Oscillation of Dirac or Majorana neutrinos produced in muon decay

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Outline

1. Introduction

2. Could we distinguish Dirac from Majorana neutrinos in a near detector ?

3. Dirac and Majorana neutrinos after oscillation in a far detector

4. Conclusions

1. Introduction

μ^- decay

For Dirac neutrinos

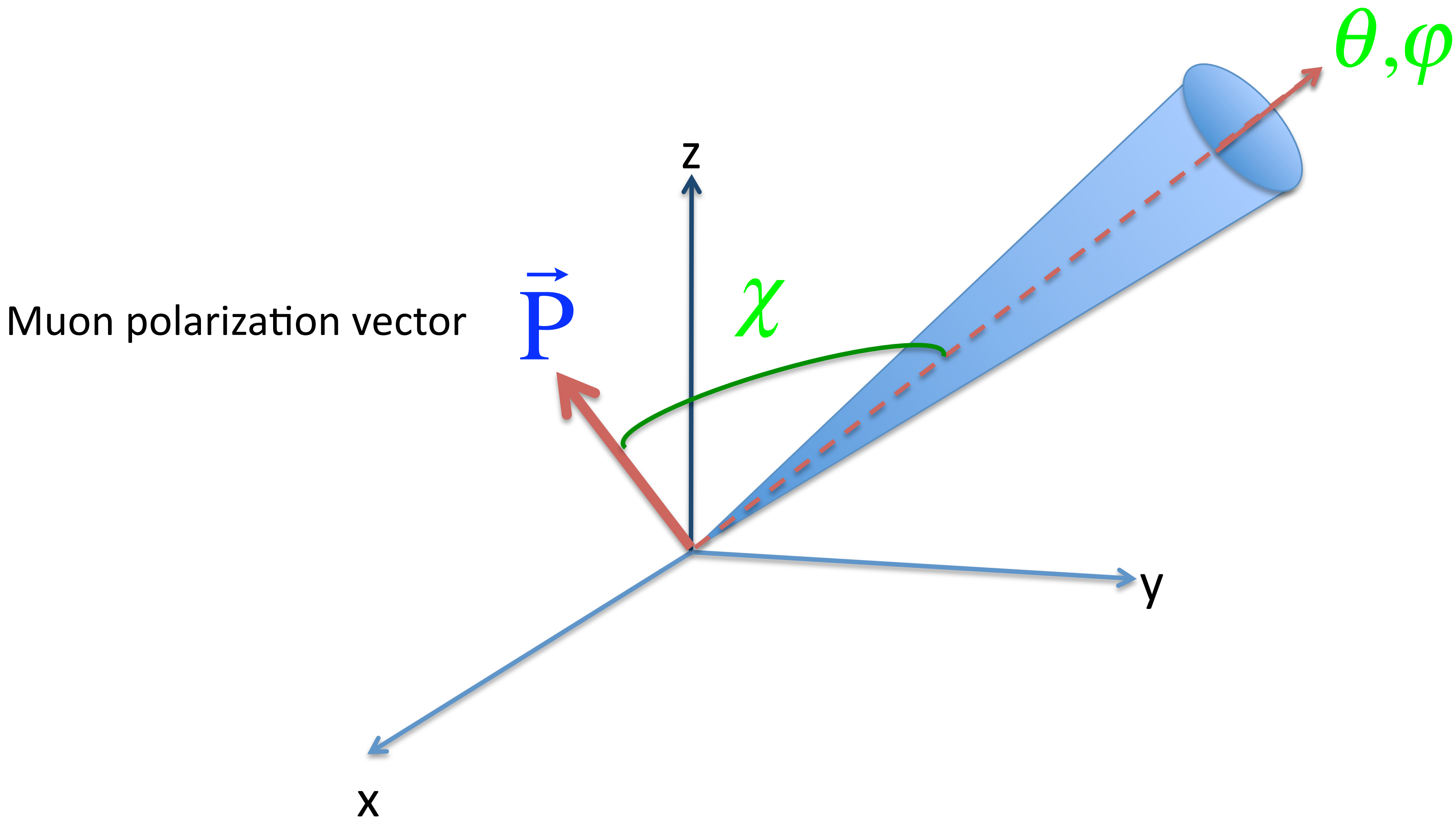
$$\mu^-(\lambda_\mu) \to e^-(\lambda_e) + \overline{\nu}_n(\lambda_n) + \nu_m(\lambda_m)$$

In the SM (+1) (-1)

Beyond the SM (+1,-1)

For Majorana neutrinos

$$\mu^-(\lambda_\mu) \to e^-(\lambda_e) + \nu_n(\lambda_n) + \nu_m(\lambda_m)$$



Number od electron neutrino and muon neutrino in the solid angle depends on $d\Omega$ direction θ,φ.
Generally both types of neutrino are observed

For Dirac neutrinos

In the SM

$$\bar{\nu}_e = \nu(\lambda = +1) \qquad \left|\bar{\nu}_e\right\rangle = \sum_{i=1}^{3} U_{e\,i}\left|\bar{\nu}_i\right\rangle$$

distingishable

$$\nu_\mu = \nu(\lambda = -1) \qquad \left|\nu_\mu\right\rangle = \sum_{i=1}^{3} U^*_{\mu i}\left|\nu_i\right\rangle$$

Pure QM STATES

Beyond the SM

$$\bar{\nu}_e \rightarrow \nu(\lambda = +1)$$

$$\nu_\mu \rightarrow \nu(\lambda = -1)$$

Mixed QM STATES

Density matrix required

For Majorana neutrinos

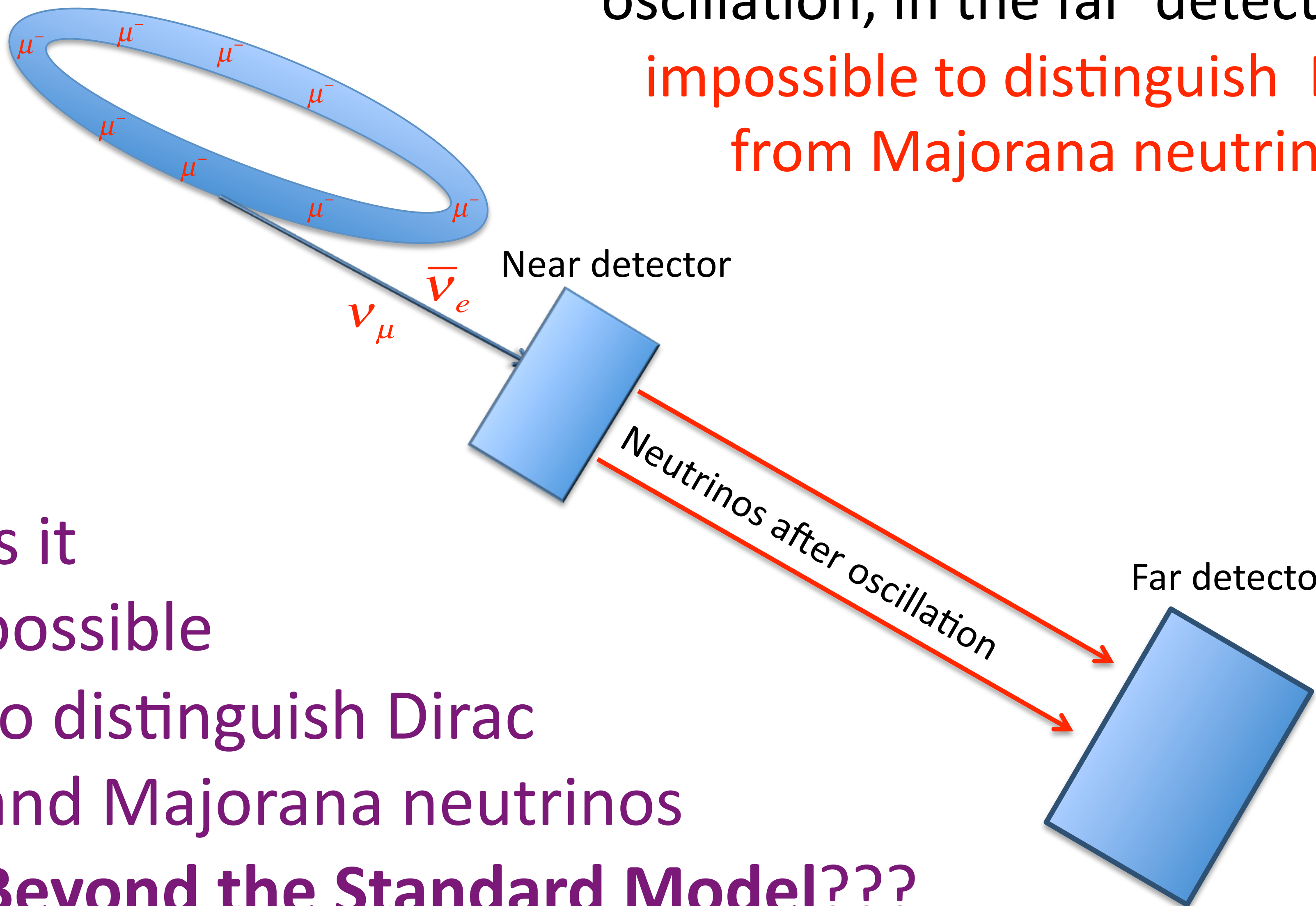
In the SM or beyond

$$\nu_e \rightarrow \nu(\lambda = +1)$$

$$\nu_\mu \rightarrow \nu(\lambda = -1)$$

QM mixed STATE

In the **Standard Model** whether in a near detector or, after oscillation, in the far detector, it is impossible to distinguish Dirac from Majorana neutrinos.



Is it possible to distinguish Dirac and Majorana neutrinos **Beyond the Standard Model**???

2. Could we distinguish Dirac from Majorana neutrinos in a near detector ?

$$\mu^-(\lambda_\mu) \to e^-(\lambda_e) + \overline{\nu}_n(\lambda_n) + \nu_m(\lambda_m)$$

$$\nu_m(\lambda_m) + e^-(\lambda_e) \to \mu^-(\lambda_\mu) + \nu_n(\lambda_n)$$

$$\overline{\nu}_n(\lambda_n) + e^-(\lambda_e) \to \mu^-(\lambda_\mu) + \overline{\nu}_m(\lambda_m)$$

In the near detector we look for mions produced by inverse muon decay processes, assuming that neutrino are Dirac or Majorana particles

We assume that neutrino interactions are described by the most general 4-fermion interaction

W.Fetscher, H,-J.Gerber and K.J.Johnson, Phys. Lett. B173(1986)102;

$$\boldsymbol{H} = \frac{4G_F}{\sqrt{2}} \sum_{\delta,\varepsilon,\varepsilon'} \sum_{i,k=1}^{3} \left[(g^{\delta}_{\varepsilon,\varepsilon'})_{i,k} (\bar{l}_{\varepsilon,e} \Gamma^{\delta} \nu_i)(\overline{\nu}_k \Gamma_{\delta} l_{\varepsilon',\mu}) + (g^{\delta\,*}_{\overline{\varepsilon},\overline{\varepsilon}'})_{i,k} (\overline{\nu}_i \Gamma^{\delta}\, l_{\overline{\varepsilon},e})(\bar{l}_{\overline{\varepsilon}',\mu} \Gamma_{\delta} \nu_k) \right]$$

$$\delta = \left(S(=1), V(=\gamma^{\mu}), T(=\frac{i}{2\sqrt{2}}[\gamma^{\mu},\gamma^{\nu}] \equiv t_{\mu\nu}) \right)$$

$$\varepsilon, \varepsilon' = L, R \left(P_{R,L} = \tfrac{1}{2}(1 \pm \gamma_5) \right)$$

$\overline{\varepsilon} = \varepsilon$ for V, $\overline{\varepsilon} = -\varepsilon$ for S and T and the same for $\overline{\varepsilon}'$.

Standard Model is recovered for

$$(g^{V}_{LL})_{i,k} = g^{V}_{LL} U_{e,i} U^{*}_{\mu,k},$$

MNSP matrix (U); g^{V}_{LL} =1

All other couplings equal zaro

In the same way we paremeterize

$$(g^{\delta}_{\varepsilon,\varepsilon'})_{i,k} = g^{\delta}_{\varepsilon,\varepsilon"}(U^{\delta}_{\varepsilon})_{e,i}(U^{\delta}_{\varepsilon'})^{*}_{\mu,k}$$

$(S_{LL}, S_{LR}, S_{RL}, S_{RR})$ (U^S_L, U^S_R) $(= V)$

$(V_{LL}, V_{LR}, V_{RL}, V_{RR})$ (U^V_L, U^V_R) $(= U_{MNSP} \equiv U)$

(T_{LR}, T_{RL}) (U^T_L, U^T_R)

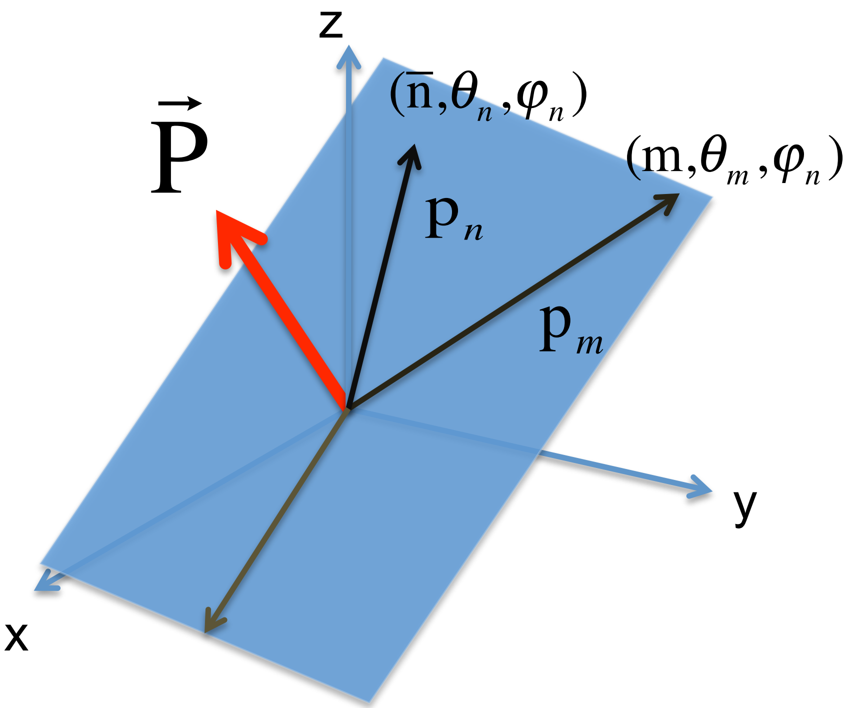
Neutrino masses are not measured,

- sumed uncoherently over final neutrino mass states,
- averaged over initial neutrino states.

$$\Gamma_{total} = \sum_{n,m=1}^{3} \Gamma_{\overline{n},m}$$

For the process:

$$\mu^-(\lambda_\mu) \to e^-(\lambda_e) + \overline{\nu}_n(\lambda_n) + \nu_m(\lambda_m)$$



Denotation: $K_n \equiv (\lambda_n, p_n, \theta_n, \varphi_n)$

$$A^D_{\bar{n},m}(K_n, K_m)$$

$$A^{AD}_{n,\bar{m}}(K_n, K_m)$$

$$A^{AD}_{n,\bar{m}}(K_n, K_m) = A^D_{\bar{m},n}(K_m, K_n)$$

$$A^M_{n,m} = A^D_{\bar{n},m}(K_n, K_m) - A^{AD}_{n,\bar{m}}(K_n, K_m)$$

$$= A^D_{\bar{n},m}(K_n, K_m) - A^D_{\bar{m},n}(K_m, K_n)$$

If neutrino masses are neglected,

crucial term ➡ $\left(-2\,\mathrm{Re}[A^D_{\bar{n},m}(K_n, K_m)\, A^{D*}_{\bar{m},n}(K_m, K_n)]\right)$

In the SM only one neutrino helicity amplitudes does not vanish:

$$A^{D}_{\bar{n},m}(\lambda_n = +1, \lambda_m = -1)$$

And because of that:

$$A^{M}_{n,m}(+,+) = A^{M}_{n,m}(-,-) = 0$$

$$A^{M}_{n,m}(+,-) = A_{\overline{n},m},\ A^{M}_{n,m}(-,+) = -A_{\overline{m},n}$$

Only one neutrino helicity configuration contribute to the spin amplitudes, interference terms do not appear,

there is no difference between Dirac and Majorana

Beyond the SM

P. Langacker and D. London, Phys. Rev. D 39(1989)266,

They conclude:

"It is not possible, even in principle, to test lepton number nonconservation in muon decay if final neutrino are masless"

The reason – Fierz identities

Dirac and Majorana amplitudes are equal after substitution:

$$
\begin{aligned}
(g^S_{LR})^D_{i,k} &\Leftrightarrow (g^S_{LR})^M_{i,k} - \frac{1}{2}(g^S_{LR})^M_{k,i} + \frac{3}{2}(g^T_{LR})^M_{k,i},\\
(g^S_{LL})^D_{i,k} &\Leftrightarrow (g^S_{LL})^M_{i,k} + 2\,(g^V_{LL})^M_{k,i},\\
(g^S_{RR})^D_{i,k} &\Leftrightarrow (g^S_{RR})^M_{i,k} + 2\,(g^V_{RR})^M_{k,i},\\
(g^S_{RL})^D_{i,k} &\Leftrightarrow (g^S_{RL})^M_{i,k} - \frac{1}{2}(g^S_{RL})^M_{k,i} + \frac{3}{2}(g^T_{RL})^M_{k,i},\\
(g^V_{RR})^D_{i,k} &\Leftrightarrow (g^V_{RR})^M_{i,k} + \frac{1}{2}(g^S_{RR})^M_{k,i},\\
(g^V_{RL})^D_{i,k} &\Leftrightarrow (g^V_{RL})^M_{i,k} + (g^V_{RL})^M_{k,i},\\
(g^V_{LR})^D_{i,k} &\Leftrightarrow (g^V_{LR})^M_{i,k} + (g^V_{LR})^M_{k,i},\\
(g^V_{LL})^D_{i,k} &\Leftrightarrow (g^V_{LL})^M_{i,k} + \frac{1}{2}(g^S_{LL})^M_{k,i},\\
(g^T_{LR})^D_{i,k} &\Leftrightarrow (g^T_{LR})^M_{i,k} + \frac{1}{2}(g^S_{LR})^M_{k,i} + \frac{1}{2}(g^T_{LR})^M_{k,i},\\
(g^T_{RL})^D_{i,k} &\Leftrightarrow (g^T_{RL})^M_{i,k} + \frac{1}{2}(g^S_{RL})^M_{k,i} + \frac{1}{2}(g^T_{RL})^M_{k,i}.
\end{aligned}
$$

If the couplings are unknown -> always we can find such couplings that these relations are satisfied

SM coupling V_{LL} mixes with the scalar S_{LL} one.

For Majorana neutrinos, but not for Dirac, there are observables linearly proportional to

g^S_{LL}

If NP scalar coupling S_{LL} is known **from other suorce**, Dirac and Majorana neutrinos are in principle distinguishable

Using general interaction we calculate for neutrinos from muon decay

1. Density matrix for final Dirac neutrinos

$$\varrho^{\nu}_{i,\lambda;k,\eta}(E,\theta,\varphi) =$$
$$\frac{1}{N_\mu}\int_0^{2\pi} d\overline{\psi} \int_{\overline{E}_{min}}^{\overline{E}_{max}} d\overline{E}_\nu \sum_{\lambda_\mu,\lambda'_\mu,\lambda_e,\overline{\lambda}_\nu,\overline{j}} A^{\overline{j},i}_{(\lambda_\mu,\lambda_e,\overline{\lambda}_\nu,\lambda)}(E,\theta,\varphi;\overline{E}_\nu,\overline{\psi})\varrho_{\lambda_\mu,\lambda'_\mu} A^{\overline{j},k\ *}_{(\lambda'_\mu,\lambda_e,\overline{\lambda}_\nu,\eta)}(E,\theta,\varphi;\overline{E}_\nu,\overline{\psi})$$

2. In the same way final density matric for Dirac antineutrinos

3. Density matrix for final Majorana neutrinos

Then we calclate cross sections for muon production processes with Dirac neutrinos

$$\nu_m(\lambda_m) + e^-(\lambda_e) \to \mu^-(\lambda_\mu) + \nu_n(\lambda_n)$$

$$\sigma^\nu = \frac{p_f}{264\pi^2 s p_i} \sum_{\lambda_e,n,\lambda_n;\lambda_\mu,i,\lambda_i,k,\lambda_k} \int d\vartheta d\phi f^{\nu}{}_{\lambda_e,n,\lambda_n;\lambda_\mu,i,\lambda_i}(E,\vartheta,\phi)\varrho^{\nu}_{i,\lambda;k,\eta} f^{\nu *}{}_{\lambda_e,n,\lambda_n;\lambda_\mu,k,\lambda_k}(E,\vartheta,\phi)$$

And similarly for Dirac antineutrin → $\sigma^{\bar{\nu}}$

and for Majorana neutrino → σ^{ν_M}

We have to know the number of Dirac neutrinos and Dirac antineutrinos flying in direction (θ,φ) ,

→ we caculate angular distribution:

$$N^{\nu}(E,\theta,\varphi)=\frac{d^3\Gamma^{\nu}}{dEd\theta d\varphi} \qquad N^{\bar{\nu}}(E,\theta,\varphi)=\frac{d^3\Gamma^{\bar{\nu}}}{dEd\theta d\varphi}$$

So number of neutrino and antineutrino in the beam is proportional respectively to:

$$\alpha(E,\theta,\varphi)=\frac{N^{\nu}}{N^{\nu}+N^{\bar{\nu}}} \qquad \beta(E,\theta,\varphi)=\frac{N^{\bar{\nu}}}{N^{\nu}+N^{\bar{\nu}}} \qquad \alpha+\beta=1$$

For Majorana neutrinos such weight factor are automaticaly included
In the density matrix.

$$\sigma^{v} = \frac{128}{3}\pi\, G_F^2\, p_i^2\, p_k \left(3\,\varepsilon_\mu \left(8\, \mathrm{ammvv1}\, (V_{LL})_{12}\, (V_{LL})_{34}\, ((V_{LL})^*)_{12}\, ((V_{LL})_{34})^*\right) + p_k \left(24\, \mathrm{ammvv1}\, (V_{LL})_{12}\, (V_{LL})_{34}\, ((V_{LL})^*)_{12}\, ((V_{LL})_{34})^*\right)\right) + O(NP\text{^}2);$$

$$\sigma_{\downarrow}^{M} = \left[\begin{array}{l}
512\, \mathrm{ammvv1}\, \pi\, G_F^2\, p_i^2\, p_k^2\, (V_{LL})_{12}\, (V_{LL})_{34}\, ((V_{LL})^*)_{12}\, ((S_{LL})_{43})^* + \\
512\, \mathrm{ammvv1}\, \pi\, G_F^2\, p_i^2\, p_k\, \varepsilon_\mu\, (V_{LL})_{12}\, (V_{LL})_{34}\, ((V_{LL})^*)_{12}\, ((S_{LL})_{43})^* + \\
1024\, \mathrm{ammI1vs1}\, \pi\, G_F^2\, p_i^2\, p_k^2\, (V_{LL})_{12}\, (V_{LL})_{34}\, ((S_{LL})^*)_{21}\, ((V_{LL})_{34})^* + \\
1024\, \mathrm{ammI1vs1}\, \pi\, G_F^2\, p_i^2\, p_k\, \varepsilon_\mu\, (V_{LL})_{12}\, (V_{LL})_{34}\, ((S_{LL})^*)_{21}\, ((V_{LL})_{34})^* + \\
1024\, \mathrm{ammI2sv2}\, \pi\, G_F^2\, p_i^2\, p_k^2\, (S_{LL})_{21}\, (V_{LL})_{34}\, ((V_{LL})^*)_{12}\, ((V_{LL})_{34})^* + \\
1024\, \mathrm{ammI2sv2}\, \pi\, G_F^2\, p_i^2\, p_k\, \varepsilon_\mu\, (S_{LL})_{21}\, (V_{LL})_{34}\, ((V_{LL})^*)_{12}\, ((V_{LL})_{34})^* + \\
1024\, \mathrm{ammvv1}\, \pi\, G_F^2\, p_i^2\, p_k^2\, (V_{LL})_{12}\, (V_{LL})_{34}\, ((V_{LL})^*)_{12}\, ((V_{LL})_{34})^* + \\
1024\, \mathrm{ammvv1}\, \pi\, G_F^2\, p_i^2\, p_k\, \varepsilon_\mu\, (V_{LL})_{12}\, (V_{LL})_{34}\, ((V_{LL})^*)_{12}\, ((V_{LL})_{34})^*
\end{array}\right.$$

For Majorana neutrinos, because the interference between particle and antiparticle, there are terms linear in NP parameters. For Dirac neutrinos there are only qudratic term in NP.

We compare $\sigma^D \equiv \alpha\, \sigma^{\nu} + \beta\, \sigma^{\bar{\nu}}$ with σ^M $[10^{-45}\,\mathrm{m}^2]$ for L=0 $\left(\frac{\sigma^D - \sigma^M}{\sigma^D}\right) 100\%$

Parameters	Polarisation	Dirac particle $\alpha\, \sigma^{\nu}$	Dirac antyparticle $\beta\, \sigma^{\bar{\nu}}$	Sum $\alpha\, \sigma^{\nu} + \beta\, \sigma^{\bar{\nu}}$	Majorana σ^M	Procentage difference
1)SM	parallel	1,7372	2,2751	4,0123	4,0123	0,00%
1)SM	orthogonal	2,7141	1,8605	4,5746	4,5746	0,00%
1)SM	antiparallel	7,0983	0,0000	7,0983	7,0983	0,00%
2)SM+(S_{LL}=0.5)	parallel	1,6434	2,1476	3,7910	3,4916	7,90%
2)SM+(S_{LL}=0.5)	orthogonal	2,5613	1,7610	4,3223	3,9810	7,90%
2)SM+(S_{LL}=0.5)	antiparallel	6,6807	0,0261	6,7068	6,1772	7,90%
3)SM+(V_{RR}=0.03)	parallel	1,7369	2,2747	4,0115	4,0128	-0,03%
3)SM+(V_{RR}=0.03)	orthogonal	2,7117	1,8588	4,5705	4,5726	-0,05%
3)SM+(V_{RR}=0.03)	antiparallel	7,0674	0,0001	7,0674	7,0739	-0,09%
4)NP	parallel	1,7330	2,2697	4,0027	3,9194	2,08%
4)NP	orthogonal	2,7070	1,8553	4,5623	4,4680	2,07%
4)NP	antiparallel	7,0725	0,0008	7,0733	6,9265	2,08%

With NP cross cections for Dirac and Majorana neutrino differ, with present boud on NP parameters, this difference can be large (>7%) for E= 20 GeV.
(NP= {V_{LL}=1, S_{LL}=0.1, and all others = 0.01})

3. Dirac and Majorana neutrinos after oscillation in a far detector

Neutrino oscillation is described by density matrix:

$$\rho \Rightarrow \rho(i,\lambda;k,\eta)$$

Density matrix is calculated in muon rest frame → Lorentz boost

→ integration over detector solid angle

For very small neutrino masses Lorentz boost does not change density matrix

$$\rho \xrightarrow{\text{Lorent boost}} \rho$$

M.Ochman,R. Szafron,MZ,
J.Phys.G35:065003,2008

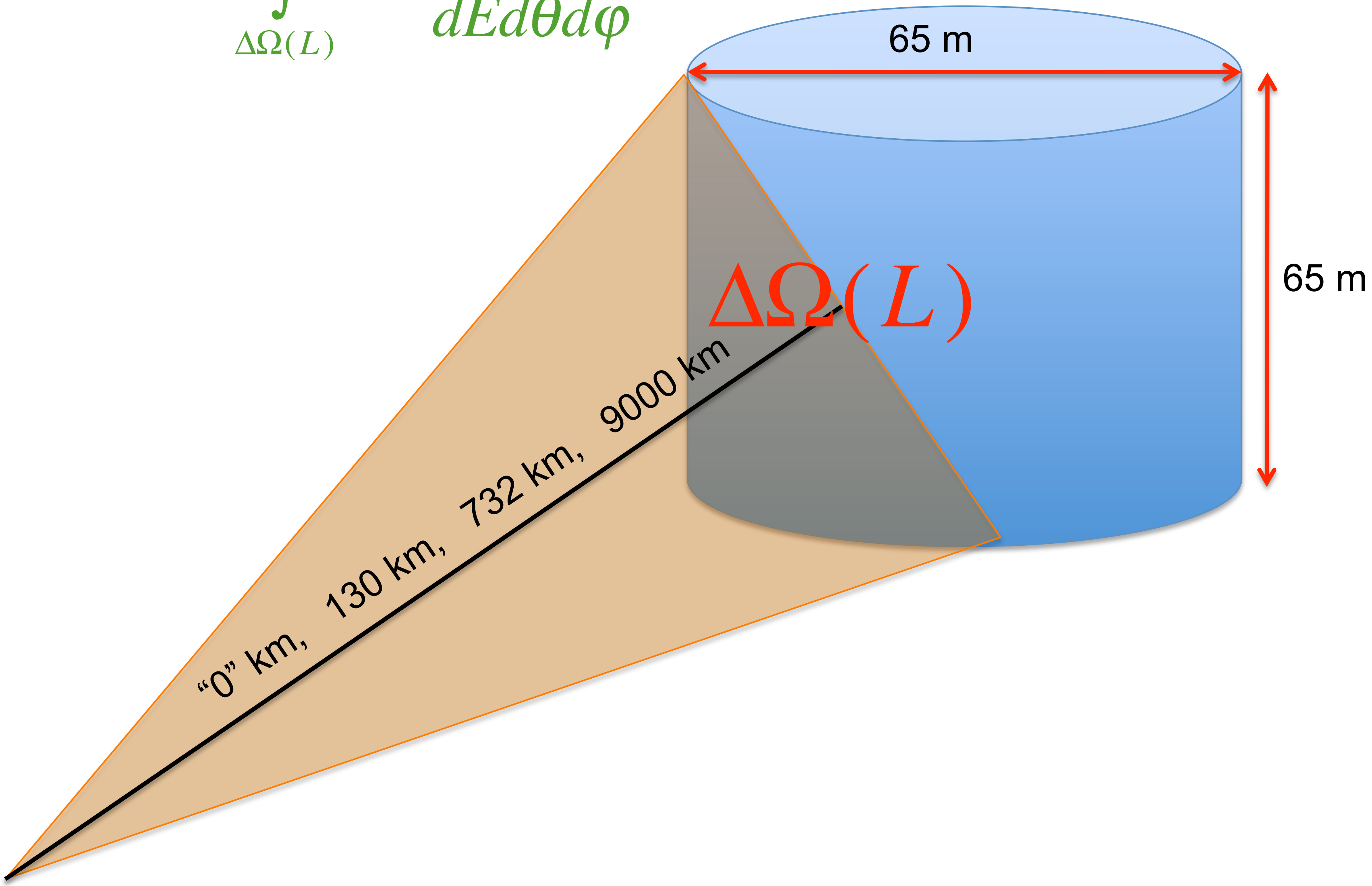
Oscillation:

$$\rho(0) \Rightarrow \rho(L) = e^{-\mathrm{iHL}} \rho \, e^{+\mathrm{iHL}}$$

$$\left(\rho(L;i,\lambda;k,\eta) = e^{i\frac{\delta m_{i,k}^2}{2E}L} \rho(i,\lambda;k,\eta) \right)$$

In vacuum

$$N^{\nu}(E,L) = \int_{\Delta\Omega(L)} d\Omega \frac{d^3\Gamma^{\nu}}{dE\,d\theta\,d\varphi}$$



We calculate neutrino detection cross section in the detector rest frame:

$$\sigma(L) = \frac{p_f}{264\pi^2 s p_i} \sum_{\lambda_e,n,\lambda_n;\lambda_\mu,i,\lambda_i,k,\lambda_k} \int d\vartheta d\phi f^{\nu}{}_{\lambda_e,n,\lambda_n;\lambda_\mu,i,\lambda_i}(E,\vartheta,\phi)\varrho(L;i,\lambda;k,\eta) f^{\nu *}{}_{\lambda_e,n,\lambda_n;\lambda_\mu,k,\lambda_k}(E,\vartheta,\phi)$$

- It is difficult to define oscillation probability,
- Generally there is no factorization for oscillation probability and detection cross section,
- Coherent or not coherent oscillation,

J. Syska,S.Zając,M.Z.,
Acta Phys.Pol.,B38:3365,2007

F.Del Aguila,J. Syska, M.Z.
J.Phys.Conf.Ser.136:042027,2008

There is important difference between elements of density matrix for Dirac and Majorana neutrinos.

$$\text{roneutr}[1,1] = \text{apps1}\,(S_{LL})_{12}\,((S_{LL})^*)_{12} + \text{appt}\,(T_{RL})_{12}\,((T_{RL})^*)_{12} + \text{appv1}\,(V_{LR})_{12}\,((V_{LR})^*)_{12} + \text{appv2}\,(V_{RR})_{12}\,((V_{RR})^*)_{12} + \text{apps2}\,(S_{RL})_{12}\,((S_{RL})^*)_{12} + \text{appst1}\,(T_{RL})_{12}\,((S_{RL})^*)_{12} + \text{appst2}\,(S_{RL})_{12}\,((T_{RL})^*)_{12};$$

$$\text{roneutr}[-1,-1] = \text{ammss1}\,(S_{RR})_{12}\,((S_{RR})^*)_{12} + \text{ammss2}\,(S_{LR})_{12}\,((S_{LR})^*)_{12} + \text{ammtt}\,(T_{LR})_{12}\,((T_{LR})^*)_{12} + \text{ammts1}\,(T_{LR})_{12}\,((S_{LR})^*)_{12} + \text{ammst2}\,(S_{LR})_{12}\,((T_{LR})^*)_{12} + \text{ammvv1}\,(V_{LL})_{12}\,((V_{LL})^*)_{12} + \text{ammvv2}\,(V_{RL})_{12}\,((V_{RL})^*)_{12};$$

$$\begin{aligned}
&\mathrm{roMajorana}[1,\,1] = \mathrm{apps1}\,(S_{LL})_{12}\,((S_{LL})^{*})_{12} + \\
&\quad \mathrm{appI2vs1}\,(V_{LL})_{21}\,((S_{LL})^{*})_{12} + \mathrm{apps2}\,(S_{RL})_{12}\,((S_{RL})^{*})_{12} + \mathrm{appI2ss1}\,(S_{RL})_{21}\,((S_{RL})^{*})_{12} + \\
&\quad \mathrm{appst1}\,(T_{RL})_{12}\,((S_{RL})^{*})_{12} + \mathrm{appI2ts2}\,(T_{RL})_{21}\,((S_{RL})^{*})_{12} + \mathrm{appI1ss1}\,(S_{RL})_{12}\,((S_{RL})^{*})_{21} + \\
&\quad \mathrm{anppss2}\,(S_{RL})_{21}\,((S_{RL})^{*})_{21} + \mathrm{appI1ts2}\,(T_{RL})_{12}\,((S_{RL})^{*})_{21} + \mathrm{anppts2}\,(T_{RL})_{21}\,((S_{RL})^{*})_{21} + \\
&\quad \mathrm{anppss1}\,(S_{RR})_{21}\,((S_{RR})^{*})_{21} + \mathrm{appI1vs1}\,(V_{RR})_{12}\,((S_{RR})^{*})_{21} + \mathrm{appst2}\,(S_{RL})_{12}\,((T_{RL})^{*})_{12} + \\
&\quad \mathrm{appI2st1}\,(S_{RL})_{21}\,((T_{RL})^{*})_{12} + \mathrm{appt}\,(T_{RL})_{12}\,((T_{RL})^{*})_{12} + \mathrm{appI2tt1}\,(T_{RL})_{21}\,((T_{RL})^{*})_{12} + \\
&\quad \mathrm{appI1st1}\,(S_{RL})_{12}\,((T_{RL})^{*})_{21} + \mathrm{anppst1}\,(S_{RL})_{21}\,((T_{RL})^{*})_{21} + \mathrm{appI1tt1}\,(T_{RL})_{12}\,((T_{RL})^{*})_{21} + \\
&\quad \mathrm{anpptt1}\,(T_{RL})_{21}\,((T_{RL})^{*})_{21} + \mathrm{appI1sv2}\,(S_{LL})_{12}\,((V_{LL})^{*})_{21} + \mathrm{anppvv1}\,(V_{LL})_{21}\,((V_{LL})^{*})_{21} + \\
&\quad \mathrm{appv1}\,(V_{LR})_{12}\,((V_{LR})^{*})_{12} + \mathrm{appI2vv1}\,(V_{LR})_{21}\,((V_{LR})^{*})_{12} + \mathrm{appI1vv1}\,(V_{LR})_{12}\,((V_{LR})^{*})_{21} + \\
&\quad \mathrm{anppvv2}\,(V_{LR})_{21}\,((V_{LR})^{*})_{21} + \mathrm{appI2sv2}\,(S_{RR})_{21}\,((V_{RR})^{*})_{12} + \mathrm{appv2}\,(V_{RR})_{12}\,((V_{RR})^{*})_{12};
\end{aligned}$$

$$\begin{aligned}
&\mathrm{roMajorana}[-1,\,-1] = \mathrm{anmmss1}\,(S_{LL})_{21}\,((S_{LL})^{*})_{21} + \\
&\quad \mathrm{ammI1vs1}\,(V_{LL})_{12}\,((S_{LL})^{*})_{21} + \mathrm{ammss2}\,(S_{LR})_{12}\,((S_{LR})^{*})_{12} + \mathrm{ammI2ss1}\,(S_{LR})_{21}\,((S_{LR})^{*})_{12} + \\
&\quad \mathrm{ammts1}\,(T_{LR})_{12}\,((S_{LR})^{*})_{12} + \mathrm{ammI2ts2}\,(T_{LR})_{21}\,((S_{LR})^{*})_{12} + \mathrm{ammI1ss1}\,(S_{LR})_{12}\,((S_{LR})^{*})_{21} + \\
&\quad \mathrm{anmmss2}\,(S_{LR})_{21}\,((S_{LR})^{*})_{21} + \mathrm{ammI1ts2}\,(T_{LR})_{12}\,((S_{LR})^{*})_{21} + \mathrm{anmmts1}\,(T_{LR})_{21}\,((S_{LR})^{*})_{21} + \\
&\quad \mathrm{ammss1}\,(S_{RR})_{12}\,((S_{RR})^{*})_{12} + \mathrm{ammI2vs1}\,(V_{RR})_{21}\,((S_{RR})^{*})_{12} + \mathrm{ammst2}\,(S_{LR})_{12}\,((T_{LR})^{*})_{12} + \\
&\quad \mathrm{ammI2st1}\,(S_{LR})_{21}\,((T_{LR})^{*})_{12} + \mathrm{ammtt}\,(T_{LR})_{12}\,((T_{LR})^{*})_{12} + \mathrm{ammI2tt1}\,(T_{LR})_{21}\,((T_{LR})^{*})_{12} + \\
&\quad \mathrm{ammI1st1}\,(S_{LR})_{12}\,((T_{LR})^{*})_{21} + \mathrm{anmmst2}\,(S_{LR})_{21}\,((T_{LR})^{*})_{21} + \mathrm{ammI1tt1}\,(T_{LR})_{12}\,((T_{LR})^{*})_{21} + \\
&\quad \mathrm{anmmtt1}\,(T_{LR})_{21}\,((T_{LR})^{*})_{21} + \mathrm{ammI2sv2}\,(S_{LL})_{21}\,((V_{LL})^{*})_{12} + \mathrm{ammvv1}\,(V_{LL})_{12}\,((V_{LL})^{*})_{12} + \\
&\quad \mathrm{ammvv2}\,(V_{RL})_{12}\,((V_{RL})^{*})_{12} + \mathrm{ammI2vv1}\,(V_{RL})_{21}\,((V_{RL})^{*})_{12} + \mathrm{ammI1vv1}\,(V_{RL})_{12}\,((V_{RL})^{*})_{21} + \\
&\quad \mathrm{anmmvv1}\,(V_{RL})_{21}\,((V_{RL})^{*})_{21} + \mathrm{ammI1sv2}\,(S_{RR})_{12}\,((V_{RR})^{*})_{21} + \mathrm{anmmvv2}\,(V_{RR})_{21}\,((V_{RR})^{*})_{21};
\end{aligned}$$

For Majorana neutinos there are linear terms in NP parameters

For Majorana neutrino dominant term – pure states

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} (a U^*_{\alpha,i} + b V^*_{\alpha,i}) |\nu_i\rangle$$

MNSP matrix

Mixing for S_{LL}

$$\rho^M_\alpha \approx |\nu_\alpha\rangle\langle\nu_\alpha| + \ldots.$$

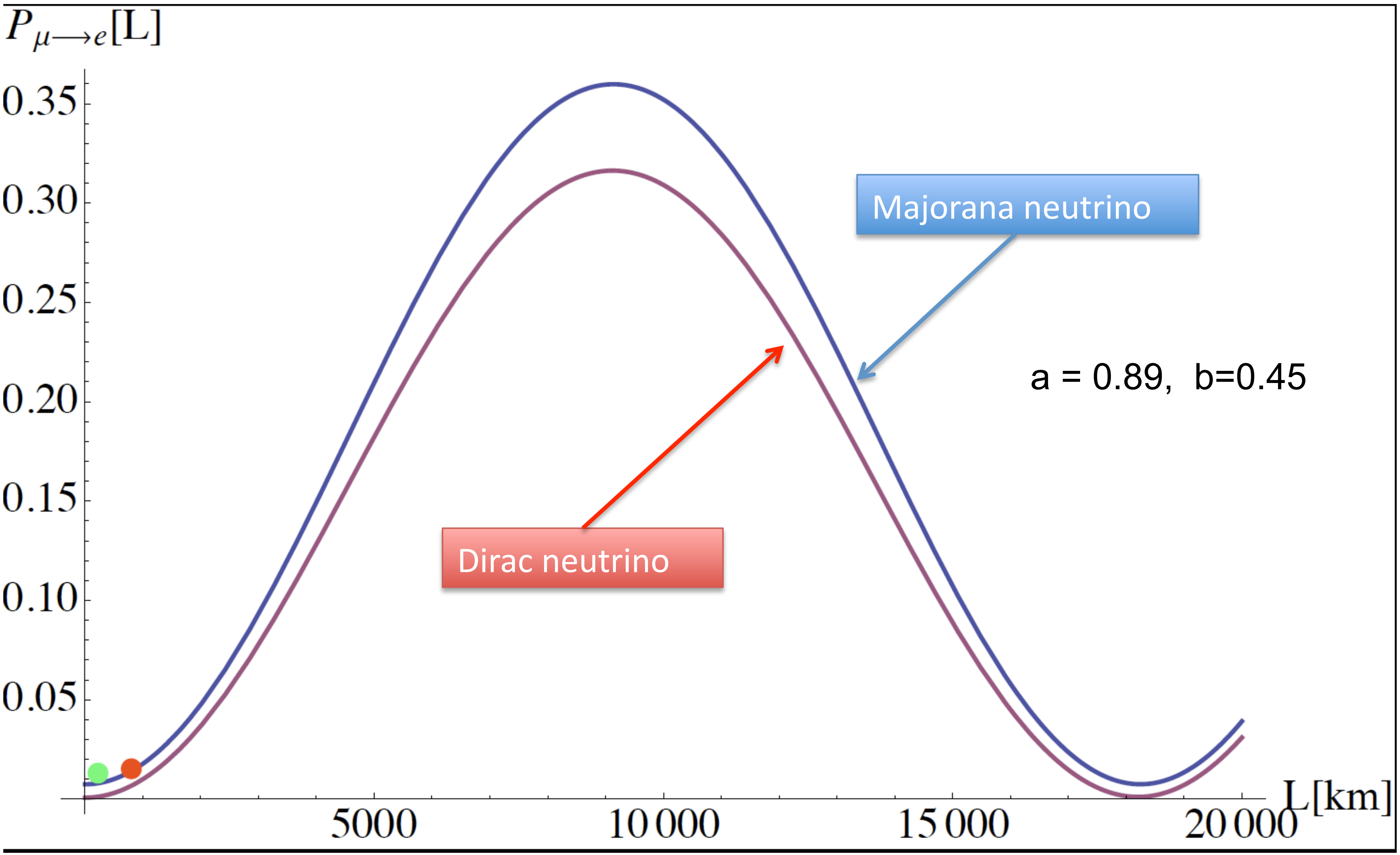
has interference terms U x V

Coherent oscillations

$$(\rho^D_\alpha)_{i,k} = |a|^2 U^*_{\alpha,i} U_{\alpha,k} + |b|^2 V^*_{\alpha,i} V_{\alpha,k} + \ldots..$$

No interfernce terms

Incoherent oscillations



$P_{\mu \longrightarrow e}[L]$
0.35
0.30
0.25
0.20
0.15
0.10
0.05
5000
10 000
15 000
20 000
L[km]
Majorana neutrino
Dirac neutrino
a = 0.89, b=0.45

Parameters	Polarization	$\alpha\sigma^{\nu}$ Dirac particle $[10^{-45}\,\mathrm{m}^2]$	$\beta\sigma^{\bar{\nu}}$ Dirac antiparticle $[10^{-45}\,\mathrm{m}^2]$	σ^D Sum $[10^{-45}\,\mathrm{m}^2]$	σ^M Majorana $[10^{-45}\,\mathrm{m}^2]$	$\frac{\sigma^D-\sigma^M}{\sigma^D}$ Percentage difference
1)SM	parallel	1,7746	2,2586	4,0332	4,0332	0,00%
1)SM	orthogonal	2,7064	1,8631	4,5694	4,5694	0,00%
1)SM	antiparallel	6,4092	0,2913	6,7005	6,7005	0,00%
2)SM+0.5SLL	parallel	1,6785	2,1323	3,8107	3,5098	7,90%
2)SM+0.5SLL	orthogonal	2,5540	1,7634	4,3175	3,9765	7,90%
2)SM+0.5SLL	antiparallel	6,0332	0,2978	6,3310	5,8310	7,90%
3)SM+0.03VRR	parallel	1,7742	2,2581	4,0323	4,0336	-0,03%
3)SM+0.03VRR	orthogonal	2,7040	1,8614	4,5653	4,5674	-0,05%
3)SM+0.03VRR	antiparallel	6,3847	0,2902	6,6749	6,6808	-0,09%
4)NP	parallel	1,7703	2,2532	4,0235	3,9398	2,08%
4)NP	orthogonal	2,6994	1,8584	4,5577	4,4630	2,08%
4)NP	antiparallel	6,3870	0,2910	6,6779	6,5393	2,08%

Dirac and Majorana neutrino oscillation in vacuum for L=130 km and E= 20 GeV

(NP= {V_{LL}=1, S_{LL}=0.1, and all others = 0.01})

Parameters	Polarization	Dirac particle	Dirac antiparticle	Sum	Majorana	Percentage difference
		$\alpha\sigma^{\nu}$	$\beta\sigma^{\bar{\nu}}$	σ^{D}	σ^{M}	$\frac{\sigma^{D}-\sigma^{M}}{\sigma^{D}}$
1)SM	parallel	1,7175	2,2642	3,9817	3,9817	0,00%
1)SM	orthogonal	2,6802	1,8524	4,5326	4,5326	0,00%
1)SM	antiparallel	6,9855	0,0108	6,9963	6,9963	0,00%
2)SM+0.5SLL	parallel	1,6248	2,1374	3,7622	3,4643	7,92%
2)SM+0.5SLL	orthogonal	2,5293	1,7534	4,2828	3,9435	7,92%
2)SM+0.5SLL	antiparallel	6,5747	0,0361	6,6108	6,0865	7,93%
3)SM+0.03VRR	parallel	1,7172	2,2638	3,9809	3,9822	-0,03%
3)SM+0.03VRR	orthogonal	2,6778	1,8507	4,5285	4,5306	-0,05%
3)SM+0.03VRR	antiparallel	6,9553	0,0107	6,9660	6,9726	-0,09%
4)NP	parallel	1,7133	2,2588	3,9721	3,8892	2,09%
4)NP	orthogonal	2,6732	1,8477	4,5209	4,4265	2,09%
4)NP	antiparallel	6,9602	0,0115	6,9717	6,8260	2,09%

Dirac and Majorana neutrino oscillation in vacuum for L=732 km and E= 20 GeV

(NP= {V_{LL}=1, S_{LL}=0.1, and all others = 0.01})

Parameters	Polarisation	Dirac particle $\alpha\sigma^{\nu}$	Dirac antyparticle $\beta\sigma^{\bar{\nu}}$	Sum σ^D	Majorana σ^M	Procentage difference $\frac{\sigma^D - \sigma^M}{\sigma^D}$
1)SM	parallel	0,2126	1,5561	1,7687	1,7687	0,00%
1)SM	orthogonal	0,3322	1,2725	1,6047	1,6047	0,00%
1)SM	antiparallel	0,8687	0,0000	0,8688	0,8688	0,00%
2)SM+0.5SLL	parallel	0,2082	1,4707	1,6789	1,4829	11,68%
2)SM+0.5SLL	orthogonal	0,3193	1,2073	1,5265	1,3189	13,60%
2)SM+0.5SLL	antiparallel	0,8176	0,0251	0,8427	0,5833	30,78%
3)SM+0.03VRR	parallel	0,2126	1,5558	1,7684	1,7695	-0,06%
3)SM+0.03VRR	orthogonal	0,3319	1,2714	1,6033	1,6058	-0,16%
3)SM+0.03VRR	antiparallel	0,8649	0,0001	0,8650	0,8743	-1,07%
4)NP	parallel	0,2121	1,5524	1,7645	1,7034	3,47%
4)NP	orthogonal	0,3314	1,2694	1,6007	1,5329	4,24%
4)NP	antiparallel	0,8659	0,0006	0,8665	0,7689	11,26%

Dirac and Majorana neutrino oscillation in vacuum for L=9000 km and E= 20 GeV

(NP= {V_{LL}=1, S_{LL}=0.1, and all others = 0.01})

For (NP= {V_{LL}=1, S_{LL}=0.1, and all others = 0.01})

L=0	ortogonal	2,7070	1,8553	4,5623	4,4680	2,07%
L=130 km	ortogonal	2,6994	1,8584	4,5577	4,4630	2,08%
L=732 km	ortogonal	2,6732	1,8477	4,5209	4,4265	2,09%
L= 9000 km	ortogonal	0,3314	1,2694	1,6007	1,5329	4,24%

For (NP= {V_{LL}=1, S_{LL}=0.5, and all others = 0.0})

L=0	antyparallel	6,6807	0,0261	6,7068	6,1772	7,90%
L= 130 km	antyparallel	6,0332	0,2978	6,3310	5,8310	7,90%
L = 732 km	antyparallel	6,5747	0,0361	6,6108	6,0865	7,93%
L = 9000 km	antyparallel	0,8176	0,0251	0,8427	0,5833	30,78%

4. Conclusions

- **Neutrino production states is not pure QM states – density matrix,**
- **Final detection rates do not factorize,**
- **It is possible to distinguish Dirac from Majorana neutrinos,**
- **Coherent and incoherent oscillation,**
- **Density matrix is useful even for the nSM neutrino oscillation.**