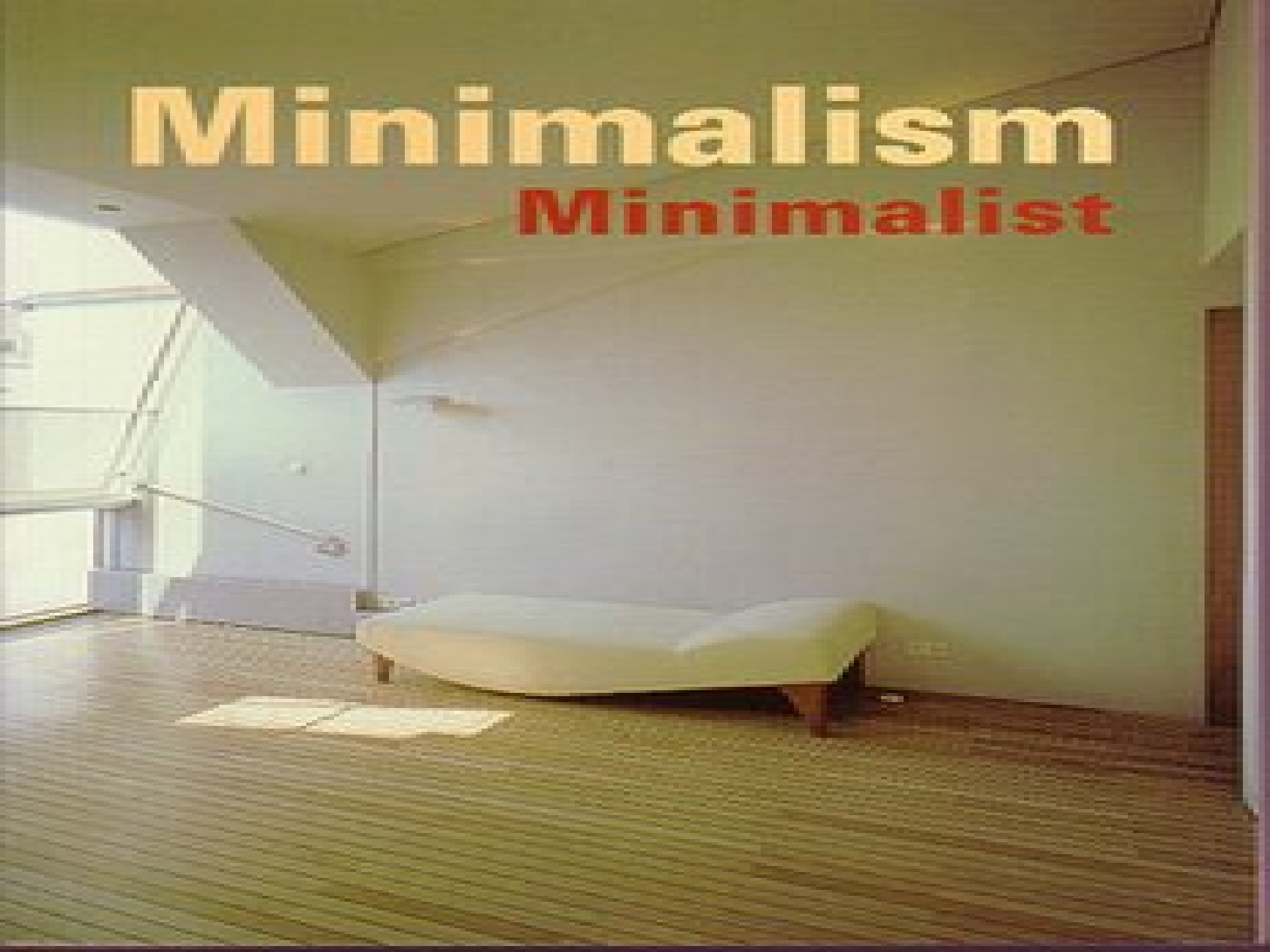


# Minimalism

## Minimalist



# Minimalism

## Minimalist

### de Sitter Vacua

Bret Underwood  
McGill University  
Pheno 2009

S. Haque, G. Shiu, *BU*, T. Van Riet, Phys. Rev. D79:086005 (2009), arXiv:0810.5328

Does dS space exist classically from a top-down construction?

# dS Space

## Bottom Up

To construct classical dS solutions,  
just add cosmological constant

$$H^2 = \frac{1}{3M_p^2} (\rho + \Lambda_{cc})$$

# dS Space

## Bottom Up

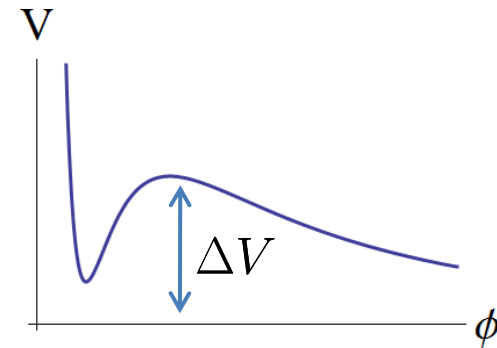
To construct classical dS solutions,  
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## Top Down

Cosmological constant comes from  
scalar potential with positive energy  
local minimum.

Constructions require moduli  
stabilization, *quantum effects*. [KKLT, 03], [LARGE volume, 05],...



**Is nature telling us that dS space is inherently quantum, or  
do *classical* top-down constructions exist?**

Other interesting features:

- Typically  $\Delta V \sim M_p^2 m_{3/2}^2$ ,  $\Rightarrow H_{inf} < m_{3/2}$
- Typically  $m_\phi \sim \mathcal{O}(\text{few} \times \text{TeV})$  non-thermal decay into DM

# No-Go Theorems

Dimensional reduction of 10d supergravity  $\rightarrow$  potential in 4D:

Two scalar fields always present  
in any compactification

$$\rho \equiv (\text{Vol})^{1/3}, \quad \tau \equiv e^{-\phi}(\text{Vol})^{1/2}$$

$$V(\rho, \tau) = a(\rho)\tau^{-2} - b(\rho)\tau^{-3} + c(\rho)\tau^{-4}$$

Internal curvature, H<sub>3</sub>  
flux, KK5-branes, NS5-  
branes, (indefinite sign)

D-branes  
O-planes  
(indefinite sign)

p-form fluxes  
(always positive)

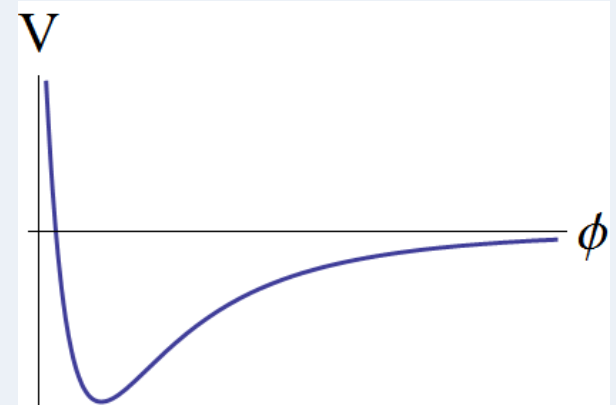
**No-Go:** [Maldacena, Nunez, 00]

“If no sources, then no dS”

$$V(\rho, \tau) = a(\rho)\tau^{-2} - \cancel{b(\rho)\tau^{-3}} + c(\rho)\tau^{-4}$$

Always has minimum with

$$\underline{V < 0}$$



# No-Go Theorems

$$V(\rho, \tau) = a(\rho)\tau^{-2} - b(\rho)\tau^{-3} + c(\rho)\tau^{-4}$$

Finding dS vacua is as easy as “a,b,c”:

dS vacua  $\iff$  Minimizing the quantity  $\frac{4ac}{b^2} \approx 1$  [Silverstein, 08]

**No-Go:** [Tegmark et al, 07]

“If IIA, fluxes, **O6/D6**, then no dS vacua”

$$a(\rho) = \frac{A_{NSNS}}{\rho^3}$$

H3 flux, (always positive)

$$b(\rho) = n_{O6}A_{O6} - n_{D6}A_{D6}$$

D-branes O-planes (indefinite sign)

$$c(\rho) = \sum_p \rho^{3-p} A_p^{RR}$$

p-form fluxes (always positive)

$$\frac{4ac}{b^2} = (\text{const}) \sum_p \rho^{-p} A_p^{RR}$$

Cannot be minimized in  $\rho$  direction:  
No dS vacua!

# Minimal dS vacua [\[Underwood et al 08\]](#)

$$V = \left( \frac{A_{curvature}}{\rho} + \frac{A_{NSNS}}{\rho^3} \right) \tau^{-2} - (n_{O6} - n_{D6}) A_6 \tau^{-3} + \left( \sum_p \rho^{3-p} A_p^{RR} \right) \tau^{-4}$$
$$A_{curvature} \sim - \int R_6$$

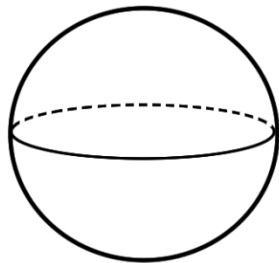
Zero Curvature:

$$A_{curvature} \sim - \int R_6 = 0$$



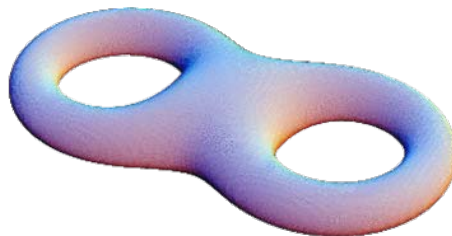
Positive Curvature:

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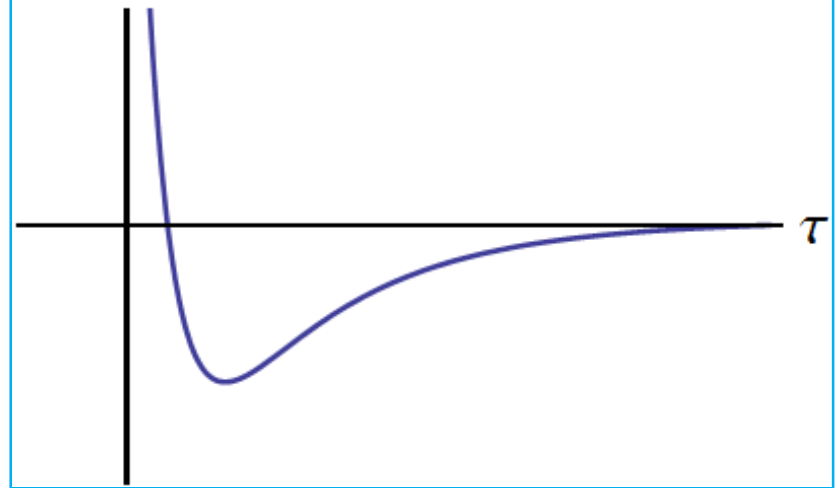


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$V(\tau)$



# Minimal dS vacua [\[Underwood et al 08\]](#)

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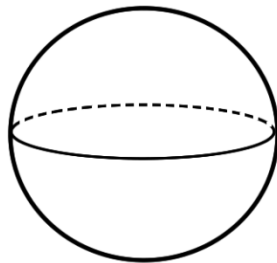
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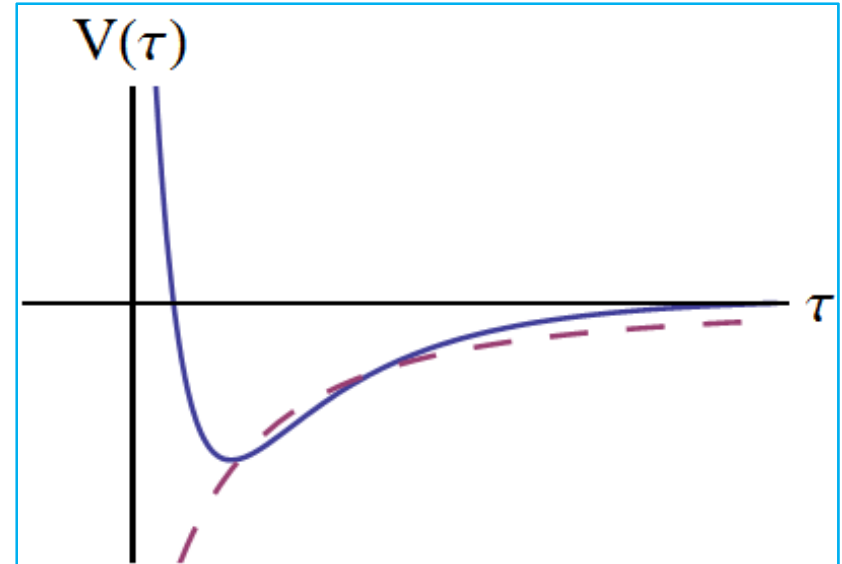
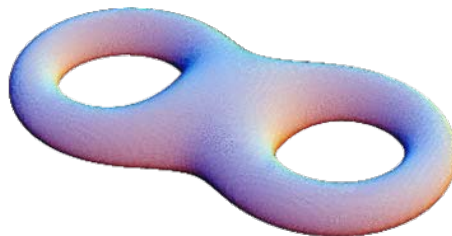
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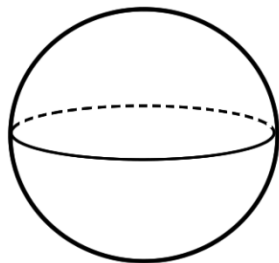
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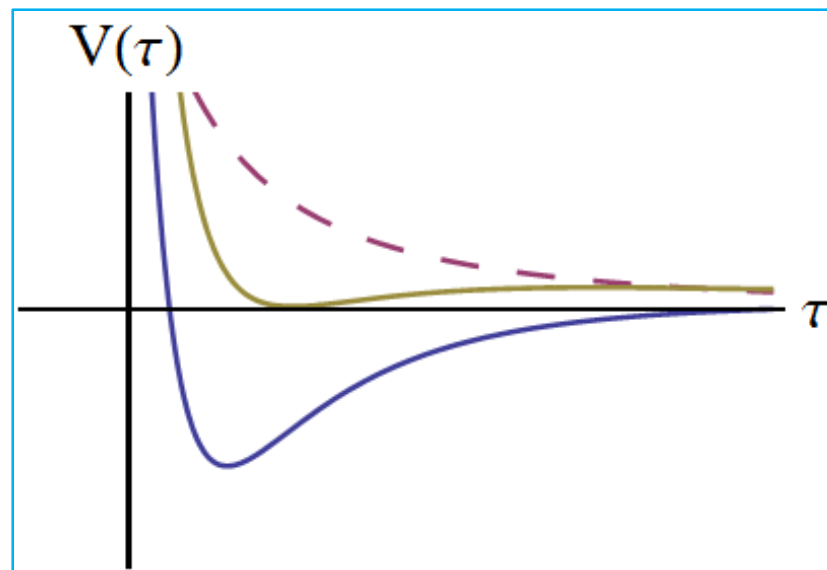
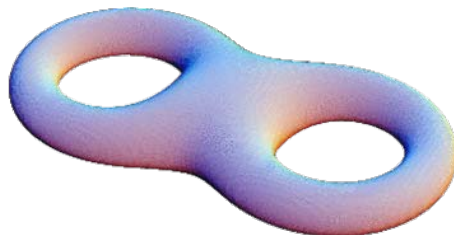
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Minimal dS vacua:  
 “If IIA, fluxes, O6/D6, **negative curvature**, then dS vacua possible”

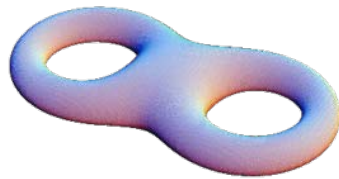
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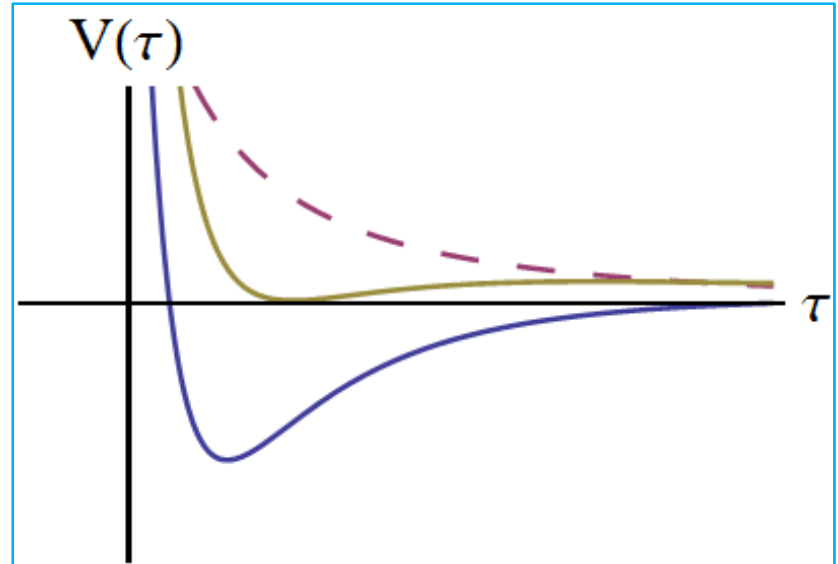
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Minimal dS vacua:

“If IIA, fluxes, O6/D6, **negative curvature**, then dS vacua possible”



$$\frac{4ac}{b^2} = (\text{const}) \sum_p A_p^{RR} [A_{curvature} \rho^{2-p} + A_{NSNS} \rho^{-p}] \quad \text{Can minimize } \frac{4ac}{b^2} \approx 1$$

# Summary

Minimal Ingredients for 10d dS (in IIA):

“If IIA, fluxes, O6/D6, **internal curvature**, then dS vacua possible”

Examples...?

- “Twisted 3-tori”: Can classify all possibilities. All **fail** – additional moduli lead to runaway directions. [Underwood et al, 08]
- “Twisted 6-tori”: simple constructions all **fail** [Caviezel et al, 08]  
[Flauger et al, 08]
- “Compact Hyperbolic Manifolds”: **Seem to work!** [Underwood et al, 08]

Lift to 10D? Solving 10D equations tricky...

[Underwood et al, in progress]

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**Q: Is nature telling us that dS vacua are inherently quantum, or do *classical* top-down constructions exist?**

**A: Minimal (necessary) ingredients known, but are they sufficient?**

# Example: Compact Hyperbolic Manifolds

Consider 6D internal space to be product of 3-dimensional compact hyperbolic spaces:

$$ds_6^2 = d\mathbb{H}_3^2 + d\tilde{\mathbb{H}}_3^2 \quad d\mathbb{H}_3^2 = \frac{6}{\Lambda}(d\varphi^2 + \sinh^2(\varphi)d\Omega_2^2)$$

O6-plane maps  $\mathbb{H}_3 \leftrightarrow \tilde{\mathbb{H}}_3$

$$f_6 \in \mathbb{Z}, f_0 = 1, 2$$

$$\frac{V(\tau, \rho)}{M_p^2 \alpha'^{-1}} = \tau^{-2} \left[ \frac{1}{\rho} + \frac{32\pi^4}{f_0^2} \rho^{-3} \right] - 4\sqrt{8}\pi\tau^{-3} + \left[ \frac{f_0^2}{16\pi^2} \rho^3 + \frac{(2\pi)^{10} f_6^2}{\rho^3} \right] \tau^{-4}$$

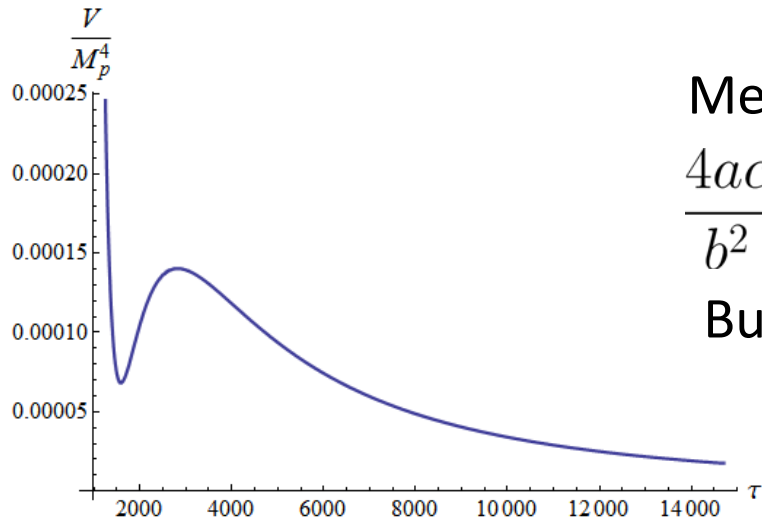
Curvature

NSNS flux

O6-plane

0-form  
flux

6-form  
flux



Meta-stable solutions exist:

$$\frac{4ac}{b^2} \approx 1.03, \quad \rho_{dS} \approx 90, \quad V_{dS}/M_p^4 \approx 8 \times 10^{-5}$$

But with tradeoffs...

$$g_s \approx \frac{\sqrt{\rho_{dS}}}{4\pi\sqrt{2}} \approx 0.56 \quad \frac{H^{-1}}{r_c} \sim 5 \quad m_{\text{moduli}} \sim m_{kk}$$