Determination of B_s^0 and B_d^0 mixing parameters using lattice QCD

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1867

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HPQCD Collaboration

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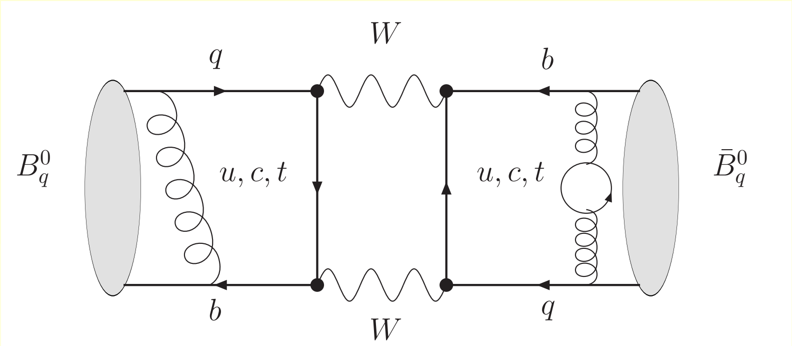
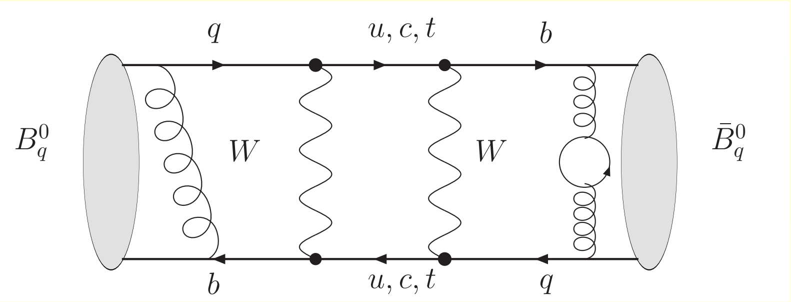
1. New Physics effects on $B^0 - \bar{B}^0$ mixing





- B_0 mixing parameters determined by the off diagonal elements of the mixing matrix

$$i\frac{d}{dt}\begin{pmatrix} |B_{s/d}(t)\rangle \\ |\bar{B}_{s/d}(t)\rangle \end{pmatrix} = \left(M^{s/d} - \frac{i}{2}\Gamma^{s/d}\right)\begin{pmatrix} |B_{s/d}(t)\rangle \\ |\bar{B}_{s/d}(t)\rangle \end{pmatrix}$$

$$\Delta M_{s/d} \propto |M_{12}^{s/d}| \qquad \Delta\Gamma_{s/d} \propto |\Gamma_{12}^{s/d}|$$

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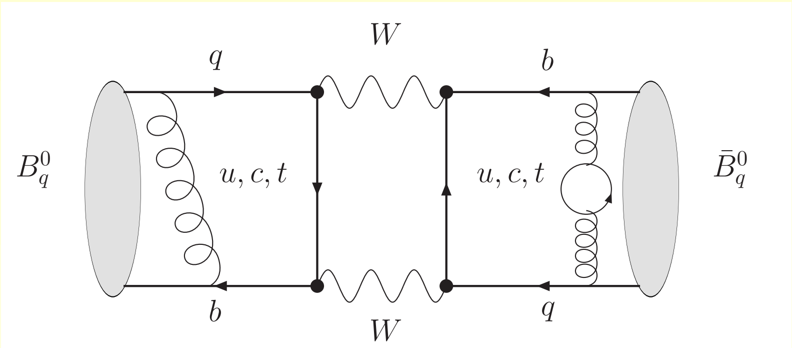

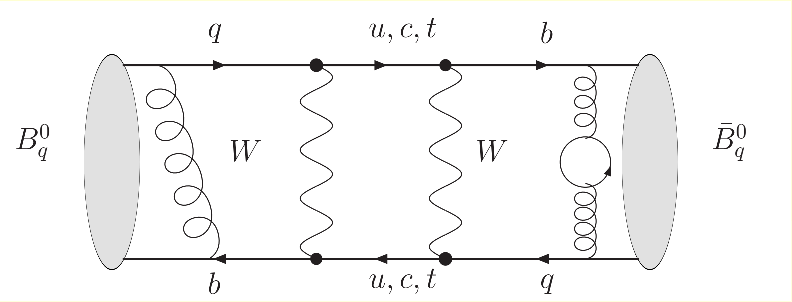

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New physics can significantly affect $M_{12}^{s/d} \propto \Delta M_{s/d}$

* Γ_{12} dominated by `CKM-favoured` $b \to c\bar{c}s$ tree-level decays.

Hints of discrepancies between SM expectations and some flavour observables (see, for example, **E. Lunghi**, talk at BEACH08)

* $sin(2\beta)$ **E. Lunghi and A. Soni**, PLB 666 (2008) 162

* B_s^0 mixing phase **Nierste and Lenz**, JHEP 0706, **UTfit coll.**, arXiv:0803.0659

* CP violating effects $B_d^0 - \bar{B}_d^0$, **Buras and Guadagnoli**, PRD78(2008)033005

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These analyses depend on several theoretical inputs:

V_{cb}, V_{ub}, $\hat{B}_K$ and `the SU(3) breaking mixing parameter` ξ:

$$\left|\frac{V_{td}}{V_{ts}}\right| = \xi\sqrt{\frac{\Delta M_d M_{B_s}}{\Delta M_s M_{B_d}}}$$

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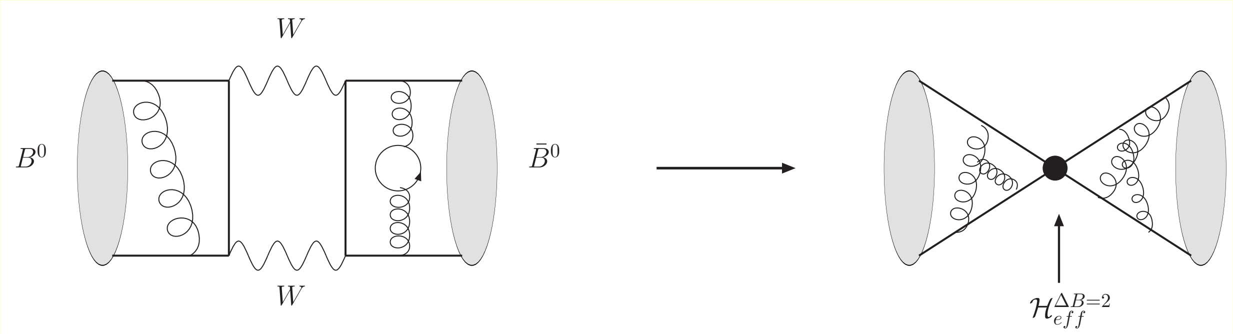
Improvement in $B^0 - \bar{B}^0$ mixing parameters which enter on those analyses is crucial.

2. Mixing parameters in the Standard Model

In the Standard Model



$$\Delta M_q|_{theor.} = \frac{G_F^2 M_W^2}{6\pi^2} |V_{tq}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_s} f_{B_q}^2 \hat{B}_{B_q}$$

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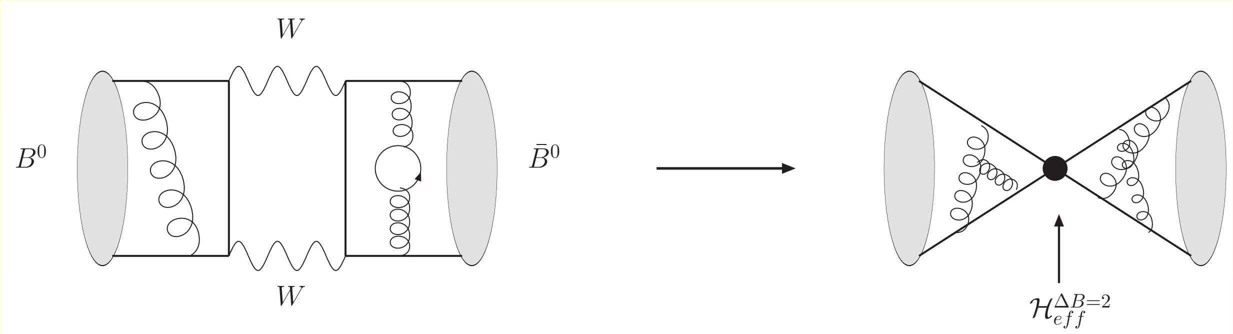

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* Non-perturbative input

$$\frac{8}{3} f_{B_q}^2 B_{B_q}(\mu) M_{B_q}^2 = \langle \bar{B}_q^0 | O_L | B_q^0 \rangle (\mu) \quad \text{with} \quad O_L \equiv [\overline{b^i}\, q^i]_{V-A} [\overline{b^j}\, q^j]_{V-A}$$

In terms of decay constants and bag parameters

$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

* Many uncertainties in the theoretical (lattice) determination cancel totally or partially in the ratio $\Longrightarrow$ **very accurate calculation**

3. Some details of the lattice formulations and simulations

Unquenched: Fully incorporate vacuum polarization effects

MILC $N_f^{sea} = 2 + 1$

u,d,s **Asqtad** action: improved staggered quarks $\Longrightarrow$ errors $\mathcal{O}(a^2\alpha_s)$, $\mathcal{O}(a^4)$

* good chiral properties
* accessible dynamical simulations

b **NRQCD**: `Non-relativistic QCD` improved through $\mathcal{O}(1/M^2)$, $\mathcal{O}(a^2)$ and leading relativistic $\mathcal{O}(1/M^3)$

* Simpler and `faster algorithms` to calculate b `propagator`

Improved gluon action

* For further reduction of discretization errors

Parameters of the simulation

Lattice spacing: Two different values $a \simeq 0.12\ fm, 0.09\ fm$.
Extracted from Υ 2S-1S splitting.

Bottom mass: Fixed to its physical value from Υ mass.

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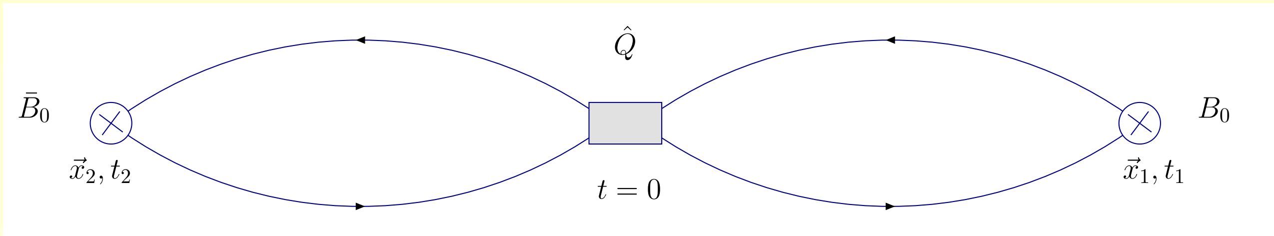
Renormalization and matching to the continuum: One-loop.

$$< O_L >^{\overline{MS}} \propto (1 + \rho_{LL}\alpha_s) < O_L >^{latt.} + \rho_{LL}\alpha_s < O_S >^{latt}$$

with $O_S = \left[\bar{b}(1-\gamma_5)q\right]\left[\bar{b}(1-\gamma_5)q\right]$.

Need 3-point (for any $\hat{Q} = Q_X, Q_X^{1j}$) and 2-point correlators



$$C^{(4f)}(t_1, t_2) = \sum_{\vec{x}_1, \vec{x}_2} \langle 0 | \Phi_{\bar{B}_q}(\vec{x}_1, t_1) \left[\hat{Q} \right] (0) \Phi^\dagger_{\bar{B}_q}(\vec{x}_2, -t_2) | 0 \rangle$$

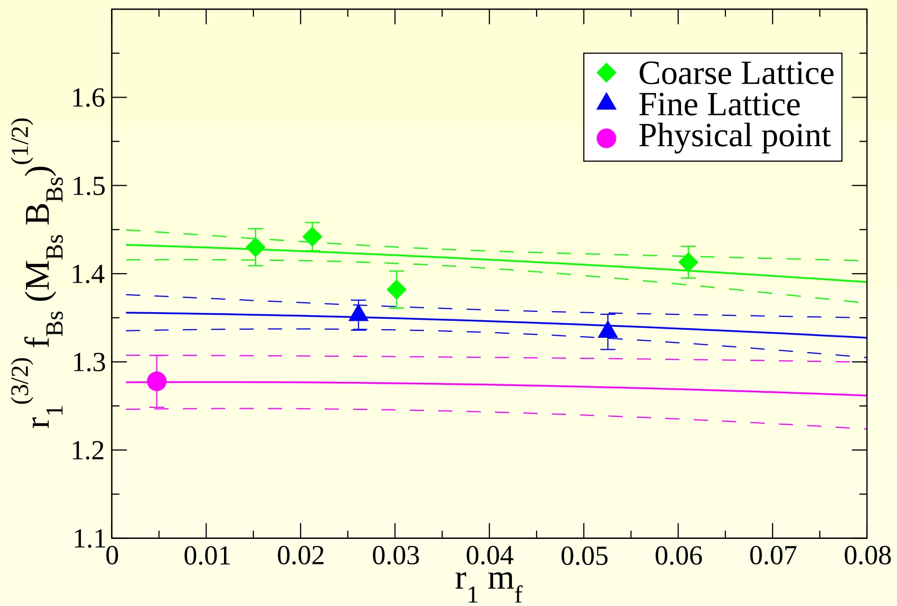
$$C^{(B)}(t) = \sum_{\vec{x}} \langle 0 | \Phi_{\bar{B}_q}(\vec{x}, t) \Phi^\dagger_{\bar{B}_q}(\vec{0}, 0) | 0 \rangle$$

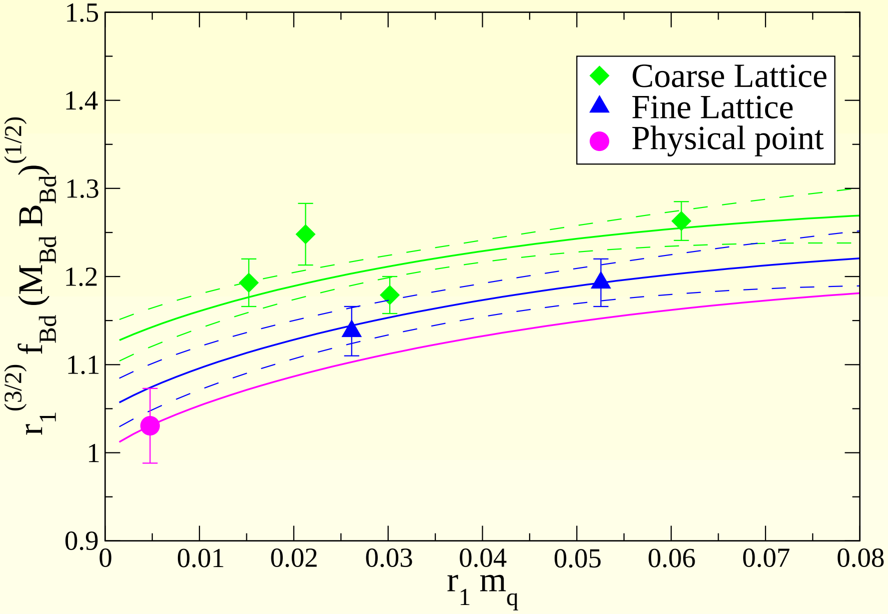
- $\Phi_{\bar{B}_q}(\vec{x}, t) = \bar{b}(\vec{x}, t) \gamma_5 q(\vec{x}, t)$ is an interpolating operator for the B_q^0 meson.

We carried out **simultaneous** fits of the 3-point and 2-point correlators using bayesian statistics to the forms → extract $< O_X >$ and $f_{B_{s(d)}}$.

4. Results

Results for $f_{B_q}\sqrt{M_{B_q} B_{B_q}}$





$$f_{B_s}\sqrt{\hat{B}_{B_s}} = 266(6)(17)\text{MeV}$$

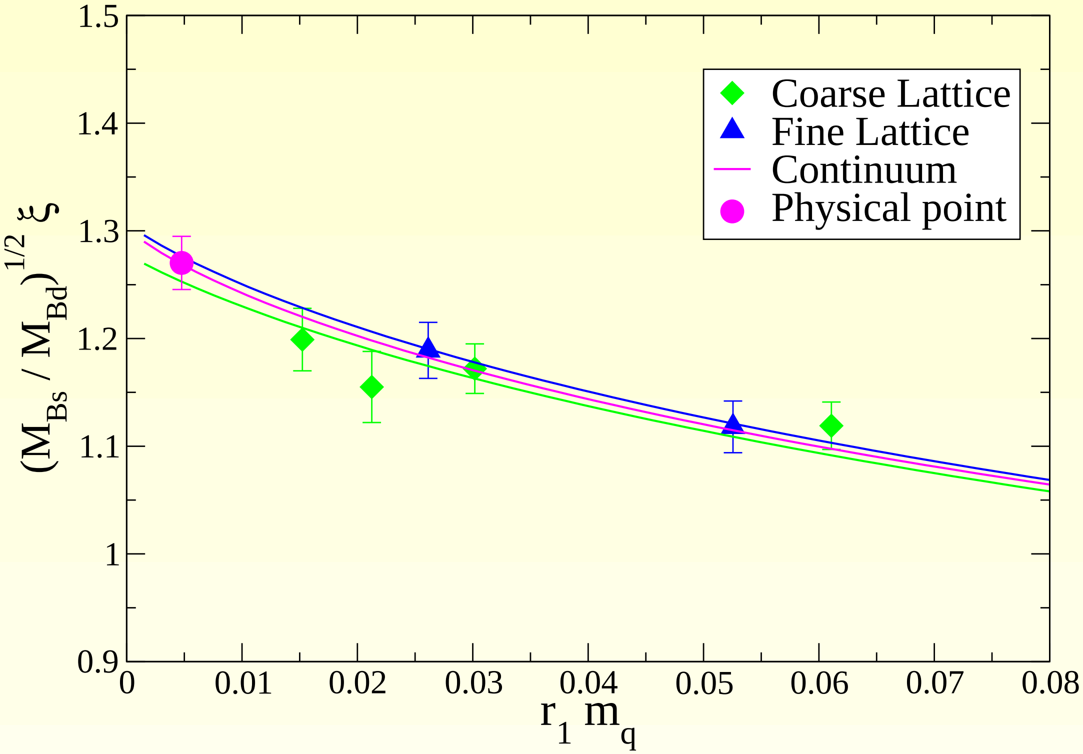
$$f_{B_d}\sqrt{\hat{B}_{B_d}} = 216(9)(12)\text{MeV}$$

Chiral+continuum extrapolations: NLO Staggered CHPT.

* accounts for NLO quark mass dependence.

* accounts for light quark discretization effects through $\mathcal{O}\left(\alpha_s^2 a^2 \Lambda_{QCD}^2\right)$
  $\rightarrow$ remove the dominant light discretization errors

Results for $\xi\sqrt{\frac{M_{B_s}}{M_{B_d}}}$

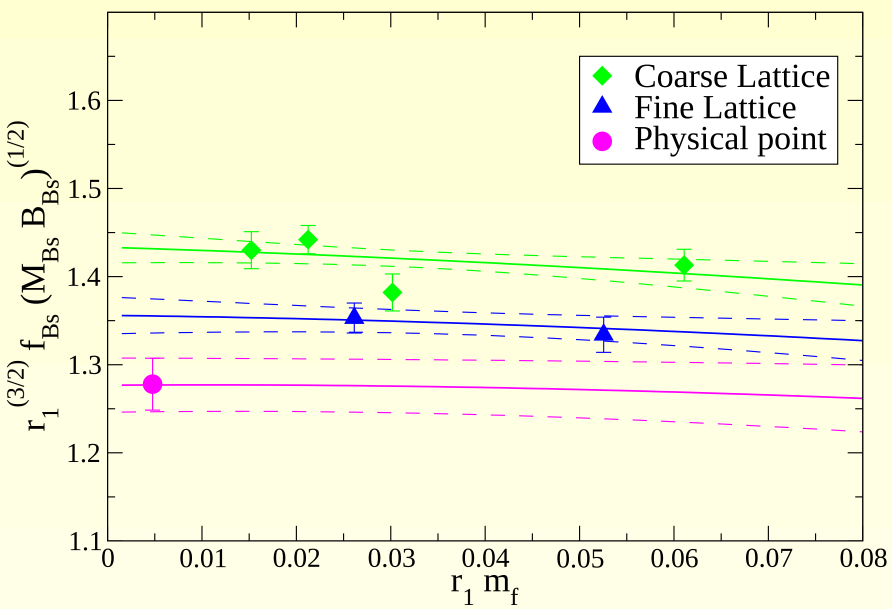


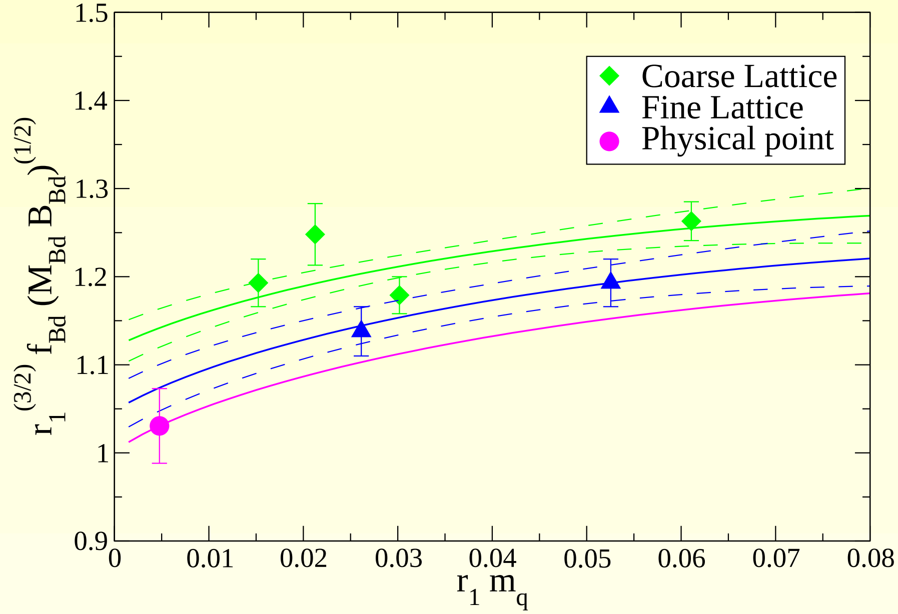
$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}} = 1.258(25)(21) \quad \Longrightarrow \quad \boxed{\left|\frac{V_{td}}{V_{ts}}\right| = 0.214(1)(5)}$$

* Previous value used in UT fits and another analyses (HPQCD/JLQCD):

$$\xi = 1.20 \pm 0.06$$

Results for $f_{B_q}\sqrt{M_{B_q}}$





$$f_{B_s} = 231(15) MeV$$

$$f_{B_d} = 190(13) MeV$$

$$\frac{f_{B_s}}{f_{B_d}} = 1.226(26)$$

* HPQCD previous numbers are $f_{B_s} = 260(29) MeV$, $f_{B_d} = 216(22) MeV$ and $f_{B_s}/f_{B_d} = 1.20(3)(1)$.

** chiral extrapolation based only on coarse lattice, no continuum extrapolation.

Bag parameters: Calculation of $\mathcal{B}r(B \to \mu^+\mu^-)$

Very interesting place to look for the effect of new operators in the effective Hamiltonian. **Hurth et al, NPB 808 (2009); Buras, arXiv:0904.4917**.

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$$\frac{\mathcal{B}r(B_q \to \mu^+\mu^-)}{\Delta M_q} = \tau(B_q)\, 6\pi \frac{\eta_Y}{\eta_B} \left(\frac{\alpha}{4\pi M_W sin^2\theta_W} \right)^2 m_\mu^2 \frac{Y^2(x_t)}{S(x_t)} \frac{1}{\hat{B}_q}$$

* Using our value of $\hat{B}_s \to$ $\boxed{\mathcal{B}r(B_s \to \mu^+\mu^-) = (3.2 \pm 0.3) \times 10^{-9}}$

to be compared with **CDF** bound $\mathcal{B}r(B_s \to \mu^+\mu^-) \leq 6 \times 10^{-8}$

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* Similarly for $\hat{B}_d \to$ $\boxed{\mathcal{B}r(B_d \to \mu^+\mu^-) = (0.98 \pm 0.12) \times 10^{-10}}$

to be compared with CDF bound $\mathcal{B}r(B_d \to \mu^+\mu^-) \leq 2 \times 10^{-8}$

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Comparison of experimental measurements and theoretical predictions can constraint some BSM parameters and help to understand BSM physics.

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Effects of heavy new particles seen in the form of effective operators built with **SM** degrees of freedom

The most general `Effective Hamiltonian` describing $\Delta B = 2$ processes is

$$\mathcal{H}_{eff}^{\Delta B=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \widetilde{C}_i \widetilde{Q}_i \qquad \text{with}$$

$$Q_1^q = \left(\bar{\psi}_b^i \gamma^\nu (\mathrm{I} - \gamma_5) \psi_q^i\right) \left(\bar{\psi}_b^j \gamma^\nu (\mathrm{I} - \gamma_5) \psi_q^j\right)$$

SM

$$Q_2^q = \left(\bar{\psi}_b^i (\mathrm{I} - \gamma_5) \psi_q^i\right) \left(\bar{\psi}_b^j (\mathrm{I} - \gamma_5) \psi_q^j\right) \qquad Q_3^q = \left(\bar{\psi}_b^i (\mathrm{I} - \gamma_5) \psi_q^j\right) \left(\bar{\psi}_b^j (\mathrm{I} - \gamma_5) \psi_q^i\right)$$

$$Q_4^q = \left(\bar{\psi}_b^i (\mathrm{I} - \gamma_5) \psi_q^i\right) \left(\bar{\psi}_b^j (\mathrm{I} + \gamma_5) \psi_q^j\right) \qquad Q_5^q = \left(\bar{\psi}_b^i (\mathrm{I} - \gamma_5) \psi_q^j\right) \left(\bar{\psi}_b^j (\mathrm{I} + \gamma_5) \psi_q^i\right)$$

$$\tilde{Q}_{1,2,3}^q = Q_{1,2,3}^q \text{ with the replacement } (\mathrm{I} \pm \gamma_5) \rightarrow (\mathrm{I} \mp \gamma_5)$$

where ψ_b is a heavy b-fermion field and ψ_q a light ($q = u, d$) fermion field.

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- $C_i, \widetilde{C}_i$ Wilson coeff. calculated for a particular BSM theory
- $\langle \bar{B}^0 | Q_i | B^0 \rangle$ calculated on the `lattice`

Some examples:

F. Gabbiani et al, Nucl.Phys.B477 (1996), **D. Bećirević et al**, Nucl.Phys.B634 (2002); `general SUSY models`

P. Ball and R. Fleischer, Eur.Phys.J. C48(2006); `extra Z' boson`; SUSY

Help to constrain the soft SUSY breaking terms and the mechanism of SUSY breaking.

M. Ciuchini and L. Silvestrini, PRL 97 (2006) 021803; `SUSY`

Constraints on the mass insertions ($|Re(\delta^d_{23})_{RR}| < 0.4$, $|(\delta^d_{23})_{LL}| < 0.1$,...)

M. Blanke et al, JHEP 12(2006) 003; `Little Higgs model with T-parity`

ΔM_q can be used to test viability of the model. To constrain and test the model in detail $\Delta M_s/\Delta M_d$ and $\Delta\Gamma_q$.

Lunghi and Soni, 0707.0212; `Top Two Higgs Doublet Model`

Constraints on β_H (ratio of vev's of the two Higgs) and m_{H^+}

M. Blanke et al, 0809.1073; `Warped Extra Dimensional Models`

Constraints on the KK mass scale (it can be as low as $M_{KK} \simeq 3TeV$)

6. Conclusions and outlook

SM results for the B_s^0 and B_d^0 mixing parameters (ΔM and $\Delta\Gamma$)

* $f_B\sqrt{B_B}$ with 7% error and ξ with 2.6% error.

SM results for decay constants, f_{B_s} with 6% error and f_{B_d} with 7%.

Important tests of the SM are possible

Improvements of the analysis: More statistics, more (and smaller) lattice spacings, improved renormalization techniques, direct extraction of bag parameters from fits . . .

Same accuracy can be achieved for the matrix elements in the general $\Delta B = 2$ effective hamiltonian **BSM**.

* The value of those matrix elements, together with experimental data, will help to constrain the parameter space in **BSM** theories

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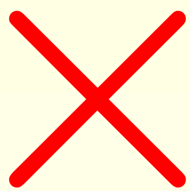
Similar analysis from the **FNAL/MILC** will be completed soon.



Error budget: B^0 mixing (in %)

	$f_{B_s}\sqrt{\hat{B}_{B_s}}$	$f_{B_d}\sqrt{\hat{B}_{B_d}}$	ξ
Statistical + chiral extrapolation	2.3	4.1	2.0
Residual a^2	3.0	2.0	0.3
$r_1^{3/2}$ uncertainty	2.3	2.3	-
$g_{BB^*\pi}$	1.0	1.0	1.0
m_s and m_b tuning	1.5	1.0	1.0
operator matching	4.0	4.0	0.7
relativistic corrections	2.5	2.5	0.4
Total	**6.7**	**7.1**	**2.6**

Error budget: Decay constants (in %)

	f_{B_s}	f_{B_d}	f_{B_s}/f_{B_d}
Statistical + chiral extrapolation	2.2	3.5	1.6
Residual a^2	3.0	3.0	0.5
$r_1^{3/2}$ uncertainty	2.3	2.3	-
$g_{BB^*\pi}$	1.0	1.0	0.3
m_s and m_b tuning	1.5	1.0	1.0
operator matching	4.0	4.0	0.7
relativistic corrections	1.0	1.0	0.2
Total	**6.3**	**6.7**	**2.1**