Precise Predictions for Hadronic Collisions from On-Shell Methods

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BlackHat and Sherpa

BlackHat:

CFB, Zvi Bern, Lance Dixon, Fernando Febres Cordero, Darren Forde, Harald Ita, David Kosower, Daniel Maitre

BlackHat: arXiv:0902.2760, PRD78 (2008) 036003. Badger: JHEP 0901 (2009) 049. Forde: PRD75 (2007) 125019. CFB, Bern, Dixon, Forde, Kosower: PRD74 (2006) 036009.

Sherpa liaison (real emissions):

Tanju Gleisberg

Gleisberg et al, JHEP 0902 (2009) 007. Gleisberg, Krauss, Eur. Phys. J C53 (2008) 501.

Outline

- **Introduction**
  Do we really need NLO?
- **What is BlackHat?**
  - Terms with logarithms (dilogs, ...) from generalized unitarity
  - Rational terms
- **Physics Results – $W + 3$ jets**
  $\Rightarrow$ Fernando’s Talk!

Precision Calculations

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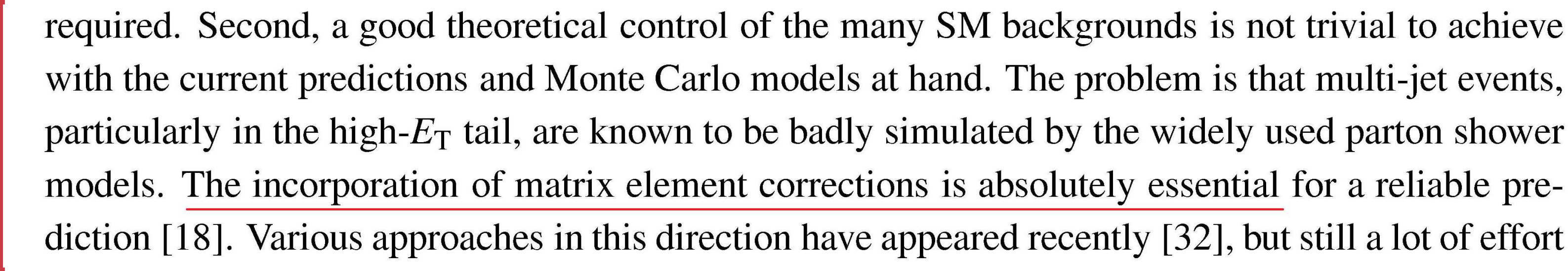
Dissertori (CMS)



required. Second, a good theoretical control of the many SM backgrounds is not trivial to achieve with the current predictions and Monte Carlo models at hand. The problem is that multi-jet events, particularly in the high-E_T tail, are known to be badly simulated by the widely used parton shower models. The incorporation of matrix element corrections is absolutely essential for a reliable prediction [18]. Various approaches in this direction have appeared recently [32], but still a lot of effort

The (In)Famous Experimenters' Wishlists

Les Houches 2005

process wanted at NLO ($V \in \{Z, W, \gamma\}$)	**background to**
1. $pp \rightarrow VV +$ jet	$t\bar{t}H$, new physics
2. $pp \rightarrow H + 2$ jets	H production by vector boson fusion (VBF)
3. $pp \rightarrow t\bar{t}b\bar{b}$	$t\bar{t}H$
4. $pp \rightarrow t\bar{t} + 2$ jets	$t\bar{t}H$
5. $pp \rightarrow VVb\bar{b}$	VBF $\rightarrow H \rightarrow VV$, $t\bar{t}H$, new physics
6. $pp \rightarrow VV + 2$ jets	VBF $\rightarrow H \rightarrow VV$
7. $pp \rightarrow V + 3$ jets	new physics
8. $pp \rightarrow VVV$	SUSY trilepton

$2 \rightarrow 3$, computed via standard methods, + one process $2 \rightarrow 4$ Bredenstein, Dittmaier, Denner, Pozzorini

$2 \rightarrow 4$, computed via on-shell methods (BlackHat and (partially) Rocket)

The (In)Famous Experimenters' Wishlists

Run II Monte Carlo Workshop 2001

Single boson	Diboson	Triboson	Heavy flavor
$W + \leq 5j$	$WW + \leq 5j$	$WWW + \leq 3j$	$t\bar{t} + \leq 3j$
$W + b\bar{b} + \leq 3j$	$WW + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} + \leq 3j$	$WW + c\bar{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 5j$	$ZZ + \leq 5j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 3j$	$ZZ + b\bar{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \leq 2j$
$Z + c\bar{c} + \leq 3j$	$ZZ + c\bar{c} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{b} + \leq 2j$
$\gamma + \leq 5j$	$\gamma\gamma + \leq 5j$		$tb\bar{b} + \leq 3j$
$\gamma + b\bar{b} + \leq 3j$	$\gamma\gamma + b\bar{b} + \leq 3j$		
$\gamma + c\bar{c} + \leq 3j$	$\gamma\gamma + c\bar{c} + \leq 3j$		
	$WZ + \leq 5j$		
	$WZ + b\bar{b} + \leq 3j$		
	$WZ + c\bar{c} + \leq 3j$		
	$W\gamma + \leq 3j$		
	$Z\gamma + \leq 3j$		

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  It took 11 years to go from 5-gluon 1-loop amplitudes to 6 gluons!

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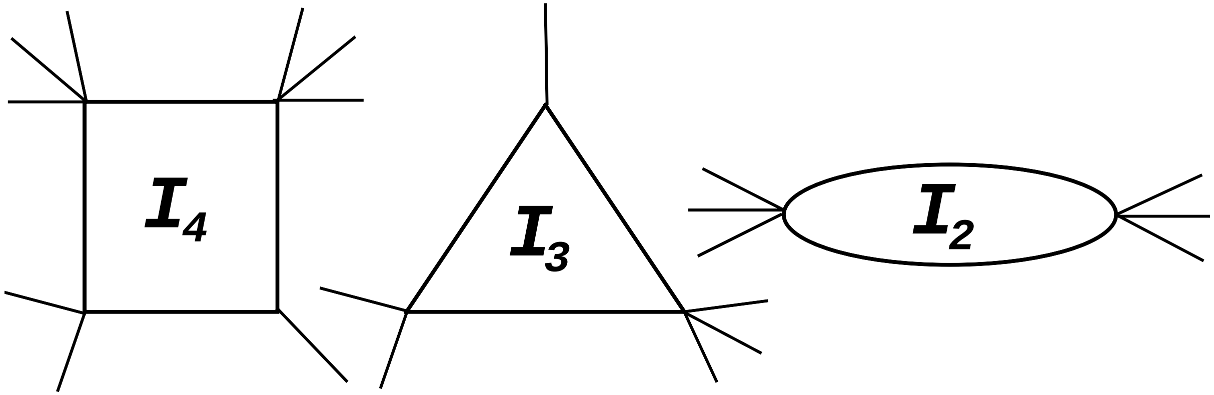
- **Real-virtual cancellations a solved problem, automated**
- **Bottleneck: 1-loop virtual amplitudes**
  It took 11 years to go from 5-gluon 1-loop amplitudes to 6 gluons!
- **New methods based on (generalized) unitarity and recursion $\Rightarrow$ new codes: BlackHat, Rocket (D-dim unitarity), CutTools/OneLOop (D-dim unitarity at integrand level)** Rocket: Ellis, Giele, Kunszt, Melnikov, Zanderighi.

  CutTools/OneLOop: van Hameren, Ossola, Papadopoulos, Pittau

One-Loop Decomposition



Any n-leg (massless) one-loop amplitude expressible in terms of scalar box, triangle and bubble integrals:

$$\mathcal{A} = c_4 I_4 + c_3 I_3 + c_2 I_2 + \text{ rational}$$

With massive partons there are additionally I_1 (tadpoles)

We know the integrals, the task is to determine the coefficients

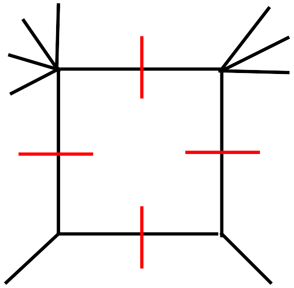
Bern, Dixon, Dunbar, Kosower

Generalized Unitarity

$$c_4 I_4 = c_4 \int d^4 l \frac{1}{l^2 (l-K_1)^2 (l-K_2)^2 (l-K_3)^2}$$

$$\frac{1}{P^2 + i\varepsilon} = \frac{1}{P^2} + i\delta^+(P^2)$$

Box integrals have unique leading singularity ⇒ generalized unitarity



$$c_4 \Delta_{LS} I_4 = \int d^4 l \delta^+(l^2) \delta^+((l-K_1)^2)$$

$$\times \delta^+((l-K_2)^2) \delta^+((l-K_3)^2)$$

$$\times A_1^{\text{tree}}(l) \times A_2^{\text{tree}}(l) \times A_3^{\text{tree}}(l) \times A_4^{\text{tree}}(l)$$

$$c_4 = A_1^{\text{tree}}(l_{\mathrm{sol}}) \times A_2^{\text{tree}}(l_{\mathrm{sol}}) \times A_3^{\text{tree}}(l_{\mathrm{sol}}) \times A_4^{\text{tree}}(l_{\mathrm{sol}})$$

Tree graphs on shell

Trees “recycled” into loops

Britto, Cachazo, Feng

On-Shell Recursion Relations at Tree Level

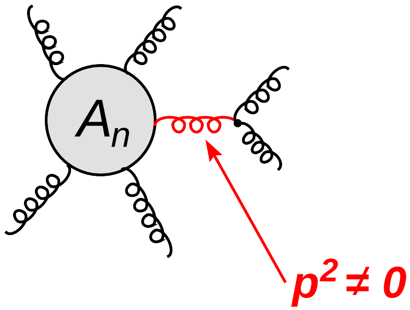
Outline

Introduction

BlackHat

- One-Loop Decomposition
- Generalized Unitarity
- Tree Level
- Proof at Tree-Level
- Rational Terms from Recursion
- Rational Terms - D-dim Unitarity

Summary

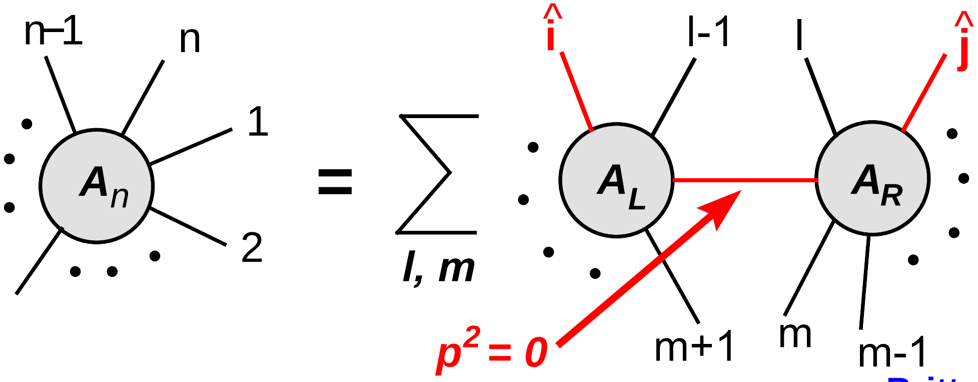


Complex continue (shift) spinors and momenta:

$$p_i \longrightarrow p_i(z) \qquad p_j \longrightarrow p_j(z)$$

$$p_i + p_j \longrightarrow p_i + p_j$$

Momentum conservation is maintained, momenta on-shell ($p_i(z)^2 = p_j(z)^2 = 0$).



Britto, Cachazo, Feng

Proof at Tree-Level

Propagators and thus amplitudes are now functions of the complex parameter:

$$1/P^2_{l\ldots j\ldots m} \quad \longrightarrow \quad 1/P^2_{l\ldots j\ldots m}(z)$$

$$A(z) = \sum_{l,m} \sum_{h} A_L^h(z) \frac{1}{P^2_{l\ldots j\ldots m}(z)} A_R^{-h}(z)$$

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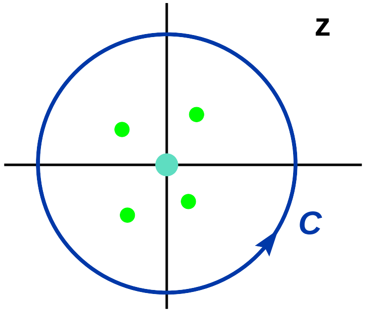
If $A(z\to\infty)\to 0$ **- Cauchy's theorem**



$$\frac{1}{2\pi i}\oint_C \frac{dz}{z}A(z) = 0$$

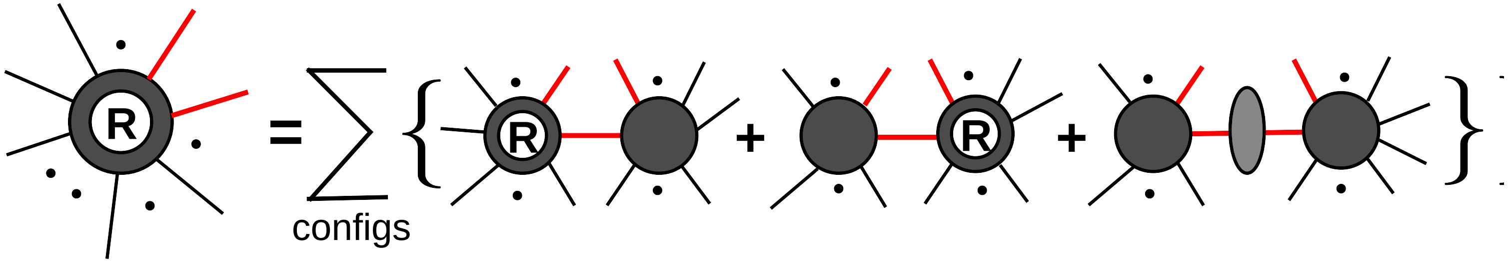
$$A(0) = -\sum_{\text{poles }\alpha}\operatorname*{Res}_{z=z_\alpha}\frac{A(z)}{z}$$

$$= \sum_{\text{poles }\alpha}\sum_{h} A_L^h(z_\alpha)\frac{1}{P^2_{l\ldots j\ldots m}}A_R^{-h}(z_\alpha)$$

Britto, Cachazo, Feng, Witten

Rational Terms from Recursion

$$\mathcal{A} = \sum_i c_i I_i + \textbf{ rational}$$



$$R = \sum_{\text{configs}} A_L \frac{1}{P^2_{l\ldots m}} A_R$$

CFB, Bern, Dixon, Forde, Kosower

Rational Terms - D-dim Unitarity

Unitarity in $D = 4 - 2\varepsilon$:
Split up into 4-D piece and (-2ε)-dim. piece (~ small "mass")

$$l_D^2 = l_4^2 + l_{[-2\varepsilon]}^2 = l_4^2 + \mu^2$$

$$\int \frac{d^D l}{(2\pi)^D} = \int \frac{d^4 l_4}{(2\pi)^4} \int \frac{d^{-\varepsilon}(\mu^2)}{(2\pi)^{-2\varepsilon}}$$

Extract rational part R by keeping track of μ-dependence in generalized unitarity cuts $\Rightarrow$ loosely speaking, 4-D unitarity, but trees are now "massive" (μ^2)

Badger, Forde. See also Ossola, Papadopoulos, Pittau (CutTools); Ellis, Giele, Kunszt, Melnikov, Zanderighi (Rocket).

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- Fully automatizable for any process (incl. BSM – just need relevant trees/low-order loops and color/coupling info)

Summary

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Physics results ⇒ Fernando’s talk next!