Determining x-dependent gluon parton distribution functions from QCD

Chris Monahan (he/him/his), for the HadStruc Collaboration

William & Mary

Results from PRD 104 (2021) 034507 and 2207.08733

CIPANP 2022

WILLIAM & MARY
CHARTERED 1693

Gluon structure – the experimental picture

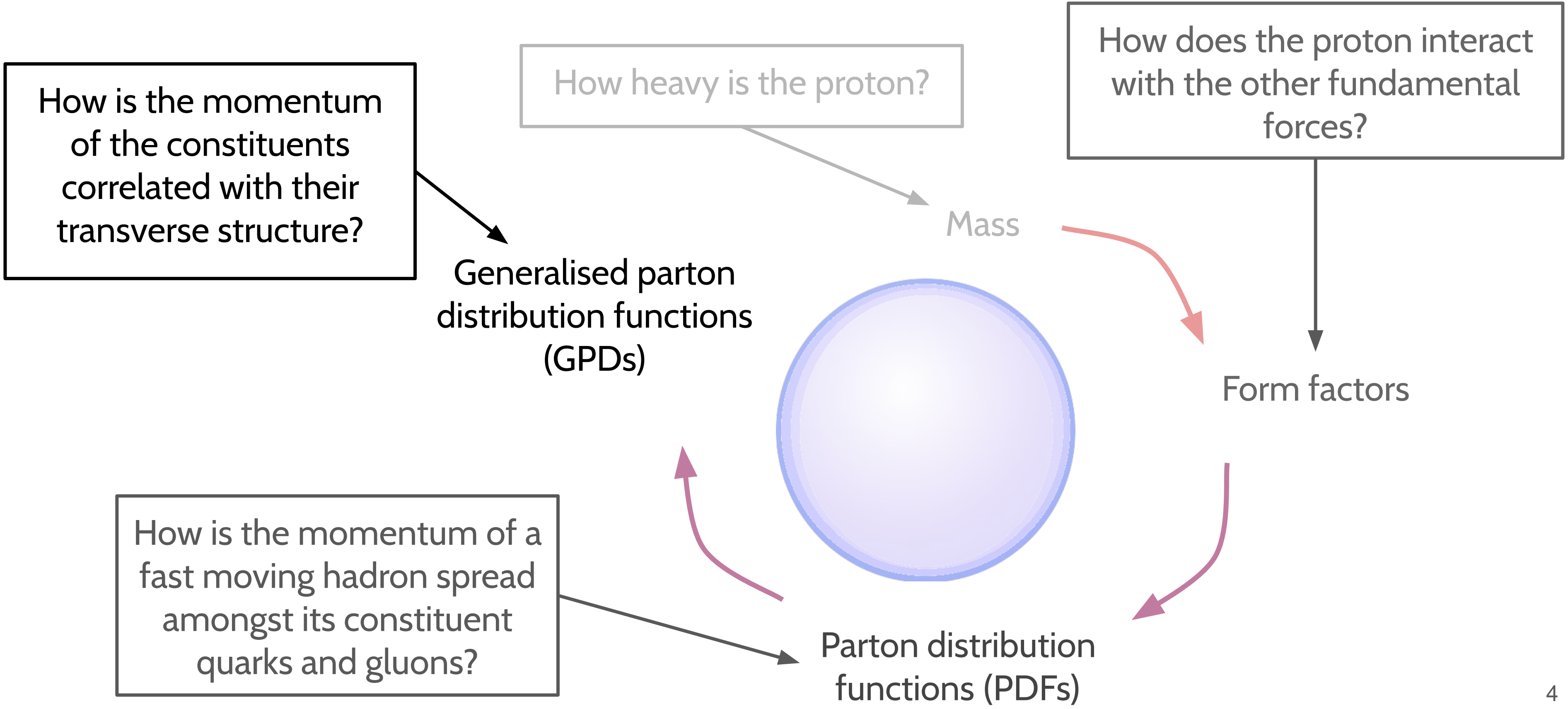
Gluons are pretty important

Gluons are key to understanding the visible universe

- Dominant contribution to mass of the visible universe
- Central to origin of hadron spin and (resolution of?) “proton spin crisis”

Complete tomography of hadrons needs detailed understanding of gluon structure

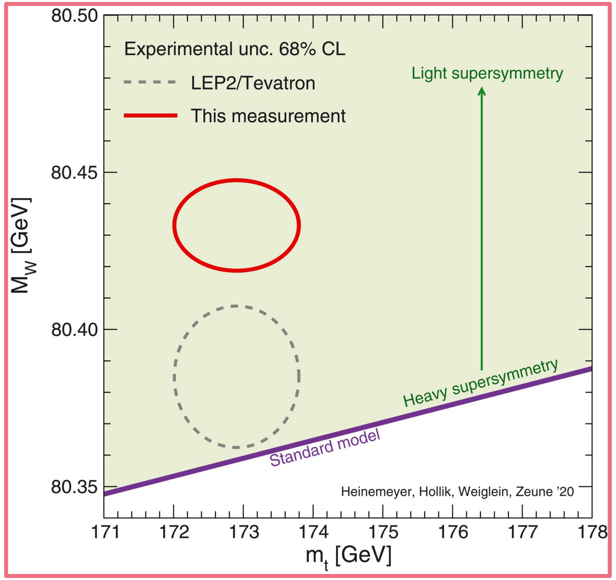
But hadron structure is pretty complicated



Unpolarised gluon PDFs

Unpolarised gluon PDFs an important source of theoretical uncertainty at LHC

- Higgs couplings
- Certain search channels for BSM particles
- Mass of the W boson



Recent measurement from Tevatron

$$m_W^{(\text{Tevatron 2022})} = 80.4335(94)\ \text{GeV}$$

Significant (7 sigma) tension with standard model expectation

$$m_W^{(\text{SM})} = 80.357(6)\ \text{GeV}$$

And previous experimental results

$$m_W^{(\text{LEP+Tevatron})} = 80.385(15)\ \text{GeV}$$

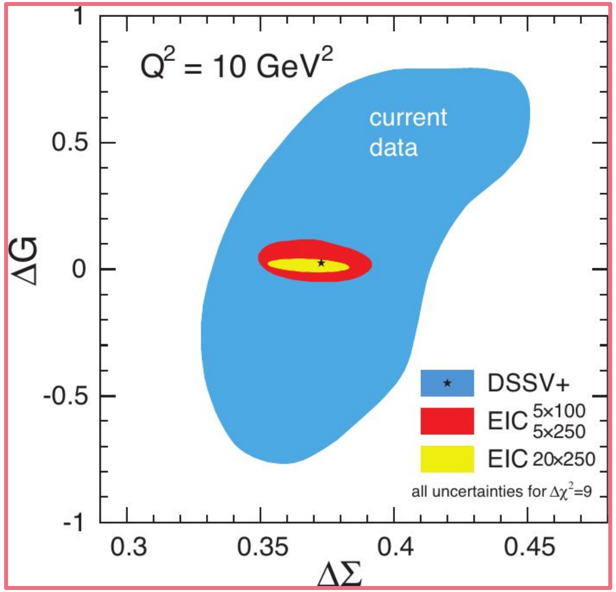
$$m_W^{(\text{ATLAS})} = 80.370(19)\ \text{GeV}$$

CDF, Science 376 (2022) 170

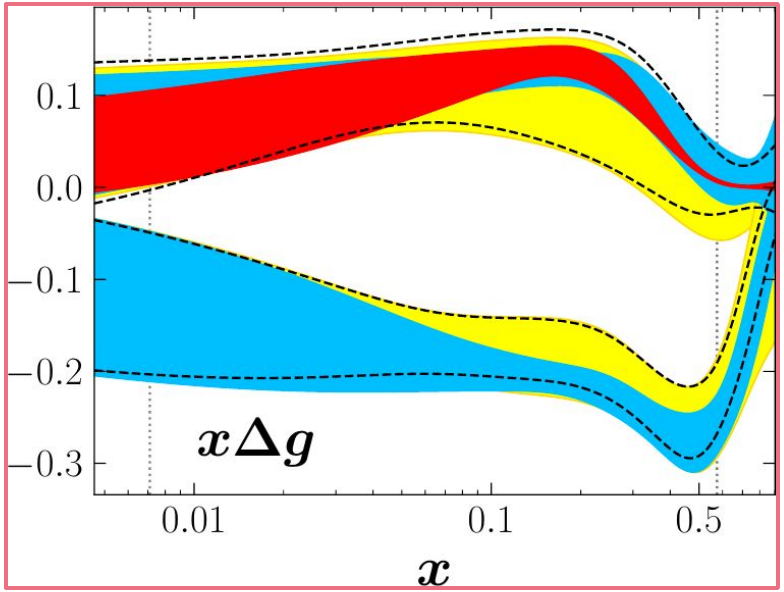
Polarised gluon structure

$$\Delta G(Q^2) = \int_0^1 \mathrm{d}x\, \Delta g(x, Q^2)$$

Polarised gluon structure central to understanding the origin of hadron spin



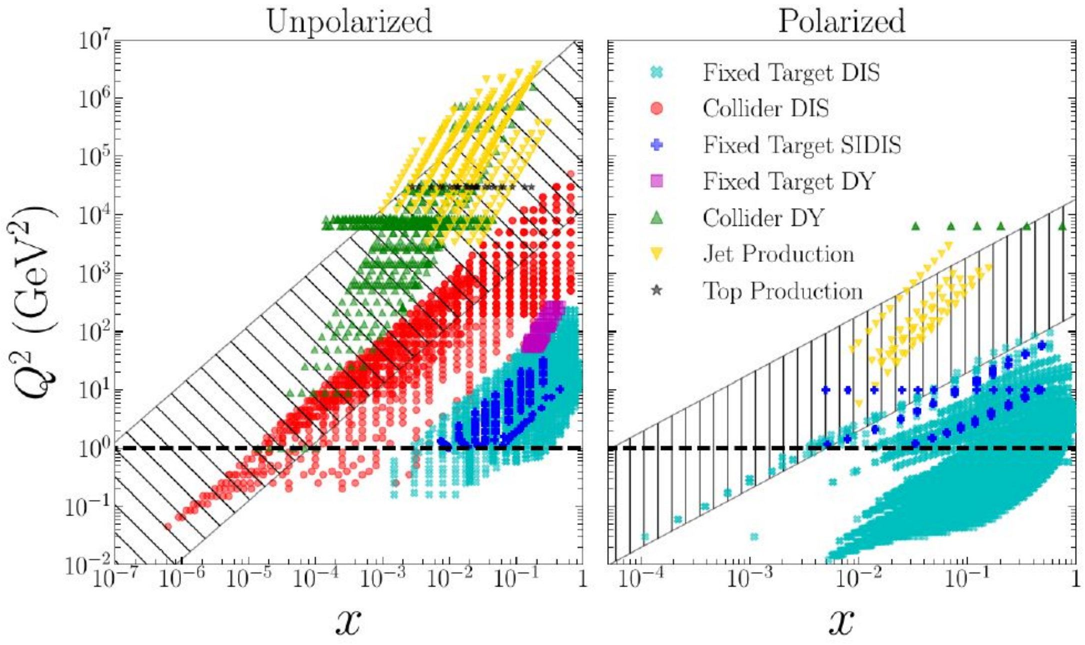
Accardi et al., Eur. Phys. J. A52 (2012) 268



Zhou et al., PRD 105 (2022) 074022

Gluon PDFs: kinematic coverage

LHC has considerably improved our knowledge of gluon PDFs



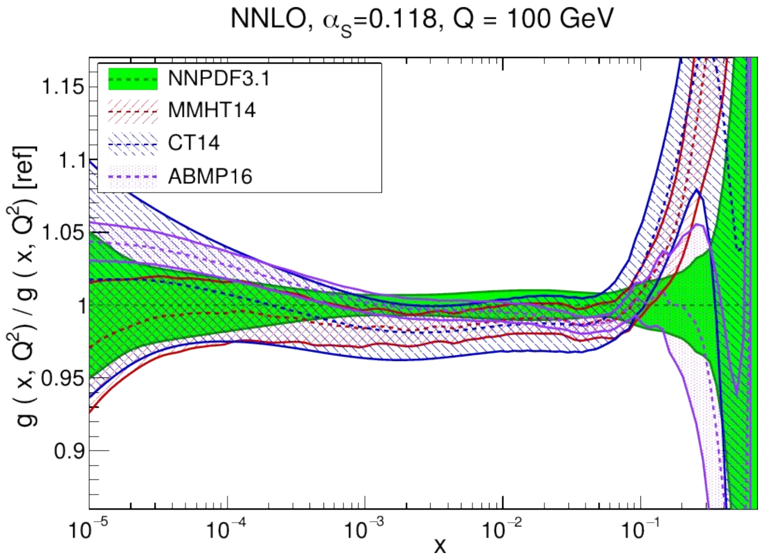
EIC and LHeC will expand this much further

Ethier and Nocera, Ann.Rev.Nucl.Part.Sci. 70 (2020) 43

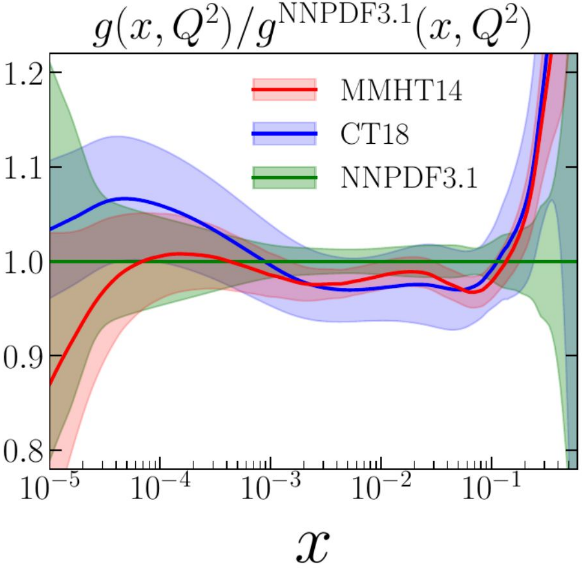
Gluon PDFs: experimental status

LHC has considerably improved our knowledge of unpolarised gluon PDFs

Large uncertainties remain at large and small Bjorken-x



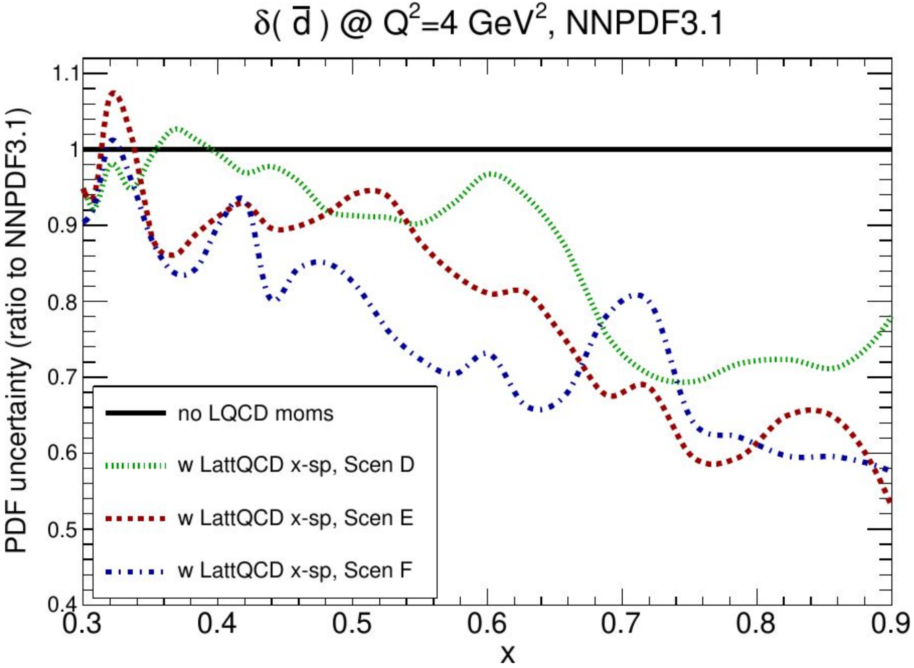
Gao, Harland-Lang, Rojo, Phys. Rep. 742 (2018) 01

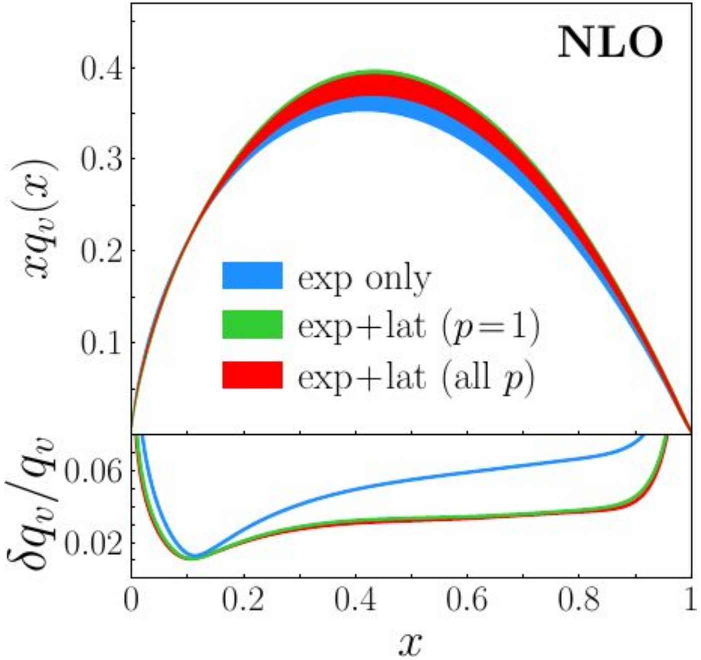


Ethier and Nocera, Ann.Rev.Nucl.Part.Sci. 70 (2020) 43

PDFs from QCD

First principles calculations complement, and inform, JLab 12 GeV, the LHC and the EIC



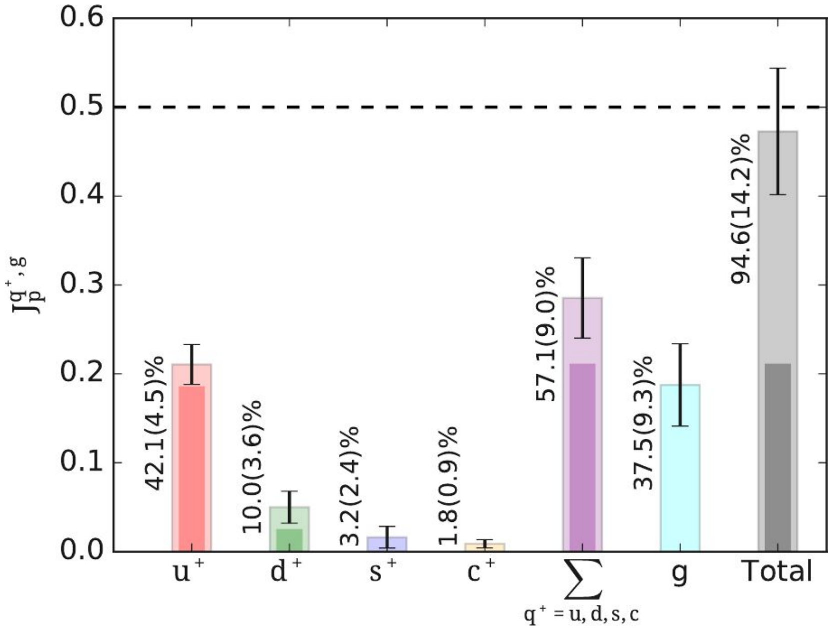


Lin et al., Prog. Nucl. Part. Phys. 100 (2018) 107

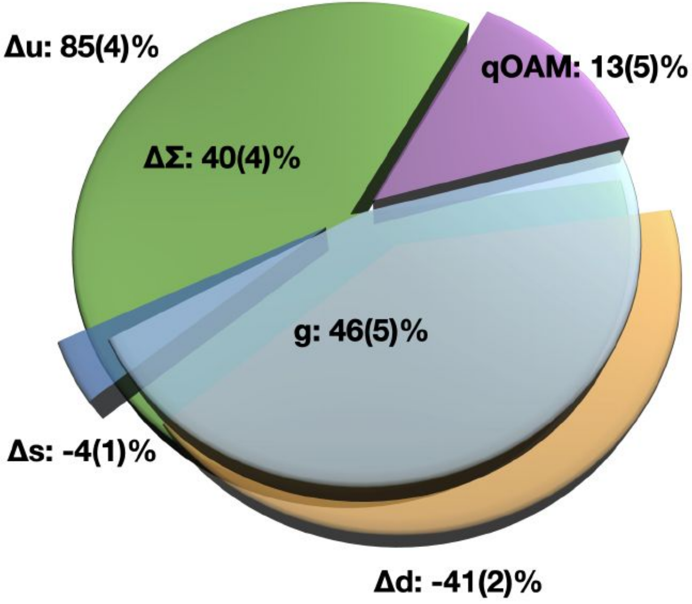
Barry et al., PRD 105 (2022) 114051

First calculations of the spin of the proton

Complete understanding of hadron structure requires theoretical insight from QCD



Alexandrou et al., PRD 101 (2020) 094513



Wang et al., PRD 106 (2022) 014512

Large experimental uncertainties = opportunities for theorists

Light-cone distributions

$$x f^{(0)}_{g/H}(x) = \int_{-1}^{1} \mathrm{d}\nu \, e^{ix\nu} \mathcal{M}^{(0)}_{g/H}(\nu, 0)$$

PDFs

$n^2 = 0$ $\qquad$ ξ^-

$$M^{(0)}_{\mu\nu\rho\sigma;H}(P, n) = \langle H(P) | G_{\mu,\nu}(n^\alpha) W^{(A)}(n^\alpha, 0) G_{\rho\sigma} | H(P) \rangle$$

Distributions galore

PDFs

Large momentum effective theory (LaMET)

Short distance factorisation

$$n^2 = 0 \quad \xi^-$$

$$M^{(0)}_{\mu\nu\rho\sigma;H}(P,n) = \langle H(P)|G_{\mu,\nu}(n^\alpha)W^{(A)}(n^\alpha,0)G_{\rho\sigma}|H(P)\rangle$$

$$P^z \quad n^2 = -z^2 \quad \zeta$$

quasi PDFs

pseudo PDFs

Ji, PRL 110 (2013) 262002

Radyushkin, PRD 96 (2017) 034025

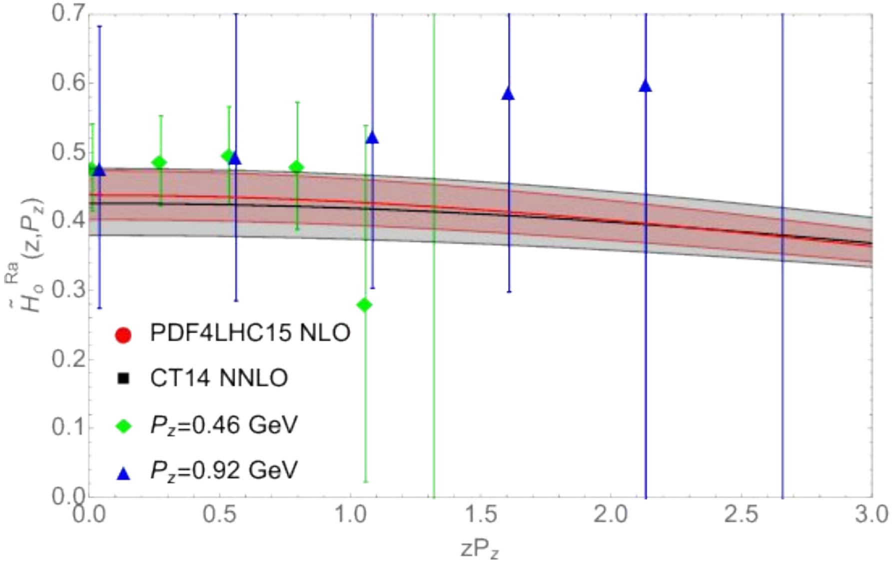
Gluon PDFs: lattice calculations

Gluon observables provide significant challenges for lattice calculations

- significant signal-to-noise issues
- nonperturbative renormalisation challenging

First proof-of-principle calculation using LaMET illustrates the challenges

Fan *et al.*, PRL 121 (2018) 242001



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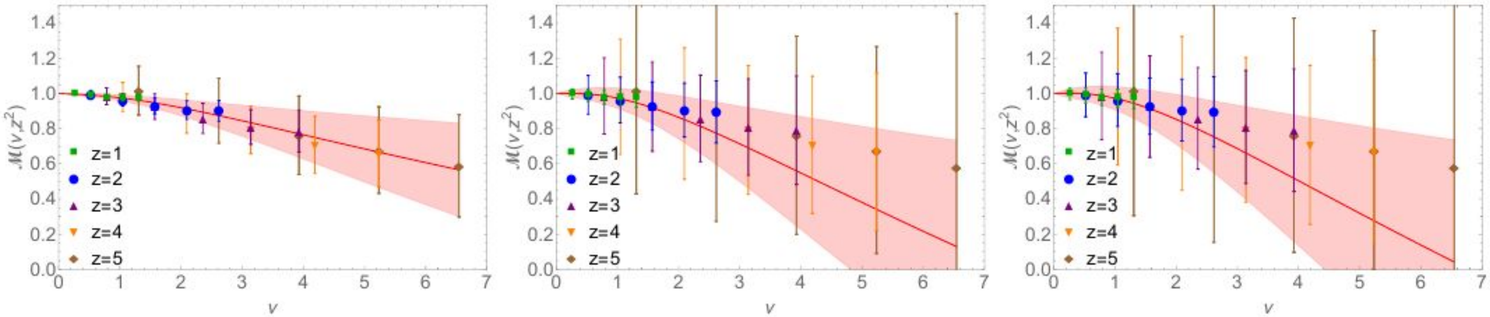
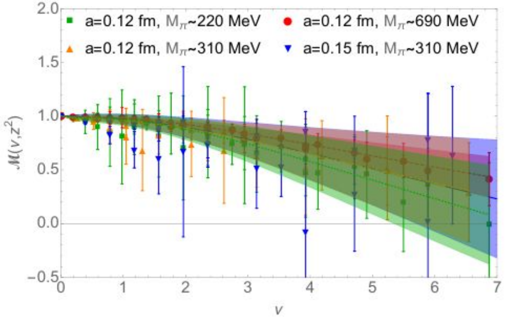


Fan et al., Int.J.Mod.Phys.A 36 (2021) 13



Fan and Lin, PLB 823 (2021) 136778

Gluon structure = hard to calculate on the lattice

Next: HadStruc Collaboration’s approach to overcoming the challenges

Khan et al., PRD 104 (2021) 034507

Gluon PDFs: pseudo-distribution formalism

Starting point:

Balitsky, Morris and Radyushkin, PLB 808 (2020) 135621

$$M^{(0)}_{\mu\nu\rho\sigma;H}(P,n) = \langle H(P)|G_{\mu,\nu}(n^\alpha)W^{(A)}(n^\alpha,0)G_{\rho\sigma}|H(P)\rangle$$

$$W^{(A)}(n^\alpha,0) = \mathcal{P}\exp\left\{ig\int_0^n \mathrm{d}y^\mu A^{(A)}_\mu(y)\right\}$$

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$n^2 = 0$

$$\mathcal{M}^{(0)}_{g/H}(\nu,0) = \frac{1}{2(P^+)^2}\left[M^{(0)}_H(P,z^-)\right]^{+\mu}{}_{+\mu}$$

PDFs ξ^-

$$x f^{(0)}_{g/H}(x) = \int_{-1}^{1} \mathrm{d}\nu\, e^{ix\nu}\mathcal{M}^{(0)}_{g/H}(\nu,0)$$

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$n^2 = -z^2$

$$\mathcal{M}^{(0)}_{g/H}(\nu,z^2) = \frac{1}{2E_P^2}\left[M^{(0)}_{0ii0;H}(P,z) - M^{(0)}_{jiij;H}(P,z)\right]$$

$$\mathcal{M}^{(\mathrm{red.})}_{g/H}(\nu,z^2) = \left(\frac{\mathcal{M}^{(0)}_{g/H}(\nu,z^2)}{\mathcal{M}^{(0)}_{g/H}(\nu,0)|_{z=0}}\right) \Bigg/ \left(\frac{\mathcal{M}^{(0)}_{g/H}(0,z^2)|_{p=0}}{\mathcal{M}^{(0)}_{g/H}(0,0)|_{p=0,z=0}}\right)$$

ζ pseudo PDFs

$$\mathcal{M}^{(\mathrm{red.})}_{g/H}(\nu,z^2) = \int_0^1 \frac{\mathrm{d}\xi\,\xi}{\langle\xi\rangle^2(\mu)}\left[c_{gg}(\xi\nu,\mu^2z^2)f_{g/H}(\xi,\mu^2) + \frac{P^z}{E_P}c_{gq}(\xi\nu,\mu^2z^2)f_{S/H}(\xi,\mu^2)\right]$$

$n^2 = 0$

$$\mathcal{M}^{(0)}_{g/H}(\nu,0) = \frac{1}{2(P^+)^2}\left[M^{(0)}_H(P,z^-)\right]^{+\mu}{}_{+\mu}$$

PDFs ξ^-

$$x f^{(0)}_{g/H}(x) = \int_{-1}^{1} \mathrm{d}\nu\, e^{ix\nu}\mathcal{M}^{(0)}_{g/H}(\nu,0)$$

HadStruc lattice implementation

Gluons provide significant signal-to-noise challenges for lattice calculations

Mitigated through three strategies

1. Gradient flow smearing reduces ultraviolet fluctuations
2. Distillation and summed GEVP method improves operator overlap and reduces excited state contamination
3. Reduced Ioffe-time distribution reduces correlated uncertainties through ratio

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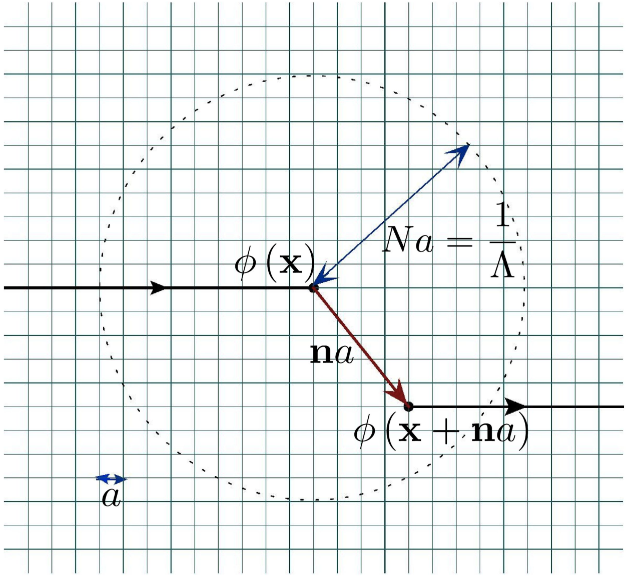
Removes need for challenging gluon operator renormalisation

But only provides shape of the PDF, not normalisation (gluon momentum fraction)

Smearing

"Smearing" partially restores rotational symmetry: widely-used lattice technique

- construct operators with improved continuum limits, *i.e.* reduced systematic uncertainties
- suppresses operator mixing
- precisely identify hadronic excited states
- reduce statistical noise



Davoudi & Savage, Phys. Rev. D 86 (2012) 054505

Gradient flow smearing

Gradient flow: deterministic evolution of fields in “flow time” τ toward classical

$$\frac{\partial}{\partial\tau}B_\mu(\tau,x) = D_\nu\Big(\partial_\nu B_\mu - \partial_\mu B_\nu + [B_\nu, B_\mu]\Big) \qquad D_\nu = \partial_\nu + [B_\nu, \cdot]$$

$$\frac{\partial}{\partial\tau}\chi(\tau,x) = D_\mu D^\mu \chi(\tau,x) \qquad D_\mu = \partial_\mu + B_\mu$$

Dirichlet boundary conditions

$$B_\mu(\tau=0,x) = A_\mu(x) \qquad \chi(\tau=0,x) = \psi(x)$$

Can be implemented on the lattice and solved nonperturbatively

$$\frac{\partial}{\partial\tau}V_\mu(\tau,x) = -g_0^2\left\{\partial_{x,\mu}S\big[V_\mu(\tau,x)\big]\right\}V_\mu(\tau,x)$$

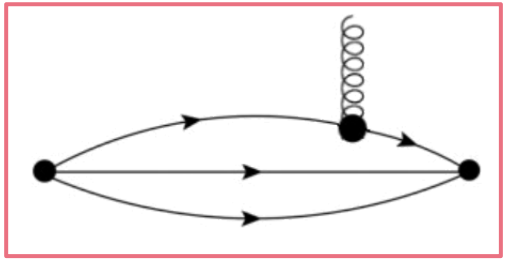
Narayanan & Neuberger, JHEP 0603 064
Lüscher, JHEP 1008 071
Lüscher, JHEP 04 (2013) 123

Correlators

Signal-to-noise ratio improved and excited state effects reduced through sGEVP

Typical lattice calculation based on 3-point function (and ratio with 2-point function)

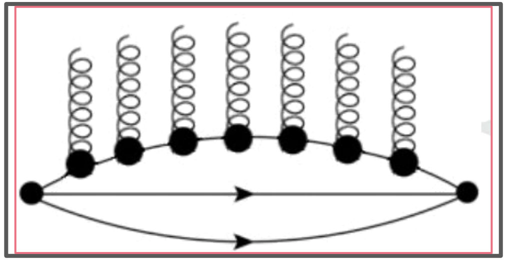
$$\langle C_{3pt}(t,t_g)\rangle = \langle 0|T\{O_N(t)\,O_g(t_g)\,\bar{O}_N(0)\}|0\rangle$$



Summation method

$$C^{i,s}_{3pt}(t) = \sum_{t_g=1}^{t-1} C^{i}_{3\mathrm{pt}}(t,t_g)$$

$$C^{i}_{3pt}(t,t_g) = \Big(C^{i}_{2pt}(t) - \langle C_{2pt}(t)\rangle\Big)\Big(O^{i}_{g}(t_g) - \langle O_g(t_g)\rangle\Big)$$



Leads to

$$\mathcal{M}^{\mathrm{eff}}(t) = A + B\,t\exp(-\Delta E\,t)$$

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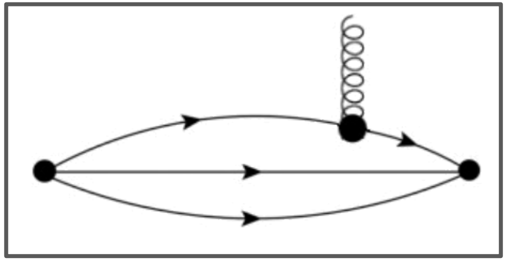

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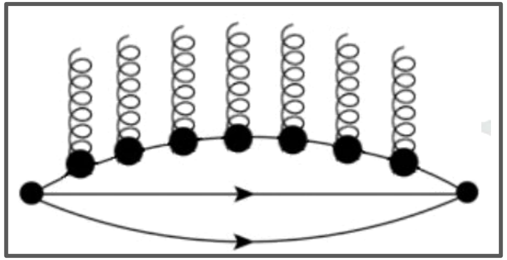

Leads to

what we want

$$\mathcal{M}^{\mathrm{eff}}(t) = A + B\, t \exp(-\Delta E\, t)$$

Correlators

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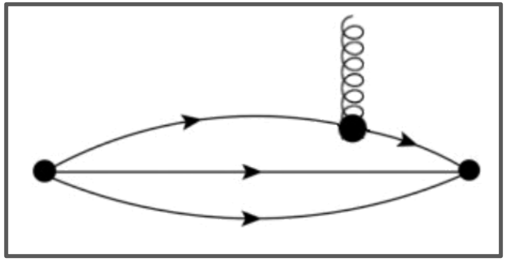

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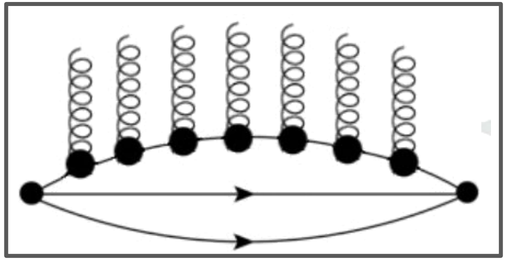


Leads to

$$\mathcal{M}^{\mathrm{eff}}(t) = A + B\, t \exp(-\Delta E\, t)$$

Combined with distillation and GEVP method for operator construction

Bouchard *et al.*, PRD 96 (2017) 014504

Reduced Ioffe-time distribution

Double ratio removes correlated uncertainties and need for renormalization

$$\mathfrak{M}(\nu, z^2) = \left(\frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu, 0)|_{z=0}}\right) \Big/ \left(\frac{\mathcal{M}(0, z^2)|_{p=0}}{\mathcal{M}(0, 0)|_{p=0,z=0}}\right)$$

Connect this ratio to the Ioffe-time distribution through factorisation

$$\begin{aligned}\mathfrak{M}(\nu, z^2) = \frac{\mathcal{I}_g(\nu, \mu^2)}{\mathcal{I}_g(0, \mu^2)} - \frac{\alpha_s N_c}{2\pi} \int_0^1 du\, \frac{\mathcal{I}_g(u\nu, \mu^2)}{\mathcal{I}_g(0, \mu^2)} \Bigg\{ \ln\left(\frac{z^2 \mu^2 e^{2\gamma_E}}{4}\right) B_{gg}(u) + 4\left[\frac{u + \ln(\bar{u})}{\bar{u}}\right]_+ \\ + \frac{2}{3}\left[1 - u^3\right]_+ \Bigg\} - \frac{\alpha_s C_F}{2\pi} \ln\left(\frac{z^2 \mu^2 e^{2\gamma_E}}{4}\right) \int_0^1 dw\, \frac{\mathcal{I}_S(w\nu, \mu^2)}{\mathcal{I}_g(0, \mu^2)}\, \mathfrak{B}_{gq}(w) \,.\end{aligned}$$

with

$$\mathcal{I}_g(\nu, \mu^2) = \frac{1}{2} \int_{-1}^{1} dx\, e^{ix\nu}\, x\, g(x, \mu^2) \qquad B_{gg}(u) = 2\left[\frac{(1 - u\bar{u})^2}{1 - u}\right]_+ \qquad \mathfrak{B}_{gq}(w) = \left[1 + (1 - w)^2\right]_+$$

Extracting the PDF

Determining the PDF from limited, discrete lattice data is an ill-posed inverse problem

Treat this inverse problem by parameterising the PDF

$$\mathfrak{M}(\nu, z^2) = \int_0^1 dx\, \mathcal{K}(x\nu, \mu^2 z^2)\, \frac{x^\alpha (1-x)^\beta}{B(\alpha+1, \beta+1)}$$

Test systematic effects by modifying parameterisation

transformed Jacobi polynomial

$$\mathfrak{M}(\nu, z^2) = \int_0^1 dx\, \mathcal{K}(x\nu, \mu^2 z^2)\, x^\alpha (1-x)^\beta \left(\frac{1}{B(\alpha+1, \beta+1)} + d_1^{(\alpha,\beta)}\, J_1^{(\alpha,\beta)}(x) \right)$$

$$\mathfrak{M}(\nu, z^2) = \int_0^1 dx\, \mathcal{K}(x\nu, \mu^2 z^2)\, \frac{x^\alpha (1-x)^\beta}{B(\alpha+1, \beta+1)} + \left(\frac{a}{|z|}\right) P_1(\nu)$$

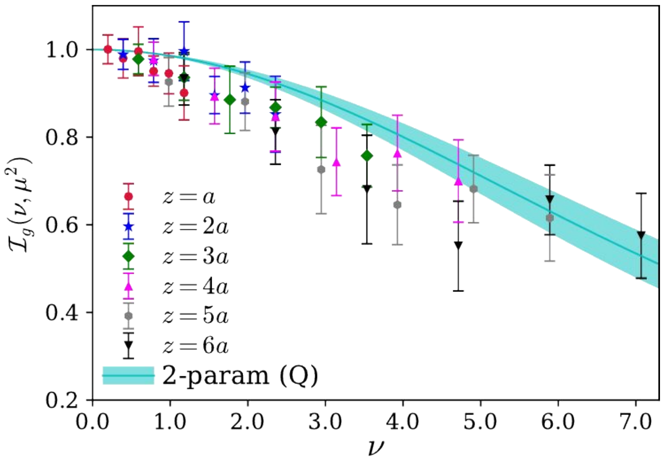
Results

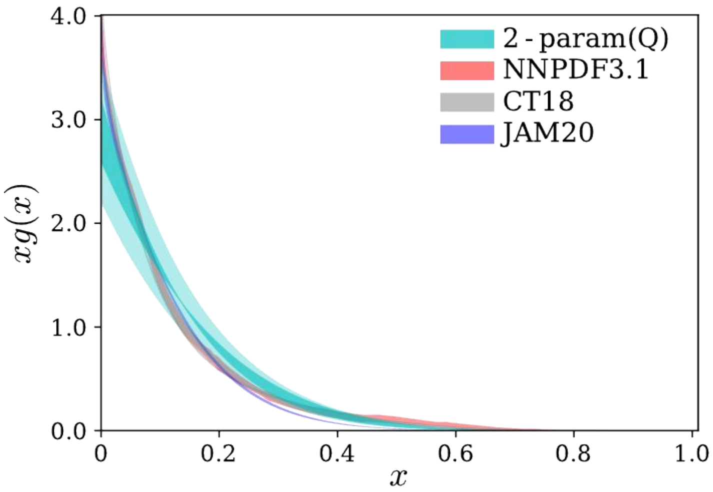
Results calculated on a single lattice ensemble of 2+1 clover fermions

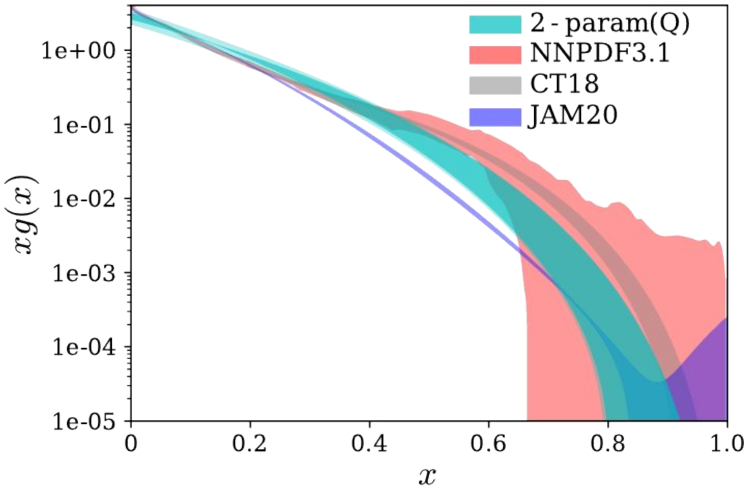
ID	a (fm)	M_π (MeV)	$L^3 \times N_t$	$N_{\rm cfg}$	N_{srcs}
$a094m358$	0.094(1)	358(3)	$32^3 \times 64$	349	64

- Implement gradient flow via the Wilson flow
- Unimproved field strength operator
- Momentum smearing for interpolating operators at nonzero momentum

Results: unpolarised gluons







Unpolarised gluon PDFs: the take-home message

Challenging calculations, requiring a suite of sophisticated approaches, but controlled extractions with moderate precision at moderate Bjorken-x feasible in the near future

Egerer et al., 2207.08733

Gluon PDFs: pseudo-distribution formalism for polarised gluons

Unpolarised case

Balitsky, Morris and Radyushkin, PLB 808 (2020) 135621

$$M^{(0)}_{\mu\nu\rho\sigma;H}(P,z) = \langle H(P)|G_{\mu\nu}(0,z,\mathbf{0}_\mathrm{T})W^{(A)}(z,0)G_{\rho\sigma}(0)|H(P)\rangle$$

becomes

Balitsky, Morris and Radyushkin, JHEP 02 (2022) 193

$$M^{(0)}_{\mu\nu\rho\sigma;H}(P,z) = \langle H(P)|G_{\mu\nu}(0,z,\mathbf{0}_\mathrm{T})W^{(A)}(z,0)\widetilde{G}_{\rho\sigma}(0)|H(P)\rangle$$

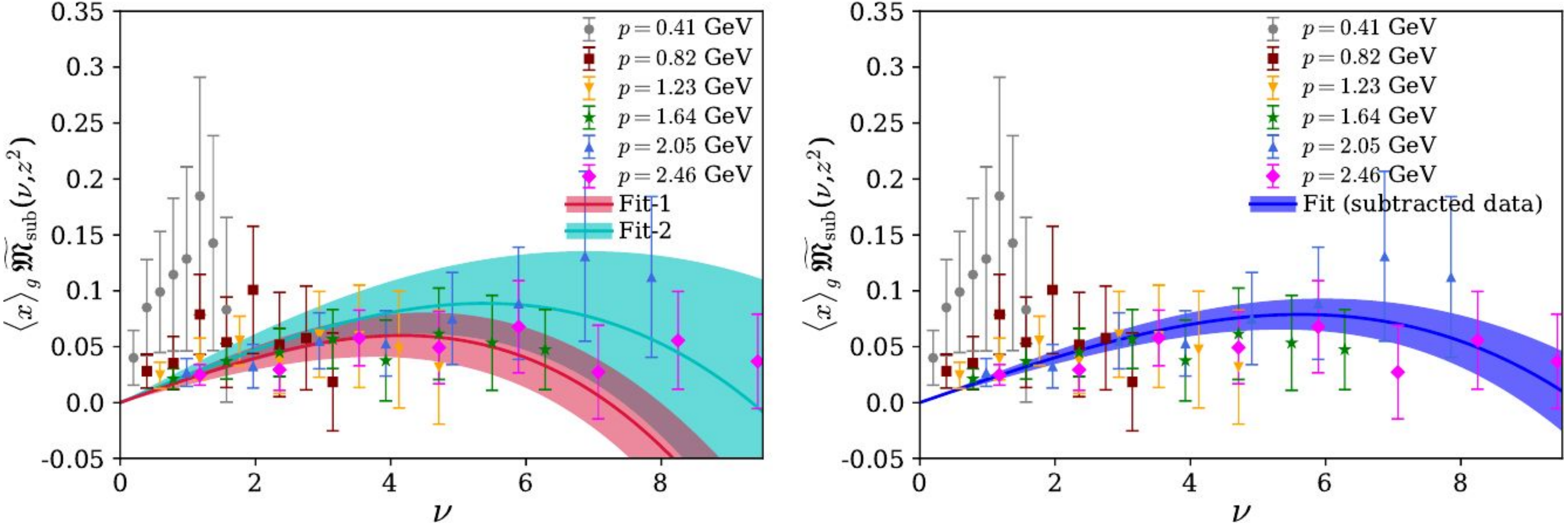
Decompose into invariant amplitudes, but a higher twist term cannot be removed

$$M^{(0)}_{0i;0i}(P,z) + M^{(0)}_{ij;ij}(P,z) = -2P^z E_H \mathcal{M}_{\Delta g/H}(\nu,z^2) + 2E_H^3 z\mathcal{M}_{pp}(\nu,z^2)$$

what we want

Complicates the analysis!

Results: polarised gluons



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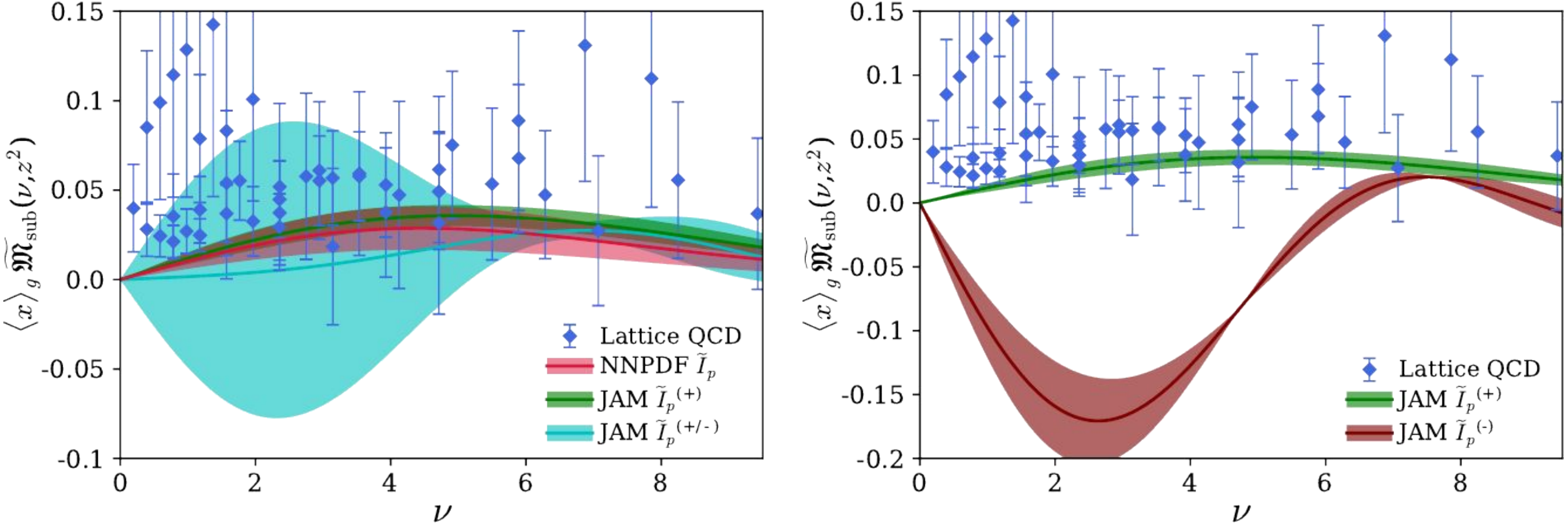

Polarised gluon PDFs: the take-home message

Very challenging calculations, requiring high statistics, but clear opportunity for meaningful contributions from controlled calculations, even with large uncertainties

Summary

Precise extraction of the unpolarised gluon PDF from pseudo-distribution framework

First extraction of the polarised gluon PDF from lattice QCD

evidence for meaningful contributions to our picture, even with large uncertainties

Significant improvement in precision using:

gradient flow smearing; distillation and summed GEVP method; ratio method

Future improvements needed:

1. Increased statistics
2. Calculation of gluon momentum fraction
3. Long-term goal - combine with isoscalar quark PDF

Thank you!

Chris Monahan

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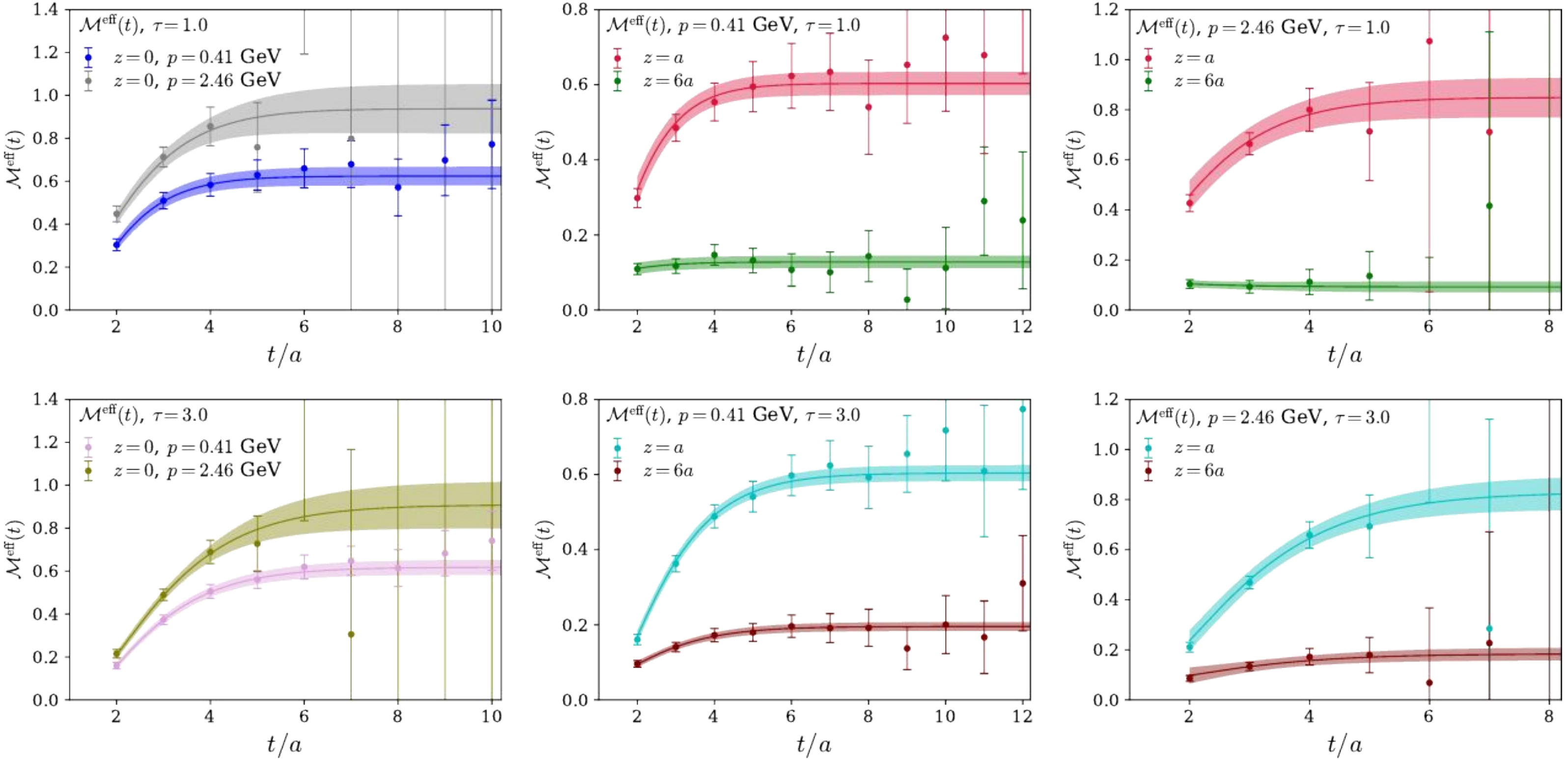
Results: unpolarised gluons



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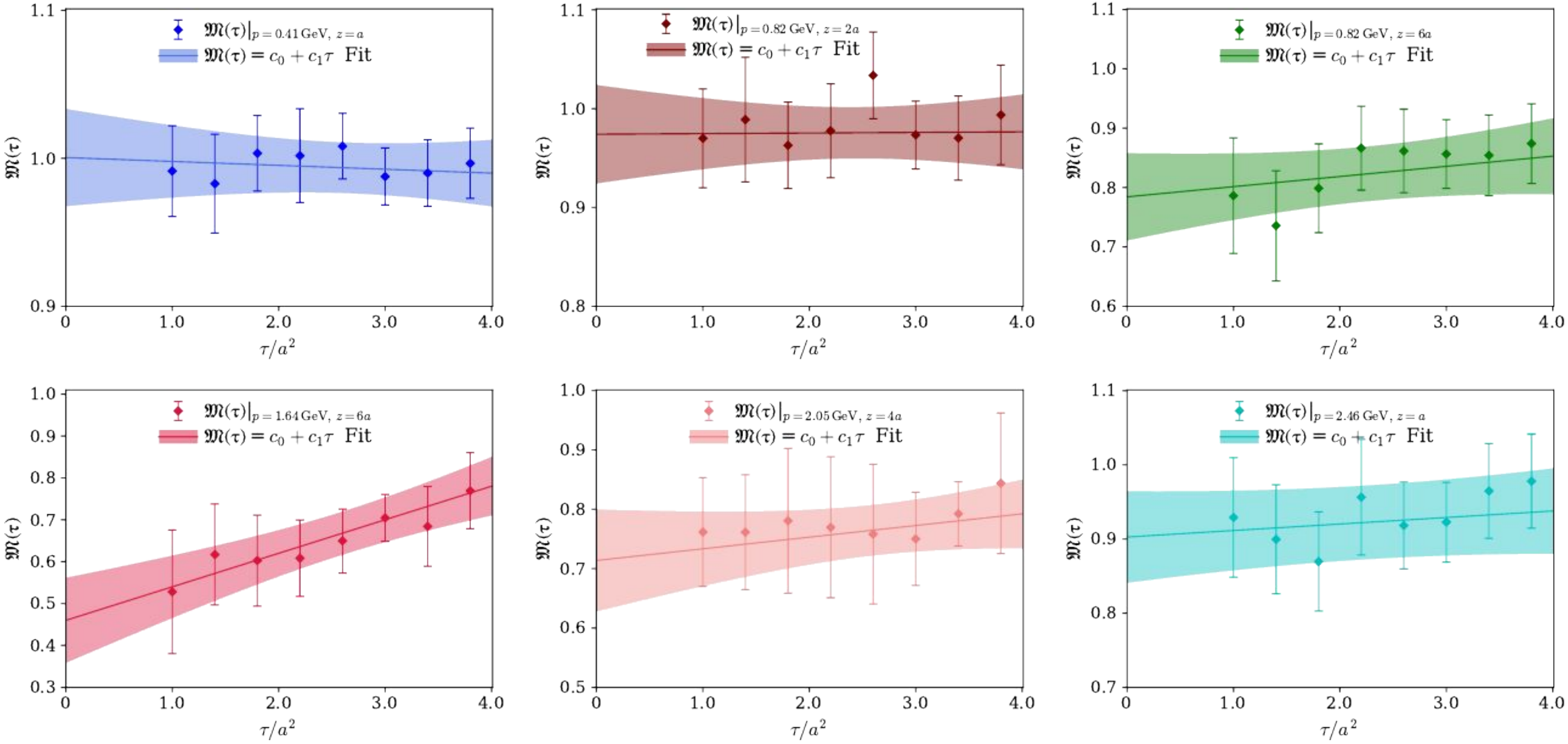
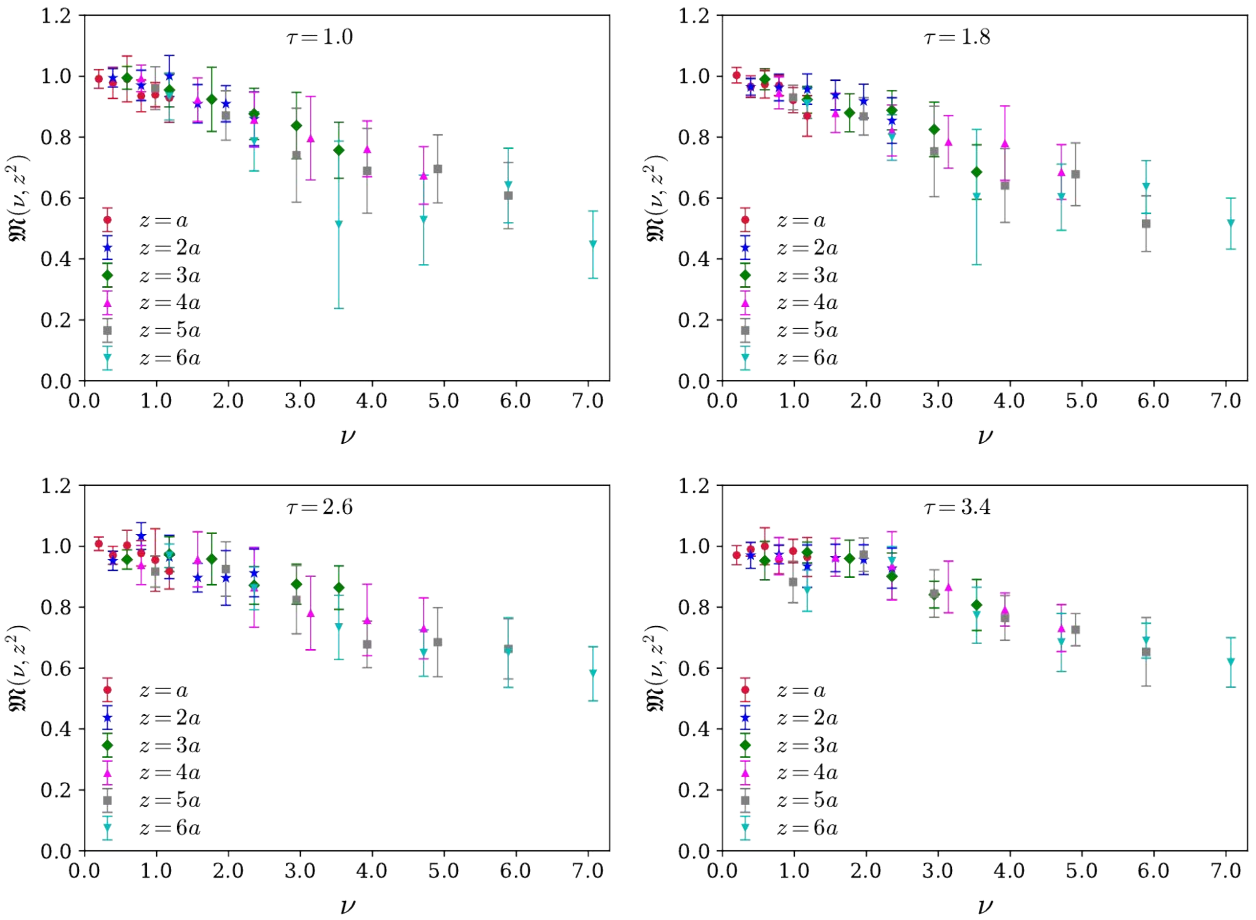
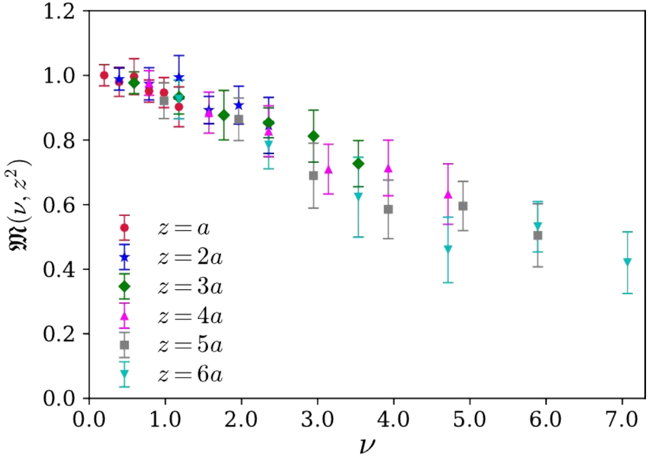


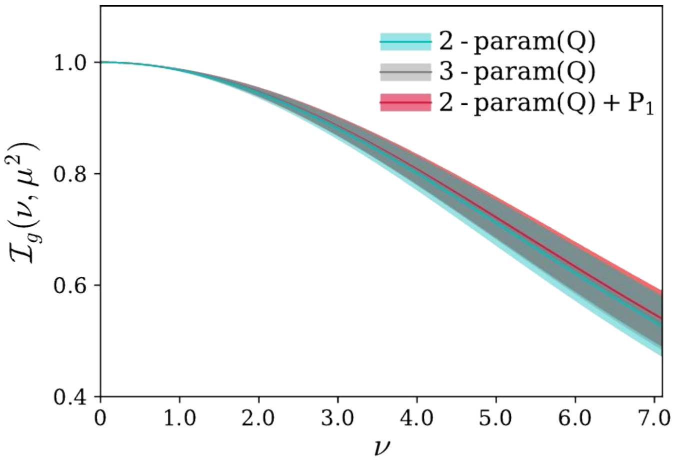
Results

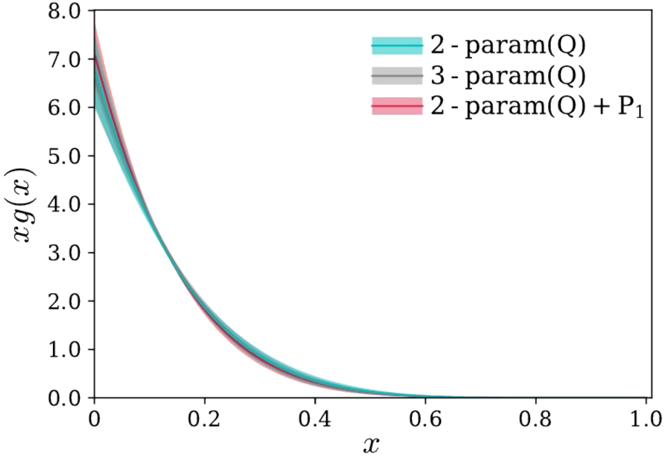


$\tau = 1.0$
$\tau = 1.8$
$\tau = 2.6$
$\tau = 3.4$
$\mathfrak{M}(\nu, z^2)$
ν
$z = a$
$z = 2a$
$z = 3a$
$z = 4a$
$z = 5a$
$z = 6a$

Results: unpolarised gluons







Gradient flow theory in D+1 dimensions

Gradient flows are, broadly speaking, any solution of steepest descent:

- energy landscape
- field configuration space
- probability measure space

Typically expressed through solutions to PDEs.

E.g. in Euclidean space, gradient flows along f:R →R are solutions to

$$\frac{\mathrm{d}x_t}{\mathrm{d}t} = -\nabla f(t)$$

Leads to the gradient descent algorithm for finding local minima

Lüscher & Weisz, JHEP 1102 (2011) 51
Lüscher, JHEP 1304 (2013) 123

Gradient flow theory in D+1 dimensions

Gradient flow for boundary theory can be implemented in a D+1 dimensional Lagrangian

$$S_{GF} = -2\int_0^\infty \mathrm{d}\tau \int \mathrm{d}^D x\, \mathrm{Tr}\left\{L_\mu(\tau,x)\big[\partial_\tau B_\mu(\tau,x) - D_\nu G_{\nu\mu}(\tau,x)\big]\right\}$$

Lagrange multiplier field couples to the boundary gauge field only through the bulk field

Variation of the action with respect to Lagrange multiplier field imposes gradient flow equation in the bulk

Formalism suited to studying formal properties of the flow; not typically practical

- Generalised BRST invariance constrains counterterms in the bulk
- Guarantees renormalised correlation functions remain finite, up to a fermionic wavefunction renormalisation

Lüscher & Weisz, JHEP 1102 (2011) 51
Lüscher, JHEP 1304 (2013) 123

Gradient flow: perturbation theory

Gradient flow in QCD

$$\frac{\partial}{\partial\tau}B_\mu(\tau,x) = D_\nu\Big(\partial_\nu B_\mu - \partial_\mu B_\nu + [B_\nu, B_\mu]\Big) \qquad D_\nu = \partial_\nu + [B_\nu, \cdot]$$

Dirichlet boundary conditions

$$B_\mu(\tau=0,x) = A_\mu(x) \qquad\qquad \chi(\tau=0,x) = \psi(x)$$

Tree-level expansion and "flow propagator"

$$B_\mu(\tau,x) = \int \mathrm{d}^4y \Big\{ K_\tau(x-y)_{\mu\nu}A_\nu(y) + \int_0^\tau \mathrm{d}\sigma K_{\tau-\sigma}(x-y)_{\mu\nu}R_\nu(\sigma,y)\Big\}$$

where

$$K_\tau(x)_{\mu\nu} = \int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{e^{ipx}}{p^2}\Big\{(\delta_{\mu\nu}p^2 - p_\mu p_\nu)e^{-\tau p^2} + p_\mu p_\nu\Big\}$$

$$R_\mu(\tau,x) = 2[B_\nu, \partial_\nu B_\mu] - [B_\nu, \partial_\mu B_\nu] - [B_\mu, \partial_\nu B_\nu] + [B_\nu, [B_\nu, B_\mu]]$$

Renormalised boundary theory requires no further renormalisation in the bulk

Lüscher & Weisz, JHEP 1102 (2011) 51
Makino & Suzuki, arXiv:1410.7538