Open Strings
In
Warped Extra Dimensions

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Outline

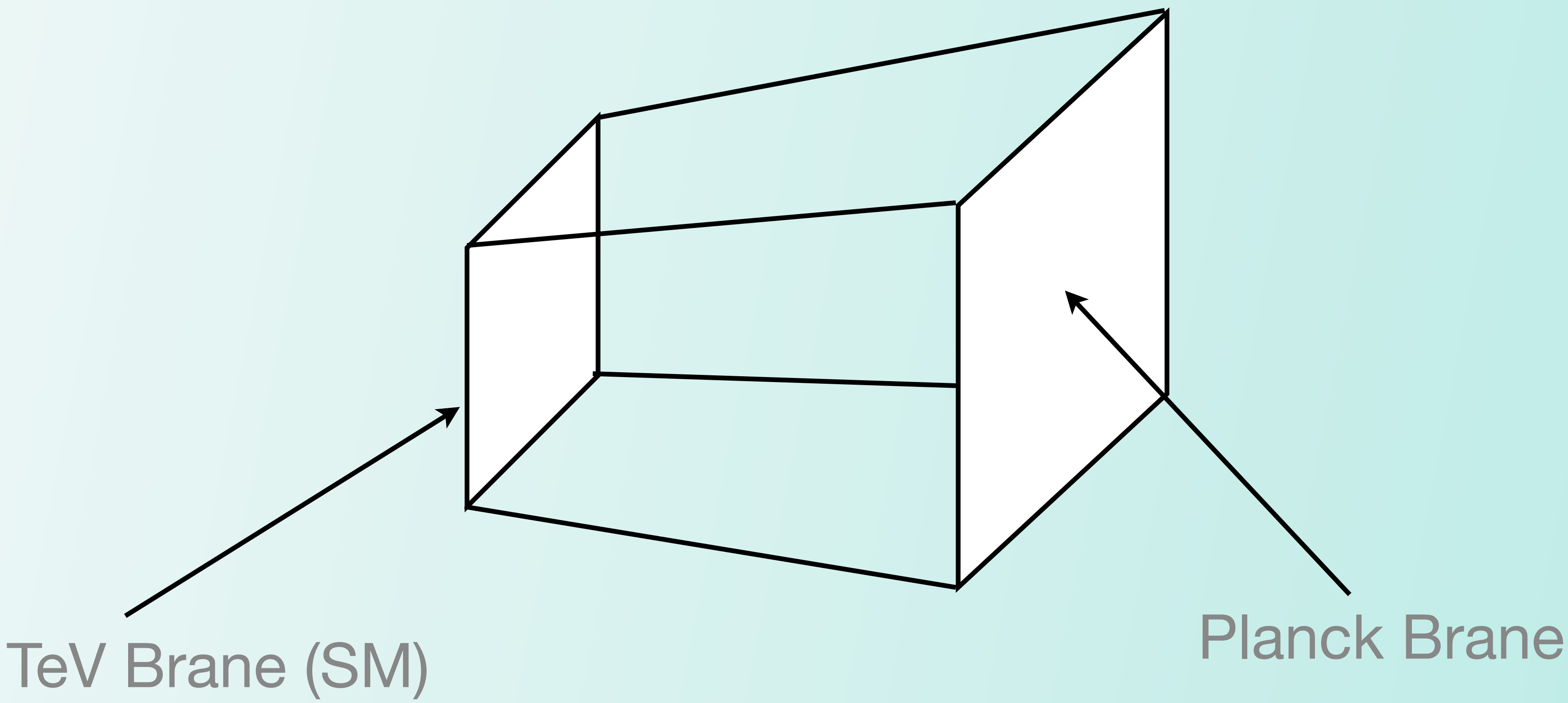
- Warped Extra Dimensions
- Open String Wavefunctions
- Kähler Metrics
- Open Strings on Intersecting D-branes (work in progress).

arXiv:0812.2247 [hep-th]

Randall-Sundrum

$$ds^2_{\mathrm{AdS}_5} = Z^{-1/2}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + Z^{1/2}\mathrm{d}r^2$$

$$Z = \frac{r^4}{R^4}$$



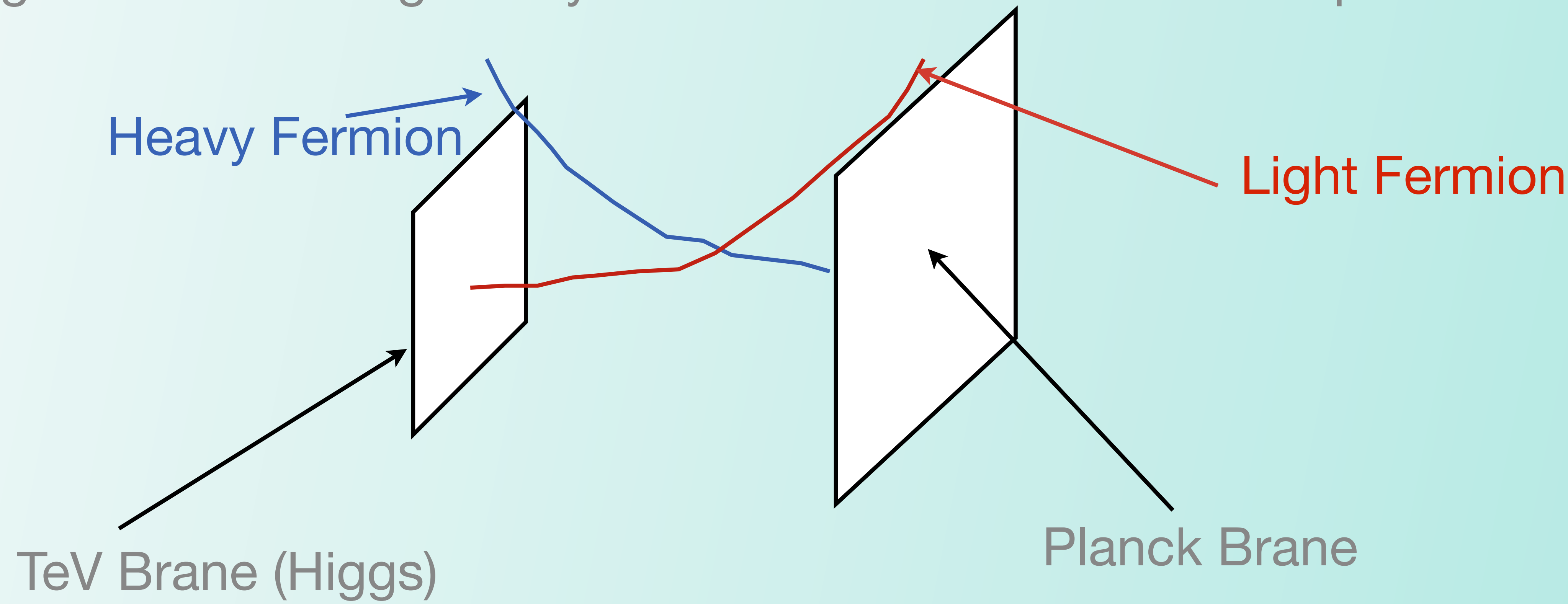
The Randall-Sundrum scenario addresses the hierarchy problem with higher dimensional geometry. The electroweak hierarchy results from redshift.

Randall-Sundrum

The original RS scenario had the standard model confined to the 4D TeV brane.

However, a very rich phenomenology results if some or all of the standard model can propagate in the 5D bulk (see e.g. Csaki hep-ph/0510275).

It may be interesting to know how this phenomenology is influenced by an embedding of RS into string theory. This talk can be seen as a step in this direction.



Warped Extra Dimensions

IIB string theory admits warped solutions

$$\mathrm{d}s_{10}^2 = Z^{-1/2}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + Z^{1/2}\tilde{g}_{mn}\mathrm{d}y^m \mathrm{d}y^n$$

Such a solution is produced by (e.g.) a stack of parallel D3-branes.

The 10D string spectrum also includes rank p anti-symmetric tensor fields that must be non-vanishing for non-trivial warping.

In particular, there is always non-zero $C_{(4)}$ with field-strength

$$F_{(5)} = \mathrm{d}C_{(4)}$$

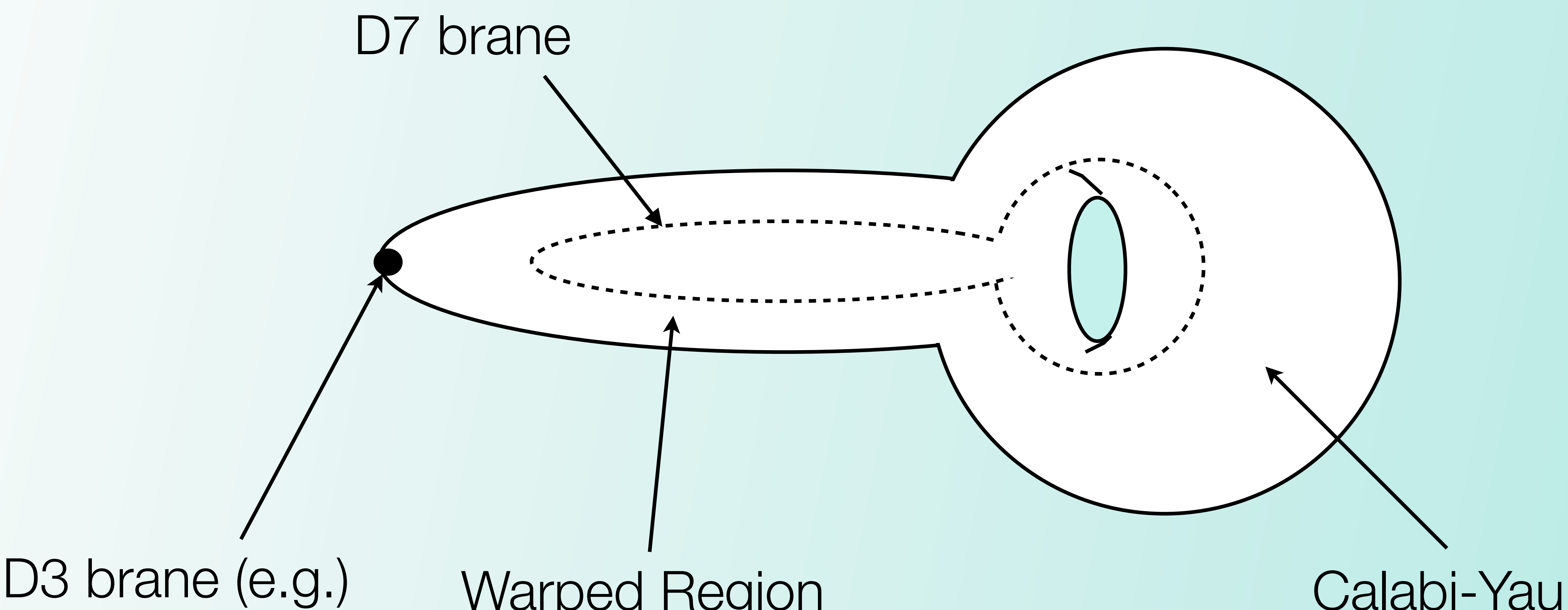
These fluxes qualitatively change the wavefunction profiles:

$$Z^{1/8} \to Z^{-1/8}, Z^{3/8}$$

Open Strings

The standard model fields come from open strings attached to D-branes living in the extra dimensions.

In this talk, I will focus on open strings coming from D7 branes that extend along the 4 large directions and 4 internal directions.



Open String Wavefunctions

We want to understand the effective 4D action for the light degrees of freedom coming from open strings on the D7 brane.

Can proceed by dimensional reduction:

8D field

Internal wavefunction

$$\Phi\left(x,y\right)=\phi\left(x\right)\psi\left(y\right)$$

4D zero mode

$$\partial_\mu\partial^\mu\phi=0$$

D-Brane Action (Bosonic)

The light bosonic degrees of freedom on a D-brane are the gauge field for the *U*(1) theory living on the brane, A_α, and the transverse fluctuations of the worldvolume, Φ^i, with effective action given by $S^{\mathrm{bos}}_{\mathrm{D}p} = S^{\mathrm{DBI}}_{\mathrm{D}p} + S^{\mathrm{CS}}_{\mathrm{D}p}$.

$$
\begin{aligned}
S^{\mathrm{DBI}}_{\mathrm{D}p} &= -\tau_{\mathrm{D}p} \int_{\mathcal{W}} \mathrm{d}^{p+1}\xi \, e^{-\Phi} \sqrt{-\det\left(\mathrm{P}\left[g_{\alpha\beta}\right] + \mathcal{F}\right)} \\
S^{\mathrm{CS}}_{\mathrm{D}p} &= \tau_{\mathrm{D}p} \int_{\mathcal{W}} \mathrm{P}\left[\mathcal{C}\right] e^{\mathcal{F}}
\end{aligned}
$$

Where $\mathcal{F} = \mathrm{P}\left[B\right] + kF$, $k = 2\pi\alpha'$ and

$$
\mathcal{C} = \sum_p C_{(p)}
$$

4D Modes: Gauge Boson A_μ, Wilson line A_a, Modulus ζ

(matter)

D-Brane Action (Fermionic)

The fermionic partners are contained within a doublet of 10D Majorana-Weyl spinors

$$\Theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

The quadratic action is (Martucci *et al*. hep-th/0504041)

$$S^{\rm fer}_{{\rm D}p} = \tau_{{\rm D}p} \int {\rm d}^{p+1}\xi \, e^{-\Phi} \sqrt{-\det\left({\rm P}\left[G\right] + \mathcal{F}\right)} \bar{\Theta} P^{{\rm D}p}_{-}\left(\mathcal{F}\right) \left(\left(\mathcal{M}^{-1}\right)^{\alpha\beta} \Gamma_\beta \mathcal{D}_\alpha - \frac{1}{2}\mathcal{O} \right) \Theta$$

$$\mathcal{M}_{\alpha\beta} = g_{\alpha\beta} + \mathcal{F}_{\alpha\beta}\Gamma_{10} \otimes \sigma_3$$

Where $\mathcal{D}_\alpha$ and $\mathcal{O}$ come from the gravitino and dilatino SUSY variations and $P^{{\rm D}p}_{-}\left(\mathcal{F}\right)$ is a particular projection operator.

4D Modes: gaugino, Wilsonini, modulino

(matter)

Fermionic Equations of Motion

Take a D7 in warped flat space (can be generalized to CY)

$$\mathrm{d}s_{10}^2 = Z^{-1/2}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + Z^{1/2}\tilde{g}_{mn}\mathrm{d}y^m \mathrm{d}y^n$$

$$F_{(5)} = (1 + *_{10})\, F_{(5)}^{\mathrm{int}} \qquad\qquad F_{(5)}^{\mathrm{int}} = \tilde{*}_6 \mathrm{d}Z$$

The operators in the fermionic action are

$$\mathcal{O} = 0$$

$$\mathcal{D}_\alpha = \nabla_\alpha + \frac{1}{16} e^{\Phi} \not{F}_{(5)} i\sigma_2 \Gamma_\alpha$$

With $\mathcal{F} = 0$

$$P_{\pm}^{\mathrm{D7}} = \frac{1}{2}\left(1 \mp \Gamma_{(8)} \otimes \sigma_2\right)$$

Fermionic Equations of Motion

The resulting* equation of motion for the 4D zero modes is

$$\left(\partial_a - \frac{1}{8}\partial_a \ln\left(Z\right)\left(1 + 2\Gamma_{\mathrm{Extra}}\right)\right)\theta = 0$$

with Γ_{Extra} the chirality operator on the 4-cycle wrapped by the D7.

Solutions:

Gaugino, Modulino ($\Gamma_{\mathrm{extra}} = +1$): $\theta = Z^{3/8}\eta$

Wilsonini ($\Gamma_{\mathrm{extra}} = -1$): $\theta = Z^{-1/8}\eta$

Kähler Metrics

Integrating over internal dimensions gives 4D kinetic terms.
Take (e.g.)

$$\Phi\left(x,y\right)=\phi\left(x\right)\psi\left(y\right)$$

Then the 8D kinetic term gives

$$\int_{\mathcal{W}} \mathrm{d}^8\xi\, \partial_a\Phi^*\partial^a\Phi$$

$$\int_{\mathbb{R}^{1,3}} \mathrm{d}^4x\, \partial_\mu\phi^*\partial^\mu\phi \boxed{\int_{\mathcal{S}_4} \mathrm{d}^4y\, \psi^*\psi}$$

~Kähler metric

Kähler Metrics

For a D7 in a warped Calabi-Yau, the wavefunctions give us:

Gauge kinetic function (also Baumann *et al* hep-th/0607050)

$$f_{\mathrm{D7}} \sim \int_{\mathcal{S}_4} \frac{\mathrm{d}\hat{\mathrm{vol}}_{\mathcal{S}_4}}{\sqrt{\hat{g}_{\mathcal{S}_4}}} \left(Z \sqrt{\hat{g}_{\mathcal{S}_4}} + i C_4^{\mathrm{int}} \right)$$

Deformation moduli (m_A harmonic (2,0) forms)

warping effects

$$\mathcal{K}_{A\bar{B}} \sim \frac{1}{\mathcal{V}_{\mathrm{w}}} \int_{\mathcal{S}_4} Z\, m_A \wedge m_{\bar{B}}$$

Wilson lines (W^I harmonic (1,0) forms) (consistent with Frey *et al* 0810.5768)

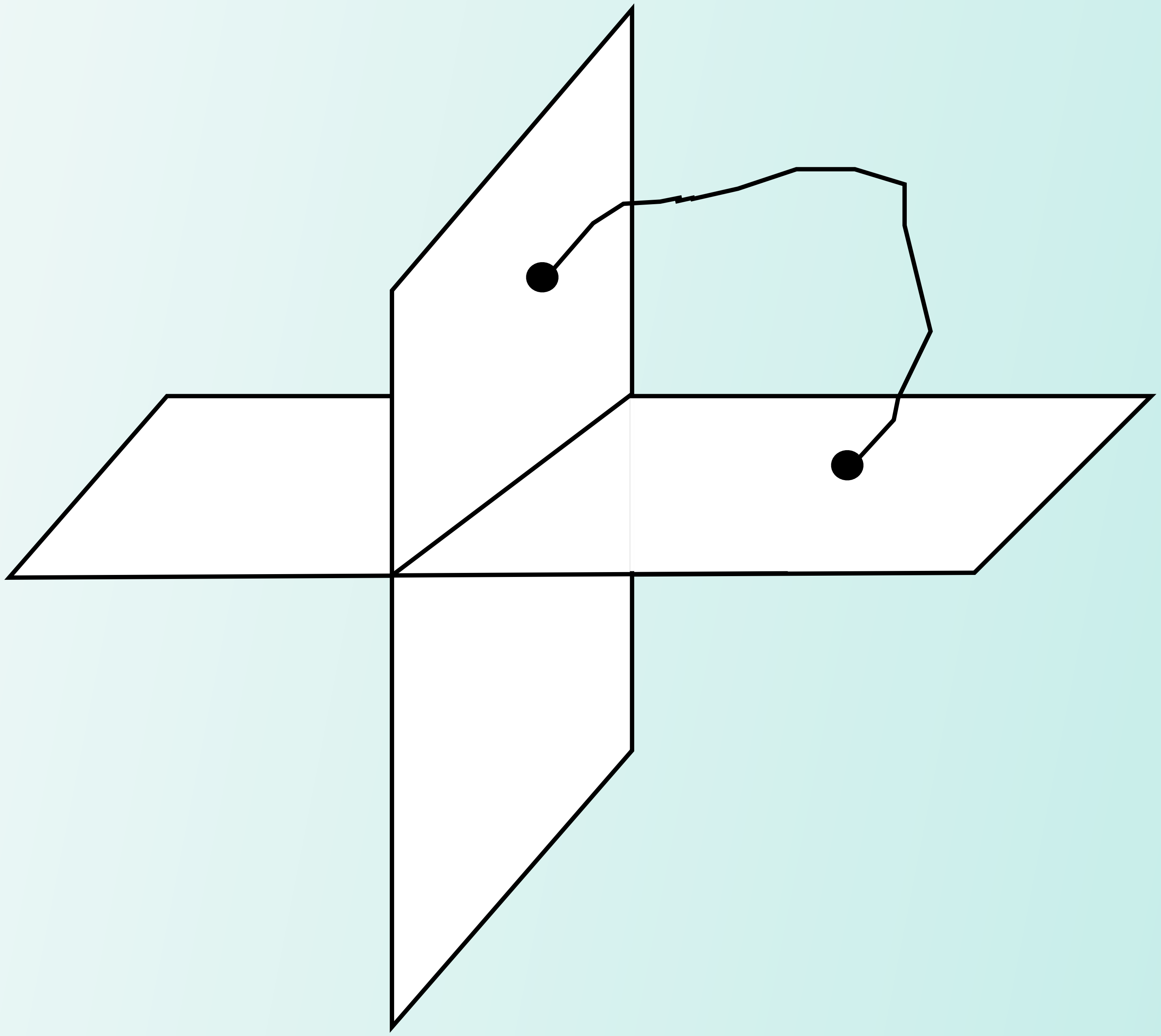
$$\mathcal{K}_{I\bar{J}} \sim \frac{1}{\mathcal{V}_{\mathrm{w}}} \int_{\mathcal{S}_4} \mathrm{P}\left[J^{\mathrm{CY}}\right] \wedge W^I \wedge W^{\bar{J}}$$

Warped volume:

$$\mathcal{V}_{\mathrm{w}} = \int Z\, J^{\mathrm{CY}} \wedge J^{\mathrm{CY}} \wedge J^{\mathrm{CY}}$$

Intersecting D7 Branes

Many interesting constructions involve the intersection of two or more D7s.



Intersecting D7 Branes

It would be useful to know the effective action for the light modes localized on the intersection.

Some progress has been made in this direction

- Worldsheet methods (Lust *et al*. hep-th/0404134)
- Constraints from SUSY (Vafa *et al*. 0802.3391)

But these do not include a non-trivial R-R sector.

Intersecting D7 Branes

Alternative method:

Use the non-Abelian generalization of DBI+CS (Myers hep-th/9910053)

$$S_{\mathrm{D}p}^{\mathrm{DBI}} = -\tau_{\mathrm{D7}} \int_{\mathcal{W}} \mathrm{d}^{p+1}\xi \, \mathrm{Str} \left\{ \sqrt{-\det\left(\mathrm{P}\left[E_{\alpha\beta} + E_{\alpha i}\left(Q^{-1}-\delta\right)^{ij} E_{j\beta} \right] + kF_{\alpha\beta} \right) \det\left(Q^i_j\right)} \right\}$$

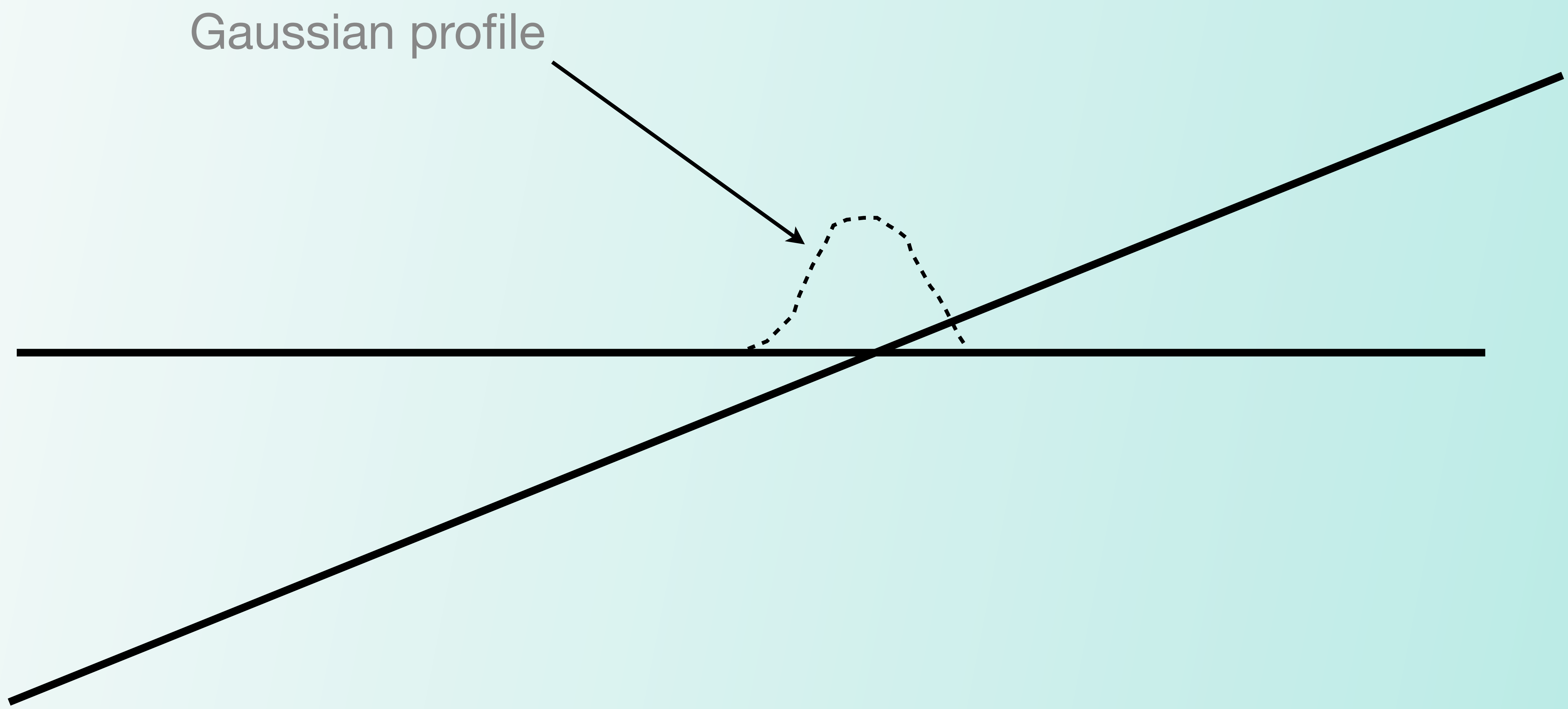
$$S_{\mathrm{D}p}^{\mathrm{CS}} = +\tau_{\mathrm{D7}} \int_{\mathcal{W}} \mathrm{Str}\left\{ \mathrm{P}\left[e^{ik \mathrm{i}_\Phi \mathrm{i}_\Phi} \mathcal{C} e^{B} \right] e^{kF} \right\}$$

$$Q^i_j = \delta^i_j + ik\left[\Phi^i, \Phi^k\right] E_{kj}$$

Giving a vev to Φ^i corresponds to tilting one of the branes.

Dimensional reduction in this open string “background” gives an action for the intersection.

Intersecting D7 Branes



Summary

- We performed a dimensional reduction of the fermionic (and bosonic) modes living on a D7 in warped extra dimensions.
- Presence of bulk flux qualitatively changes the behavior of the profiles.
- This allows low energy physics (e.g. Kähler potentials, Yukawa couplings) to be inferred.
- Can also be done for Calabi-Yaus and in the presence of magnetic flux.
- Can extend this process to modes on the intersection of branes (work in progress).