A basis approach of heavy quarkonia wavefunctions on light-front

Guangyao Chen

Jacksonville University

In collaboration with:
James Vary, Meijian Li, Yang Li, and Tuomas Lappi

Outlines

- Heavy quarkonium in Basis Light Front Quantization
- Heavy quarkonium in small-basis approach

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- ❑Heavy quarkonium in Basis Light Front Quantization
- ❑Heavy quarkonium in small-basis approach

Basis Light-front Quantization

- Finding spectrum using light-front Hamiltonian

$$H_{LF}|\psi_h\rangle = M_h^2|\psi_h\rangle, \quad (H_{LF} \equiv P^+\hat{P}^-_{LF} - \vec{P}^2_\perp)$$

- Adopting basis according to the symmetry of system
- Advantages:
  - Boost Invariant Amplitude
  - Parton Interpretation
  - Fully relativistic
  - Moore's Law

General Procedures of BLFQ

- ❑Derive LF-Hamiltonian from Lagrangian
- ❑Construct basis states $|\alpha\rangle$, and truncation scheme
- ❑Evaluate Hamiltonian in the basis
- ❑Diagonalize Hamiltonian and obtain its eigen states and their LF-amplitudes
- ❑Evaluate observables using LF-amplitudes
- ❑Extrapolate to continuum limit

Vary et al ‘10, Honkanen et al ‘11

Heavy Quarkonium in BLFQ

❑Effective Hamiltonian

$$H_{\text{eff}} = \underbrace{\frac{\vec{k}_\perp^2 + m_q^2}{z(1-z)}}_{\text{LF kinetic energy}} + \underbrace{\kappa^4 \zeta_\perp^2 - \frac{\kappa^4}{4m_q^2}\partial_z\big[z(1-z)\partial_z\big]}_{\text{confinement}} - \underbrace{\frac{C_F 4\pi\alpha_s}{Q^2}\bar{u}_{s'}(k')\gamma_\mu u_s(k)\bar{v}_{\bar{s}}(\bar{k})\gamma^\mu v_{\bar{s}'}(\bar{k}')}_{\text{one-gluon exchange}}$$

- Inspired by holographic AdS/QCD.

Teramond and Brodsky, ‘09

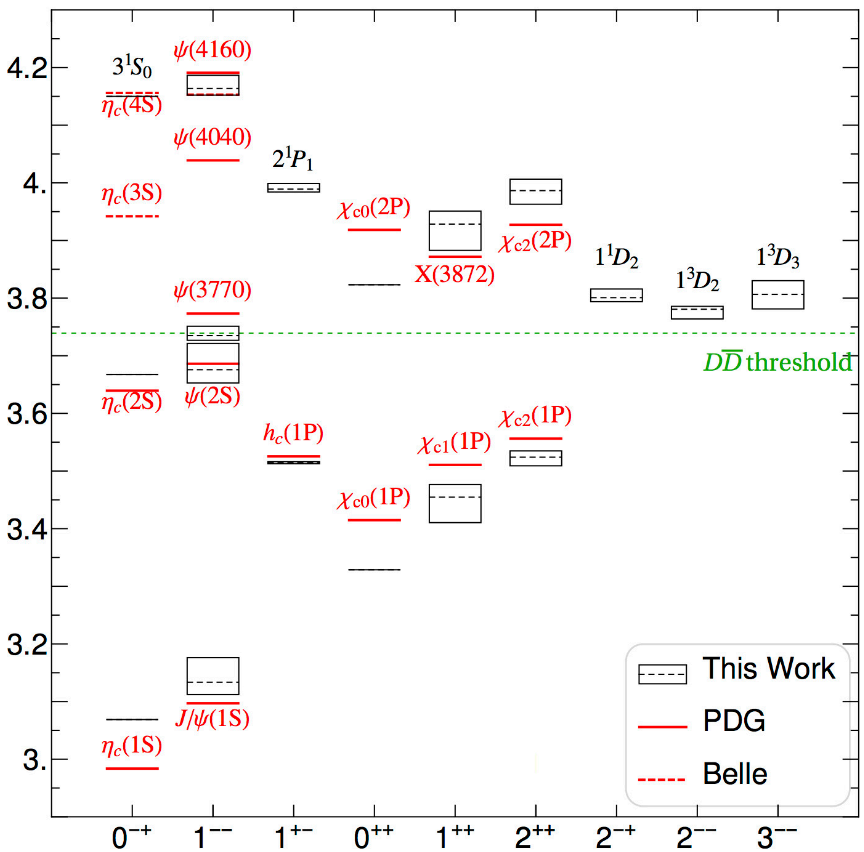
- Two parameters fitted to spectra.

Li et al., PLB 758, 118 (2016)

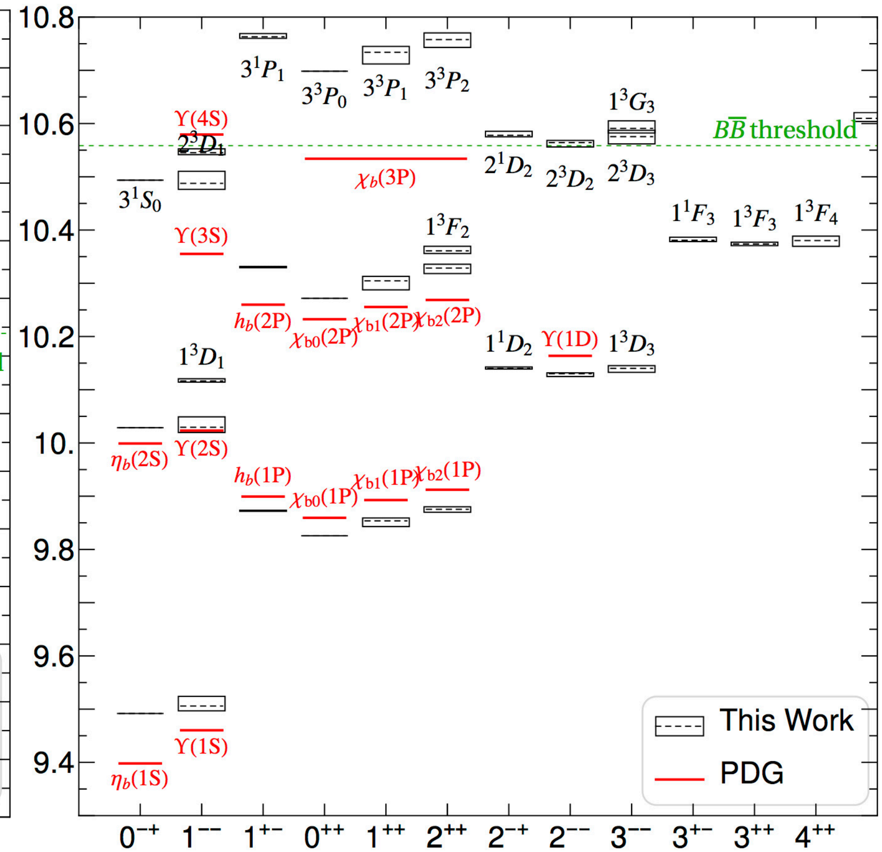
Li et al., PRD96, 016022, (2017)

Heavy Quarkonia Spectra

Li et al., PLB 758, 118 (2016)



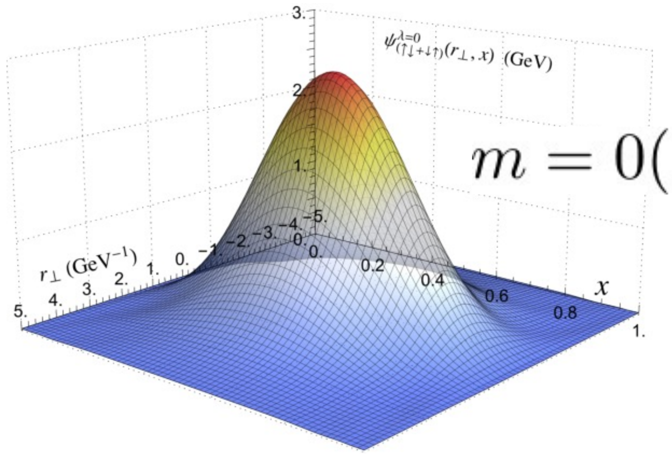
$$\delta\overline{M} = 52\ \mathrm{MeV}$$



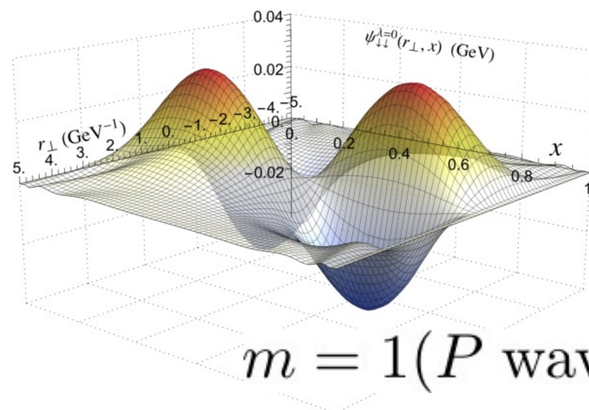
$$\delta\overline{M} = 50\ \mathrm{MeV}$$

Visualizing LFWF

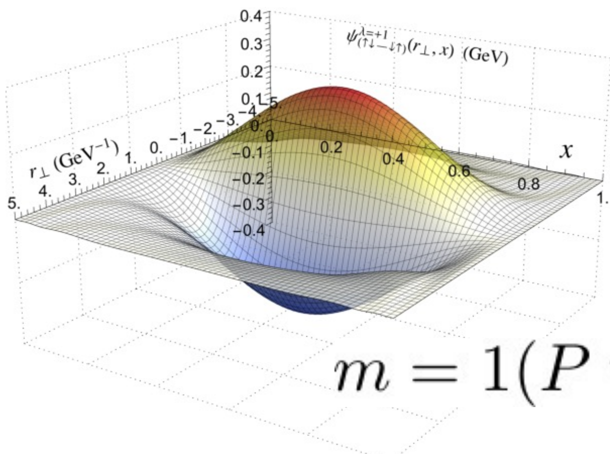
Li et al., PRD96, 016022, (2017)



$m = 0(S \text{ wave})$

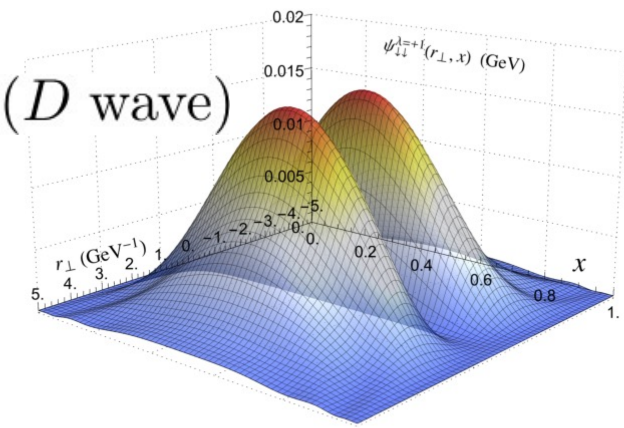


$m = 1(P \text{ wave})$



$m = 1(P \text{ wave})$

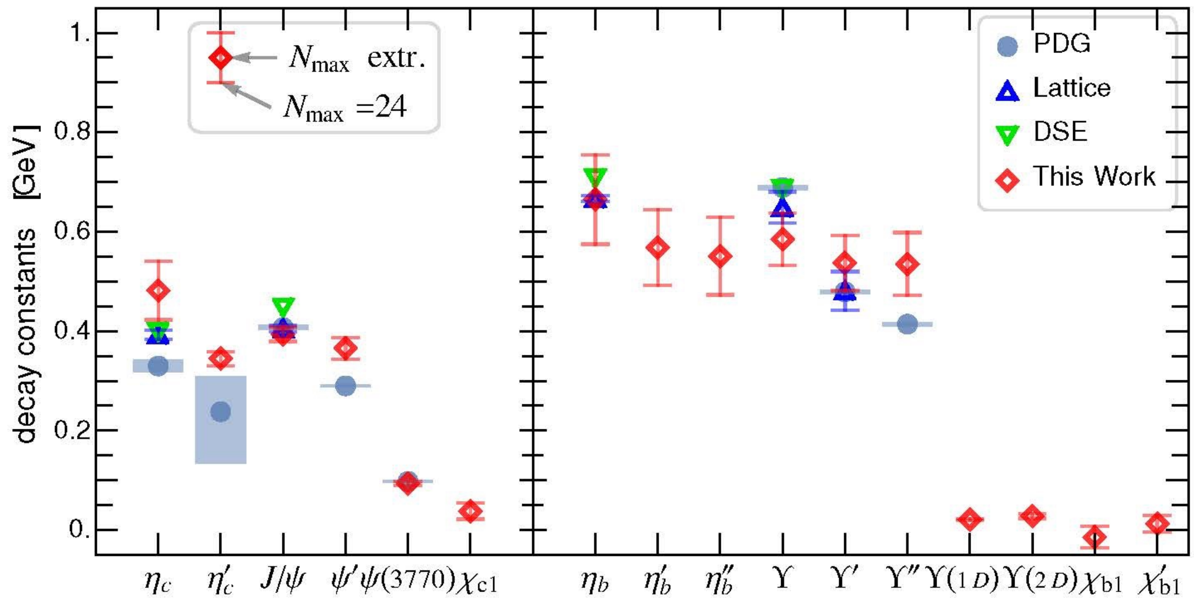
$m = 2(D \text{ wave})$



BLFQ LFWF predictions

Li et al., PLB 758, 118, (2016)

- Decay constants



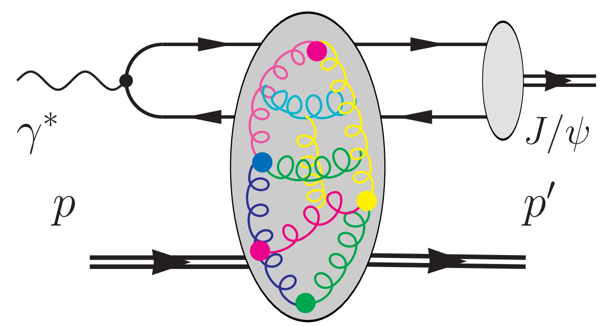
- Also predict radii and charge form factor!

Dipole picture of diffractive processes

- The exclusive VM production amplitude:

$$\mathcal{A}_{T,L}^{\gamma^* p \to Ep}(x, Q, \Delta) = \mathrm{i} \int d^2\vec{r} \int_0^1 \frac{dz}{4\pi} \int d^2\vec{b} \, (\Psi_E{}^* \Psi)_{T,L} \times \mathrm{e}^{-\mathrm{i}[\vec{b}-(1-z)\vec{r}]\cdot\vec{\Delta}} \, \frac{d\sigma_{q\bar{q}}}{d^2\vec{b}}$$

- Ψ_E : LFWF of vector meson
- Ψ : Photon LFWF
- $\frac{d\sigma_{q\bar{q}}}{d^2\vec{b}}$: dipole cross section



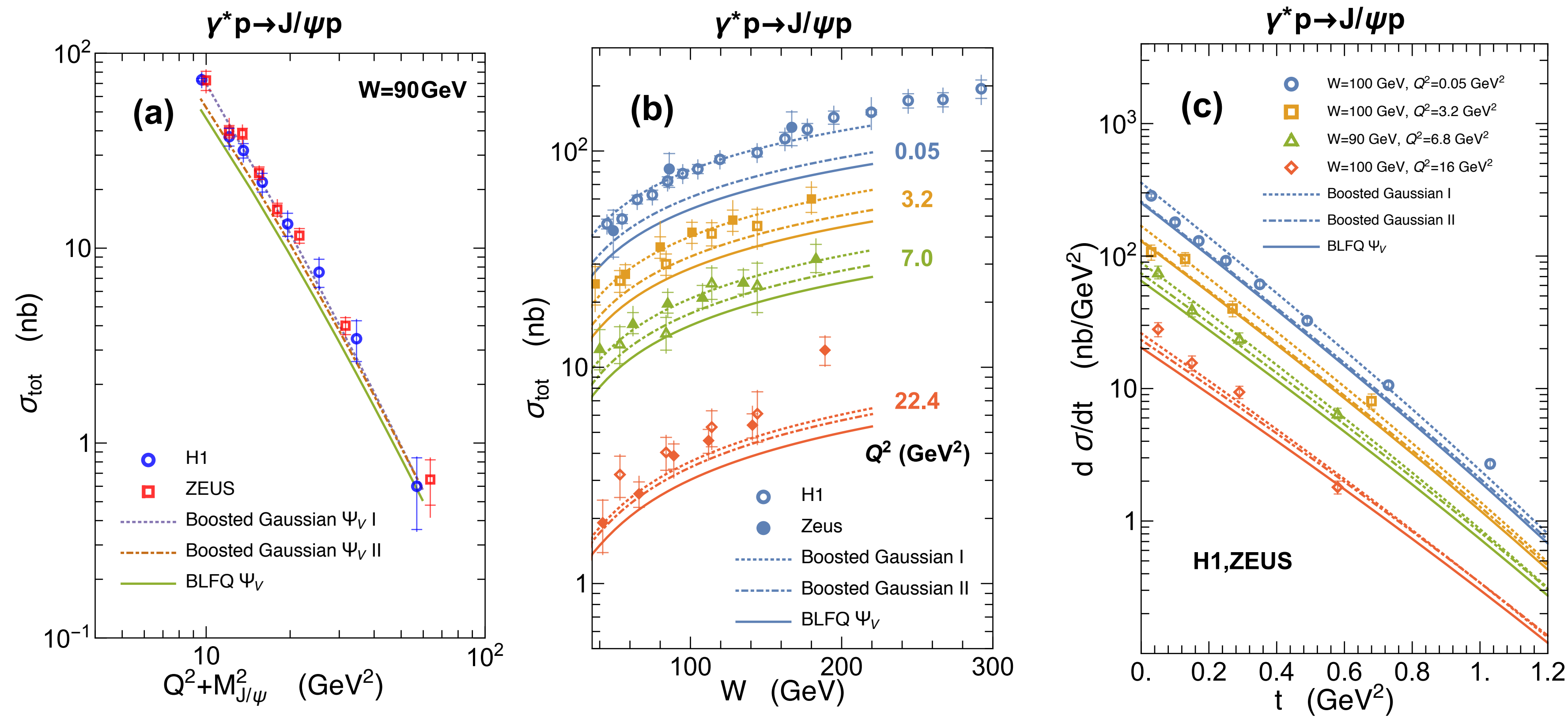
- Description of vector meson on the Light-front is the KEY!

Golec-Biernat and Wusthoff , 1999
Kowalski and Teaney , 2001

HERA: cross section

GC et al., PLB 769, 477, 2017

ZEUS, 2004.
H1, 2006.



J/Ψ from Pb-Pb UPC at LHC

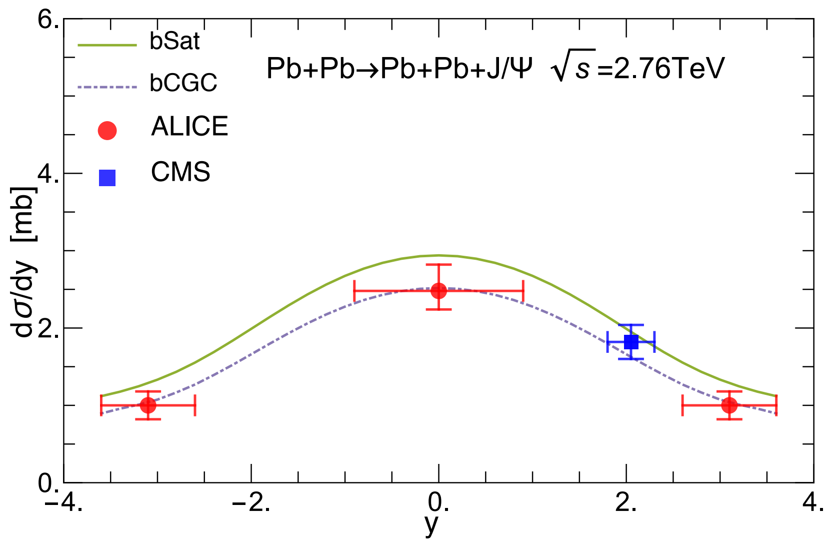
GC et al., PLB 769, 477, 2017

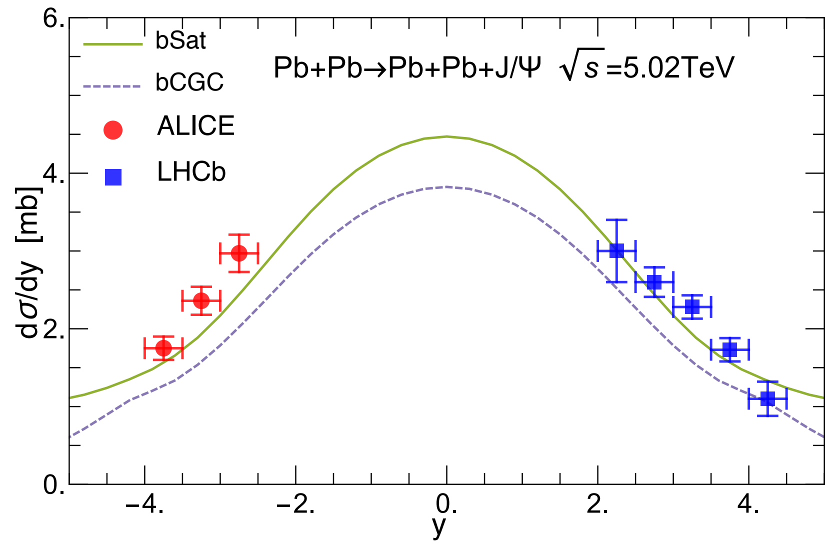
GC et al., PRC 100, 025208, 2019

ALICE, 2013, 2017.

CMS, 2016.

LHCb, 2018

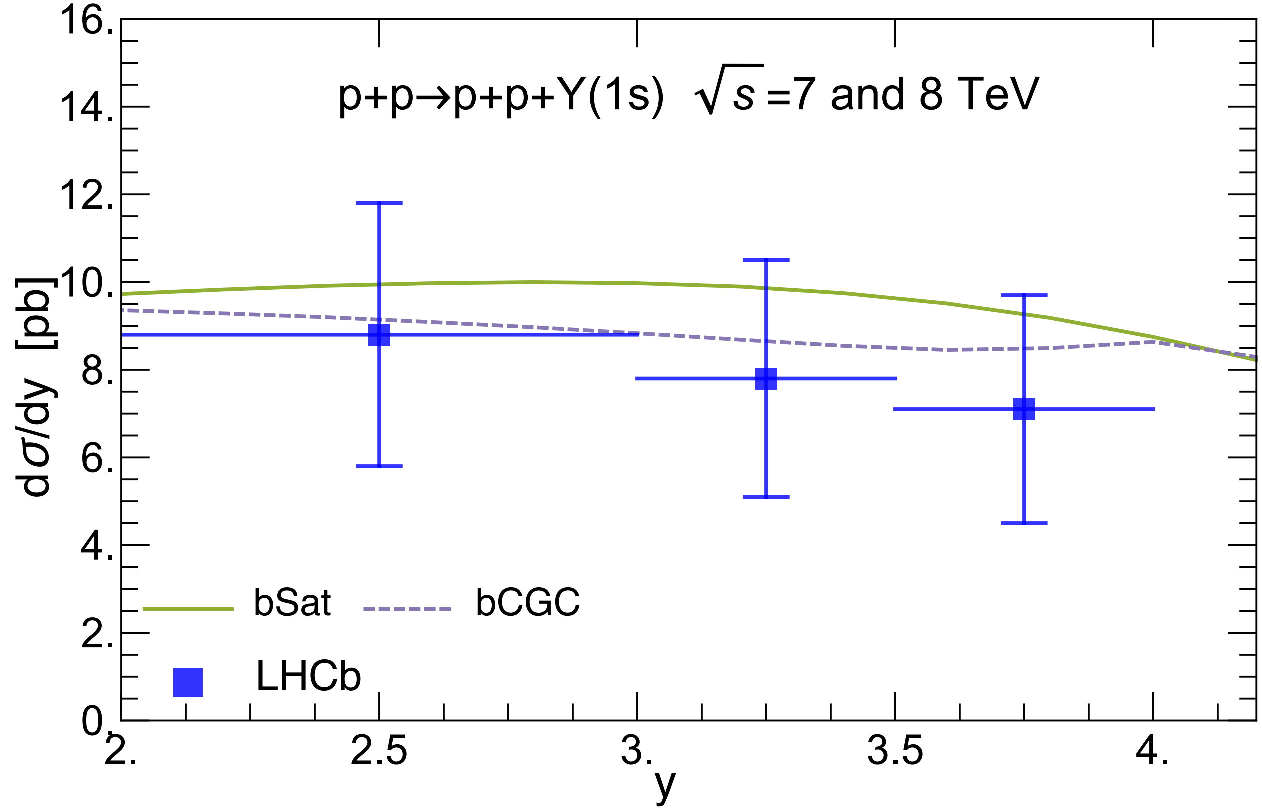




$\Upsilon(1s)$ from pp UPC at LHC

GC et al., PRC 100, 025208, 2019

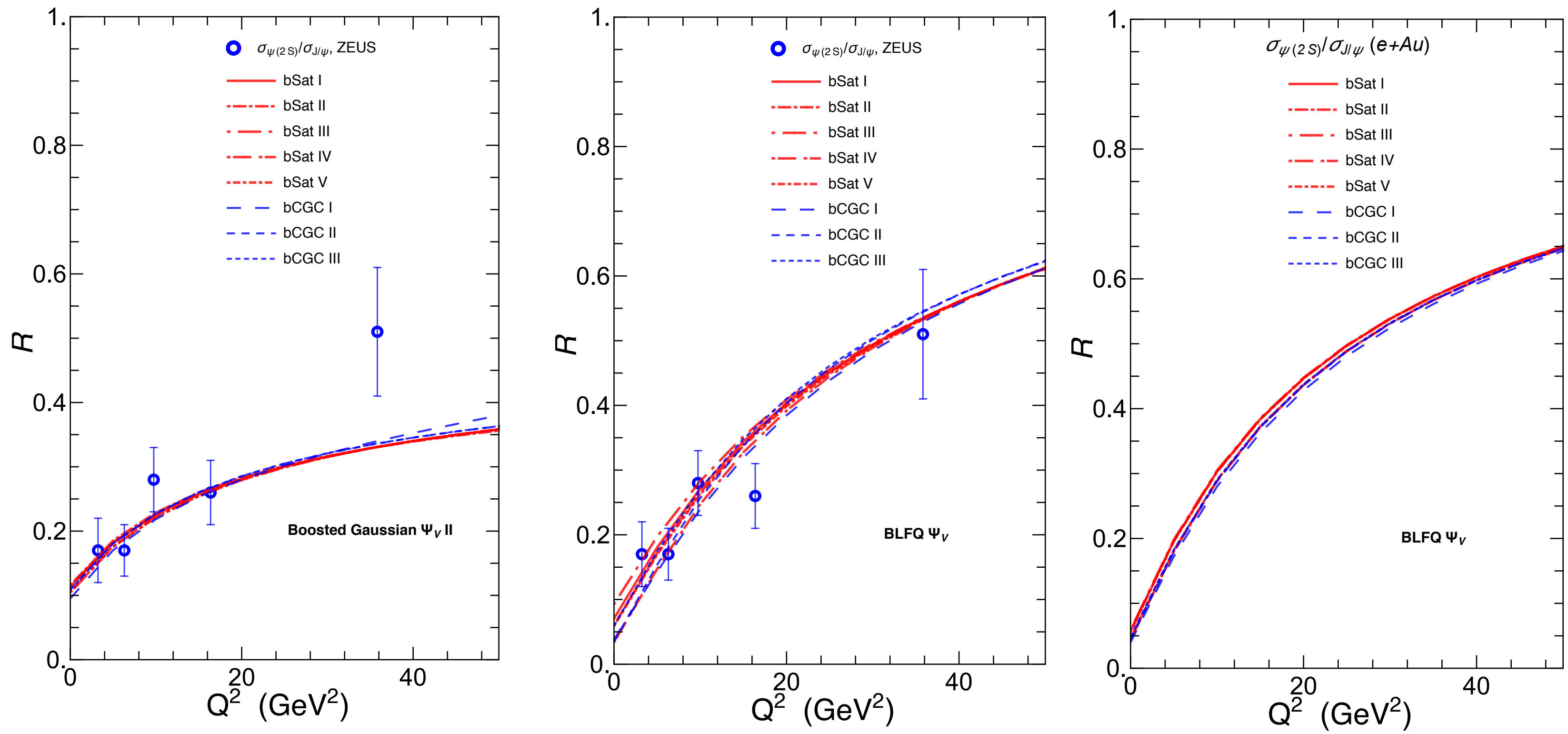
LHCb, 2015



Cross section ratio

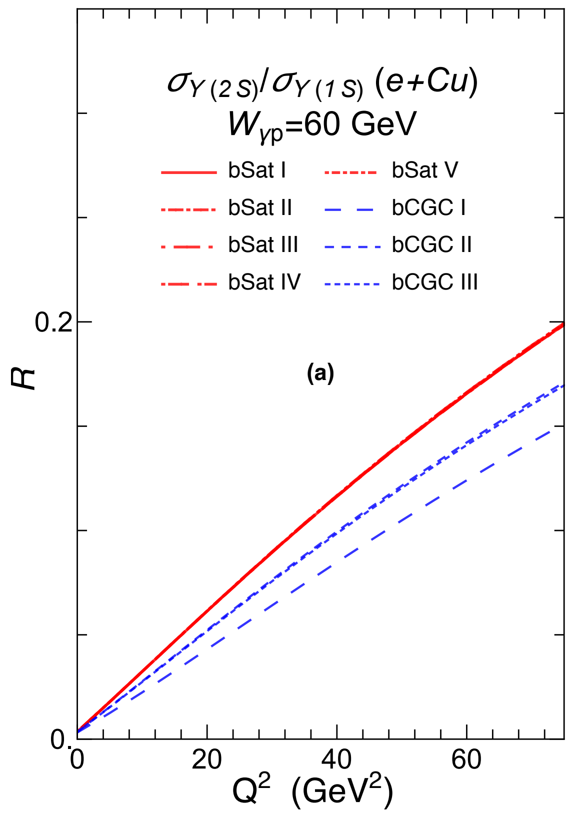
ZEUS, 2016.

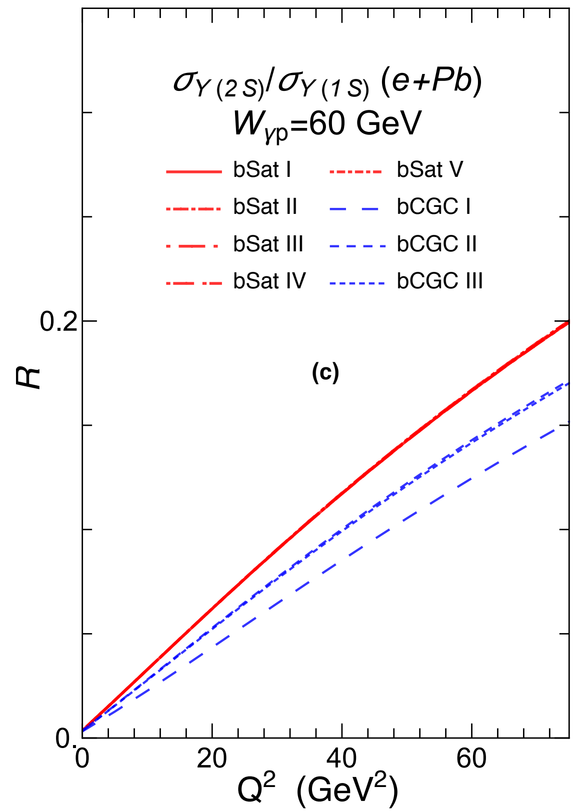
GC et al., PLB 769, 477, 2017



Cross section ratio, Upsilons

GC et al., PRC 100, 025208, 2019





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- ❑ Heavy quarkonium in Basis Light Front Quantization
- ❑ Heavy quarkonium in small-basis approach

Introduction

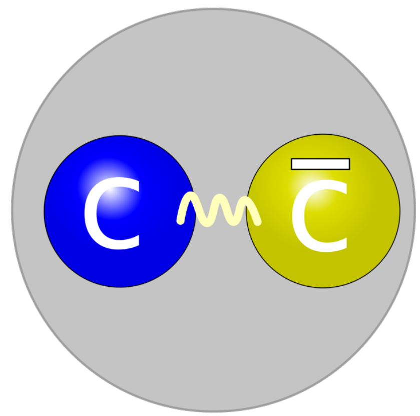
- Goals: simple-function charmonium LFWFs with few parameters!

i. Approximation to QCD.

ii. Retain more symetries.

iii. Matching the NR limit.

iv. Emphsis on decay width.



- We designed LFWFs for η_c, J/ψ, ψ' and $\psi(3770)$.

Basis functions

- LF holography/Basis LF Quantization Hamiltonian.

$$H_0 = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x) r_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x (x(1-x)\partial_x) \, ,$$

i. Two parameters: m_q and κ.

ii. One-gluon interactions were treated perturbatively.

- The basis function representation.

$$\tilde{\psi}^{(m_j)}_{ss'/h}(\vec{r}_\perp, x) = \sqrt{x(1-x)} \sum_{n,m,l} \psi_h^{(m_j)}(n, m, l, s, s') \tilde{\phi}_{nm}(\sqrt{x(1-x)}\vec{r}_\perp)\chi_l(x) \, ,$$

Teramond and Brodsky, '09

Li et al., PLB 758, 118 (2016)

Basis functions

- Small basis for charmonium states:

$$\psi_{\mathrm{LF}-1S} = \psi_{0,0,0} \, .$$

$$\psi_{\mathrm{LF}-1P0} = \psi_{0,0,1} \, ,$$

$$\psi_{\mathrm{LF}-1P\pm1} = -\psi_{0,\pm1,0} \, .$$

$$\psi_{\mathrm{LF}-2S} = \sqrt{\frac{2}{3}}\psi_{1,0,0} - \sqrt{\frac{1}{3}}\psi_{0,0,2} \, .$$

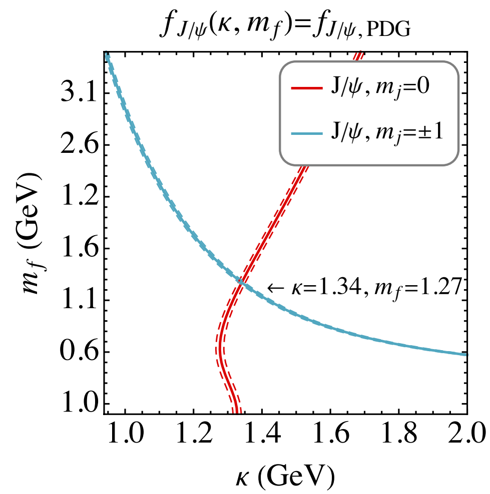
$$\psi_{\mathrm{LF}-1D0} = \sqrt{\frac{1}{3}}\psi_{1,0,0} + \sqrt{\frac{2}{3}}\psi_{0,0,2} \, ,$$

$$\psi_{\mathrm{LF}-1D\pm1} = -\psi_{0,\pm1,1} \, ,$$

$$\psi_{\mathrm{LF}-1D\pm2} = \psi_{0,\pm2,0} \, .$$

J/ψ as a 1^{--} state

- We assume a 100% LF-1S state for J/ψ.
- Matching J/ψ decay constant to the PDG value:



- We fix m_c and κ using the J/ψ decay constant.

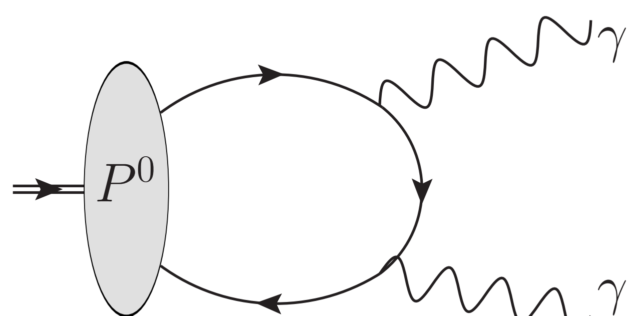
Li et al., arXiv: 2002.09757

η_c as a 0^{-+} state

- η_c predominantly LF-1S+LF-2S and LF-1P.

$$\psi_{\eta_c} = C_{\eta_c,1S}\,\psi_{\mathrm{LF-1}S,0-+} + C_{\eta_c,2S}\,\psi_{\mathrm{LF-2}S,0-+} + C_{\eta_c,1P}\,\psi_{\mathrm{LF-1}P,0-+}\ .$$

- Basis coeffecients are determined using the diphoton decay width $\Gamma(\eta_c \to \gamma\gamma)$.

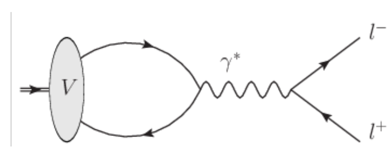


ψ' as a 1^{--} state

- A mix of LF-1S and LF-2S states for ψ'.

$$\psi_{\psi'}^{(m_j=0)} = C_{\psi',1S}^{(m_j=0)} \psi_{\mathrm{LF-1}S,1--}^{(m_j=0)} + C_{\psi',2S}^{(m_j=0)} \psi_{\mathrm{LF-2}S,1--}^{(m_j=0)} ,$$
$$\psi_{\psi'}^{(m_j=1)} = C_{\psi',1S}^{(m_j=1)} \psi_{\mathrm{LF-1}S,1--}^{(m_j=1)} + C_{\psi',2S}^{(m_j=1)} \psi_{\mathrm{LF-2}S,1--}^{(m_j=1)} ,$$
$$\psi_{\psi'}^{(m_j=-1)} = C_{\psi',1S}^{(m_j=-1)} \psi_{\mathrm{LF-1}S,1--}^{(m_j=-1)} + C_{\psi',2S}^{(m_j=-1)} \psi_{\mathrm{LF-2}S,1--}^{(m_j=-1)} .$$

- Basis coeffecients are determined using the dilepton decay constant.



$$\left|f_{\mathcal{V}}|_{m_j=0}\right| = \left|f_{\mathcal{V}}|_{m_j=\pm 1}\right| = f_{\mathcal{V},\text{experiment}} \ .$$

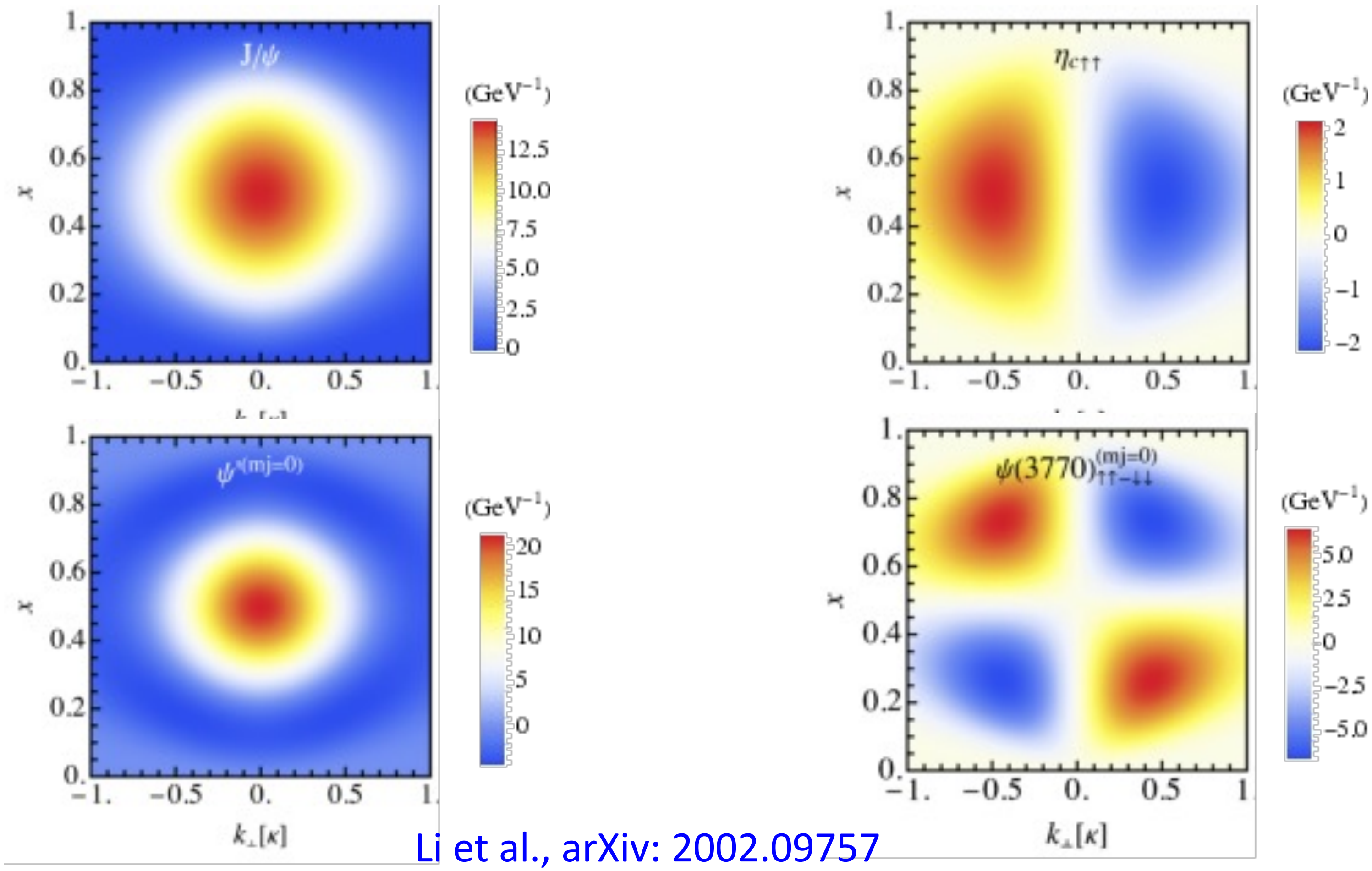
$\psi(3770)$ as a 1^{--} state

- A mix of LF-1S, LF-2S, LF-1D states for $\psi(3770)$, LF-1D is dominating.

$$\begin{aligned}\psi_{\psi(3770)}^{(m_j=0)} =& C_{\psi(3770),1S}^{(m_j=0)} \psi_{\mathrm{LF}-1S,1--}^{(m_j=0)} \\ &+ C_{\psi(3770),2S}^{(m_j=0)} \psi_{\mathrm{LF}-2S,1--}^{(m_j=0)} \\ &+ C_{\psi(3770),1D}^{(m_j=0)} \psi_{\mathrm{LF}-1D,1--}^{(m_j=0)} ,\end{aligned}$$

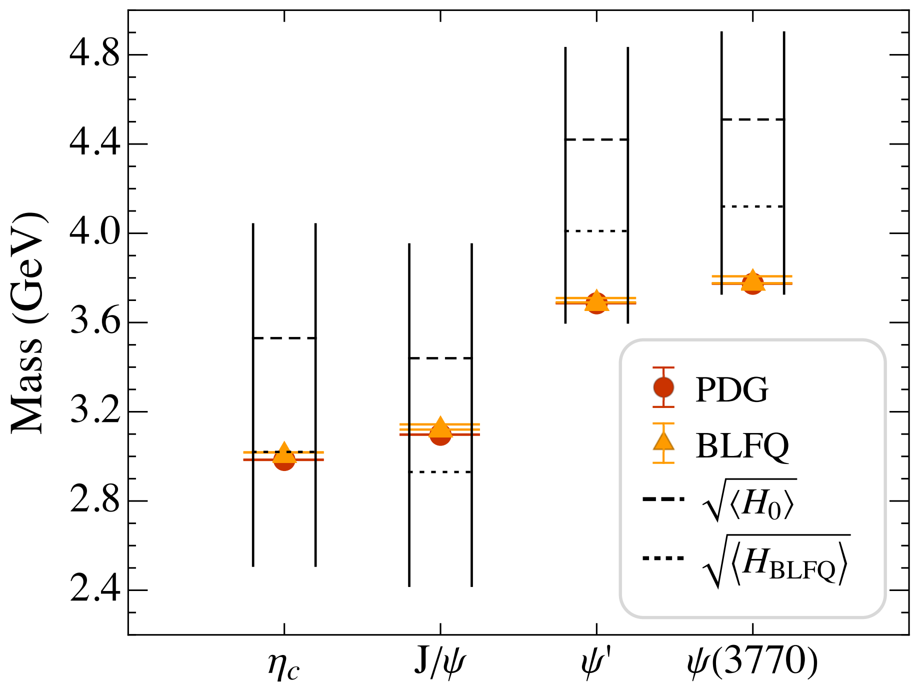
- Basis coeffecients are determined by requiring orthogonality between ψ' and $\psi(3770)$.

LFWFs by design



Li et al., arXiv: 2002.09757

The mass spectrum



$$
\begin{aligned}
\left(\tilde{M}_h^{(m_j)}\right)^2 = \sum_{n,m,l,s,\bar{s}} \sum_{n',m',l',s',\bar{s}'} & \psi_h^{(m_j)}(n,m,l,s,\bar{s}) \\
& \times \psi_h^{(m_j)*}(n',m',l',s',\bar{s}') \\
& \times \Big[M^2_{n,m,l} \delta_{n,n'} \delta_{m,m'} \delta_{l,l'} \delta_{s,s'} \delta_{\bar{s},\bar{s}'} \\
& + \langle \beta_{n',m',l',s',\bar{s}'} | \Delta H | \beta_{n,m,l,s,\bar{s}} \rangle \Big] .
\end{aligned}
$$

$$
V_{\mathrm{OGE}} = -\frac{C_F 4\pi \alpha_s(q^2)}{q^2} \bar{u}_{s'}(k') \gamma_\mu u_s(k) \bar{v}_{\bar{s}}(\bar{k}) \gamma^\mu v_{\bar{s}'}(\bar{k}') ,
$$

Masses calculated from small-basis LFWFs should be regarded as Estimated!

Li et al., arXiv: 2002.09757

Li et al., PLB 758, 118 (2016)

The charge radii

- Defined in terms of the slope of the charge form factor at zero momentum transfer.

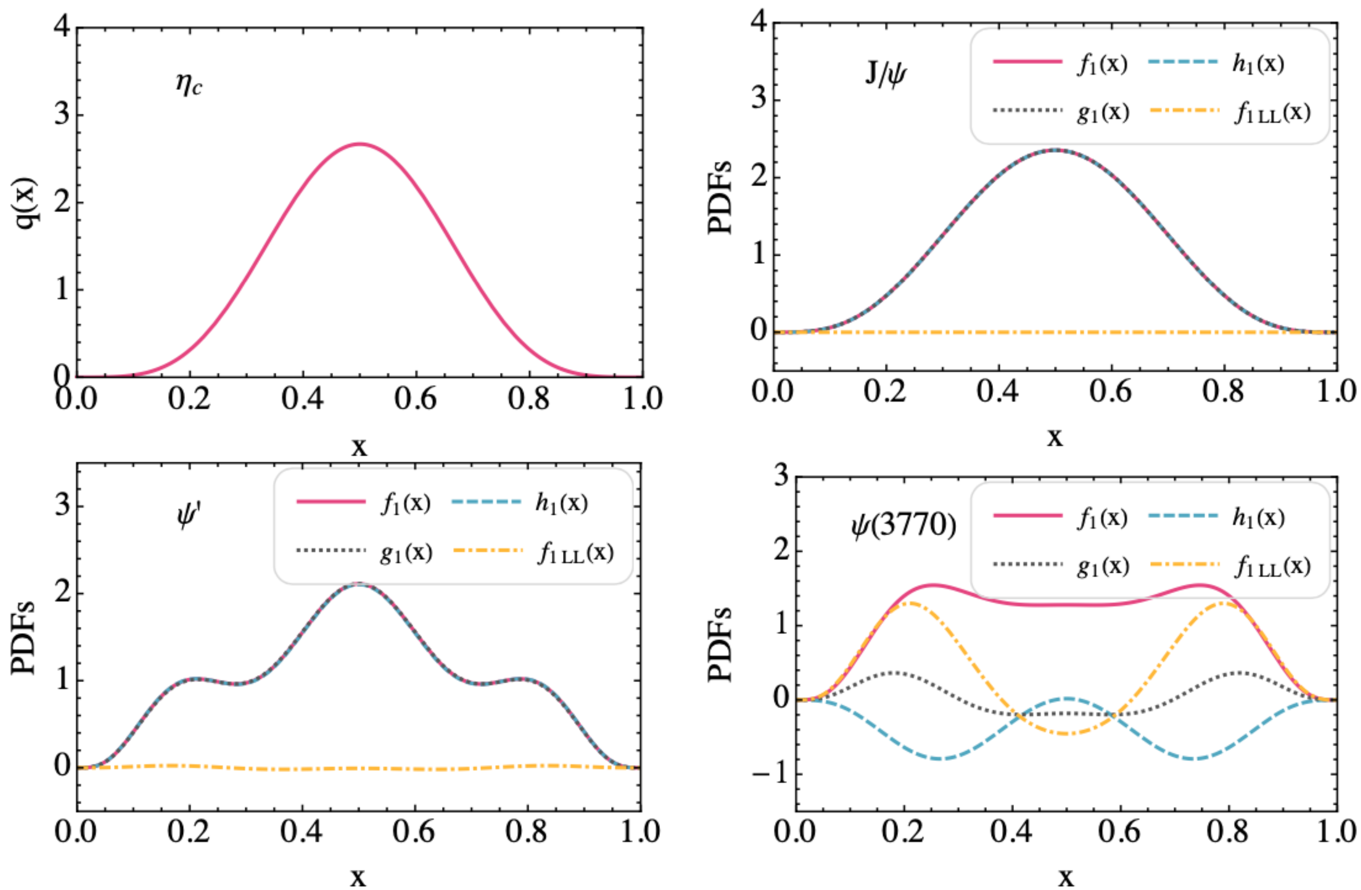
$$\langle r_h^2 \rangle = -6 \frac{\partial}{\partial Q^2} G_0(Q^2)\Big|_{Q\to 0} .$$

(fm^2)	$\langle r_{\eta_c}^2 \rangle$	$\langle r_{J/\psi}^2 \rangle$	$\langle r_{\psi'}^2 \rangle$	$\langle r_{\psi(3770)}^2 \rangle$
this work	0.098	0.046	0.154	0.138
BLFQ [27]	0.029(1)	0.0402(2)	0.13(0)	0.13(0)

- J/ψ, ψ' and $\psi(3770)$ radii consistent with BLFQ calculations.
- A large size η_c!

Li et al., arXiv: 2002.09757
Li et al., PLB 758, 118 (2016)

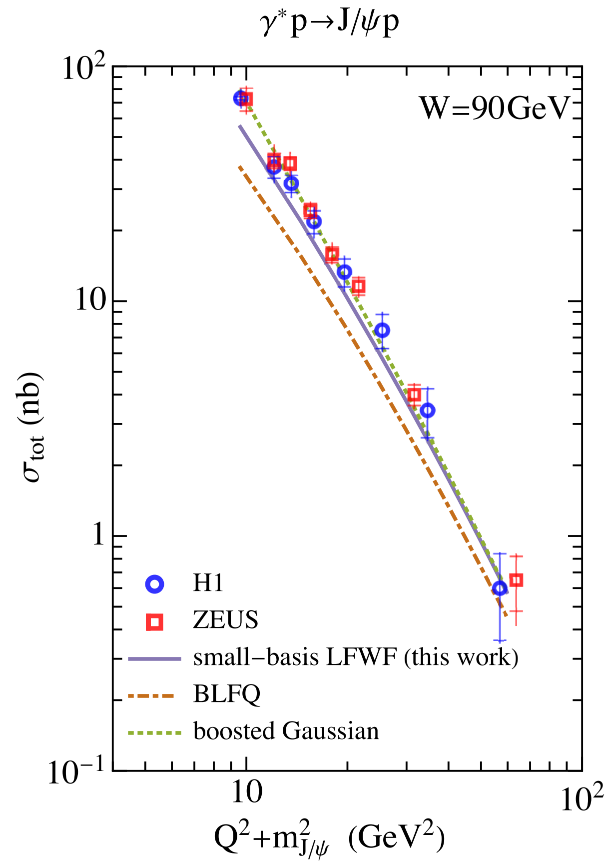
Parton Distribution Functions (PDFs)

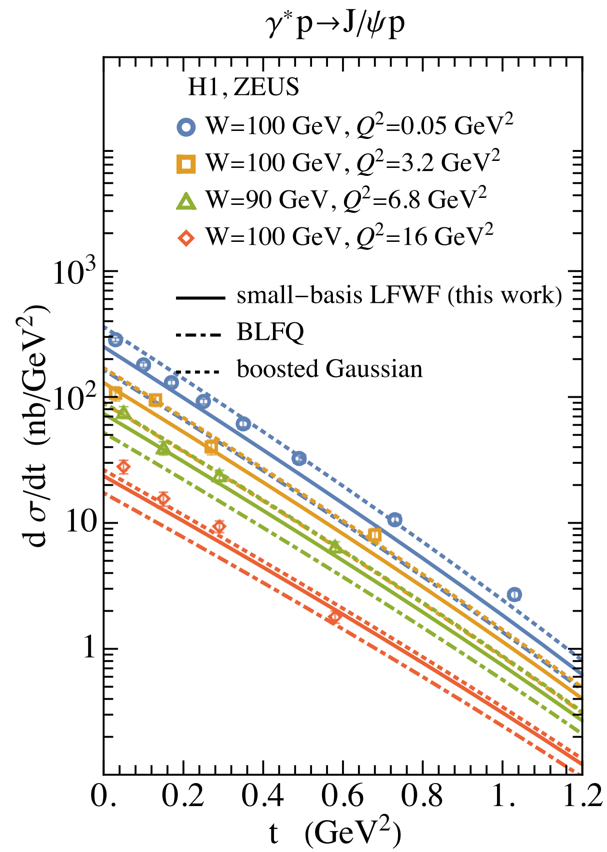


Li et al., arXiv: 2002.09757

J/ψ production at HERA

ZEUS, 2004.
H1, 2006.



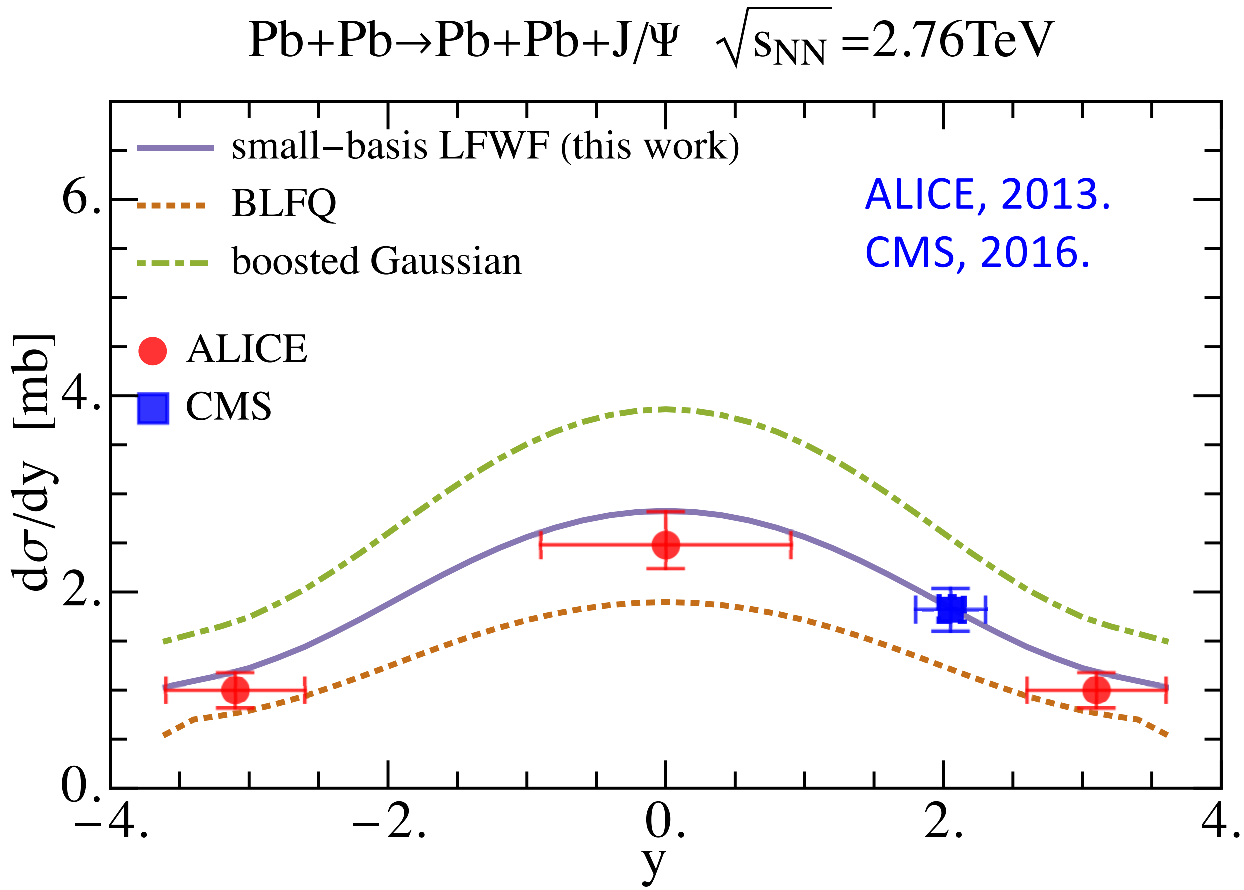


GC et al., PLB 769, 477, 2017

GC et al., PRC 100, 025208, 2019

Li et al., arXiv: 2002.09757

J/ψ production at LHC

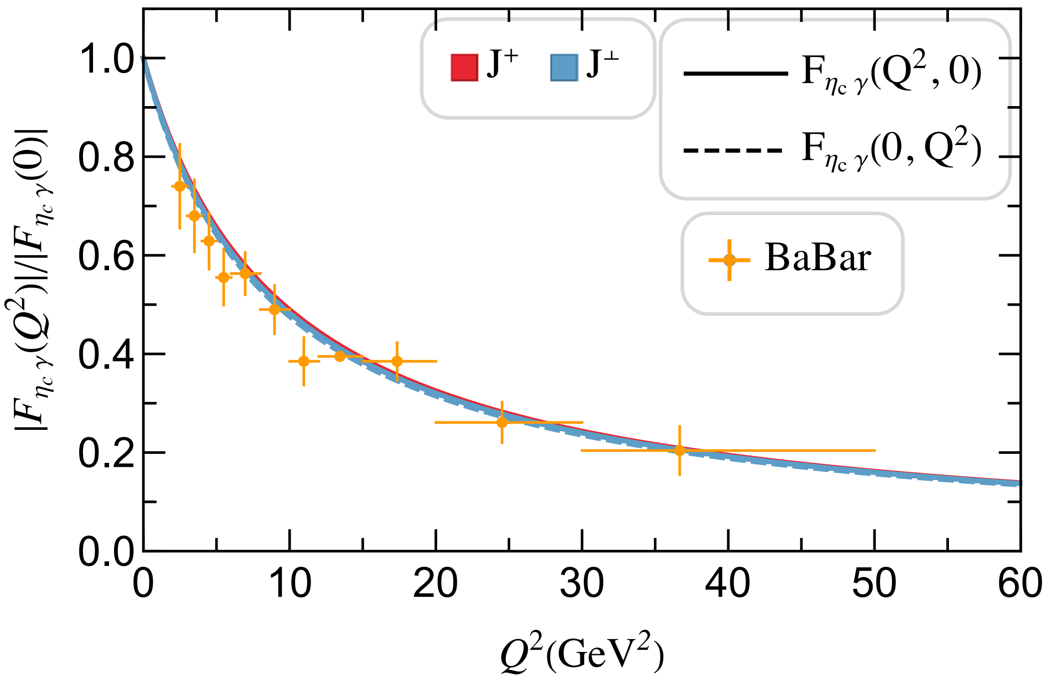


GC et al., PLB 769, 477, 2017

GC et al., PRC 100, 025208, 2019

Li et al., arXiv: 2002.09757

$\gamma^*\gamma \to \eta_c$ Transition Form Factor



$$I^{\mu}_{\lambda_1}(P,q_1) \equiv \langle \gamma^*(q_1,\lambda_1)|\, J^{\mu}(0)\, |\mathcal{P}(P)\rangle$$
$$= -ie^2 F_{\mathcal{P}\gamma}(Q_1^2,Q_2^2)\epsilon^{\mu\alpha\beta\sigma}P_\alpha q_{1\beta}\epsilon^*_{\sigma,\lambda_1}(q_1)\ ,$$

BaBar, 2010.

Li et al., arXiv: 2002.09757

Summary

- η_c , J/ψ, ψ' and ψ(3770) LFWFs in two approaches: BLFQ (HPC), small-basis (analytical).
- Physical observables calculated:

i. Masses and charge radii.

ii. PDFs.

iii. J/ψ production at HERA and LHC.

iv. η_c diphoton transition form factor.

- Outlook: analytical LFWFs with simultaneous global analysis.

Thank you!

- Collaborators: Meijian Li, Yang Li, Tuomas Lappi, James Vary
- This work is supported by Department of Energy, USA, and European Research Council

Backup Slides

Basis Function

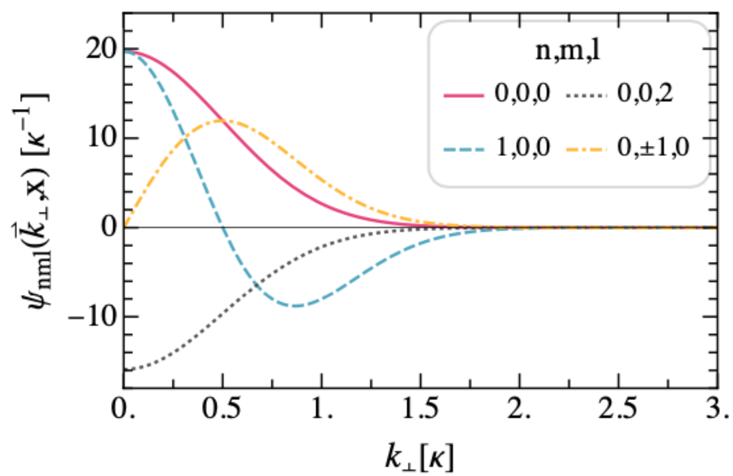
- Transverse:

$$\phi_{nm}(\vec{k}_\perp) = \kappa^{-1}\sqrt{\frac{4\pi n!}{(n+|m|)!}}\left(\frac{k_\perp}{\kappa}\right)^{|m|}\exp(-k_\perp^2/(2\kappa^2))$$
$$L_n^{|m|}(k_\perp{}^2/\kappa^2)\exp(im\theta_k)\ ,$$

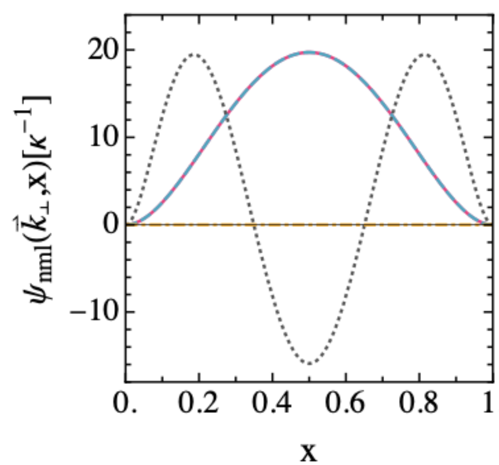
- Longitudinal:

$$\chi_l(x) = \sqrt{4\pi(2l+\alpha+\beta+1)}\sqrt{\frac{\Gamma(l+1)\Gamma(l+\alpha+\beta+1)}{\Gamma(l+\alpha+1)\Gamma(l+\beta+1)}}$$
$$x^{\beta/2}(1-x)^{\alpha/2}P_l^{(\alpha,\beta)}(2x-1)\ ,$$

Sample Basis Function



(a) $\psi_{nml}(k_\perp, \theta_k = 0, x = 0.5)$



(b) $\psi_{nml}(\vec{k}_\perp = \vec{0}_\perp, x)$

Mirror Parity and Chargeconjugation

TABLE I. Basis states $\psi_{nml}\sigma_{s\bar{s}}$ categorized by the mirror parity m_{P} according to Eq. (17).

m_j	m	$m_{\mathsf{P}} = 1$	$m_{\mathsf{P}} = -1$
0	0	$\psi_{n,0,l}\sigma_+$	$\psi_{n,0,l}\sigma_-$
	± 1	$\frac{1}{\sqrt{2}}(\psi_{n,-1,l}\sigma_{\uparrow\uparrow} - \psi_{n,1,l}\sigma_{\downarrow\downarrow})$	$\frac{1}{\sqrt{2}}(\psi_{n,-1,l}\sigma_{\uparrow\uparrow} + \psi_{n,1,l}\sigma_{\downarrow\downarrow})$
$1, -1$	0	$\psi_{n,0,l}\sigma_{\uparrow\uparrow}, \psi_{n,0,l}\sigma_{\downarrow\downarrow}$	$\psi_{n,0,l}\sigma_{\uparrow\uparrow}, -\psi_{n,0,l}\sigma_{\downarrow\downarrow}$
	± 1	$\psi_{n,1,l}\sigma_\pm, \mp\psi_{n,-1,l}\sigma_\pm$	$\psi_{n,1,l}\sigma_\pm, \pm\psi_{n,-1,l}\sigma_\pm$
	± 2	$\psi_{n,2,l}\sigma_{\downarrow\downarrow}, \psi_{n,-2,l}\sigma_{\uparrow\uparrow}$	$\psi_{n,2,l}\sigma_{\downarrow\downarrow}, -\psi_{n,-2,l}\sigma_{\uparrow\uparrow}$

TABLE II. Basis states $\psi_{nml}\sigma_{s\bar{s}}$ categorized by the charge conjugation C according to Eq. (18).

$m + l$	$\mathsf{C} = 1$	$\mathsf{C} = -1$
even	$\psi_{n,m,l}\sigma_-$	$\psi_{n,m,l}\sigma_+, \psi_{n,m,l}\sigma_{\uparrow\uparrow}, \psi_{n,m,l}\sigma_{\downarrow\downarrow}$
odd	$\psi_{n,m,l}\sigma_+, \psi_{n,m,l}\sigma_{\uparrow\uparrow}, \psi_{n,m,l}\sigma_{\downarrow\downarrow}$	$\psi_{n,m,l}\sigma_-$

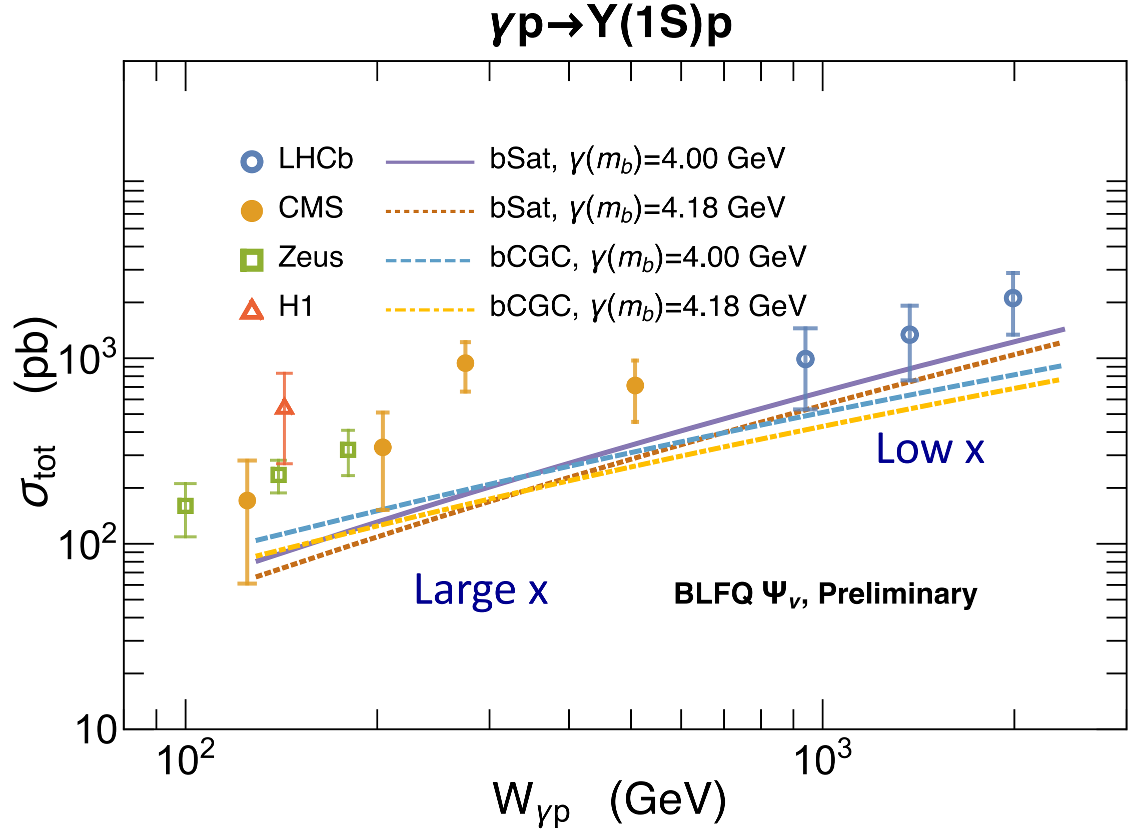
J/ψ Decay Constant

$$f_{\mathcal{V}}|_{m_j=0} = \sqrt{2N_c}\int_0^1 \frac{\mathrm{d}x}{\sqrt{x(1-x)}}\int\frac{\mathrm{d}^2k_\perp}{(2\pi)^3}$$
$$\psi_{+/\mathcal{V}}^{(m_j=0)}(\vec{k}_\perp, x)\ ,$$

$$f_{\mathcal{V}}|_{m_j=1} = \frac{\sqrt{N_c}}{2m_{\mathcal{V}}}\int_0^1 \frac{\mathrm{d}x}{[x(1-x)]^{3/2}}\int\frac{\mathrm{d}^2k_\perp}{(2\pi)^3}$$
$$\Big\{k^L[(1-2x)\psi_{+/\mathcal{V}}^{(m_j=1)}(\vec{k}_\perp, x) - \psi_{-/\mathcal{V}}^{(m_j=1)}(\vec{k}_\perp, x)]$$
$$- \sqrt{2}m_f\psi_{\uparrow\uparrow/\mathcal{V}}^{(m_j=1)}(\vec{k}_\perp, x)\Big\}\ ,$$

$\Upsilon(1s)$ in γp at LHC

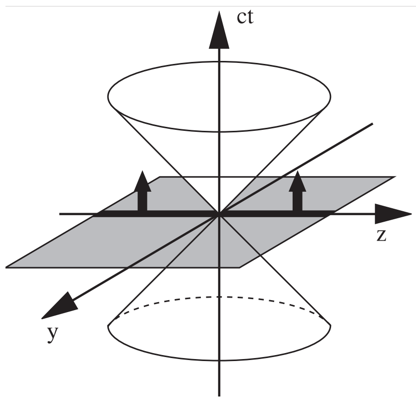
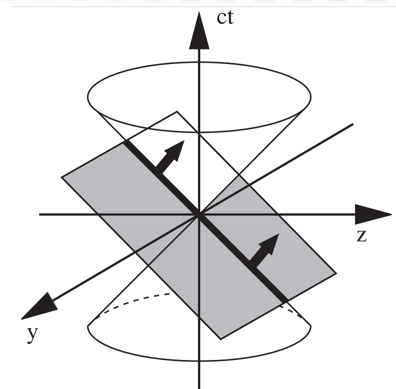
GC et al., in preparation



ZEUS, 2009.
H1, 2012.
CMS, 2016.
LHCb, 2016.

$$x \sim \frac{M_V^2}{W_{\gamma p}^2}$$

Equal time vs. Light-front Quantization

$t \equiv x^0$	$t \equiv x^+ = x^0 + x^3$
	 Dirac (1949)
$i\frac{\partial}{\partial t}\left\|\varphi(t)\right\rangle = H\left\|\varphi(t)\right\rangle$	$i\frac{\partial}{\partial x^+}\left\|\varphi(x^+)\right\rangle = \frac{1}{2}P^-\left\|\varphi(x^+)\right\rangle$
$H = P^0$	$P^- = P^0 - P^3$