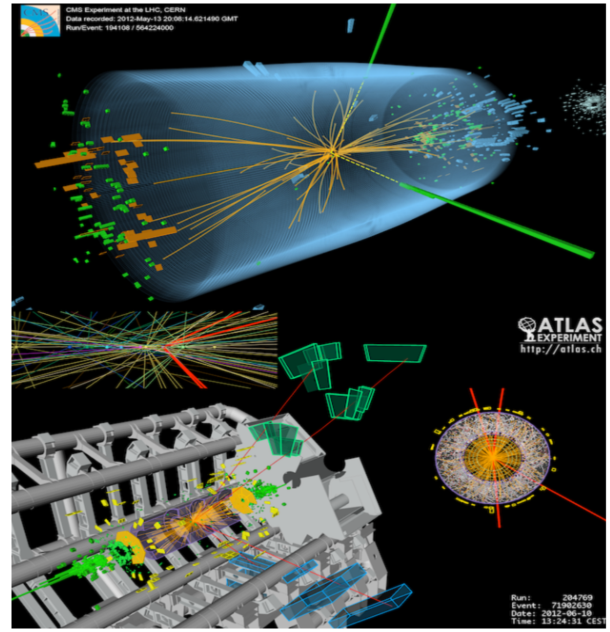
Hadrons, Type II Superconductors, and Cosmological Constant

PRD104,076010 (2021)
[arXiv:2103.15768]

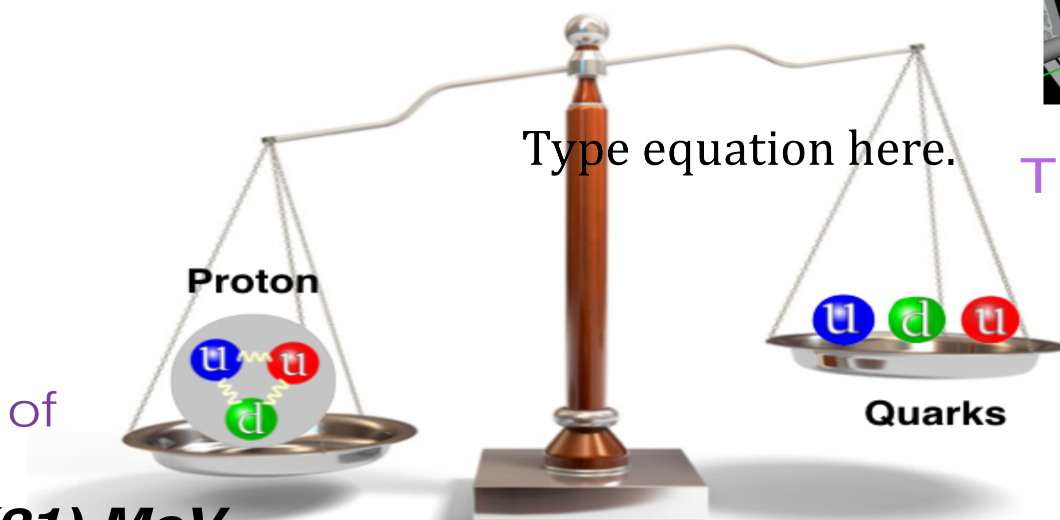
- Common features of these systems in terms of their condensate and confinement.
- Rest energy and equilibrium correspondence principle
- Puzzle of pion mass and trace anomaly

Aug. 30, 2022
Orlando

Motivation



Where does the proton mass come from, and how ?



Type equation here.

The Higgs boson make the u/d quark having masses (2GeV MS-bar):

$m_u = 2.08(9)$ *MeV*

$m_d = 4.73(12)$ *MeV*

Laiho, Lunghi, & Van de Water, Phys.Rev.D81:034503,2010

But the mass of the proton is ***938.272046(21) MeV.***

~100 times of the sum of the quark masses!

Mass cannot be decomposed in terns of its constituents.

Example: $e^+e^- \rightarrow \gamma\gamma$ (Okun doi:10.1134/1.1358478)

Mass is not additive, but energy is.

Mass and Rest Energy

- $E_0 = m c^2$ (Einstein 1905, $m^2 = E^2 - p^2$)
- $e^+ e^- \rightarrow \gamma\gamma$ ($m_{\gamma\gamma} = 2m_e$)
- E and p are additive, not mass
- Mass from trace of energy-momentum tensor

$$\langle p|T^{\mu\nu}|p\rangle = 2p^\mu p^\nu$$
$$\langle p|T^{\mu\mu}|p\rangle / \langle p|p\rangle = M$$
$$T^\mu_\mu = \sum_f m_f \bar{\psi}_f \psi_f + \left[\sum_f m_f \gamma_m(g) \bar{\psi}_f \psi_f + \frac{\beta(g)}{2g} F^{\alpha\beta} F_{\alpha\beta}\right]$$

scale and frame independent

- Rest energy from Hamiltonian or gravitational form factors

- L. Okun
doi:10.1134/1.1358478

Rest Energy - Experimentally Measurable Decomposition

- Separate the EMT into traceless part and trace part

$$T^{\mu\nu} = \overline{T}^{\mu\nu} + \frac{1}{4}\eta^{\mu\nu}(T^{\rho}_{\rho})$$

[Ji]

- Hamiltonian -- $H = \int d^3\vec{x}\, T^{00}(x)$

$$H_q(\mu) = \int d^3\vec{x}\,\Big(\frac{i}{4}\sum_f \bar{\psi}_f \gamma^{\{0}\overleftrightarrow{D}^{0\}}\psi_f - \frac{1}{4}T^{\mu}_{q\,\mu}\Big),$$

Quark momentum fraction (scale dependent)

$$H_g(\mu) = \int d^3\vec{x}\,\frac{1}{2}(B^2 + E^2),$$

Glue momentum fraction (scale dependent)

$$H_{tr} = \int d^3\vec{x}\,\frac{1}{4}T^{\mu}_{\mu}.$$

- Rest energy -- $E_0 = M = \langle H_{q_f}(\mu)\rangle + \langle H_g(\mu)\rangle + \langle H_{tr}\rangle,$

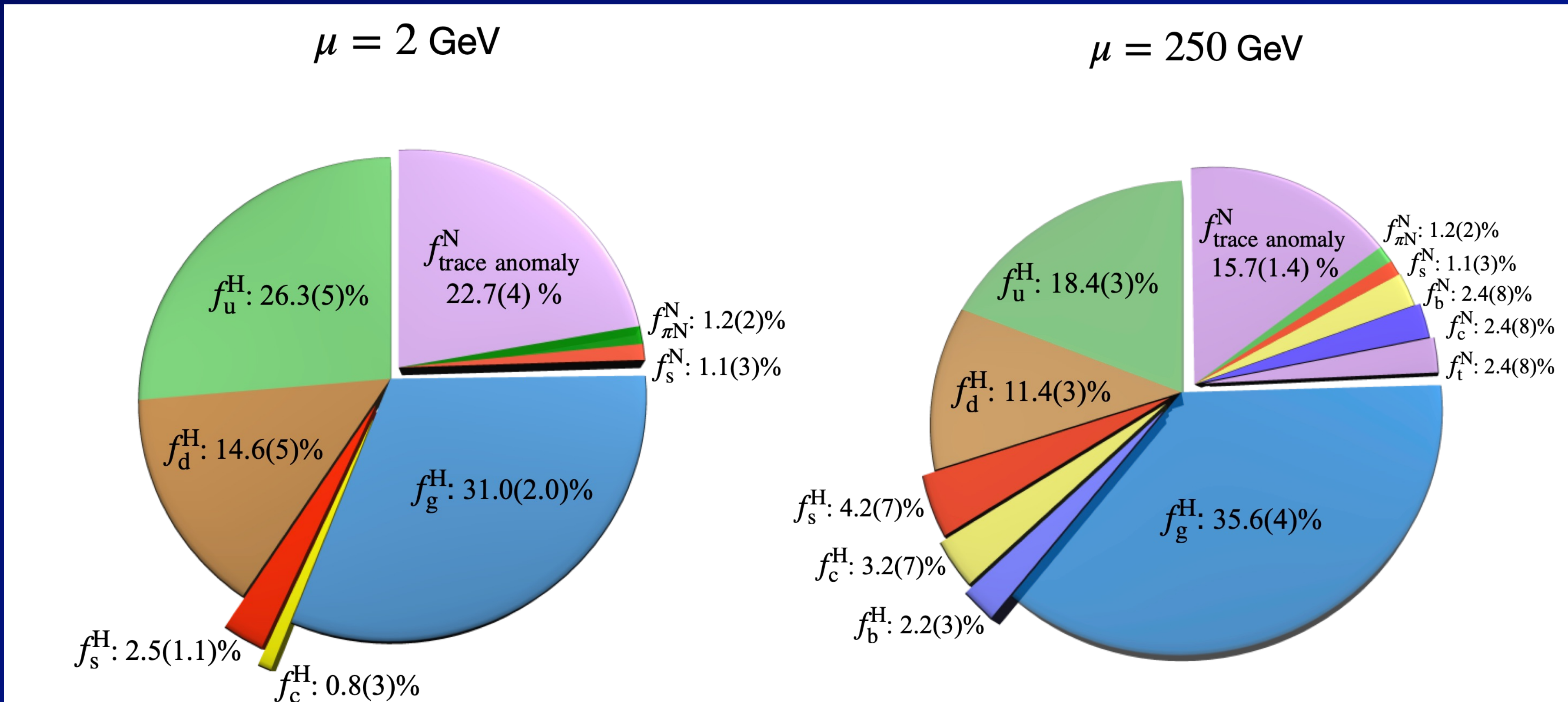
$$\langle H_{q_f}(\mu)\rangle = \frac{3}{4}\sum_f \langle x\rangle_f(\mu) M, \qquad \langle H_g(\mu)\rangle = \frac{3}{4}\langle x\rangle_g(\mu) M,$$

<x> - momentum fraction

$$\langle H_{tr}\rangle = \frac{1}{4}M = \frac{1}{4}\Big[\sum_f m_f(1+\gamma_m)\langle\bar{\psi}\psi\rangle + \frac{\beta}{2g}\langle F^{\alpha\beta}F_{\alpha\beta}\rangle\Big].$$

Y.B. Yang et al (χQCD), PRL 121, 212001 (2018)
Physic 11, 118 (2018);
ScienceNews, Nov. 16 (2018)

Rest Energy Decomposition from Hamiltonian



$$f_f^H = \langle H_q \rangle / M = \frac{3}{4}\langle x \rangle_f(\mu), \qquad f_g^H = \langle H_g \rangle / M = \frac{3}{4}\langle x \rangle_g(\mu),$$

$$f_{\pi N}^N = \frac{1}{4}\frac{\sigma_{\pi N}}{M}, \qquad f_s^N = \frac{1}{4}\frac{\sigma_s}{M}, \qquad f_{\text{trace anomaly}}^N = \frac{1}{4}\frac{\langle H_{\text{ta}} \rangle}{M}$$

Y.B. Yang et al (χ QCD) [arXiv: 1511.15089]

Momentum fractions from CT18 (T.J. Hou et al, PRD, arXiv:1912.10053) at $\mu = 2$ GeV and 250 GeV.

Rest Energy from Gravitational FF

- Gravitational Form factors from the EMT matrix elements

$$\begin{aligned}\langle P'|T^{\mu\nu}_{q,g}(\mu)|P\rangle/2M_N &= \bar{u}(P')[T_{1_{q,g}}(q^2,\mu)\gamma^{(\mu}\bar{P}^{\nu)} + T_{2_{q,g}}(q^2,\mu)\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}q_\alpha}{2M_N} \\ &+ D_{q,g}(q^2,\mu)\frac{q^\mu q^\nu - g^{\mu\nu}q^2}{M_N} + \bar{C}_{q,g}(q^2,\mu)M_N\eta^{\mu\nu}]u(P)\end{aligned}$$

  - T_1 and T_2 : momentum and angular momentum fraction [Ji]
  - D term: deformation of space = elastic property - [Polyakov]
  - C-bar term: pressure-volume work - [Lorce, Liu]

$$E_0(q,g) = \langle P|(T^{00}_{q,g})_{\mathrm{M}}(\mu)|P\rangle|_{\vec{P}=0}/2M_N = \langle x\rangle_{q,g}(\mu)M_N + \bar{C}_{q,g}(0,\mu)M_N$$

$$\langle P|T^{ii}_{q,g}(0,\mu)|P\rangle = -3\bar{C}_{q,g}(0,\mu) = -\langle P|(T_{q,g})^\mu_\mu|P\rangle + \langle P|T^{00}_{q,g}(\mu)|P\rangle$$

$$\bar{C}_q + \bar{C}_g = \tfrac{1}{4}(\textstyle\sum_f f_f^N + f_a^N) - \tfrac{1}{4}(\langle x\rangle_q + \langle x\rangle_q) = 0$$

$$\partial_\nu T^{\mu\nu} = 0$$

$$E_0 = \tfrac{3}{4}(\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))M_N + \tfrac{1}{4}(\textstyle\sum_f f_f^N + f_a^N)M_N$$

Same as from Hamiltonian

<x> -- momentum fraction, f_f – sigma term fraction,
f_a – trace anomaly fraction

Trace Anomaly and String Tension in Charmonium

- Heavy quarkonium is confined by a linear potential.
- Constant vacuum energy density and flux tube

$$V(r) = |\epsilon_{vac}|\, A\, r = \sigma\, r$$

- Infinitely heavy quark with Wilson loop

Dosch, Nachtmann, Rueter - 9503386; Rothe - 9504102

$$V(r) + r\frac{\partial V(r)}{\partial r} = \frac{\langle \frac{\beta}{2g}(\int d^3\vec{x}\, F^2)\, W_L(r,T)\rangle}{\langle W_L(r,T)\rangle}.$$

- For charmonium

$$2\sigma\langle r\rangle = \langle H_\beta\rangle_{\bar{c}c} = \frac{\langle \bar{c}c|\frac{\beta}{2g}\int d^3\vec{x}\, F^2|\bar{c}c\rangle}{\langle \bar{c}c|\bar{c}c\rangle}$$

$$\langle H_\beta\rangle_{\bar{c}c} = M_{\bar{c}c} - (1+\gamma_m)\langle H_m\rangle_{\bar{c}c}.$$

- Lattice calculation of charmonium (W. Sun et al.,2012.06228)

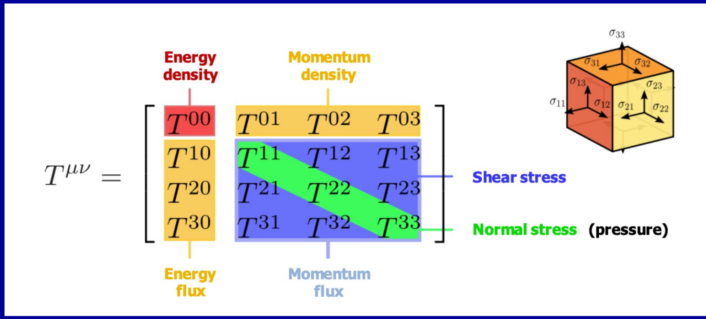
- $\langle H_\beta\rangle_{\bar{c}c} = 199\,\mathrm{MeV}$ → σ = 0.153 GeV^2

- Cornell potential fit of charmonium → σ = 0.164(11) GeV^2

Mateu:2018zym

Trace Anomaly and Gluon Condensate

- What is trace anomaly? What dynamical role does it play, if any?
  - Note $\langle P|(T^{ii}_{q,g})_{\rm M}(\mu)|P\rangle|_{\vec{P}=0}/2M_N = -3\bar{C}_{q,g}(0,\mu)M_N$
  - $\frac{1}{3}\langle P|(T^{ii}_{q,g})|P\rangle|_{\vec{P}=0}$ is pressure-volume work at equilibrium
  - The pressure balance equation -- equilibrium

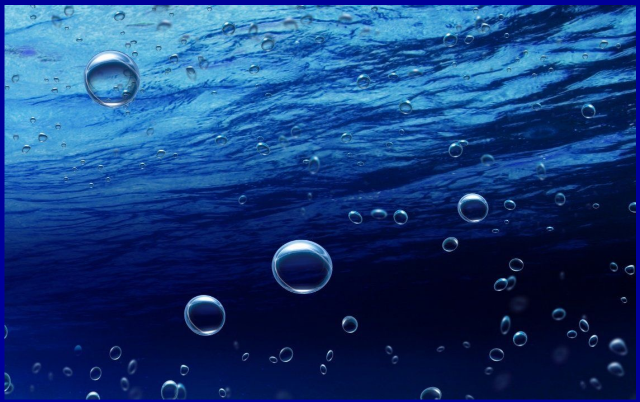


$$PV = -\frac{dE}{dV}V = -(\bar{C}_q + \bar{C}_g) = \frac{1}{4}(\langle x\rangle_q + \langle x\rangle_q) - \frac{1}{4}(\sum_f f_f^N + f_a^N) = 0$$

- Nucleon is a bubble in the sea of gluon condensate, where

$$\langle H_a\rangle = -\epsilon_{vac}V,$$

$$\epsilon_{vac} = \frac{\beta(g)}{2g}\langle 0|F^{\alpha\beta}F_{\alpha\beta}|0\rangle < 0$$



$$E_0 = E_T + E_S,$$

$$E_S = \frac{1}{4}[\langle H_m\rangle + \langle H_a\rangle] \propto V,$$

$$E_T = \langle H_{q_f}(\mu)\rangle + \langle H_g(\mu)\rangle \propto V^{-1/3}$$

$$P_{\rm total}V = -V\frac{d\,E_0}{d\,V} = -E_S + \frac{1}{3}E_T = 0$$

$$-\epsilon_{\rm vac}V$$

- (MIT bag model, E_0 = BV+ $\Sigma_{q,g}$/R – Equation of State)

Rest Energy-Equilibrium Correspondence Principle

- Rest energy $E_0 = \frac{3}{4}(\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))M_N + \frac{1}{4}(\sum_f f_f^N + f_a^N)M_N$
- Equilibrium -- Pressure-volume work balanced

$$PV = \frac{1}{4}(\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))M_N - \frac{1}{4}(\sum_f f_f^N + f_a^N)M_N = 0$$

  - MIT bag model (E_0 = BV+ $\Sigma_{q,g}$/R), pressure: $\partial E_0/\partial R = 0$
  - Skyrmionium:

    Derrick's theorem ($r \to \lambda r$)

$$\mathcal{L}_2 = f_\pi^2 Tr\left[\partial_\mu U^\dagger \partial^\mu U\right] \longrightarrow \lambda$$

$$\mathcal{L}_4 = \frac{1}{e^2} Tr\left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U\right]^2 \longrightarrow 1/\lambda$$

  - Potential models: kinetic energy ↘ r, potential energy ↗ r.
- Other decomposition formulas
  - Gravitational FF without $\overline{C}$

$$E_0 = (\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))M_N$$

No potential energy, no relation to pressure

- Virial Theorem
  - Coulomb potential: <H >= <T> + <V>
  - E = - <T>, E = ½ <V> are not good physics explanation of the decomposition of the binding energy.
  - Equation of state – E = a/R^2 + b/R → 2<T> + <V> = 0 (equilibrium)

Trace Anomaly and Cosmological Constant

- Vacuum energy density is indeed a constant which is analogous to the cosmological constant in the $g^{\mu\nu}$ term as Einstein introduced for a static universe.

$$R_{\mu\nu} + \frac{1}{2} R\, g_{\mu\nu} - \Lambda\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$$\Lambda = 4\pi G \rho$$

for a static Universe

- Friedman equation for the accelerating expansion of the universe

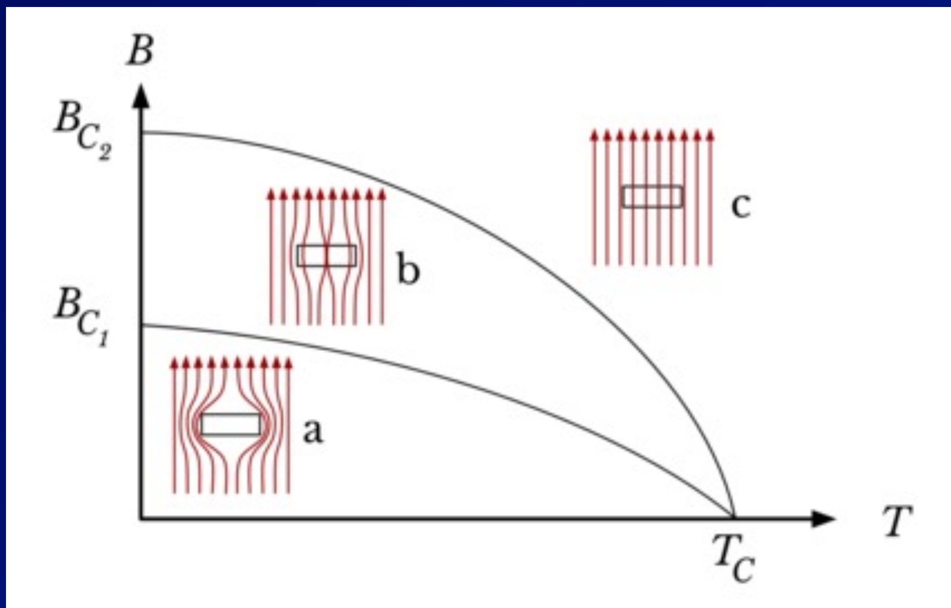
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3}$$

$$g^{00}\Lambda \longrightarrow \rho_{\mathrm{vac}} = \frac{\Lambda}{8\pi G}$$

$$g^{ii}\Lambda \longrightarrow P_{\mathrm{vac}} = -\frac{\Lambda}{8\pi G}$$

- Pressure: anti-gravity

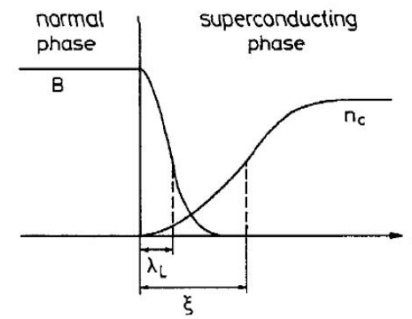
Type II Superconductor



Physics of type I and II superconductors

- "London Penetration Depth" λ_L
  is the e-fold decay length of the magnetic field from the superconductor skin due to the Meissner effect (in the range of 10 to 10^3 nm)

- "Coherence Length" ξ
  the average size of Cooper in the superconductor (in the range of 10 to 100 nm, I.e. much larger than the inter-atomic distance typically of 0.1 to 0.3 nm.



Ginzburg-Landau Parameter κ

$$\kappa = \lambda_L/\xi \Rightarrow \begin{cases} k < 1/\sqrt{2} \Leftrightarrow \text{type I} \\ k > 1/\sqrt{2} \Leftrightarrow \text{type II} \end{cases}$$

material	In	Pb	Sn	Nb
λ_L [nm]	24	32	≈ 30	32
ξ [nm]	360	510	≈ 170	39

Ginzburg–Landau equations

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m}(-i\hbar\nabla - 2e\mathbf{A})^2\psi = 0$$

$$\nabla \times \mathbf{B} = \mu_0\mathbf{j} \;\; ; \;\; \mathbf{j} = \frac{2e}{m}\mathrm{Re}\{\psi^*(-i\hbar\nabla - 2e\mathbf{A})\psi\}$$

$$|\psi|^2 = n_s$$

London penetration depth

$$\lambda_L = \sqrt{\frac{m}{4\mu_0 e^2 \psi_0^2}}$$

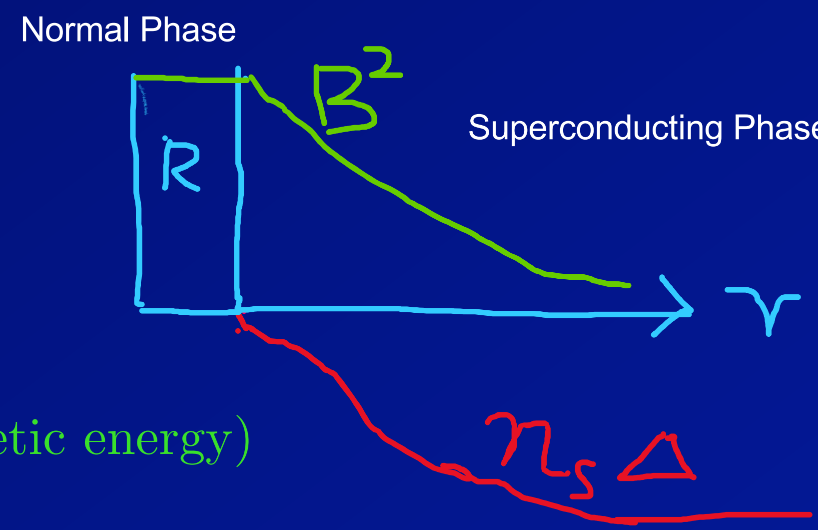
Coherent length ξ

$$\kappa = \lambda_L/\xi$$

Energetics and Pressure

- Type II superconductor

$$F = F_s + F_B + F_{sc}$$
$$F_s = \text{ cost of condensation energy}$$
$$F_B = \int dv\, B^2/2\mu_0 \ \text{(magnetic energy)}$$
$$F_{sc} = 1/2 \int dv\, \lambda_L^2 J_s \cdot J_s \ \text{(supercurrent kinetic energy)}$$



- Variational model (J.R. Clem, Jour. Low Temp. Phys. 18, 5/6 (1975))

$$\frac{|\psi|^2}{n_0} = \frac{n_s}{n_0} = \frac{\rho^2}{\rho^2 + R^2} \underset{\rho\to\infty}{\longrightarrow} 1$$

$$\frac{1}{\sqrt{2}H_c}\frac{E}{l} = \phi_0 H_c'/4\pi \ \text{ where } \phi_0 = hc/2e,\ \sqrt{2}H_c = \kappa\phi_0/2\pi\lambda_L^2$$

$$H_c' = \underbrace{\kappa R'/8}_{F_s} + \underbrace{1/8\kappa + K_0(R')/2\kappa R'\, K_1(R')}_{F_B + F_{sc}}, \ \text{ where } \ R' = R/\lambda_L$$

$$\partial H_c'/\partial R' = 0$$

- Equation of State

Equilibrium

SC, Hadrons, Cosmos

- Type II Superconductor

$$P_s = -\frac{\partial F_s}{\partial V} < 0, \quad P_{B+sc} = -\frac{\partial F_B + F_{sc}}{\partial V} > 0$$

- Hadrons

$$P_{tr} = -\frac{\partial E_S}{\partial V} < 0, \quad P_{q+g} = -\frac{\partial E_T}{\partial V} > 0$$

- Cosmos $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3}, \quad \Lambda > 0 \longrightarrow \frac{\ddot{a}}{a} > 0$

- The common theme is the existence of a condensate.
- Hadrons: condensates from breaking of conformal and chiral symmetries. SC: Cooper pair condensate from gauge symmetry breaking. Cosmos: ?

Pion Mass Puzzle

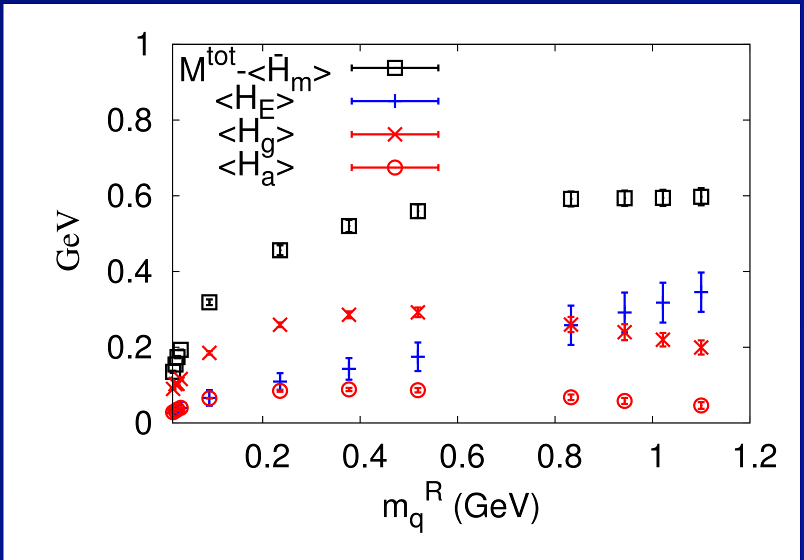
- Pion mass in terms of trace of EMT

$$m_\pi = m\langle\pi|\bar\psi\psi|\pi\rangle + \langle\pi|\frac{\beta}{2g}G^2_{\mu\nu} + m\gamma_m\bar\psi\psi|\pi\rangle$$

- Gellmann-Oakes-Renner relation

$$m_\pi^2 = -2m\langle\bar\psi\psi\rangle/f_\pi^2, \quad m_\pi^2 \propto m$$

- $\langle\pi|\bar\psi\psi|\pi\rangle \propto 1/\sqrt{m}$ But, why should the trace anomaly be proportional to $\sqrt{m}$? V → 0 ?



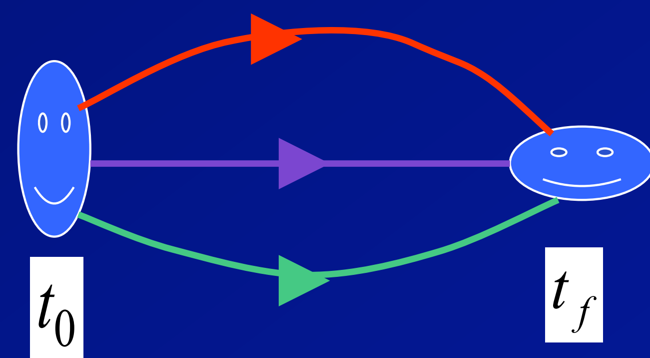
Y.B. Yang et al. (χQCD), PRD (2015); 1405.4440

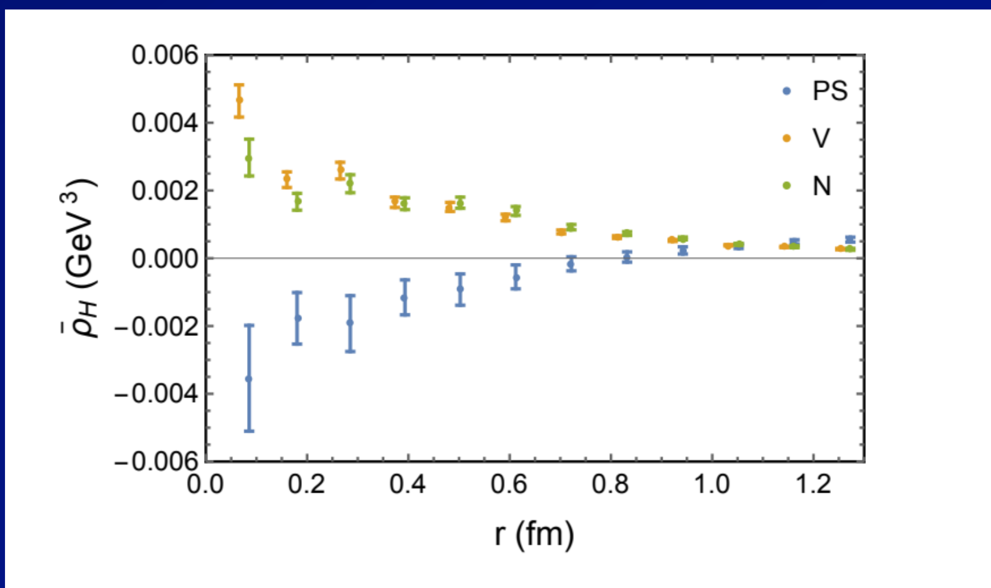
Trace anomaly Distribution

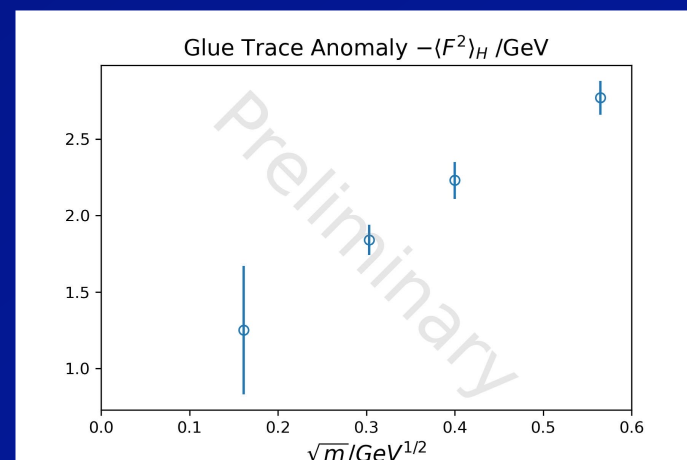
- Distribution as a function of the relative distance between the glue operator and the sink positions.

- F. He, P. Sun and Y.B. Yang (χQCD) (PRD 2021, 2101.04942)

$$F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2$$



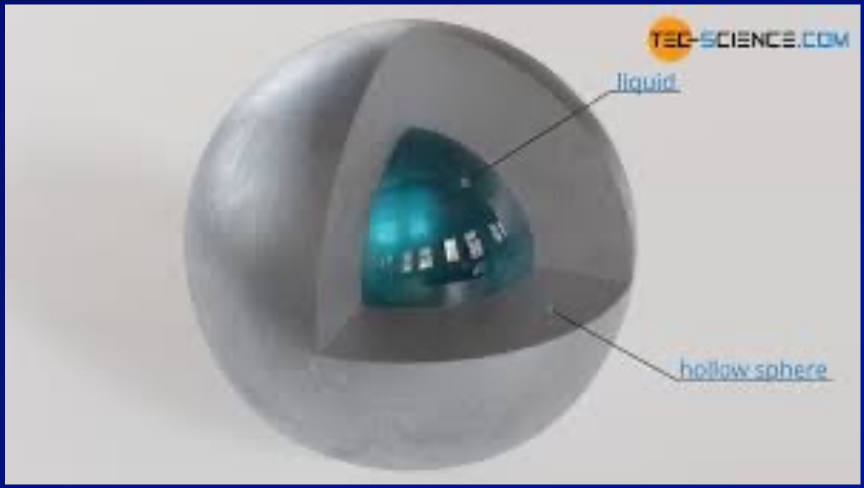


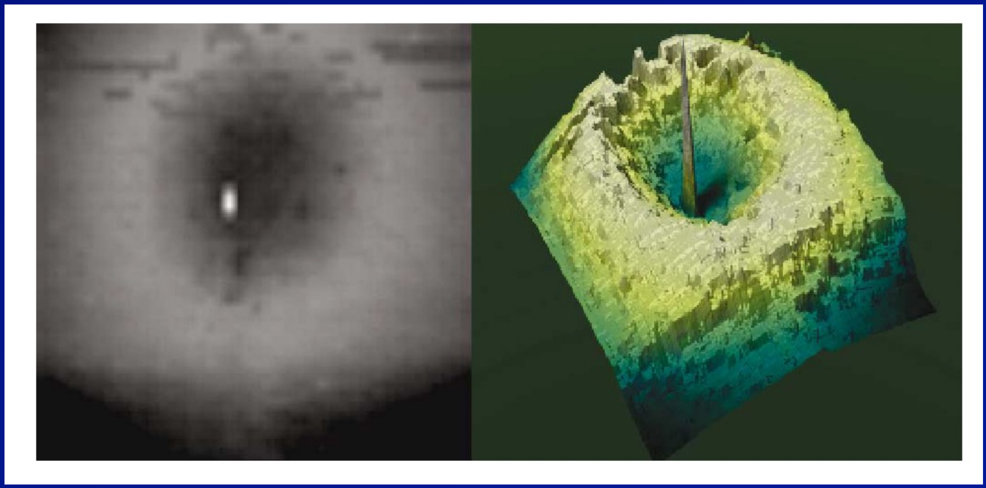


- It changes sign in pion so that the integral approaches zero at the chiral limit.

Pion as a Ring-shaped Type II Superconductor

A. Groeger et al., PRL 90, 237004 (2003)



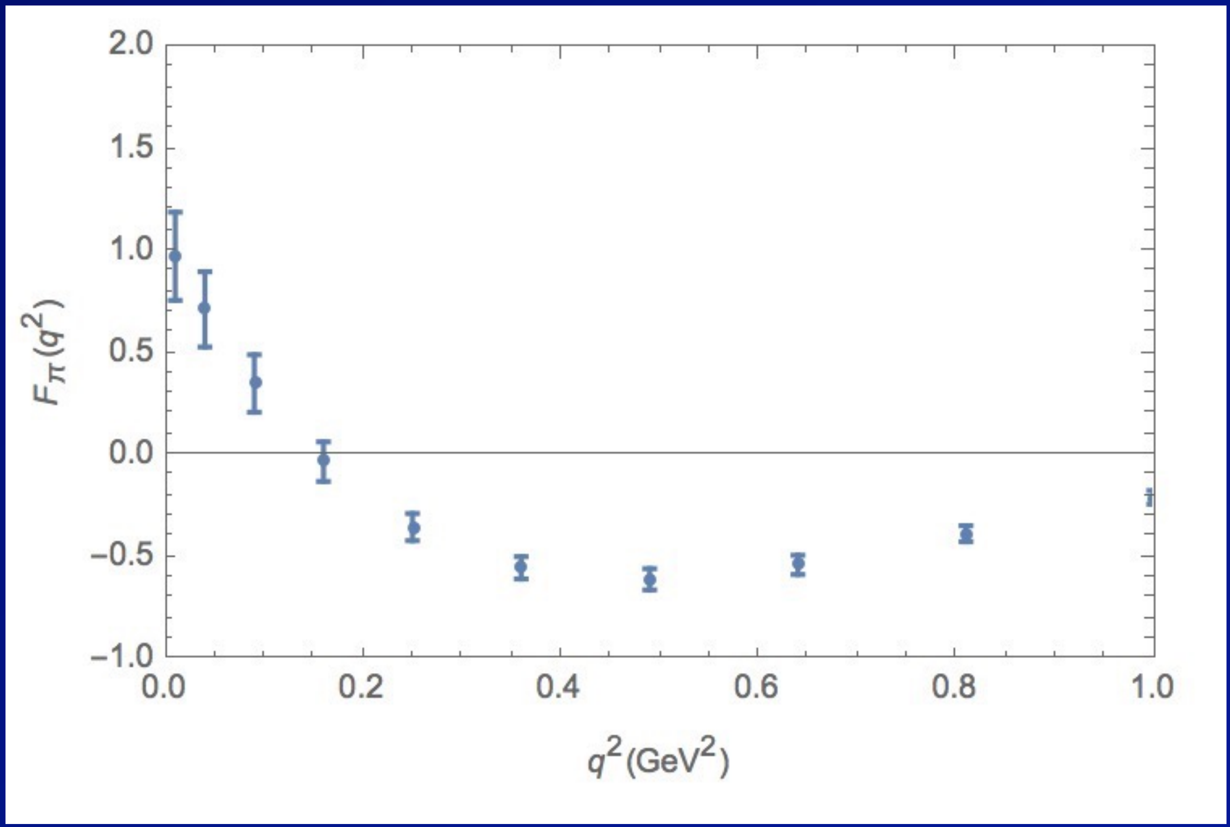


Pion with shell of positive trace anomaly and an inner core with gluon condensate (negative trace anomaly.)

Niobium, normal conducting vortex ring around a superconducting region,

Pion trace anomaly FF

Preliminary



m_π = 340 MeV

X.B. Tong, J.P. Ma and F. Yuan
arXiv:2203.13493

Summary and Challenges

- From femto-scale to micro-scale to that of the cosmos, Nature seems to choose the same mechanism for confinement or acceleration. Rest energy-stability correspondence.
- $m_q \leftarrow$ Higgs mechanism
- Quark condensate $\leftarrow$ chiral symmetry breaking (restoration at T and μ)
- Trace anomaly (confinement) $\leftarrow$ conformal symmetry breaking (conformal phases with multi-flavors and SU(N); finite $T > T_c$)
- Chiral symmetry breaking and conformal symmetry breaking are linked in the case of the pion trace anomaly distribution.
- String theory invented in hadron physic finds its home in quantum gravity.
- Cosmological constant introduced in general relativity is relevant to hadron physics.
- Challenges for EIC is to measure the trace anomaly form factors for the proton and, particularly, the pion.
- Glue condensate (trace anomaly) is an order parameter for confinement – deconfinement transition.