x-dependent GPDs from lattice QCD

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Supported by the National Science Center of Poland
SONATA BIS grant No. 2016/22/E/ST2/00013 (2017-2022)
OPUS grant No. 2021/43/B/ST2/00497 (2022-2026)

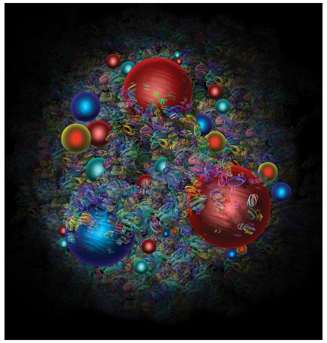
Outline:

Introduction
GPDs from lattice:
– how to access
– twist-2 GPDs
– twist-3 GPDs
Prospects/conclusion

Many thanks to my Collaborators for work presented here:

C. Alexandrou, S. Bhattacharya, M. Constantinou, J. Dodson, X. Gao, K. Hadjiyiannakou, K. Jansen, A. Metz, S. Mukherjee, A. Scapellato, F. Steffens, Y. Zhao

Generalized parton distributions (GPDs)



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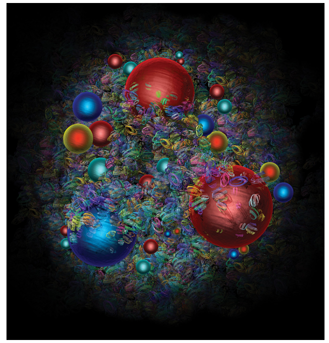
to understand details of 3D nucleon structure.

Particularly important in the context of EIC launch.



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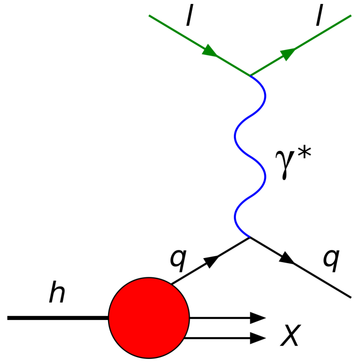
- related to matrix elements with same incoming/outgoing hadron state,
- probed in deep inelastic scattering (DIS) – $ep \longrightarrow eX$.



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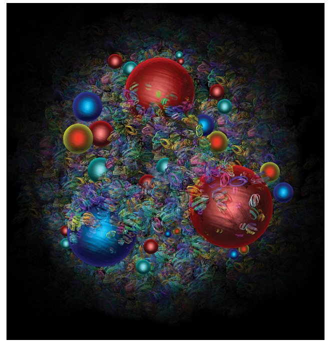

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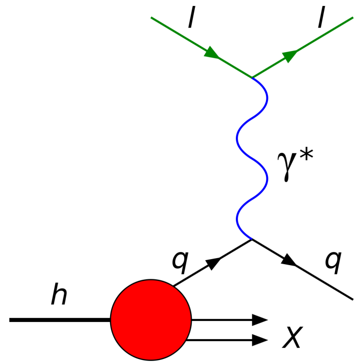

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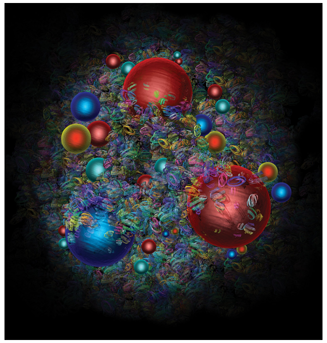

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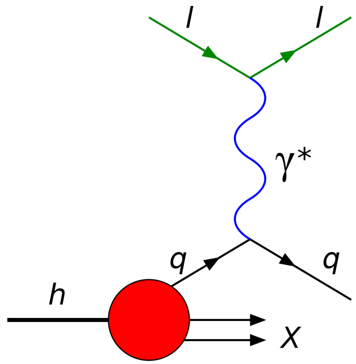

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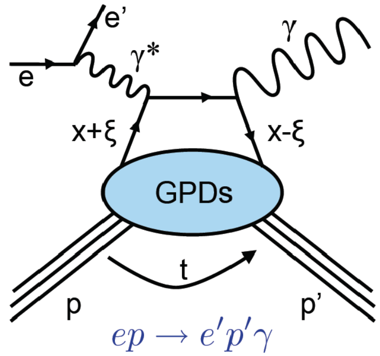
Adding momentum transfer is a natural generalization, leading to **generalized parton distributions** (GPDs):

- experimentally, require exclusive processes like deeply virtual Compton scattering (DVCS) – $ep \longrightarrow e'p'\gamma$,
- reflect spatial distribution of partons in the transverse plane,
- contain information on mechanical properties of hadrons,
- wealth of information on the hadron spin,
- reduce to PDFs in the forward limit, e.g. $H(x, 0, 0) = q(x)$,
- moments of GPDs are form factors, e.g. $\int dx H(x, \xi, t) = F_1(t)$.



GPDs from Lattice QCD

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  1. Set of gauge field configurations on which to measure observables.
     QCD d.o.f.'s put on a **Euclidean** lattice
     ⋆ quarks $\rightarrow$ sites
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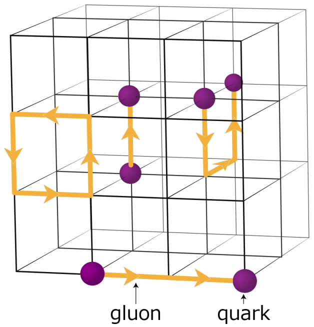
     typical lattice parameters:
     $L/a = [32, 96]$, $a \in [0.04, 0.15]$ fm, $m_\pi \in [135, 500]$ MeV
     $\Rightarrow$ ∞-dim QCD path integral $\rightarrow$ $10^8 - 10^9$-dim integral
     Monte Carlo simulations to evaluate the discretized path integral feasible, but still requires huge computational resources!



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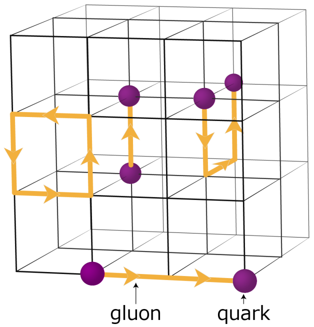

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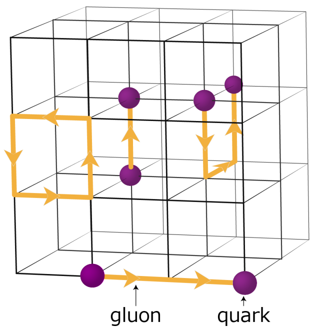

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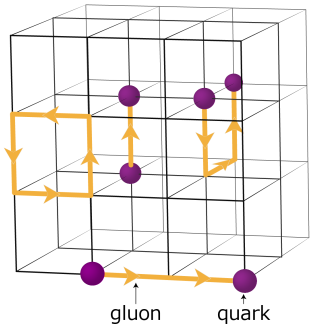

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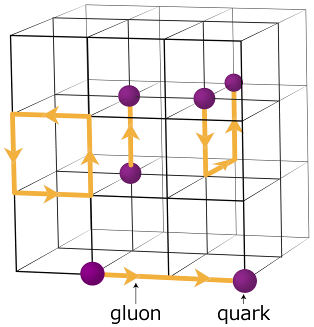


  2. Suitable definition of lattice observables (LCSs).
  3. Optimized computation setup.
  4. A lot of computing time!
  5. Ingenious analysis techniques, with inputs from perturbation theory.

Lattice PDFs/GPDs: dynamical progress

results @ physical pion mass
results extrapolated to physical pion mass
results @ non-physical pion mass

Quasi-distributions

- Nucleon twist-2 PDF
  - unpolarized: ETMC, PRL121(2018)112001; ETMC, PRD99(2019)114504; LP3, arXiv:1803.04393; LPC, PRD101(2020)034020; BNL/MSU, PRD102(2020)074504; ETMC, PRD103(2021)094512; MSU/NTU, 2011.14971
  - helicity: ETMC, PRL121(2018)112001; ETMC, PRD99(2019)114504; LP3, PRL121(2018)242003; BNL/MSU, PRD102(2020)074504; ETMC, PRD103(2021)094512
  - transversity: ETMC, PRD98(2018)091503(R); ETMC, PRD99(2019)114504; LP3, arXiv:1810.05043
- Nucleon twist-3 PDF: ETMC/Temple PRD102(2020)111501(R), PRD104(2021)114510
- Nucleon GPD: ETMC, PRL125(2020)262001; ETMC, PRD105(2022)034501; MSU, PRL127(2021)182001
- Nucleon singlet PDF: ETMC, PRL126(2021)102003; ETMC, PRD104(2021)054503; MSU/LANL, PRD104(2021)094511
- Δ PDF: ETMC/PKU PRD102(2020)014508
- Meson DA: LP3, PRD95(2017)094514; LP3, NPB939(2019)429; MSU/NTU, PRD102(2020)094519; LPC, PRL127(2021)062002
- Pion/Kaon PDF: LP3, PRD100(2019)034505; BNL, PRD100(2019)034516; MSU/NTU/BNU, PRD103(2021)014516; CCNU/BNL/ANL, PRL128(2022)142003
- Pion GPD: MSU/NTU/UR, NPB952(2020)114940

Pseudo-distributions

- Nucleon PDF: HadStruc, PRD96(2017)094503; HadStruc, JHEP12(2019)081; HadStruc, PRL125(2020)232003; ETMC, PRD103(2021)034510; HadStruc, JHEP11(2021)024; HadStruc, JHEP11(2021)148; adStruc, PRD105(2022)034507
- Gluon PDF: MSU, IJMPA36(2021)13; MSU, PLB823(2021)136778; HadStruc, PRD104(2021)094516; MSU, arXiv:2112.03124
- Pion PDF: HadStruc, PRD100(2019)114512; BNL, PRD102(2020)094513

Current-current

- Pion PDF: HadStruc, PRD99(2019)074507; HadStruc, PRD102(2020)054508

Auxiliary light quark

- Pion DA: UR, EPJC78(2018)217; UR, PRD98(2018)094507

Auxiliary heavy quark

- Pion DA: Taiwan/MIT, arXiv:1810.12194; Taiwan/MIT, PRD105(2022)034506

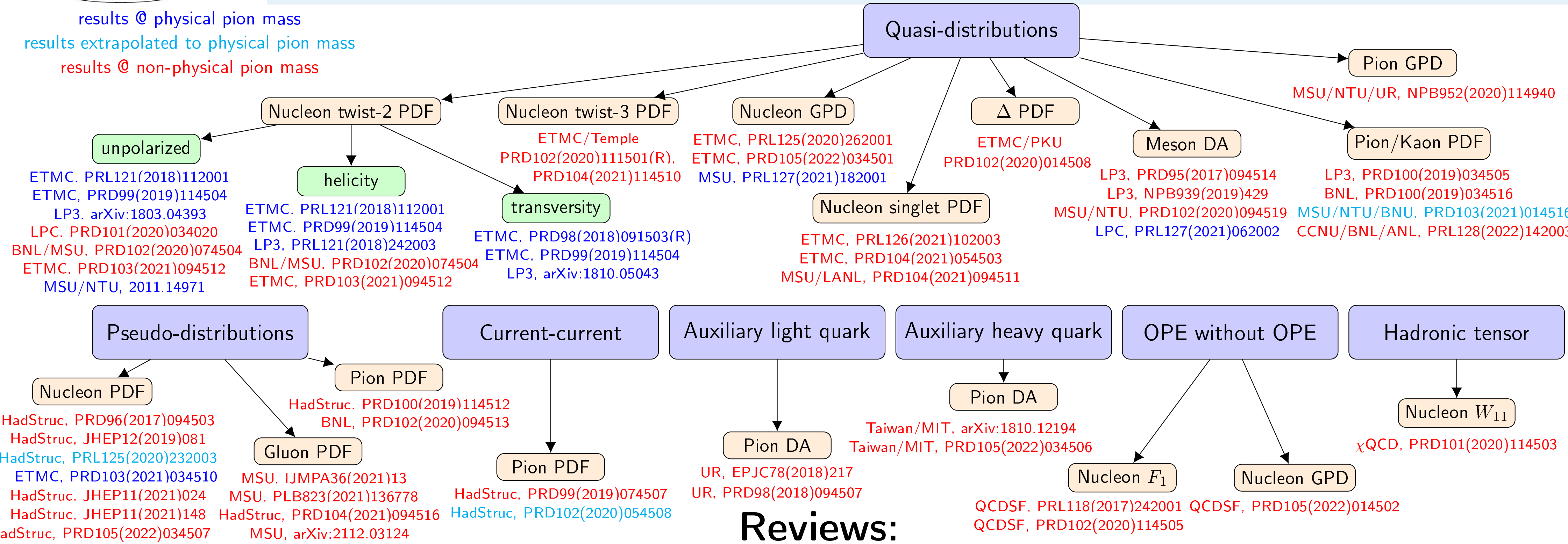
OPE without OPE

- Nucleon F_1: QCDSF, PRL118(2017)242001; QCDSF, PRD102(2020)114505
- Nucleon GPD: QCDSF, PRD105(2022)014502

Hadronic tensor

- Nucleon W_{11}: χQCD, PRD101(2020)114503

Lattice PDFs/GPDs: dynamical progress



Reviews:

K. Cichy, *Progress in x-dependent partonic distributions from lattice QCD*, plenary talk LATTICE 2021, 2110.07440

K. Cichy, *Overview of lattice calculations of the x-dependence of PDFs, GPDs and TMDs*, plenary talk of Virtual Tribute to Quark Confinement 2021, 2111.04552

K. Cichy, M. Constantinou, *A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results*, invited review for a special issue of Adv. High Energy Phys. 2019 (2019) 3036904, 1811.07248

M. Constantinou, *The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD* (would-be) plenary talk of LATTICE 2020, EPJA 57 (2021) 77, 2010.02445

X. Ji et al., *Large-Momentum Effective Theory*, Rev. Mod. Phys. 93 (2021) 035005

M. Constantinou et al., *Parton distributions and LQCD calculations: toward 3D structure*, PPNP 121 (2021) 103908

Quasi-PDFs

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

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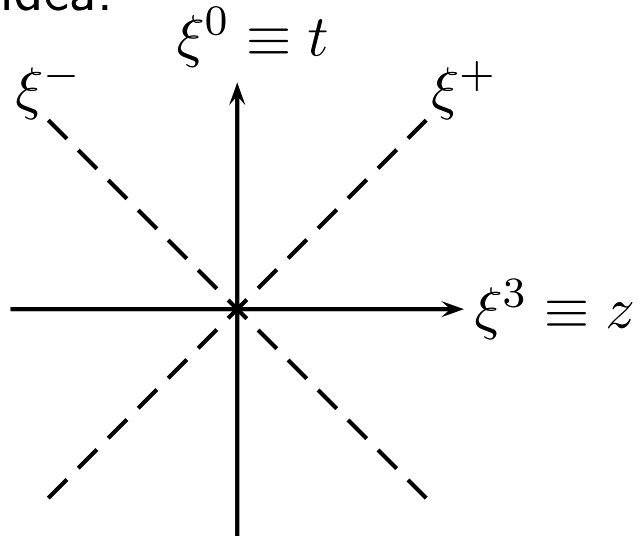

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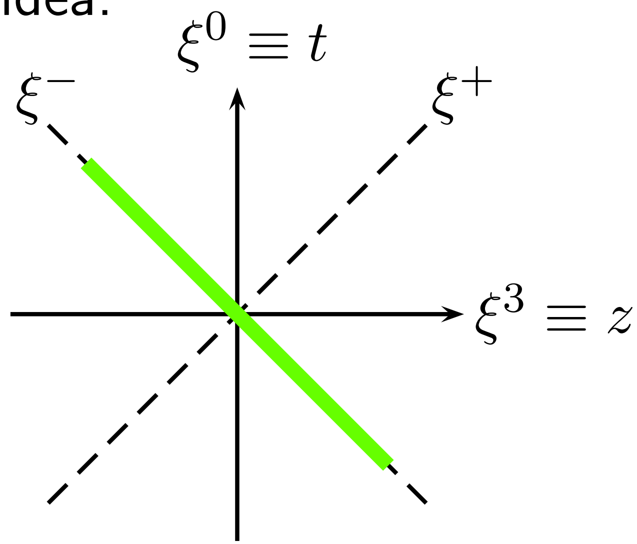

Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

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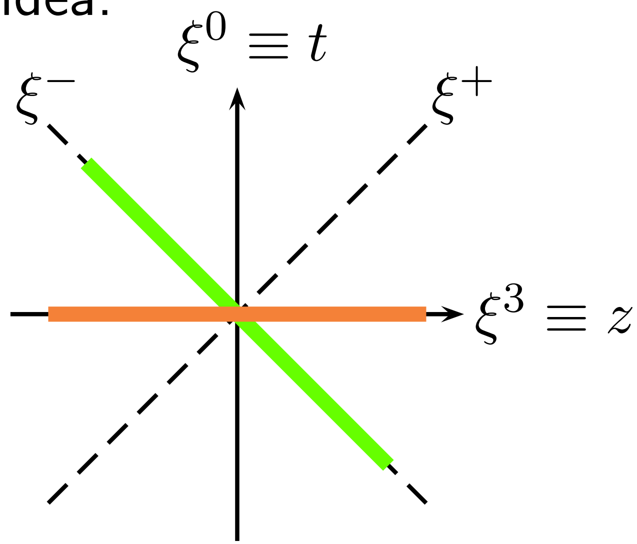

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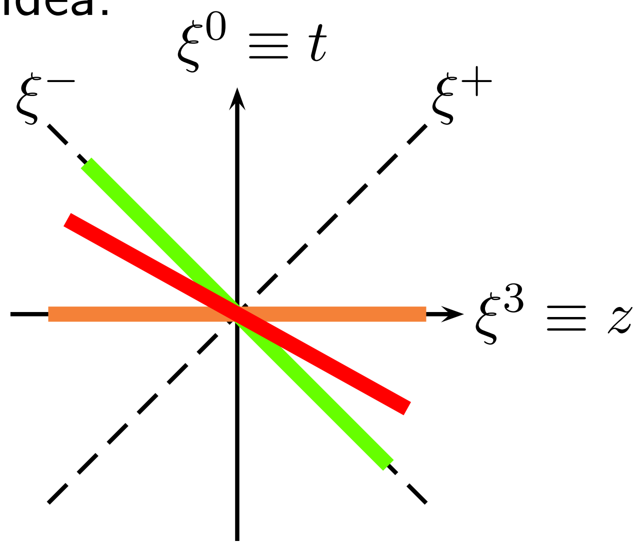

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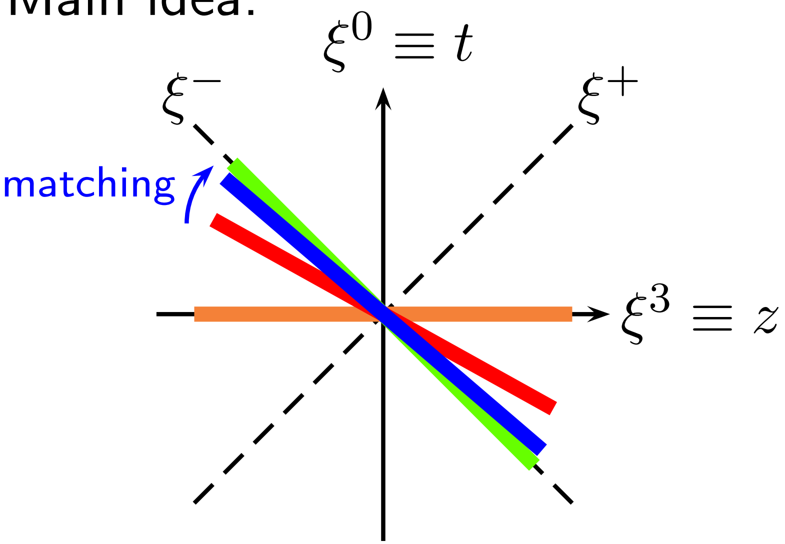

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Matching (Large Momentum Effective Theory (LaMET)

X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, Sci.China Phys.Mech.Astron. 57 (2014) 1407

$\to$ brings quasi-distribution to the light-cone distribution, up to power-suppressed effects:

$$\underbrace{\tilde{q}(x,\mu,P_3)}_{\text{quasi-PDF}} = \int_{-1}^{1} \frac{dy}{|y|} \underbrace{C\left(\frac{x}{y}, \frac{\mu}{P_3}\right)}_{\text{pert.kernel}} \underbrace{q(y,\mu)}_{\text{PDF}} + \underbrace{\mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)}_{\text{higher-twist effects}}$$

Quasi-GPDs lattice procedure

spatial correlation in a boosted nucleon

$\langle N(\vec{P'})|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$

$\vec{P'} = \vec{P} + \vec{Q}$, $\vec{Q}$ – momentum transfer

lattice computation of bare ME

↓

renormalization

intermediate RI scheme

conversion to $\mathrm{M\overline{MS}}$ scheme

(incl. evolution to $\mu = 2$ GeV)

↓

reconstruction of x-dependence

z-space$\rightarrow$$x$-space

Backus-Gilbert

↓

matching to light cone

$\mathrm{M\overline{MS}} \rightarrow \mathrm{\overline{MS}}$

↓

light-cone GPD

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light-cone GPD

most costly part of the procedure!
needs several $\vec{Q}$ vectors
Breit frame: separate calculations for each $\vec{Q}$

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also: one needs to disentangle 2/4 GPDs types

unpol./hel.: $H/\tilde{H}$ and $E/\tilde{E}$-GPDs

transv.: H_T, E_T, $\tilde{H}_T$ and $\tilde{E}_T$-GPDs

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↓

reconstruction of x-dependence

z-space→x-space

Backus-Gilbert

↓

matching to light cone

$\mathrm{M\overline{MS}} \to \overline{\mathrm{MS}}$

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light-cone GPD

most costly part of the procedure!

needs several $\vec{Q}$ vectors

Breit frame: separate calculations for each $\vec{Q}$

logarithmic and power divergences in bare matrix elements

also: one needs to disentangle 2/4 GPDs types

unpol./hel.: $H/\tilde{H}$ and $E/\tilde{E}$-GPDs

transv.: H_T, E_T, $\tilde{H}_T$ and $\tilde{E}_T$-GPDs

non-trivial aspect: reconstruction of a continuous distribution from a finite set of ME ("inverse problem")

Quasi-GPDs lattice procedure

spatial correlation in a boosted nucleon

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$\vec{P}' = \vec{P} + \vec{Q}$, $\quad\vec{Q}$ – momentum transfer

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light-cone GPD

the final desired object!

Setup

Lattice setup:

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 1.778$
- gauge field configurations generated by ETMC
- lattice spacing $a \approx 0.093$ fm,
- $32^3 \times 64 \Rightarrow L = 3$ fm,
- $m_\pi \approx 260$ MeV.

P_3	P_3 [GeV]	$N_{\rm meas}$
$4\pi/L$	0.83	4152
$6\pi/L$	1.25	42080
$8\pi/L$	1.67	112192

ETMC, Phys. Rev. Lett. 125 (2020) 262001
ETMC, Phys. Rev. D105 (2022) 034501
S. Bhattacharya et al., 2112.05538

Always: $u - d$ flavor combination

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Bare matrix elements

Lattice matrix elements need to be computed with 2 different projections (unpolarized/polarized).
Below for the unpolarized Dirac insertion (for unpolarized GPDs)

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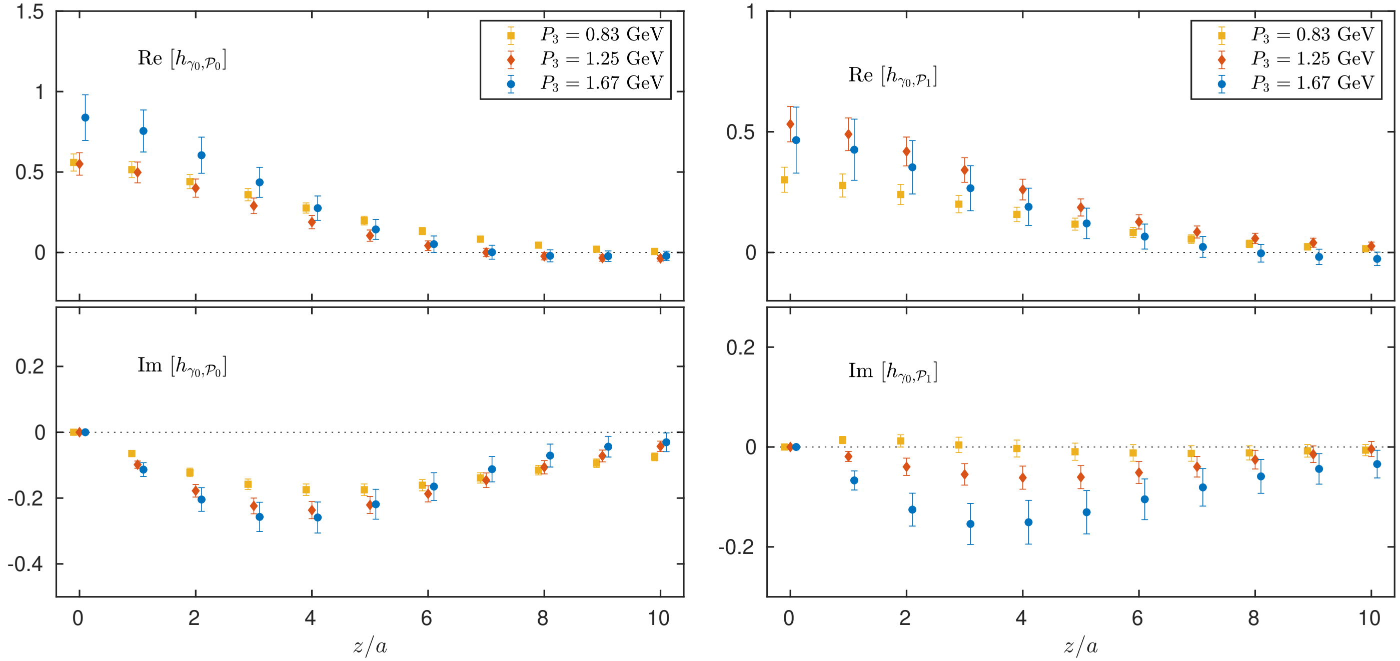
↓

light-cone GPD

unpolarized projector | polarized projector



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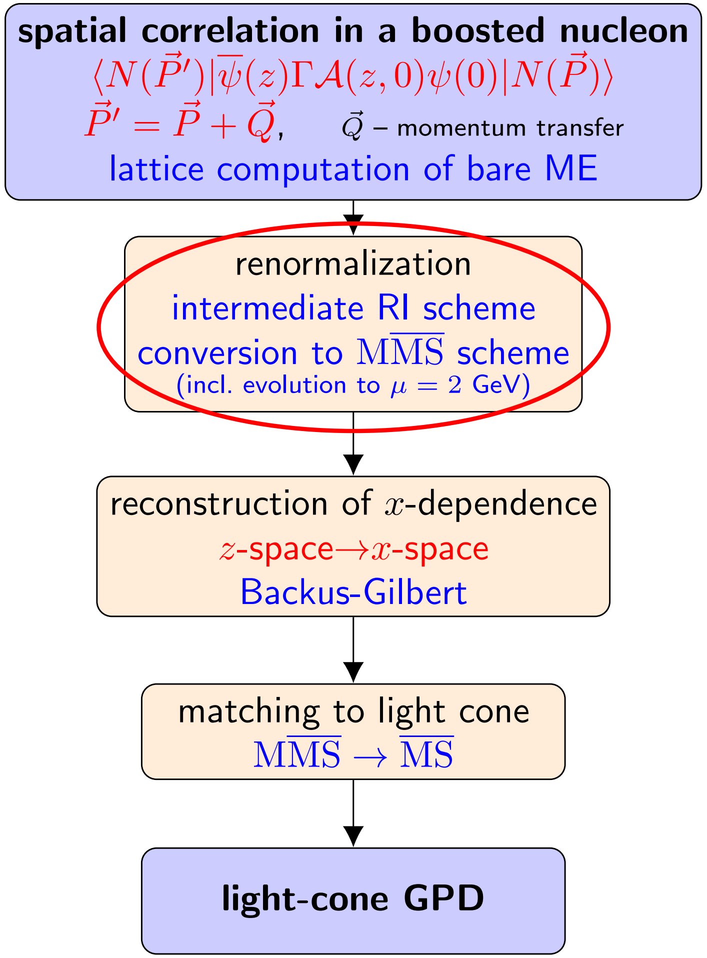
Zero skewness: $\xi = 0$

ETMC, Phys. Rev. Lett. 125 (2020) 262001

Removal of divergences and disentangling of H- and E-GPDs.
Unpolarized Dirac insertion (for unpolarized GPDs)



Disentangled renormalized matrix elements

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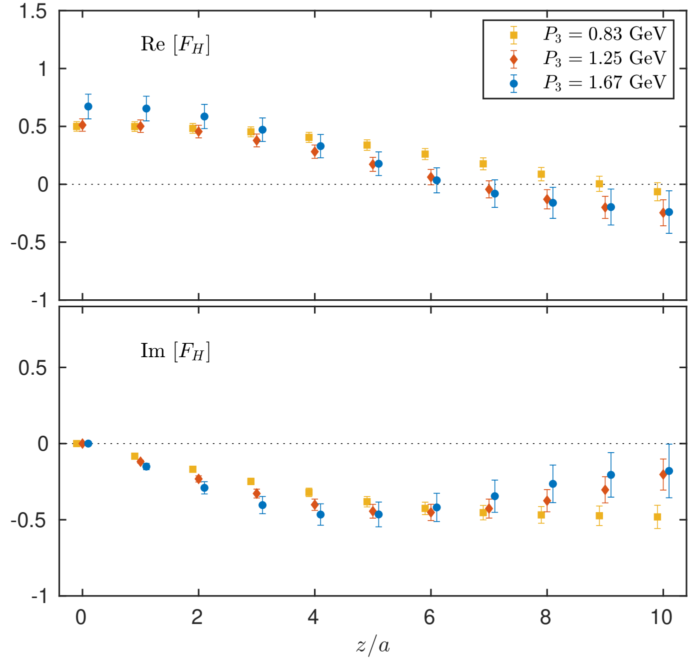
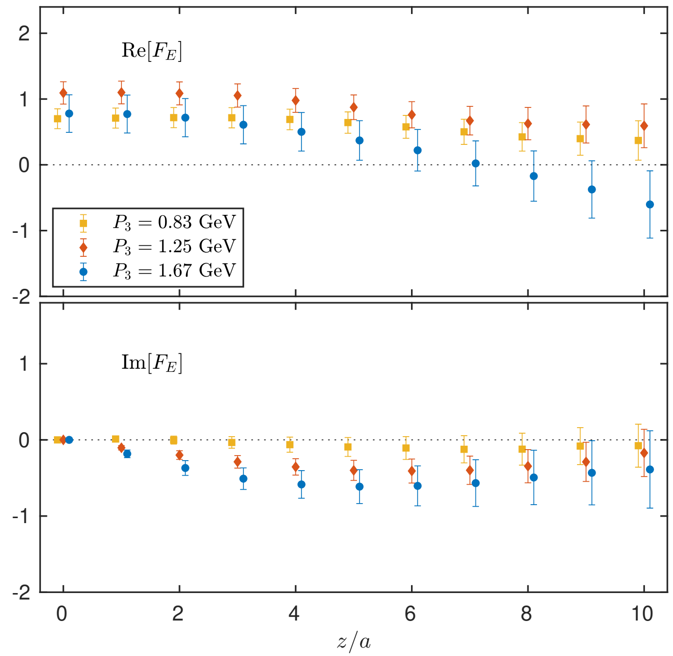
light-cone GPD

ME of H-function



ME of E-function



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ETMC, Phys. Rev. Lett. 125 (2020) 262001

Light-cone distributions

Reconstruction of x-dependence and matching to light cone.
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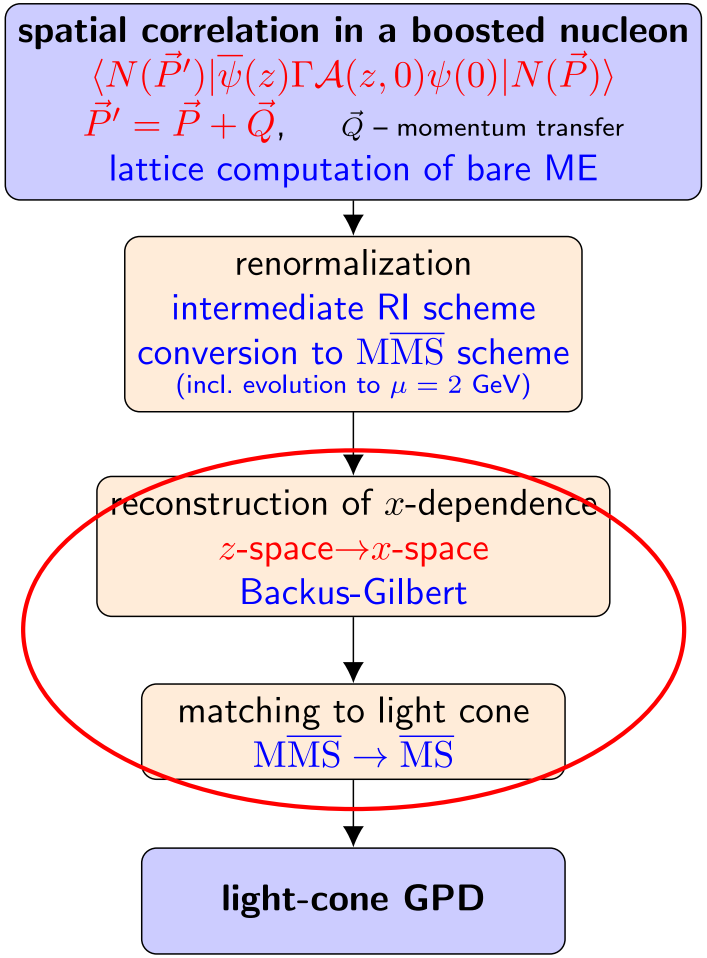

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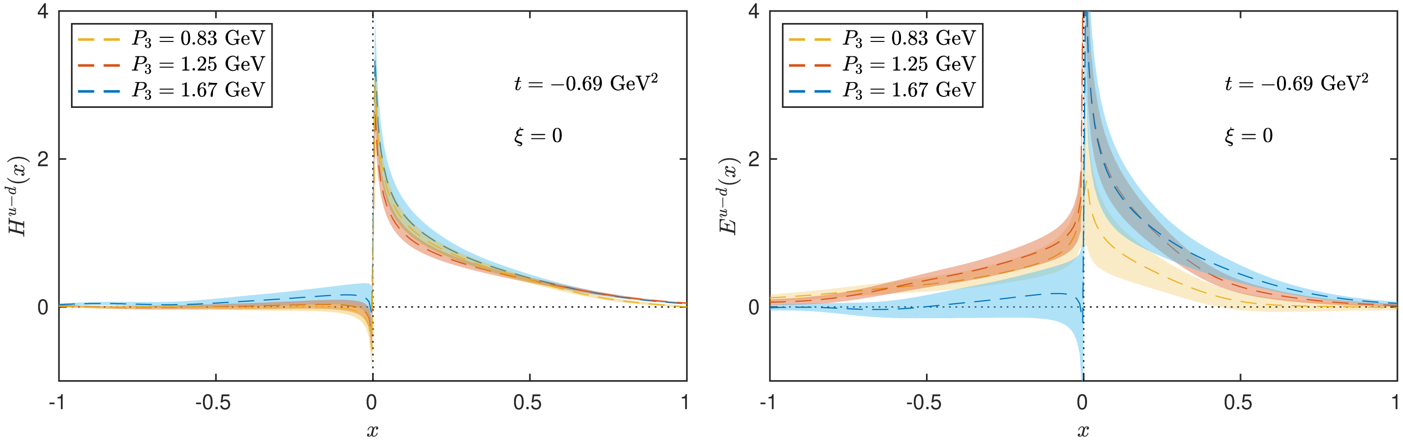
light-cone GPD

H-GPD | E-GPD



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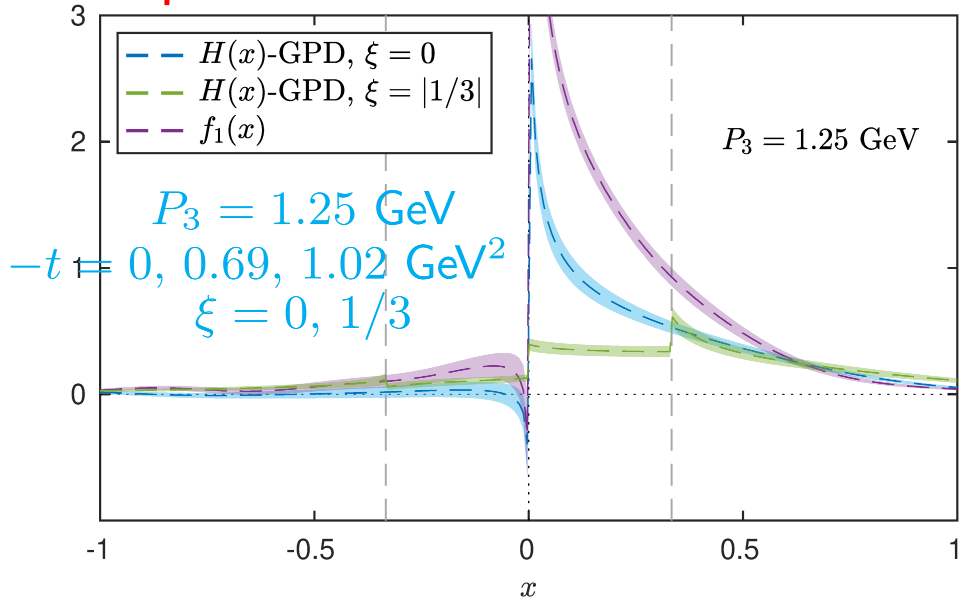
Comparison of PDFs and H-GPDs

unpolarized

ETMC, Phys. Rev. Lett. 125 (2020) 262001



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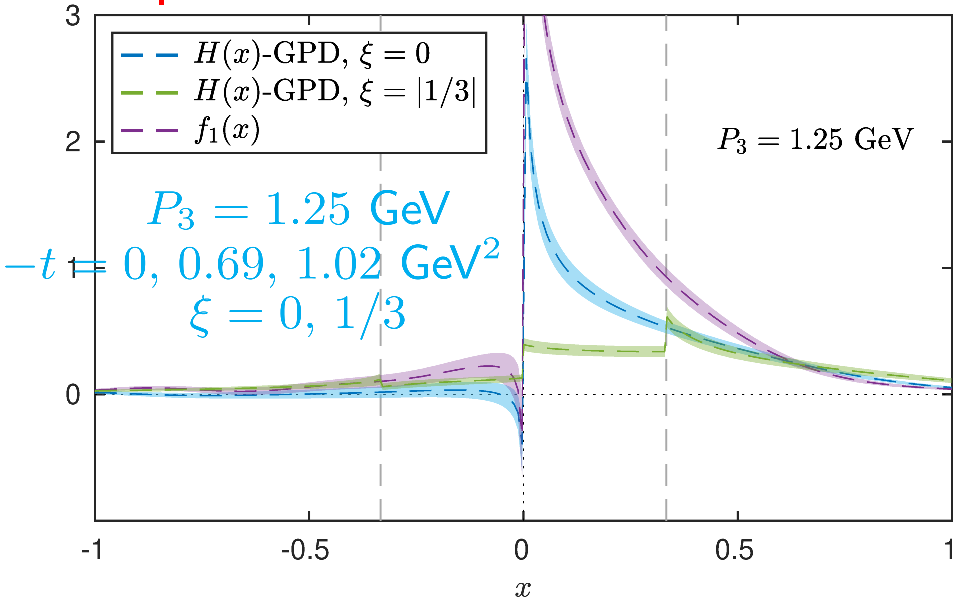

Important insights from models:

S. Bhattacharya, C. Cocuzza, A. Metz
Phys. Lett. B788 (2019) 453
Phys. Rev. D102 (2020) 054201

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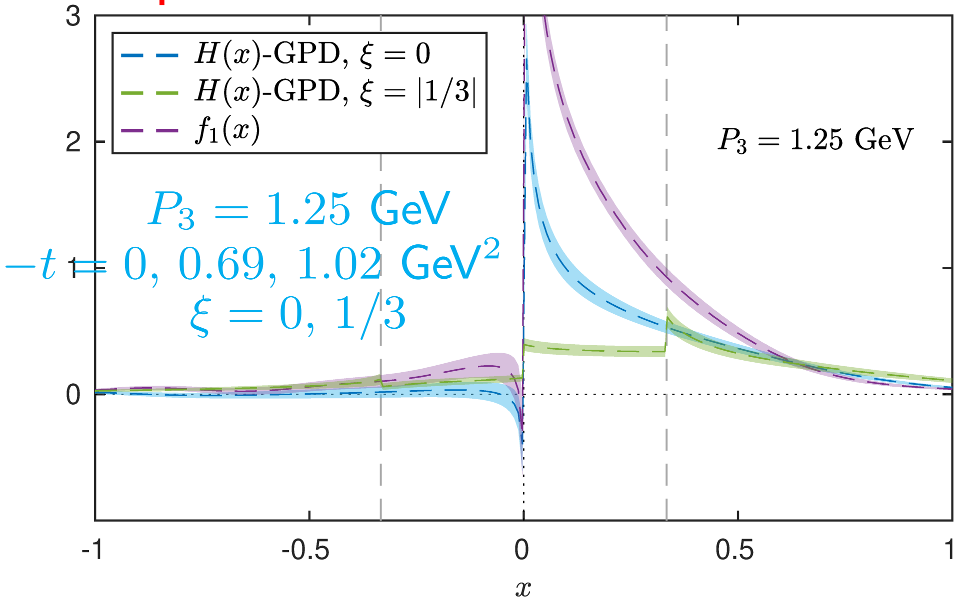
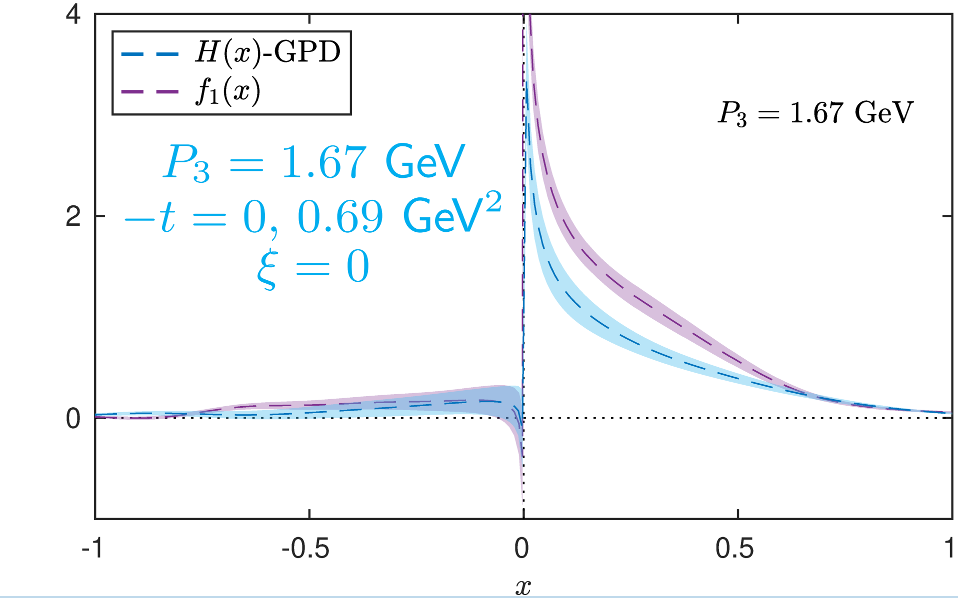
ETMC, Phys. Rev. Lett. 125 (2020) 262001





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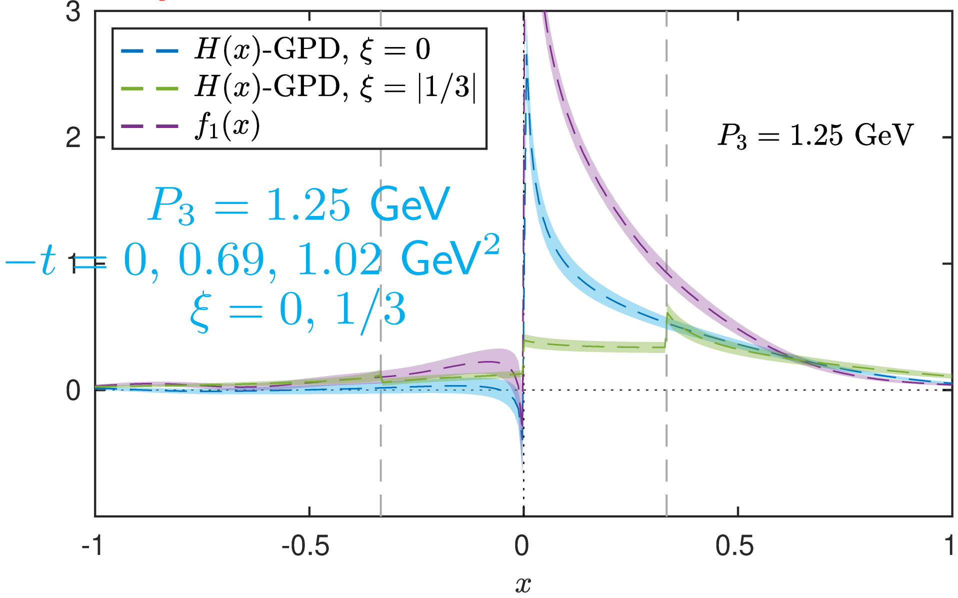
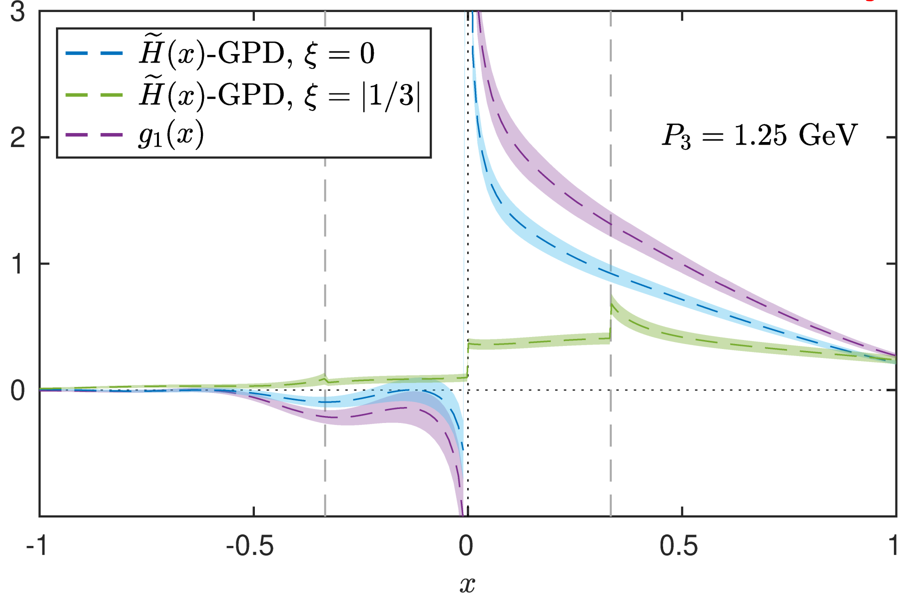
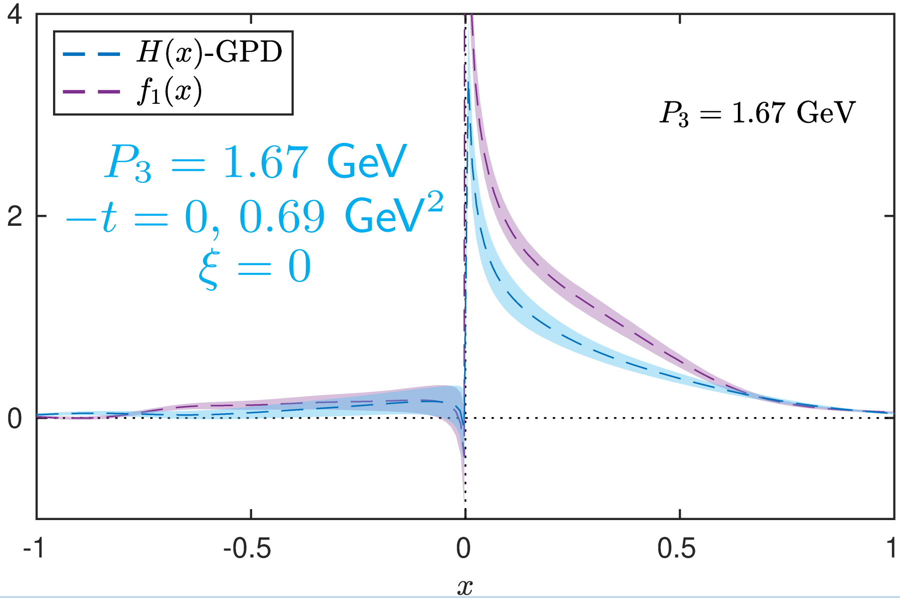
ETMC, Phys. Rev. Lett. 125 (2020) 262001

helicity







Comparison of PDFs and H-GPDs

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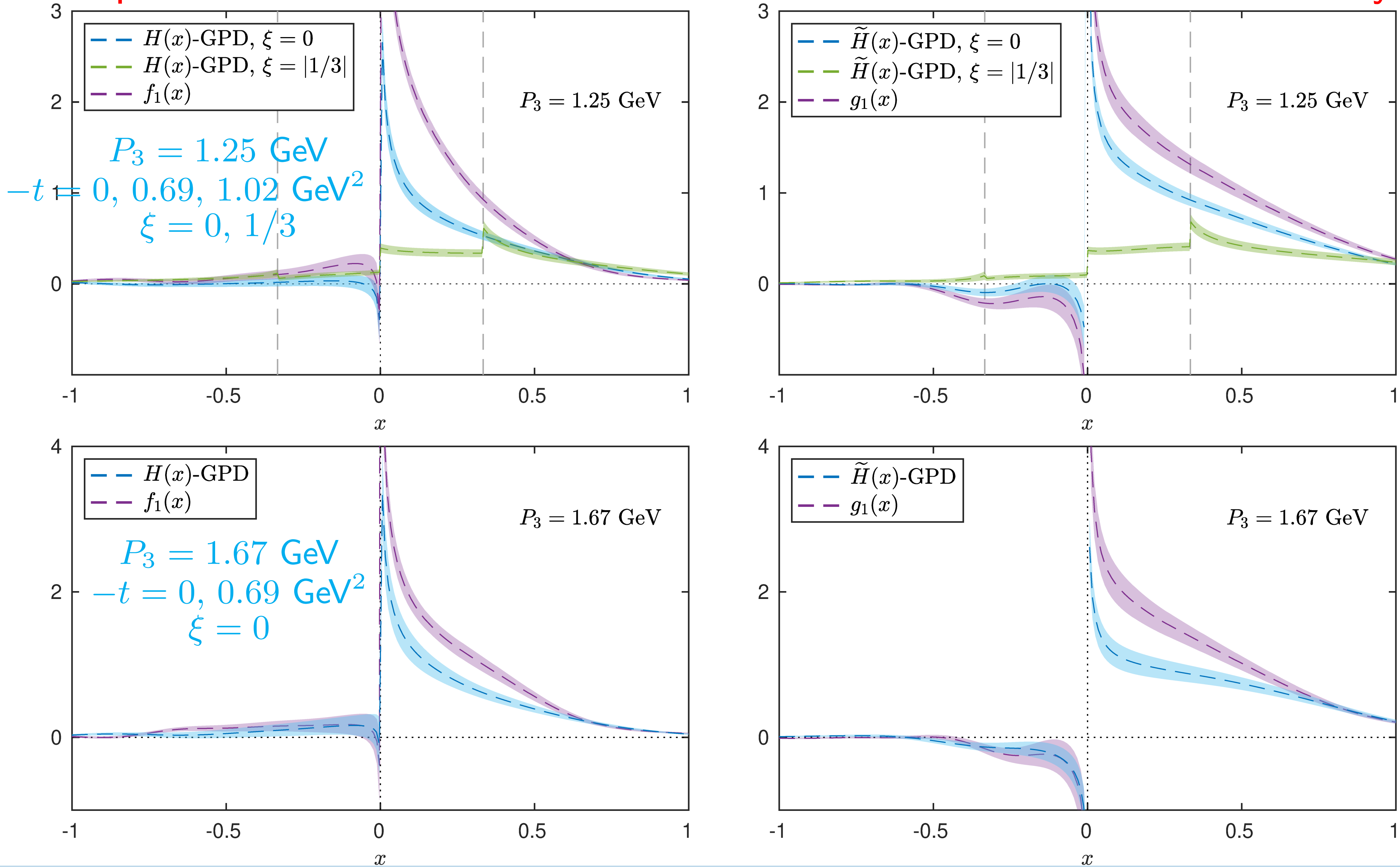

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Drawback on the lattice:
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Main theoretical tool:
Lorentz-covariant parametrization of matrix elements (e.g. vector case):

$$F^{\mu}(z,P,\Delta) = \bar{u}(p',\lambda')\left[\frac{P^\mu}{m}A_1 + m z^\mu A_2 + \frac{\Delta^\mu}{m}A_3 + i m \sigma^{\mu z}A_4 + \frac{i\sigma^{\mu\Delta}}{m}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{m}A_6 + \frac{z^\mu i\sigma^{z\Delta}}{m}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{m}A_8\right]u(p,\lambda),$$

(inspired by: S. Meissner, A. Metz, M. Schlegel, JHEP08(2009)056).

- most general parametrization in terms of 8 linearly-independent Lorentz structures,
- 8 Lorentz-invariant amplitudes $A_i(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$.

Example

The relation between lattice-calculated matrix elements and the Lorentz-invariant amplitudes A_i is different in the symmetric and the non-symmetric frame.

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For example: (γ_0 insertion, unpolarized projector)

symmetric frame:

$$\Pi_0^s(\Gamma_0) = C\left(\frac{E\left(E(E+m)-P_3^2\right)}{2m^3}A_1 + \frac{(E+m)\left(-E^2+m^2+P_3^2\right)}{m^3}A_5 + \frac{EP_3\left(-E^2+m^2+P_3^2\right)z}{m^3}A_6\right),$$

asymmetric frame:

$$\begin{aligned}\Pi_0^a(\Gamma_0) &= C\Bigg(-\frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^3}A_1 - \frac{(E_f-E_i-2m)(E_f+m)(E_f-E_i)}{4m^3}A_3 + \frac{(E_i-E_f)P_3 z}{4m}A_4 \\ &+ \frac{(E_f+E_i)(E_f+m)(E_f-E_i)}{4m^3}A_5 + \frac{E_f(E_f+E_i)P_3(E_f-E_i)z}{4m^3}A_6 + \frac{E_f P_3(E_f-E_i)^2 z}{2m^3}A_8\Bigg).\end{aligned}$$

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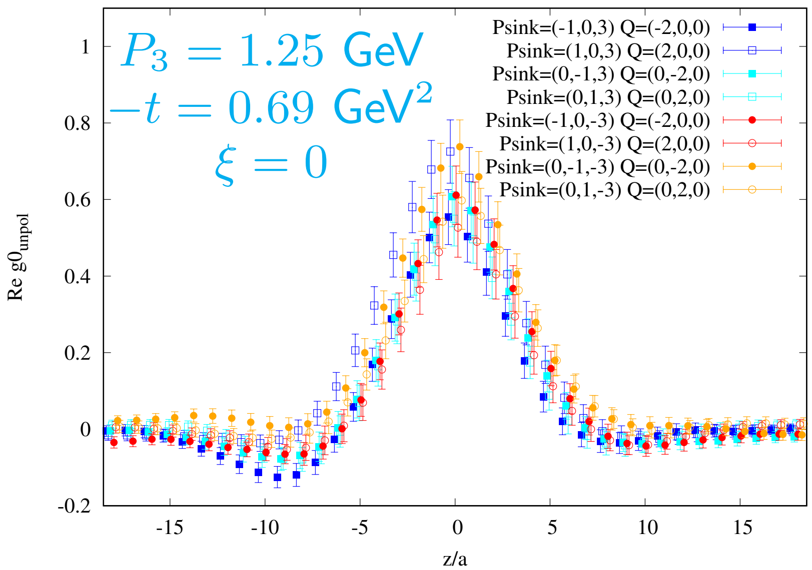
Thus,

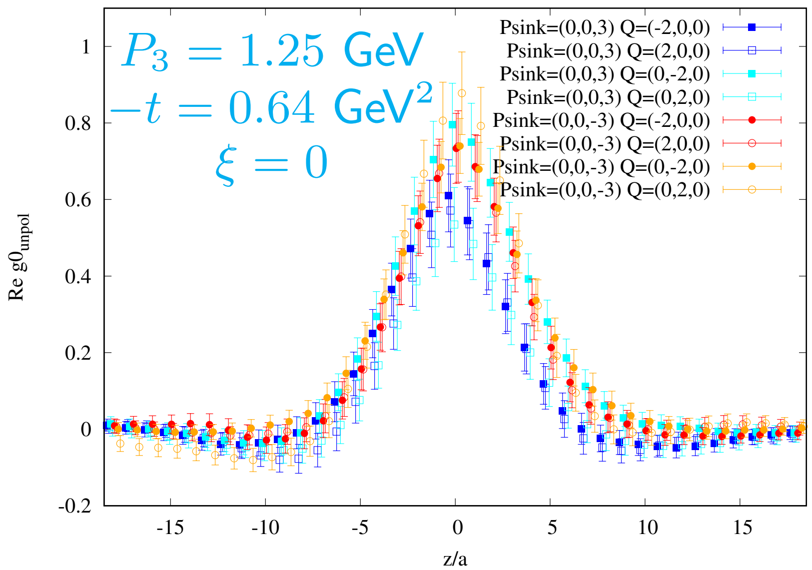
- matrix elements $\Pi_\mu(\Gamma_\nu)$ are frame-dependent,
- but the amplitudes A_i are frame-invariant.

Bare matrix elements of $\Pi_0(\Gamma_0)$

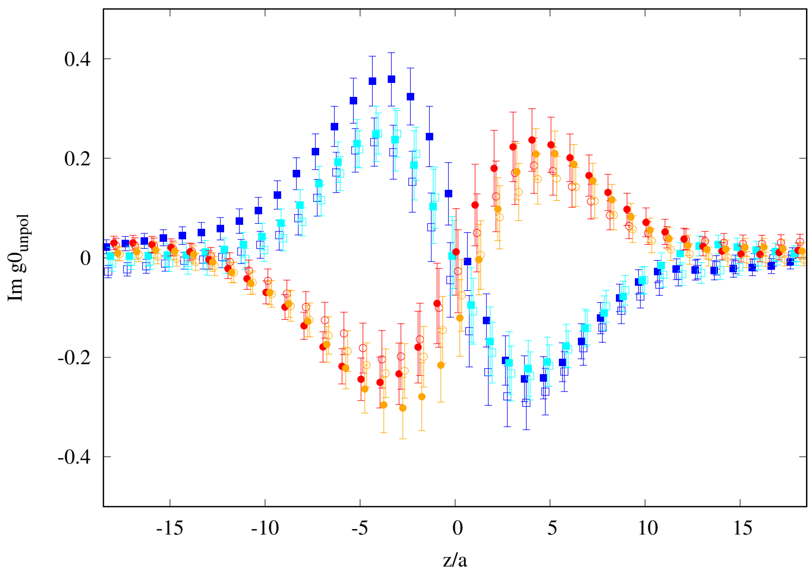
symmetric frame / non-symmetric frame

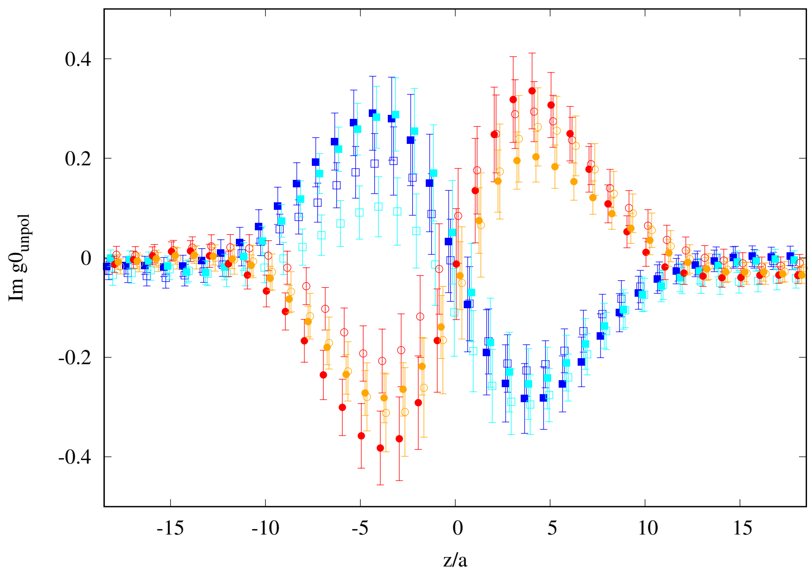
Re





Im

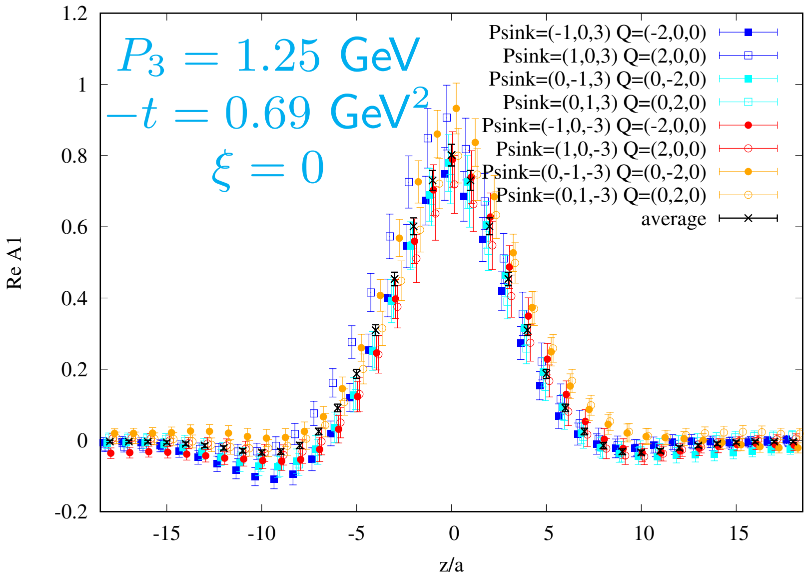


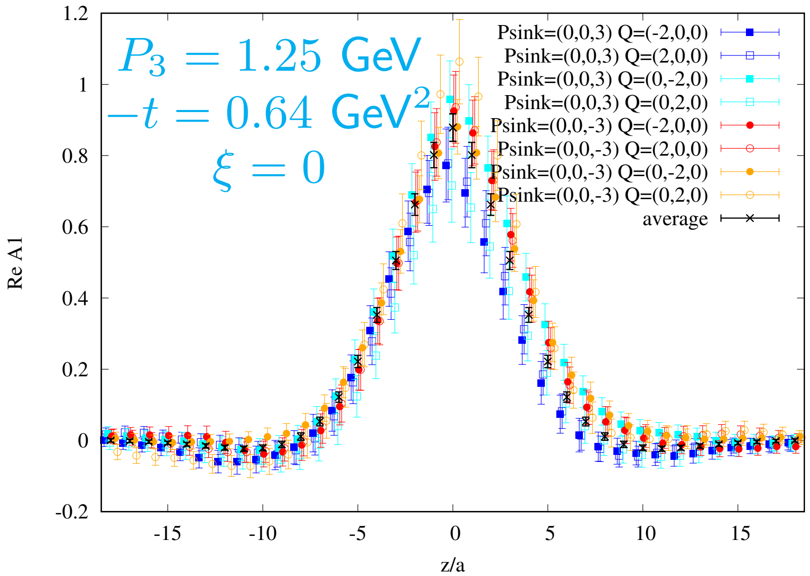


Example amplitude A_1

symmetric frame / non-symmetric frame

Re





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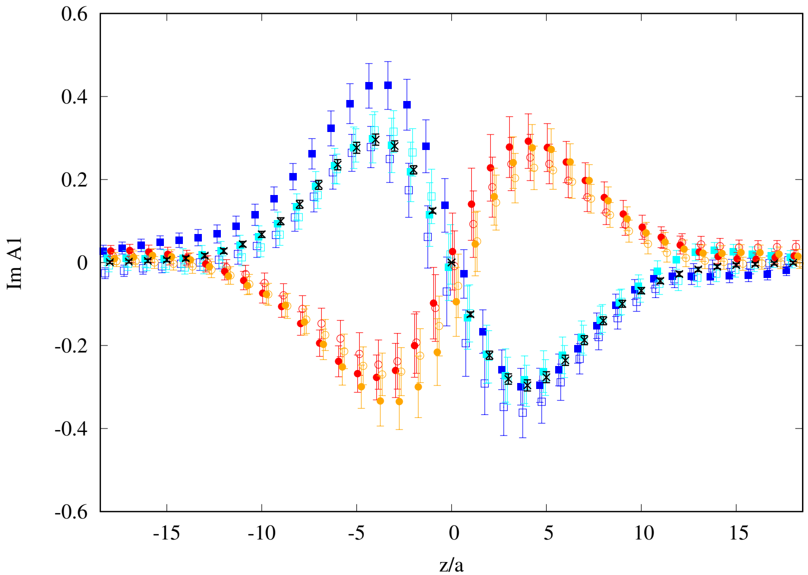
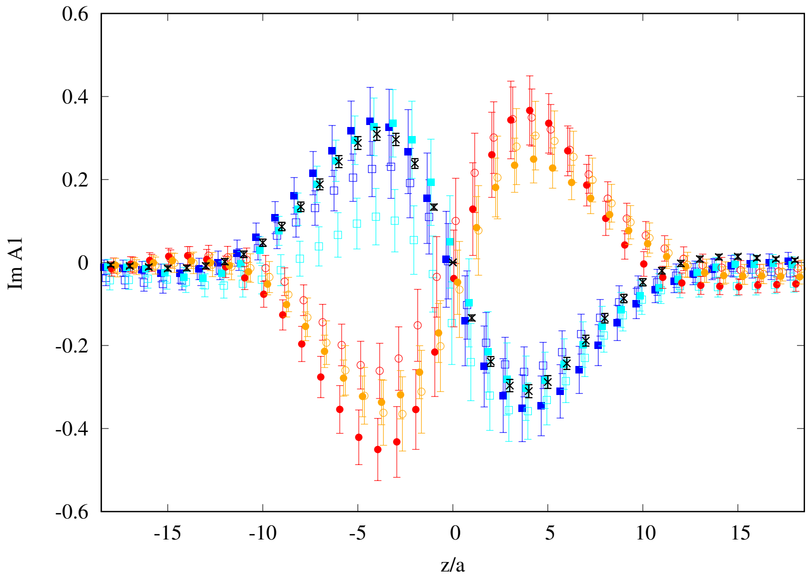
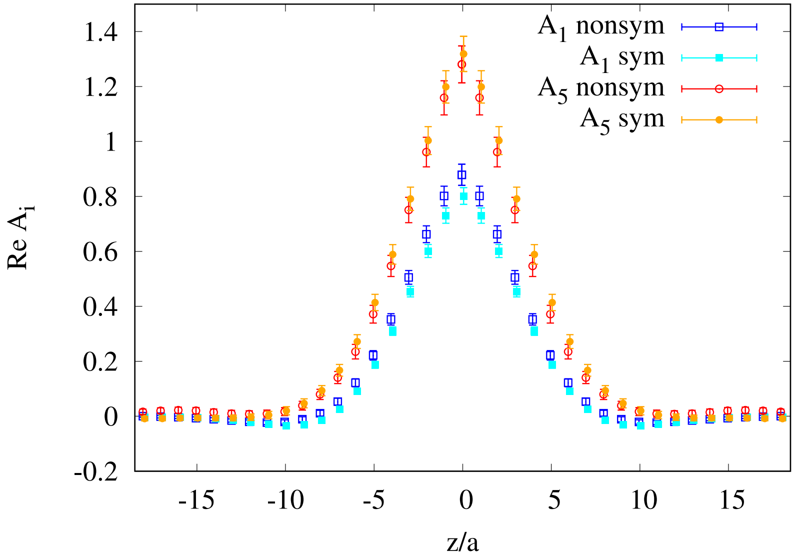
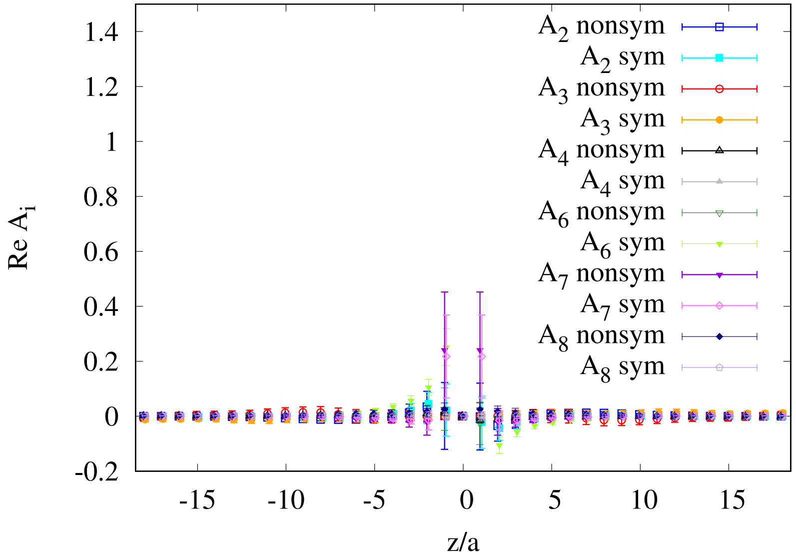




Comparison of amplitudes between frames

A_1, A_5 (leading ones) A_2, A_3, A_4, A_6, A_7, A_8 (subleading ones)





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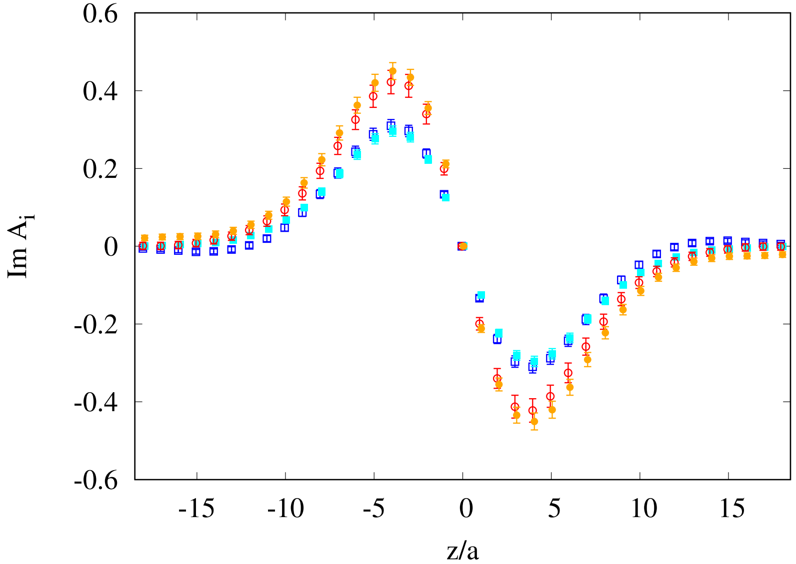

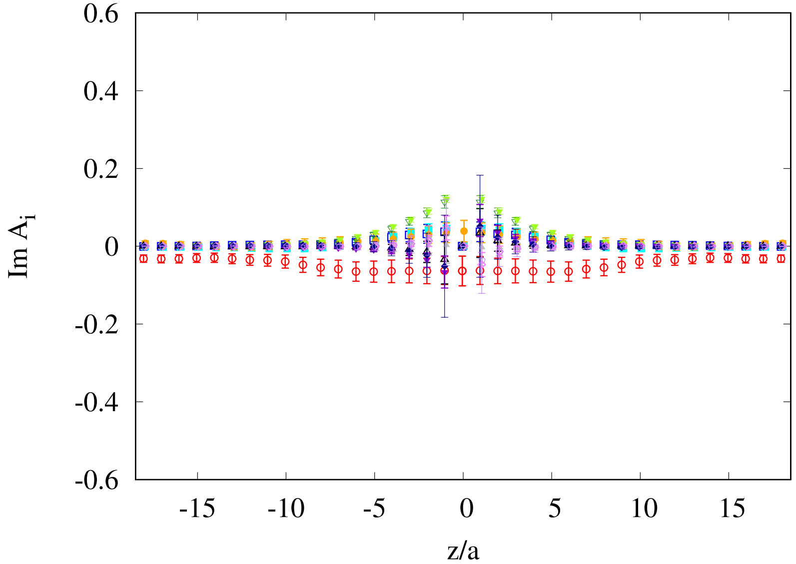

Im

H and E GPDs – standard definition

The standard definition of H and E GPDs:

$$F^0(z,P,\Delta)=\bar{u}(p',\lambda')\left[\gamma^0 F_{H^{(0)}}(z,P,\Delta)+\frac{i\sigma^{0\mu}\Delta_\mu}{2m}F_{E^{(0)}}(z,P,\Delta)\right]u(p,\lambda)\,.$$

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Thus-defined GPDs are obviously frame-dependent! In terms of A_i's ($\xi=0$ case):

symmetric frame:

$$F_{H(0)} = A_1 + \frac{z(Q_1^2+Q_2^2)}{2P_3}A_6\,,$$

$$F_{E(0)} = -A_1 - \frac{m^2 z}{P_3}A_4 + 2A_5 - \frac{z\left(4E^2+Q_1^2+Q_2^2\right)}{2P_3}A_6\,.$$

asymmetric frame:

$$F_{H(0)} = A_1 + \frac{Q_0}{P_0}A_3 + \frac{m^2 z Q_0}{2P_0P_3}A_4 + \frac{z(Q_0^2+Q_\perp^2)}{2P_3}A_6 + \frac{z(Q_0^3+Q_0Q_\perp^2)}{2P_0P_3}A_8\,,$$

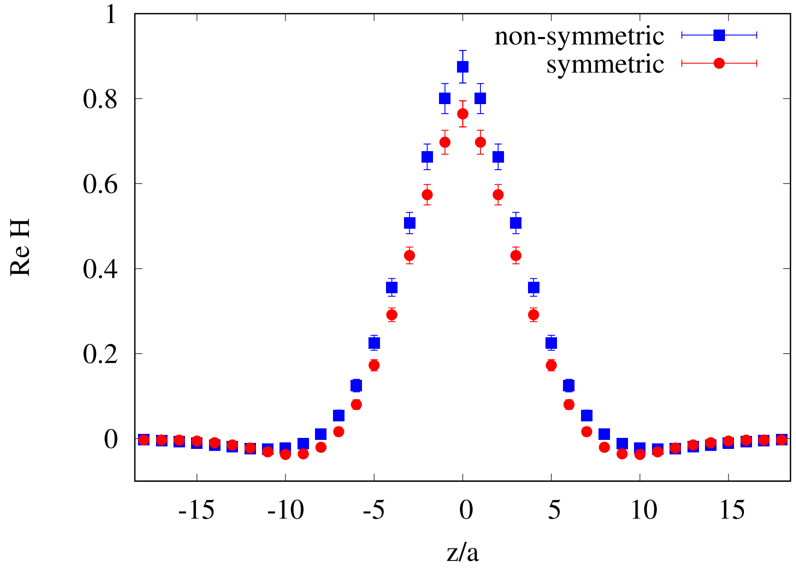
$$F_{E(0)} = -A_1 - \frac{Q_0}{P_0}A_3 - \frac{m^2 z(Q_0+2P_0)}{2P_0P_3}A_4 + 2A_5 - \frac{z\left(Q_0^2+2P_0Q_0+4P_0^2+Q_\perp^2\right)}{2P_3}A_6 - \frac{zQ_0\left(Q_0^2+2Q_0P_0+4P_0^2+Q_\perp^2\right)}{2P_0P_3}A_8\,.$$

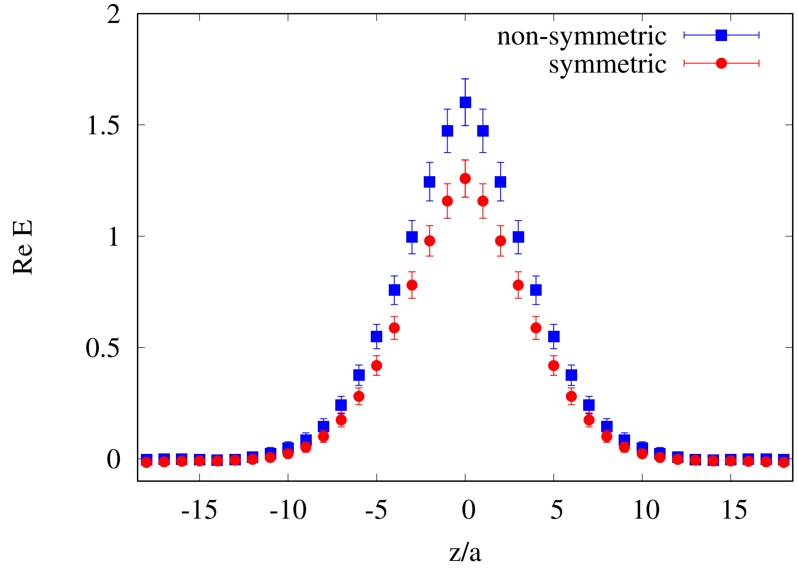
Note: the standard definition is frame-dependent, but still valid in the sense of approaching the correct GPDs in the light-cone limit.

H and E GPDs – standard definition

H-GPD

E-GPD





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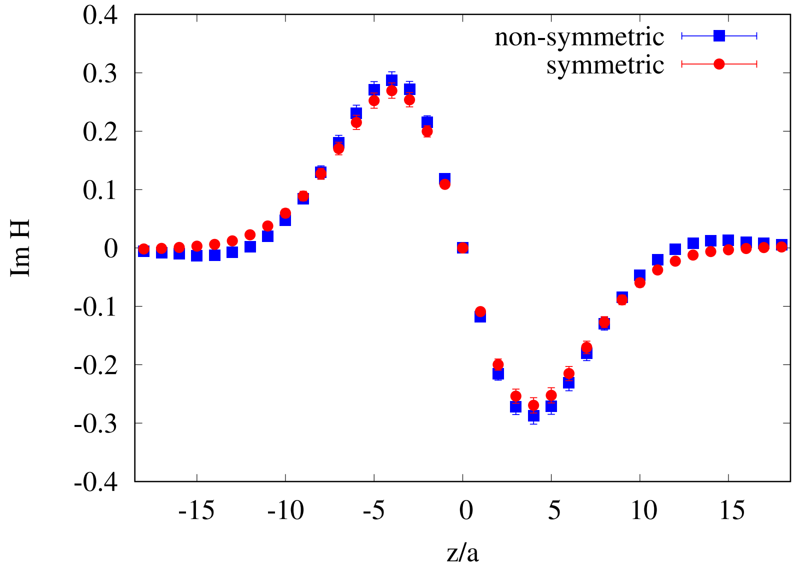

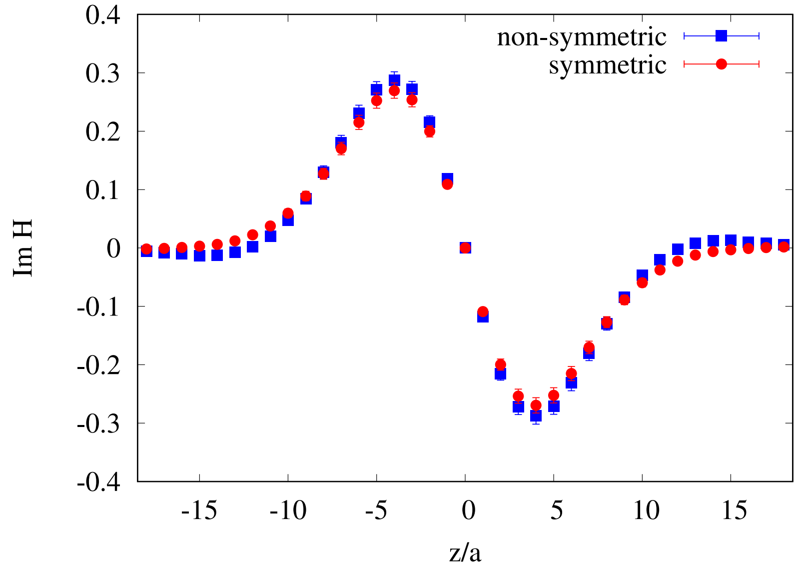

Im

H and E GPDs – improved definition

The definition of H and E GPDs can be made Lorentz-invariant in the following way:

$$F_H = A_1 + \frac{\Delta \cdot z}{P \cdot z} A_3 \,,$$

$$F_E = -A_1 - \frac{\Delta \cdot z}{P \cdot z} A_3 + 2A_5 + 2P \cdot z A_6 + 2\Delta \cdot z A_8 \,.$$

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At zero-skewness:

$$F_H = A_1 \,,$$

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With respect to the standard definition, removed/reduced contribution from A_3, A_4, A_6, A_8.

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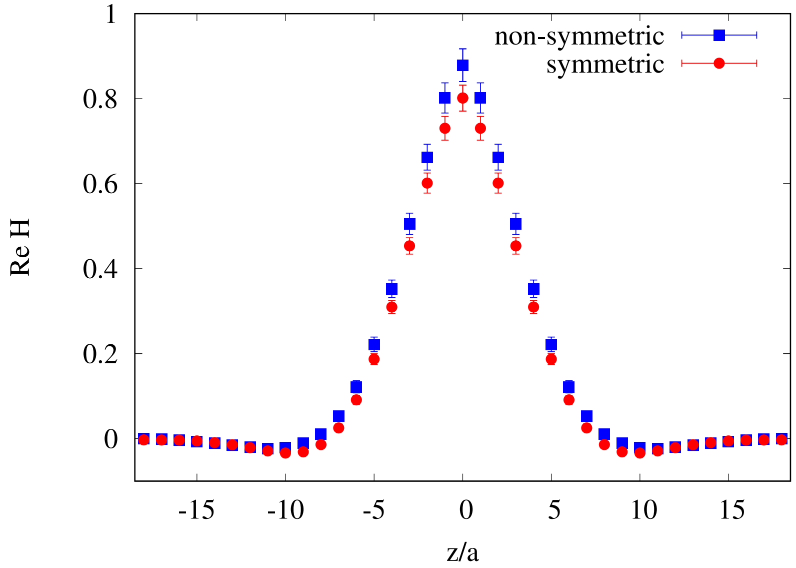
In terms of matrix elements:

- standard definition – only $\Pi_0(\Gamma_0)$, $\Pi_0(\Gamma_{1/2})$,
- improved definition – additionally:
  - ⋆ symmetric: $\Pi_{1/2}(\Gamma_3)$,
  - ⋆ non-symmetric: $\Pi_{1/2}(\Gamma_3)$, $\Pi_{1/2}(\Gamma_0)$, $\Pi_1(\Gamma_2)$, $\Pi_2(\Gamma_1)$.

Thus, adding info from additional MEs removes some kinematic contaminations!

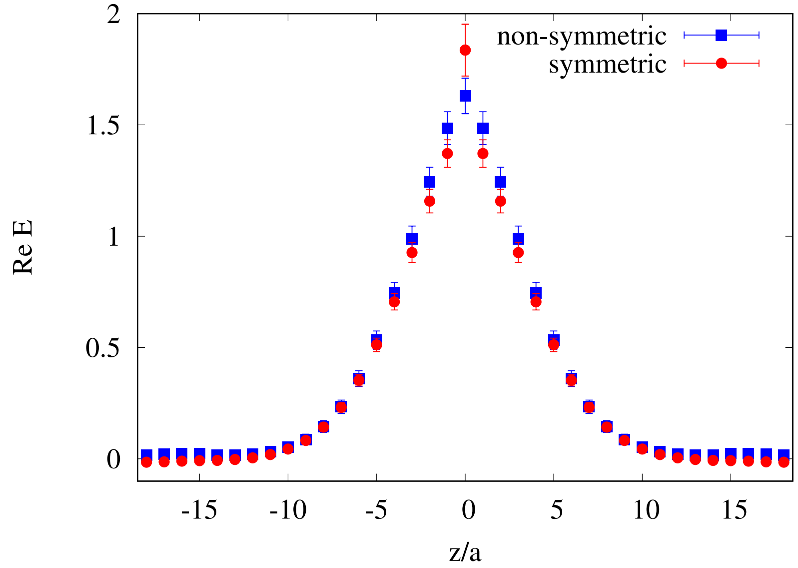
H and E GPDs – improved definition

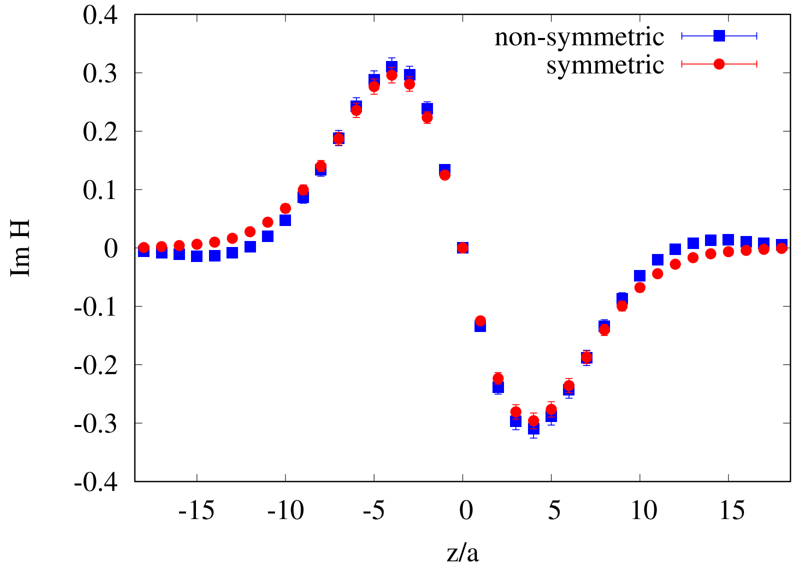
H-GPD



Re

E-GPD





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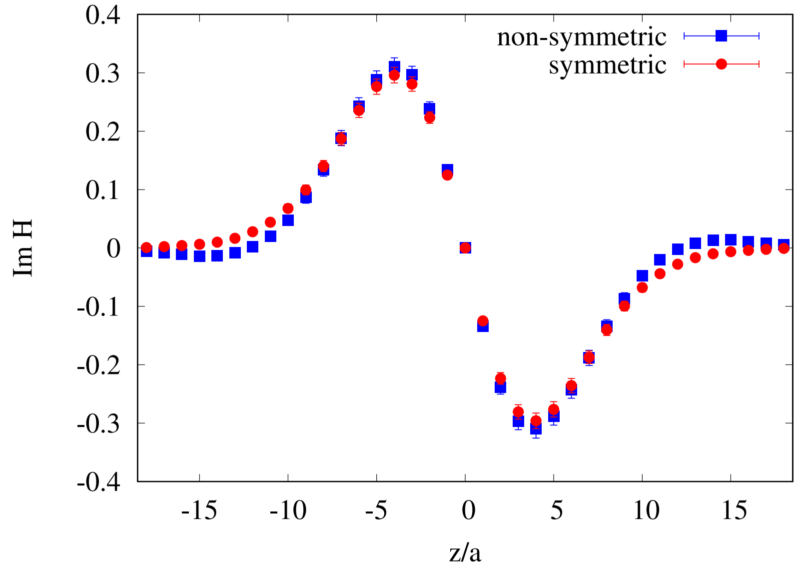
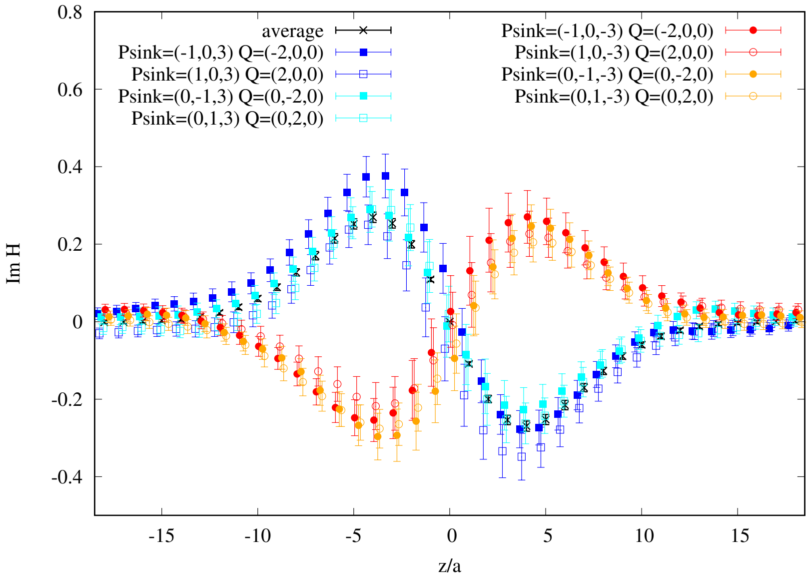
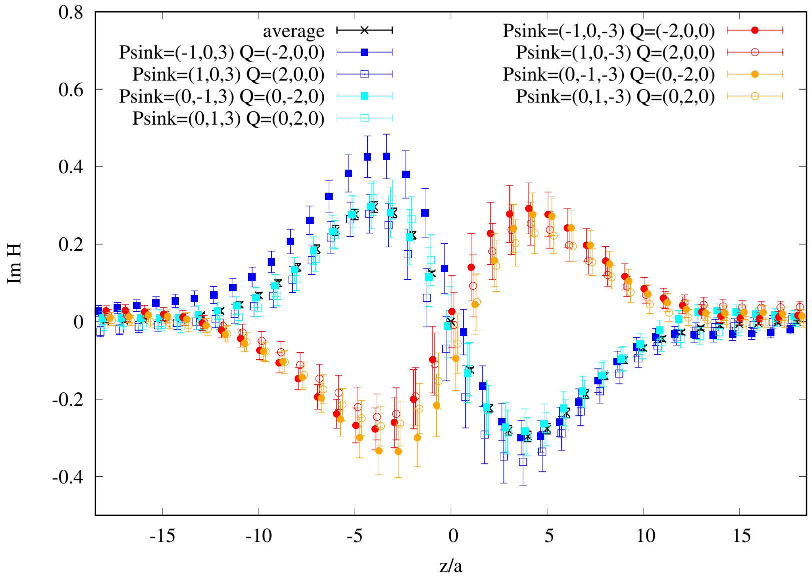


H and E GPDs – signal improvement

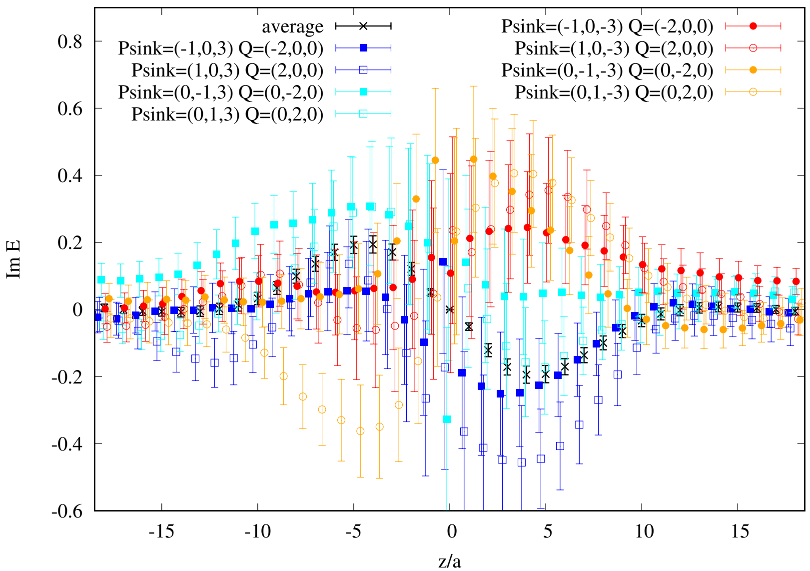
standard

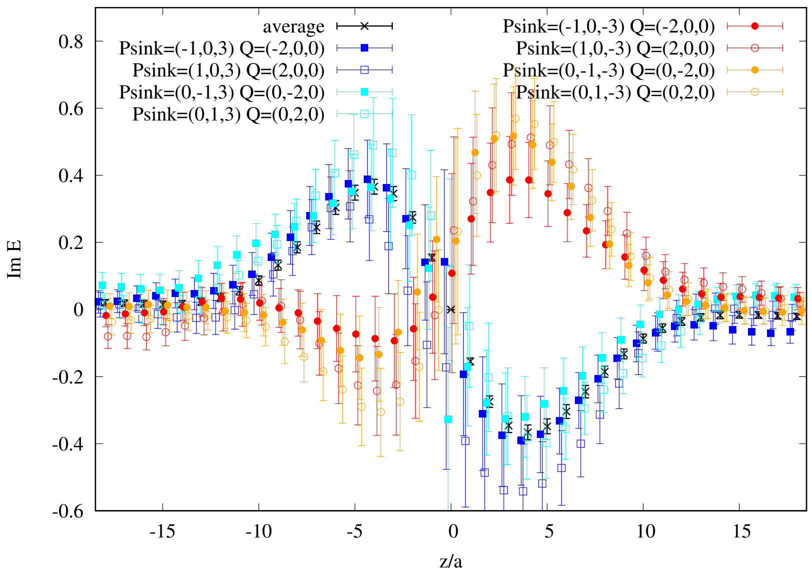
improved





Im H

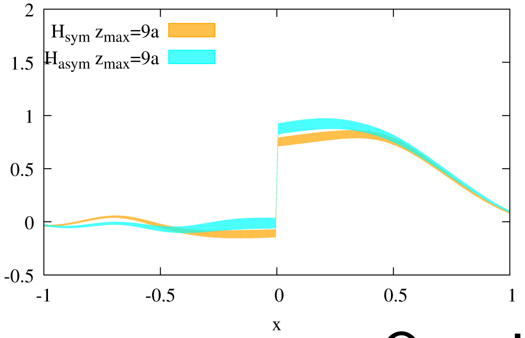


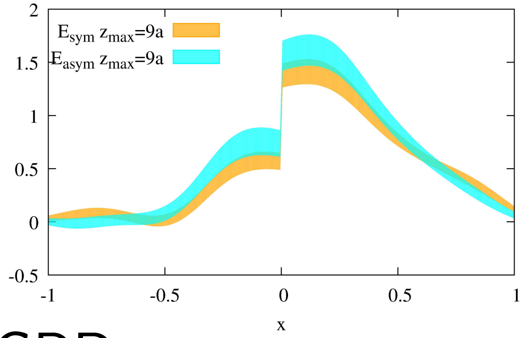


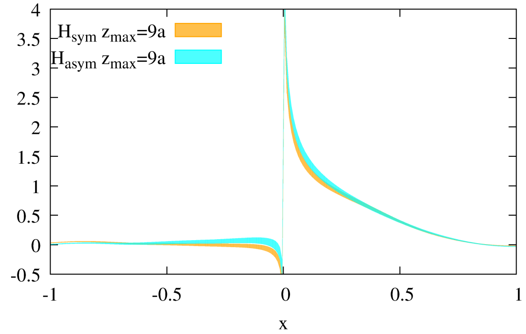
Im E

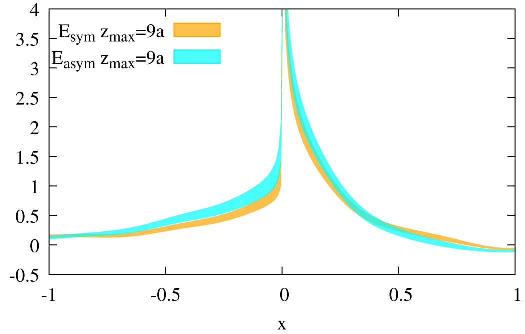
Quasi- and matched H and E GPDs

STANDARD DEFINITION









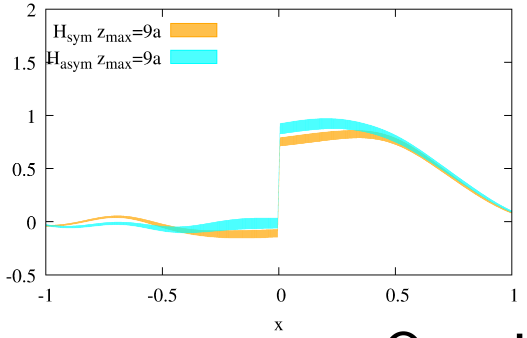
Quasi-GPDs

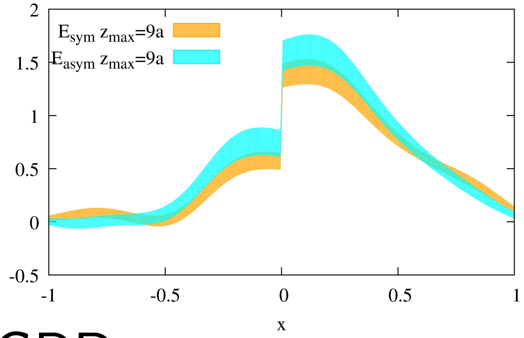
Matched GPDs

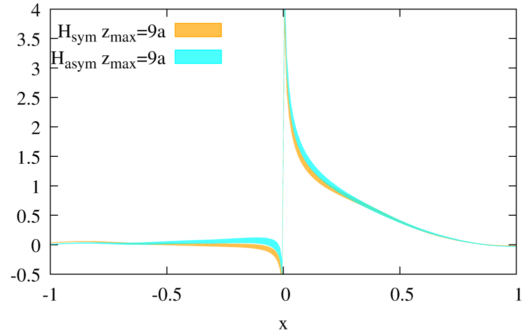
H-GPD E-GPD H-GPD E-GPD

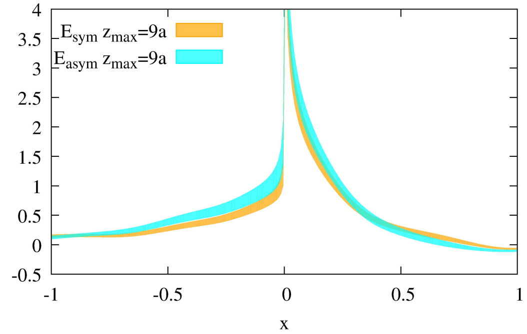
Quasi- and matched H and E GPDs

STANDARD DEFINITION









Quasi-GPDs

Matched GPDs

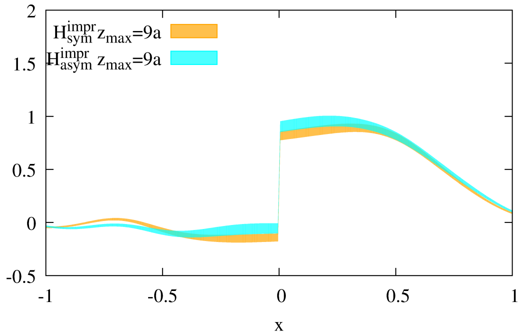
H-GPD

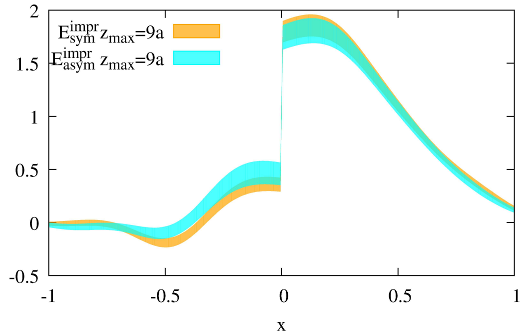
E-GPD

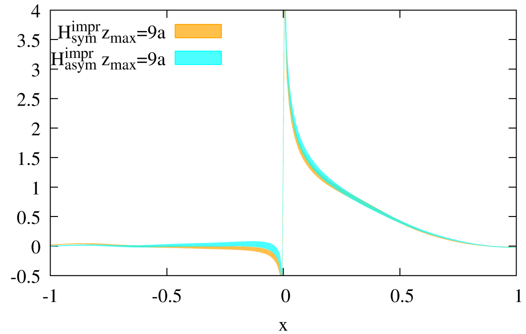
H-GPD

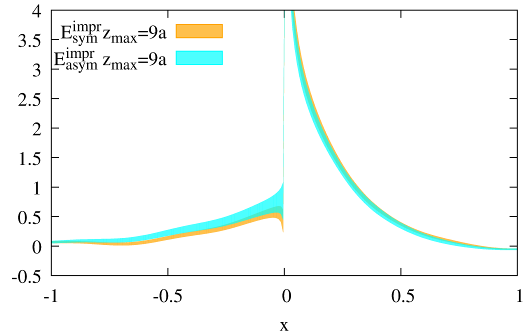
E-GPD

IMPROVED DEFINITION









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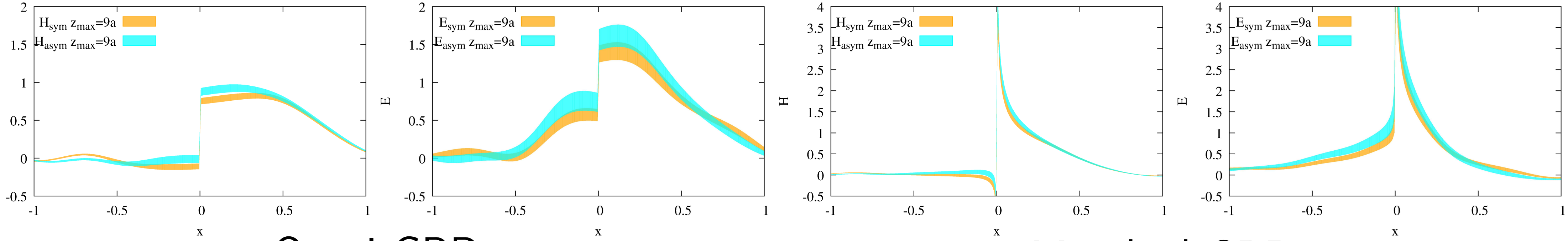
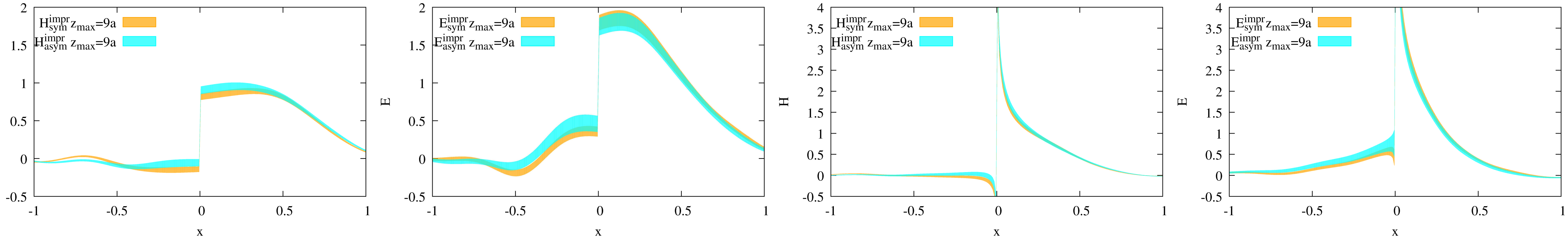


Quasi-GPDs | Matched GPDs

H-GPD | E-GPD | H-GPD | E-GPD

IMPROVED DEFINITION



Main conclusions:

– GPDs can be computed in non-symmetric frames, reducing the computational cost

– GPDs can be made frame-independent by using a Lorentz-invariant definition

Overall, it gives much better perspectives for lattice GPDs!

Transversity GPDs

Transversity GPDs:
4 GPDs: H_T, E_T, $\tilde{H}_T$, $\tilde{E}_T$

spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
$\vec{P}' = \vec{P} + \vec{Q}$, $\vec{Q}$ – momentum transfer
lattice computation of bare ME

↓

renormalization
intermediate RI scheme
conversion to $\mathrm{M}\overline{\mathrm{MS}}$ scheme
(incl. evolution to $\mu = 2$ GeV)

↓

reconstruction of x-dependence
z-space→x-space
Backus-Gilbert

↓

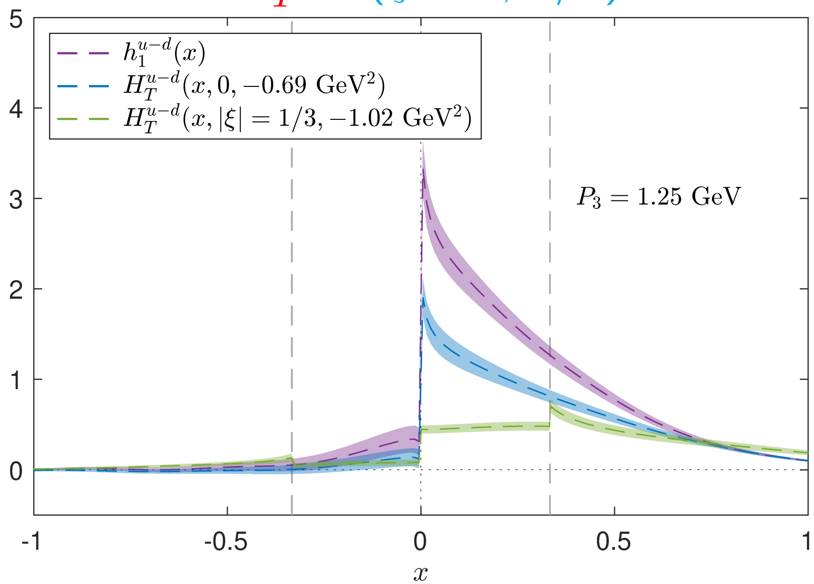
matching to light cone
$\mathrm{M}\overline{\mathrm{MS}} \to \overline{\mathrm{MS}}$

↓

light-cone GPD

ETMC, Phys. Rev. D105 (2022) 034501

H_T^{u-d} $(\xi = 0,\ 1/3)$



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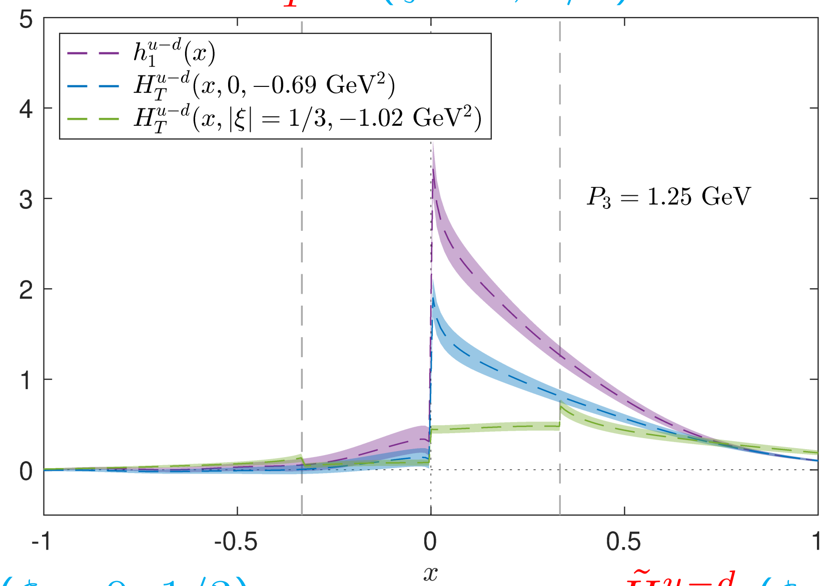
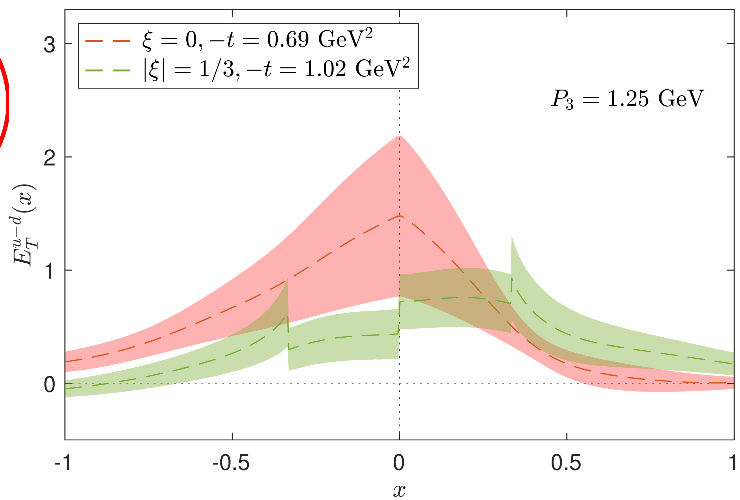
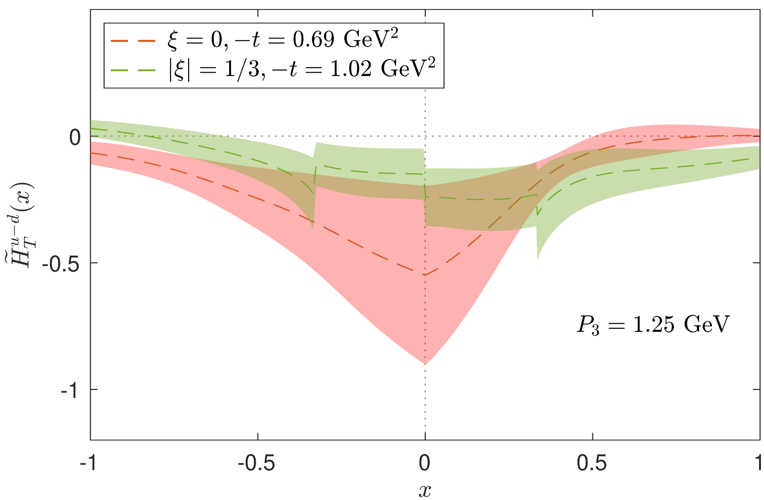


E_T^{u-d} ($\xi = 0,\ 1/3$)



$\tilde{H}_T^{u-d}$ ($\xi = 0,\ 1/3$)



Comparison of different types of PDFs/GPDs

ETMC, Phys. Rev. Lett. 125 (2020) 262001

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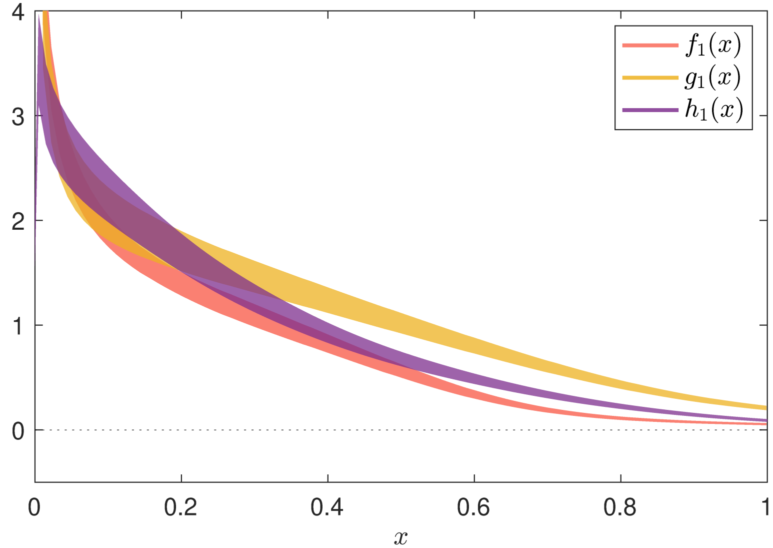

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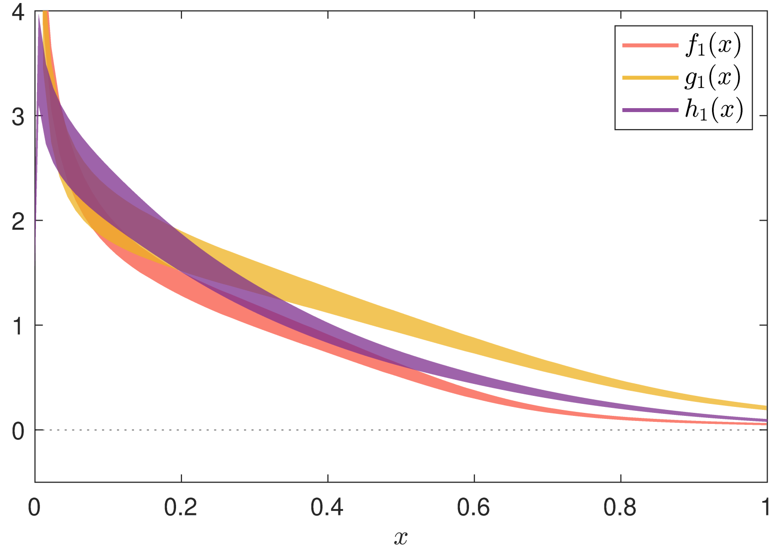

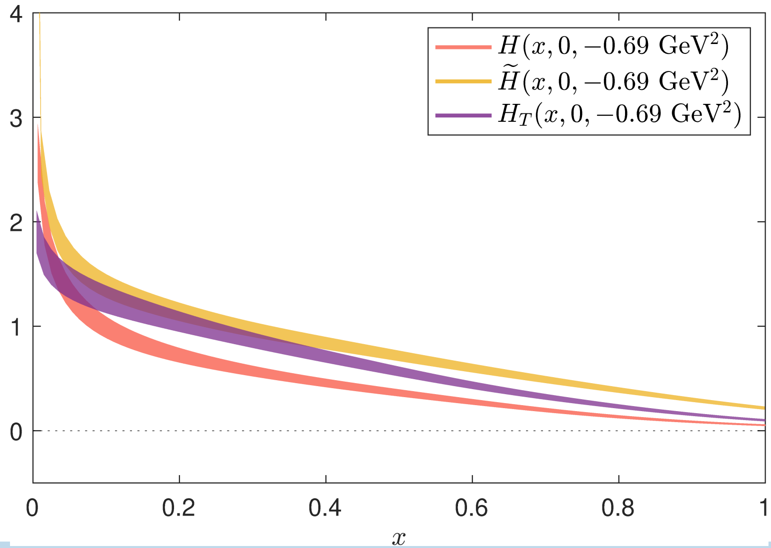

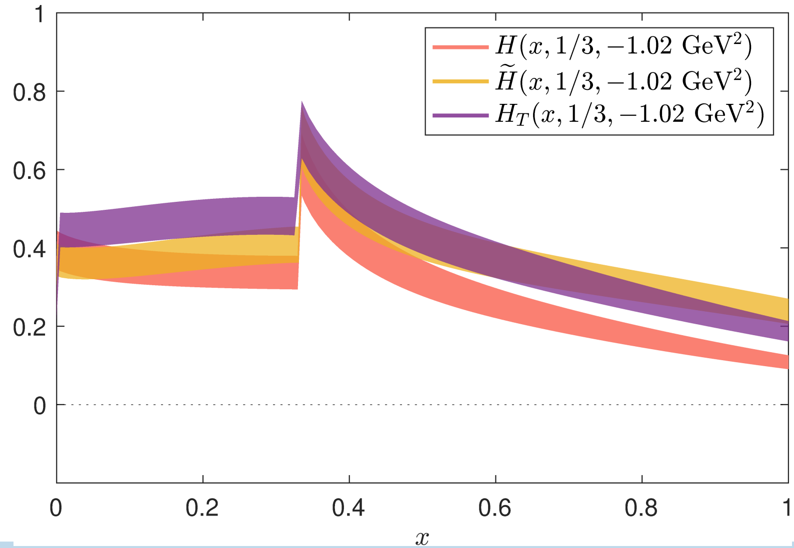

Twist-3

PDFs/GPDs can be classified according to their twist, which describes the order in $1/Q$ at which they appear in the factorization of structure functions.
LT: **twist-2** – probability densities for finding partons carrying fraction x of the hadron momentum.

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Twist-3:

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Exploratory studies:

- matching for twist-3 PDFs: g_T, h_L, e
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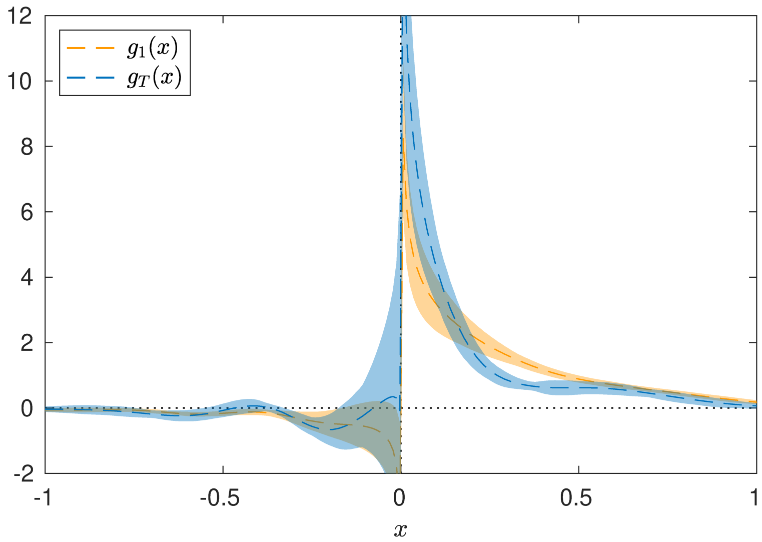

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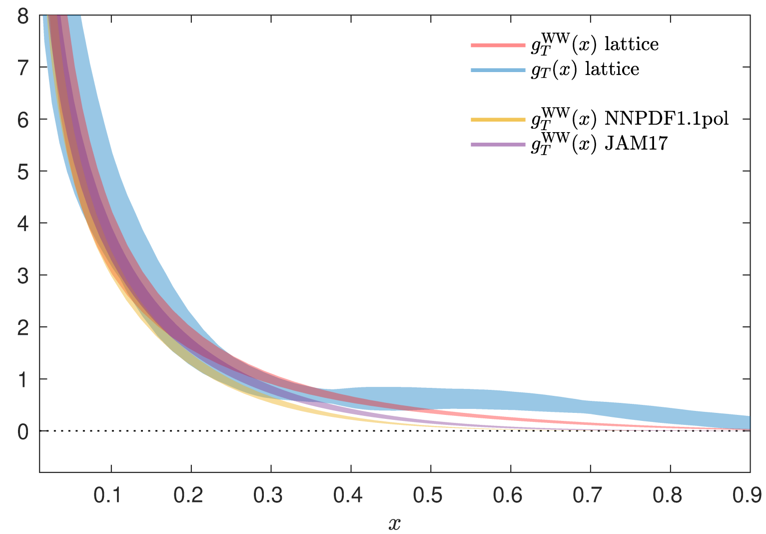

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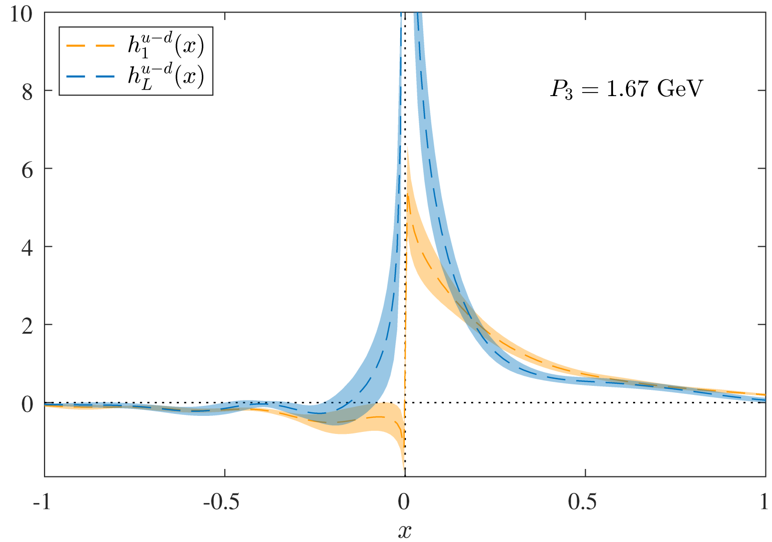

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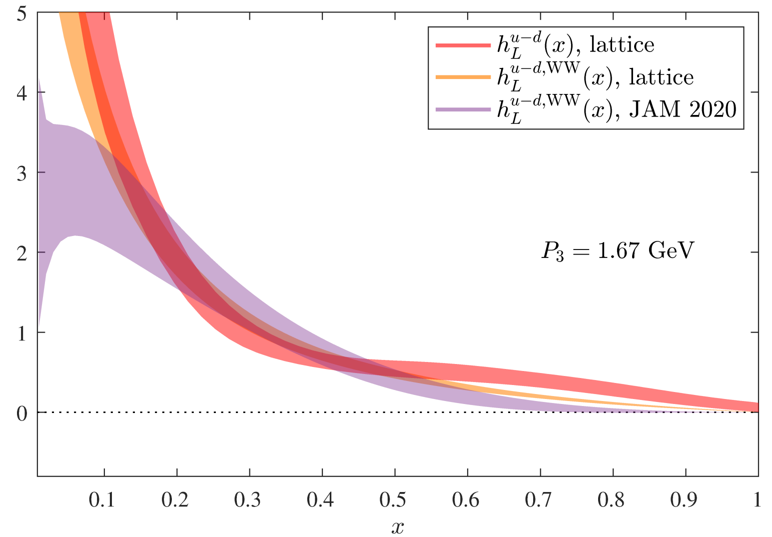

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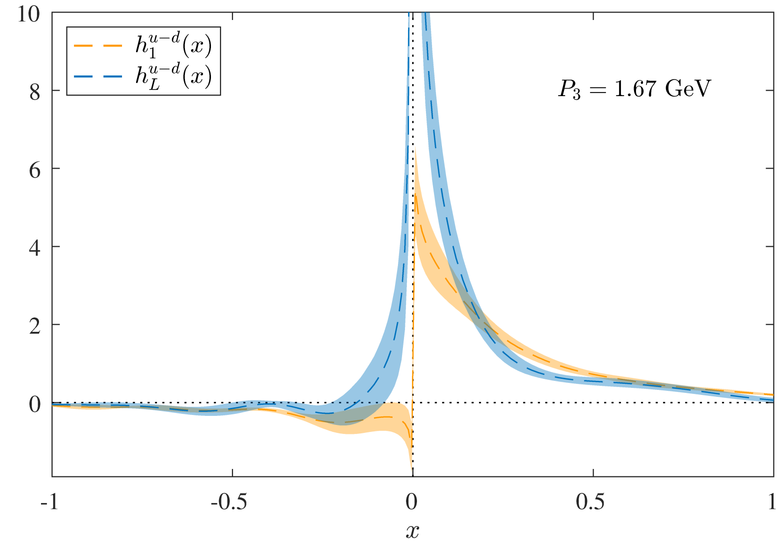

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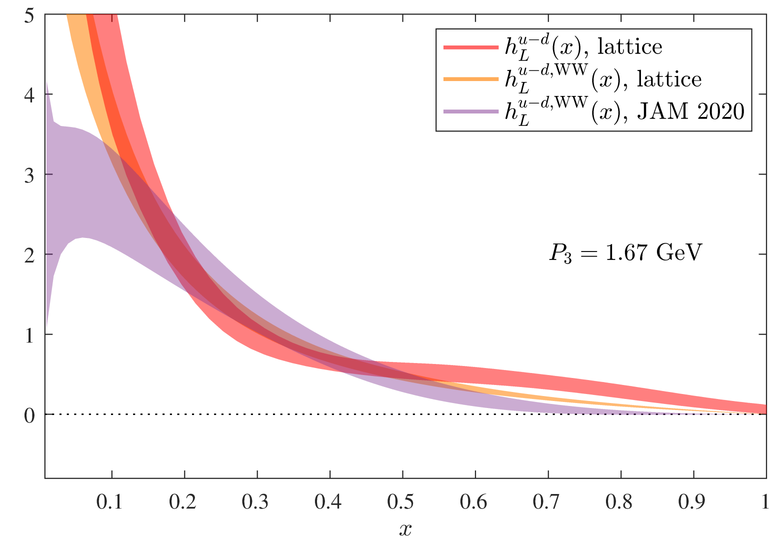


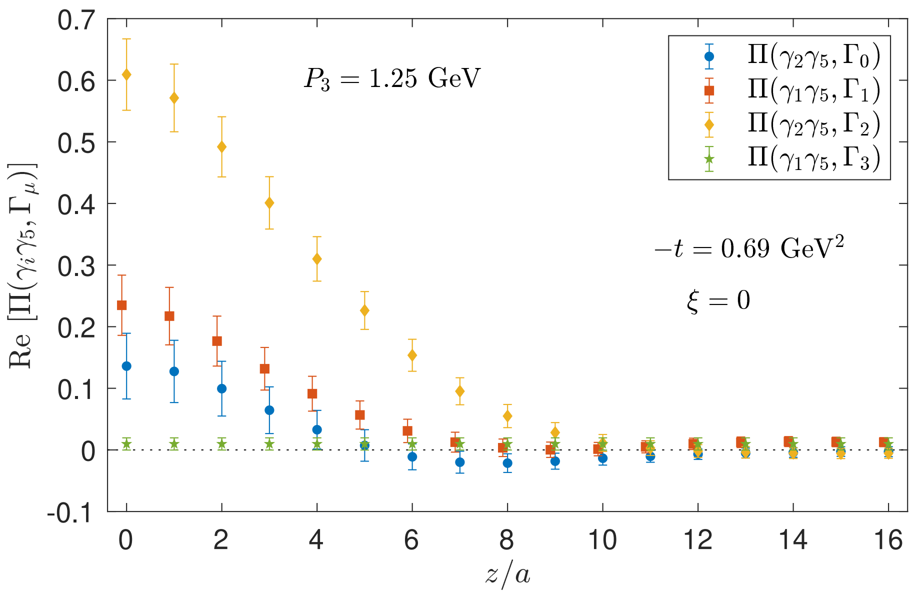
First exploration of twist-3 GPDs

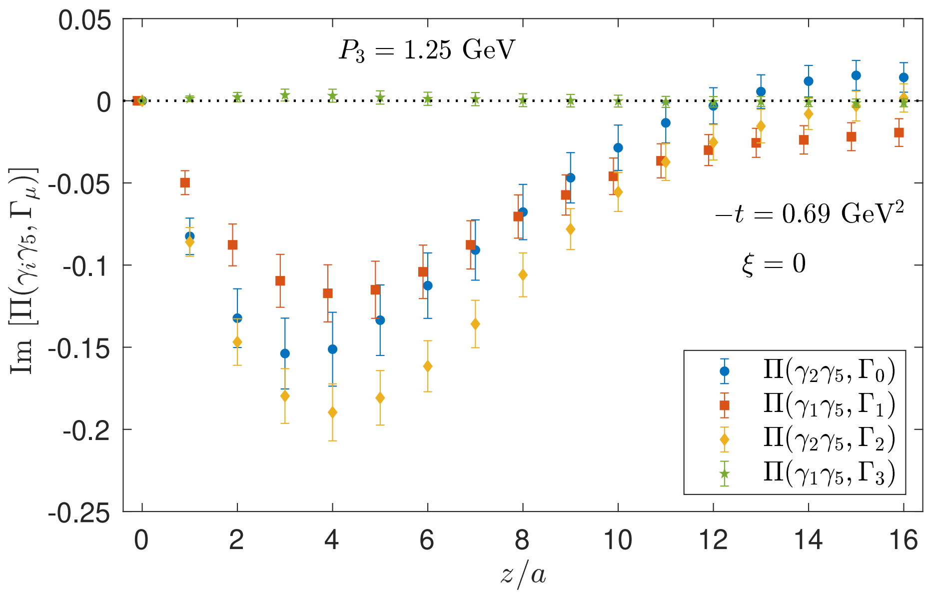
Very recently, we combined our explorations of GPDs and of twist-3 distributions
S. Bhattacharya et al., 2112.05538

Twist-3 axial GPDs: $\widetilde{G}_1$, $\widetilde{G}_2$, $\widetilde{G}_3$, $\widetilde{G}_4$

$$h_{\gamma^j\gamma_5} = \langle\langle \frac{g_\perp^{j\rho}\Delta_\rho\gamma_5}{2m} \rangle\rangle [F_{\widetilde{E}} + F_{\widetilde{G}_1}] + \langle\langle g_\perp^{j\rho}\gamma_\rho\gamma_5 \rangle\rangle [F_{\widetilde{H}} + F_{\widetilde{G}_2}] + \langle\langle \frac{g_\perp^{j\rho}\Delta_\rho\gamma^+\gamma_5}{P^+} \rangle\rangle F_{\widetilde{G}_3} + \langle\langle \frac{i\epsilon_\perp^{j\rho}\Delta_\rho\gamma^+}{P^+} \rangle\rangle F_{\widetilde{G}_4} \, .$$

Bare ME: (same lattice setup)



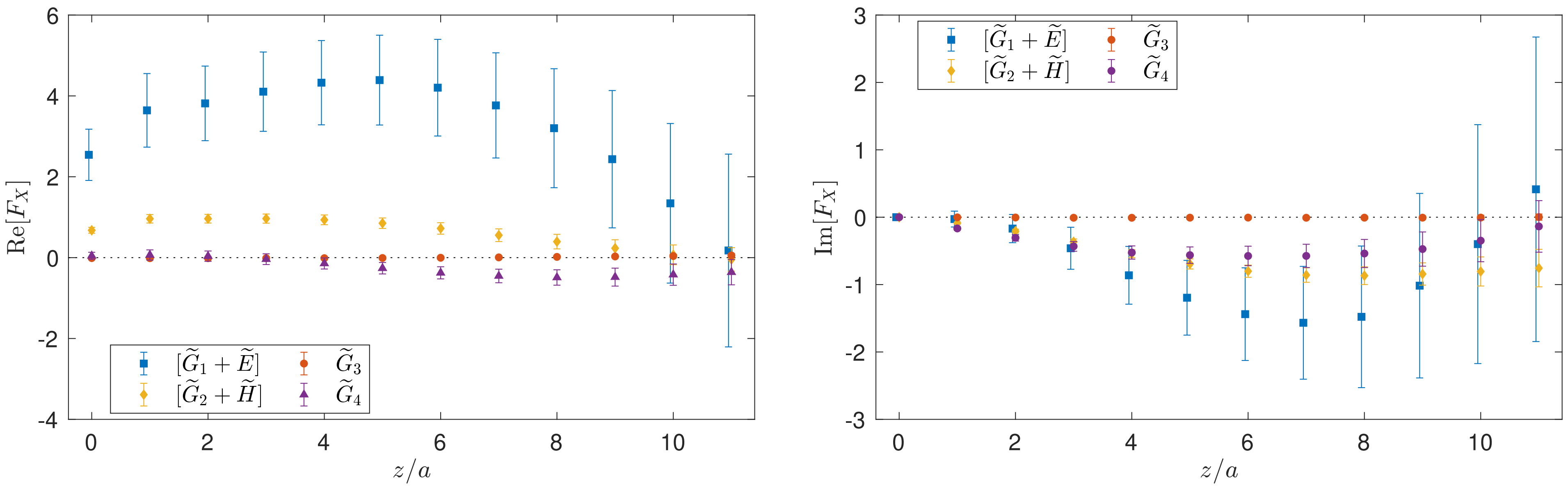


First exploration of twist-3 GPDs

Contributions from different insertions and projectors ($\vec{Q} = (Q_x, 0, 0)$):

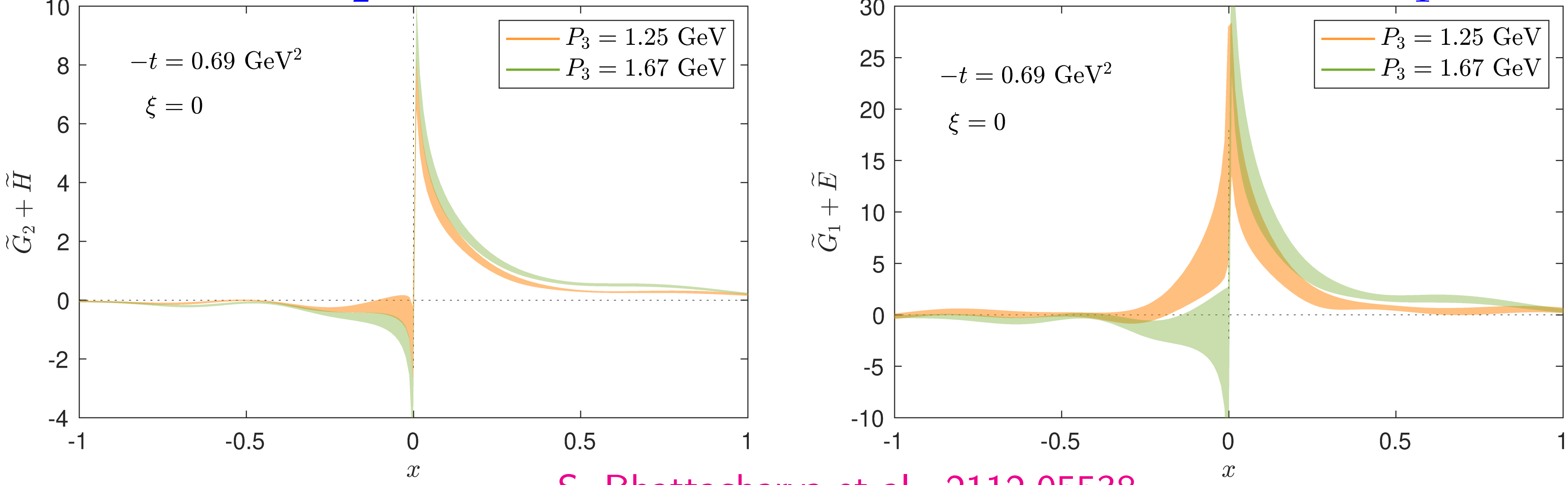
$\Pi(\gamma^2\gamma^5, \Gamma_0)$: $\widetilde{H} + \widetilde{G}_2$ and $\widetilde{G}_4$,
$\Pi(\gamma^2\gamma^5, \Gamma_2)$: $\widetilde{H} + \widetilde{G}_2$ and $\widetilde{G}_4$,
$\Pi(\gamma^1\gamma^5, \Gamma_1)$: $\widetilde{H} + \widetilde{G}_2$ and $\widetilde{E} + \widetilde{G}_1$,
$\Pi(\gamma^1\gamma^5, \Gamma_3)$: $\widetilde{G}_3$.

S. Bhattacharya et al., 2112.05538



First exploration of twist-3 GPDs

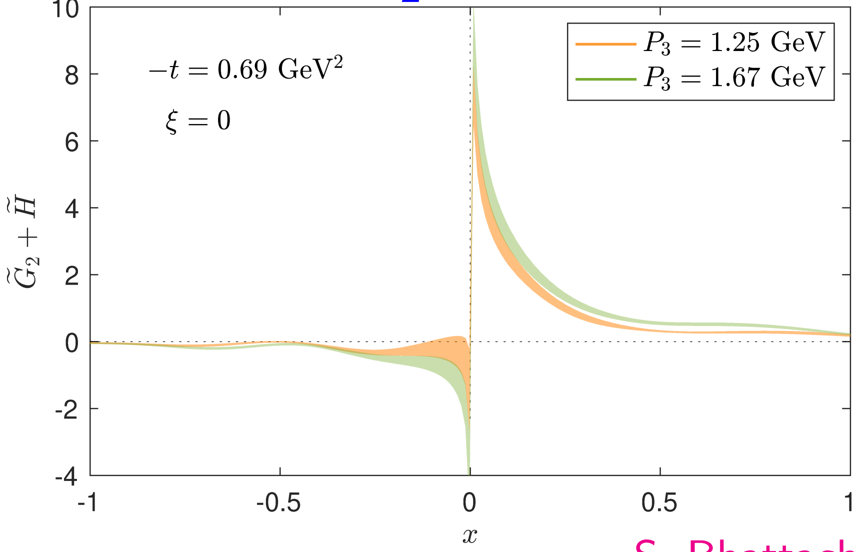
$\widetilde{H}+\widetilde{G}_2$ ($\xi=0, -t=0.69$ GeV^2) $\widetilde{E}+\widetilde{G}_1$

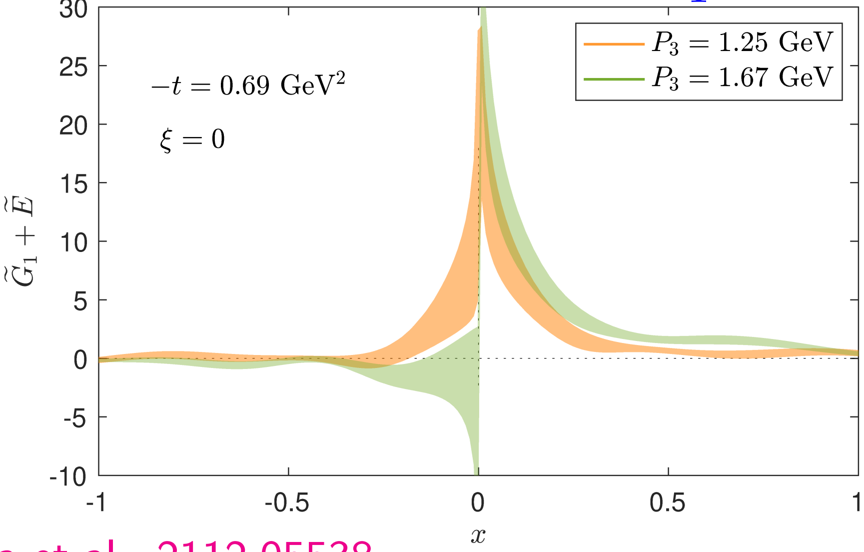


S. Bhattacharya et al., 2112.05538

First exploration of twist-3 GPDs

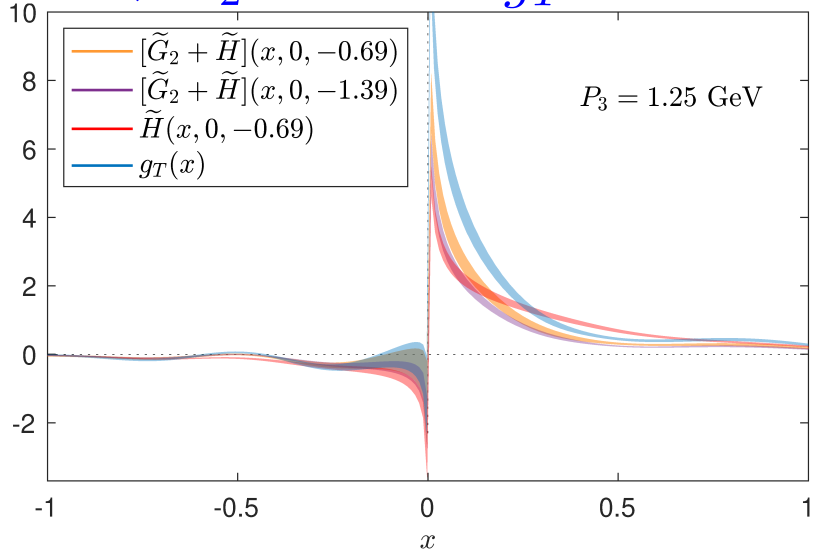
$\widetilde{H} + \widetilde{G}_2$ $(\xi = 0, -t = 0.69 \text{ GeV}^2)$ $\widetilde{E} + \widetilde{G}_1$

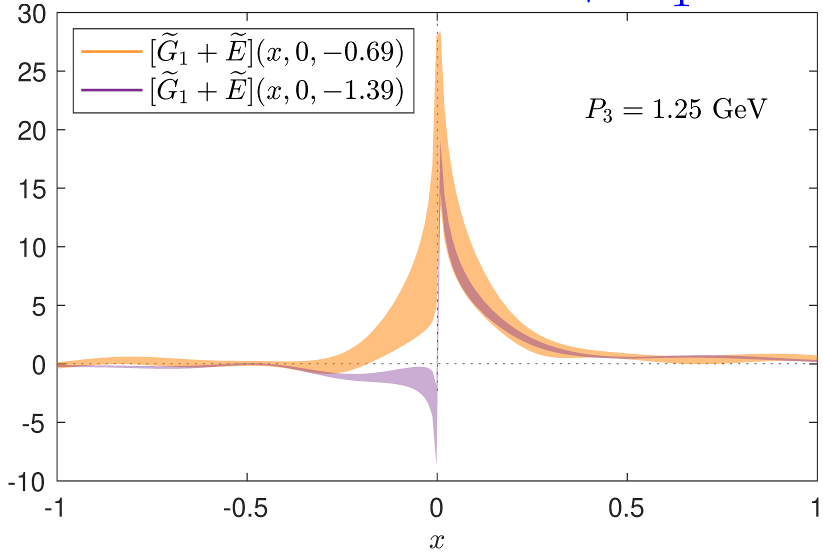




S. Bhattacharya et al., 2112.05538

$\widetilde{H} + \widetilde{G}_2$ vs. $\widetilde{H}$ vs. g_T $(\xi = 0, -t = 0.69,\ 1.39 \text{ GeV}^2)$ $\widetilde{E} + \widetilde{G}_1$





Conclusions and prospects

- **Huge progress in lattice calculations of GPDs!**
- Recent breakthrough:
  - ⋆ computationally more efficient calculations in non-symmetric frames,
  - ⋆ with, additionally, faster convergence to the light-cone.
- Overall very encouraging results!
- Still several challenges to overcome (control of systematics).
- Obviously, GPDs much more challenging than PDFs.
- Expect slow, but consistent progress and complementary role to pheno.

Thank you for your attention!

Backup slides

Transversity GPDs: ETMC, Phys. Rev. D105 (2022) 034501

4 GPDs: H_T, E_T, $\tilde{H}_T$, $\tilde{E}_T$

Three nucleon boosts ($\xi = 0$): $P_3 = 0.83, 1.25, 1.67$ GeV
Nucleon boost ($\xi \neq 0$): $P_3 = 1.25$ GeV

Momentum transfer ($\xi = 0$): $-t = 0.69$ GeV^2
Momentum transfer ($\xi \neq 0$): $-t = 1.02$ GeV^2

Transversity GPDs

Transversity GPDs: ETMC, Phys. Rev. D105 (2022) 034501

4 GPDs: H_T, E_T, $\tilde{H}_T$, $\tilde{E}_T$

spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
$\vec{P}' = \vec{P} + \vec{Q}$, $\vec{Q}$ – momentum transfer
lattice computation of bare ME

↓

renormalization
intermediate RI scheme
conversion to $\mathrm{M\overline{MS}}$ scheme
(incl. evolution to $\mu = 2$ GeV)

↓

reconstruction of x-dependence
z-space→x-space
Backus-Gilbert

↓

matching to light cone
$\mathrm{M\overline{MS}} \to \overline{\mathrm{MS}}$

↓

light-cone GPD

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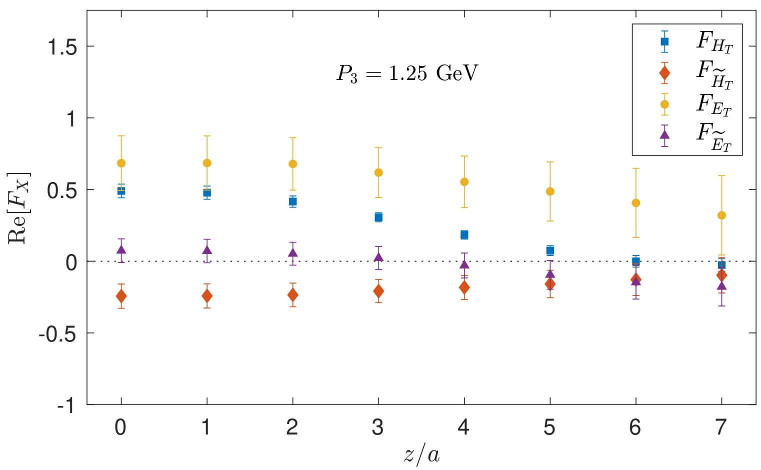
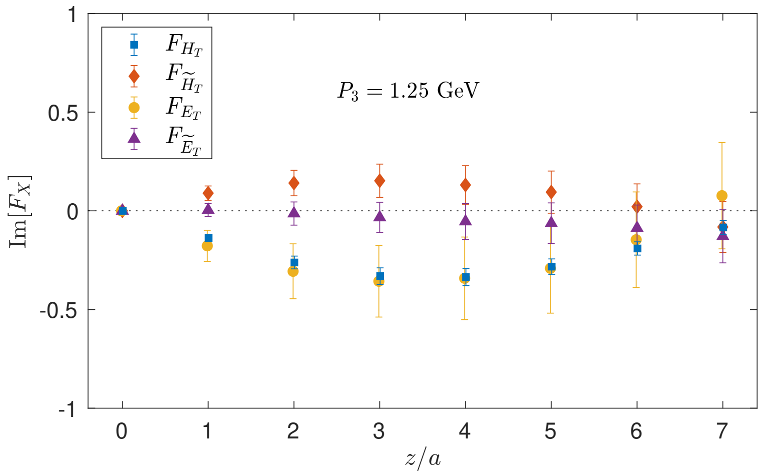
Renormalized ME

Real part | Imaginary part

$\xi = 1/3$





Transversity GPDs

ETMC, Phys. Rev. D105 (2022) 034501

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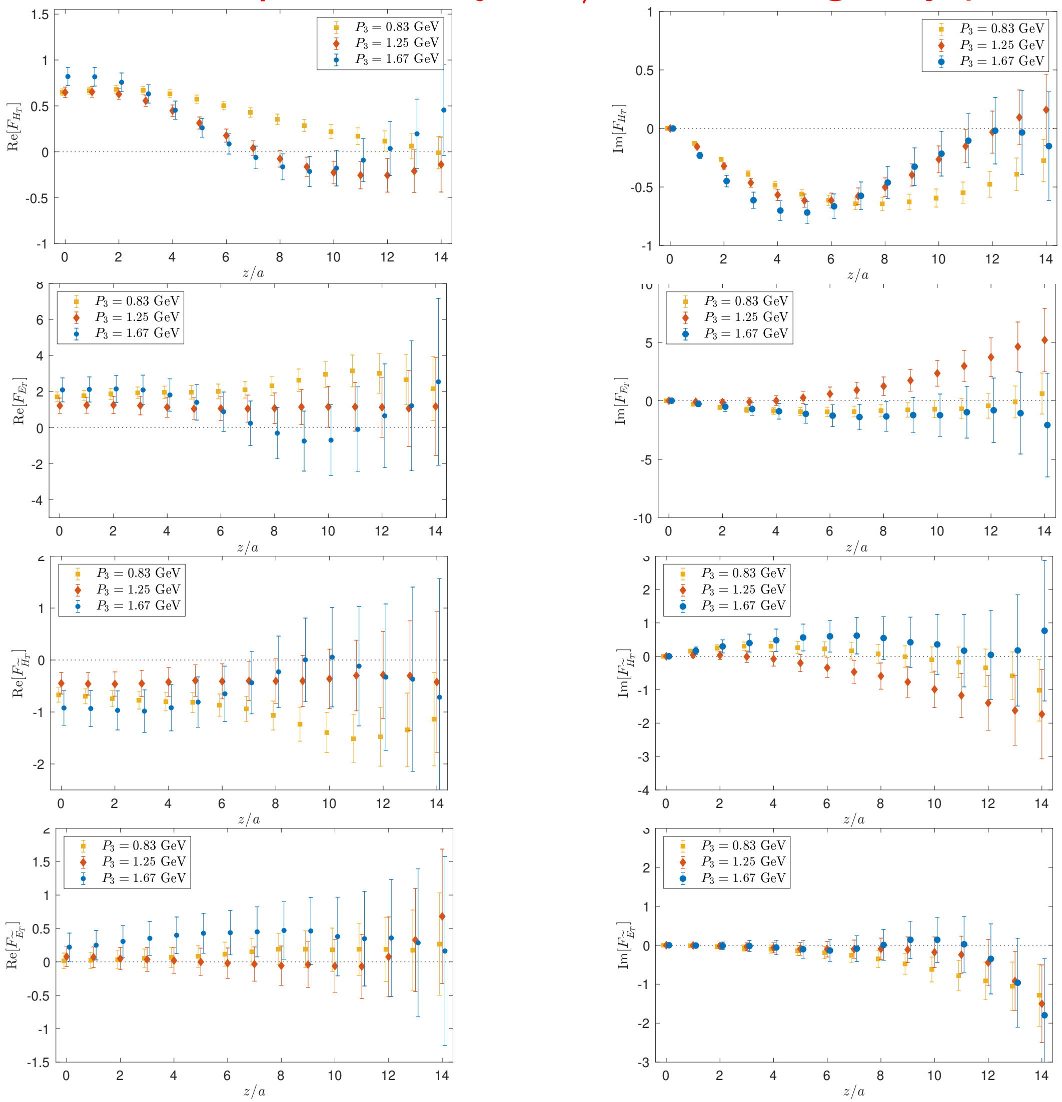
light-cone GPD

Real part $\xi = 1/3$ Imaginary part



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ETMC, Phys. Rev. D105 (2022) 034501

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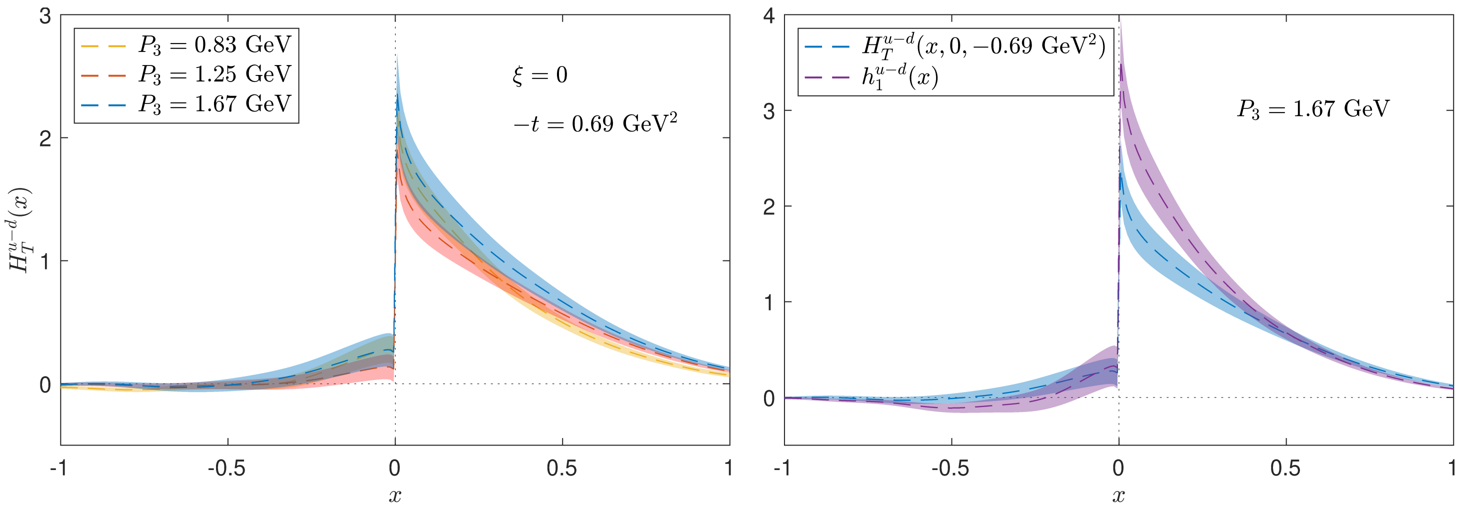
light-cone GPD

H_T^{u-d} $(\xi = 0)$



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ETMC, Phys. Rev. D105 (2022) 034501

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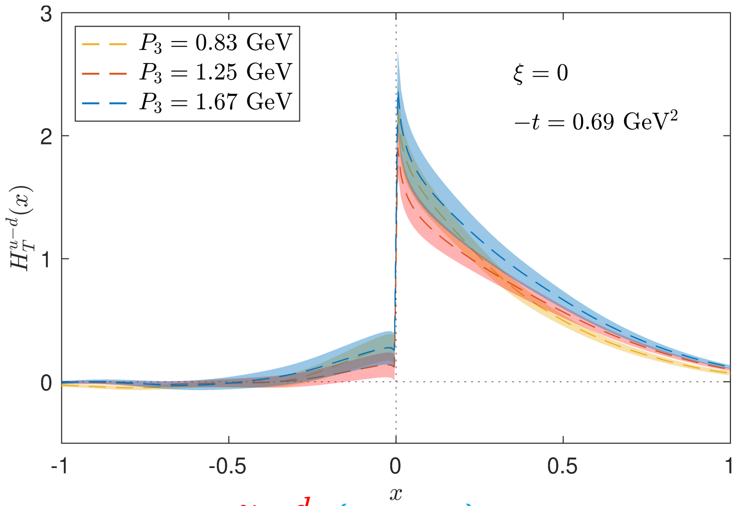
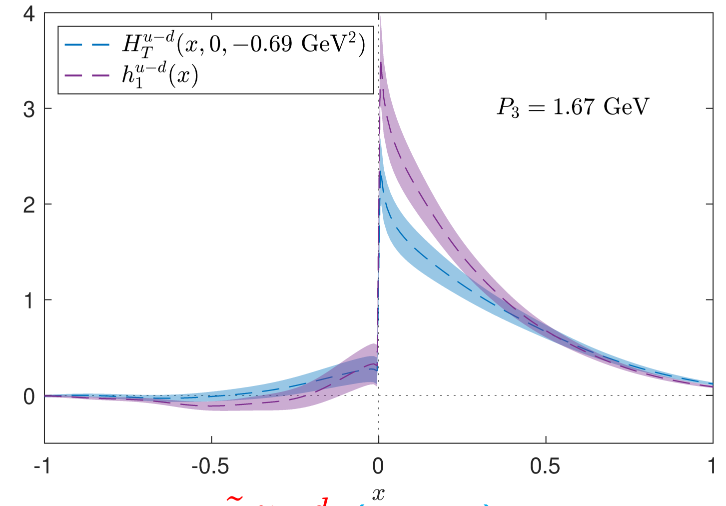
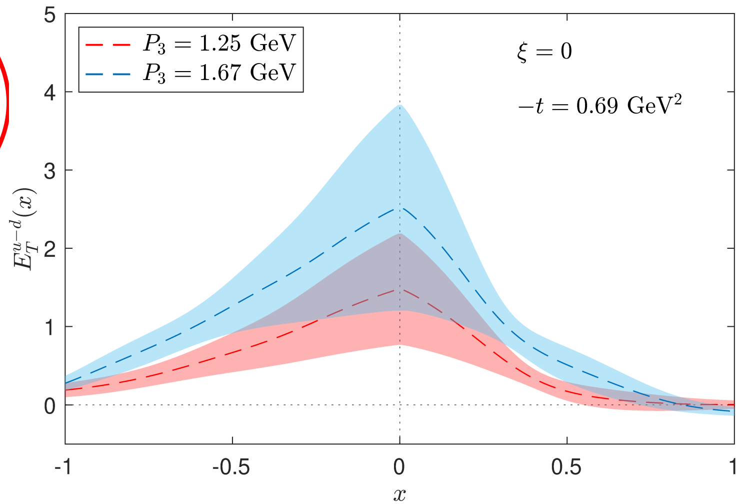
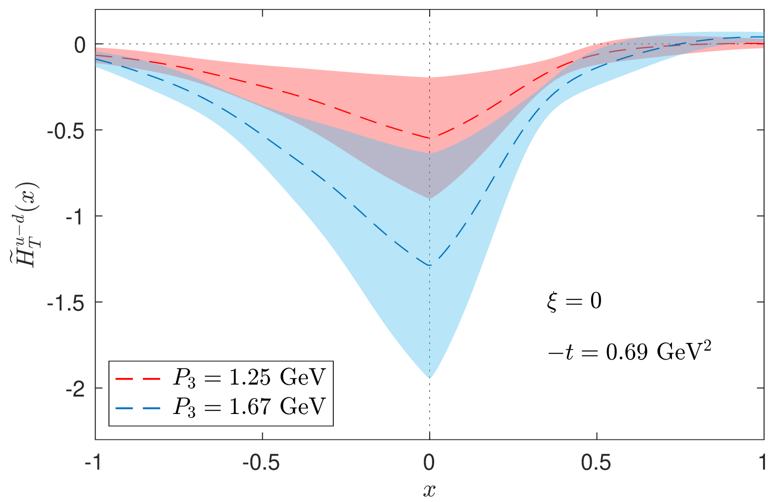




E_T^{u-d} $(\xi = 0)$



$\tilde{H}_T^{u-d}$ $(\xi = 0)$



Transversity GPDs

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light-cone GPD

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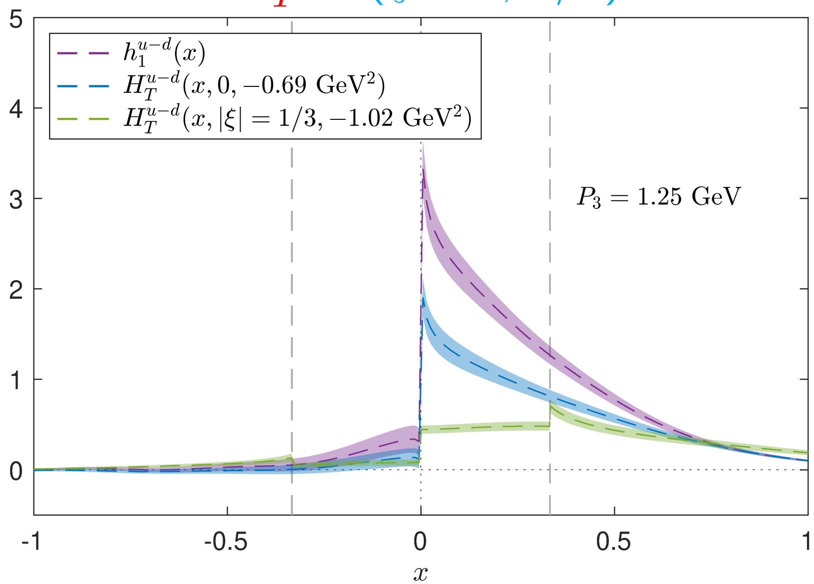
H_T^{u-d} $(\xi = 0,\ 1/3)$



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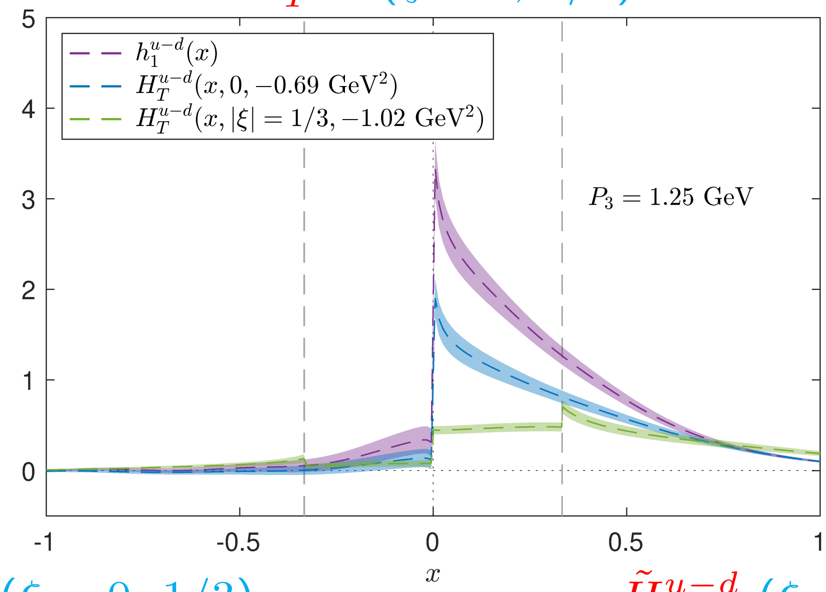
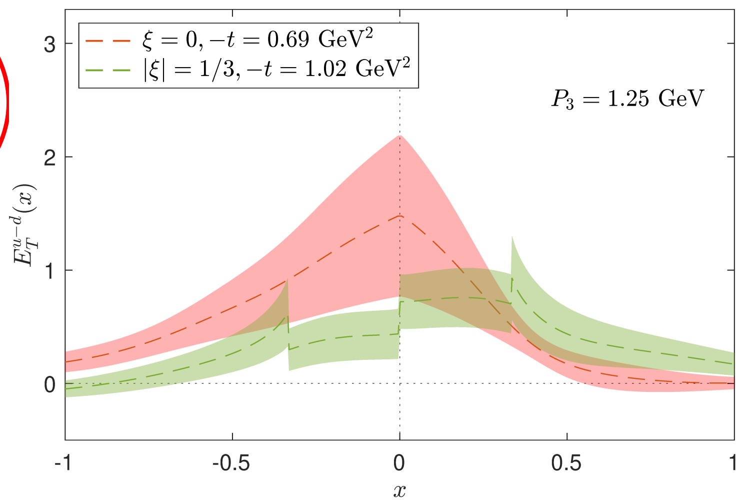
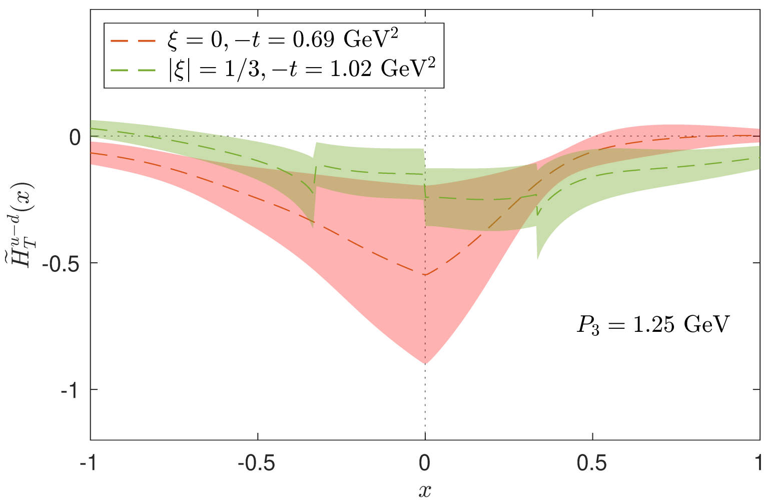


E_T^{u-d} $(\xi = 0,\ 1/3)$



$\tilde{H}_T^{u-d}$ $(\xi = 0,\ 1/3)$



Transversity GPDs

ETMC, Phys. Rev. D105 (2022) 034501

Transversity GPDs:
4 GPDs: H_T, E_T, $\tilde{H}_T$, $\tilde{E}_T$

More fundamental quantity: $E_T + 2\tilde{H}_T$

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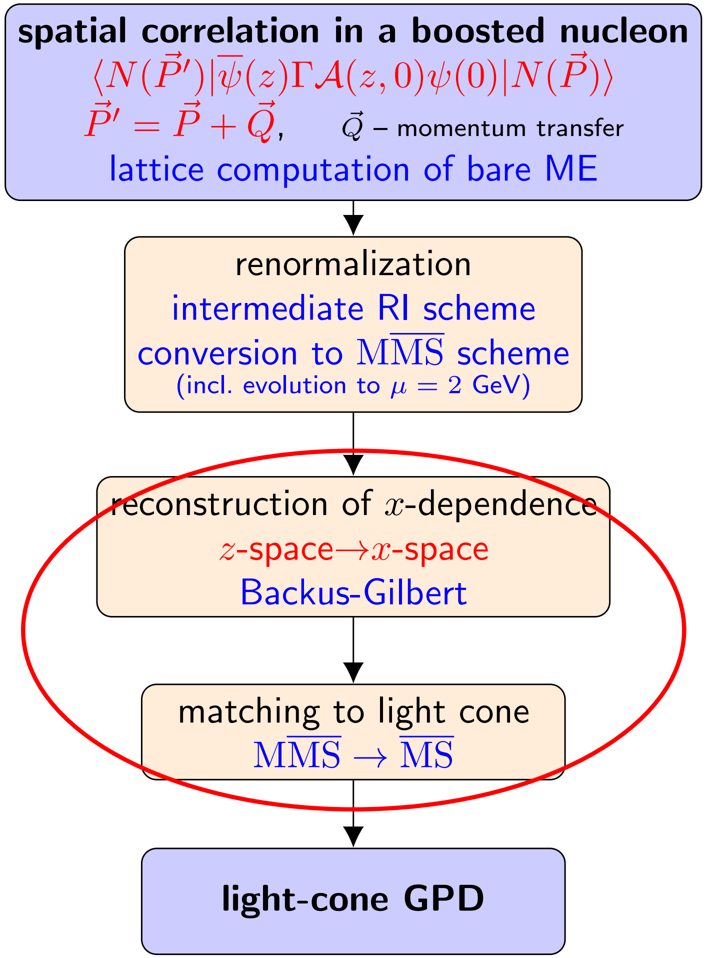

ETMC, Phys. Rev. D105 (2022) 034501

More fundamental quantity: $E_T + 2\tilde{H}_T$

– related to the transverse spin structure of the proton
– physically interpreted as lateral deformation in the distribution of transversely polarized quarks in an unpolarized proton
– lowest Mellin moment in the forward limit: transverse spin-flavor dipole moment in an unpolarized target (k_T)
– second moment related to the transverse-spin quark angular momentum in an unpolarized proton

Transversity GPDs

ETMC, Phys. Rev. D105 (2022) 034501

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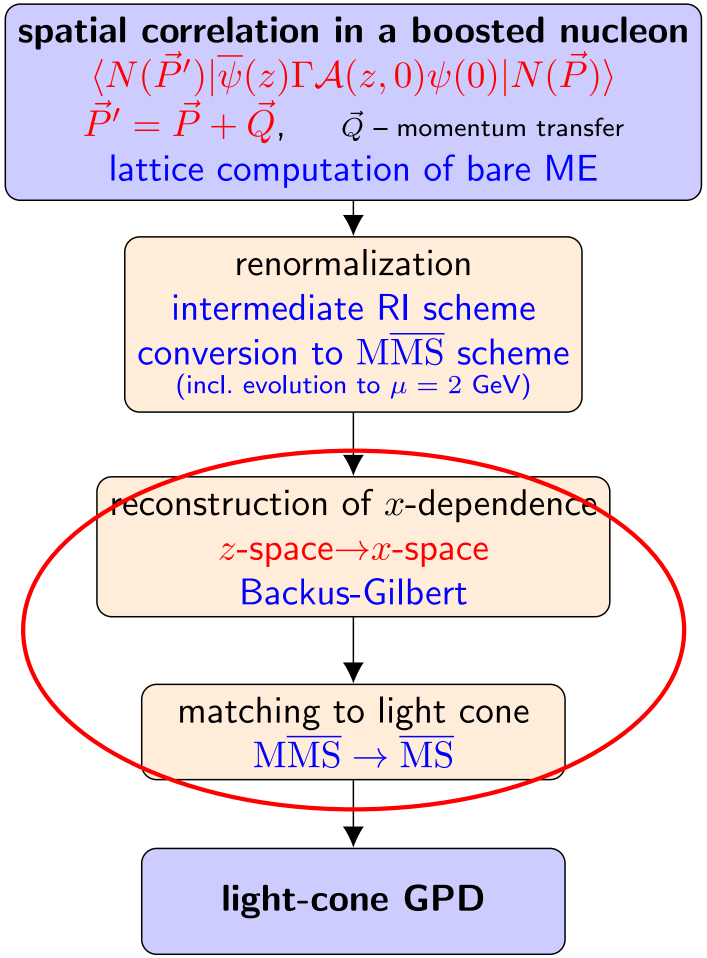

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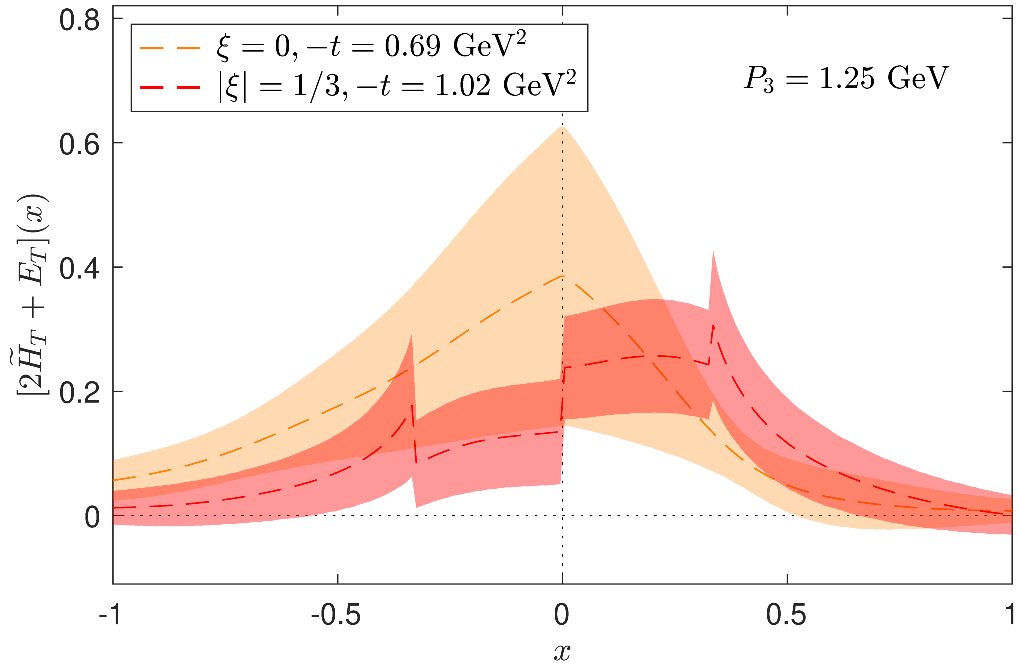

Moments of transversity GPDs

$n = 0$ Mellin moments:

$$\begin{aligned}
\int_{-1}^{1} dx\, H_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, H_{Tq}(x,\xi,t,P_3) = A_{T10}(t)\,, \\
\int_{-1}^{1} dx\, E_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, E_{Tq}(x,\xi,t,P_3) = B_{T10}(t)\,, \\
\int_{-1}^{1} dx\, \widetilde{H}_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, \widetilde{H}_{Tq}(x,\xi,t,P_3) = \widetilde{A}_{T10}(t)\,, \\
\int_{-1}^{1} dx\, \widetilde{E}_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, \widetilde{E}_{Tq}(x,\xi,t,P_3) = 0\,,
\end{aligned} \quad (1)$$

- lowest moments of GPDs skewness-independent,
- lowest moments of quasi-GPDs boost-independent.

$n = 1$ Mellin moments (related to GFF of one-derivative tensor operator):

$$\begin{aligned}
\int_{-1}^{1} dx\, x\, H_T(x,\xi,t) &= A_{T20}(t)\,, \\
\int_{-1}^{1} dx\, x\, E_T(x,\xi,t) &= B_{T20}(t)\,, \\
\int_{-1}^{1} dx\, x\, \widetilde{H}_T(x,\xi,t) &= \widetilde{A}_{T20}(t)\,, \quad (3) \\
\int_{-1}^{1} dx\, x\, \widetilde{E}_T(x,\xi,t) &= 2\xi \widetilde{B}_{T21}(t)\,, \quad (2)
\end{aligned}$$

- skewness-dependence only in for $\widetilde{E}_T$ (only ξ-odd GPD).

Moments of transversity GPDs

Moments of	$H_T(x, \xi = 0, t = -0.69\,\mathrm{GeV}^2)$			$H_T(x, \xi = 1/3, t = -1.02\,\mathrm{GeV}^2)$
	$P_3 = 0.83$ GeV	$P_3 = 1.25$ GeV	$P_3 = 1.67$ GeV	$P_3 = 1.25$ GeV
H_{Tq}	0.65(4)	0.64(6)	0.81(10)	0.49(5)
H_T	0.69(4)	0.67(6)	0.84(10)	0.45(4)
xH_T	0.20(2)	0.21(2)	0.24(3)	0.15(2)
A_{T10} $(z = 0)$	0.65(4)	0.65(6)	0.82(10)	0.49(5)

Moments of transversity GPDs

Moments of	$H_T(x, \xi = 0, t = -0.69\,\mathrm{GeV}^2)$			$H_T(x, \xi = 1/3, t = -1.02\,\mathrm{GeV}^2)$
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Mellin moments P_3-independent, preserved by matching, suppressed with increasing $-t$.

Moments of transversity GPDs

Moments of	$H_T(x,\xi=0,t=-0.69\,\text{GeV}^2)$			$H_T(x,\xi=1/3,t=-1.02\,\text{GeV}^2)$
	$P_3 = 0.83$ GeV	$P_3 = 1.25$ GeV	$P_3 = 1.67$ GeV	$P_3 = 1.25$ GeV
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Mellin moments P_3-independent, preserved by matching, suppressed with increasing $-t$.

Moments of	$E_T(x,\xi=0,t=-0.69\,\text{GeV}^2)$			$H_T(x,\xi=1/3,t=-1.02\,\text{GeV}^2)$
	$P_3 = 0.83$ GeV	$P_3 = 1.25$ GeV	$P_3 = 1.67$ GeV	$P_3 = 1.25$ GeV
E_{Tq}		1.20(42)	2.05(65)	0.67(19)
E_T		1.15(43)	2.10(67)	0.73(19)
xE_T		0.06(4)	0.13(5)	0.11(11)
B_{T10} $(z=0)$	1.71(28)	1.22(43)	2.10(67)	0.68(19)
Moments of	$\widetilde{H}_T(x,\xi=0,t=-0.69\,\text{GeV}^2)$			$\widetilde{H}_T(x,\xi=1/3,t=-1.02\,\text{GeV}^2)$
	$P_3 = 0.83$ GeV	$P_3 = 1.25$ GeV	$P_3 = 1.67$ GeV	$P_3 = 1.25$ GeV
$\widetilde{H}_{Tq}$		-0.44(20)	-0.90(32)	-0.26(9)
$\widetilde{H}_T$		-0.42(21)	-0.92(33)	-0.27(9)
$x\widetilde{H}_T$		-0.17(8)	-0.30(10)	-0.05(5)
$\widetilde{A}_{T10}$ $(z=0)$	-0.67(14)	-0.45(21)	-0.92(33)	-0.24(8)

Similar conclusions (but very large errors).