QCD radiative correction to pair-annihilation of spin-1 Dark Matter

PRD 78 094022 (2008)[hep-ph/0807.2459]

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PHENO09

INTRODUCTION

- **Spin-1 dark matter**
  - Spin-1 dark matter candidate *B'* : Z_2 odd partner of the hypercharge gauge boson *B*.
  - Interesting models : UED, LH etc
    - KK-parity in UED : generic KK number conservation, i.e. momentum conservation in the extra dimensions. LKP is odd of KK-parity and becomes stable.

B'

Z_2 **symmetry**

T. Appelquist et al., PRD 64, 035002 (2001)
G. Servant, T.M.P. Tait, NPB 650,391 (2003)

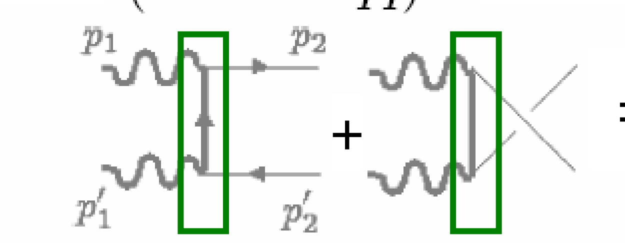
    - T-parity in LH : solution of little hierachy problem. LTP is odd of T-parity and becomes stable.

N. Arkani-Hamed et al., JHEP 07, 034 (2002)
H.C. Cheng, I.Low, JHEP 10, 067 (2004)

Leading order

Born diagrams

$(B^{'}B^{'} \to q\bar{q})$



$$\gamma^\mu \frac{\not{p}_2 - \not{p}_1 + \widetilde{M}}{t - \widetilde{M}^2}\gamma^\nu + \gamma^\nu \frac{\not{p}_2 - \not{p}_1' + \widetilde{M}}{u - \widetilde{M}^2}\gamma^\mu$$

V~0

$$= \frac{(p_2 - p_2')^\mu \gamma^\nu + (p_2 - p_2')^\nu \gamma^\mu}{M^2 + \widetilde{M}^2}$$

$p_1 + p_1' = p_2 + p_2'$

$p_1^\mu \epsilon_\mu(p_1) = 0$

$p_1'^\nu \epsilon_\nu(p_1') = 0$

$$\left.\begin{array}{l} s \simeq 4M^2 \\ t \simeq -M^2 \\ u \simeq -M^2 \end{array}\right\} \text{for } v\text{~}0$$

Corresponding annihilation rate in $d = 4 - 2\epsilon$ dimensions :

ϵ prescription **Model dependent**

$$\sigma_B v = \boxed{\frac{(1-\epsilon)\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)}\left(\frac{4\pi\mu^2}{4M^2}\right)^\epsilon} \underline{\frac{2g_Y^4(\widetilde{Y}_L^4 + \widetilde{Y}_R^4)N_c}{9\pi} \frac{M^2}{\left(M^2 + \widetilde{M}^2\right)^2}}$$

g_Y : hypercharge coupling

$N_C = 3$: number of colors

$\widetilde{Y}$: $B^{'}$ gauge charge

annihilation rate in *d=4* dimensions

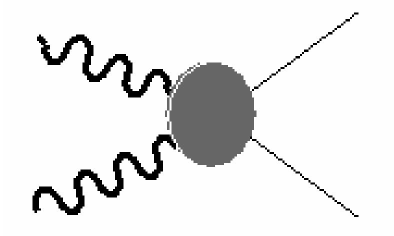
Annihilation rate is very sensitive to the gauge couplings

Branching ratio

$$\mathrm{Br}(B'B' \to q\bar{q}) \simeq \left\{\begin{array}{ll} \text{32\% for UED} & \widetilde{Y}_L(u,d) = \frac{1}{6},\ \widetilde{Y}_R(u) = \frac{2}{3},\ \widetilde{Y}_R(d) = -\frac{1}{3} \\ \text{0\% for LH} & \underline{\widetilde{Y}_L(u,d) = \frac{1}{10},\ \widetilde{Y}_R = 0} \end{array}\right.$$

Highly suppressed

PURPOSE and STRATEGY



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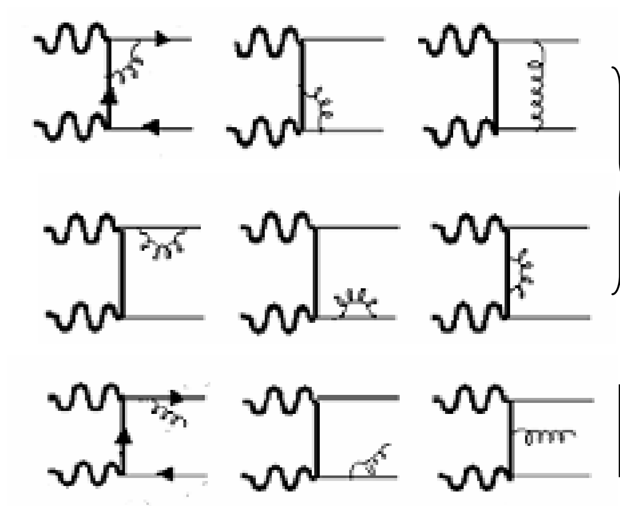


Virtual corrections

+ crossed diagrams

Real corrections

Propagator corrections ~
for UV

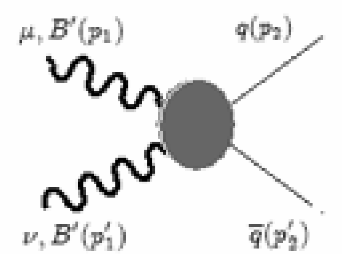
A real gluon emission ~
for IR

Our purpose

$$\boxed{\delta_{QCD} = \frac{\alpha_s}{\pi}\left(\text{costant} + O\left(\frac{\Delta M^2}{4M^2}\right)\right), \quad \Delta M^2 = \widetilde{M}^2 - M^2}$$

- Dimensional Regularization : Both UV and IR singularities, i.e. the same regulators for both singularities in the renormalizable Feynman gauge.
- Form factor approach : virtual corrections.
- Mass of new particles : same mass as *B'*.

NLO CORRECTIONS



- **Virtual corrections :**

$$\left\{\begin{array}{l} (\not p_2 \pm \not p_2')g^{\mu\nu} \\ (p_2+p_2')^\mu\gamma^\nu + (p_2+p_2')^\nu\gamma^\mu \\ \boxed{(p_2-p_2')^\mu\gamma^\nu + (p_2-p_2')^\nu\gamma^\mu} \end{array}\right.$$

: massless outgoing particles (quarks)

: *B'* polarizability in static limit (*v~0*)

neglected

$$\mathcal{M}_1 = F_{B'}\mathcal{M}_B \xrightarrow{S^\dagger S = 1} \delta_{QCD} = 2\,\mathrm{Re}(F_{B'})$$

Actual measurement

Simplified in the common integral
: shifted momentum and Feynman parameters

$$\int \frac{d^d\ell dx_i}{(2\pi)^d}\delta(\sum x_i - 1)\frac{(\ell^2)^r}{(\ell^2 - C)^m} = \frac{i(-1)^{r-m}}{(4\pi)^{d/2}}\frac{\Gamma(r+d/2)\Gamma(m-r-d/2)}{\Gamma(d/2)\Gamma(m)}\int dx_i\delta\left(\sum x_i - 1\right)C^{r-m+d/2}$$

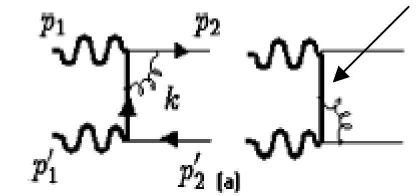
W.J. Marciano, PRD 12, 3861 (1975)

UV divergences

IR divergences

Feynman diagrams (+ crossed diagrams)

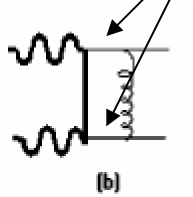
heavy quark line



No IR divergences

$$F_{B'}^{(a)} = \frac{\alpha_s C_F}{2\pi}\left(\frac{4\pi\mu^2}{4M^2}\right)^{\epsilon}\Gamma(1+\epsilon)\left(\frac{1}{\epsilon_{UV}}+1+\log 2\right)$$

massless quark line



No UV divergences

$$F_{B'}^{(b)} = \frac{\alpha_s C_F}{2\pi}\left(\frac{4\pi\mu^2}{4M^2}\right)^{\epsilon}\Gamma(1+\epsilon)$$

$$\left(-\frac{1}{\epsilon_{IR}^2}-\frac{2}{\epsilon_{IR}}-\frac{14}{3}+\frac{2}{3}\log 2+\frac{2\pi^2}{3}+i\pi\left(\frac{1}{\epsilon_{IR}}+3+\frac{\pi}{6}\right)\right)$$

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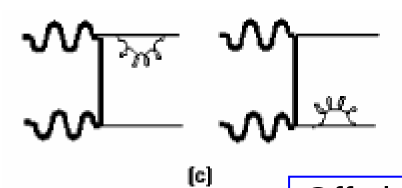
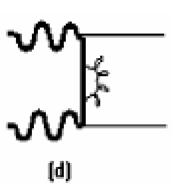
massless on shell quark



$$F_{B'}^{(c)} = \frac{\alpha_s C_F}{2\pi}\left(\frac{4\pi\mu^2}{4M^2}\right)^{\epsilon}\Gamma(1+\epsilon)\left(-\frac{1}{2\epsilon_{UV}}+\frac{1}{2\epsilon_{IR}}\right)$$

Off shell heavy quark

Two vertices



No IR divergences

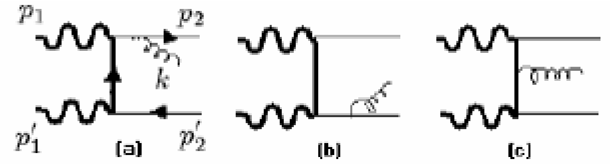
$$F_{B'}^{(d)} = 2\frac{d\Sigma_2}{d\not{p}}\Big|_{p^2=-M^2}$$

$$= \frac{\alpha_s C_F}{2\pi}\left(\frac{4\pi\mu^2}{4M^2}\right)^{\epsilon}\Gamma(1+\epsilon)\left(-\frac{1}{2\epsilon_{UV}}-1+\log 2\right)$$

Real correction

: the ratio of annihilation rates for two and three body final states.

$$\boxed{B'B' \to q\bar{q}g}$$



+ crossed diagrams

$$\sigma v = \frac{1}{4M^2} \cdot \frac{N_C}{9} \int d\Phi_3 |\mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c|^2$$

New dimensionless parameters

$$x_1 = \frac{p_2 \cdot k}{2M^2}, \qquad x_2 = \frac{p_2' \cdot k}{2M^2}, \qquad x_3 = \frac{p_2 \cdot p_2'}{2M^2}$$

Lorentz-invariant three body phase space w.r.t. new parameters

$$\int d\Phi_3 = \frac{4M^2}{(4\pi)^3 \Gamma(2-2\epsilon)} \left(\frac{4\pi\mu^2}{4M^2}\right)^{2\epsilon} \int dx_1 dx_2 dx_3 (x_1 x_2 x_3)^{-\epsilon} \delta(\sum x_i - 1)$$

$$\boxed{1/\epsilon \text{ poles at } x_1 = 0 \text{ and/or } x_2 = 0}$$

$$|\mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c|^2 \begin{cases} |\mathcal{M}_a|^2, |\mathcal{M}_b|^2 \text{ and } 2\,\mathrm{Re}(\mathcal{M}_a^* \mathcal{M}_b) \\ 2\mathrm{Re}(\mathcal{M}_a^* \mathcal{M}_c) \text{ and } 2\mathrm{Re}(\mathcal{M}_b^* \mathcal{M}_c) \\ |\mathcal{M}_c|^2 \end{cases}$$

Soft and collinear singularities

No singularities as combining the numerator and denominator

Totally free of IR divergences

$$\delta_{QCD}^{(\mathrm{real})} = \frac{\alpha_s C_F}{\pi} \left(\frac{4\pi\mu^2}{4M^2}\right)^{\epsilon} \Gamma(1+\epsilon) \left(\frac{1}{\epsilon_{IR}^2} + \frac{3}{2\epsilon_{IR}} + \frac{39}{4} - \frac{7}{6}\pi^2\right)$$

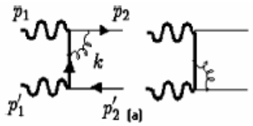
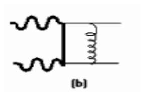
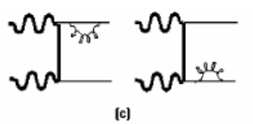
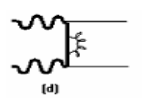
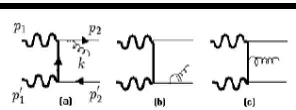
Corrections by the heavy gluon

- **Virtual corrections** :
- ✓ The corresponding Feynman diagrams are similar to the massless gluon. Just interchange the bold and light quark lines as well as the usual gluon is replaced by the heavy gluon.
- ✓ There is **NO** IR divergences.
- ✓ The UV divergences are cancelled among diagrams.

- **Real correction** : **No** real correction since a single heavy gluon cannot be produced by $\boldsymbol{Z_2}$ symmetry.

$$\delta_{QCD}^{(\mathrm{gluon})} = \frac{\alpha_s C_F}{\pi}\left(-5 + \frac{9\pi^2}{8} - 10\log 2 + \frac{7\pi}{6\sqrt{3}}\right)$$

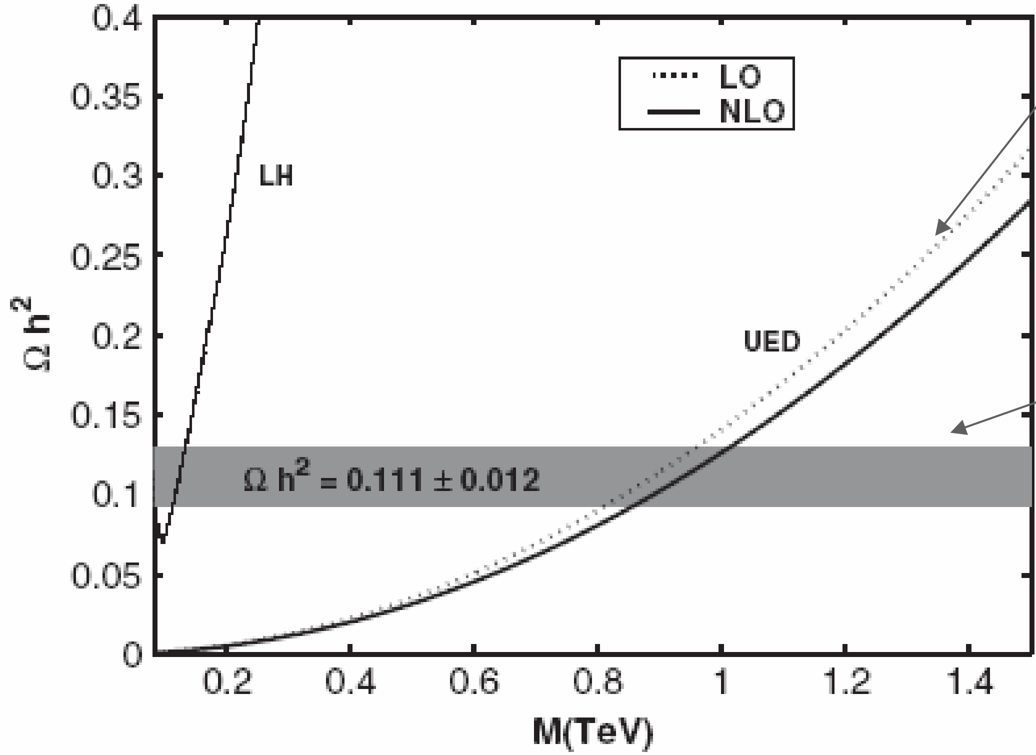
Overall QCD corrections

Virtual corrections		$\frac{\alpha_s C_F}{2\pi}\left(\frac{4\pi\mu^2}{4M^2}\right)^\epsilon \Gamma(1+\epsilon)\left(\frac{1}{\epsilon_{UV}}+1+\log 2\right)$
		$\frac{\alpha_s C_F}{2\pi}\left(\frac{4\pi\mu^2}{4M^2}\right)^\epsilon \Gamma(1+\epsilon)\left(-\frac{1}{\epsilon_{IR}^2}-\frac{2}{\epsilon_{IR}}-\frac{14}{3}+\frac{2}{3}\log 2+\frac{2\pi^2}{3}\right)$
		$\frac{\alpha_s C_F}{2\pi}\left(\frac{4\pi\mu^2}{4M^2}\right)^\epsilon \Gamma(1+\epsilon)\left(-\frac{1}{2\epsilon_{UV}}+\frac{1}{2\epsilon_{IR}}\right)$
		$\frac{\alpha_s C_F}{2\pi}\left(\frac{4\pi\mu^2}{4M^2}\right)^\epsilon \Gamma(1+\epsilon)\left(-\frac{1}{2\epsilon_{UV}}-1+\log 2\right)$
Sum of Virtual corrections		$\delta_{QCD}^{(\mathrm{virutal})} = 2\,\mathrm{Re}(F_{B'})$ **UV divergences canceled** $= \frac{\alpha_s C_F}{\pi}\left(\frac{4\pi\mu^2}{4M^2}\right)^\epsilon \Gamma(1+\epsilon)\left(-\frac{1}{\epsilon_{IR}^2}-\frac{3}{2\epsilon_{IR}}+\frac{2\pi^2}{3}-\frac{14}{3}+\frac{8}{3}\log 2\right)$
real correction		**IR divergences canceled** $\delta_{QCD}^{(\mathrm{real})} = \frac{\alpha_s C_F}{\pi}\left(\frac{4\pi\mu^2}{4M^2}\right)^\epsilon \Gamma(1+\epsilon)\left(\frac{1}{\epsilon_{IR}^2}+\frac{3}{2\epsilon_{IR}}+\frac{39}{4}-\frac{7}{6}\pi^2\right)$
Sum of all the corrections		$\delta_{QCD} = \frac{\alpha_s C_F}{\pi}\left(\frac{61}{12}+\frac{8}{3}\log 2-\frac{\pi^2}{2}\right)$ — **8% enhancement**
Correction by Heavy gluon		$\delta_{QCD}^{(\mathrm{gluon})} = \frac{\alpha_s C_F}{\pi}\left(-5+\frac{9\pi^2}{8}-10\log 2+\frac{7\pi}{6\sqrt{3}}\right)$ — **5% enhancement**

$\alpha_s(1\mathrm{TeV}) = 0.09$ and $C_F = 4/3$

APPLICATION TO RELIC DENSITY

G. Servant, T.M.P. Tait,
NPB 650,391 (2003)
K. Kong, K.T. Matchev,
JHEP 01, 038 (2006)



D.N. Spergel et al. [WMAP Collaboration],
APJ Suppl. Ser. 170, 377 (2007)

- WIMP mass in the window is shifted about 50 GeV for UED.
- No difference for LH because of the negligible annihilation fractions into quarks

CONCLUSION

- The finite NLO QCD corrections can be calculated for pair annihilation of spin-1 dark matter by dimensionally regularizing both UV and IR singularities in the non relativistic limit.

- The NLO correction amounts to about 8% and can enhance to 13% , when including heavy gluon.

- The NLO QCD correction could give the sizable shift to the DM mass constrained by relic density measurements.