ER, Haxton, and McElvain, arXiv:2109.13503
Haxton, **ER**, McElvain, and Ramsey-Musolf, arXiv:2208.07945

Nuclear Effective Theory of $\mu \to e$ Conversion

Evan Rule | 14th CIPANP | August 30, 2022

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N3AS Network for Neutrinos, Nuclear Astrophysics, and Symmetries PHYSICS FRONTIER CENTER

NSF

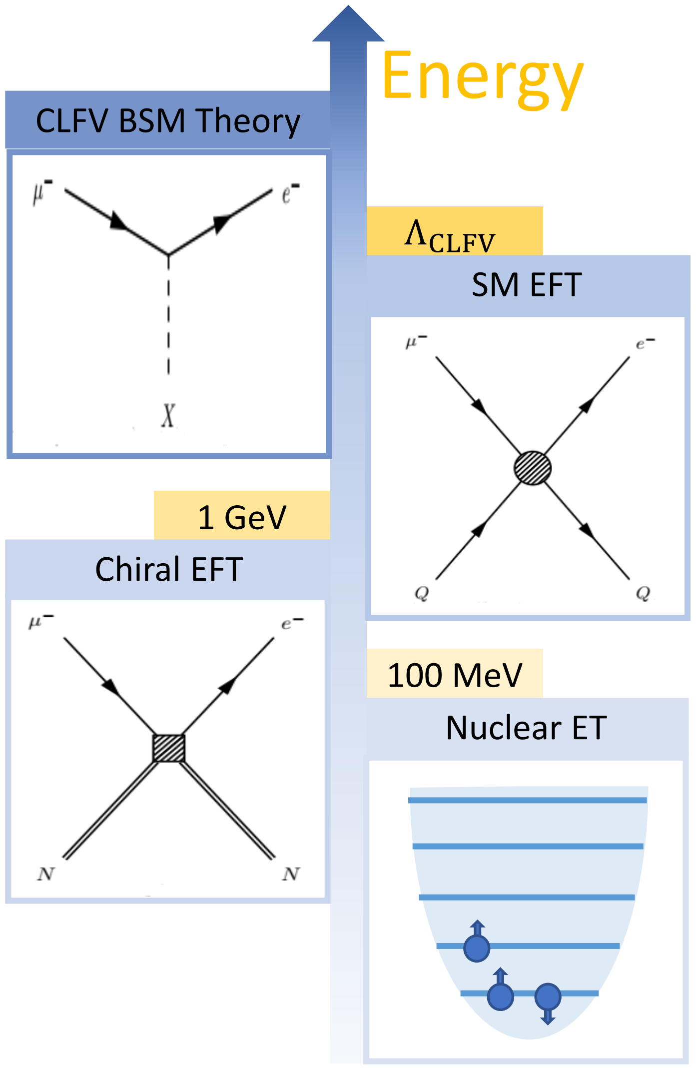
Background

$$B\left(\mu^- + (A,Z) \to e^- + (A,Z)\right) \equiv \frac{\Gamma\left(\mu^- + (A,Z) \to e^- + (A,Z)\right)}{\Gamma\left(\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right)}$$

- Charged lepton flavor violation can test BSM physics at scales beyond the reach of direct searches
- Next-generation experiments Mu2e and COMET can improve current limits by four orders of magnitude†, probing scales $\lesssim 10^4$ TeV
- Experiments take place on atomic nucleus $^{27}\mathrm{Al}$
- Low-energy, highly-exclusive process

How can we extract the most information about underlying CLFV operators from observations of elastic $\mu \to e$ conversion in nuclei?

†: Mu2e Collaboration, R. J. Abrams et al., arXiv:1211.7019
COMET Collaboration, Y. G. Cui et al



Energy
CLFV BSM Theory
μ^-
e^-
X
Λ_{CLFV}
SM EFT
μ^-
e^-
Q
Q
1 GeV
Chiral EFT
μ^-
e^-
N
N
100 MeV
Nuclear ET

Coherent Response

- If CLFV couples to nuclear charge, rate enhanced by $\approx A^2$
- Only nuclear operator is charge monopole M_0
- Numerical Dirac wave functions for $1s$ muon and $\kappa = \pm 1$ electron

Weinberg & Feinberg, *Phys. Rev. Lett.* **3**, 111 and 245 (E) (1959)

Marciano & Sanda, *Phys. Rev. Lett.* **38**, 1512 (1977)

Shanker, *Phys. Rev. D* **20**, 1608 (1979)

Kitano, Koike, & Okada, *Phys. Rev. D* **66**, 096002 (2002)

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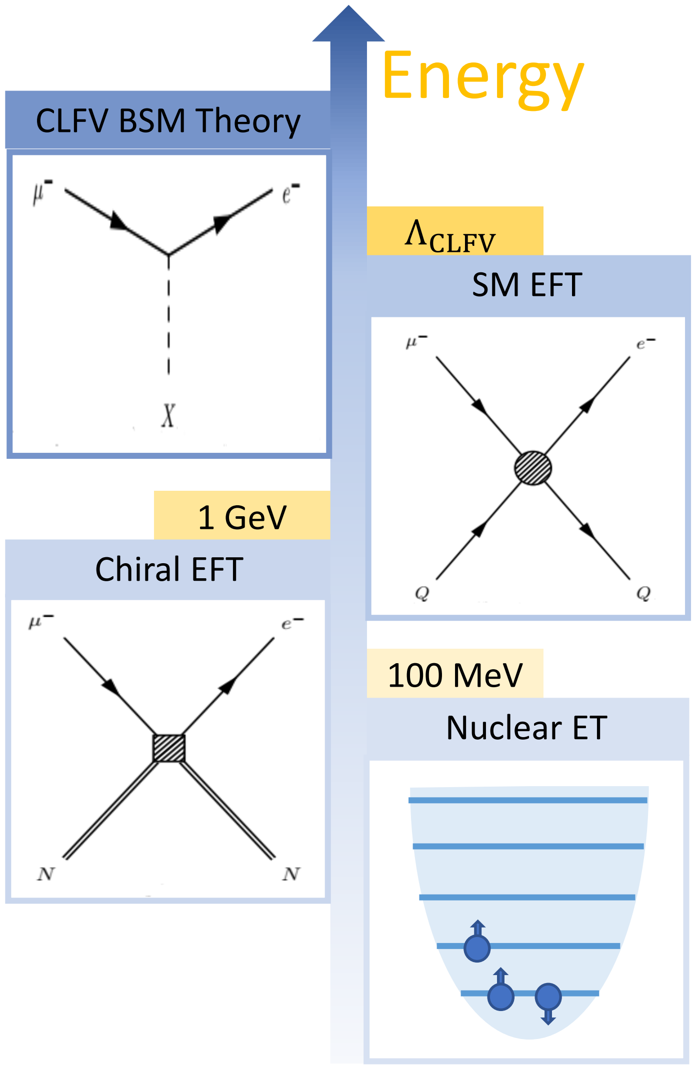

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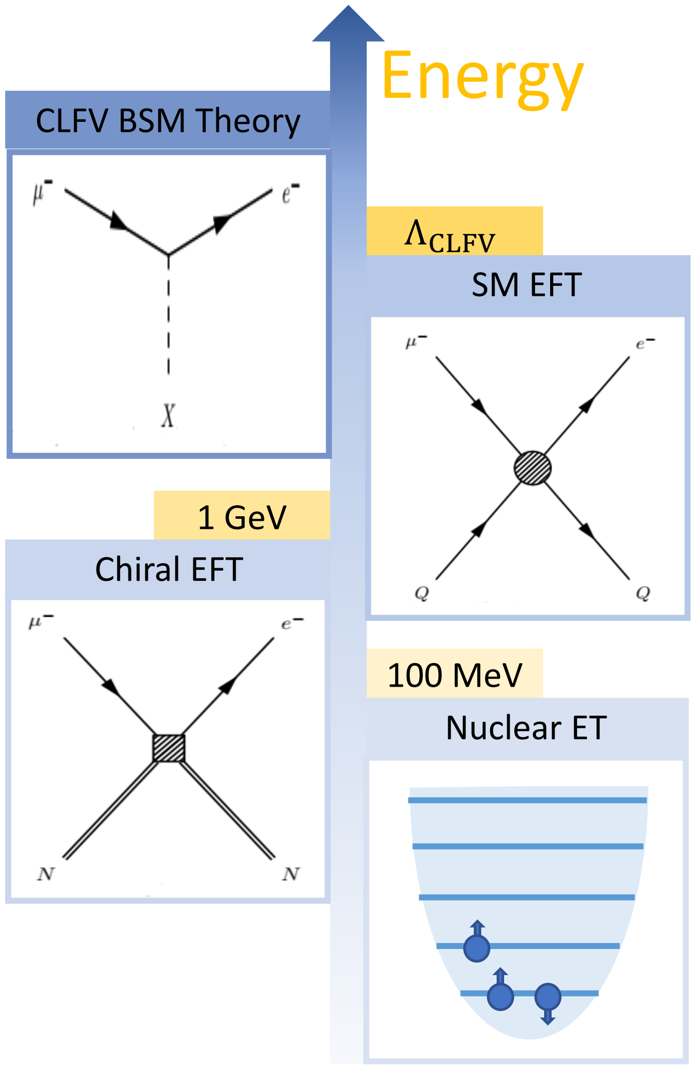
- NOT GENERAL
- Tedious to extend to higher partial waves, spin- and velocity-dependent operators



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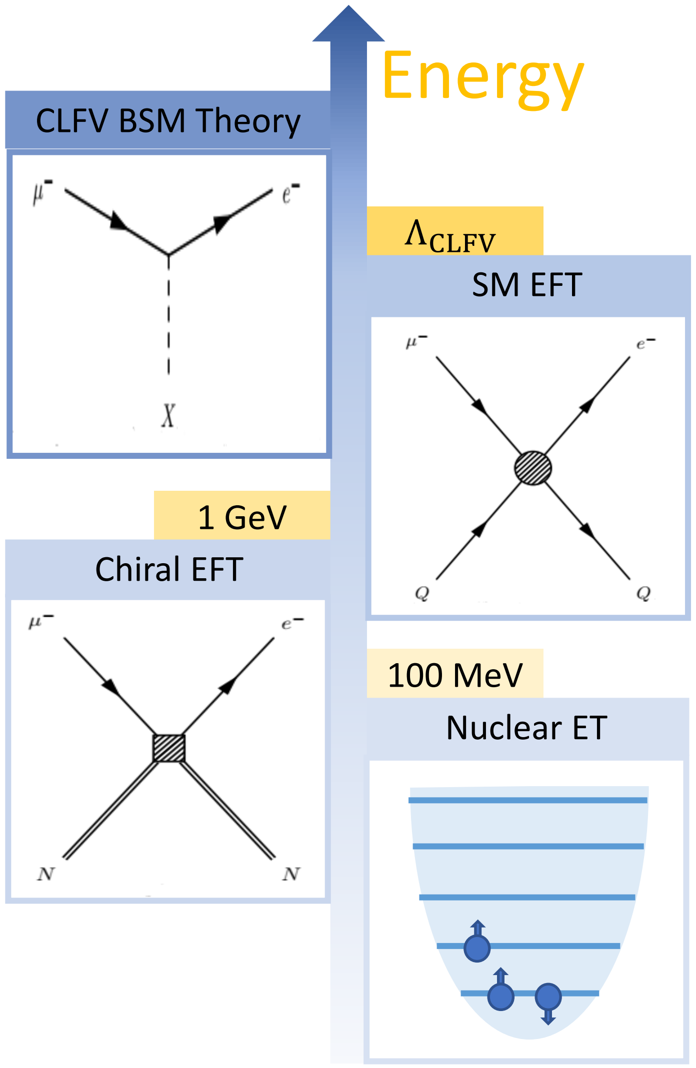


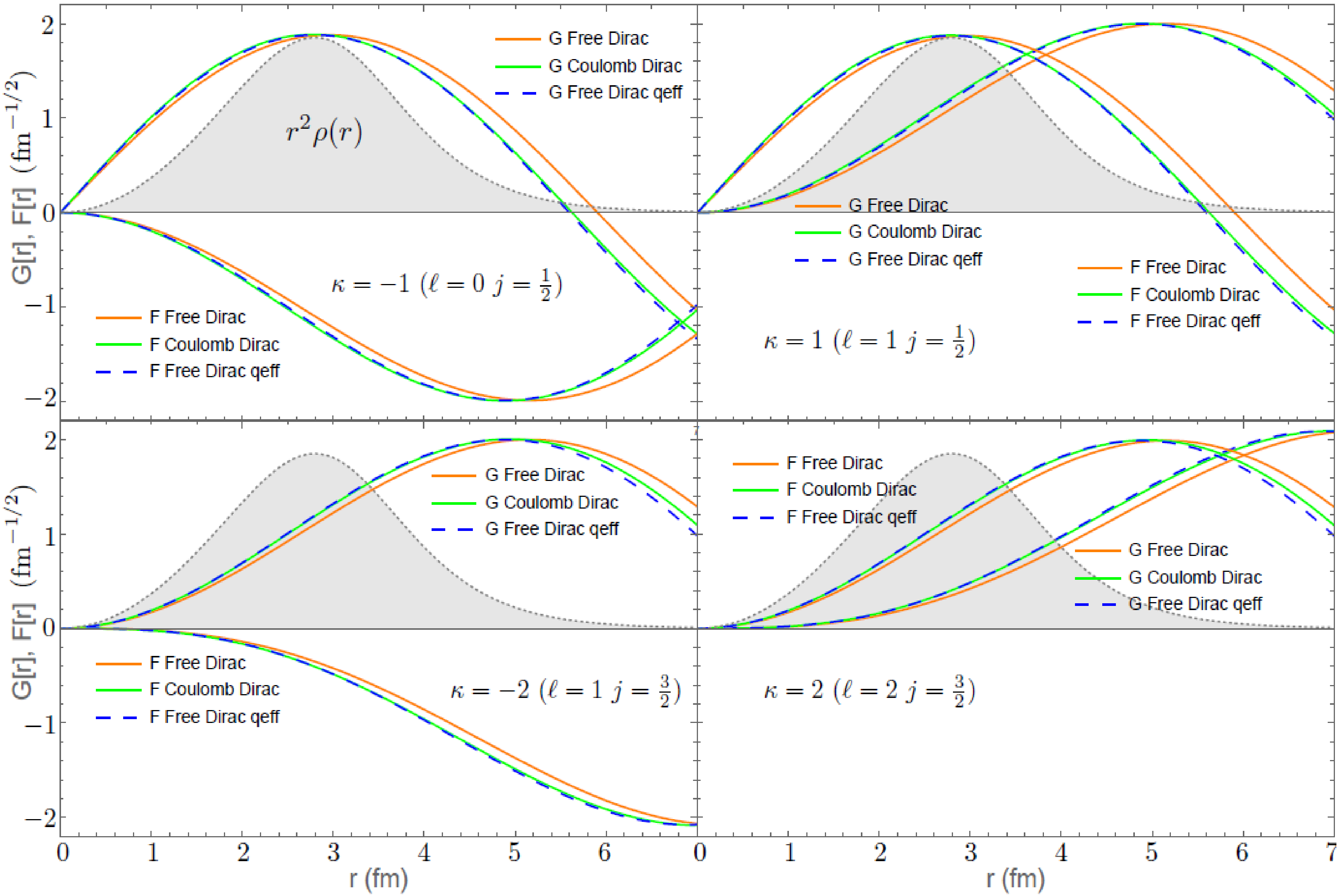
Nuclear Effective Theory

- Most general response consistent with nuclear angular momentum, isospin, parity, and time-reversal
- Factors the CLFV leptonic physics from the nuclear physics
- CLFV LECs probed directly by experiment
- Simple and accurate electron wave function
- Analogous to nuclear effective theory of WIMP dark matter†

†: Fitzpatrick, Haxton, Katz, Lubbers, and Xu, *JCAP* **02**, 004 (2013)

Electron Wave Functions

- Outgoing electron has $E_e \approx m_\mu$ and is ultra-relativistic
- Electron wave function resembles a plane wave but is distorted by the Coulomb field of the nucleus
- We borrow from high-energy electron scattering studies and replace $e^{i\vec{q}\cdot\vec{x}} \rightarrow \frac{q_{\text{eff}}}{q} e^{i\vec{q}_{\text{eff}}\cdot\vec{x}}$
- $\vec{q}_{\text{eff}} = \vec{q} - \bar{V}_C \hat{q}$
- All Dirac spinor currents of electron and muon wave functions can be reduced in terms of Pauli spinors



Effective Theory Variations

Coherent Response
$1_N, 1_L, \hat{q}, \vec{\sigma}_L$

2 Nucleon-level Operators

1 Response function

M_0

- Only lowest multipole M_0 included

$\mathcal{O}_1 = 1_L 1_N$

$\mathcal{O}_{11} = i\hat{q} \cdot \vec{\sigma}_L \, 1_N$

Effective Theory Variations

Coherent Response
$1_N, 1_L, \hat{q}, \vec{\sigma}_L$

2 Nucleon-level Operators

1 Response function

M_0

Allowed Response
$1_N, 1_L, \hat{q}, \vec{\sigma}_L, \vec{\sigma}_N$

6 Nucleon-level operators

3 Response functions

$M_J, \Sigma'_J, \Sigma''_J$

- Most general CLFV response of point-like nucleus

$\mathcal{O}_1 = 1_L 1_N$

$\mathcal{O}_4 = \vec{\sigma}_L \cdot \vec{\sigma}_N$

$\mathcal{O}_6 = i\hat{q} \cdot \vec{\sigma}_L \, i\hat{q} \cdot \vec{\sigma}_N$

$\mathcal{O}_9 = \vec{\sigma}_L \cdot (i\hat{q} \times \vec{\sigma}_N)$

$\mathcal{O}_{10} = 1_L \, i\hat{q} \cdot \vec{\sigma}_N$

$\mathcal{O}_{11} = i\hat{q} \cdot \vec{\sigma}_L \, 1_N$

Effective Theory Variations

Coherent Response	Allowed Response	General Response
$1_N, 1_L, \hat{q}, \vec{\sigma}_L$	$1_N, 1_L, \hat{q}, \vec{\sigma}_L, \vec{\sigma}_N$	$1_N, 1_L, \hat{q}, \vec{\sigma}_L, \vec{\sigma}_N, \vec{v}_N$
2 Nucleon-level Operators	6 Nucleon-level operators	16 Nucleon-level operators
1 Response function	3 Response functions	6 Response functions
M_0	$M_J, \Sigma'_J, \Sigma''_J$	$\boxed{M_J, \Sigma'_J, \Sigma''_J, \Delta_J, \tilde{\Phi}'_J, \Phi''_J}$

- All nuclear responses allowed by P and T symmetries

$$\mathcal{O}_1 = 1_L 1_N$$
$$\mathcal{O}'_2 = 1_L\, i\hat{q} \cdot \vec{v}_N$$
$$\mathcal{O}_3 = 1_L\, i\hat{q} \cdot (\vec{v}_N \times \vec{\sigma}_N)$$
$$\mathcal{O}_4 = \vec{\sigma}_L \cdot \vec{\sigma}_N$$
$$\mathcal{O}_5 = \vec{\sigma}_L \cdot (i\hat{q} \times \vec{v}_N)$$
$$\mathcal{O}_6 = i\hat{q} \cdot \vec{\sigma}_L\, i\hat{q} \cdot \vec{\sigma}_N$$
$$\mathcal{O}_7 = 1_L\, \vec{v}_N \cdot \vec{\sigma}_N$$
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$$\mathcal{O}_{11} = i\hat{q} \cdot \vec{\sigma}_L\, 1_N$$
$$\mathcal{O}_{12} = \vec{\sigma}_L \cdot (\vec{v}_N \times \vec{\sigma}_N)$$
$$\mathcal{O}'_{13} = \vec{\sigma}_L \cdot [i\hat{q} \times (\vec{v}_N \times \vec{\sigma}_N)]$$
$$\mathcal{O}_{14} = i\hat{q} \cdot \vec{\sigma}_L\, \vec{v}_N \cdot \vec{\sigma}_N$$
$$\mathcal{O}_{15} = i\hat{q} \cdot \vec{\sigma}_L\, i\hat{q} \cdot (\vec{v}_N \times \vec{\sigma}_N)$$
$$\mathcal{O}'_{16} = i\hat{q} \cdot \vec{\sigma}_L\, i\hat{q} \cdot \vec{v}_N$$

Effective Theory Variations

Coherent Response	Allowed Response	General Response
$1_N, 1_L, \hat{q}, \vec{\sigma}_L$	$1_N, 1_L, \hat{q}, \vec{\sigma}_L, \vec{\sigma}_N$	$1_N, 1_L, \hat{q}, \vec{\sigma}_L, \vec{\sigma}_N, \vec{v}_N$
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- All nuclear responses allowed by P and T symmetries

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$$\mathcal{O}'_{16} = i\hat{q} \cdot \vec{\sigma}_L\, i\hat{q} \cdot \vec{v}_N$$

$$\mathcal{L}_{\text{eff}} \sim \sqrt{2} G_F \sum_{\tau=0,1} \sum_{i=1}^{16} \tilde{c}_i^\tau\, \mathcal{O}_i t^\tau$$

$$\tilde{R}_M^{\tau\tau'} = \tilde{c}_1^\tau \tilde{c}_1^{\tau'*} + \tilde{c}_{11}^\tau \tilde{c}_{11}^{\tau'*}$$
$$\tilde{R}_{\Phi''}^{\tau\tau'} = \tilde{c}_3^\tau \tilde{c}_3^{\tau'*} + (\tilde{c}_{12}^\tau - \tilde{c}_{15}^\tau)\left(\tilde{c}_{12}^{\tau'*} - \tilde{c}_{15}^{\tau'*}\right)$$
$$\tilde{R}_{\Phi''M}^{\tau\tau'} = \mathrm{Re}\left[\tilde{c}_3^\tau \tilde{c}_1^{\tau'*} - (\tilde{c}_{12}^\tau - \tilde{c}_{15}^\tau)\,\tilde{c}_{11}^{\tau'*}\right]$$
$$\tilde{R}_{\tilde{\Phi}'}^{\tau\tau'} = \tilde{c}_{12}^\tau \tilde{c}_{12}^{\tau'*} + \tilde{c}_{13}^\tau \tilde{c}_{13}^{\tau'*}$$
$$\tilde{R}_{\Sigma''}^{\tau\tau'} = (\tilde{c}_4^\tau - \tilde{c}_6^\tau)(\tilde{c}_4^{\tau'*} - \tilde{c}_6^{\tau'*}) + \tilde{c}_{10}^\tau \tilde{c}_{10}^{\tau'*}$$
$$\tilde{R}_{\Sigma'}^{\tau\tau'} = \tilde{c}_4^\tau \tilde{c}_4^{\tau'*} + \tilde{c}_9^\tau \tilde{c}_9^{\tau'*}$$
$$\tilde{R}_{\Delta}^{\tau\tau'} = \tilde{c}_5^\tau \tilde{c}_5^{\tau'*} + \tilde{c}_8^\tau \tilde{c}_8^{\tau'*}$$
$$\tilde{R}_{\Delta\Sigma'}^{\tau\tau'} = \mathrm{Re}\left[\tilde{c}_5^\tau \tilde{c}_4^{\tau'*} + \tilde{c}_8^\tau \tilde{c}_9^{\tau'*}\right]$$

Charge or Current	Allowed Projection	Multipole Operator	LECs tested
1_N	Charge	$M_{J=0,2,\ldots}$	c_1, c_{11}
$\vec{v}_N \cdot \vec{\sigma}_N$	none	none	none
$\vec{\sigma}_N$	Longitudinal	$\Sigma''_{J=1,3,\ldots}$	c_4, c_6, c_{10}
$\vec{\sigma}_N$	Trans Electric	$\Sigma'_{J=1,3,\ldots}$	c_4, c_9
$\vec{v}_N$	Trans Magnetic	$\Delta_{J=1,3,\ldots}$	c_5, c_8
$\vec{v}_N \times \vec{\sigma}_N$	Longitudinal	$\Phi''_{J=0,2,\ldots}$	c_3, c_{12}, c_{15}
$\vec{v}_N \times \vec{\sigma}_N$	Trans Electric	$\tilde{\Phi}'_{J=2,4,\ldots}$	c_{12}, c_{13}

CLFV Decay Rate

$$\omega = \frac{G_F^2}{\pi}\frac{q_{\mathrm{eff}}^2}{1+\frac{q}{M_T}}|\phi_{1s}^{Z_{\mathrm{eff}}}(\vec{0})|^2 \sum_{\tau=0,1}\sum_{\tau'=0,1}\Big\{\left[\tilde{R}_M^{\tau\tau'} W_M^{\tau\tau'}(q_{\mathrm{eff}}) + \tilde{R}_{\Sigma''}^{\tau\tau'} W_{\Sigma''}^{\tau\tau'}(q_{\mathrm{eff}}) + \tilde{R}_{\Sigma'}^{\tau\tau'} W_{\Sigma'}^{\tau\tau'}(q_{\mathrm{eff}})\right]$$
$$+\frac{q_{\mathrm{eff}}^2}{m_N^2}\left[\tilde{R}_{\Phi''}^{\tau\tau'} W_{\Phi''}^{\tau\tau'}(q_{\mathrm{eff}}) + \tilde{R}_{\tilde{\Phi}'}^{\tau\tau'} W_{\tilde{\Phi}'}^{\tau\tau'}(q_{\mathrm{eff}}) + \tilde{R}_{\Delta}^{\tau\tau'} W_{\Delta}^{\tau\tau'}(q_{\mathrm{eff}})\right]$$
$$-\frac{2q_{\mathrm{eff}}}{m_N}\left[\tilde{R}_{\Phi''M}^{\tau\tau'} W_{\Phi''M}^{\tau\tau'}(q_{\mathrm{eff}}) + \tilde{R}_{\Delta\Sigma'}^{\tau\tau'} W_{\Delta\Sigma'}^{\tau\tau'}(q_{\mathrm{eff}})\right]\Big\}$$

- Factorization of leptonic and nuclear physics
- Directly analogous to other semi-leptonic processes
- c_2, c_7, c_{14}, c_{16} not probed by elastic $\mu \to e$ conversion

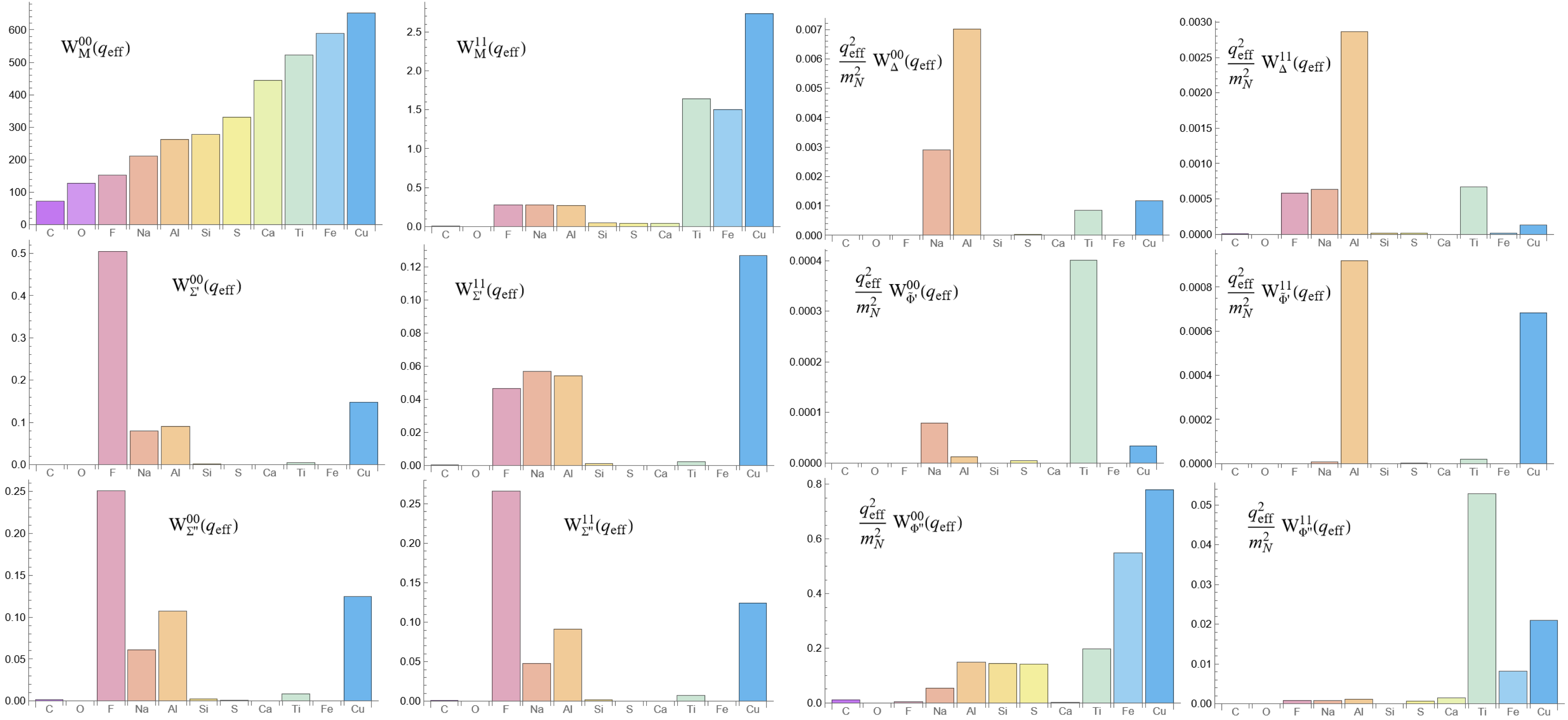
$W_i^{\tau\tau'}(q_{\mathrm{eff}}) \leftrightarrow$ “Nuclear dials”

Velocity-independent

Isoscalar | Isovector

Velocity-dependent

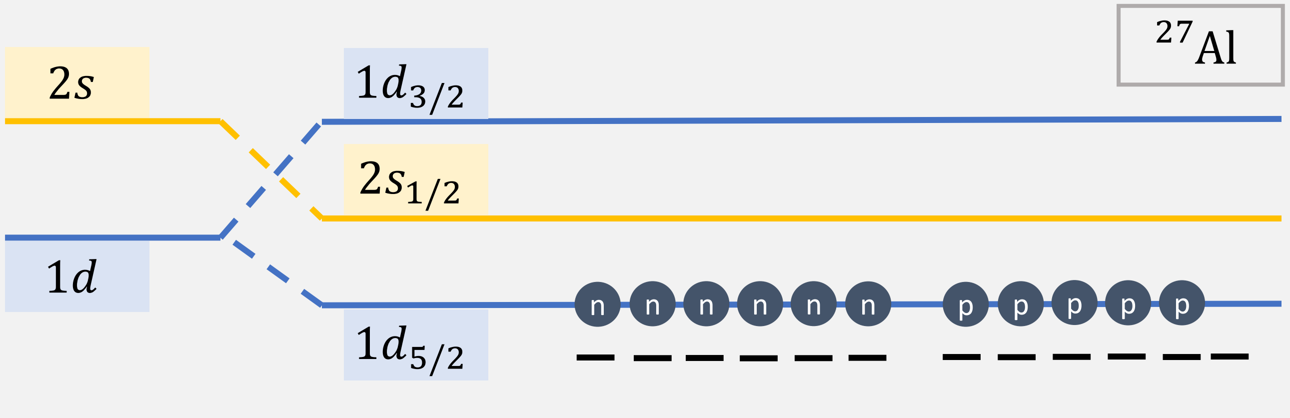
Isoscalar | Isovector



A Novel Form of Coherence

$$\Phi''_{00}(0) = -\frac{1}{6\sqrt{\pi}} \sum_{i=1}^{A} \vec{\sigma}(i) \cdot \vec{\ell}(i)$$

Sums coherently over $j = \ell + \frac{1}{2}$ and $j = \ell - \frac{1}{2}$ subshells but vanishes when both subshells are filled



Inert core

A Novel Form of Coherence

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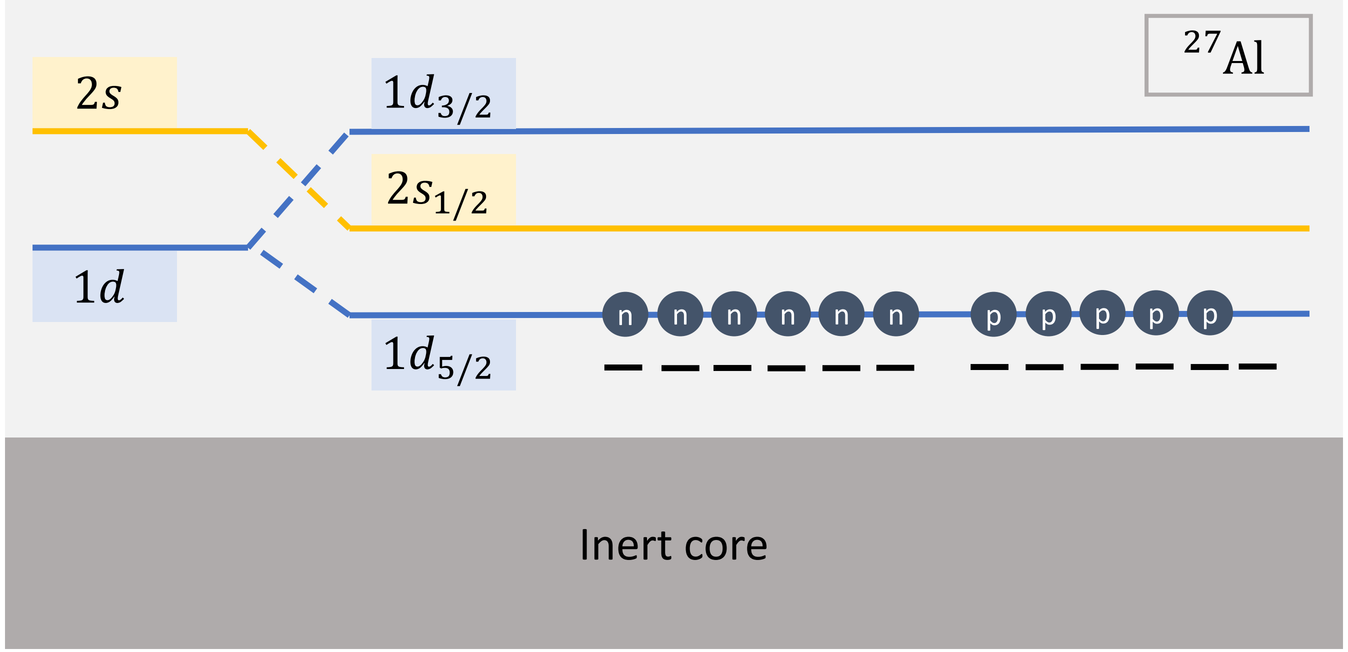
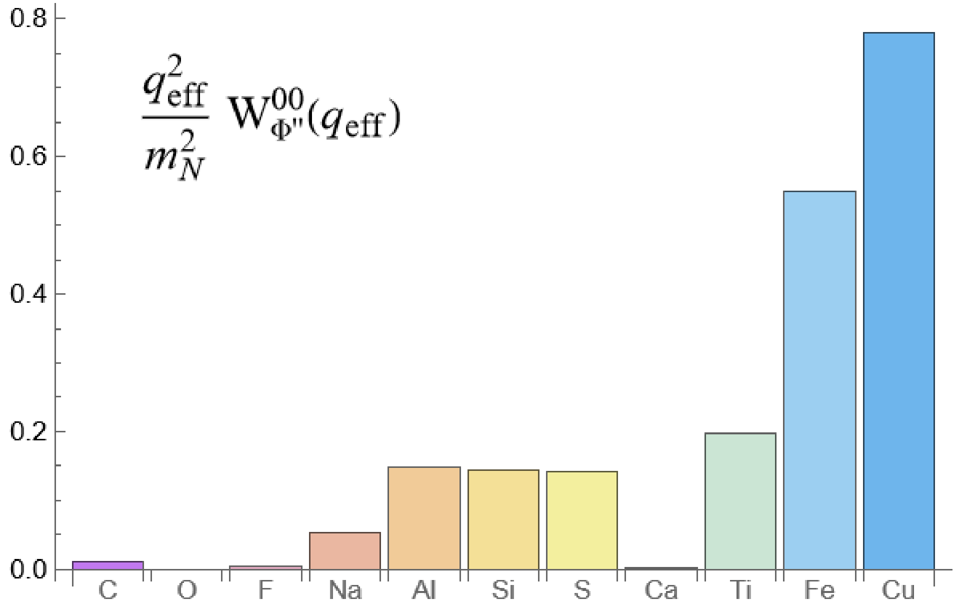


$$\bar{\chi}_e i\sigma^{\mu\nu}\gamma^5\chi_\mu \bar{N} i\sigma_{\mu\nu}\gamma^5 N$$

$$\omega \propto \left\langle \frac{q}{2m_N} M_{00} - \frac{q_{\text{eff}}}{m_N} \Phi''_{00} \right\rangle^2$$

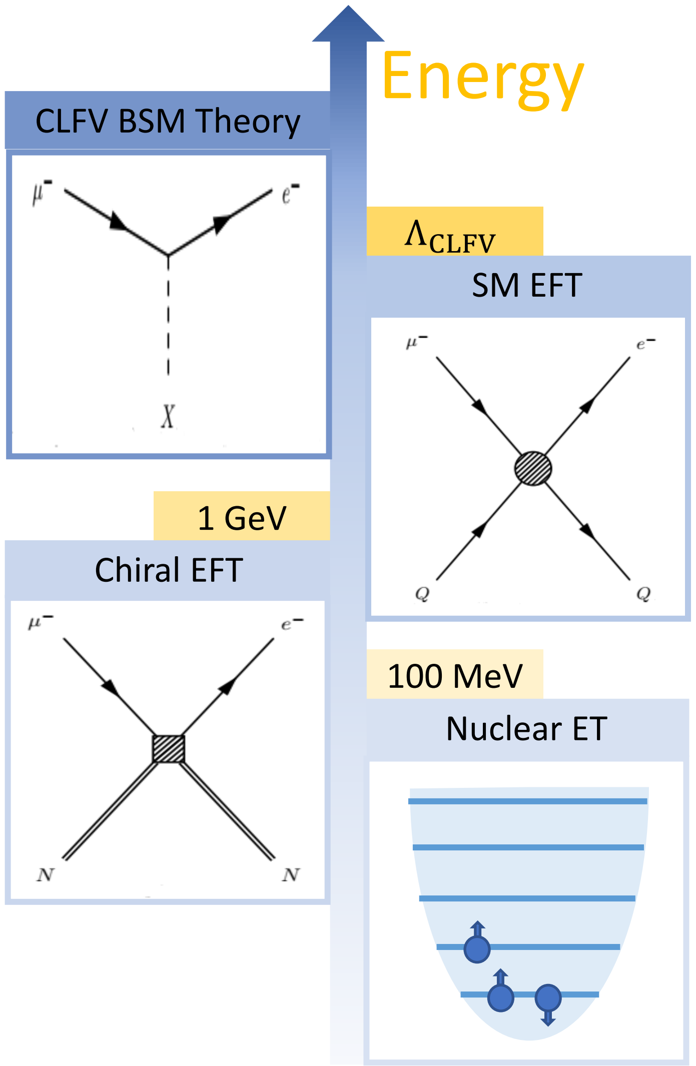
- Velocity-dependent contribution same order as charge monopole
- Certain targets are sensitive to exotic CLFV



Summary and Future Work

- Nuclear ET identifies six CLFV response functions + two interference terms probed by elastic $\mu \rightarrow e$ conversion
- Publicly-available Python & Mathematica codes for $\mu \rightarrow e$ effective theory (see arXiv:2208.07945 for details)
- Matching to quark-level EFTs in progress
- Inelastic $\mu \rightarrow e$ conversion probes 4 LECs that elastic cannot

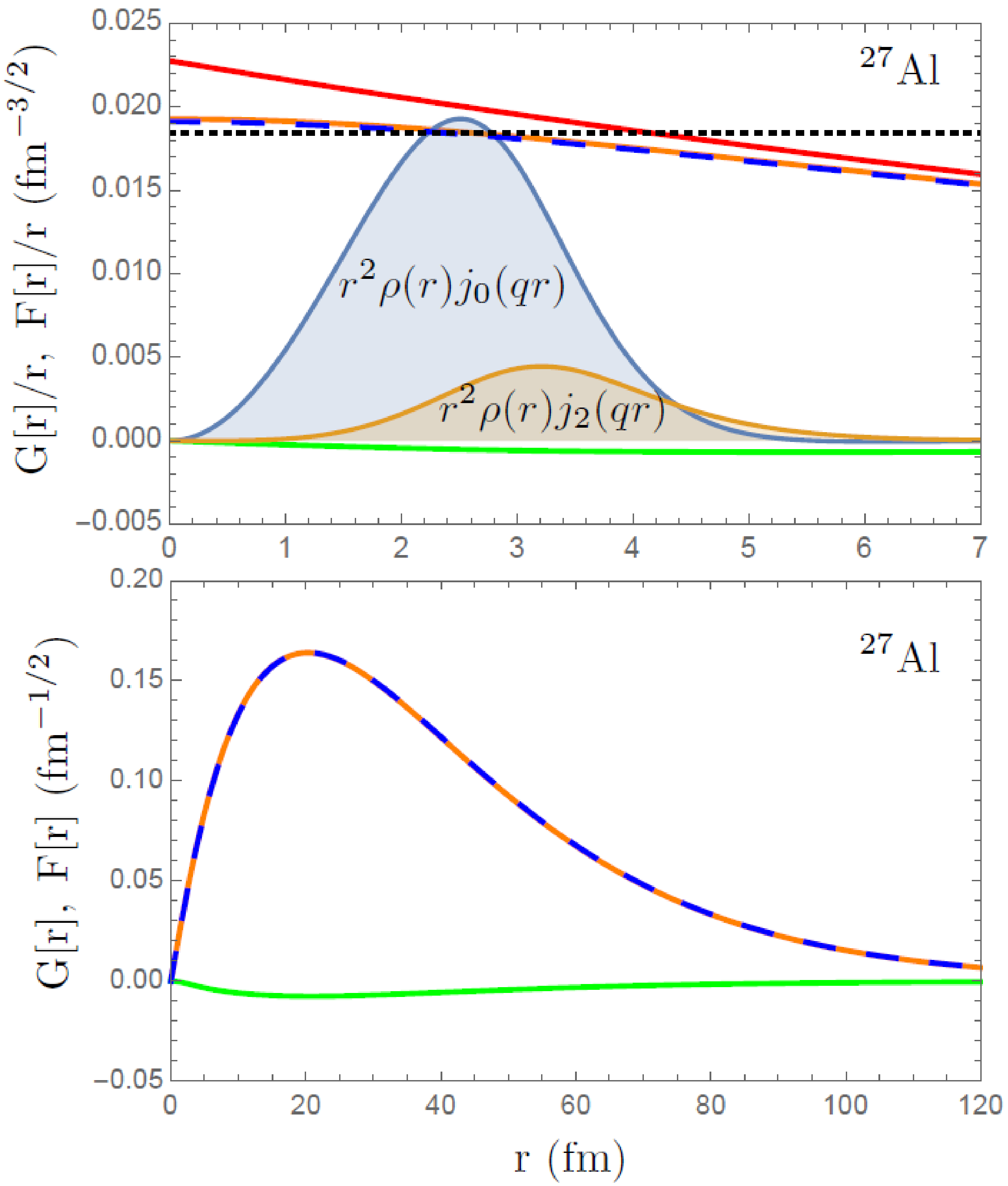
ER, Haxton, and McElvain, arXiv:2109.13503
Haxton, **ER**, McElvain, and Ramsey-Musolf, arXiv:2208.07945



Backup Slides

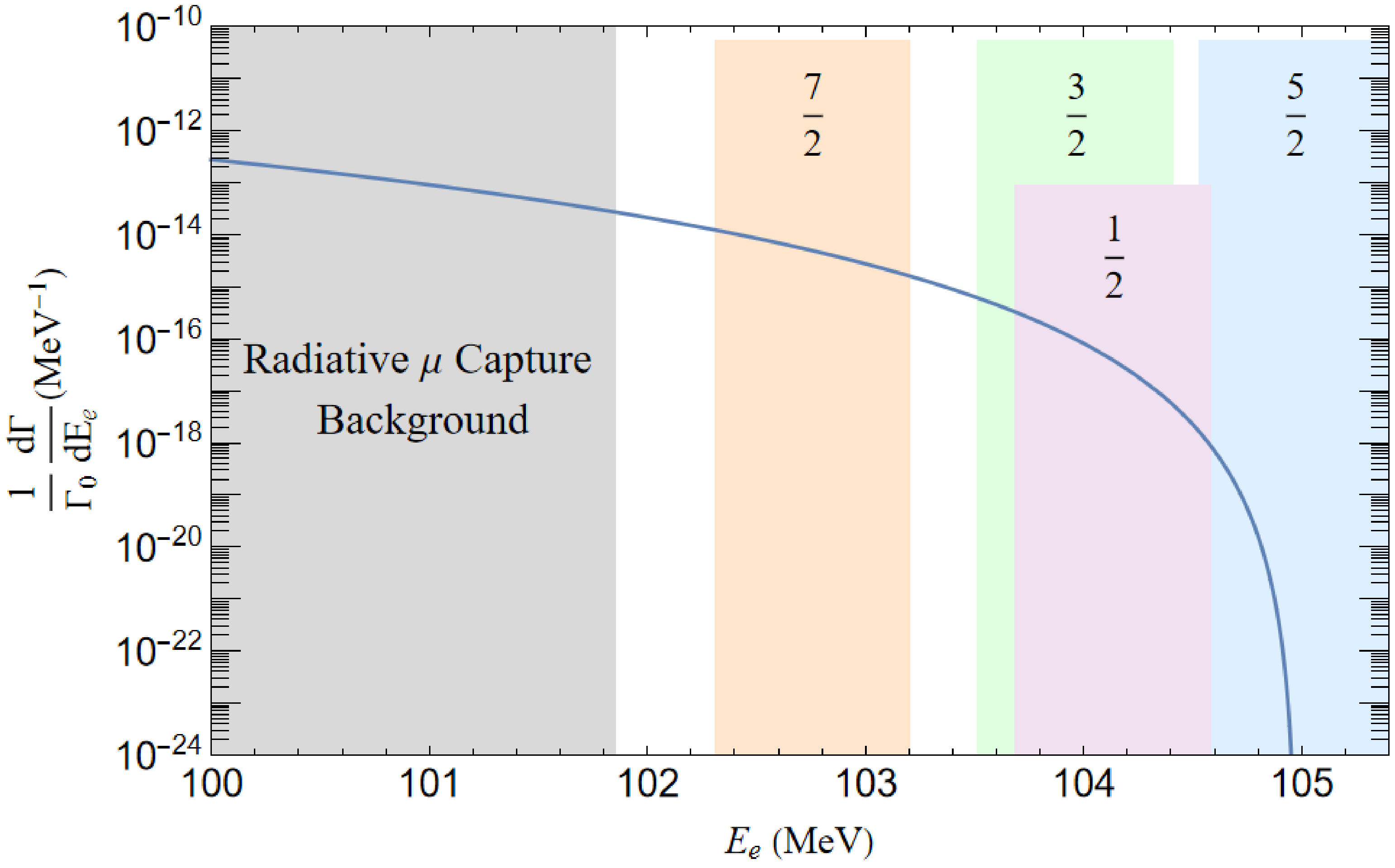
Muon Wave Functions

- Captured muons quickly cascade to 1s orbital of the nuclear Coulomb field
- Non-relativistic (lower component always a correction)
- Muon wave function varies slowly over the scale of the nucleus: $a_0^\mu \approx 20$ fm, $r_N^{RMS} \approx 3.1$ fm
- We replace full muon wave function with an average value $\left|\phi_{1s}^{Z_{\rm eff}}(\vec{0})\right|$
- Resulting errors in decay rate are $\lesssim 3\%$ for $^{27}\mathrm{Al}$



Inelastic $\mu \to e$ Conversion

- 4 CLFV operators not probed by elastic conversion due to P and T
- Transition to excited nuclear state avoids these constraints
- Background is worse from $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$
- Can interesting limits be set?



Effective Theory Variations

Coherent Response	Allowed Response	General Response
$1_N, 1_L, \hat{q}, \vec{\sigma}_L, \vec{v}_\mu$	$1_N, 1_L, \hat{q}, \vec{\sigma}_L, \vec{\sigma}_N, \vec{v}_\mu$	$1_N, 1_L, \hat{q}, \vec{\sigma}_L, \vec{\sigma}_N, \vec{v}_N, \vec{v}_\mu$
2 Nucleon-level Operators	6 Nucleon-level operators	16 Nucleon-level operators
1 Response function	3 Response functions	6 Response functions
$M_0, M_0^{(2)}$	$M_J, \Sigma_J', \Sigma_J''$	$M_J, \Sigma_J', \Sigma_J'', \Delta_J, \tilde{\Phi}_J', \Phi_J''$

- Relativistic muon effects always subleading
- Nuclear velocity can contribute at leading order

$$\mathcal{O}_2^{f\prime} = i\hat{q} \cdot \vec{v}_\mu 1_N$$
$$\mathcal{O}_3^{f} = i\hat{q} \cdot (\vec{v}_\mu \times \vec{\sigma}_L)\, 1_N$$
$$\mathcal{O}_5^{f} = (i\hat{q} \times \vec{v}_\mu) \cdot \vec{\sigma}_N$$
$$\mathcal{O}_7^{f} = \vec{v}_\mu \cdot \vec{\sigma}_L\, 1_N$$
$$\mathcal{O}_8^{f} = \vec{v}_\mu \cdot \vec{\sigma}_N$$
$$\mathcal{O}_{12}^{f} = (\vec{v}_\mu \times \vec{\sigma}_L) \cdot \vec{\sigma}_N$$
$$\mathcal{O}_{13}^{f\prime} = [i\hat{q} \times (\vec{v}_\mu \times \vec{\sigma}_L)] \cdot \vec{\sigma}_N$$
$$\mathcal{O}_{14}^{f} = \vec{v}_\mu \cdot \vec{\sigma}_L\, i\hat{q} \cdot \vec{\sigma}_N$$
$$\mathcal{O}_{15}^{f} = i\hat{q} \cdot (\vec{v}_\mu \times \vec{\sigma}_L)\, i\hat{q} \cdot \vec{\sigma}_N$$
$$\mathcal{O}_{16}^{f\prime} = i\hat{q} \cdot \vec{v}_\mu\, i\hat{q} \cdot \vec{\sigma}_N$$

Limits on LECs

- Using our expression for the decay rate, we can constrain LECs using existing and future branching ratio limits for various nuclei
- We can also estimate the energy scale probed by each operator
- Assume only one operator is responsible for CLFV
- Mathematica + Python scripts to compute $\mathrm{B}(\mu \rightarrow e)$ in terms of $\tilde{c}_i^\tau$ for a selection of nuclear targets

	Al $(B < 10^{-17})$		Ti $(B < 6.1 \times 10^{-13})^{\dagger}$	
Coupling	LEC Limit	~ Scale Probed	LEC Limit	~ Scale Probed
$\tilde{c}_1^0, \tilde{c}_{11}^0$	3.994E-10	10,000 TeV	7.380E-8	900 TeV
$\tilde{c}_1^1, \tilde{c}_{11}^1$	1.238E-8	2,000 TeV	1.316E-6	200 TeV
$\tilde{c}_3^0, \tilde{c}_{15}^0$	1.608E-8	2,000 TeV	3.801E-6	100 TeV
$\tilde{c}_3^1, \tilde{c}_{15}^1$	1.860E-7	1,000 TeV	7.344E-6	100 TeV
$\tilde{c}_4^0$	1.418E-8	2,000 TeV	1.504E-5	60 TeV
$\tilde{c}_4^1$	1.713E-8	2,000 TeV	1.718E-5	60 TeV
$\tilde{c}_5^0, \tilde{c}_8^0$	7.774E-8	1,000 TeV	5.802E-5	30 TeV
$\tilde{c}_5^1, \tilde{c}_8^1$	1.164E-7	1,000 TeV	6.521E-5	30 TeV
$\tilde{c}_6^0, \tilde{c}_{10}^0$	1.954E-8	2,000 TeV	1.794E-5	60 TeV
$\tilde{c}_6^1, \tilde{c}_{10}^1$	2.151E-8	2,000 TeV	1.999E-5	60 TeV
$\tilde{c}_9^0$	2.061E-8	2,000 TeV	2.758E-5	50 TeV
$\tilde{c}_9^1$	2.833E-8	1,000 TeV	3.360E-5	40 TeV
$\tilde{c}_{12}^0$	1.608E-8	2,000 TeV	3.797E-6	100 TeV
$\tilde{c}_{12}^1$	1.388E-7	700 TeV	7.342E-6	100 TeV
$\tilde{c}_{13}^0$	1.787E-6	200 TeV	8.422E-5	30 TeV
$\tilde{c}_{13}^1$	2.085E-7	500 TeV	3.718E-4	10 TeV

†: P. Wintz, Proc. 1st Int. Symp. on Lepton and Baryon Number Violation