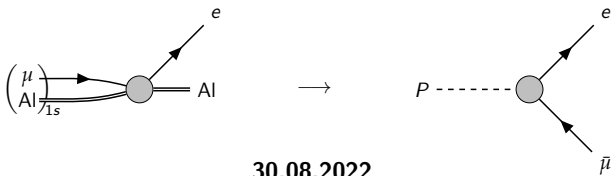


# Improved limits on LFV decays of light pseudoscalars via spin-dependent $\mu \rightarrow e$ conversion in nuclei

Frederic Noël

Universität Bern  
Institute for Theoretical Physics



30.08.2022

14th Conference on the Intersections of Particle and Nuclear Physics  
(CIPANP 2022)

based on

[Hoferichter, Menéndez, Noël, 2022, arXiv:2204.06005]

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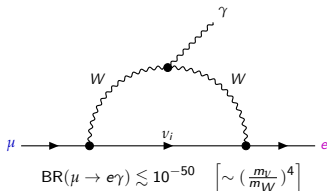
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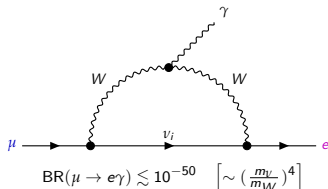


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- Observation of CLFV would be NP

Lepton Flavours

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Very **clean BSM signal** (no competing SM)

# LFV Experiments and current limits

LFV process	current limit	(planned) experiments
$\mu \rightarrow e\gamma$	$< 4.2 \cdot 10^{-13}$ [MEG]	MEG II
$\mu \rightarrow 3e$	$< 1.0 \cdot 10^{-12}$ [SINDRUM]	Mu3e
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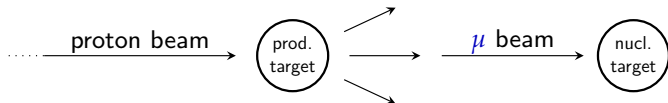
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Goal: Relate  $\mu \rightarrow e$  conversion and  $P \rightarrow \mu e$

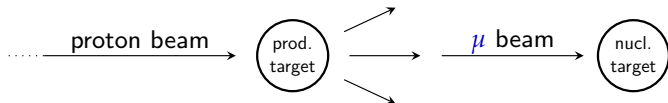
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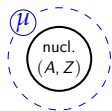


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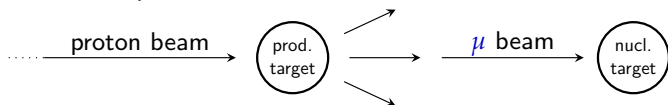


- Conversion process:

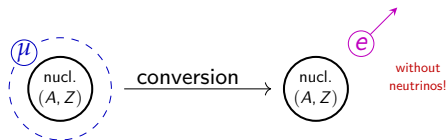


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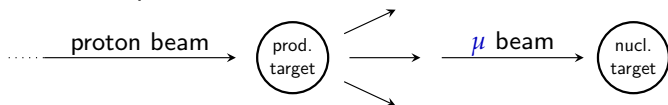


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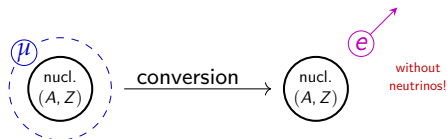


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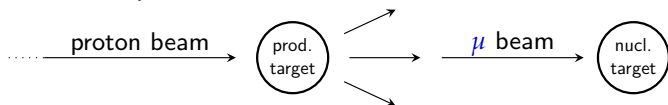
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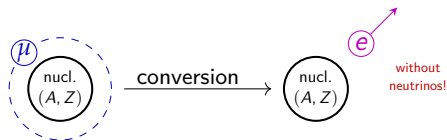
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- Distinction between:
  - Spin-independent** (SI)  $\rightarrow$  coherent enhancement ( $\sim \#N$ )
  - Spin-dependent** (SD)  $\rightarrow$  non-coherent (only  $J_{\text{nucl.}} \neq 0$ )



# Effective Theory Framework

Effective description by separation of the appearing scales




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- EFT operators given within effective Lagrangian:


$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2} \sum_{\substack{Y=L,R \\ q=u,d,s}} \left[ C_Y^{P,q} (\bar{e}\gamma\mu)(\bar{q}\gamma_5 q) + C_Y^{A,q} (\bar{e}\gamma\gamma^{\mu}\mu)(\bar{q}\gamma_{\mu}\gamma_5 q) \right] + \frac{i\alpha_s}{\Lambda^3} \sum_{Y=L,R} C_Y^{G\tilde{G}} (\bar{e}\gamma\mu) G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} + \text{h.c.}$$



$P$



$A$



$G\tilde{G}$

(only relevant part for SD  $\mu \rightarrow e$  conversion shown here and omitting tensor contributions)

pseudoscalar  $P$ , axialvector  $A$  and gluonic  $G\tilde{G}$  operators


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


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


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See also: [\[Talk \(Saturday\): E. Mereghetti\]](#)

# Master Formula: $P \rightarrow \mu e$

Conversion  
Rate

=



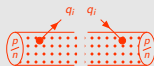
bound state physics

⊗



nuclear response

⊗



hadronic matrix elements

⊗



(short distance) EFT operator

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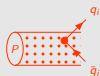
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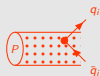
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- hadronic matrix elements from lattice-QCD and phenomenology
- Ward identity:

$$b_q f_P^q M_P^2 = b_q h_P^q - a_P$$

	$\pi$	$\eta$		$\eta'$	
		Pheno	Lattice	Pheno	Lattice
$\frac{b_u f_P^u}{F_\pi}$	1	0.80	0.77	0.66	0.56
$\frac{b_d f_P^d}{F_\pi}$	-1	0.80	0.77	0.66	0.56
$\frac{b_s f_P^s}{F_\pi}$	0	-1.26	-1.17	1.45	1.50
$a_P$ [GeV <sup>3</sup> ]	0	-	-0.017	-	-0.038
$a_P^{\text{FKS}}$ [GeV <sup>3</sup> ]	0	-0.022	-0.021	-0.056	-0.048
$h_P^q$		Ward identity			

Phenomenology: [Escribano et al., 2016]

Lattice-QCD: [Bali et al., 2021]

# Master Formula: SD $\mu \rightarrow e$ conversion

Conversion  
Rate =



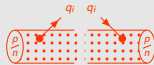
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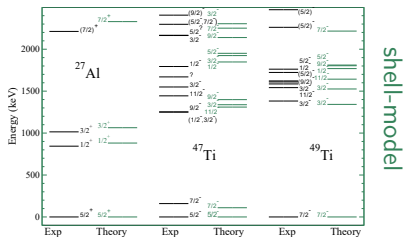
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$$\text{Br}_{\mu \rightarrow e}^{\text{SD}} = \frac{4m_{\mu}^5 \alpha^3 Z^3}{\pi \Gamma_{\text{cap}} (2J+1)} \left( \frac{Z_{\text{eff}}}{Z} \right)^4 \times \sum_{\substack{Y=L,R \\ \tau=L,T}} \left[ C_Y^{\tau,00} S_{00}^{\tau} + C_Y^{\tau,11} S_{11}^{\tau} + C_Y^{\tau,01} S_{01}^{\tau} \right]$$

- numerical solution of Dirac equation:

$$Z_{\text{eff}}^{\text{Al}} = 11.64, \quad Z_{\text{eff}}^{\text{Ti}} = 17.65 \quad [\text{Kitano et al., 2002}]$$



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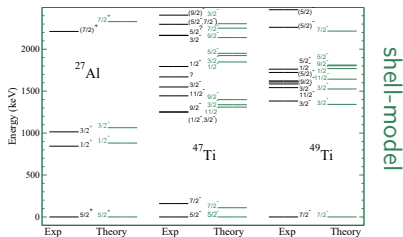
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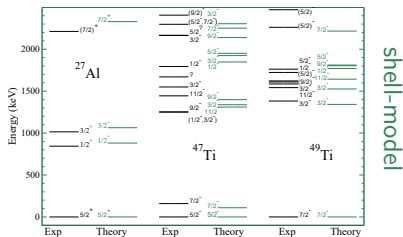
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$g_A^{u,p}$	$g_A^{d,p}$	$g_A^{s,N}$	$\bar{a}_N$ [GeV]	$g_5^{q,N}$
0.842(12)	-0.427(13)	-0.085(18)	-0.39(12) [ $N_C \rightarrow \infty$ ]	Ward identity
[HERMES, 2007]				



# Outlook - Master Formula: general $\mu \rightarrow e$ conversion

- other works: SI [Kitano et al., 2002], SD [Cirigliano et al., 2017; Davidson et al., 2018], scalar at NLO [Cirigliano et al., 2022], nucleon-level EFT [Haxton et al., 2022/Talk: E. Rule]

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- effective Lagrangian with all possible **quark and gluon operators**:

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$$\mathcal{M} \sim \sum_{\Gamma, q} C_q^\Gamma$$



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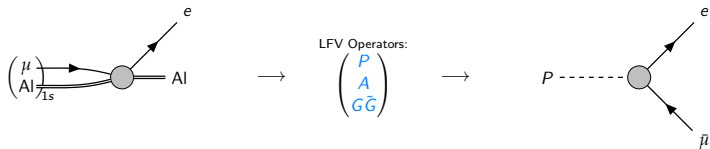
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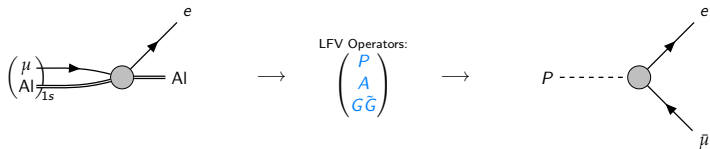
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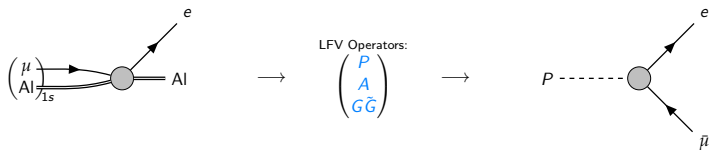
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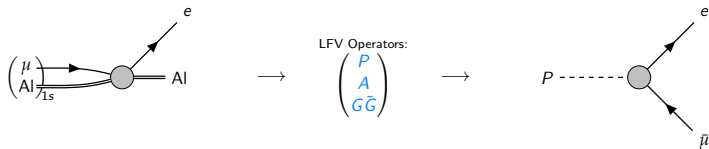
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$\mu \rightarrow e$ (exp.)	$P \rightarrow \bar{\mu}e$ (derived)	current limit
$BR_{Ti} < 6.1 \times 10^{-13}$	$BR_{\pi^0} \lesssim 4 \times 10^{-17}$	$< 3.6 \times 10^{-10}$
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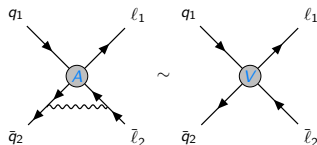
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- easily spoilt by **RG corrections**
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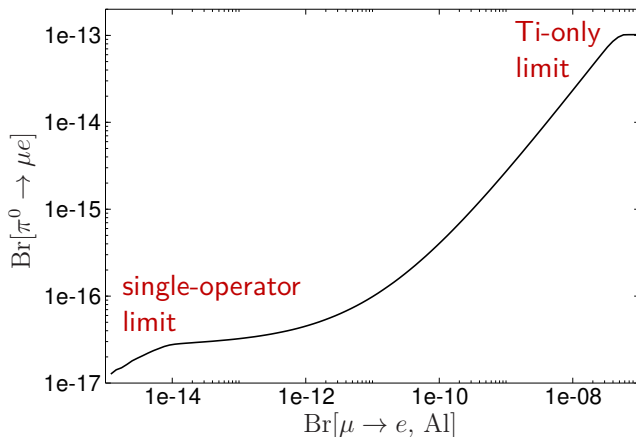
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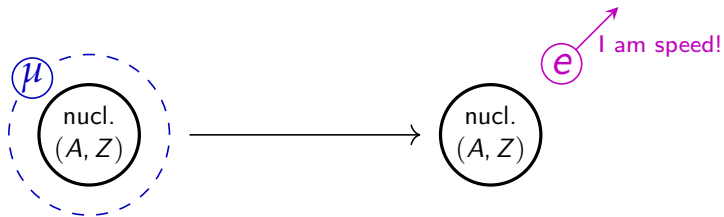
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- Future results from Mu2e and COMET can **further improve these limits**
- **General treatment of  $\mu \rightarrow e$  conversion:**  
beyond SI or SD, combining nuclear **and** bound state physics

Thank you for your attention!



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# Backup-Slides

# Standard Model EFT

- **Model-independent** effective field theory description of BSM physics with **higher dimensional operators** obeying all SM symmetries:

$$\mathcal{L}^{\text{SM EFT}} = \mathcal{L}^{\text{SM}} + \frac{1}{\Lambda} \mathcal{L}^{(5)} + \frac{1}{\Lambda^2} \mathcal{L}^{(6)} + \dots$$

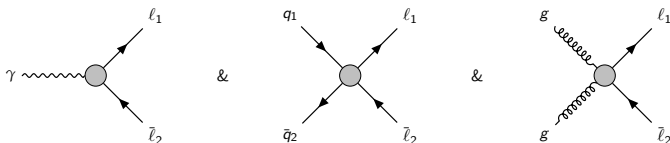
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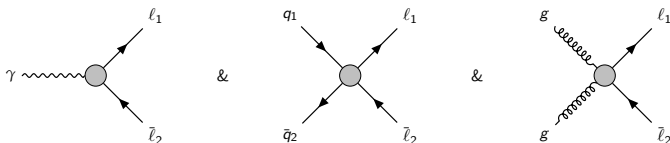


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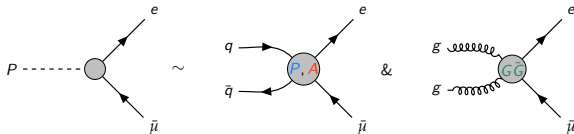
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Can be used to describe **all LFV processes** in a **model-independent** way

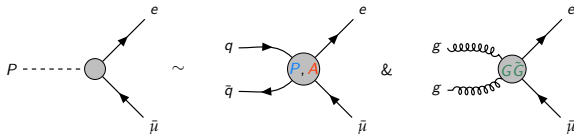
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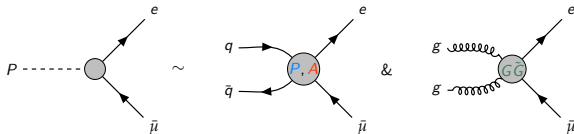
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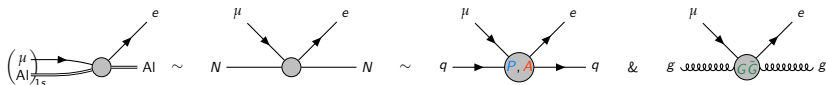
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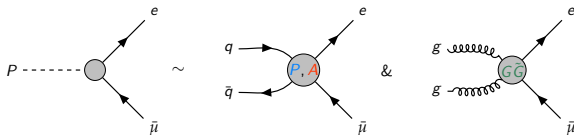
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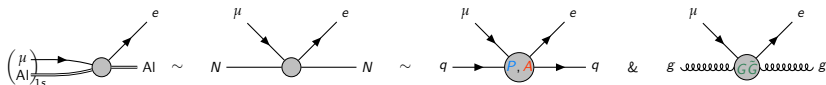
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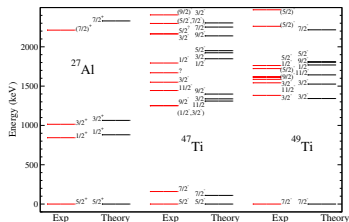
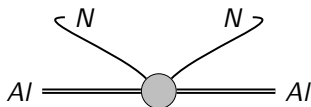
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- ... and **nuclear responses** ...



- ... in terms of **multipoles**, calculated in the **shell-model**

# Formulas I

$$\langle 0 | \bar{q} \gamma^\mu \gamma_5 q | P(k) \rangle = i b_q f_P^q k^\mu, \quad (1)$$

$$\langle 0 | m_q \bar{q} i \gamma_5 q | P(k) \rangle = \frac{b_q h_P^q}{2}, \quad (2)$$

$$\langle 0 | \frac{\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} | P(k) \rangle = a_P, \quad (3)$$

$$\langle N | \bar{q} \gamma^\mu \gamma_5 q | N \rangle = g_A^{q,N} \langle N | \bar{N} \gamma^\mu \gamma_5 N | N \rangle, \quad (4)$$

$$m_q \langle N | \bar{q} i \gamma_5 q | N \rangle = m_N g_5^{q,N} \langle N | \bar{N} i \gamma_5 N | N \rangle, \quad (5)$$

$$\langle N | \bar{q} \sigma^{\mu\nu} q | N \rangle = f_{1,T}^{q,N} \langle N | \bar{N} \sigma^{\mu\nu} N | N \rangle, \quad (6)$$

$$\langle N | \frac{\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} | N \rangle = \tilde{a}_N \langle N | \bar{N} i \gamma_5 N | N \rangle, \quad (7)$$

# Formulas II

$$\text{Br}_{\text{SI}}[\mu \rightarrow e] = \frac{4m_\mu^5}{\Gamma_{\text{cap}}} \sum_{Y=L,R} \left| \sum_{\substack{N=p,n \\ O=S,V}} \bar{c}_Y^{O,N} \mathcal{O}^{(N)} \right|^2, \quad (8)$$

$$\bar{c}_Y^{S,N} = \frac{1}{\Lambda^2} \sum_q C_Y^{S,q} \frac{m_N}{m_q} f_q^N + \frac{4\pi}{\Lambda^3} C_Y^{\text{GG}} a_N, \quad (9)$$

$$\bar{c}_Y^{V,N} = \frac{1}{\Lambda^2} \sum_q C_Y^{V,q} f_{V_q}^N, \quad (10)$$

$$S^{(N)} = V^{(N)} = \frac{(\alpha Z)^{3/2}}{4\pi} \left( \frac{Z_{\text{eff}}}{Z} \right)^2 \mathcal{F}_N^M(m_\mu^2), \quad (11)$$

## Formulas III

$$\bar{c}^0 = \frac{\bar{c}^p + \bar{c}^n}{2}, \quad \bar{c}^1 = \frac{\bar{c}^p - \bar{c}^n}{2}, \quad (12)$$

$$g_A^{q,N} = g_5^{q,N} - \frac{\tilde{a}_N}{2m_N}, \quad (13)$$

$$\tilde{a}_N = -2m_N g_A^{u,0} = -0.39(12) \text{ GeV}, \quad (14)$$

# Formulas IV

$$C_Y^{A,u} = C_Y^{A,d}, \quad C_Y^{A,s} = -\frac{2C_Y^{A,u}g_A^{u,0}}{g_A^{s,N}}, \quad (15)$$

$$\frac{C_Y^{P,u}}{m_u} = \frac{C_Y^{P,d}}{m_d}, \quad \frac{C_Y^{P,s}}{m_s} = \frac{4\pi}{\Lambda} C_Y^{G\tilde{G}} \frac{2g_A^{u,0}}{g_A^{u,0} - g_A^{s,N}}. \quad (16)$$

# Formulas V

$$S_{00}^{\mathcal{T}} = \sum_L \left[ \mathcal{F}_+^{\Sigma'_L}(q^2) \right]^2, \quad S_{00}^{\mathcal{L}} = \sum_L \left[ \mathcal{F}_+^{\Sigma''_L}(q^2) \right]^2, \quad (17)$$

$$S_{11}^{\mathcal{T}} = \sum_L \left[ \mathcal{F}_-^{\Sigma'_L}(q^2) \right]^2, \quad S_{11}^{\mathcal{L}} = \sum_L \left[ \mathcal{F}_-^{\Sigma''_L}(q^2) \right]^2, \quad (18)$$

$$S_{01}^{\mathcal{T}} = \sum_L 2\mathcal{F}_+^{\Sigma'_L}(q^2) \mathcal{F}_-^{\Sigma'_L}(q^2), \quad (19)$$

$$S_{01}^{\mathcal{L}} = \sum_L 2\mathcal{F}_+^{\Sigma''_L}(q^2) \mathcal{F}_-^{\Sigma''_L}(q^2), \quad (20)$$



## Table

	$\pi^0$	$\eta$	$\eta'$
$C_Y^{A,3}$	$1.3 \times 10^{-17}$	–	–
$C_Y^{A,8}$	–	$1.5 \times 10^{-17}$	$4.0 \times 10^{-20}$
$C_Y^{A,0}$	–	$2.9 \times 10^{-19}$	$2.1 \times 10^{-19}$
$C_Y^{P,3}$	$4.1 \times 10^{-17}$	–	–
$C_Y^{P,8}$	–	$1.6 \times 10^{-12}$	$2.1 \times 10^{-14}$
$C_Y^{P,0}$	–	$4.1 \times 10^{-12}$	$5.4 \times 10^{-13}$
$C_Y^{G\check{G}}$	–	$5.8 \times 10^{-15}$	$4.7 \times 10^{-16}$