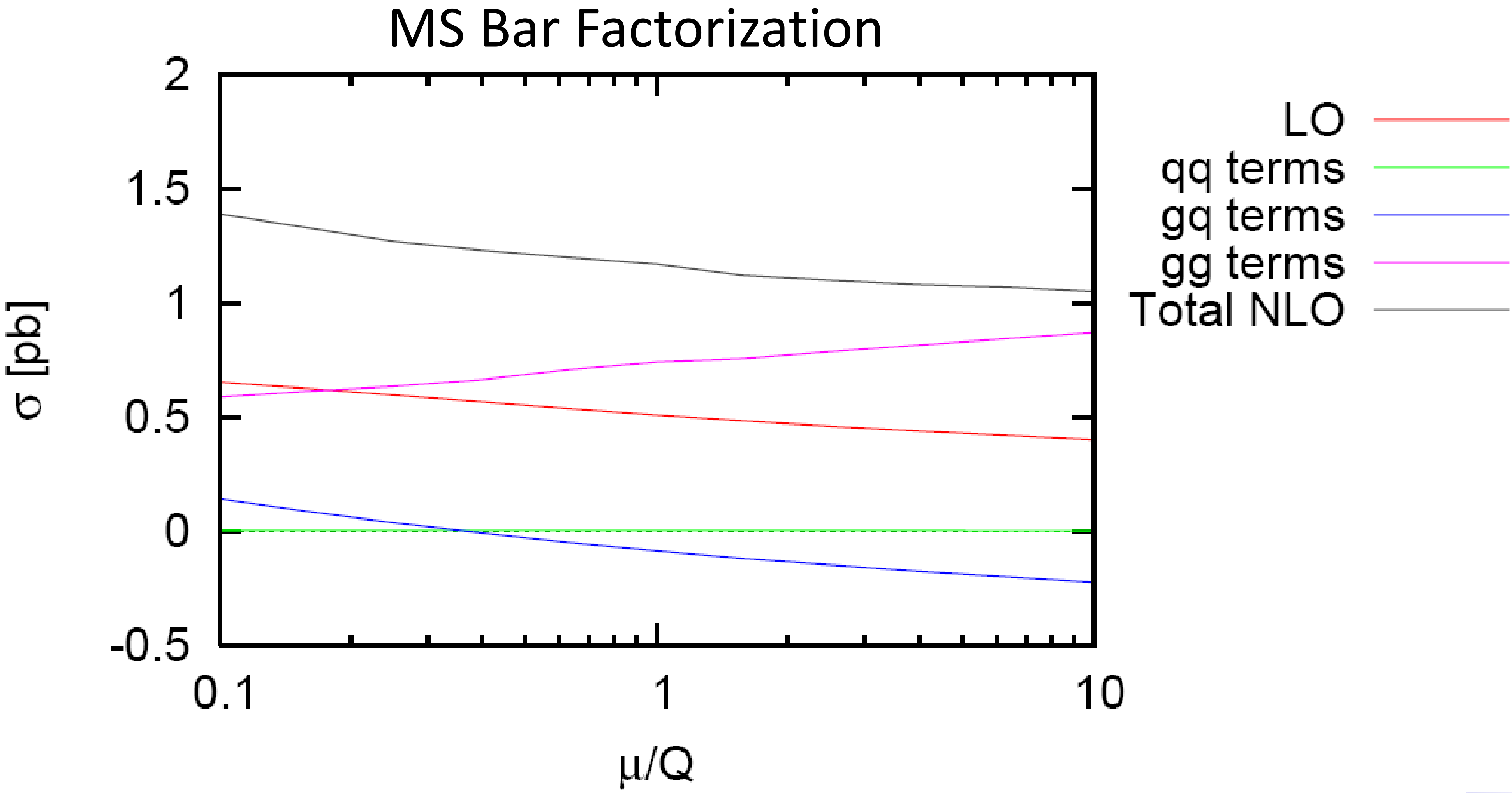


Results

gg → H at the Tevatron, M_h = 100 GeV



Results

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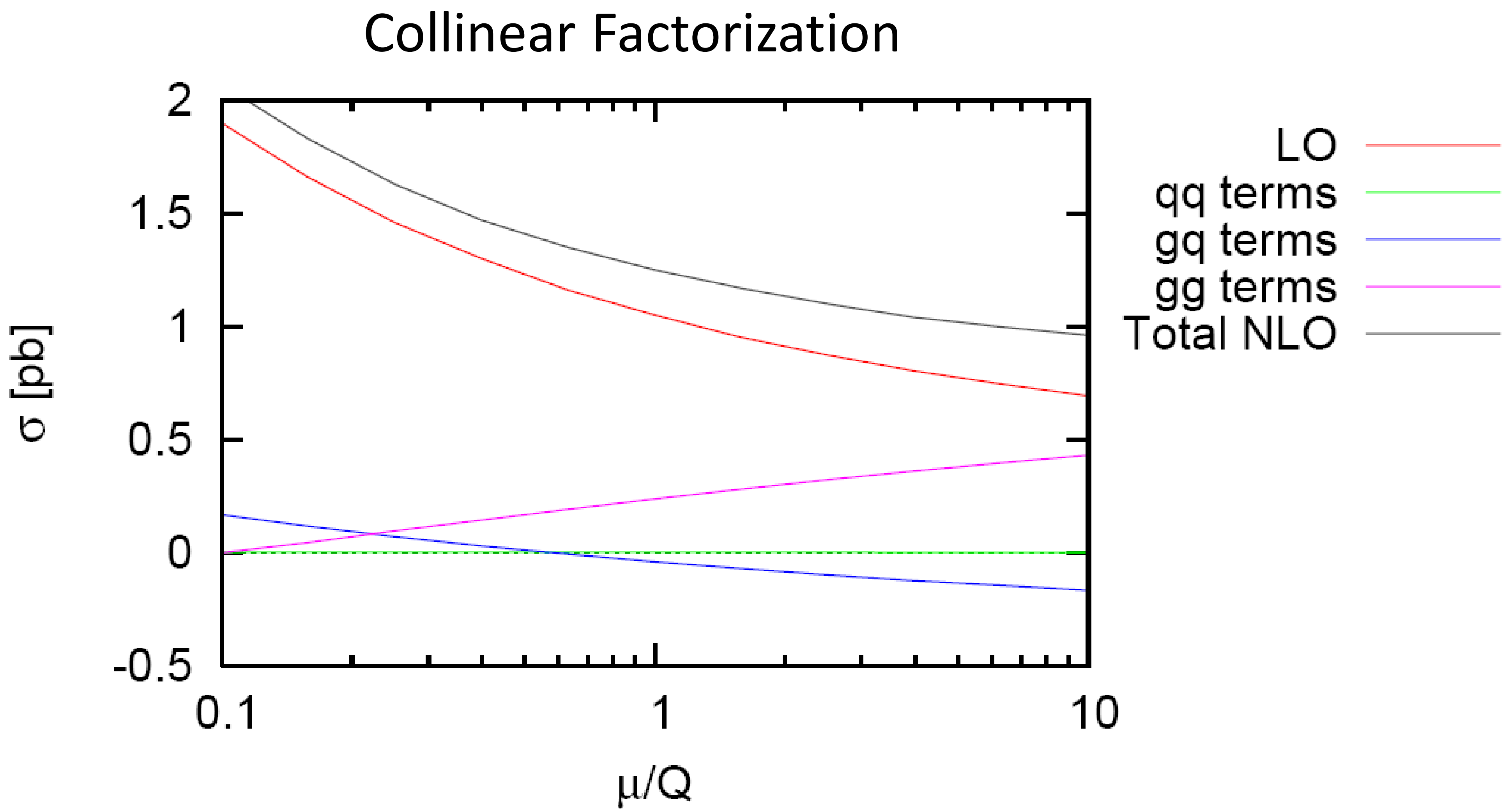
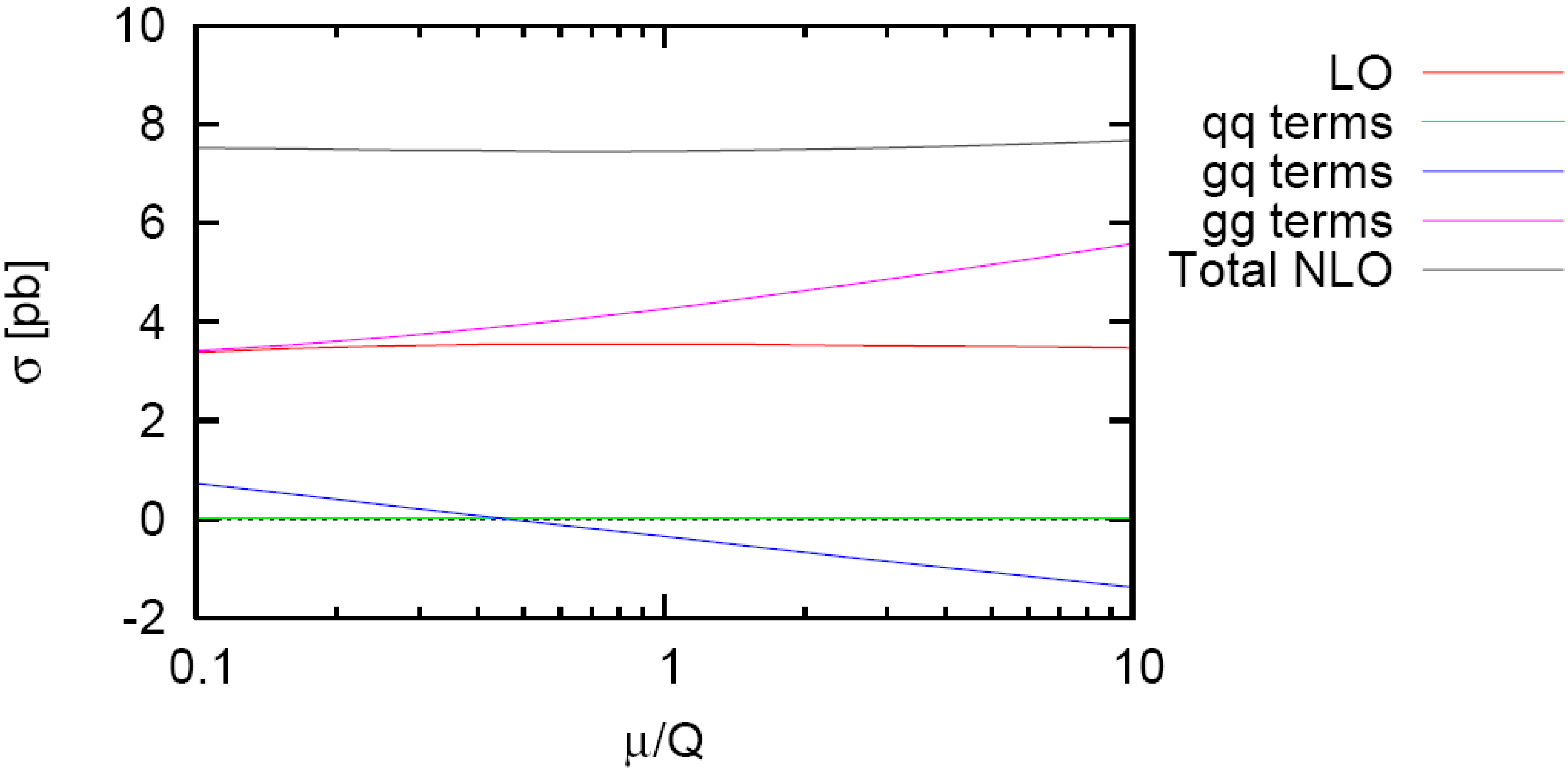


Results

gg → H at the LHC, M_h = 250 GeV

MS Bar Factorization



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Results

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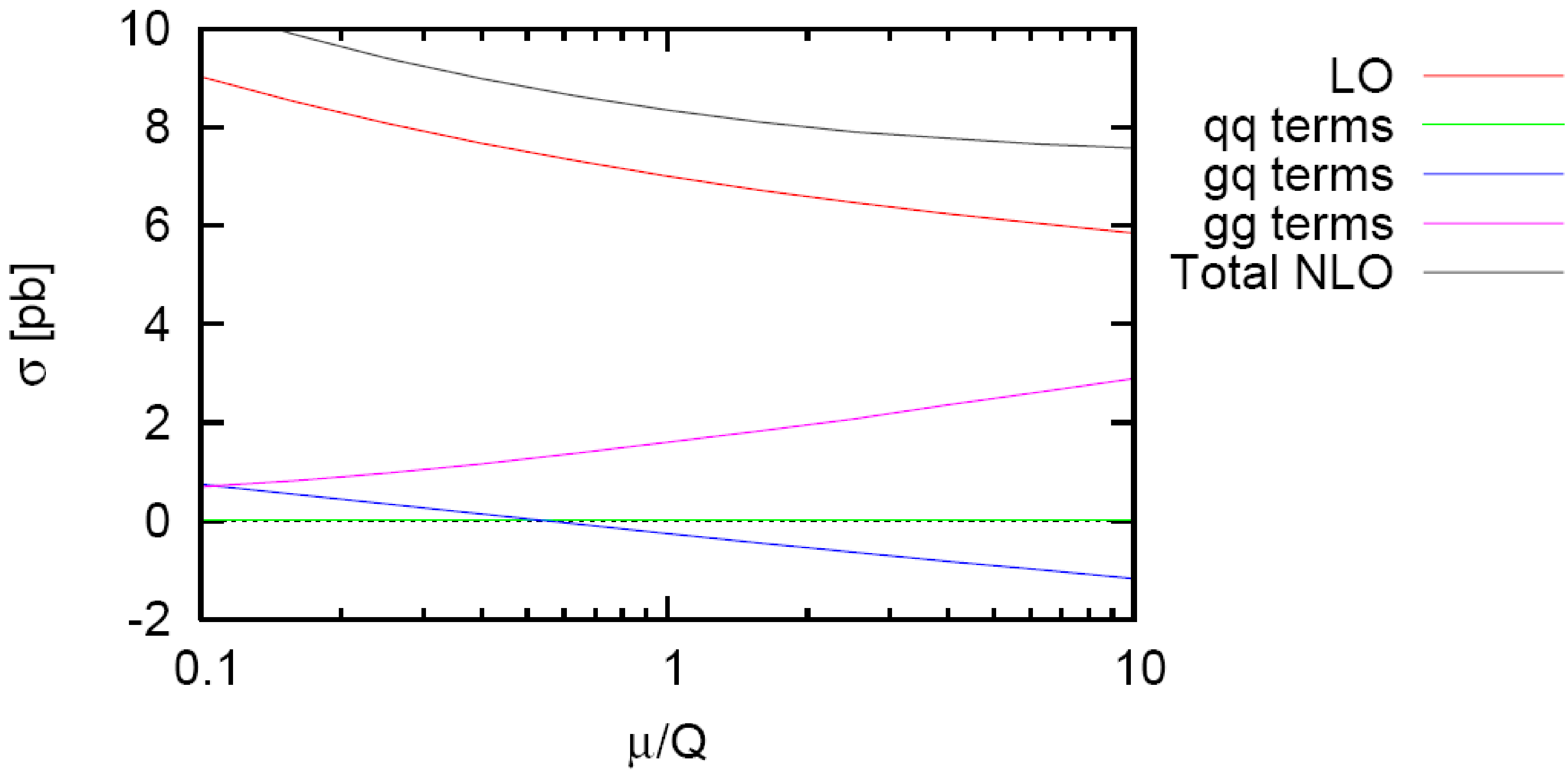
Collinear Factorization



Summary & Outlook

- Examining the collinear limit allows one to define a factorization scheme based on the collinear physics.
- This scheme subtracts the divergent terms arising from the collinear divergence, but also the ϵ^0 terms that arise from this divergence.
- In this scheme the factorization scale can be pinned down.

Summary & Outlook

- When using this factorization scheme and scale the convergence of cross sections in pQCD is significantly improved.
- Factorization is clearly important and deserves further work and thought:
  - What about a 2 → 2 or 2 → n process?
  - Can we somehow relate this to MS Bar?