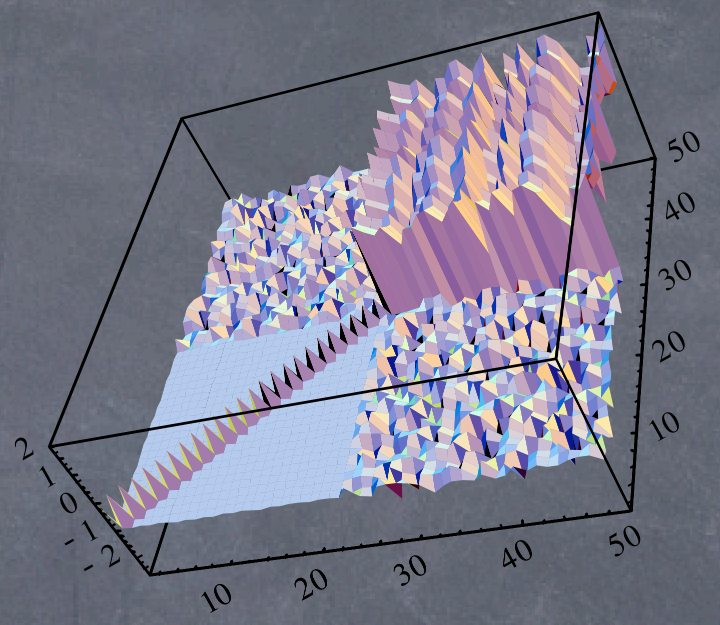
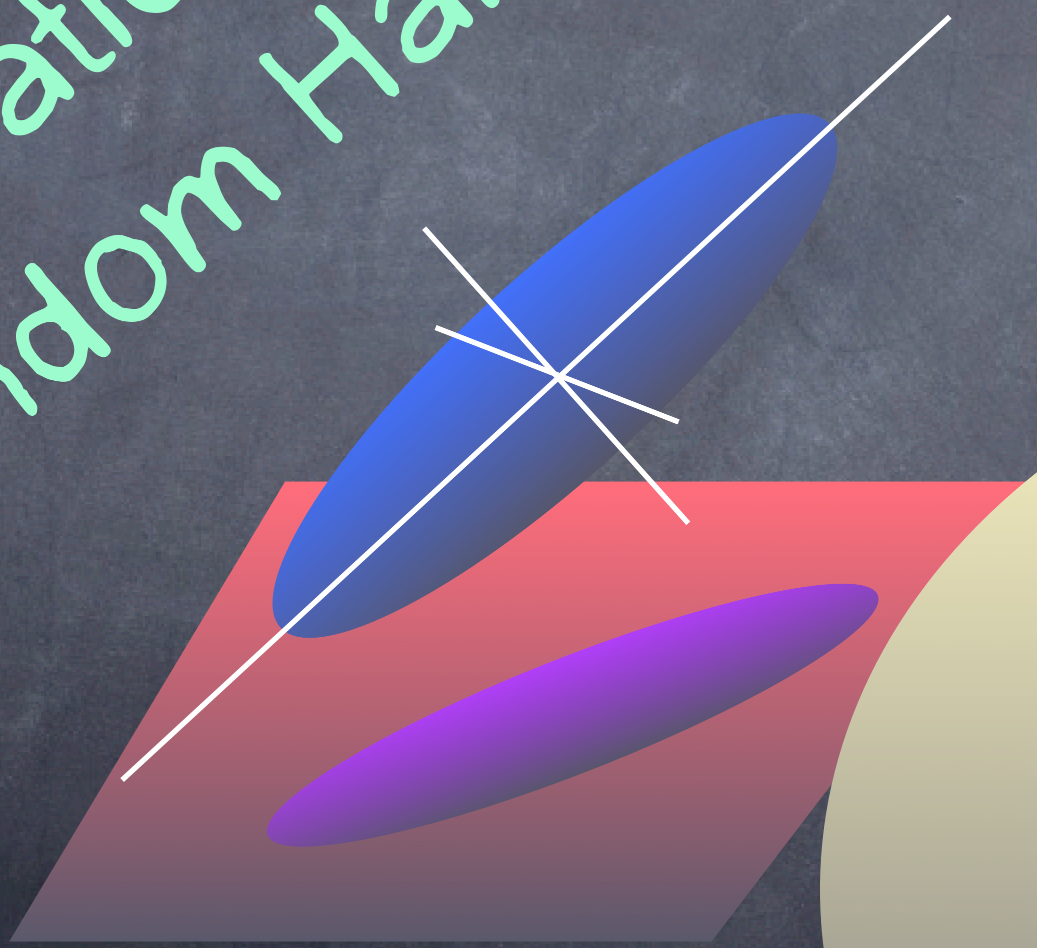
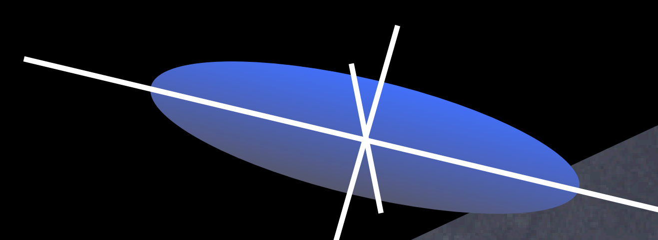
Grand Anarchy: unification patterns of random Hamiltonians





The quantum mechanical coupling
of a low energy system to a high energy one
invariably leads to perturbative instability,
naturally interpreted as
dynamical
instability.

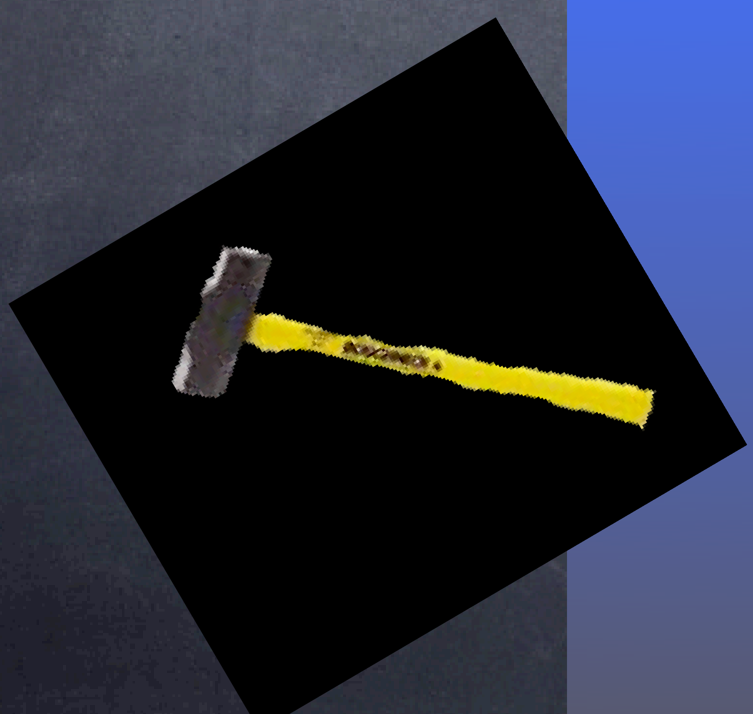


Let dim(H)= N x N

define random $H = U H_0 U^\dagger$,

with U = random $\in U(N)$, by Haar measure

case 1: H_0 has 1 "gutscale" eigenvalue,
$N-1$ "small" eigenvalues $O(1)$



What are generic features of random submatrix H_{PP} ?

random projectors P+Q=1,

$P^2=P$, $Q^2=Q$, PQ=QP=0.

dimensions dim P+ dim Q=N

Random subsystem "P" (dim P)

has Hamiltonian H_{PP} = PHP

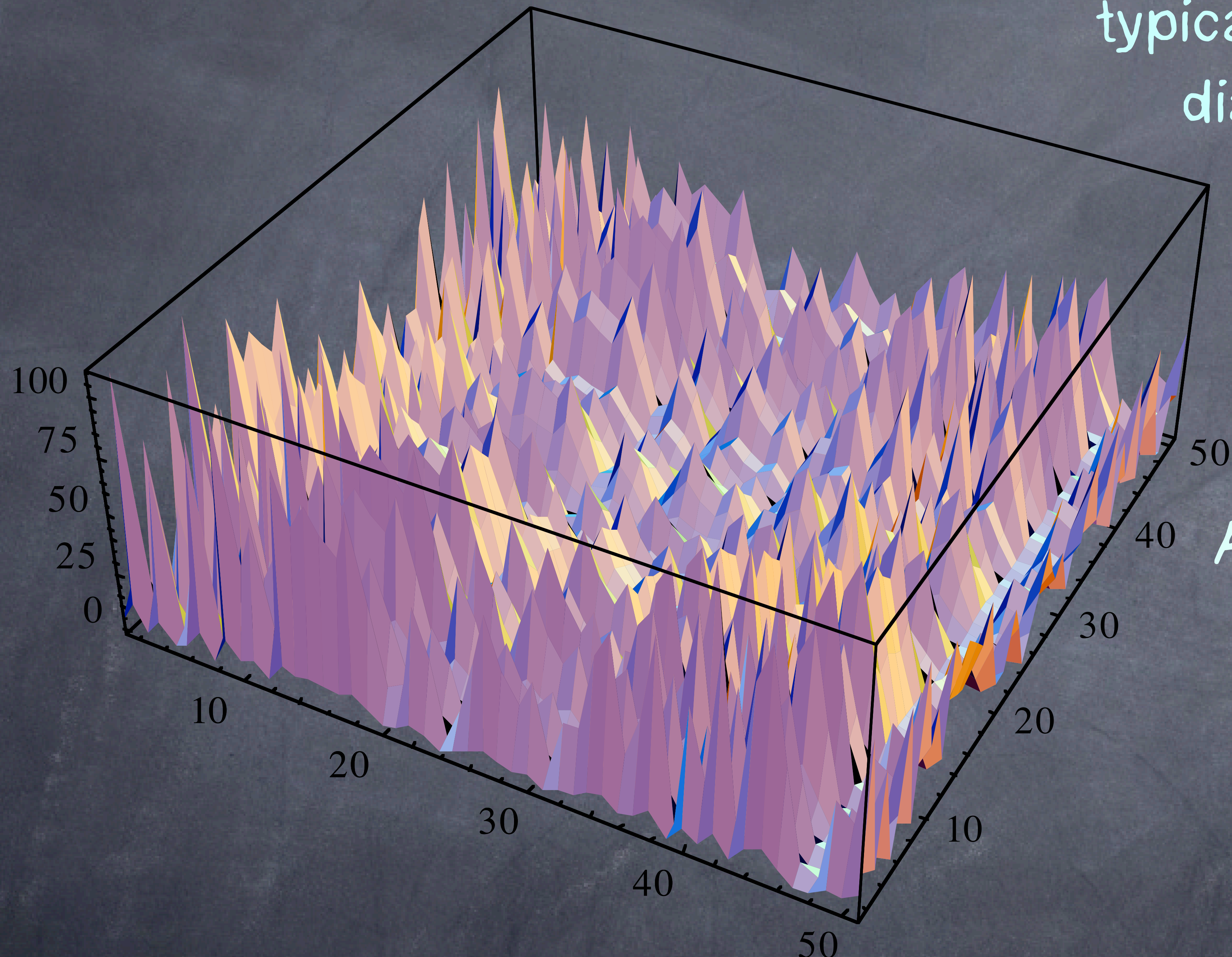
In the diagonal frame of H_{PP} we have

$$H_{tot} \quad \begin{pmatrix} E_1^{(0)} & 0 & 0... & \\ 0 & ... & 0... & H_{PQ} \\ 0 & 0 & E_P^{(0)}... & \\ & H_{QP} & & H_{QQ} \end{pmatrix}.$$

random gut–Hamiltonian in random frame

typical $H_{ij} \sim 0 \pm O(E_{gut}/N)$, $i \neq j$

diagonals $H_{ii} \sim O(E_{gut}/N)$



Abs[H_{ij}] shown;
$E_{gut}=10^3$

Three dimensional plot of random matrix elements $|H_{ij}|$. One eigenvalue $E_{gut}=10^3$; remaining eigenvalues random (0,1)

Observation 1, case 1:

If H has 1 gutscale eigenvalue, ALMOST EVERY random submatrix H_{PP} has $P-1$ small eigenvalues, plus 1 large one

exact evals= 0.586, 0.231, 0.219, 0.161, 0.119, 0.109, 999

2 x 2 evals = 0.22073, 88.1463

$$\begin{pmatrix} 80.7 & -24.48 & 165.6 & 67.91 & 115.5 & 165.7 & -21.58 \\ -24.48 & 7.667 & -50.22 & -20.6 & -35.01 & -50.13 & 6.495 \\ 165.6 & -50.22 & 340.8 & 139.8 & 237.6 & 341. & -44.55 \\ 67.91 & -20.6 & 139.8 & 57.52 & 97.5 & 140. & -18.33 \\ 115.5 & -35.01 & 237.6 & 97.5 & 165.9 & 237.8 & -31.08 \\ 165.7 & -50.13 & 341. & 140. & 237.8 & 341.7 & -44.77 \\ -21.58 & 6.495 & -44.55 & -18.33 & -31.08 & -44.77 & 6.148 \end{pmatrix}$$

4 x 4 evals = 0.300463, 0.183553, 0.118482, 486.088.

hierarchy !

Choose Any of an Infinity of Subspaces

If H has 1 gutscale eigenvalue, EVERY random submatrix H_{PP} has $P-1$ small eigenvalues, plus 1 large one

3 x 3 evals = 0.155178, 0.286777, 428.725

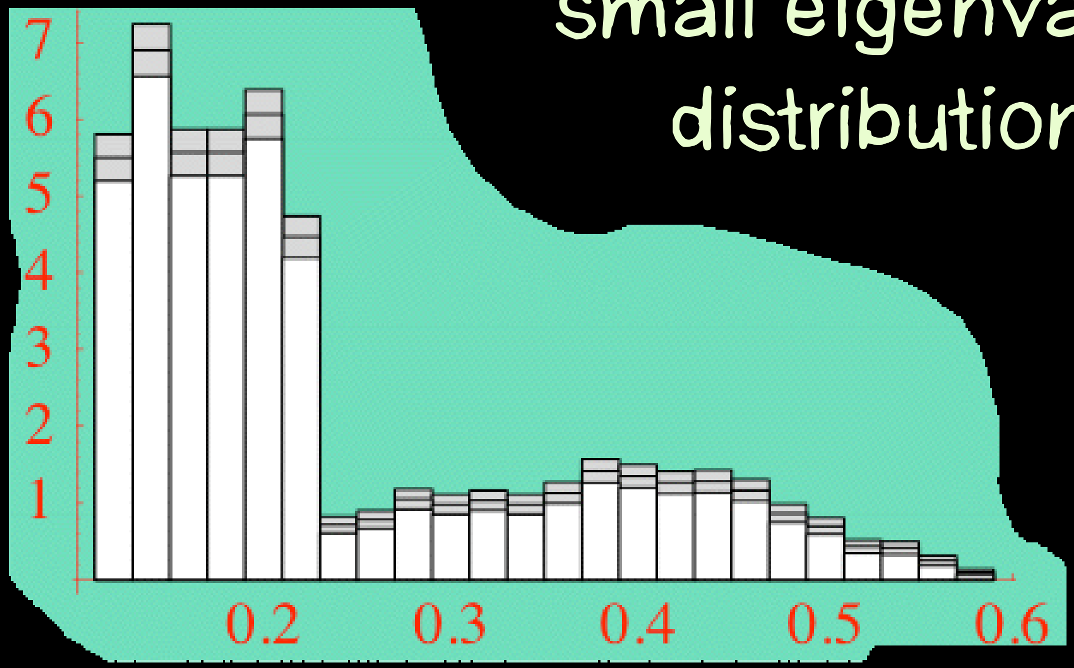
same hierarchy !

$$\begin{pmatrix} 80.7 & -24.48 & 165.6 & 67.91 & 115.5 & 165.7 & -21.58 \\ -24.48 & 7.667 & -50.22 & -20.6 & -35.01 & -50.13 & 6.495 \\ 165.6 & -50.22 & 340.8 & 139.8 & 237.6 & 341. & -44.55 \\ 67.91 & -20.6 & 139.8 & 57.52 & 97.5 & 140. & -18.33 \\ 115.5 & -35.01 & 237.6 & 97.5 & 165.9 & 237.8 & -31.08 \\ 165.7 & -50.13 & 341. & 140. & 237.8 & 341.7 & -44.77 \\ \ldots 58 & 6.495 & -44.55 & -18.33 & -31.08 & -44.77 & 6.148 \end{pmatrix}$$

hierarchy !

new 4 x 4 evals = 0.366832, 0.194968, 0.142922, 570.548

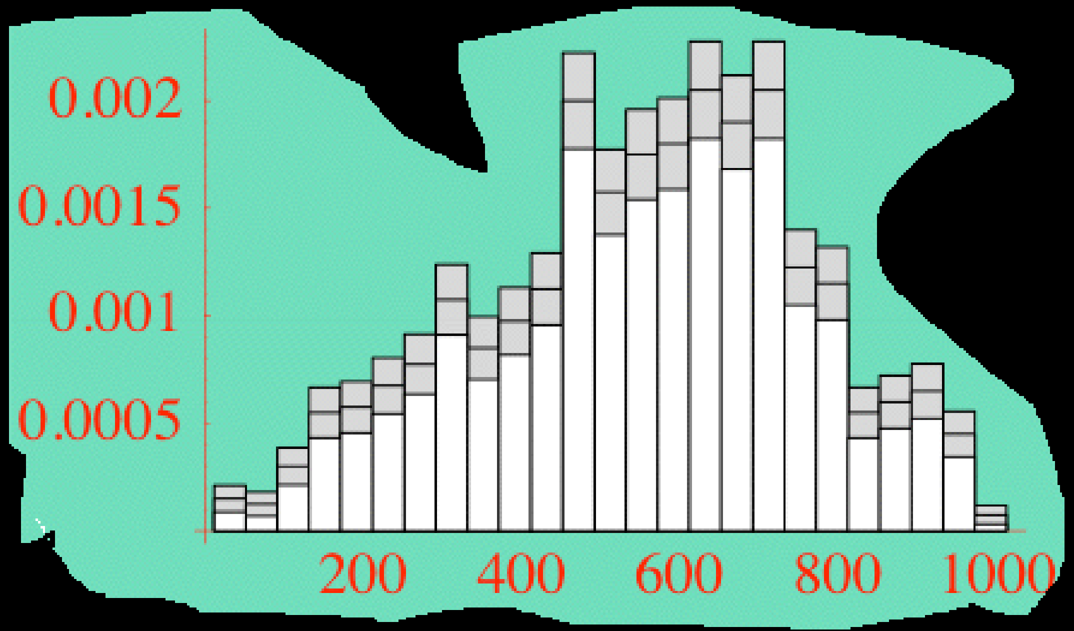
there's almost always a hierarchy: highly bimodal distributions on any dimension



small eigenvalue distribution

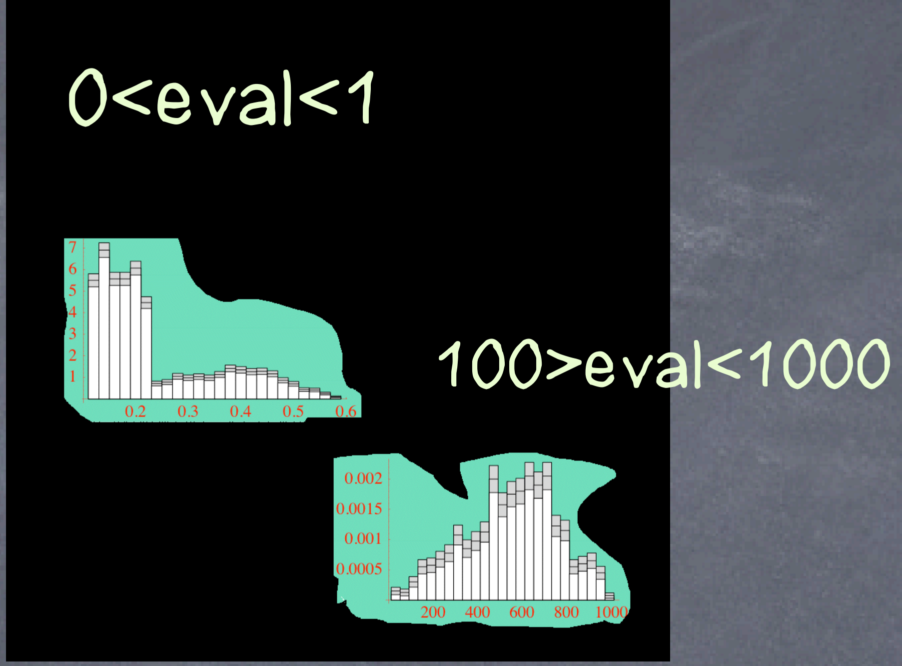
$E_{gut}=10^3$

large eigenvalue distribution



Why ?

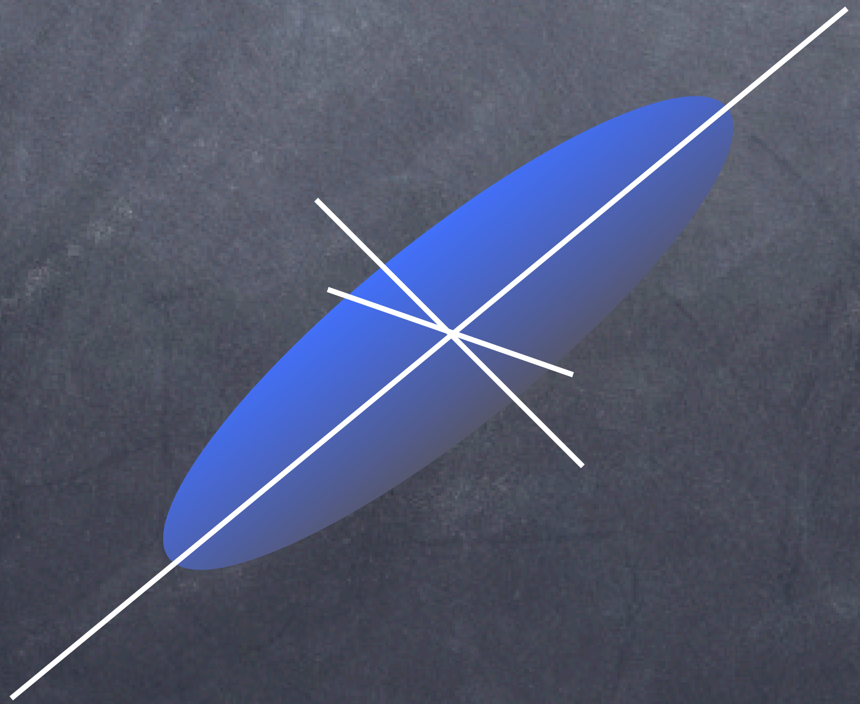
"entropy of the Hamiltonian"



$H_r = H / tr(H)$

$S_H = -tr(H_r \log(H_r));$ $\quad 0 \leq S_H \leq \log(N)$

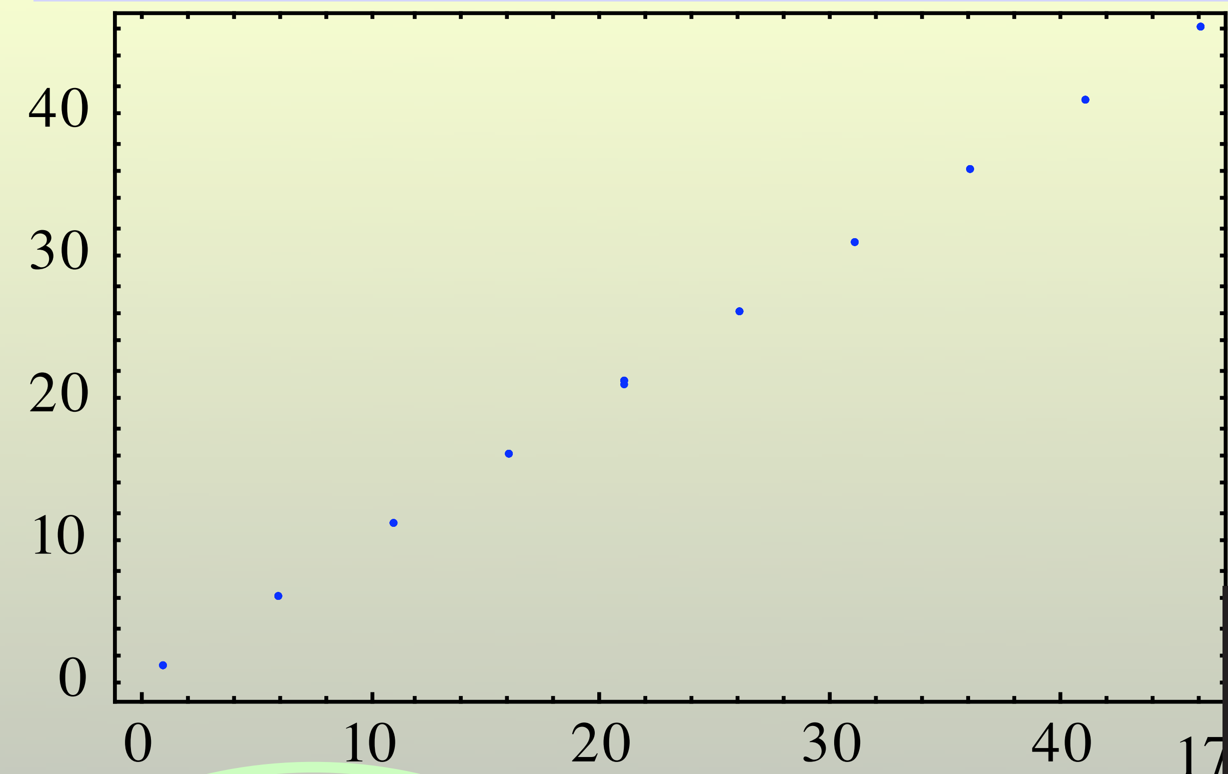
$\exp(S_H)$ = effective dimension of the eigenspace



(an invariant geometrical measure)

entropy S_{PP} scales like entropy S_H

$\exp(S_H)$ = effective dimension of the eigenspace



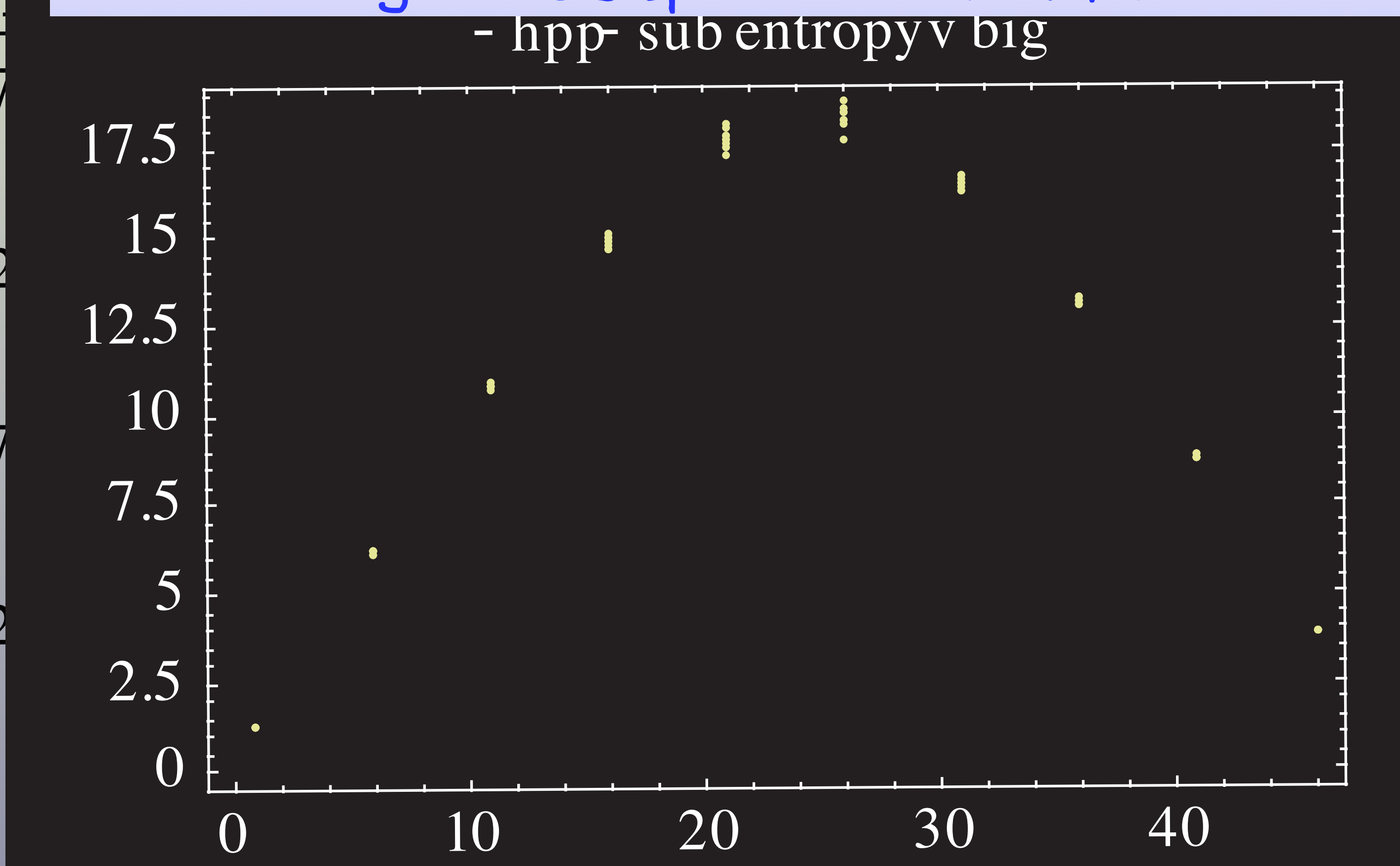
number of big evals

$$H_r = H / \mathrm{tr}(H)$$

$$S_H = -\mathrm{tr}(\, H_r \, \log(H_r) \,)$$

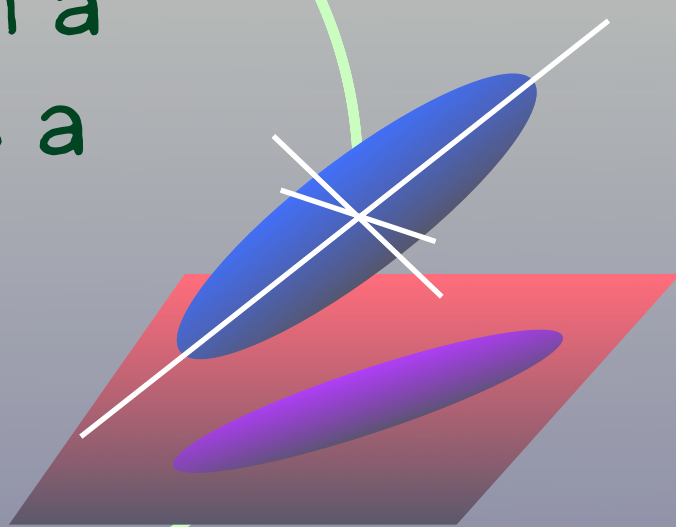
$$S_{PP} = -\mathrm{tr}(\, H_{rPP} \, \log(H_{rPP}) \,)$$

$\exp(S_{pp})$ = effective dimension of the eigenSUBspace of dim N/2



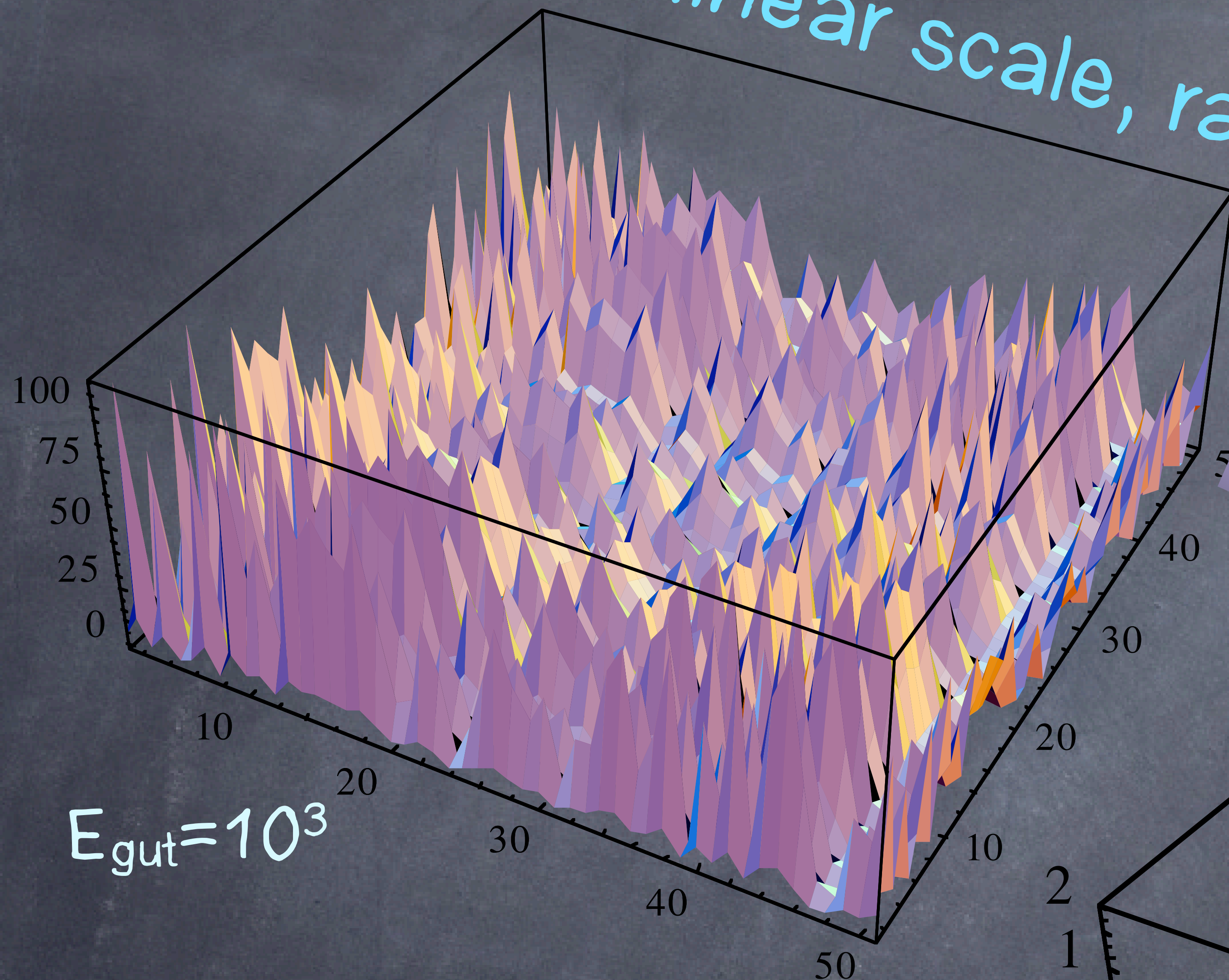
number of big evals

"almost all shadows of a needle are a needle"



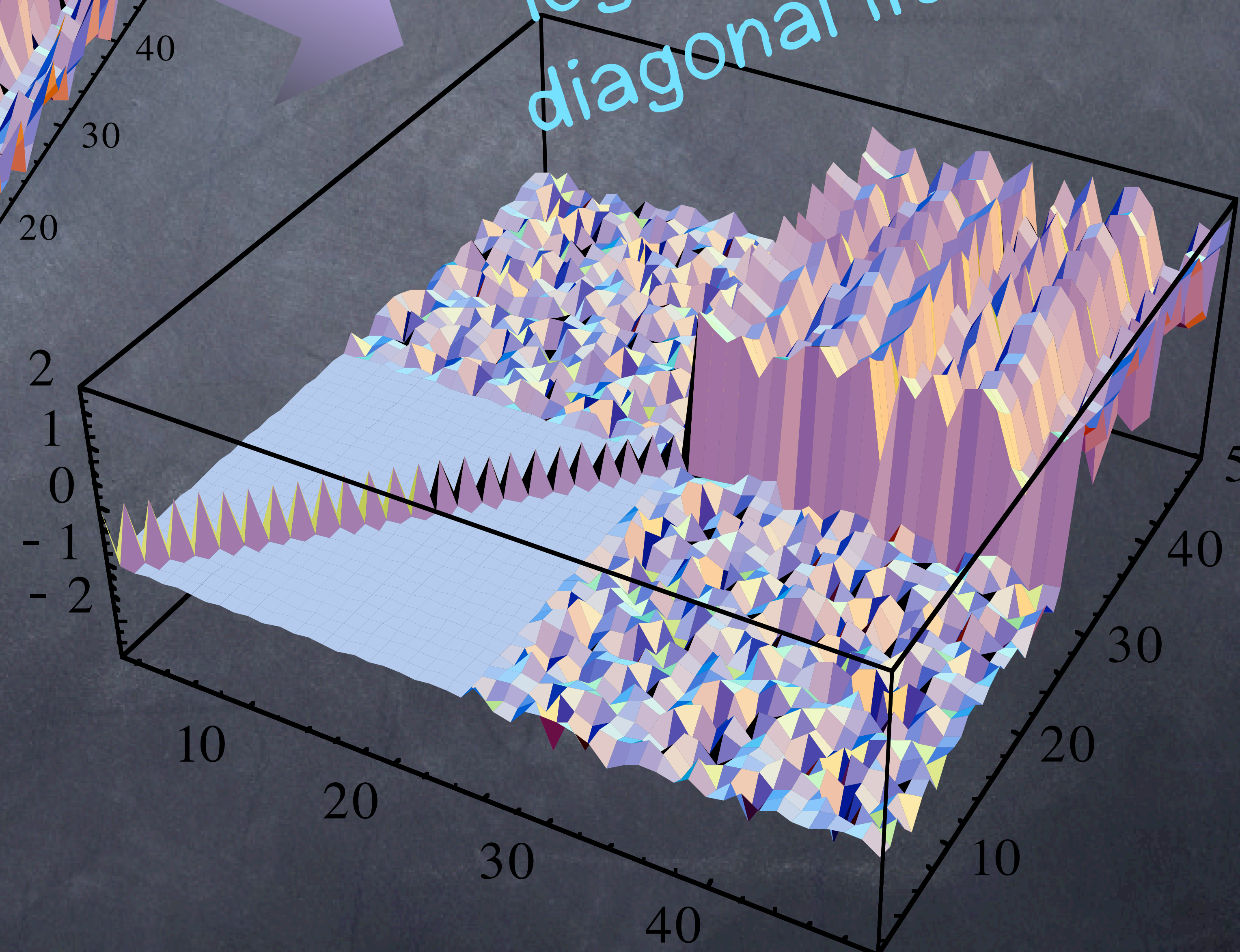
things are different in the diagonal frame of H_{pp}

linear scale, random frame



$E_{gut}=10^3$

log– 10 scale
diagonal frame

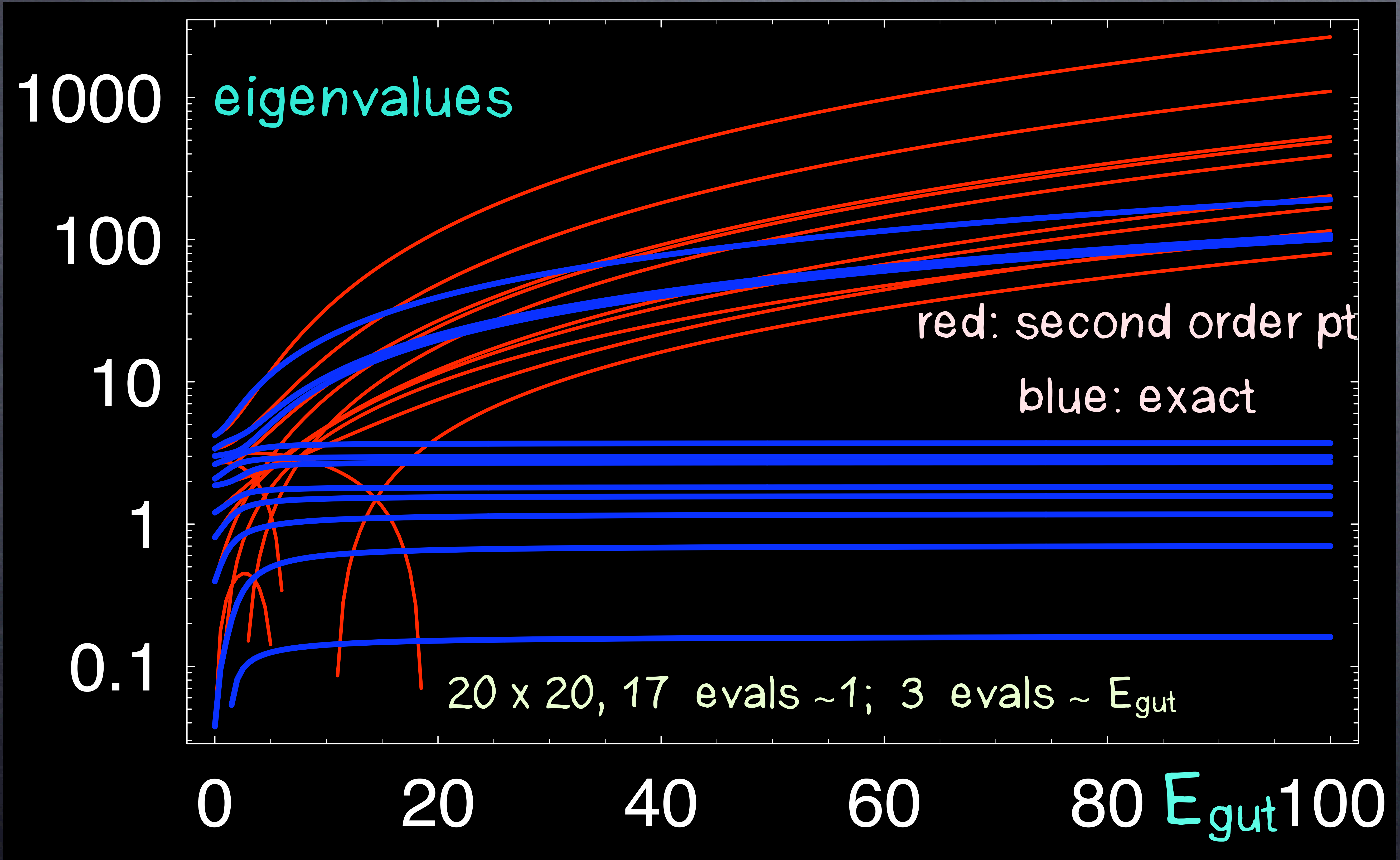


" decoupling
anarchy "

it's non-perturbative:

not available
from ordinary
perturbation theory

Rayleigh-Schroedinger perturbation theory fails with a hierachy



Brillouin–Wigner Perturbation Theory...
...the alternative at strong coupling

$$E \sim E_1 + \sum_{mn} V_{pn} \frac{1}{E - E_n^{(0)}} V_{np} + O(V^3)$$

...implicit, transcendental equations for E...but not exact

Our procedure is related...but exists EXACT "gap equation identity"

(new: see preprint):

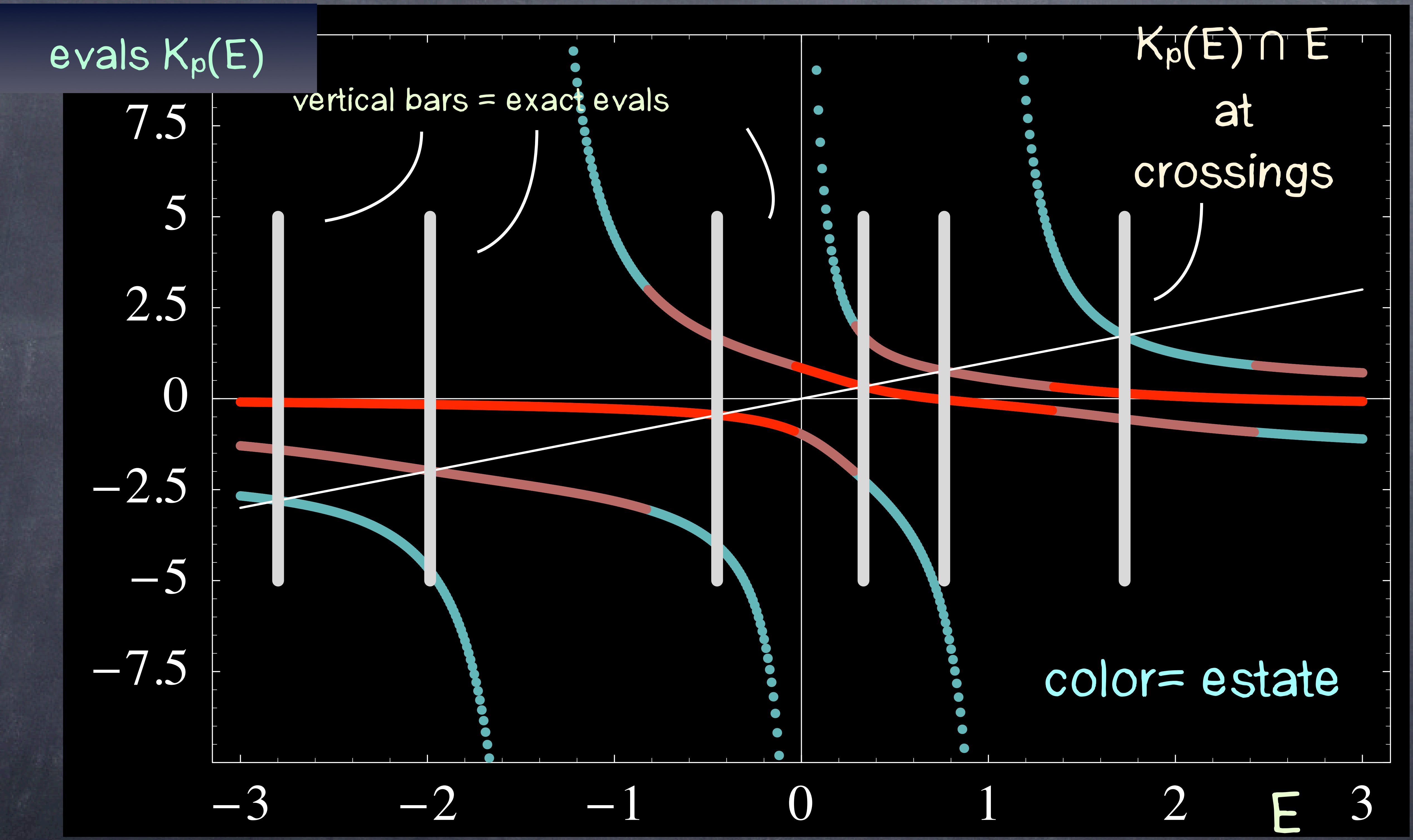
$$det(E - H) = (-1)^P E^P det(E - K_P(E)) det(E - H_{QQ})$$

$$K_p(E) = H_{PP} + H_{PQ} \frac{1}{E - H_{QQ}} H_{QP}$$

on subspace P, { $K_p(E) \cap E$} has the same spectrum as H on space P⊕Q

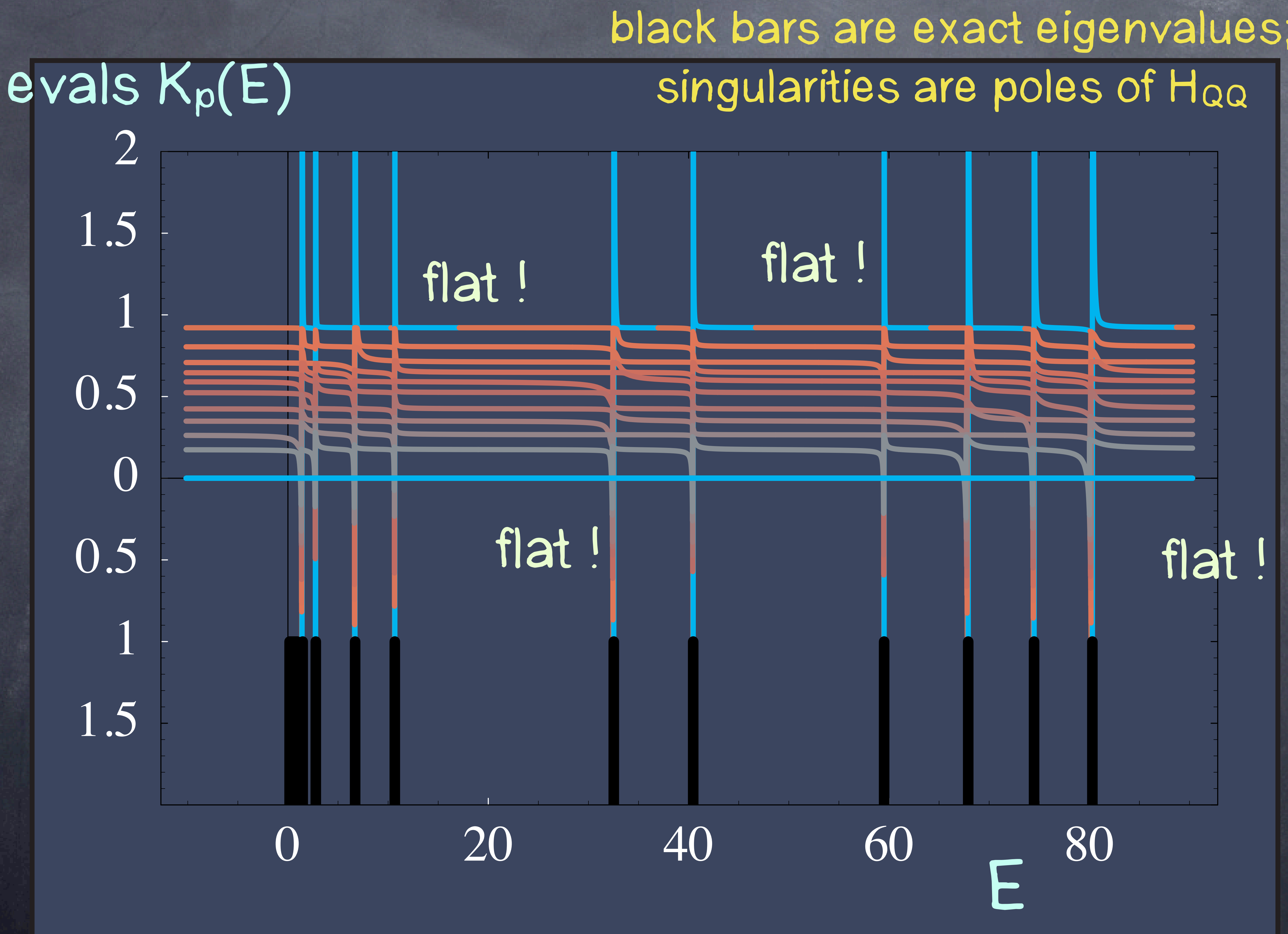
fano; feshbach

P+Q =6 exact eigenvalues on a P =3 dimensional subsystem

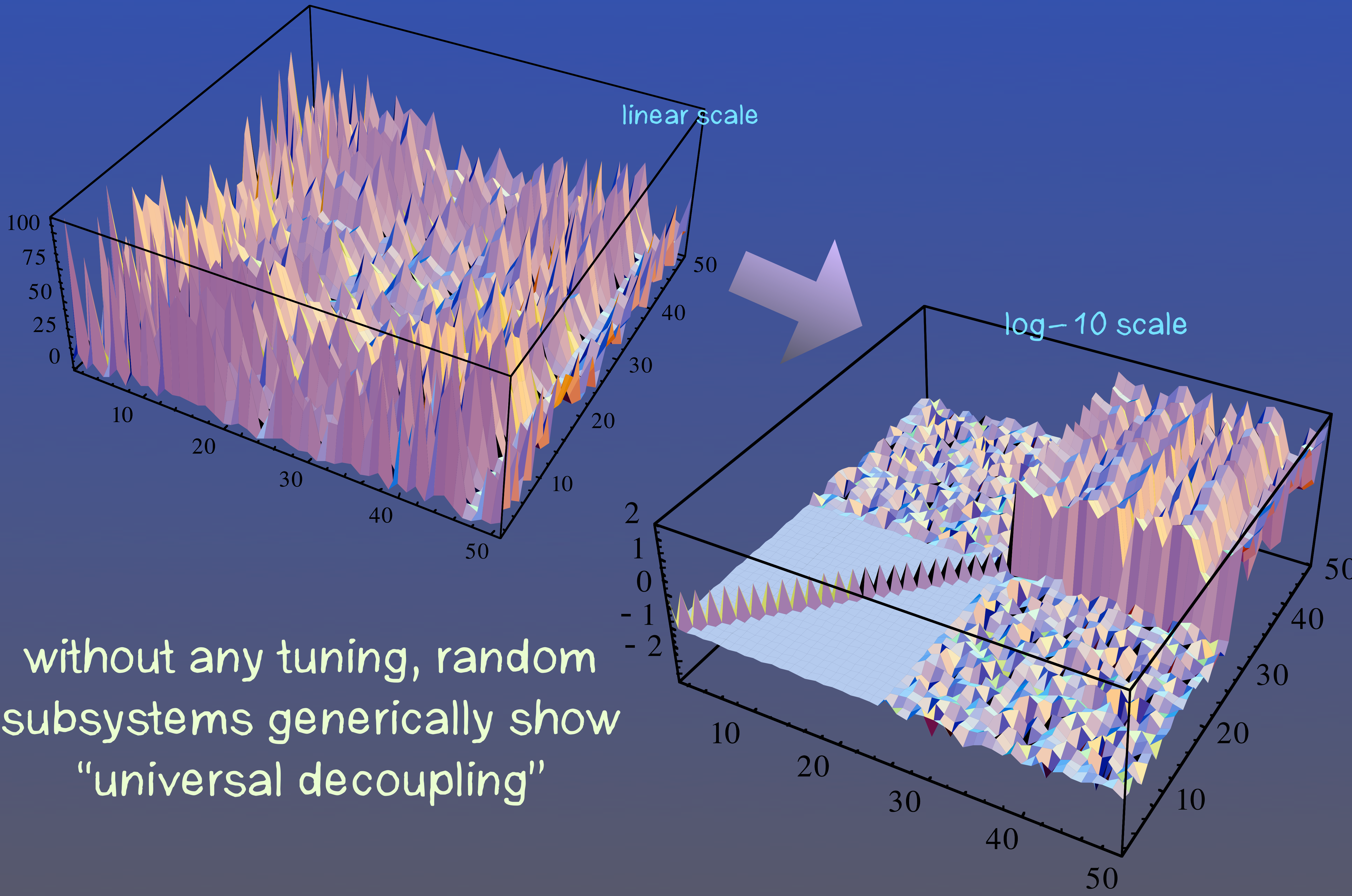


Solving the nonlinear eigenvalue equation Kp(E)=E. An N=6 dimensional system has been partitioned into direct sums of orthogonal P=3 and Q=3 dimensional subspaces

Stability of Procedure Under Hierarchy



the perception of dynamical instability is deceptive.



without any tuning, random subsystems generically show "universal decoupling"

Grand Anarchy

Given a big hierachy,
there's almost always a hierarchy
on a random subspace.

Diagonalizing on most
random subspaces,
Rayleigh Schroedinger
perturbation theory signals a
deceptive instability

...yet the Exact Gap Equation is Stable:

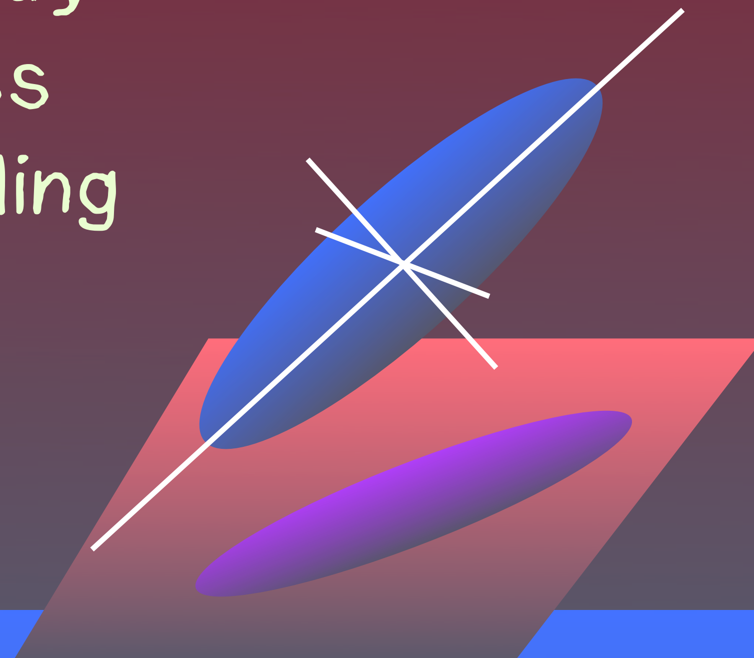
$$K_p(E) = H_{pp} + H_{pq} \frac{1}{E - H_{qq}} H_{qp}$$

$$K_p(E)|\psi_p >= \lambda(E)|\psi_p >$$

$$with \quad \lambda(E) \to E$$

Grand Anarchy

geometry
implies
decoupling



equivalant to cancellations
by symmetries?
perhaps !

..yet the Exact Gap Equation is Stable:

$$K_p(E) = H_{pp} + H_{pq}\frac{1}{E - H_{qq}}H_{qp}$$

$$K_p(E)|\psi_p >= \lambda(E)|\psi_p >$$

$$with \quad \lambda(E) \rightarrow E$$