Transverse-Momentum-Dependent Hadron Structures from Lattice QCD

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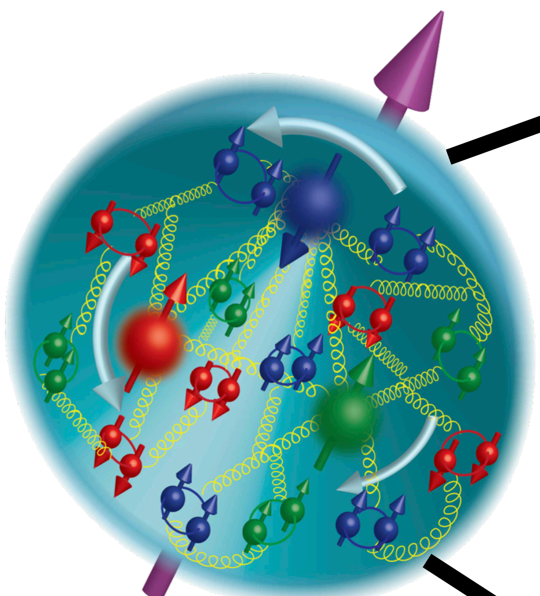
SEP 2, 2022

Argonne NATIONAL LABORATORY

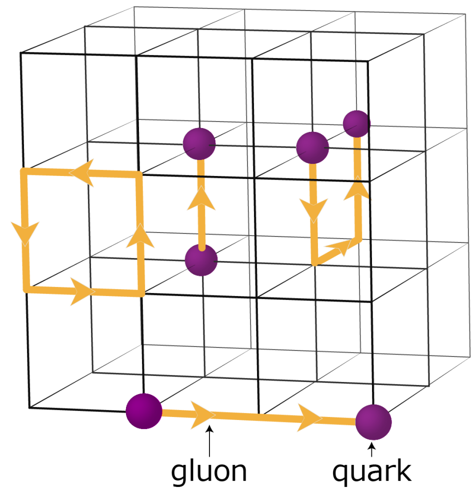
Outline

- **TMDs in the non-perturbative region**
- **Lattice calculations from quasi-TMDs**
- **Outlook**

3D Tomography of the Proton

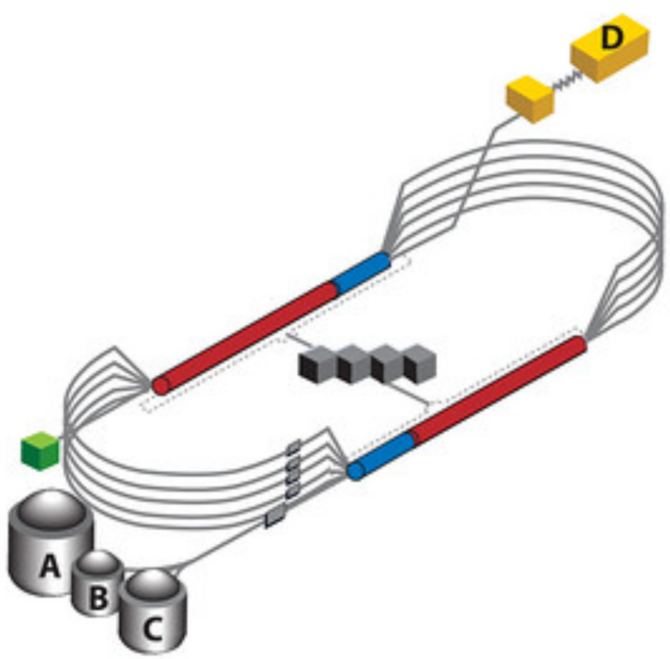


Lattice QCD

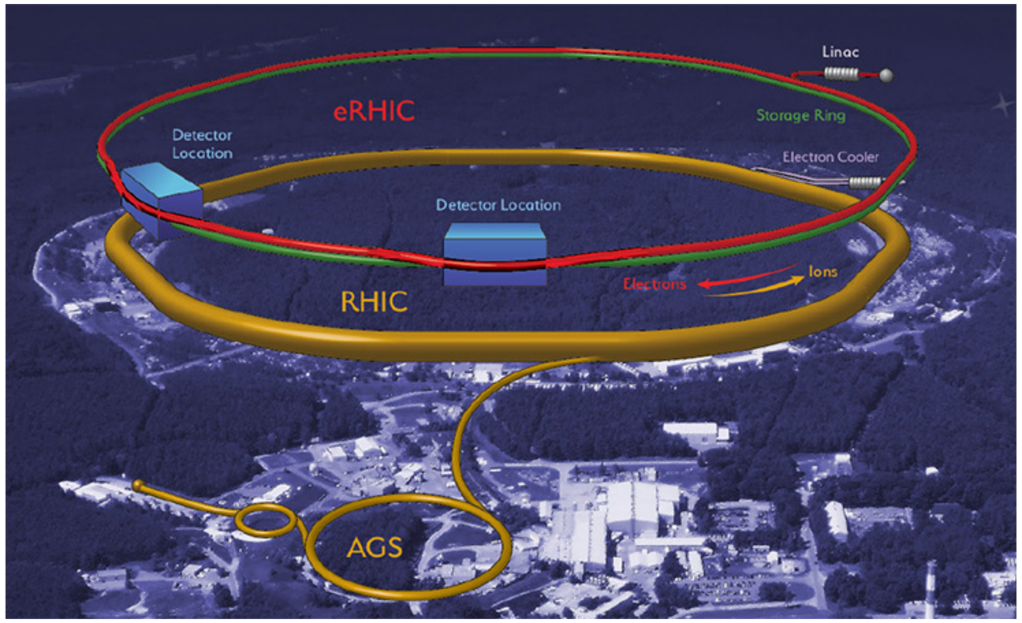


+

Hard Scattering

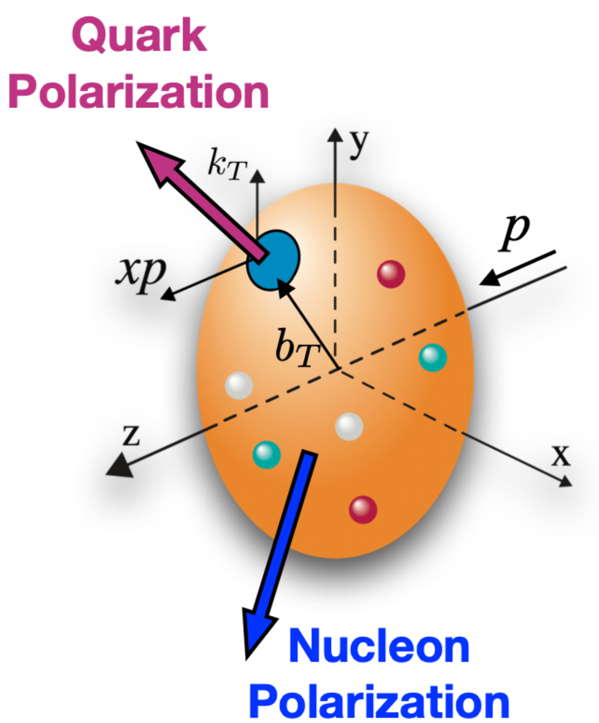


Jefferson Lab 12 GeV



The Electron-Ion Collider

3D Tomography of the Proton



Leading Quark TMDPDFs

Nucleon Spin   Quark Spin

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^\perp$ = Boer-Mulders
L		g_1 = Helicity	$h_{1L}^\perp$ = Worm-gear
T	$f_{1T}^\perp$ = Sivers	$g_{1T}^\perp$ = Worm-gear	h_1 = Transversity $h_{1T}^\perp$ = Pretzelosity

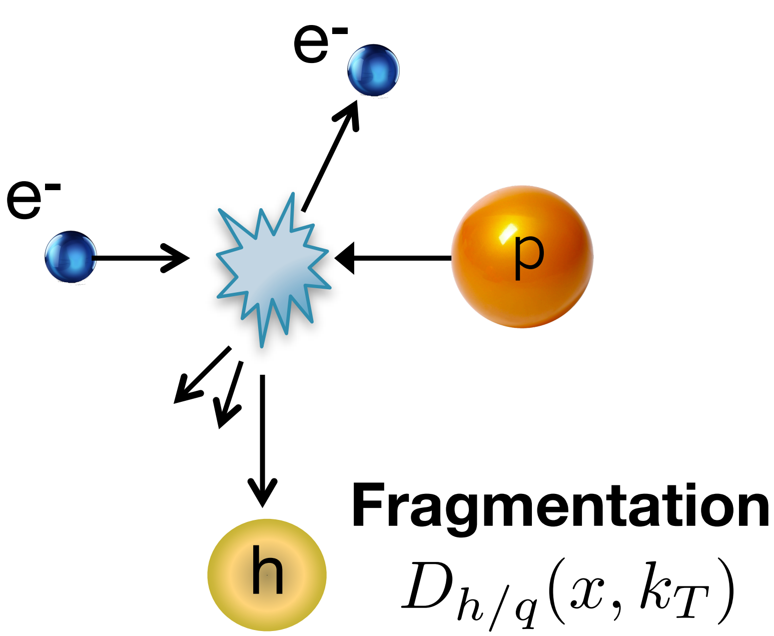
From TMD Handbook, TMD Topical Collaboration, to appear soon.

TMDs from experiments

TMD processes:

Semi-Inclusive DIS

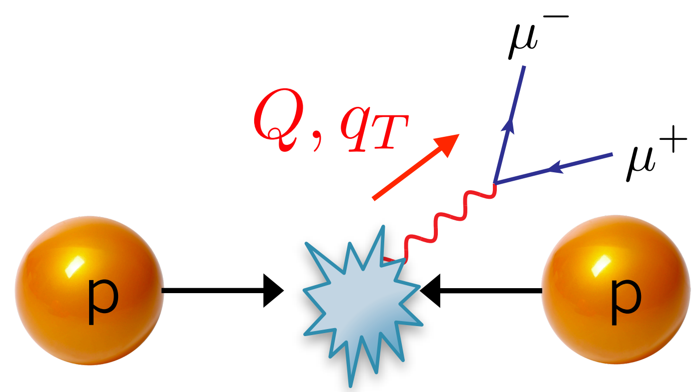
$$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(x, k_T)$$



HERMES, COMPASS, JLab, EIC, …

Drell-Yan

$$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$$

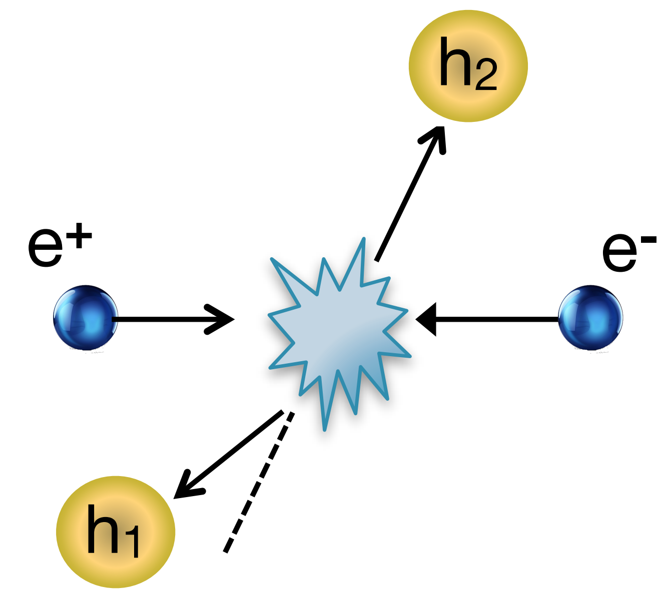


$$q_T \ll Q$$

Fermilab, RHIC, LHC, …

Dihadron in e^+e^-

$$\sigma \sim D_{h_1/q}(x, k_T) D_{h_2/q}(x, k_T)$$



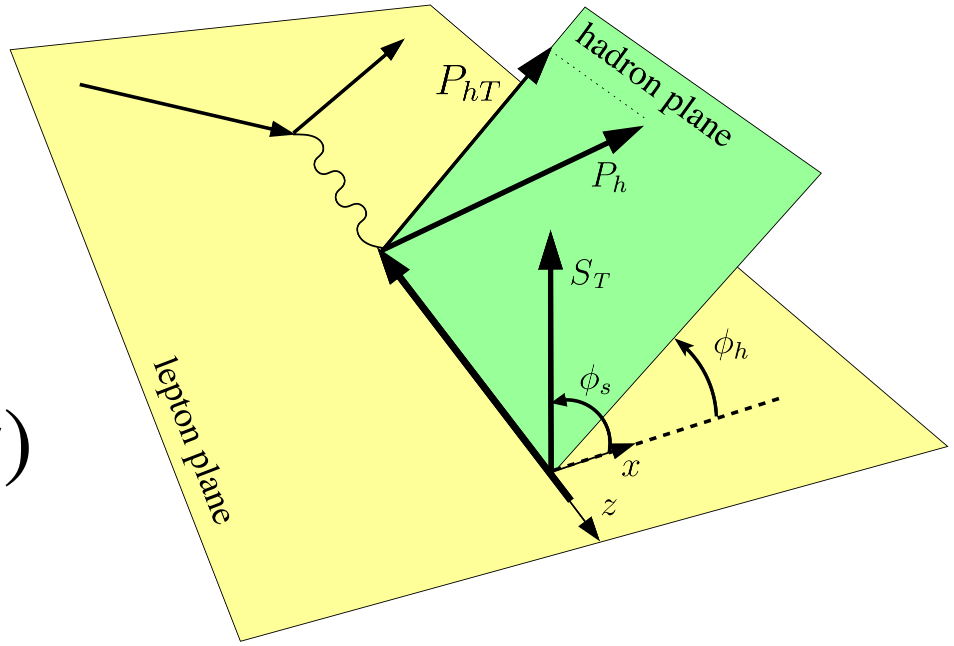
Babar, Belle, BESIII, …

TMDs from global analyses

Semi-inclusive deep inelastic scattering: $l + p \longrightarrow l + h(P_h) + X$

$$\frac{d\sigma^W}{dxdydz_h d^2\mathbf{P}_{hT}} \sim \int d^2\mathbf{b}_T \; e^{i\mathbf{b}_T\cdot\mathbf{P}_{hT}/z}$$

$$\times f_{i/p}(x, \mathbf{b}_T, Q, Q^2) \; D_{h/i}(z_h, \mathbf{b}_T, Q, Q^2)$$



Kang, Prokudin, Sun and Yuan, PRD **93**, 014009 (2016)

$$f_{i/p}(x, \mathbf{b}_T, \mu, \zeta) = f^{\text{pert}}_{i/p}(x, b^*(b_T), \mu, \zeta)$$

$$\times \left(\frac{\zeta}{Q_0^2}\right)^{g_K(b_T)/2}$$

$g_K(b_T)/2$ ⟶ **Collins-Soper kernel (NP part)**

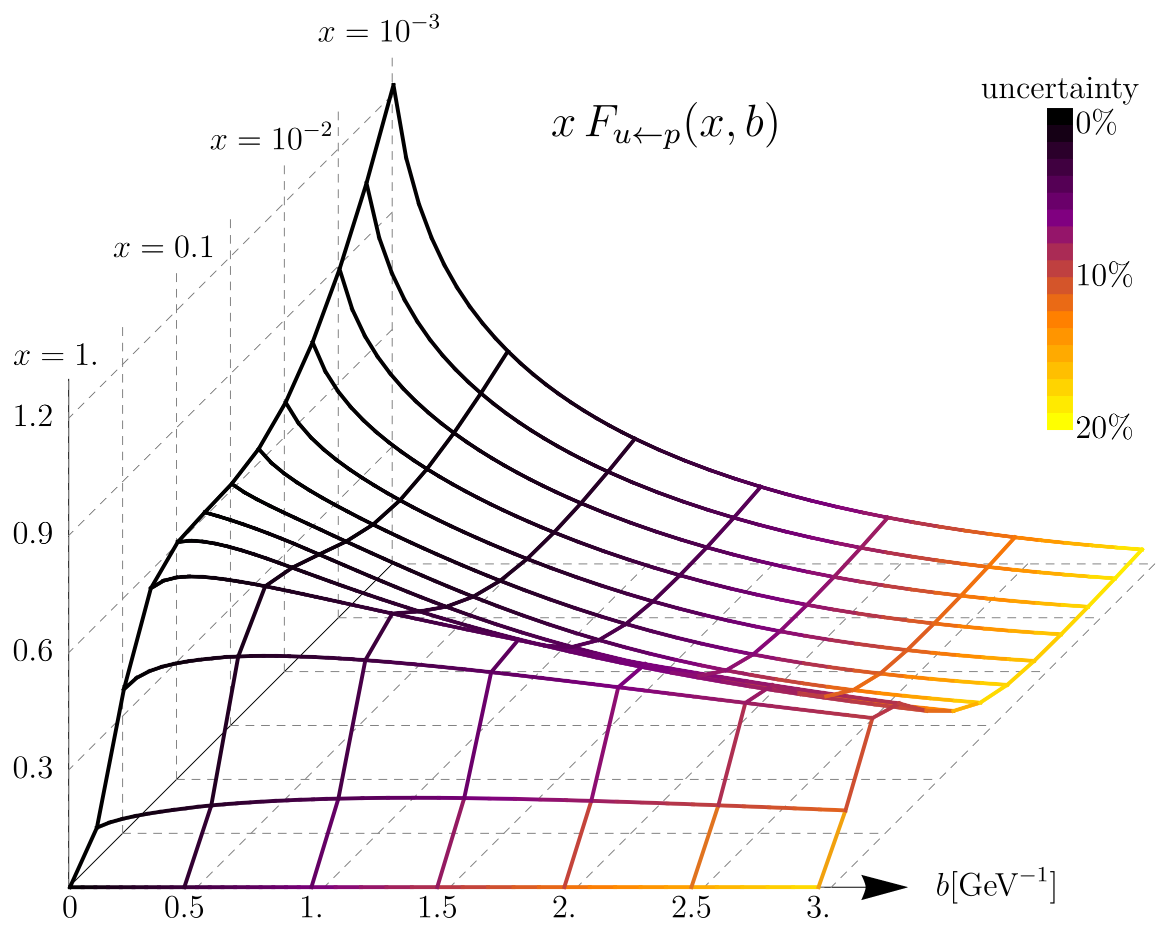
$f^{\text{NP}}_{i/p}(x, b_T)$ ⟶ **Intrinsic TMD**

$Q_0 \sim 1$ GeV

Non-perturbative when $b_T \sim 1/\Lambda_{\text{QCD}}$!

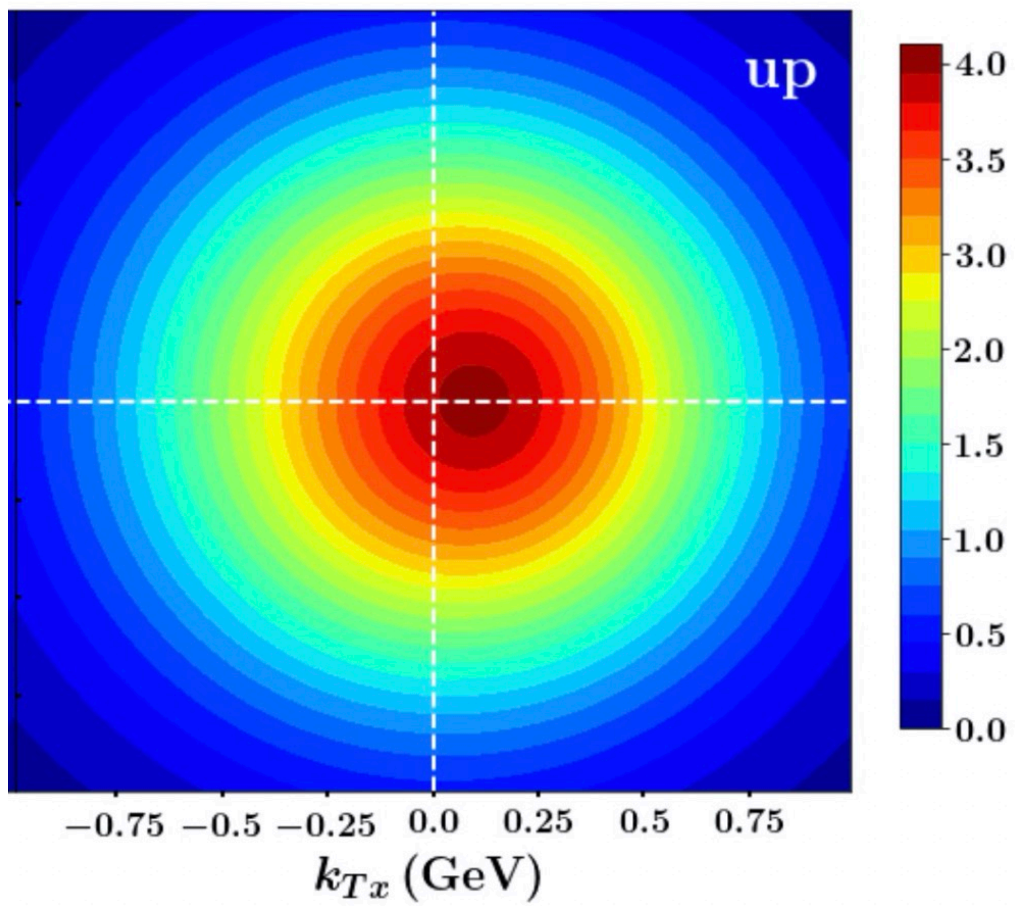
TMDs from global analyses

Unpolarized quark TMD



Scimemi and Vladimirov, JHEP **06** (2020).

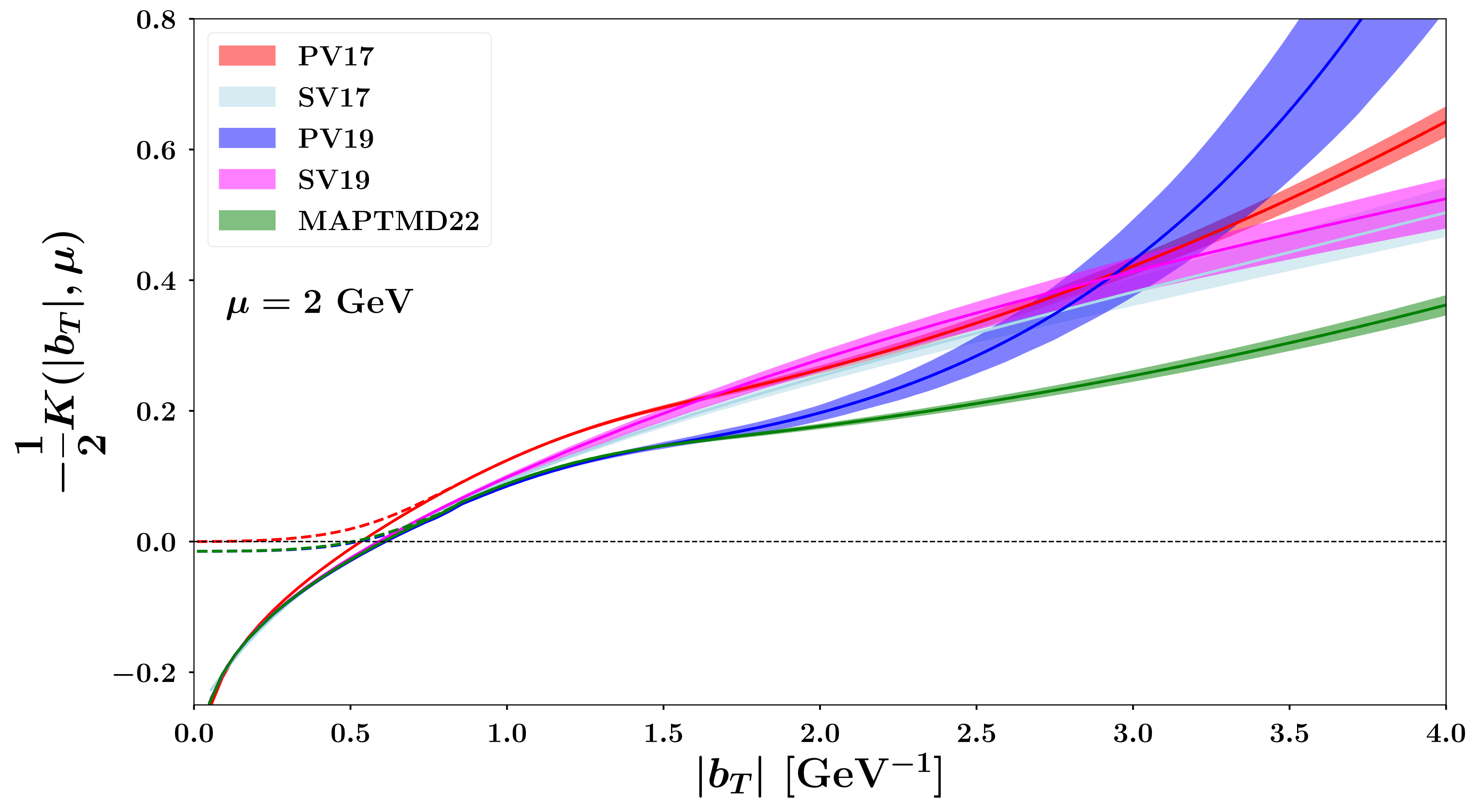
Quark Sivers function



Cammarota, Gamberg, Kang et al. (JAM Collaboration), PRD **102** (2020).

TMDs from global analyses

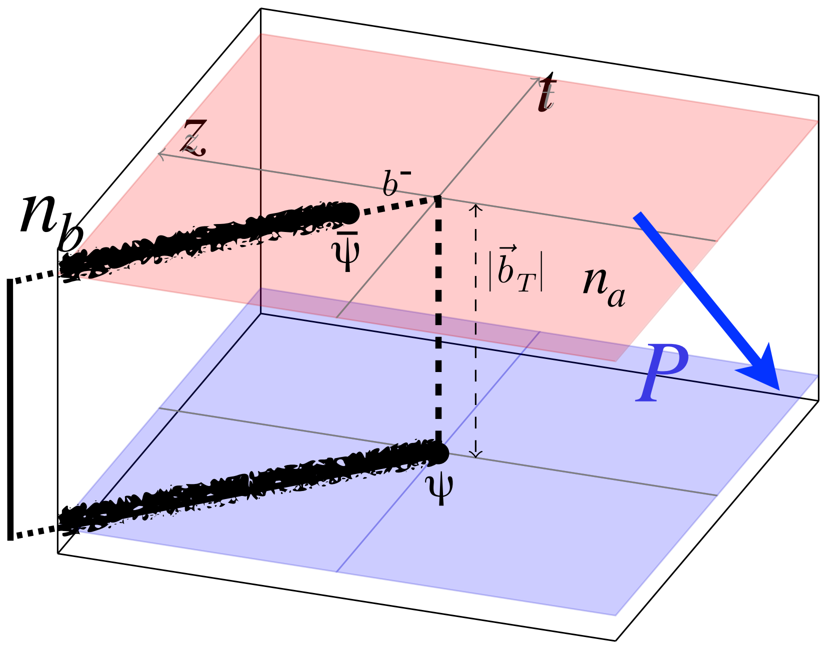
Collins-Soper Kernel $K(b_T, \mu) = K^{\text{pert}}(b_T, \mu) + g_K(b_T)$



Bacchetta, Bertone, Bissolotti, et al., MAP Collaboration, 2206.07598

TMD definition

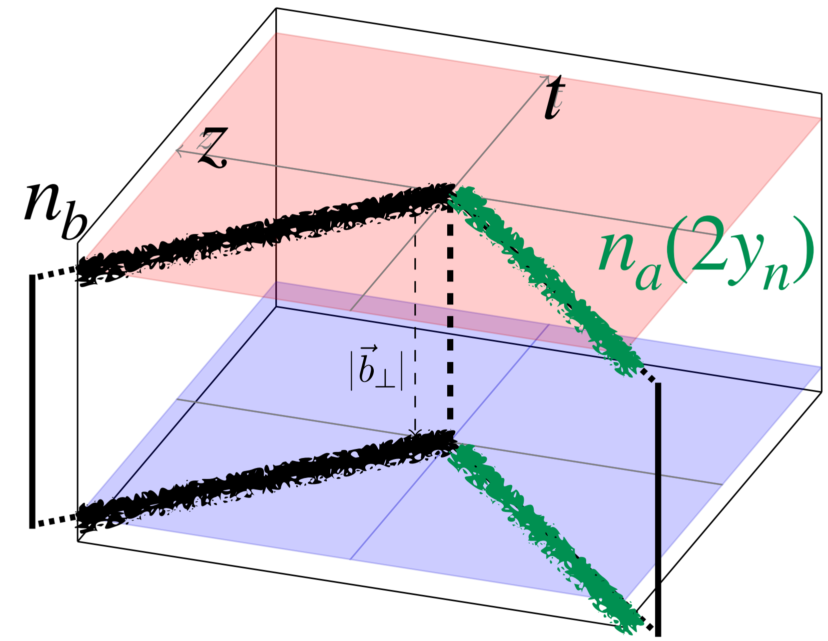
- Beam function:



Hadronic matrix element

$n_b^2 = 0$

- Soft function :



Vacuum matrix element

$$f_i(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon \to 0} Z_{\mathrm{UV}} \lim_{\tau \to 0} \frac{B_i}{\sqrt{S^q}}$$

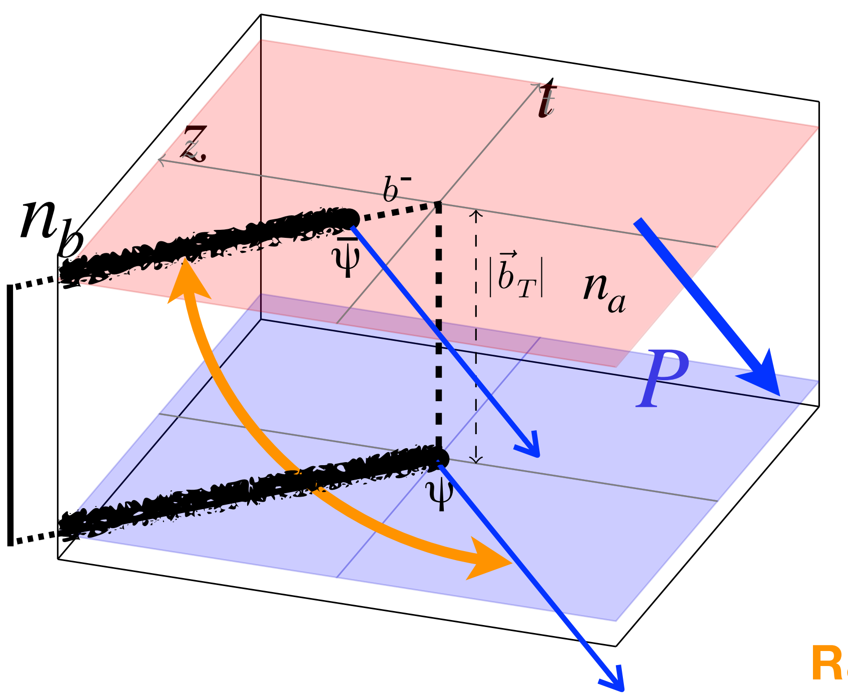
Collins-Soper scale: $\zeta = 2(xP^+ e^{-y_n})^2$

Rapidity divergence regulator

First principles calculation of TMDs from the above matrix elements would greatly complement global analyses!

TMD definition

- Beam function:

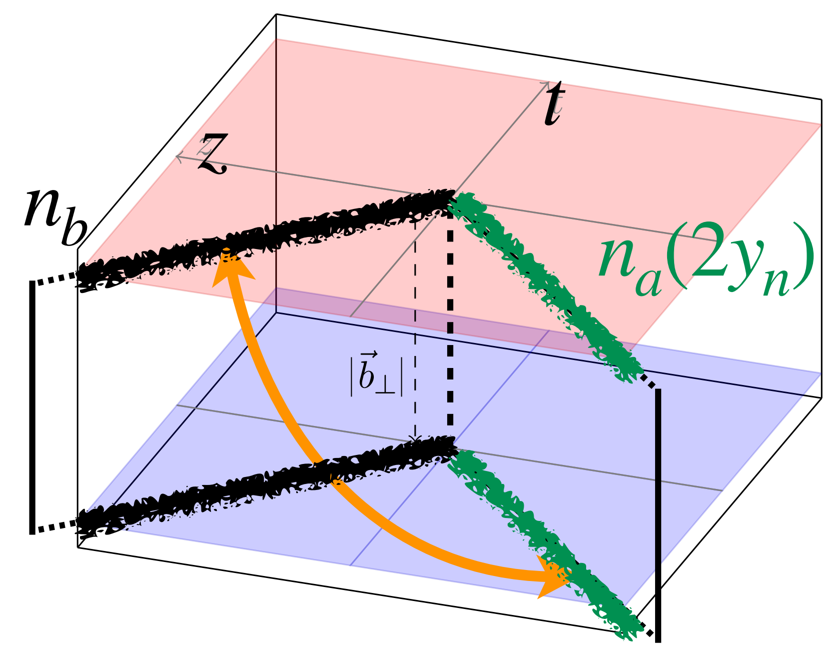


Hadronic matrix element

$$n_b^2 = 0$$

Rapidity divergences

- Soft function :



Vacuum matrix element

$$f_i(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon \to 0} Z_{\text{UV}} \lim_{\tau \to 0} \frac{B_i}{\sqrt{S^q}}$$

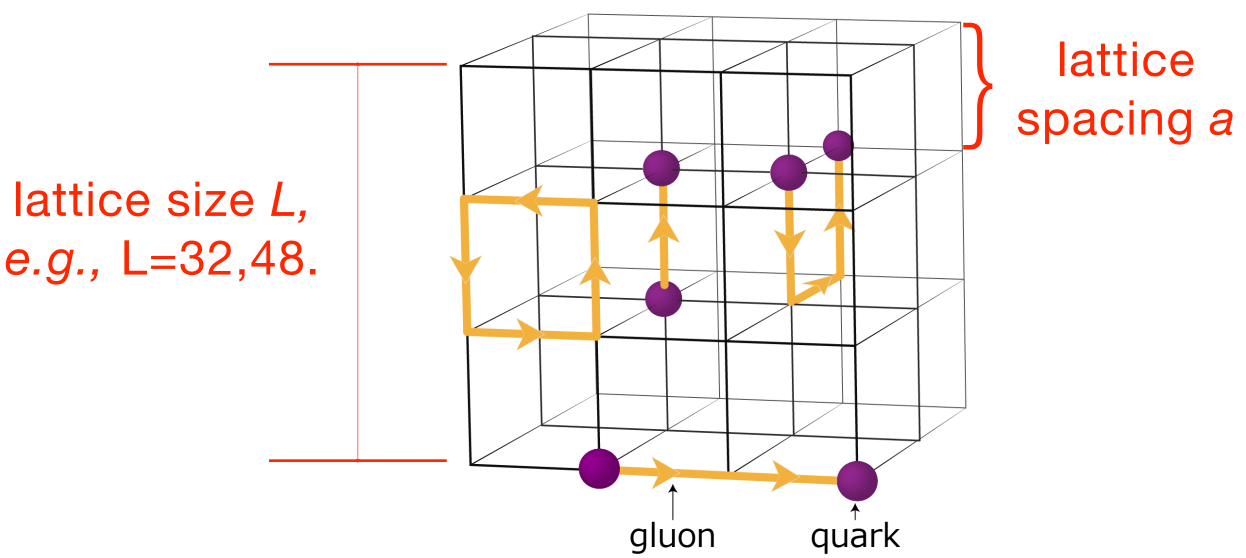
Collins-Soper scale: $\zeta = 2(xP^+ e^{-y_n})^2$

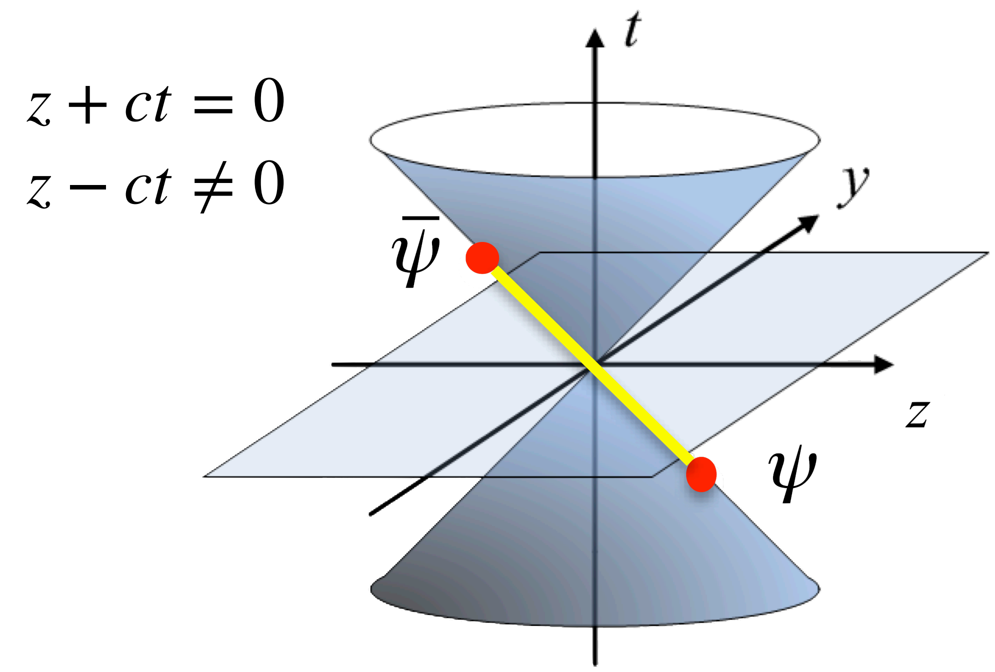
Rapidity divergence regulator

First principles calculation of TMDs from the above matrix elements would greatly complement global analyses!

Lattice QCD

Lattice gauge theory: a systematically improvable approach to solve non-perturbative QCD.





Imaginary time: $t \to i\tau$ $\quad O(i\tau) \overset{?}{\to} O(t)$

Simulating real-time dynamics has been extremely difficult due to the issue of analytical continuation. 🙁

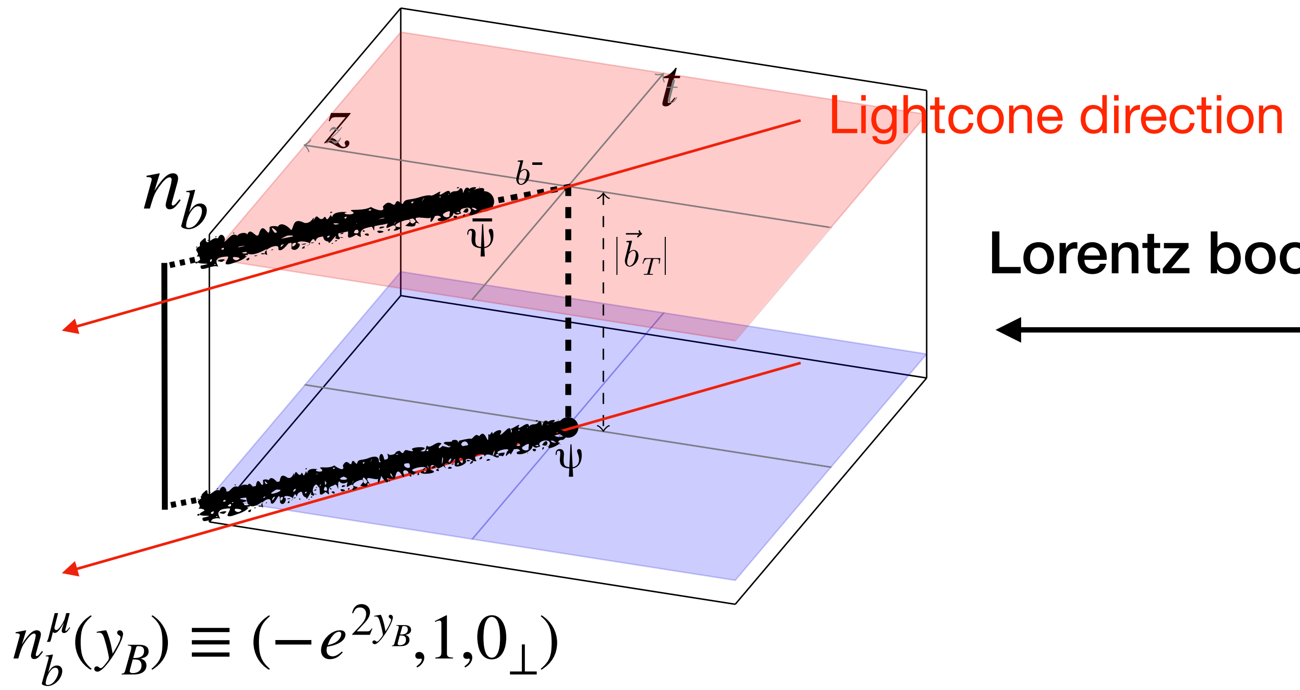
Progress in the lattice study of TMDs

- **Lorentz invariant method**
  - Musch, Hägler, Engelhardt, Negele and Schäfer et al.
  - Primary efforts focused on ratios of TMD x-moments (w/o soft function) (2009—)

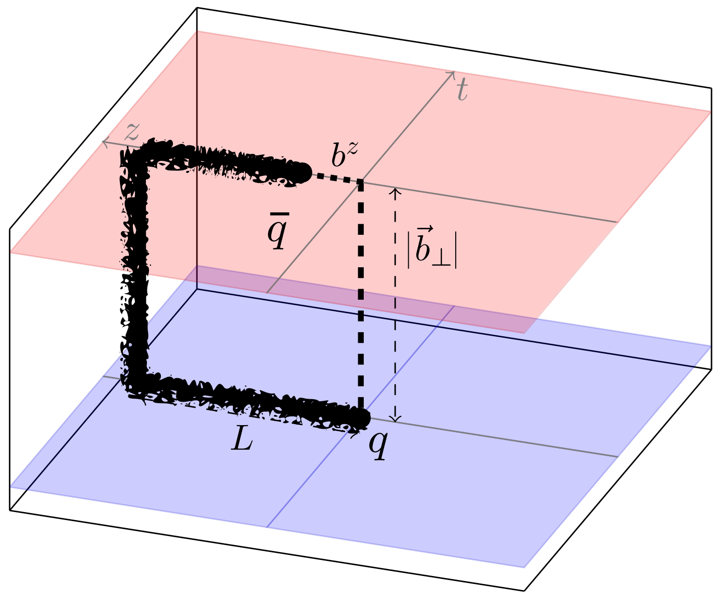
- **Quasi-TMDs**
  - Large-momentum effective theory (Ji, 2013, 2014; Ji, Liu, Liu, Zhang and YZ, 2021)
  - One-loop studies of quasi beam and soft functions (Ji, Yuan, Scäfer, Liu, Liu, Ebert, Stewart, YZ, Vladimirov, Wang, …, 2015-2022)
  - Method to calculate the Collins-Soper kernel (Ji, Yuan et al., 2015; Ebert, Stewart and YZ, 2018)
  - Method to calculate the soft function, and thus the x and b_T dependence of TMDs (Ji, Liu and Liu, 2019)
  - Derivation of factorization formula (Ebert, Schindler, Stewart and YZ, 2022)
  - First lattice results (SWZ, LPC, ETMC/PKU, SVZES, 2020—)

Quasi TMD in the LaMET formalism

- **Beam function in Collins scheme:**
- **Quasi beam function :**



Lorentz boost and $L \to \infty$



Spacelike but close-to-lightcone ($y_B \to -\infty$) Wilson lines, not calculable on the lattice ☹️

Equal-time Wilson lines, directly calculable on the lattice🙂

Related by Lorentz invariance, equivalent in the large $\tilde{P}^z$ or $(-y_B)$ expansion.

Ebert, Schindler, Stewart and YZ, JHEP **04**, 178 (2022).

TMDs from lattice QCD

$$\frac{\tilde{f}_{i/p}^{\text{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{\sqrt{S_r^q(b_T, \mu)}} = C\left(\mu, x\tilde{P}^z\right) \exp\left[\frac{1}{2}K(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right] \times f_{i/p}^{[s]}(x, \mathbf{b}_T, \mu, \zeta)\left\{1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda_{\text{QCD}}^2}{(x\tilde{P}^z)^2}\right]\right\}$$

Reduced soft function ✓

Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020).

- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ji, Jin, Yuan, Zhang and YZ, PRD99 (2019);
- Ebert, Stewart, YZ, PRD99 (2019), JHEP09 (2019) 037;
- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Vladimirov and Schäfer, PRD 101 (2020);
- Ebert, Schindler, Stewart and YZ, JHEP 04, 178 (2022).

Matching coefficient:

- Independent of spin;
  - Vladimirov and Schäfer, PRD 101 (2020);
  - Ebert, Schindler, Stewart and YZ, JHEP 09 (2020);
  - Ji, Liu, Schäfer and Yuan, PRD 103 (2021).
- No quark-gluon or flavor mixing, which makes gluon calculation much easier.

One-loop matching for gluon TMDs:

Ebert, Schindler, Stewart and YZ, 2205.12369.

TMDs from lattice QCD

$$\frac{\tilde{f}^{\text{naive}[s]}_{i/p}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{\sqrt{S^q_r(b_T, \mu)}} = C\left(\mu, x\tilde{P}^z\right) \exp\left[\frac{1}{2}K(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right] \times f^{[s]}_{i/p}(x, \mathbf{b}_T, \mu, \zeta)\left\{1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda^2_{\text{QCD}}}{(x\tilde{P}^z)^2}\right]\right\}$$

- ✳ Collins-Soper kernel;

$$K(\mu, b_T) = \frac{d}{d\ln\tilde{P}^z}\ln\frac{\tilde{f}^{\text{naive}[s]}_{i/p}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{C\left(\mu, x\tilde{P}^z\right)}$$

- ✳ Flavor separation;

$$\frac{f^{[s]}_{i/p}(x, \mathbf{b}_T)}{f^{[s']}_{j/p}(x, \mathbf{b}_T)} = \frac{\tilde{f}^{\text{naive}[s]}_{i/p}(x, \mathbf{b}_T)}{\tilde{f}^{\text{naive}[s']}_{j/p}(x, \mathbf{b}_T)}$$

- ✳ Spin-dependence, e.g., Sivers function;
- ✳ Full TMD kinematic dependence.

Ji, Liu, Schäfer and Yuan, PRD 103 (2021).

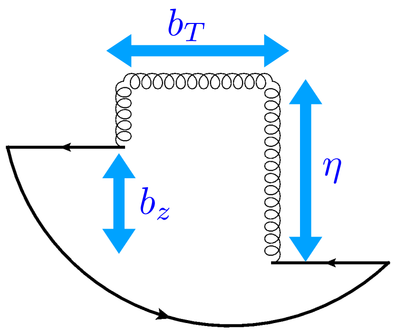
- ✳ Twist-3 PDFs from small b_T expansion of TMDs.

Collins-Soper (CS) kernel from lattice QCD

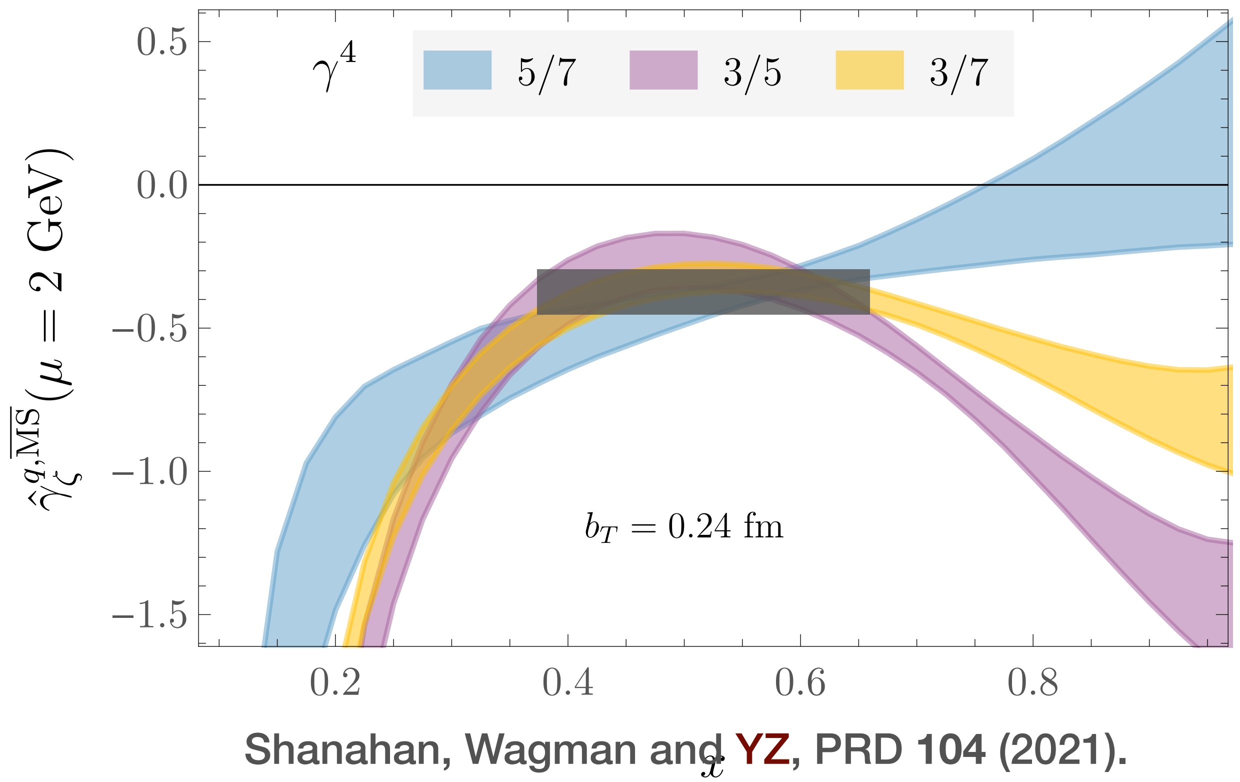
$$K^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{C(\mu, xP_2^z) \int db^z\, e^{ib^z xP_1^z}\, \tilde{Z}'(b^z,\mu,\tilde{\mu})\tilde{Z}_{\rm UV}(b^z,\tilde{\mu},a)\tilde{B}_{\rm ns}(b^z,\mathbf{b}_T,a,\eta,P_1^z)}{C(\mu, xP_1^z) \int db^z\, e^{ib^z xP_2^z}\, \tilde{Z}'(b^z,\mu,\tilde{\mu})\tilde{Z}_{\rm UV}(b^z,\tilde{\mu},a)\tilde{B}_{\rm ns}(b^z,\mathbf{b}_T,a,\eta,P_2^z)}$$

Perturbative matching

Renormalization (and operator mixing)



$$\times \left\{ 1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda_{\rm QCD}^2}{(x\tilde{P}^z)^2} \right] \right\}$$



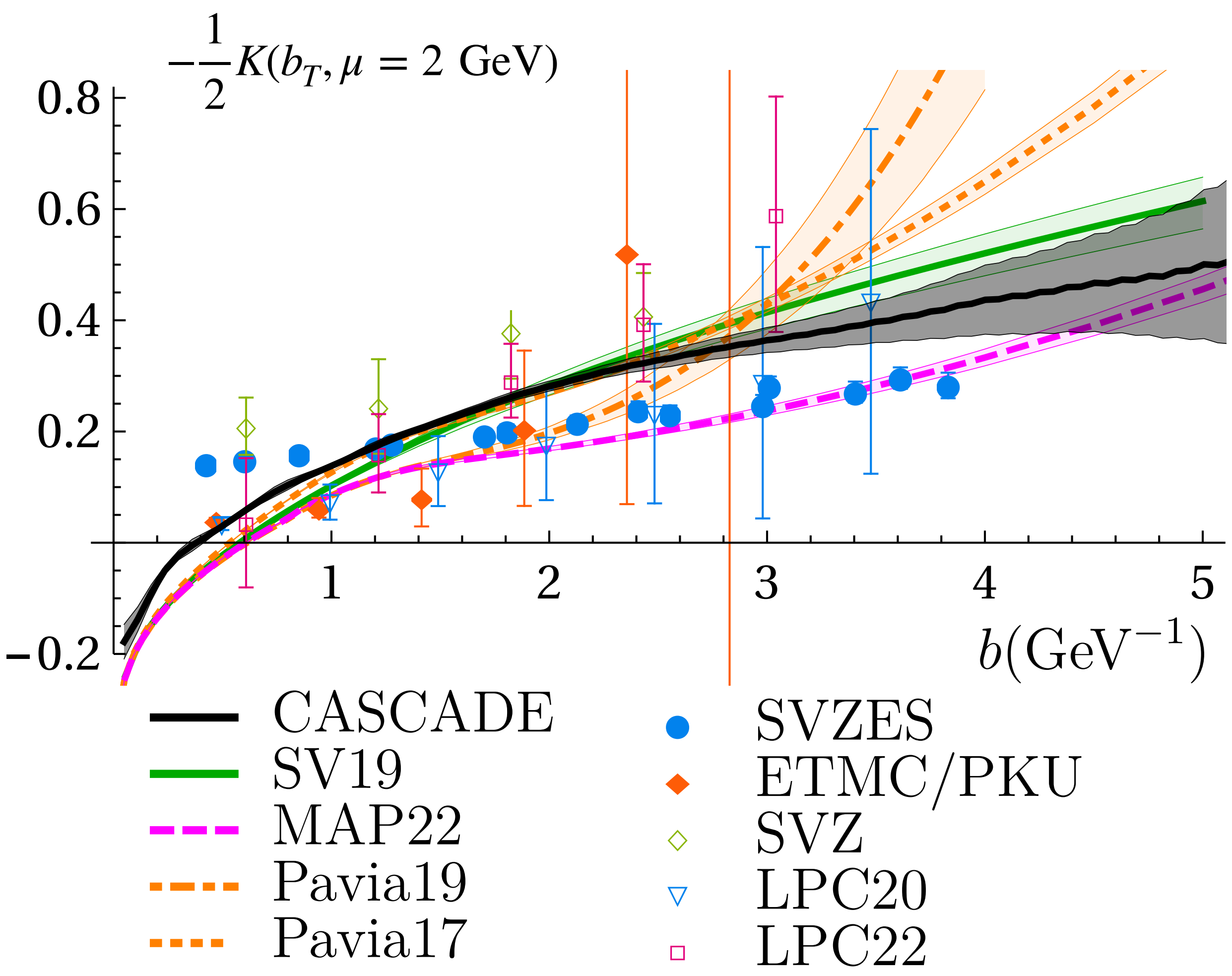
Shanahan, Wagman and **YZ**, PRD **104** (2021).

Current status for the Collins-Soper kernel

	Lattice setup	Renormalization	Operator mixing	Fourier transform	Matching	x-plateau search
SWZ20 PRD 102 (2020) Quenched	$a = 0.06$ fm, $m_\pi = 1.2$ GeV, $P^z_{\max} = 2.6$ GeV	Yes	Yes	Yes	LO	Yes
LPC20 PRL 125 (2020)	$a = 0.10$ fm, $m_\pi = 547$ MeV, $P^z_{\max} = 2.11$ GeV	N/A	No (small)	N/A	LO	N/A
SVZES 21 JHEP 08 (2021)	$a = 0.09$ fm, $m_\pi = 422$ MeV, $P^+_{\max} = 2.27$ GeV	N/A	No	N/A	NLO	N/A
PKU/ETMC 21 PRL 128 (2022)	$a = 0.09$ fm, $m_\pi = 827$ MeV, $P^z_{\max} = 3.3$ GeV	N/A	No	N/A	LO	N/A
SWZ21 PRD 106 (2022)	$a = 0.12$ fm, $m_\pi = 580$ MeV, $P^z_{\max} = 1.5$ GeV	Yes	Yes	Yes	NLO	Yes
LPC22 2204.00200	$a = 0.12$ fm, $m_\pi = 670$ MeV, $P^z_{\max} = 2.58$ GeV	Yes	No (small)	Yes	NLO	Yes

Collins Soper kernel

Comparison between lattice results and global fits



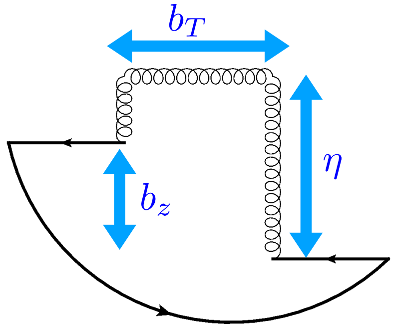
Approach	Collaboration
Quasi beam functions	P. Shanahan, M. Wagman and YZ (SWZ21), Phys. Rev.D **104** (2021)
Quasi TMD wavefunctions	Q.-A. Zhang, et al. (LPC20), Phys.Rev.Lett. **125** (2020).
	Y. Li et al. (ETMC/PKU 21), Phys.Rev.Lett. **128** (2022).
	M.-H. Chu et al. (LPC22), Phys.Rev.D 106 (2022)
Moments of quasi TMDs	Schäfer, Vladmirov et al. (SVZES21), JHEP **08** (2021)

MAP22: Bacchetta, Bertone, Bissolotti, et al., 2206.07598
SV19: I. Scimemi and A. Vladimirov, JHEP 06 (2020) 137
Pavia19: A. Bacchetta et al., JHEP 07 (2020) 117
Pavia 17: A. Bacchetta et al., JHEP 06 (2017) 081
CASCADE: Martinez and Vladimirov, 2206.01105

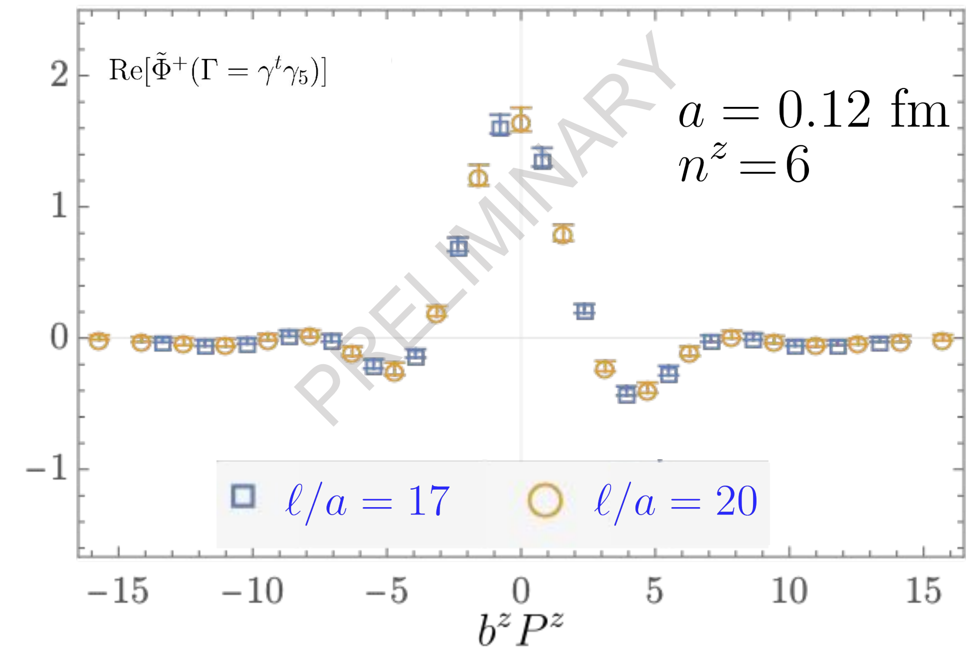
Improved calculation with TMD wave function

Φ: Quasi-TMD wave function

A. Avkhadiev, P. Shanahan, M. Wagman and YZ,
work in progress.

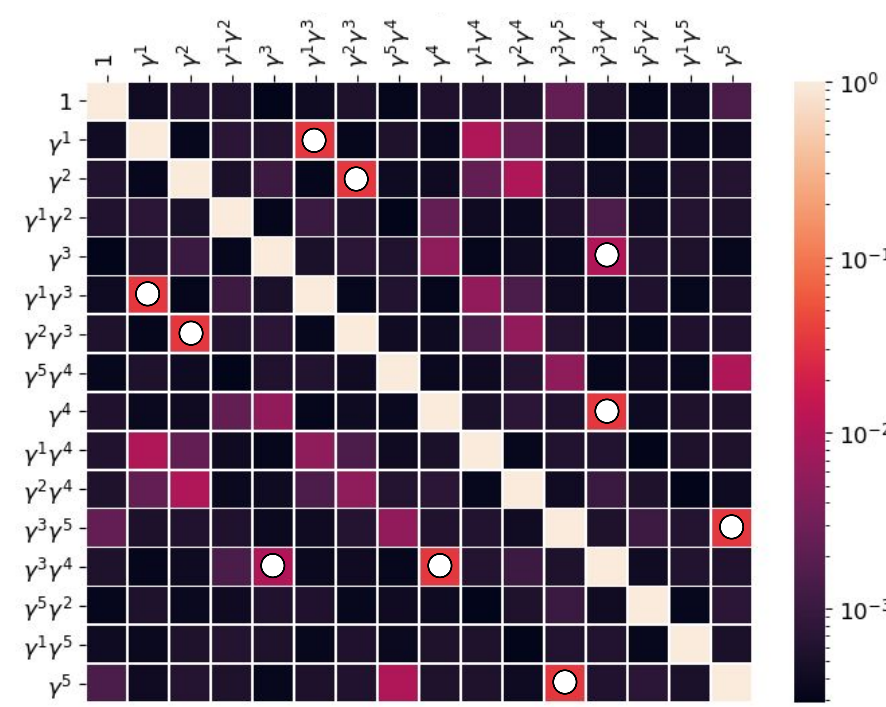
$\tilde{\Phi} = \langle 0|$  $|\pi(P)\rangle$

Q.-A. Zhang, et al. (LPC), PRL 125 (2020);
Y. Li et al., PRL 128 (2022);
M.-H. Chu et al. (LPC22), Phys.Rev.D 106 (2022).



- **Physical pion mass** and reduced systematics from Fourier transform
  - Better suppressed power correction
  - More stable extraction of x-dependence

- Renormalization of nonlocal operator
  - Systematic treatment of operator mixing using the RI-xMOM scheme



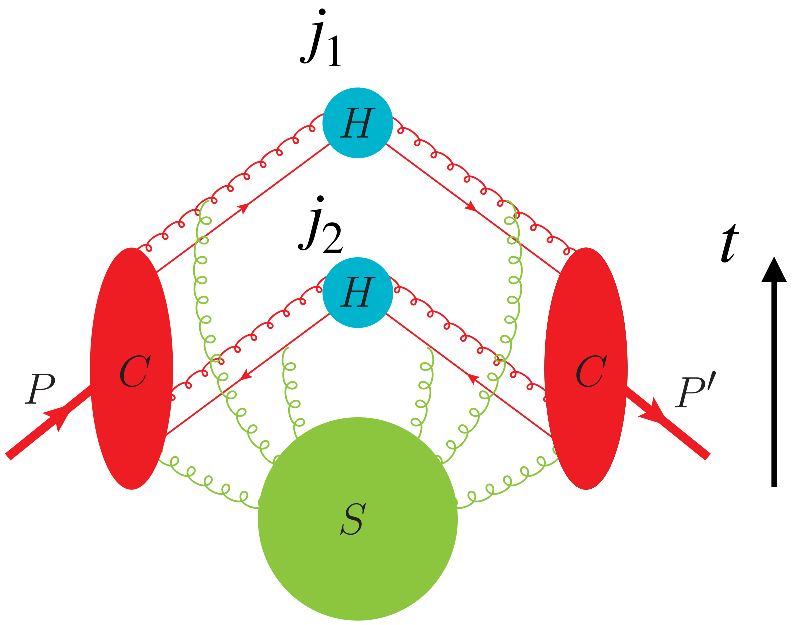
- Green, Jansen, and Steffens, Phys.Rev.Lett. **121** (2018) and PRD **101** (2020).
- Constantinou, Panagopoulos, and Spanoudes, PRD **99** (2019).

Reduced soft function from LaMET

Light-meson form factor:

$$F(b_T, P^z) = \langle \pi(-P) \,|\, j_1(b_T) j_2(0) \,|\, \pi(P) \rangle$$

$$\overset{P^z \gg m_N}{=} S_q^r(b_T, \mu) \int dx dx' \; H(x, x', \mu) \times \Phi^\dagger(x, b_T, P^z) \Phi(x', b_T, P^z)$$



- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Ji and Liu, PRD 105, 076014 (2022);
- Deng, Wang and Zeng, 2207.07280.

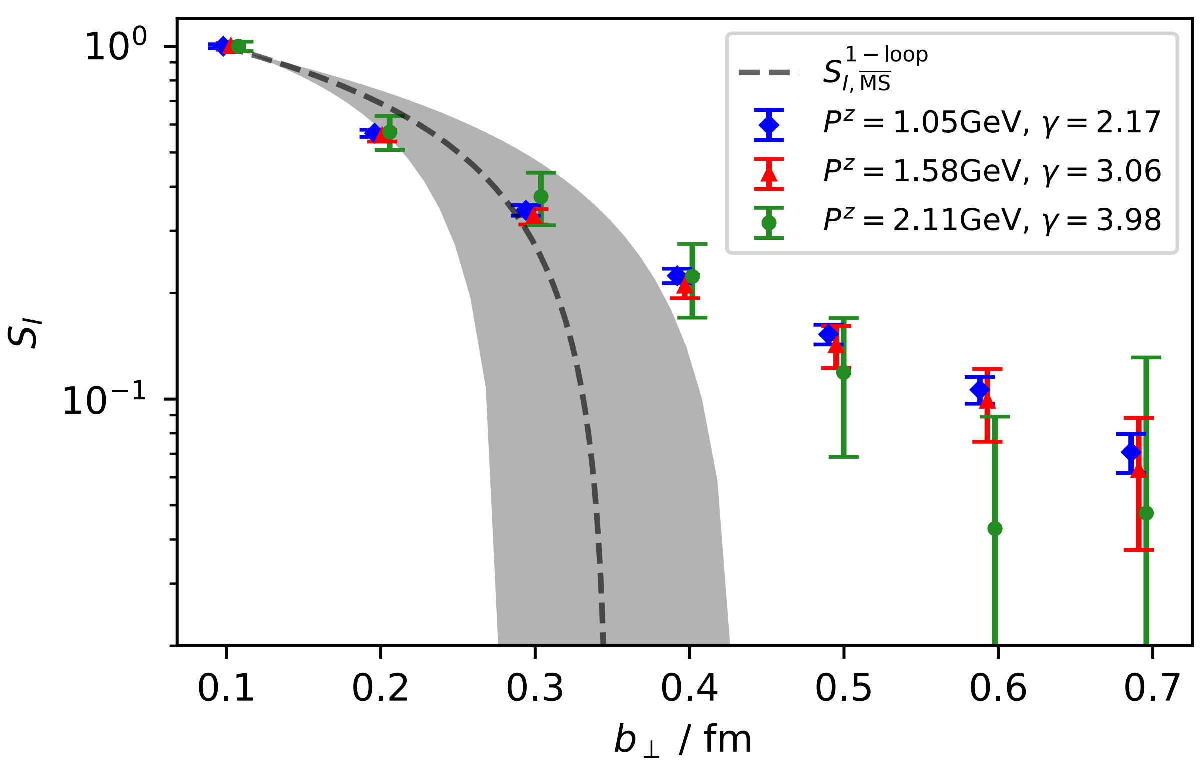
Tree-level approximation:

$$H(x, x', \mu) = 1 + \mathcal{O}(\alpha_s)$$

$$\Rightarrow S_q^r(b_T) = \frac{F(b_T, P^z)}{[\tilde{\Phi}(b^z = 0, b_T, P^z)]^2}$$

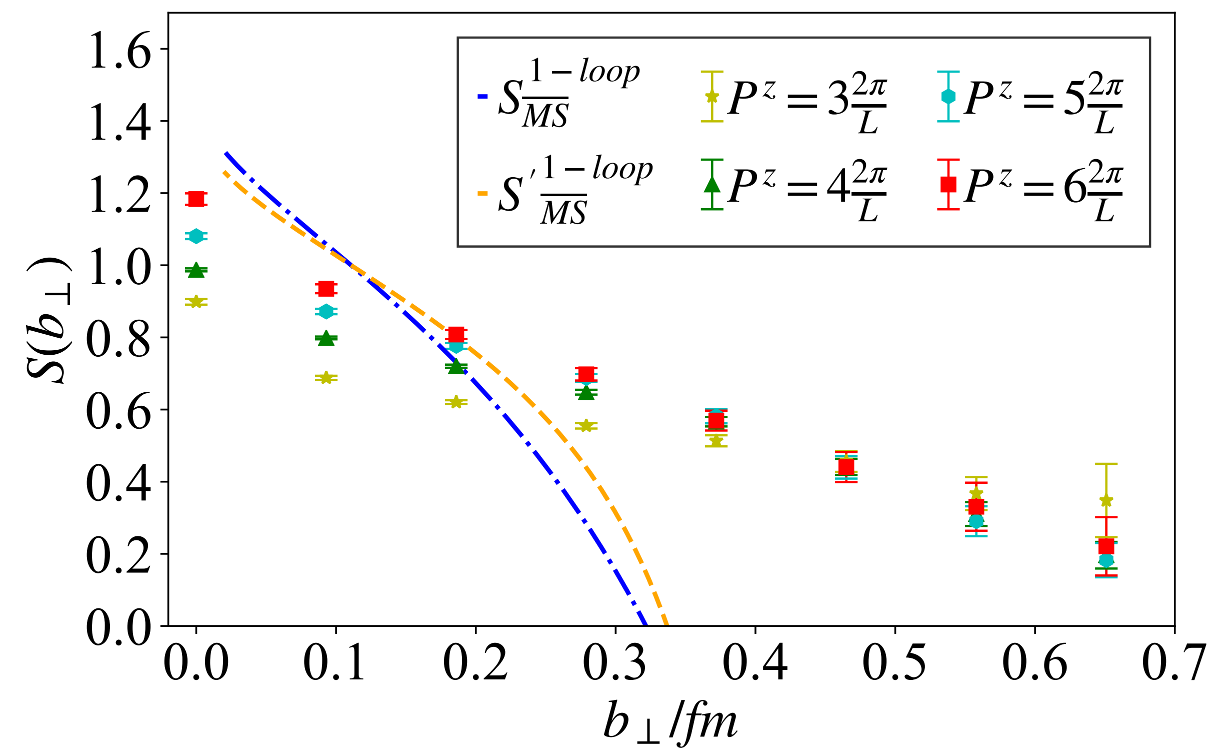
First lattice results with tree-level matching

$a = 0.10$ fm,
$m_\pi = 547$ MeV,
$P^z_{\max} = 2.11$ GeV



Q.-A. Zhang, et al. (LPC), PRL 125 (2020).

$a = 0.09$ fm,
$m_\pi = 827$ MeV,
$P^z_{\max} = 3.3$ GeV



Y. Li et al., PRL 128 (2022).

Beyond tree-level, it is necessary to obtain the *x*-dependence to carry out the convolution.

Conclusion

- The quark and gluon quasi TMDs can be related to the new LR scheme, which can be factorized into the physical TMDs;

- There is no mixing between quarks of different flavors, quark and gluon channels, or different spin structures.

- The method for calculating all the leading-power TMDs is complete;

- Lattice results for the Collins-Soper kernel and soft function are promising, but systematics need to be under control.

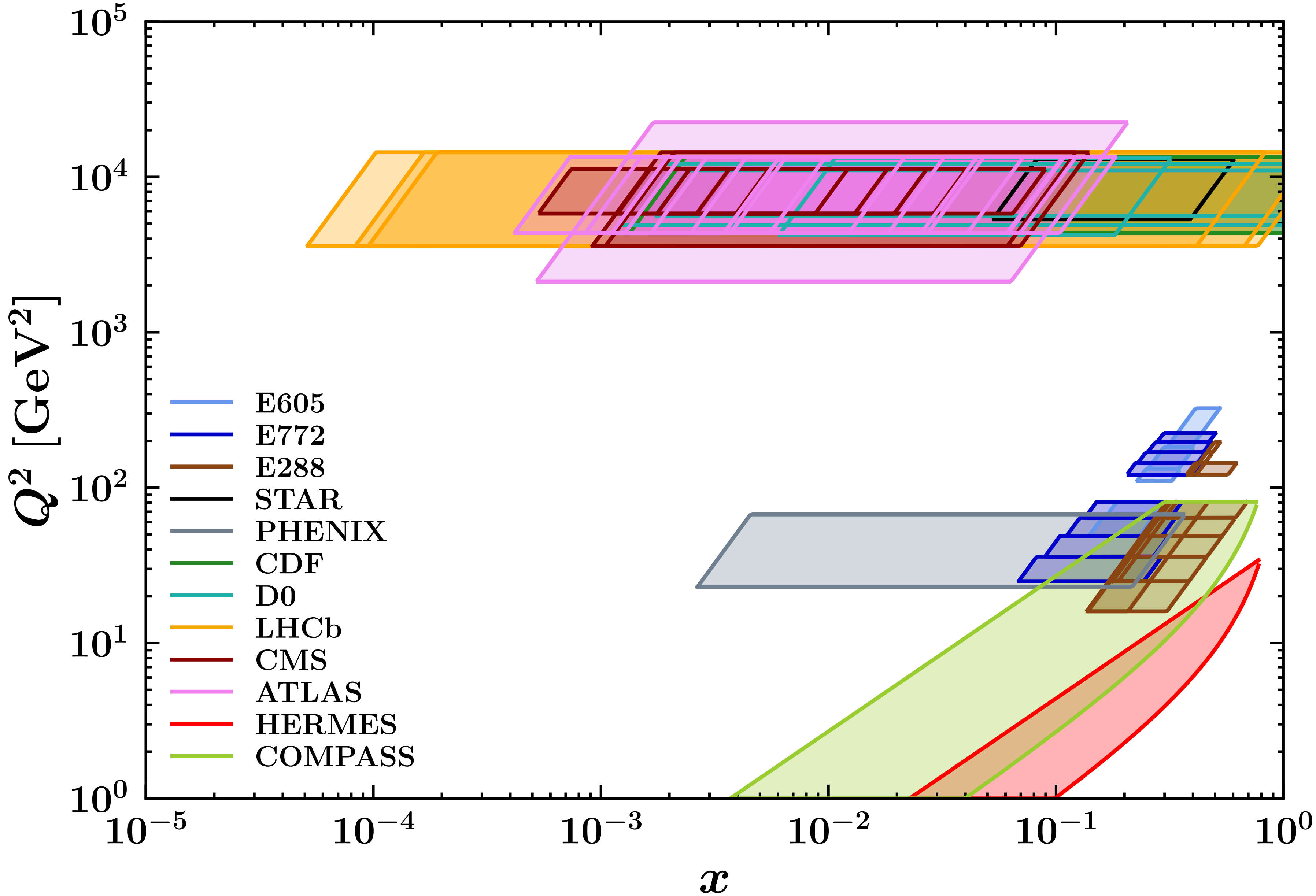
Outlook

Targets for lattice QCD studies:

Observables	Status
Non-perturbative Collins-Soper kernel	✔, keep improving the systematics
Soft factor	✔, to be under systematic control
Info on spin-dependent TMDs (in ratios)	In progress
Proton v.s. pion TMDs, (x, b_T) (in ratios)	In progress
Flavor dependence of TMDs, (x, b_T) (in ratios)	to be studied
TMDs and TMD wave functions, (x, b_T)	In progress
Gluon TMDs (x, b_T)	to be studied
Wigner distributions/GTMDs (x, b_T)	to be studied

Backup slides

Data used by the MAP collaboration in 2206.07598

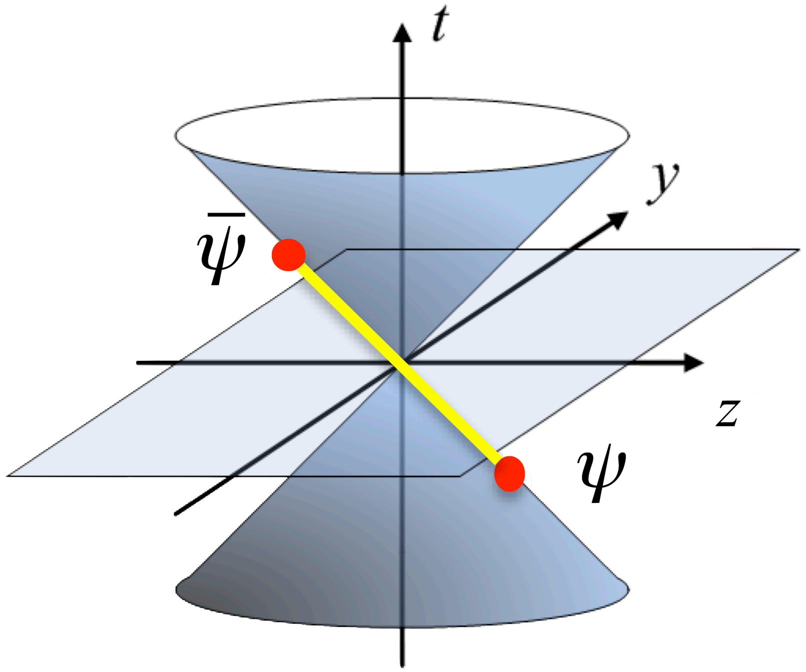


Bacchetta, Bertone, Bissolotti, et al., MAP Collaboration, 2206.07598

Large-Momentum Effective Theory (LaMET)

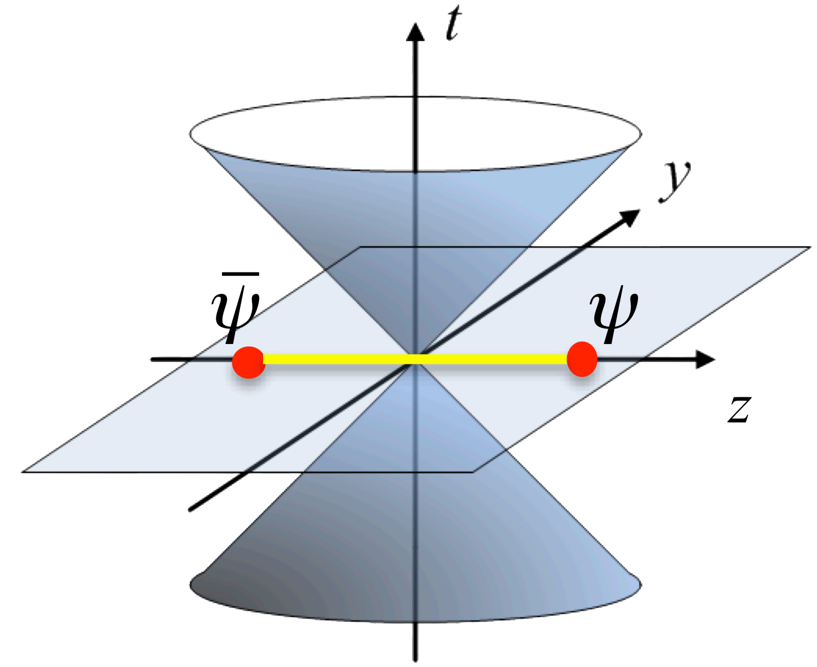
X. Ji, PRL 110 (2013)

$z + ct = 0, \ \ z - ct \neq 0$



PDF $f(x)$:
Cannot be calculated on the lattice

$t = 0, \ \ z \neq 0$



Quasi-PDF $\tilde{f}(x, P^z)$:
Directly calculable on the lattice

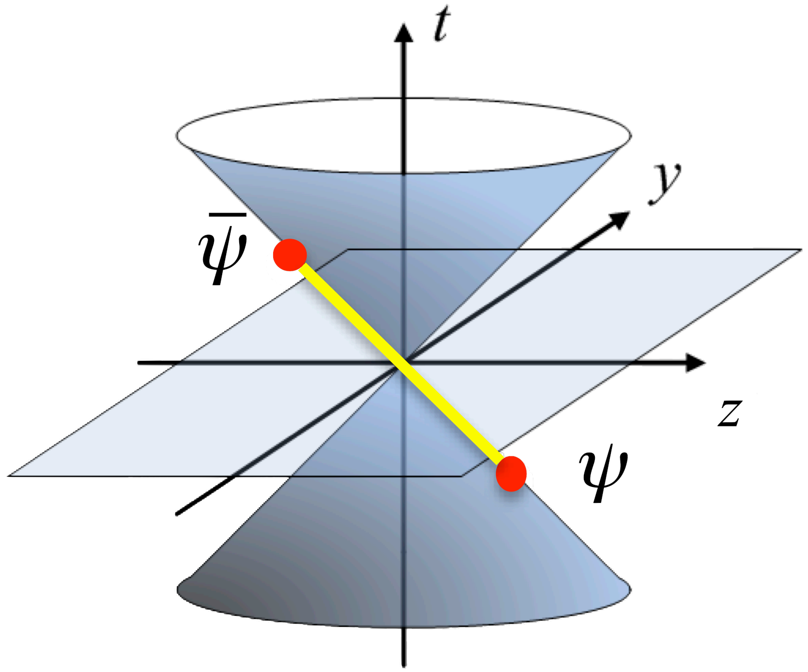
Large-Momentum Effective Theory (LaMET)

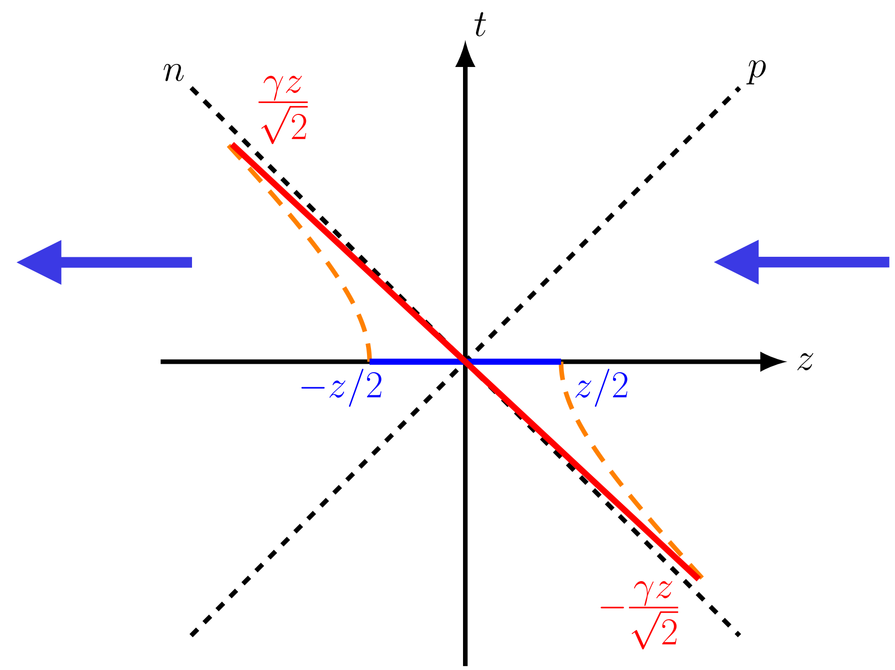
X. Ji, PRL 110 (2013)

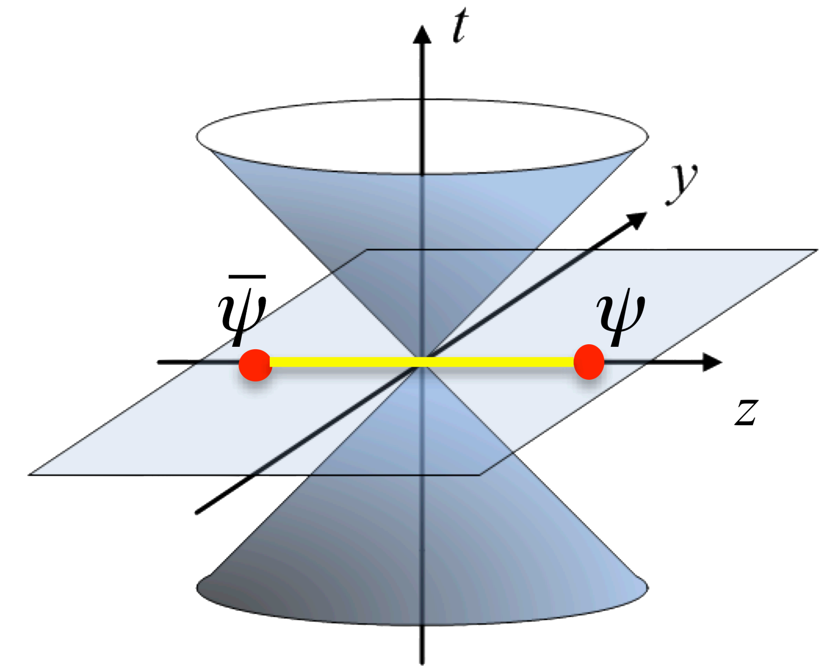
$z + ct = 0, \quad z - ct \neq 0$

Related by Lorentz boost

$t = 0, \quad z \neq 0$







PDF $f(x)$:
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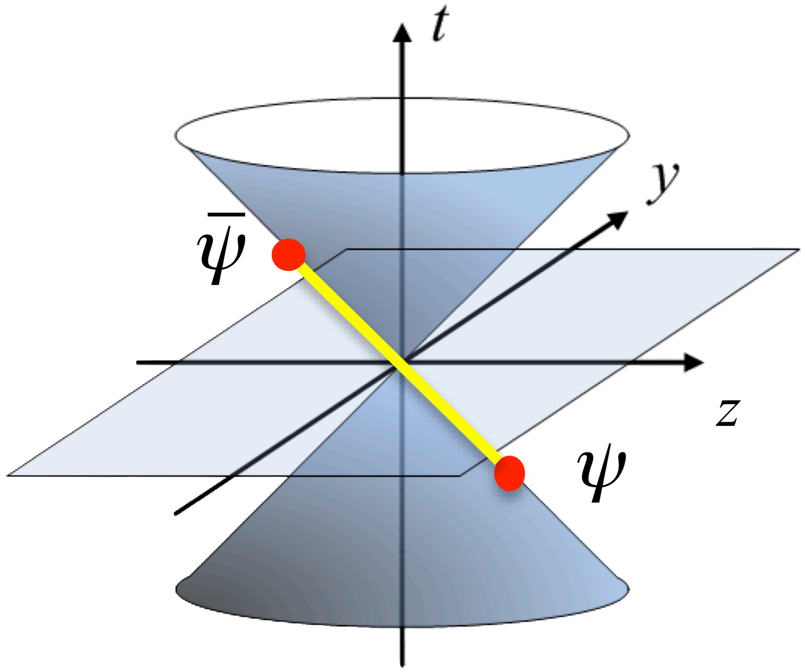

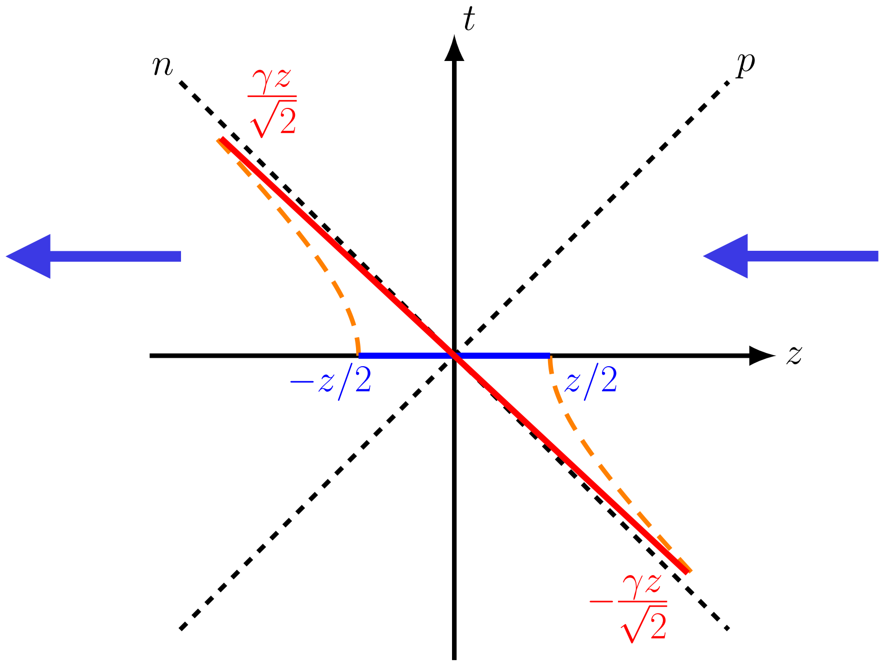

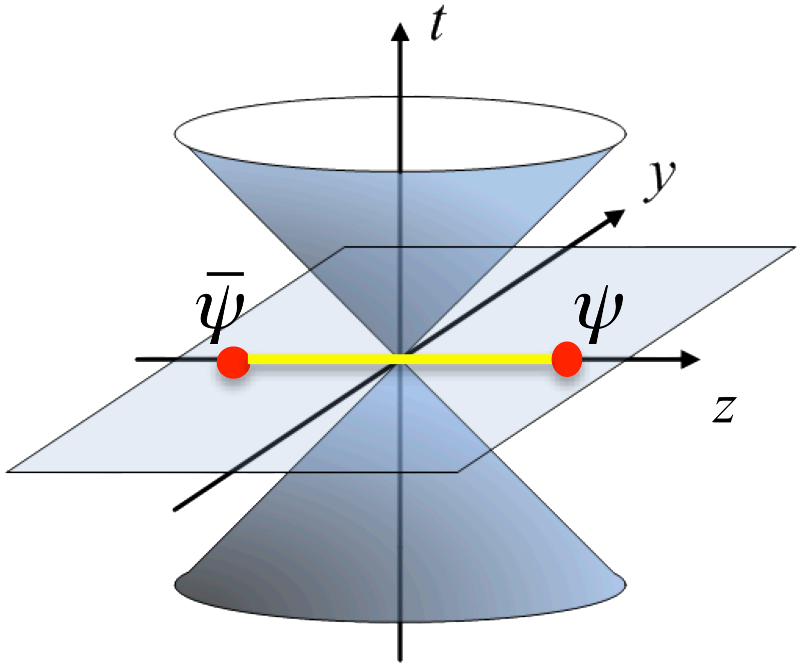

PDF $f(x)$:
Cannot be calculated on the lattice

$$\lim_{P^z \to \infty} \tilde{f}(x, P^z) \stackrel{?}{=} f(x)$$

Quasi-PDF $\tilde{f}(x, P^z)$:
Directly calculable on the lattice

Large-Momentum Effective Theory (LaMET)

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$z + ct = 0, \quad z - ct \neq 0$

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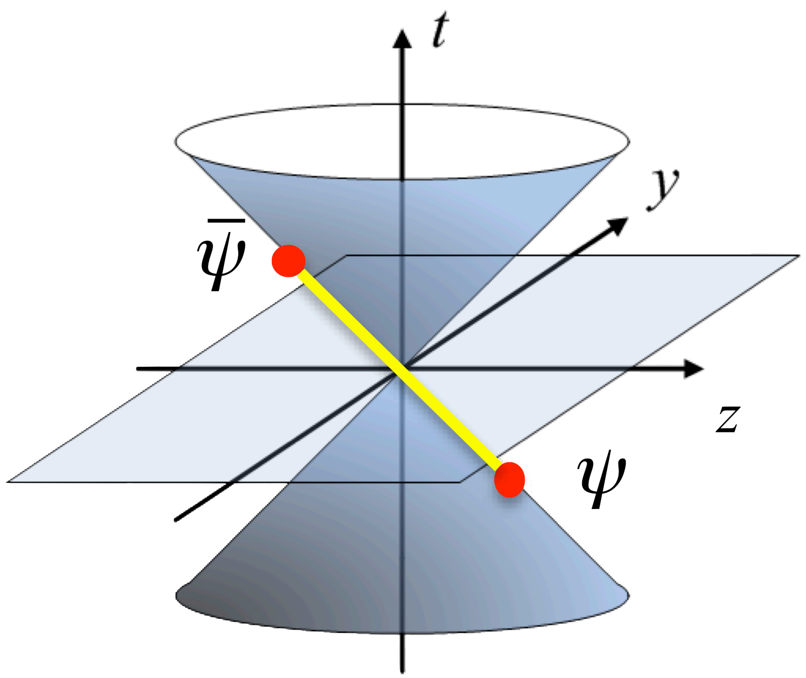

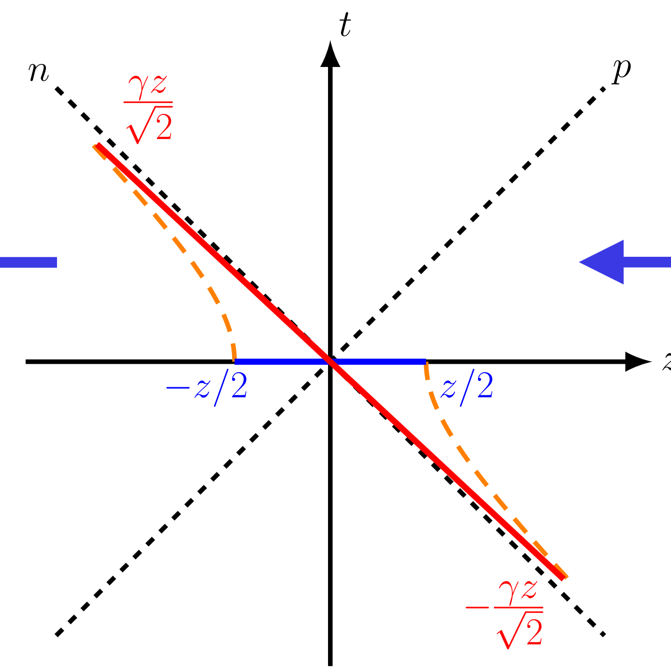

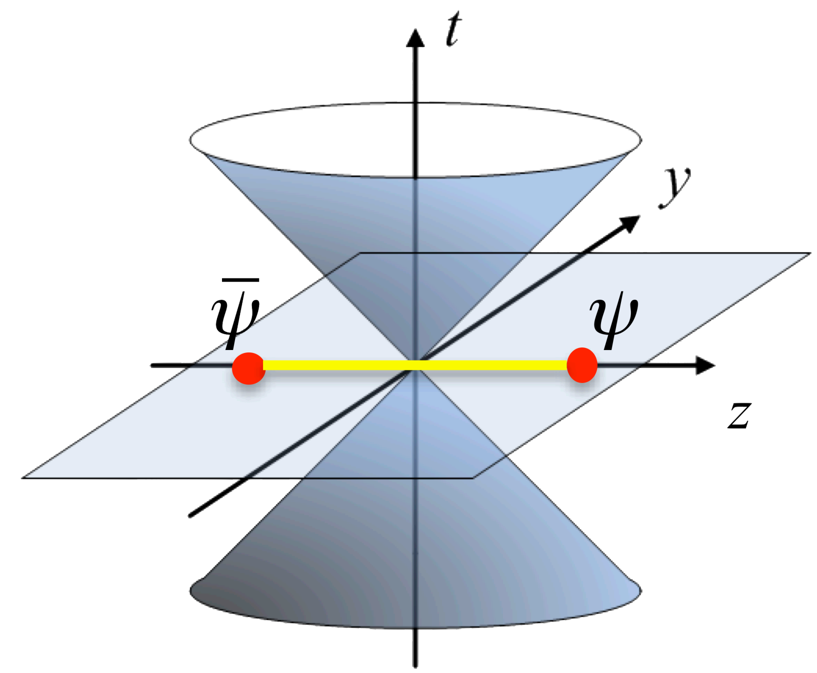

PDF $f(x)$:
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$$\lim_{P^z \to \infty} \tilde{f}(x, P^z) \stackrel{?}{=} f(x)$$

✘

Quasi-PDF $\tilde{f}(x, P^z)$:
Directly calculable on the lattice

Large-Momentum Effective Theory (LaMET)

- Quasi-PDF: $P^z \ll \Lambda$;

Λ: the ultraviolet lattice cutoff, $\sim 1/a$

- PDF: $P^z = \infty$, implying $P^z \gg \Lambda$.
    - The limits $P^z \ll \Lambda$ and $P^z \gg \Lambda$ are not usually exchangeable;
    - For $P^z \gg \Lambda_{\rm QCD}$, the infrared (nonperturbative) physics is not affected, which allows for an effective field theory matching.

$$\tilde{f}(x, P^z, \Lambda) = C\left(x, P^z/\mu, \Lambda/P^z\right) \otimes f(x, \mu) + O\left(\frac{\Lambda_{\rm QCD}^2}{P_z^2}\right)$$

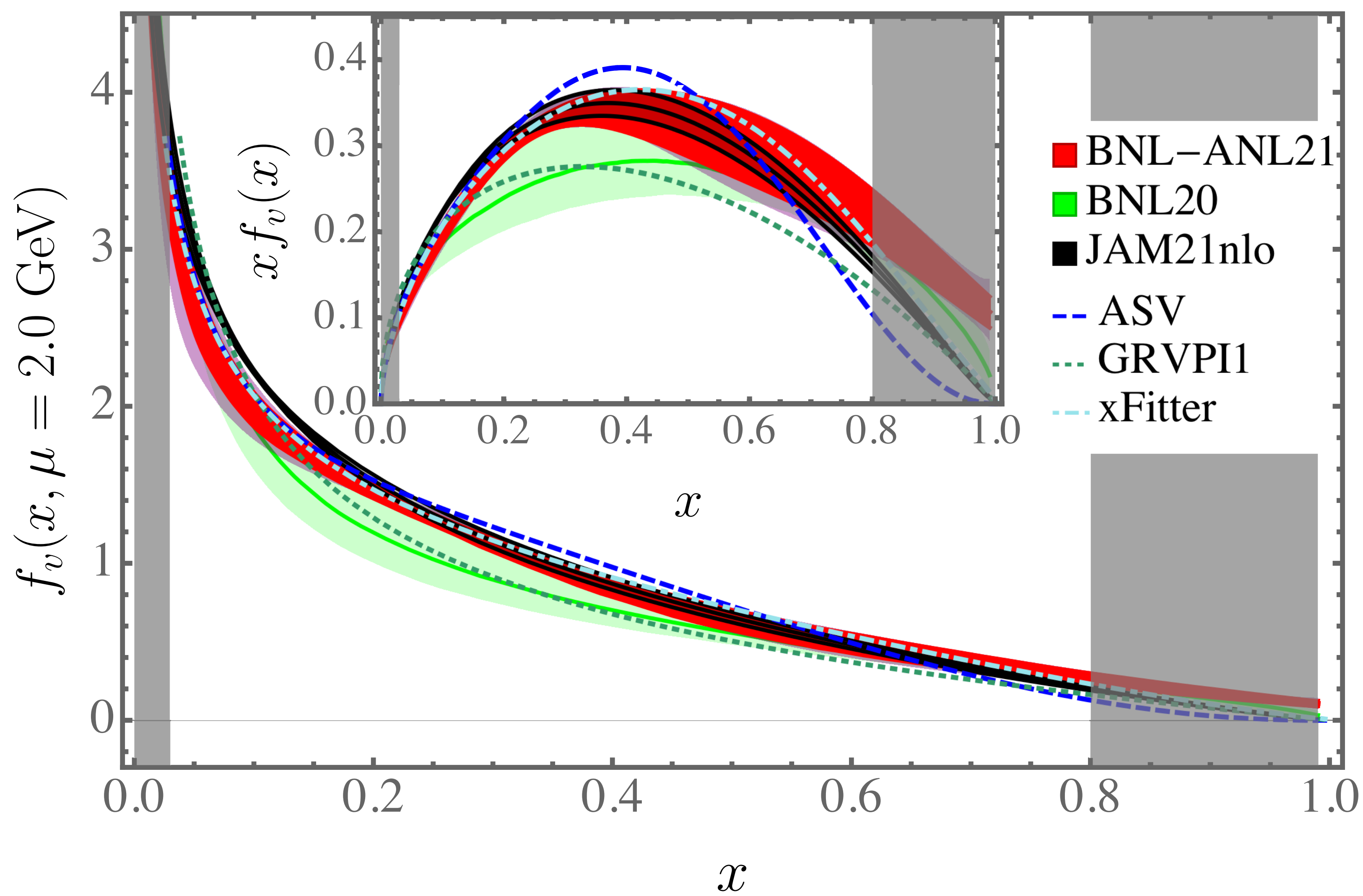
Perturbative matching

Power corrections

- X. Ji, PRL 110 (2013); SCPMA57 (2014).
- X. Xiong, X. Ji, J.-H. Zhang and YZ, PRD 90 (2014);
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, RMP 93 (2021).

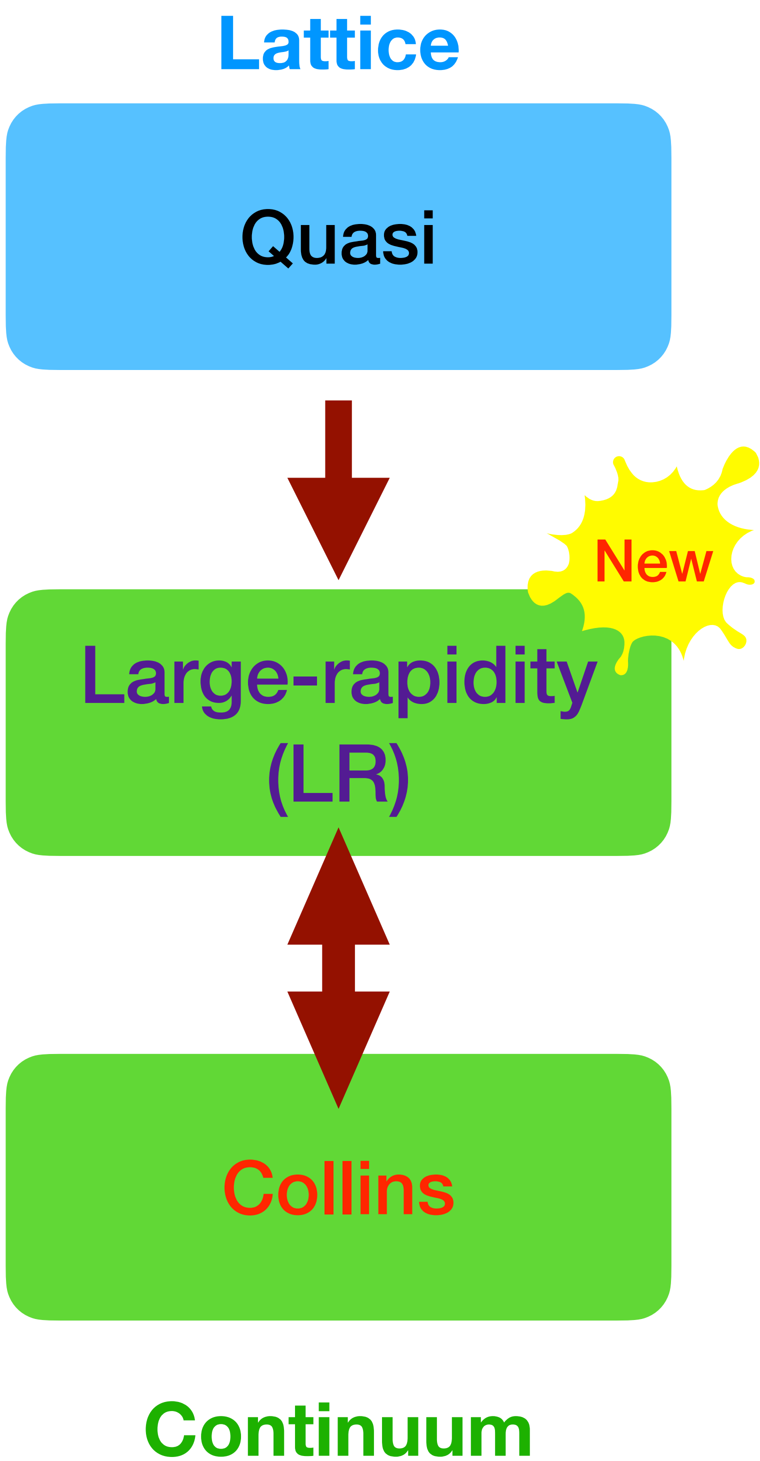
LaMET calculation of the collinear PDFs

A state-of-the-art calculation of the pion valence quark PDF with fine lattices, large momentum and NNLO matching:



Gao, Hanlon, Mukherjee, Petreczky, Scior, Syritsyn and YZ, PRL **128**, 142003 (2022).

Factorization relation with the TMDs



$$\tilde{f}_i(x, \mathbf{b}_T, \mu, \tilde{\zeta}, \tilde{P}^z) = \lim_{\tilde{P}^z \gg m_N} \lim_{a \to 0} \tilde{Z}_{\mathrm{UV}} \frac{\tilde{B}_i}{\sqrt{S^q}}$$

Lorentz invariance $\quad y_{\tilde{P}} = y_P - y_B$

$$f_i^{\mathrm{LR}}(x, \mathbf{b}_T, \mu, \zeta, y_P - y_B) = \lim_{-y_B \gg 1} \lim_{\epsilon \to 0} Z_{\mathrm{UV}}^{\mathrm{LR}} \frac{B_i}{\sqrt{S^q}}$$

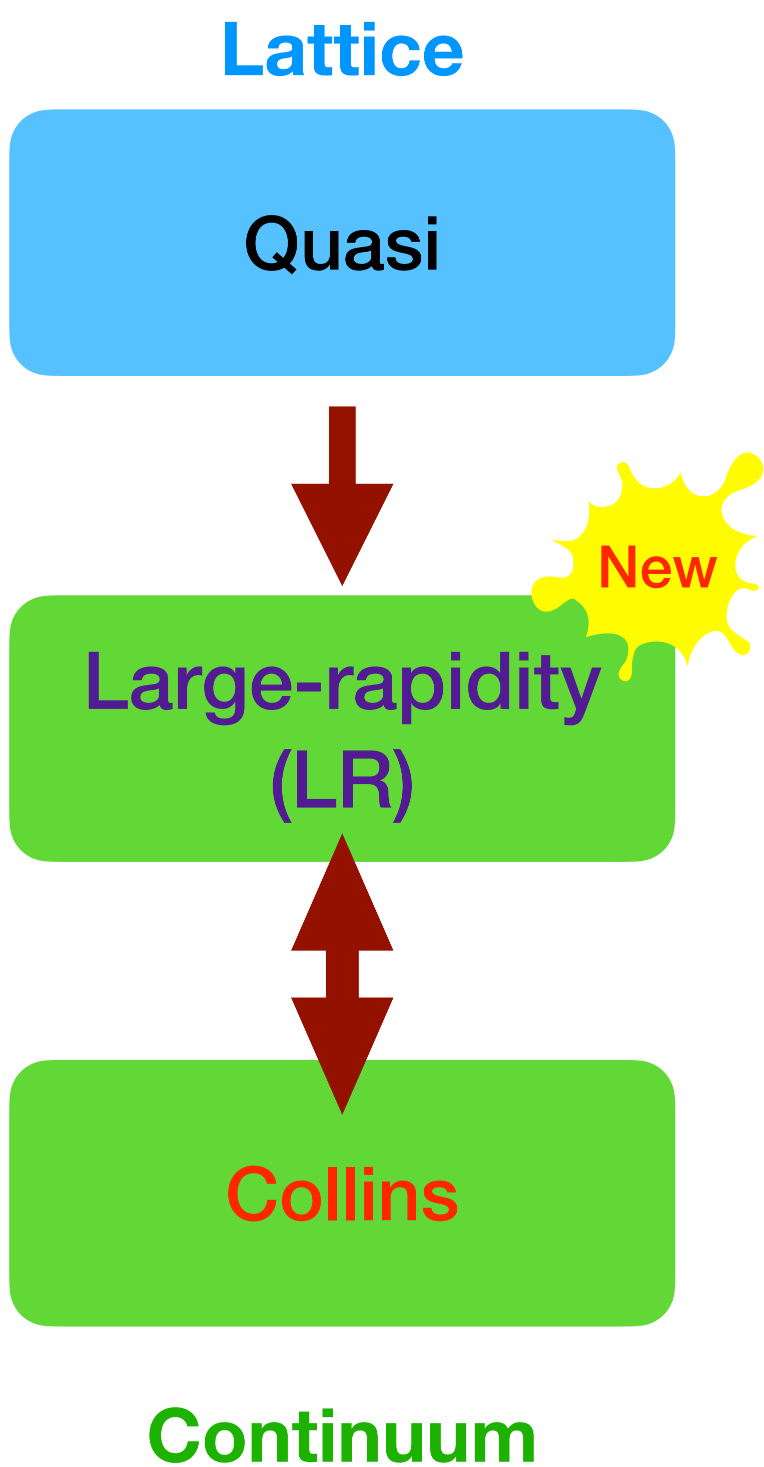
Same matrix elements, but different orders of UV limits

Perturbative matching in LaMET!

$$f_i(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon \to 0} Z_{\mathrm{UV}} \lim_{y_B \to -\infty} \frac{B_i}{\sqrt{S^q}}$$

Ebert, Schindler, Stewart and YZ, JHEP **04**, 178 (2022).

Factorization relation with the TMDs



$$\tilde{f}_i(x, \mathbf{b}_T, \mu, \tilde{\zeta}, \tilde{P}^z) = \lim_{\tilde{P}^z \gg m_N} \lim_{a \to 0} \tilde{Z}_{\rm UV} \frac{\tilde{B}_i}{\sqrt{S^q}}$$

Lorentz invariance $\quad y_{\tilde{P}} = y_P - y_B$

$$f_i^{\rm LR}(x, \mathbf{b}_T, \mu, \zeta, y_P - y_B) = \lim_{-y_B \gg 1} \lim_{\epsilon \to 0} Z_{\rm UV}^{\rm LR} \frac{B_i}{\sqrt{S^q}}$$

Same matrix elements, but different orders of UV limits

Perturbative matching in LaMET!

$$f_i(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon \to 0} Z_{\rm UV} \lim_{y_B \to -\infty} \frac{B_i}{\sqrt{S^q}}$$

Ebert, Schindler, Stewart and YZ, JHEP **04**, 178 (2022).

Factorization relation with the TMDs

- **Factorization of quasi-TMD:**

$$\tilde{f}_{q/h}(x, \mathbf{b}_T, \mu, \tilde{\zeta}, x\tilde{P}^z) = C(x\tilde{P}^z, \mu)\exp\left[\frac{1}{2}K^q(\mu, b_T)\ln\frac{\tilde{\zeta}}{\zeta}\right] f_{i/h}(x, \mathbf{b}_T, \mu, \zeta) + \mathcal{O}(y_{\tilde{P}}^{-k} e^{-y_{\tilde{P}}})$$

$\tilde{\zeta} = x^2 m_N^2 e^{2\tilde{y}_P + 2y_B - 2y_n}$

Matching coefficient

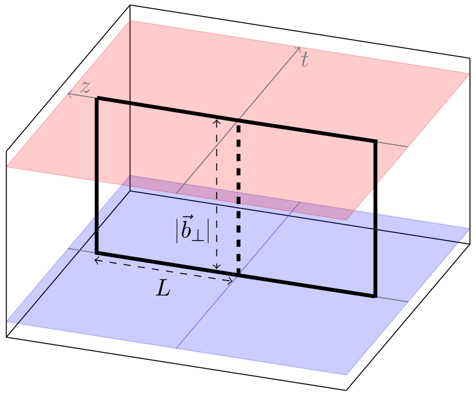
Warning:
soft function still not calculable on the lattice

- **Factorization of naive quasi-TMD:**

$$\tilde{f}_{i/h}^{\text{naive}} = \lim_{a\to 0} \tilde{Z}_{\text{uv}} \tilde{B}_{i/h} \Big/ \sqrt{\tilde{S}^q_{\text{naive}}}$$

$$\frac{\tilde{f}_{i/h}^{\text{naive}}}{\sqrt{S_r^q(b_T, \mu)}} = C(x\tilde{P}^z, \mu)\exp\left[\frac{1}{2}K^q(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right] \times f_{i/h}\left\{1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda_{\text{QCD}}^2}{(x\tilde{P}^z)^2}\right]\right\}$$

$\tilde{S}^q_{\text{naive}}$:



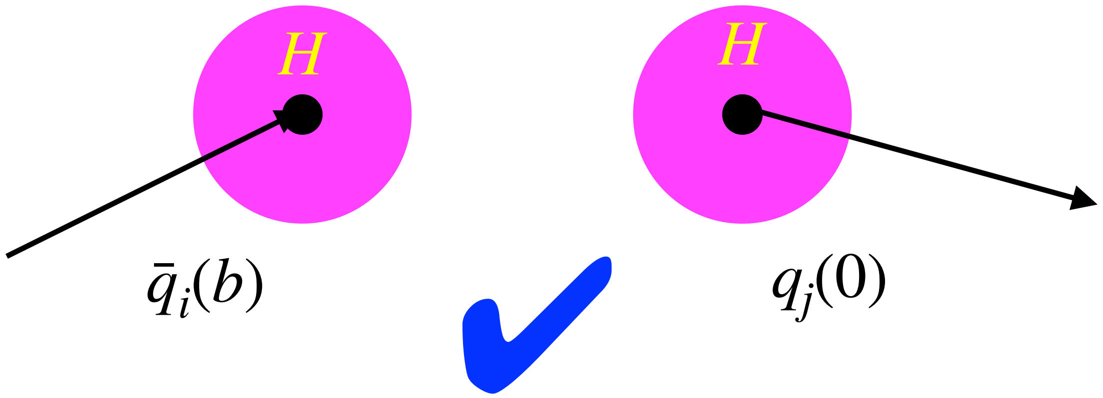
Directly calculable on the lattice!

Reduced soft function ✓

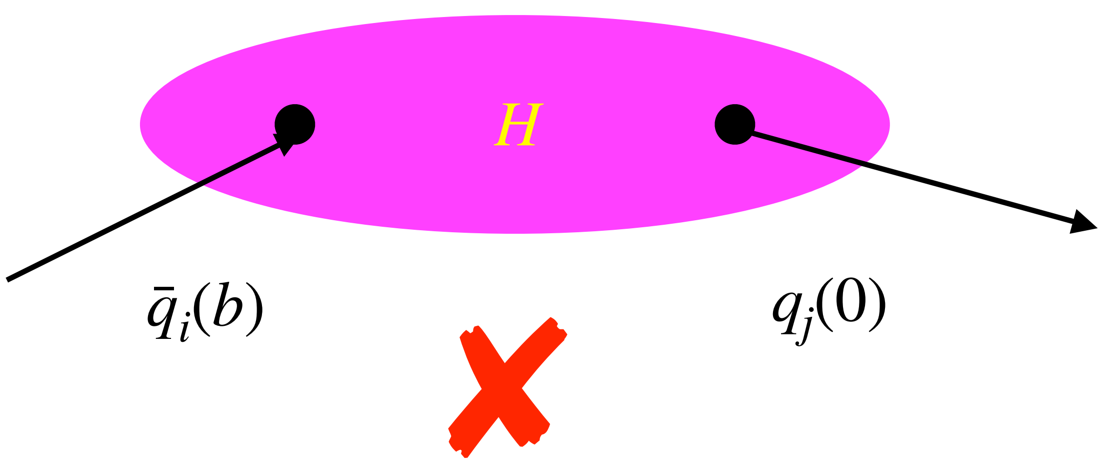
Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020).

- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ji, Jin, Yuan, Zhang and YZ, PRD99 (2019);
- Ebert, Stewart, YZ, PRD99 (2019), JHEP09 (2019) 037;
- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Vladimirov and Schäfer, PRD 101 (2020);
- **Ebert, Schindler, Stewart and YZ, JHEP 04, 178 (2022).**

Backup slides



i, j **(including spinor indices) remain intact**



$\propto \delta_{ij}$ **Can mix with singlet channel and with gluons**

$$b^2 = -b_z^2 - b_T^2 < b_T^2 \sim 1/\Lambda_{\rm QCD}^2$$

Hard particles cannot propagate that far!

Collins-Soper kernel from lattice QCD

$$K^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{C_{\rm ns}(\mu, xP_2^z)\, \tilde{B}_{\rm ns}(x, \mathbf{b}_T, \mu, P_1^z)}{C_{\rm ns}(\mu, xP_1^z)\, \tilde{B}_{\rm ns}(x, \mathbf{b}_T, \mu, P_2^z)} + \text{power corrections}$$

Studying CS kernel through quasi-TMDs suggested in

- Ji, Sun, Xiong and Yuan, PRD91 (2015);

The concrete formalism first derived in

- Ebert, Stewart and YZ, PRD 99 (2019).

Does not depend on the external hadron state, could be calculated with pion TMD or wave function (vacuum to pion amplitude) for simplicity;

- Shanahan, Wagman and YZ, PRD 102 (2020);
- Ebert, Stewart and YZ, PRD 99 (2019);
- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020).