Electromagnetic Radiation from Axion Condensates in a Time Dependent Magnetic Field

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Outline of the Talk

- Axions and Axion-like Particles
- The modified Maxwell's Equations
- Axion Condensates
- Electromagnetic Radiation from Axion Condensates in
  - Static External Magnetic Field
  - Oscillating External Magnetic Field
  - Oscillating External Magnetic Field with Time Varying Frequency
- Take Home Message

Axions and Axion-like Particles

- QCD Lagrangian is not CP symmetric
- Measurements of neutron magnetic dipole moment

  $\rightarrow$ CP violation extremely small
- Peccei and Quinn showed in 1977 that introducing a pseudo scalar field $\phi(\boldsymbol{x}, t)$ cancels the CP-violating term dynamically
- $m_a f_a \approx m_\pi f_\pi \sim \Lambda_{\mathrm{QCD}}^2$, $m_a <$ eV

R. D. Peccei and Helen R. Quinn. “CP Conservation in the Presence of Pseudoparticles”. In: *Phys. Rev. Lett.* 38 (25 1977), pp. 1440–1443. DOI: 10.1103/PhysRevLett.38.1440. URL: https://link.aps.org/doi/10.1103/PhysRevLett.38.1440

Axions and Axion-like Particles

- Particles with axion like properties are predicted by string theory
- Multiple different axions spanning many orders of magnitude
- f_a determined by string compactifications
- This motivates the search for axion-like particles with $m_a f_a \neq \Lambda_{\mathrm{QCD}}^2$
- Axions and axion-like particles are also an excellent candidate for dark matter

The Modified Maxwell's Equations

- Axion-photon Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + J_m^{\mu}A_{\mu} + \frac{C\beta}{4\pi f_a}\phi\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho} + \frac{1}{2}(\partial_{\mu}\phi)(\partial^{\mu}\phi) - V(\phi) \quad (1)$$

- To first order in $\frac{C\beta}{\pi f_a}\phi$ ($\phi \lesssim f_a$)

$$\Box\boldsymbol{A}_{\mathrm{r}}(\boldsymbol{x},t) = -\frac{C\beta}{\pi f_a}\big(\partial_t\phi(\boldsymbol{x},t)\big)\boldsymbol{B}_0 \equiv \boldsymbol{J}_a(\boldsymbol{x},t) \quad (2)$$

$$\Box\Phi_{\mathrm{r}}(\boldsymbol{x},t) = \frac{C\beta}{\pi f_a}\nabla\phi(\boldsymbol{x},t)\cdot\boldsymbol{B}_0 \equiv \rho_a(\boldsymbol{x},t) \quad (3)$$

$$(\Box + m_a^2)\phi(\boldsymbol{x},t) + \partial_{\phi}V(\phi) = -\frac{C\beta}{\pi f_a}\boldsymbol{E}_{\mathrm{r}}(\boldsymbol{x},t)\cdot\boldsymbol{B}_0(\boldsymbol{x},t) \quad (4)$$

Axion Condensates

- Axions can form spherically symmetric, coherently oscillating lumps of Bose-Einstein condensates $\phi(\boldsymbol{x}, t) \approx \phi_0 \mathrm{sech}(r/R) \cos(\omega t)$.

- Can be dense ($m_a R \sim 1$)
  - Dominated by self interactions
  - $\omega \lesssim m_a$
  - $\phi(\boldsymbol{x} = 0, t) \sim f_a$

- Or dilute ($m_a R \gg 1$)
  - Dominated by gravitational interactions
  - $\omega \approx m_a$
  - $\phi(\boldsymbol{x} = 0, t) \ll f_a$

Axion Condensates

Hong Zhang. "Axion Stars". In: *Symmetry* 12.1 (2019), p. 25. DOI: 10.3390/sym12010025. arXiv: 1810.11473 [hep-ph]

Luca Visinelli et al. "Dilute and dense axion stars". In: *Phys. Lett. B* 777 (2018), pp. 64–72. DOI: 10.1016/j.physletb.2017.12.010. arXiv: 1710.08910 [astro-ph.CO]

Electromagnetic Radiation from Axion Condensates in External Magnetic Field

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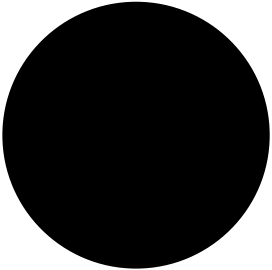

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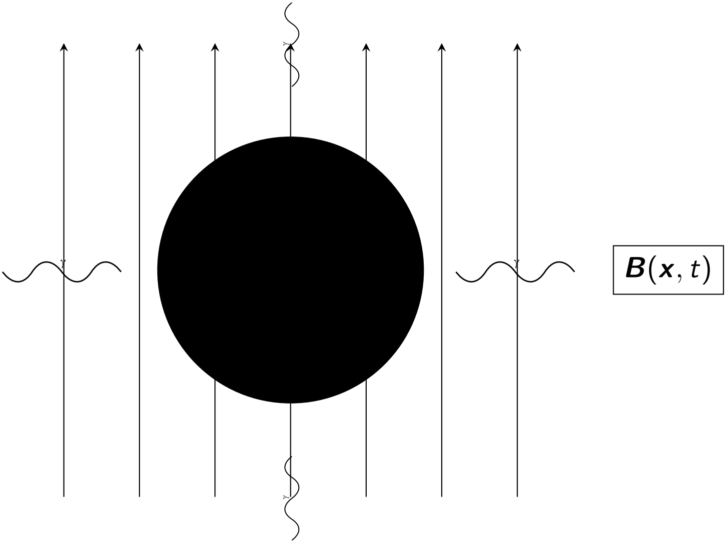

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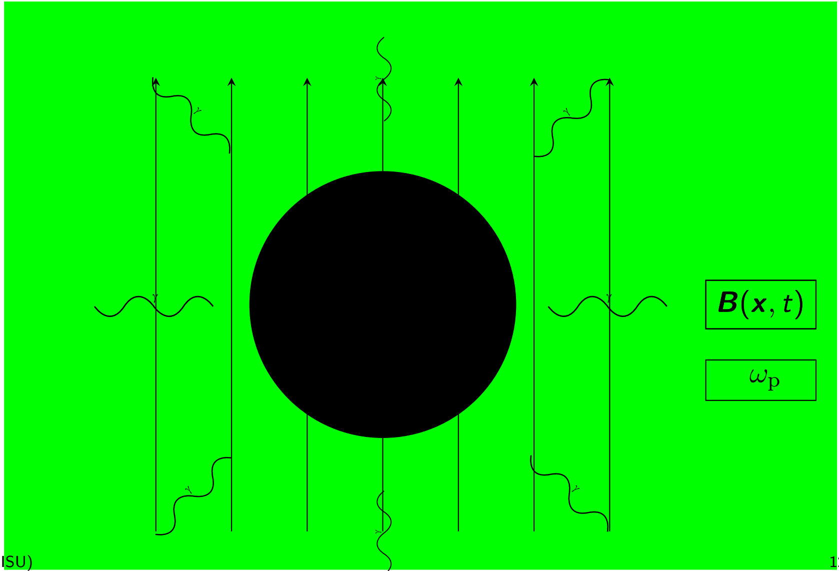

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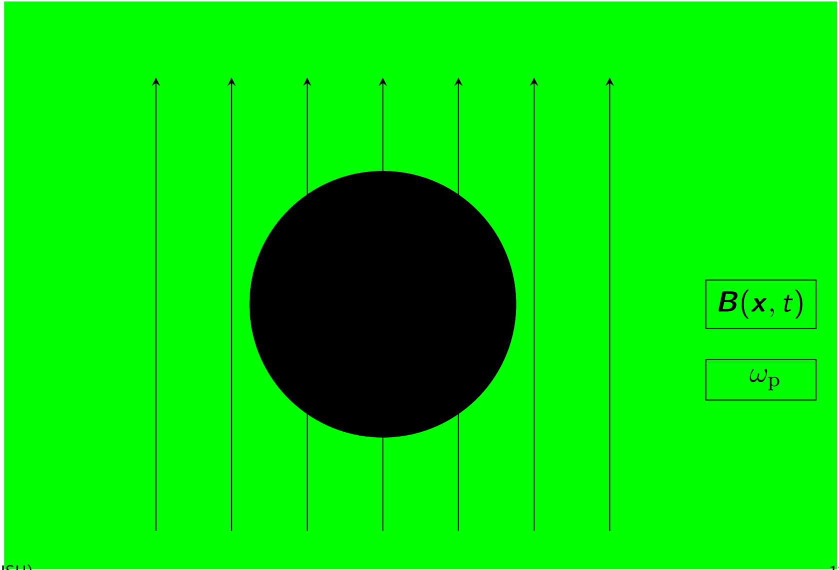
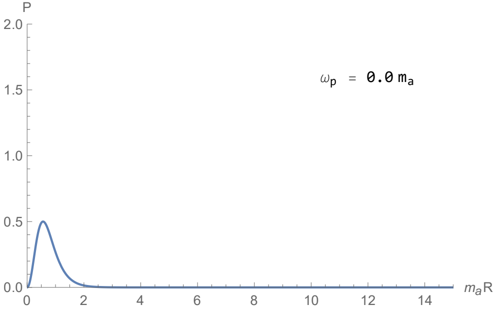


- Already found by Amin et al. to be ($k_\omega = \sqrt{\omega^2 - \omega_{\rm p}^2}$)

$$\langle P(t)\rangle_T = \left(\frac{C\beta}{\pi f_a}\right)^2 \left(\frac{\phi_0^2 B_0^2 \omega^3 R^4 \pi^5}{12 k_\omega}\right) \left(\frac{\tanh(\pi k_\omega R/2)}{\cosh(\pi k_\omega R/2)}\right)^2 \quad (5)$$



Mustafa A Amin et al. "Dipole radiation and beyond from axion stars in electromagnetic fields". In: *Journal of High Energy Physics* 2021.6 (June 2021), p. 182

Static External Magnetic Field

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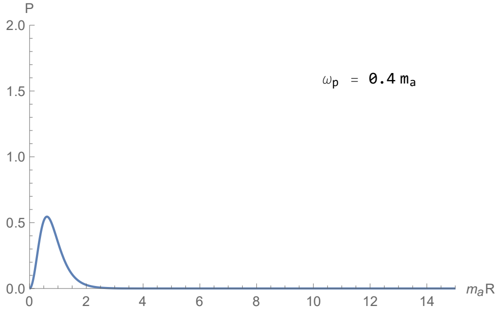

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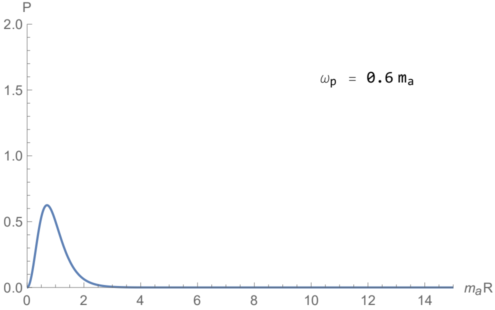

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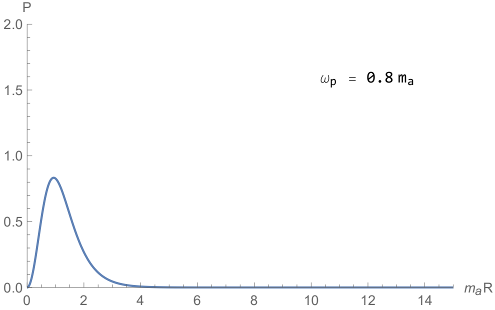

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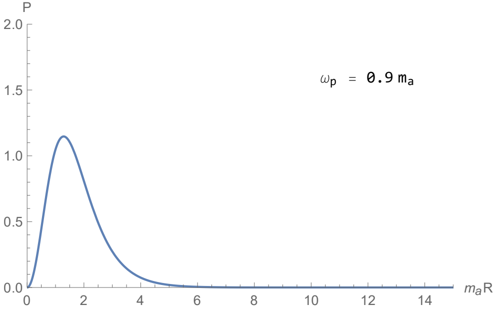

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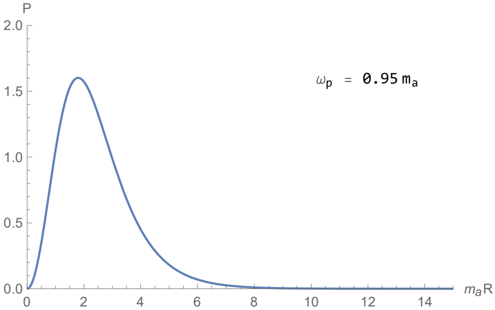

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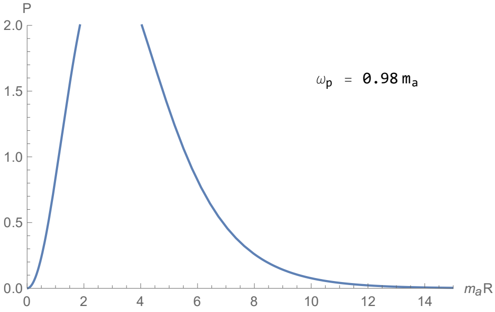

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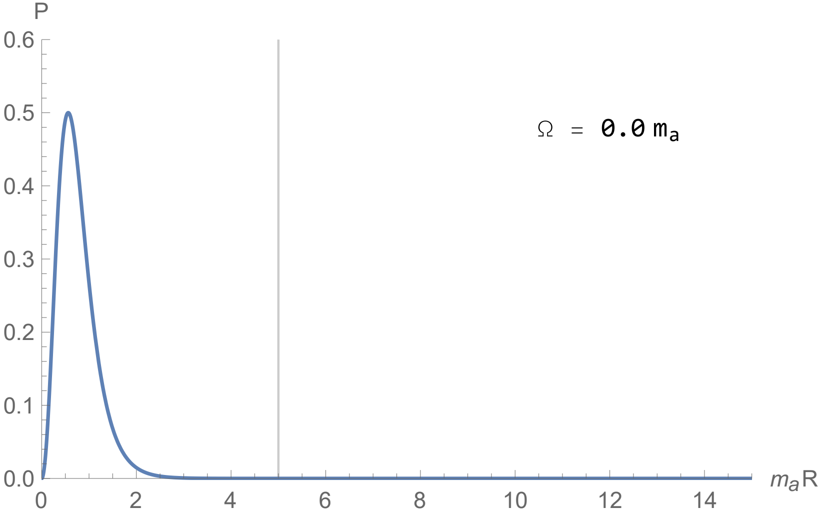
Static External Magnetic Field, takeaways

- Radiated power vanishes exponentially with system size ωR
- Tuning plasma frequency ω_{p} allows larger condensates to radiate efficiently
- Want to see if we can have similar effects by making the external magnetic field oscillate

Oscillating External Magnetic Field

- When $\boldsymbol{B}_0 \to \boldsymbol{B}_0 \cos(\Omega t)$, the effective frequency splits in two $\omega \to \omega \pm \Omega$, wavenumber $k_{\pm} = |\Omega \pm \omega|$



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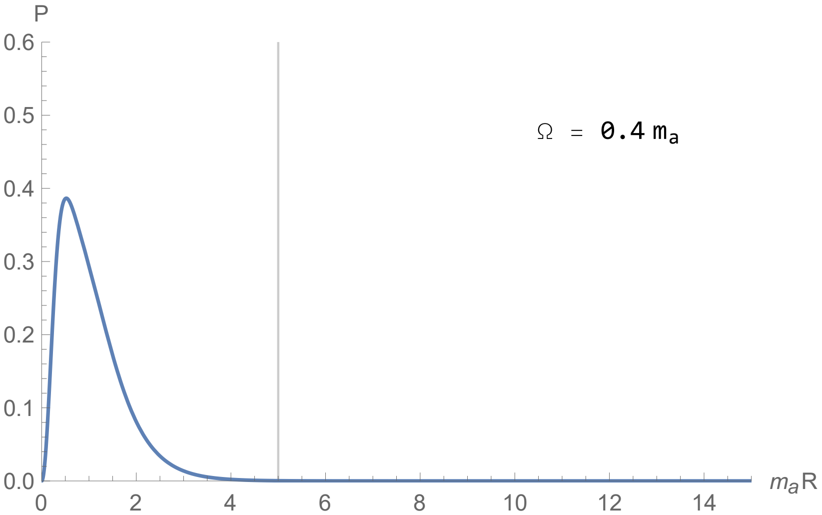

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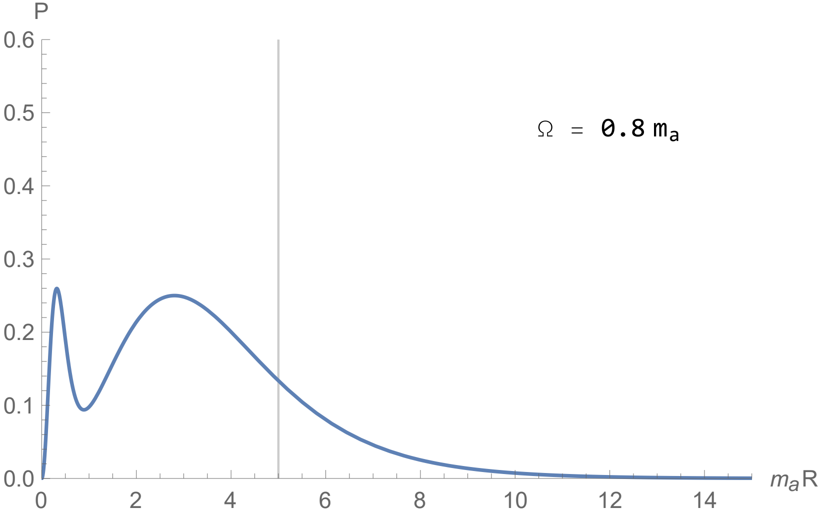

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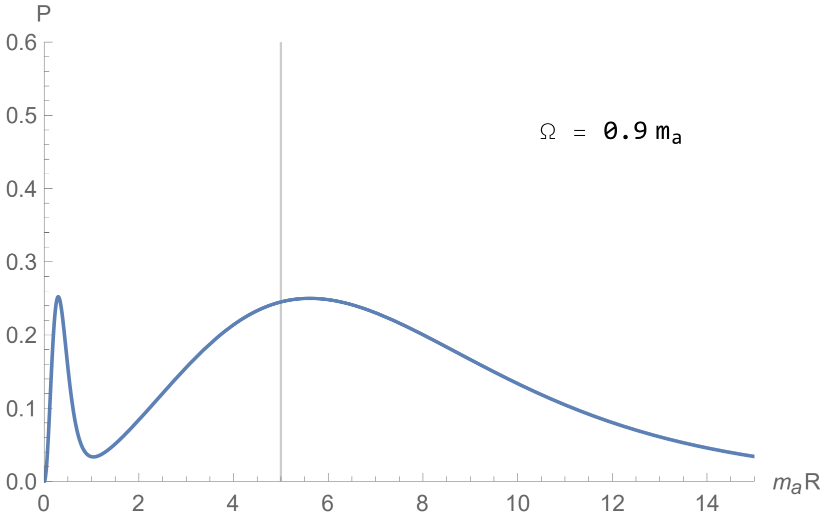

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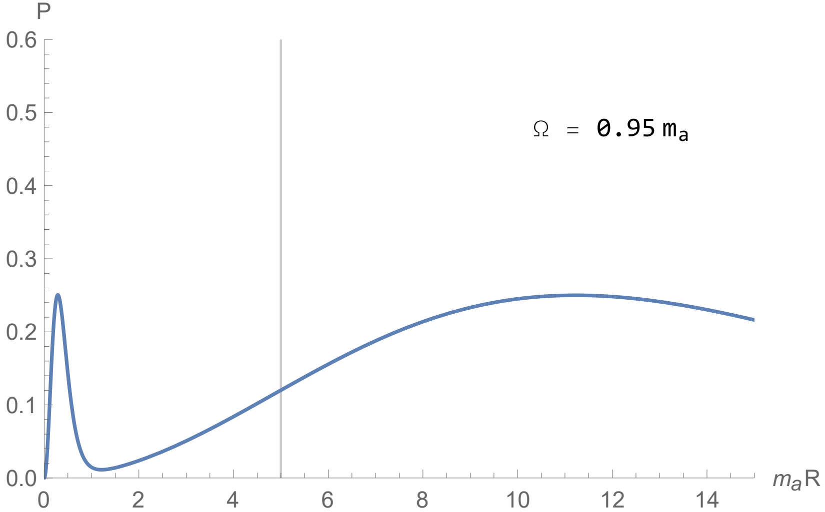

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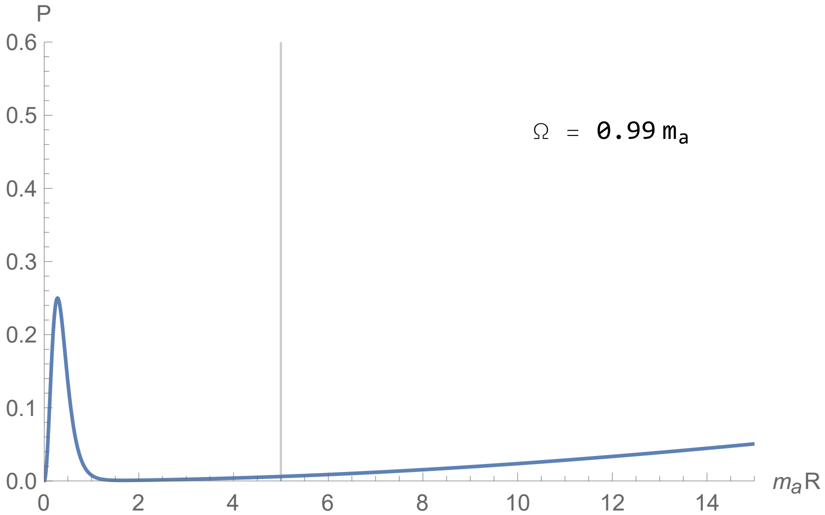

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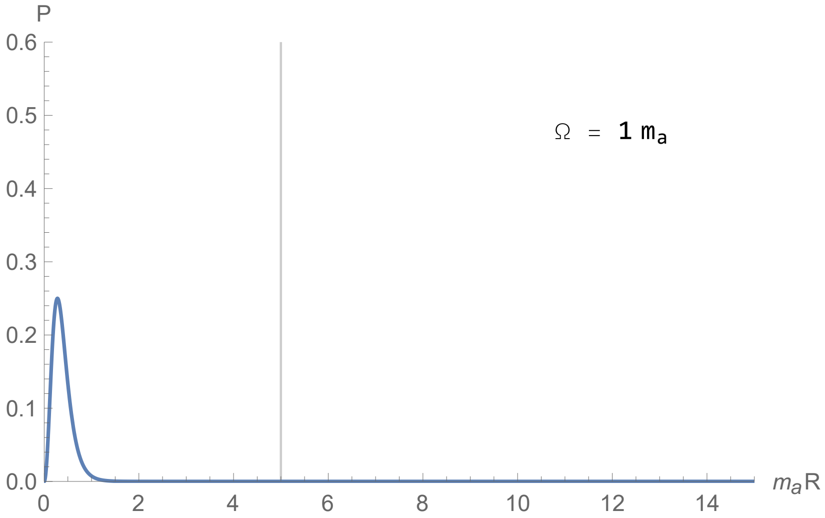

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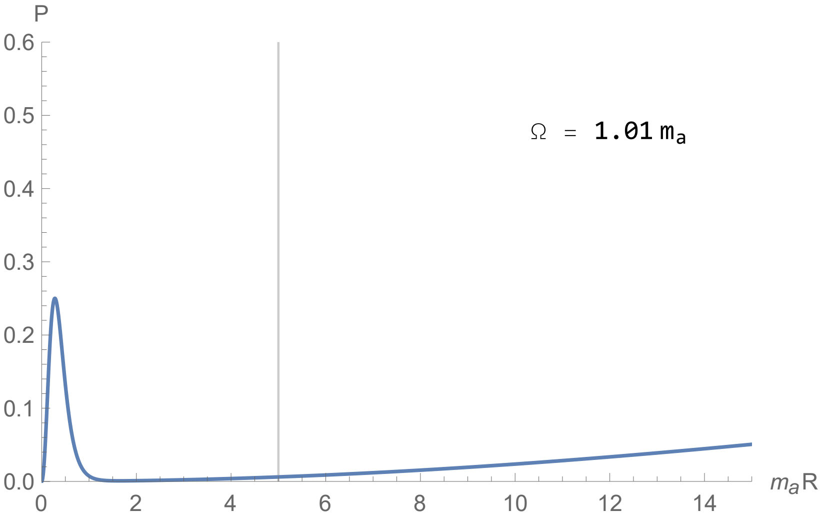

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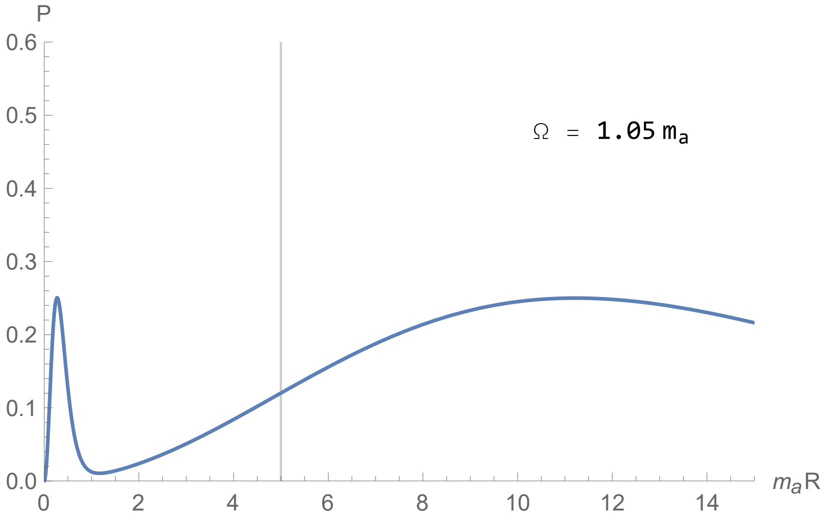

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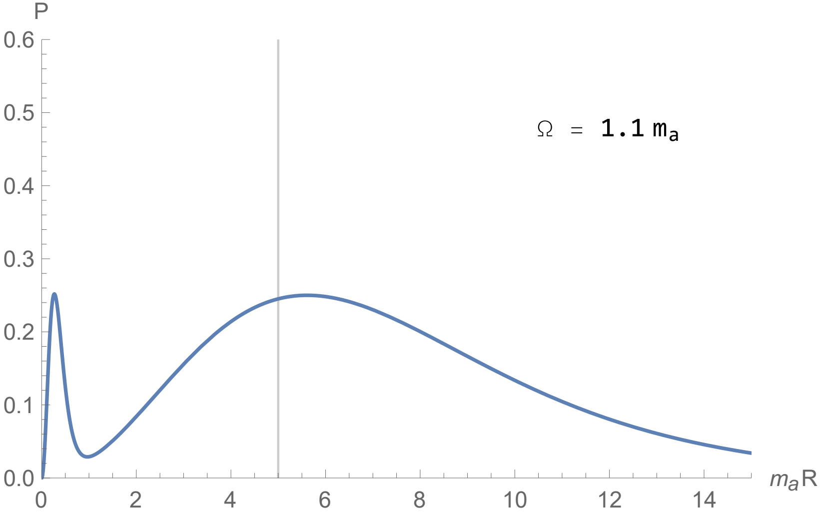

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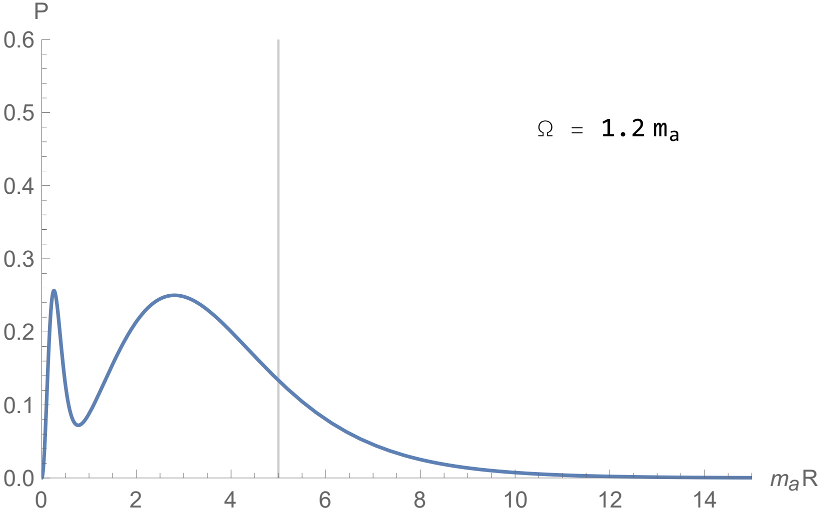

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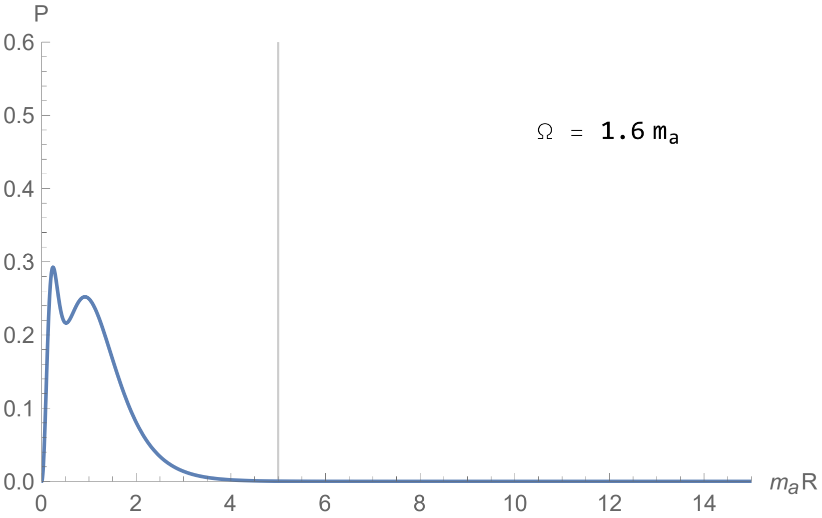

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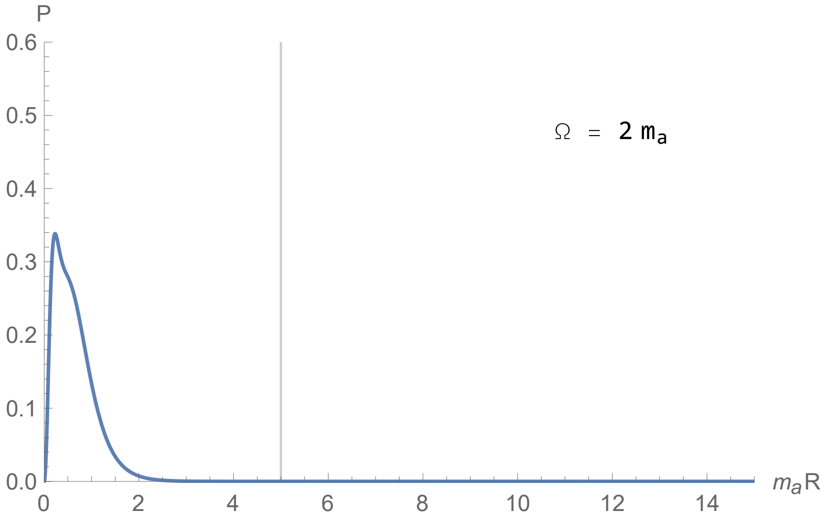
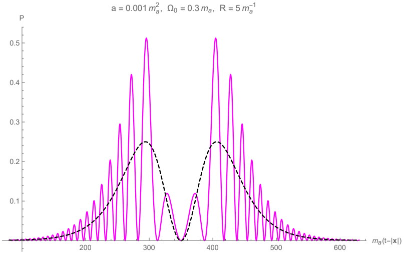
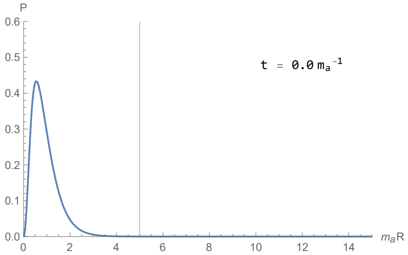


Oscillating Magnetic Background with Time Dependent frequency

- Axion condensates radiate efficiently when $k_\omega R \sim 1$
- No reason why $\omega, \omega_{\mathrm{p}}, \Omega$ should combine to make this true
- Let $\Omega \to \Omega(t) = \Omega_0 + at \to$ can scan over a set of axion masses and condensate sizes that radiates significantly

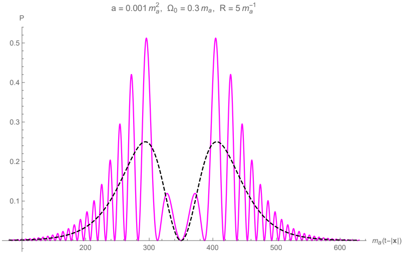
Oscillating Magnetic Background with Time Dependent Frequency: Radiated Power

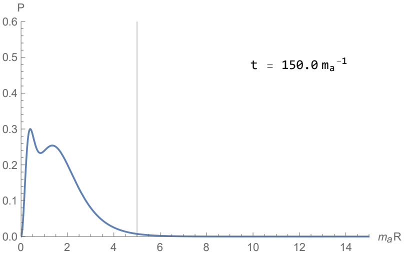




- Time in resonant region $\Delta t \sim \frac{1}{aR}$

Oscillating Magnetic Background with Time Dependent Frequency: Radiated Power





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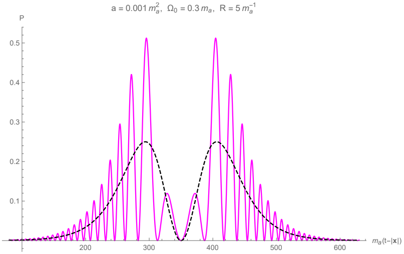

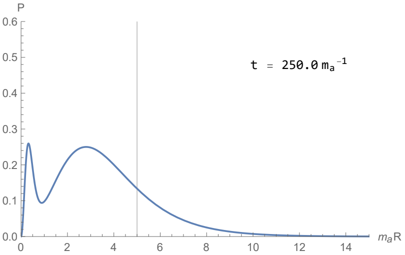

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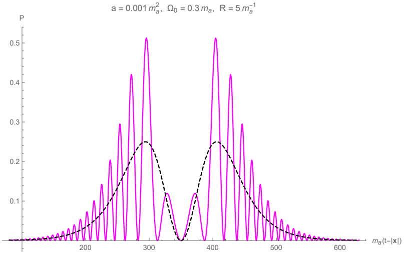

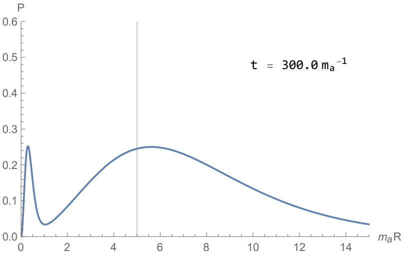

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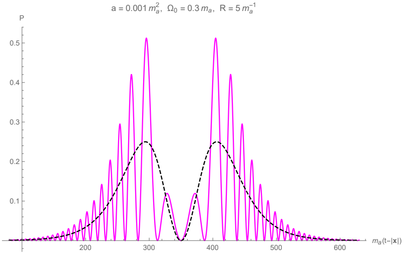

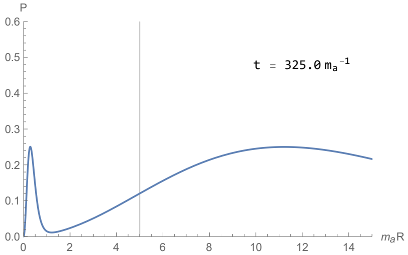

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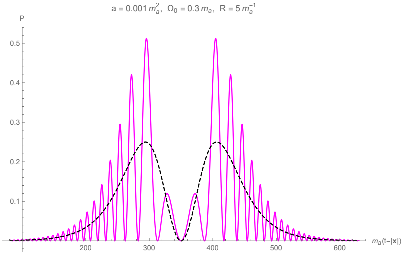

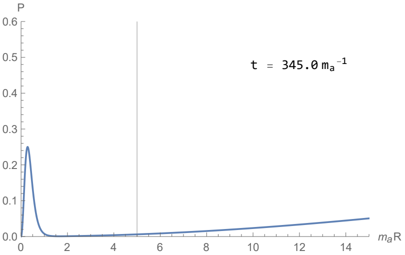

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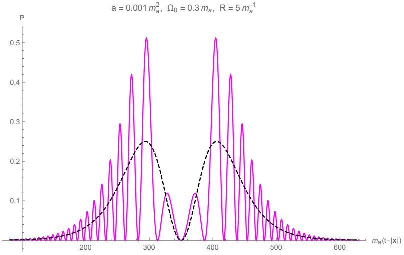

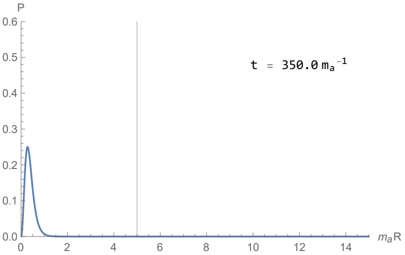

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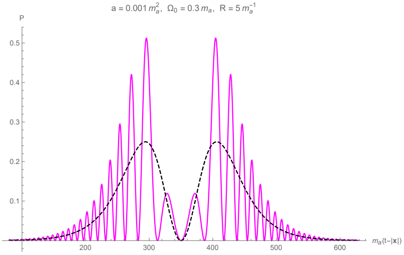

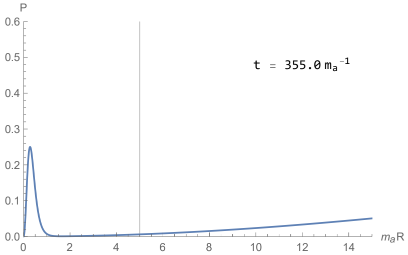

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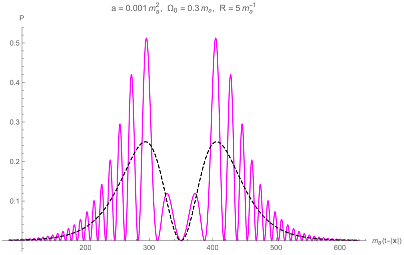

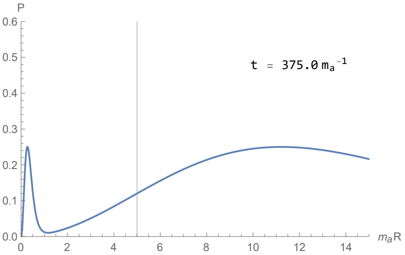

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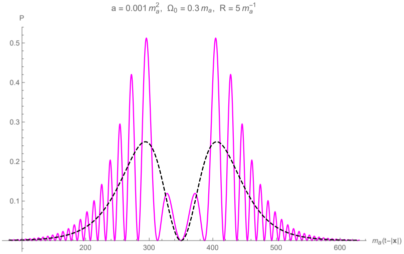
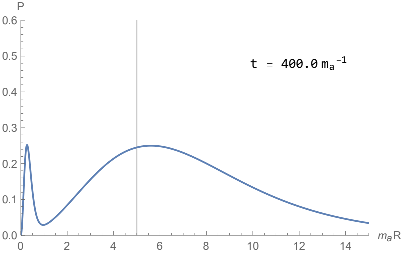
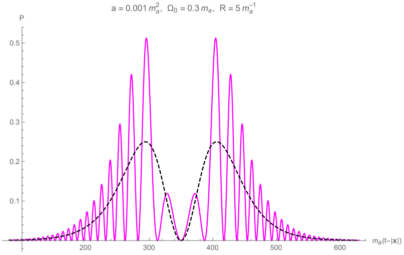
Oscillating Magnetic Background with Time Dependent Frequency: Radiated Power

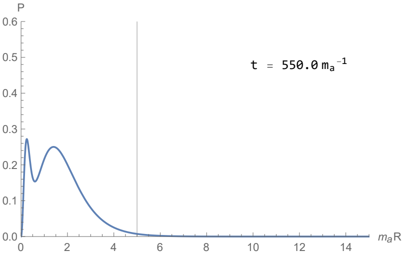




- Time in resonant region $\Delta t \sim \frac{1}{aR}$

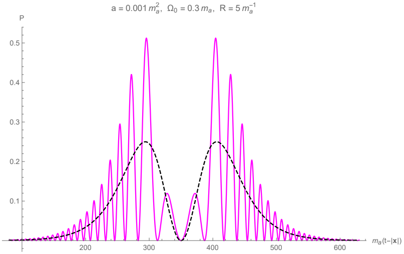
Oscillating Magnetic Background with Time Dependent Frequency: Radiated Power

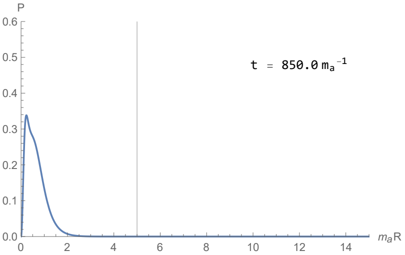




- Time in resonant region $\Delta t \sim \frac{1}{aR}$

Oscillating Magnetic Background with Time Dependent Frequency: Radiated Power

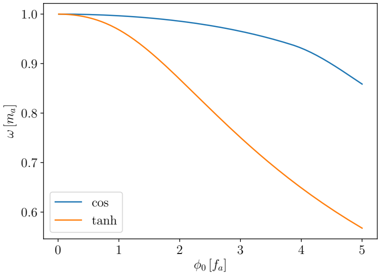


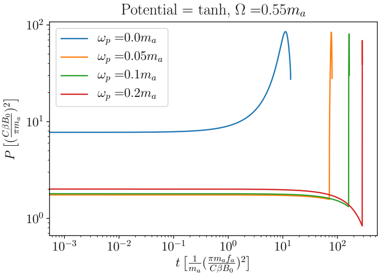


- Time in resonant region $\Delta t \sim \frac{1}{aR}$

Including Backreaction

Preliminary





Take Away Message

- Axion BEC start to give of electromagnetic radiation when subject to an external magnetic field
- There is a resonance that happens if the plasma frequency is similar to the axion mass
- A similar resonance occurs for oscillating magnetic fields when the magnetic field frequency is close to the axion mass
- A time dependent magnetic frequency allows for scanning a larger parameter space where condensates radiate efficiently
- Including spatial dependence for ω_{p} and $\boldsymbol{B}_0$ can allow BEC that normally do not radiate, to radiate efficiently as they pass through

Takk for at du kom!

Thanks for listening!

Extra stuff

The next slides are extra

Axion Equations of Motion

- Modified Maxwell equations

$$\begin{aligned}\nabla \times \boldsymbol{B}(\boldsymbol{x},t) - \partial_t \boldsymbol{E}(\boldsymbol{x},t) - \boldsymbol{J}_m(\boldsymbol{x},t) \\ = -\frac{C\beta}{\pi f_a}\Big[\big(\partial_t \phi(\boldsymbol{x},t)\big)\boldsymbol{B}(\boldsymbol{x},t) + \nabla\phi(\boldsymbol{x},t) \times \boldsymbol{E}(\boldsymbol{x},t)\Big] \quad (12)\end{aligned}$$

$$\nabla \times \boldsymbol{E}(\boldsymbol{x},t) = -\partial_t \boldsymbol{B}(\boldsymbol{x},t) \quad (13)$$

$$\nabla \cdot \boldsymbol{E}(\boldsymbol{x},t) = \rho_m(\boldsymbol{x},t) + \frac{C\beta}{\pi f_a}\nabla\phi(\boldsymbol{x},t) \cdot \boldsymbol{B}(\boldsymbol{x},t) \quad (14)$$

$$\nabla \cdot \boldsymbol{B}(\boldsymbol{x},t) = 0 \quad (15)$$

- Axion Equation of motion

$$(\Box + m_a^2)\phi(\boldsymbol{x},t) + \partial_\phi V(\phi) = -\frac{C\beta}{\pi f_a}\boldsymbol{E}(\boldsymbol{x},t) \cdot \boldsymbol{B}(\boldsymbol{x},t) \quad (16)$$

Estimation of Energy Radiated

- Energy stored in condensates $R \sim m_a^{-1}$

$$E_\phi \sim m_a^2 \phi_0^2 R^3 \sim m_a^2 f_a^2 R^3 \sim \frac{f_a^2}{m_a} \tag{17}$$

- A few orders of magnitude above or below a solar mass depending on the axion mass

- Energy stored in condensates $R \gg m_a^{-1}$

$$E_\phi \sim m_a^2 \phi_0^2 R^3 \sim \frac{m_{\mathrm{P}}^2}{m_a} \frac{1}{(m_a R)} \tag{18}$$

Estimation of Energy Radiated

- For $m_a R \sim 100$, E_ϕ is about the same as in the case $R \sim m_a^{-1}$
- Dense and Large condensates can radiate with similar efficiency
- For a time dependent frequency we need to estimate Δt
- Define resonant conversion as at least a fraction $\epsilon = 0.1$ of the maximum

$$\left(\frac{\tanh(\pi R k_-^{(2)}/2)}{\cosh(\pi R k_-^{(2)})}\right)^2 \geq \epsilon \left(\frac{\tanh(\log(\sqrt{2}+1))}{\cosh(\log(\sqrt{2}+1))}\right)^2 \tag{19}$$

- $\rightarrow \Delta t \approx \frac{4.71}{\pi a R} \sim \frac{1}{aR}$

Estimation of Energy Radiated

- If we want to exhaust the condensate, we need

$$\Delta t \sim \frac{1}{aR} \geq \tau \sim \frac{E_\phi}{\langle P(t) \rangle} \tag{20}$$

$$\rightarrow a \leq m_a^2 \left(\frac{C\beta B_0}{\pi m_a f_a} \right)^2 \tag{21}$$

- For ultralight axion in $B_0 \sim 10^{15}\mathrm{G}$

$$a \leq 10^0 - 10^4 \mathrm{s}^{-2} \tag{22}$$

- Comparable to that of inspiraling neutron stars

Axion Equations of Motion: Axion Condensates

- Interested in localized solutions of the source free equation

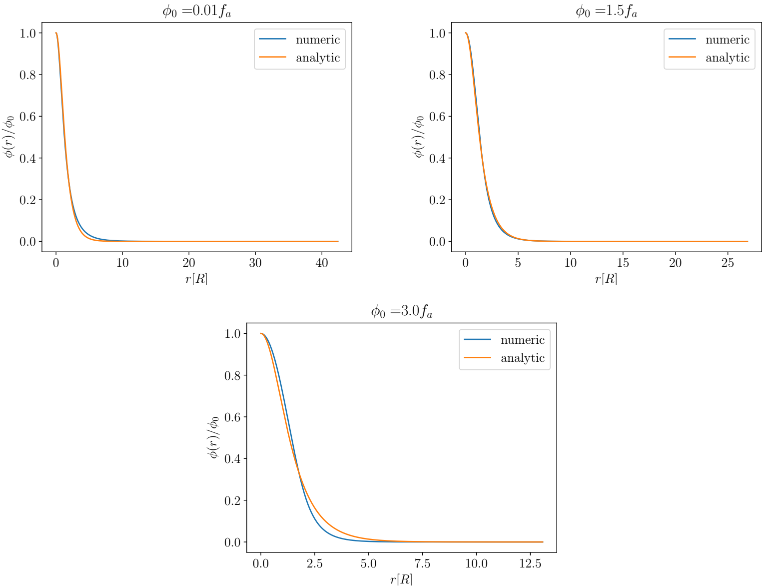
$$(\Box + m_a^2)\phi(\boldsymbol{x}, t) + \partial_\phi V(\phi) = 0 \tag{23}$$

- Good approximation

$$\phi(\boldsymbol{x}, t) = \phi_0 \,\mathrm{sech}(r/R) \cos(\omega t) \tag{24}$$

- $\omega \sim m_a$

- R is a free parameter


$\phi_0 = 0.01 f_a$
$\phi_0 = 1.5 f_a$
$\phi_0 = 3.0 f_a$
numeric
analytic
$\phi(r)/\phi_0$
$r[R]$

Axion Equations of Motion: External magnetic field

- We write $\boldsymbol{E} = \boldsymbol{E}_0 + \boldsymbol{E}_\mathrm{r}$ and $\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{B}_\mathrm{r}$
- Assume $\boldsymbol{E}_0$ and $\boldsymbol{B}_0$ satisfy usual Maxwell equations

$$\nabla \times \boldsymbol{B}_0(\boldsymbol{x}, t) - \partial_t \boldsymbol{E}_0(\boldsymbol{x}, t) - \boldsymbol{J}_m(\boldsymbol{x}, t) = 0 \tag{25}$$

$$\nabla \times \boldsymbol{E}_0(\boldsymbol{x}, t) = -\partial_t \boldsymbol{B}_0(\boldsymbol{x}, t) \tag{26}$$

$$\nabla \cdot \boldsymbol{E}_0(\boldsymbol{x}, t) = \rho_m(\boldsymbol{x}, t) \tag{27}$$

$$\nabla \cdot \boldsymbol{B}_0(\boldsymbol{x}, t) = 0 \tag{28}$$

- Set $\boldsymbol{E}_0 = 0$ and $\boldsymbol{B}_0 = B_0 \hat{r}_\mathrm{z}$
- Assume free space and expand in $\frac{C\beta}{\pi f_a}\phi$

Axion Equations of Motion: External magnetic field

- To leading order in $\frac{C\beta}{\pi f_a}\phi$

$$\nabla \times \boldsymbol{B}_\mathrm{r}(\boldsymbol{x}, t) - \partial_t \boldsymbol{E}_\mathrm{r}(\boldsymbol{x}, t) = -\frac{C\beta}{\pi f_a}\big(\partial_t \phi(\boldsymbol{x}, t)\big)\boldsymbol{B}_0 \tag{29}$$

$$\nabla \times \boldsymbol{E}_\mathrm{r}(\boldsymbol{x}, t) = -\partial_t \boldsymbol{B}_\mathrm{r}(\boldsymbol{x}, t) \tag{30}$$

$$\nabla \cdot \boldsymbol{E}_\mathrm{r}(\boldsymbol{x}, t) = \frac{C\beta}{\pi f_a}\nabla\phi(\boldsymbol{x}, t) \cdot \boldsymbol{B}_0 \tag{31}$$

$$\nabla \cdot \boldsymbol{B}_\mathrm{r}(\boldsymbol{x}, t) = 0 \tag{32}$$

$$(\Box + m_a^2)\phi(\boldsymbol{x}, t) + \partial_\phi V(\phi) = -\frac{C\beta}{\pi f_a}\boldsymbol{E}_\mathrm{r}(\boldsymbol{x}, t) \cdot \boldsymbol{B}_0 \tag{33}$$

- In particular, ϕ only coupled to $\boldsymbol{B}_0$

Axion Equations of Motion: Gauge Field

- Focus on

$$\nabla \times \boldsymbol{B}_{\mathrm{r}}(\boldsymbol{x}, t) - \partial_t \boldsymbol{E}_{\mathrm{r}}(\boldsymbol{x}, t) = -\frac{C\beta}{\pi f_a}\big(\partial_t \phi(\boldsymbol{x}, t)\big)\boldsymbol{B}_0 \tag{34}$$

$$\nabla \cdot \boldsymbol{E}_{\mathrm{r}}(\boldsymbol{x}, t) = \frac{C\beta}{\pi f_a}\nabla\phi(\boldsymbol{x}, t) \cdot \boldsymbol{B}_0 \tag{35}$$

- Insert the gauge field

$$\boldsymbol{B}_{\mathrm{r}}(\boldsymbol{x}, t) = \nabla \times \boldsymbol{A}(\boldsymbol{x}, t) \tag{36}$$

$$\boldsymbol{E}_{\mathrm{r}}(\boldsymbol{x}, t) = -\nabla\Phi(\boldsymbol{x}, t) - \partial_t \boldsymbol{A}(\boldsymbol{x}, t) \tag{37}$$

(38)

- Choosing the Lorenz gauge

$$\nabla \cdot \boldsymbol{A}(\boldsymbol{x}, t) + \partial_t \Phi(\boldsymbol{x}, t) = 0 \tag{39}$$

Axion Equations of Motion: Gauge Field

- Equations simplify to

$$\Box \boldsymbol{A}(\boldsymbol{x}, t) = -\frac{C\beta}{\pi f_a}\left(\partial_t \phi(\boldsymbol{x}, t)\right)\boldsymbol{B}_0 \equiv \boldsymbol{J}_a(\boldsymbol{x}, t) \tag{40}$$

$$\Box \Phi(\boldsymbol{x}, t) = \frac{C\beta}{\pi f_a}\nabla\phi(\boldsymbol{x}, t)\cdot\boldsymbol{B}_0 \equiv \rho_a(\boldsymbol{x}, t) \tag{41}$$

- Using the standard free retarded Green's function

$$G(\boldsymbol{x}, t; \boldsymbol{x}', t') = -\frac{\delta(t - t' - |\boldsymbol{x} - \boldsymbol{x}'|)}{4\pi|\boldsymbol{x} - \boldsymbol{x}'|} \tag{42}$$

- we can solve this

$$\boldsymbol{A}(\boldsymbol{x}, t) = \int \mathrm{d}^4 x \, G(\boldsymbol{x}, t; \boldsymbol{x}', t')\boldsymbol{J}_a(\boldsymbol{x}', t') \tag{43}$$

$$\Phi(\boldsymbol{x}, t) = \int \mathrm{d}^4 x \, G(\boldsymbol{x}, t; \boldsymbol{x}', t')\rho(\boldsymbol{x}', t') \tag{44}$$

Equations of Motion: Gauge Field

- Insert ansatz for $\phi(\boldsymbol{x}, t)$

$$\boldsymbol{J}_a(\boldsymbol{x}, t) = \omega B_0 \left(\frac{C\beta}{\pi f_a}\right) \sin(\omega t) \mathrm{sech}(r/R) \hat{r}_{\mathrm{z}} \tag{45}$$

$$\rho_a(\boldsymbol{x}, t) = -\frac{B_0}{R} \left(\frac{C\beta}{\pi f_a}\right) \left(\frac{\tanh(r/R)}{\cosh(r/R)}\right) \cos(\omega t) \cos\theta \tag{46}$$

- Note that far away from the source

$$\nabla \Phi(\boldsymbol{x}, t) \sim \hat{r} \tag{47}$$

- Can ignore $\Phi(\boldsymbol{x}, t)$ when discussing radiation since

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega_s} = r^2 \hat{r} \cdot (\boldsymbol{E}_{\mathrm{r}}(\boldsymbol{x}, t) \times \boldsymbol{B}_{\mathrm{r}}(\boldsymbol{x}, t)) \tag{48}$$

Constant Magnetic Background: Radiated Fields

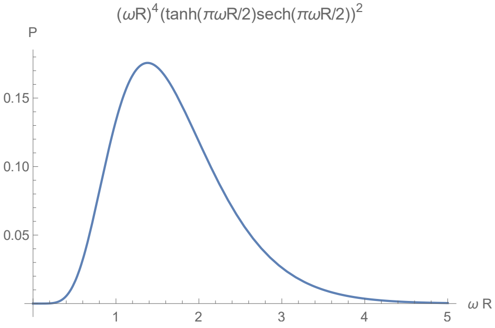
- To leading order in r^{-1}

$$\boldsymbol{E}_\mathrm{r}(\boldsymbol{x},t) = \left(\frac{C\beta}{\pi f_a}\right)\left(\frac{\phi_0 B_0 \pi^2 \omega R^2}{4r}\right) \times \left(\frac{\tanh(\pi\omega R/2)}{\cosh(\pi\omega R/2)}\right)\cos(\omega t - \omega r)\hat{r}_\mathrm{z} + (\mathrm{terms} \sim \hat{r}) \tag{49}$$

$$\boldsymbol{B}_\mathrm{r}(\boldsymbol{x},t) = \left(\frac{C\beta}{\pi f_a}\right)\left(\frac{\phi_0 B_0 \pi^2 \omega R^2}{4r}\right) \times \left(\frac{\tanh(\pi\omega R/2)}{\cosh(\pi\omega R/2)}\right)\cos(\omega t - \omega r)(\hat{r} \times \hat{r}_\mathrm{z}) \tag{50}$$

- Radiated power is given by

$$\langle P \rangle_T = \left\langle \int \mathrm{d}\Omega_s \, \hat{r} \cdot (\boldsymbol{E}_{\mathrm{r}}(\boldsymbol{x}, t) \times \boldsymbol{B}_{\mathrm{r}}(\boldsymbol{x}, t)) \right\rangle_T$$
$$= \left(\frac{C\beta}{\pi f_a}\right)^2 \left(\frac{\phi_0^2 B_0^2}{12\omega^2}\right) (\omega R)^4 \left(\frac{\tanh(\pi\omega R/2)}{\cosh(\pi\omega R/2)}\right)^2 \qquad (51)$$



Oscillating Magnetic Background: Average Power Radiated

- When $\omega \ll \Omega$ we can average over a time $\frac{2\pi}{\Omega}$

$$\langle P(t)\rangle_T = \left(\frac{C\beta}{\pi f_a}\right)^2 \left(\frac{B_0^2\phi_0^2\omega^2 R^4\pi^5}{12}\right)\left(\frac{\tanh(\pi\Omega R/2)}{\cosh(\pi\Omega R/2)}\right)^2 \quad (52)$$

- Maximum for smaller radii R

- When $\omega \gg \Omega$ we average over $T = \frac{2\pi}{\omega}$

$$\langle P(t)\rangle_T = \left(\frac{C\beta}{\pi f_a}\right)^2 \left(\frac{B_0^2\phi_0^2\omega^2 R^4\pi^5}{12}\right)\left(\frac{\tanh(\pi\omega R/2)}{\cosh(\pi\omega R/2)}\right)^2 \quad (53)$$

- Back where we started

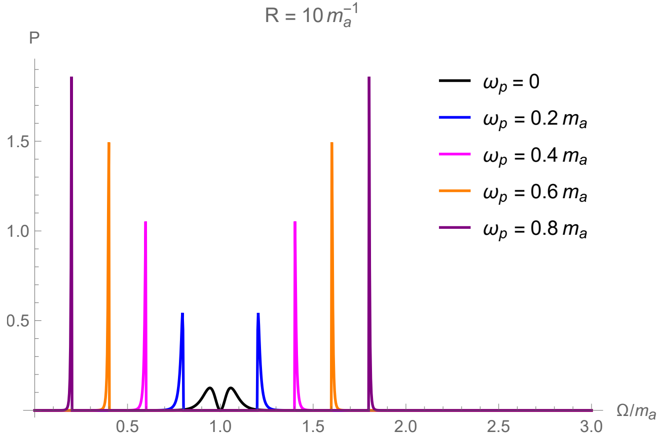
Oscillating Magnetic Background: Average Power Radiated

- Most interesting when $\omega \sim \Omega$
- If we consider pulsars, $\omega \approx m_a \sim 10^{-12}\text{eV}$
- Power radiated averaged over a time $T \gg \frac{2\pi}{\omega}$ $(\gg \frac{2\pi}{\Omega})$

$$\langle P(t) \rangle_T = \left(\frac{C\beta}{\pi f_a}\right)^2 \left(\frac{B_0^2 \phi_0^2 \omega^2 R^4 \pi^5}{24}\right) \times \left[\left(\frac{\tanh(\pi\omega_+ R/2)}{\cosh(\pi\omega_+ R/2)}\right)^2 + \left(\frac{\tanh(\pi\omega_- R/2)}{\cosh(\pi\omega_- R/2)}\right)^2\right] \tag{54}$$

- Second term allows us to realize enhancement for larger R

- Power averaged over 500 periods of the axion condensate measured in units $\left(\frac{C\beta}{\pi f_a}\right)^2\left(\frac{\phi_0^2 B_0^2 \omega^2 R^4 \pi^5}{48}\right)$



Oscillating Magnetic Background with Time Dependent Frequency

- Assume linear growth in frequency

$$\Omega(t) = \Omega_0 + at \tag{55}$$

- For now ignore plasma

- Axion current is on the same form

$$\boldsymbol{J}(\boldsymbol{x}, t) = \frac{\omega B_0}{2}\left(\frac{C\beta}{\pi f_a}\right)\mathrm{sech}(r/R)\Big[\sin(\omega_- t) + \sin(\omega_+ t)\Big]\hat{r}_z \tag{56}$$

$$\omega_- = \omega - \Omega_0 - at, \qquad \omega_+ = \omega + \Omega_0 + at \tag{57}$$

Oscillating Magnetic Background with Time Dependent Frequency: Radiated Power

- Define

$$k_{\pm}^{(1)} = \omega \pm \left[\Omega_0 + a(t - r)\right] \tag{58}$$

$$k_{\pm}^{(2)} = \omega \pm \left[\Omega_0 + 2a(t - r)\right] \tag{59}$$

- Assume late time $t - r \gg R$

$$
\begin{aligned}
P(t) =\Bigg[&\left(\frac{C\beta}{\pi f_a}\right)^2 \left(\frac{\phi_0^2 B_0^2 \omega^2 R^4 \pi^5}{24(k_-^{(2)})^2}\right) \mathrm{sech}\left[\frac{\pi R k_-^{(2)}}{2}\right] \\
&\times \Bigg\{ a\pi R k_-^{(2)} \mathrm{sech}^2\left[\frac{\pi R k_-^{(2)}}{2}\right] \sin\left[(t-r)k_-^{(1)}\right] \\
&\quad - (k_-^{(2)})^2 \tanh\left[\frac{\pi R k_-^{(2)}}{2}\right] \cos\left[(t-r)k_-^{(1)}\right] \Bigg\} \\
&\quad + 2a \tanh\left[\frac{\pi R k_-^{(2)}}{2}\right] \sin\left[(t-r)k_-^{(1)}\right] \\
&\quad - a\pi R k_-^{(2)} \tanh^2\left[\frac{\pi R k_-^{(2)}}{2}\right] \sin\left[(t-r)k_-^{(1)}\right] \Bigg\} \\
&\quad + (a \leftrightarrow -a, \Omega_0 \leftrightarrow -\Omega_0) \Bigg]^2 \qquad (60)
\end{aligned}
$$