LATTICE QCD COMPUTATION OF THE HVP CONTRIBUTION TO MUON G-FACTOR

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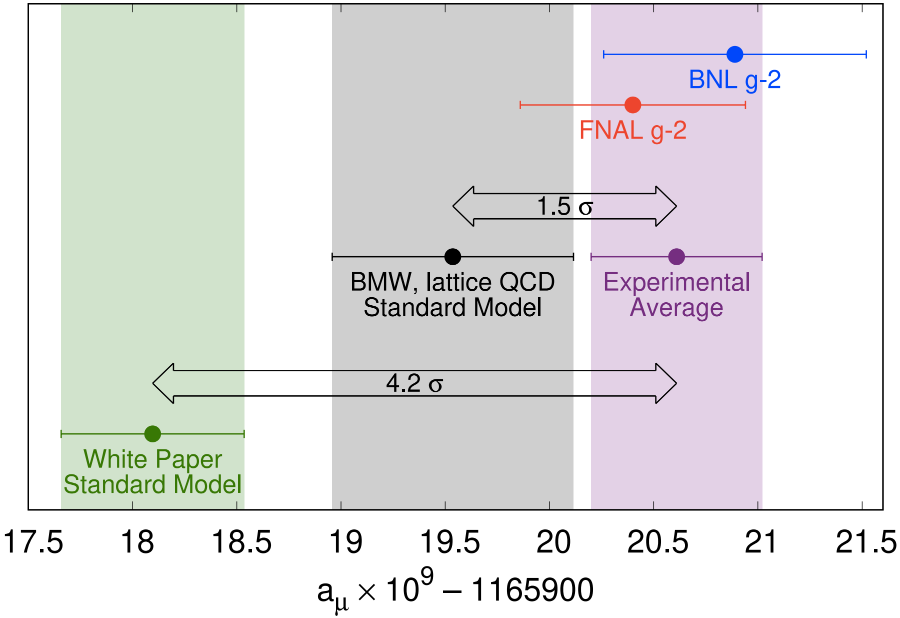
Budapest-Marseille-Wuppertal collaboration

Borsanyi,Fodor,Guenther,Hoelbling,Katz,Lellouch,Lippert Miura,
Parato, Stokes, Toth, Torok, Varnhorst

[2002.12347,Nature (2021)] Leading-order hadronic vacuum polariz ...

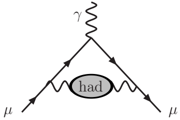
Outline



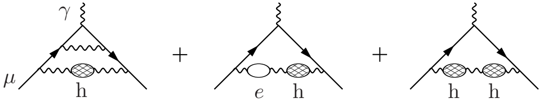
BNL g-2
FNAL g-2
1.5 σ
BMW, lattice QCD
Standard Model
Experimental
Average
4.2 σ
White Paper
Standard Model
17.5
18
18.5
19
19.5
20
20.5
21
21.5
$a_\mu \times 10^9 - 1165900$

Strong contributions

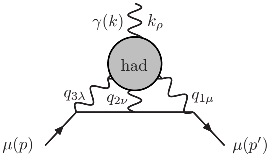
LO hadron vacuum polarization (LO-HVP, $(\frac{\alpha}{\pi})^2$)



NLO hadron vacuum polarization (NLO-HVP, $(\frac{\alpha}{\pi})^3$)



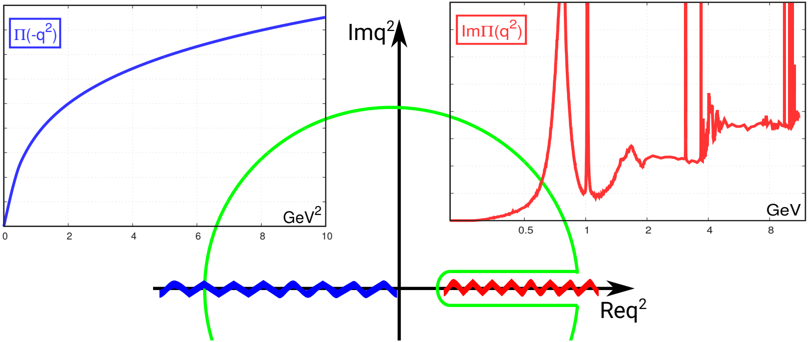
Hadronic light-by-light (HLbL, $(\frac{\alpha}{\pi})^3$)



- pheno+latt $a_\mu^{\mathrm{HLbL}} = 9.2(1.9)$
  [WhitePaper '20]
- lattice a_μ^{HLbL}=7.9(3.1)(1.8) or 10.7(1.5)
  [RBC/UKQCD '19] and [Mainz '21]

Hadronic vacuum polarization

- $\Pi_{\mu\nu}(q) = \left(q_\mu q_\nu - g_{\mu\nu} q^2\right) \Pi(q^2)$ analytic + branch-cut



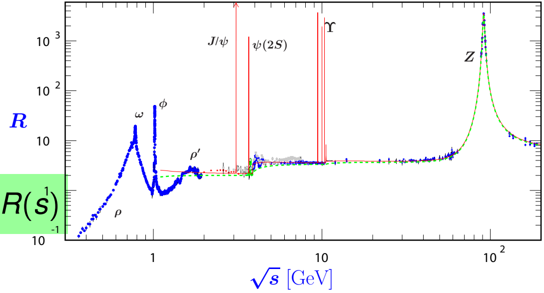
- Minkowski from R-ratio experiments
- Euclidean from lattice QCD or exp. like MUonE
- Minkowski → Euclidean via dispersion relation ($Q^2 = -q^2$)

$$\Pi(Q^2) = \int_{s_{\text{th}}}^{\infty} ds \, \frac{Q^2}{s(s+Q^2)} \frac{1}{\pi} \text{Im}\Pi(s)$$

a_μ^{HVP} from R-ratio

Use $e^+e^- \to$ had data of CMD, SND, BES, KLOE, BABAR, ...

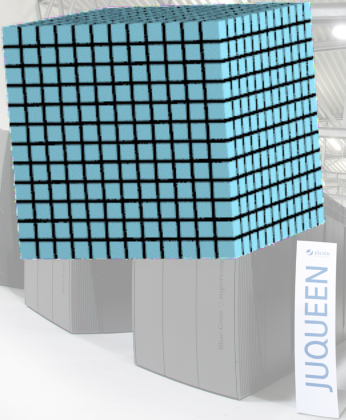
$$a_\mu^{\text{LO-HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int \frac{ds}{s^2} K_\mu(s) R(s)$$



LO	[Jegerlehner '18]	688.1(4.1)	0.60%
LO	[Davier et al '19]	693.9(4.0)	0.58%
LO	[Keshavarzi et al '19]	692.78(2.42)	0.35%
LO	[Hoferichter et al '19]	692.3(3.3)	0.48%
NLO	[Kurz et al '14]	-9.87(0.09)	
NNLO	[Kurz et al '14]	1.24(0.01)	

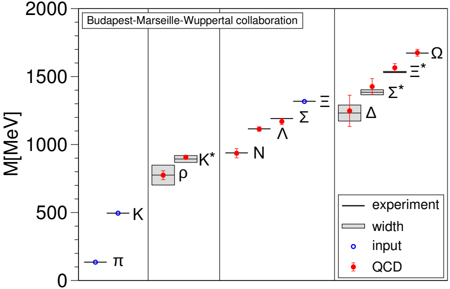
$$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{QED}} - a_\mu^{\mathrm{weak}} - a_\mu^{\mathrm{HLbL}} - a_\mu^{\mathrm{HVP}} = 25.1(5.9)$$

Lattice QCD

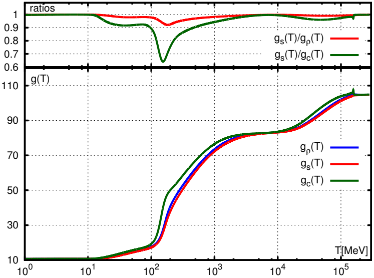


- ab-initio: use equations of QCD and nothing more
- discretize equations on space-time lattice and solve numerically ($\rightarrow$ supercomputer)
- sum over all Feynman-diagrams at once (and beyond)
- only works in imaginary time (no problem for a_μ)

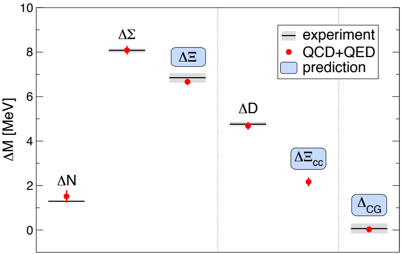
Lattice highlights



hadron spectrum [BMWc'08]



equation of state [WB '16]



neutron-proton difference [BMWc'14]

- isospin symmetry breaking (strong and QED)

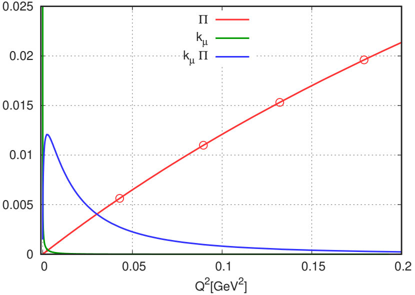
a_μ^{HVP} from lattice QCD

- get Π from Euclidean current-current correlator [Blum '02]

$$\Pi_{\mu\nu} = \int dx\, e^{iQx} \langle J_\mu(x) J_\nu(0) \rangle = \left(Q_\mu Q_\nu - \delta_{\mu\nu} Q^2 \right) \Pi(Q^2)$$

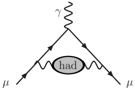
<u>problems:</u> we need $\Pi(Q^2) - \Pi(0)$, but $\Pi(0)$ is not directly accessible; also Q is available at discrete momenta only

- smooth interpolation in Q and prescription for $\Pi(0)$ [Bernecker,Meyer '11], [HPQCD'14], ...

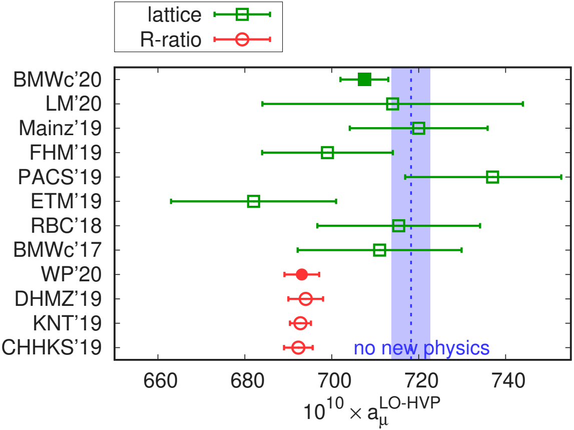


$$a_\mu^{\mathrm{HVP}} = \frac{\alpha^2}{\pi^2} \int dQ^2\, k_\mu(Q^2)\, \Pi(Q^2)$$

$k_\mu(Q^2)$ describes the leptonic part of diagram

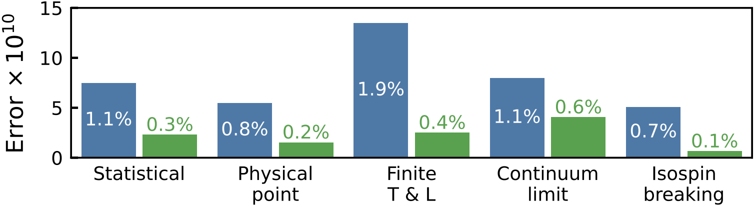


Comparison with other determinations



- $a_\mu^{\text{LO-HVP}} = 707.5(2.3)(5.0)[5.5]$ with 0.8% accuracy
- first lattice computation with all contributions
- uncertainty reduced by factor $3.4\times$ compared to [BMWc'17]
- compatible with other lattice calculations

Key improvements



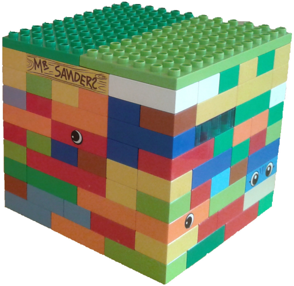
- incorporated recent algorithmic improvements to reduce statistical noise
- physical input is mass Omega baryon with 0.2% error
- dedicated finite-size study in 11 fm box (typical is $\lesssim$ 6 fm), good agreement with model computations
- six lattice spacings and improved approach towards cont.limit
- included all relevant isospin breaking effects

Finite-size effects

Typical runs use $L < 6$ fm, earlier model estimates gave $O(2)\%$.

- perform continuum limit in $L = 6$ fm, reference box
- on a coarse lattice compute finite size effect with $L = 11$ fm

“ref” $\rightarrow$



$\leftarrow$ “big”

- compare “big” - “ref” on the lattice to various models

	lattice	NNLO	MLLGS	HP	RHO
“big” - “ref”	18.1(2.0)(1.4)	15.3	18.3	16.3	15.2

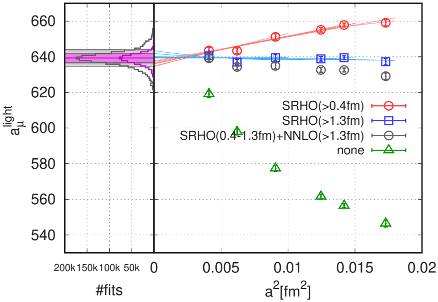
- use models for remnant finite-size effect of “big” $\sim 0.1\%$

Continuum limit

Improve continuum extrapolations by models ([HPQCD'16])

$$a_\mu(a) \to a_\mu(a) + \left[a_\mu^{\mathrm{RHO}} - a_\mu^{\mathrm{SRHO}}(a)\right]$$

$\to$ reduced sensitivity to β-cuts; improved slope and curvature

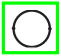


Systematic error:

- skip coarse lattices
- change the point, where we start applying the improvement
- use $a^2\alpha^\Gamma[1/a]$ instead of a^2
- use linear or quadratic extrapolations
- consider different models (SMLLGS/SRHO/SXPT)

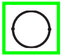
Overview of contributions

Isospin symmetric



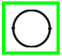
connected light

633.7(2.1)(4.2)



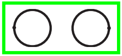
connected strange

53.393(89)(68)



connected charm

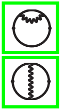
14.6(0)(1)



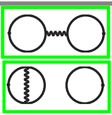
disconnected

-13.36(1.18)(1.36)

QED isospin-breaking: valence



connected -1.23(40)(31)

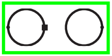


disconnected -0.55(15)(10)

Strong isospin-breaking

connected

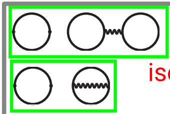
6.60(63)(53)



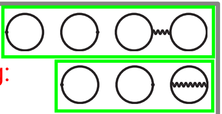
disconnected

-4.67(54)(69)

QED isospin-breaking: sea



connected 0.37(21)(24)



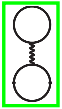
disconnected -0.040(33)(21)

Etc.

bottom; higher order; perturbative

0.11(4)

QED isospin-breaking: mixed



connected -0.0093(86)(95)

disconnected 0.011(24)(14)

Finite-size effects

isospin-symmetric

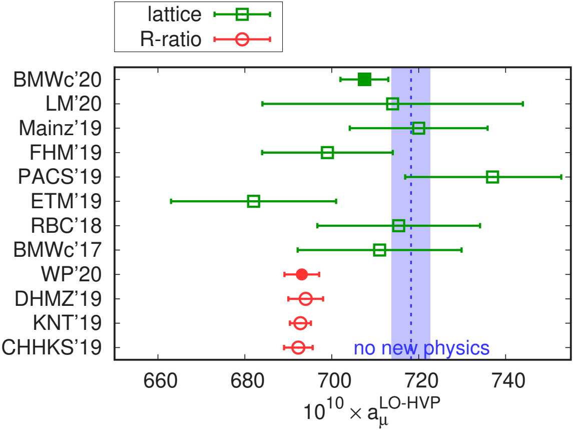
18.7(2.5)

isospin-breaking

0.0(0.1)

$$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$

Final result



- $a_\mu^{\text{LO-HVP}} = 707.5(2.3)(5.0)[5.5]$ with 0.8% accuracy
- consistent with FNAL and 1.5σ away from BNL+FNAL
- 2.0σ larger than [DHMZ19], 2.5σ than [KNT19]

Outlook

1. See if other lattice groups confirm or refute.
   - Fermilab-MILC-HPQCD, ETMC, Mainz, RBC-UKQCD, . . .
   - window observable

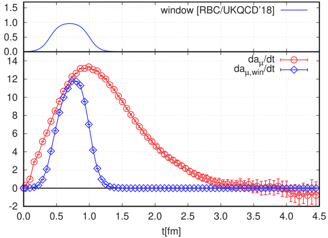
2. Improve on these computations.
   - finer and bigger lattices to improve cont. limit
   - improved analysis techniques
   - target: $4\times$ increase in precision

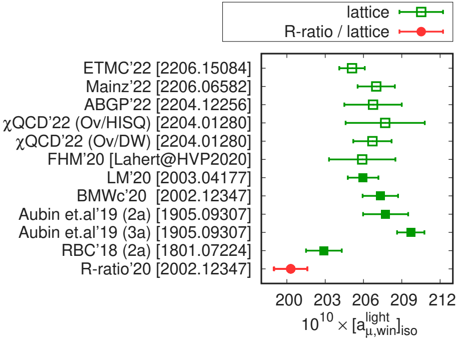
3. Find energy region causing the tension with the R-ratio.

Other groups

Window observable

- restrict correlator to window 0.4 – 1.0 fm [RBC/UKQCD'18]
- fewer difficulties





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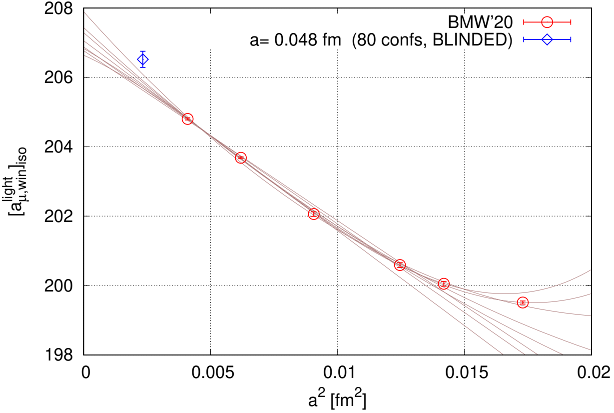
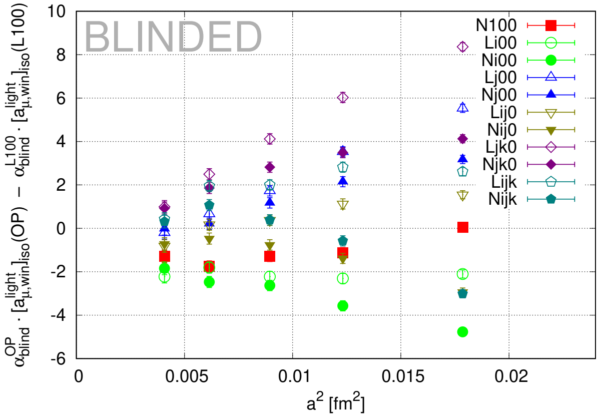
Improvement - finer lattice

$a = 0.048$ fm $128^3 \times 192$ (previously $a = 0.064$ fm $96^3 \times 144$)



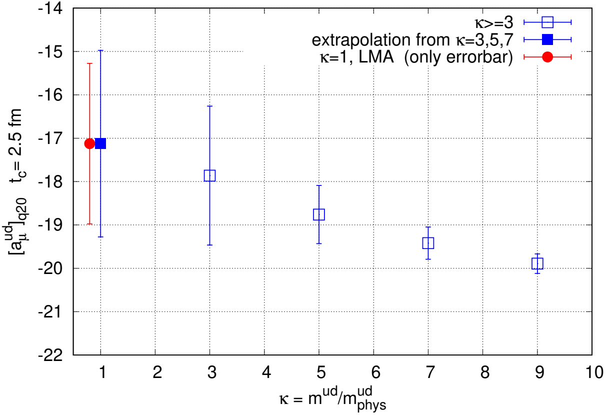
Improvement - more operators

current operator can be discretized in different ways
different result at finite lattice spacing, more control over continuum extrapolation



Improvement - QED contribution

eliminating a chiral extrapolation by direct computation at the physical mass



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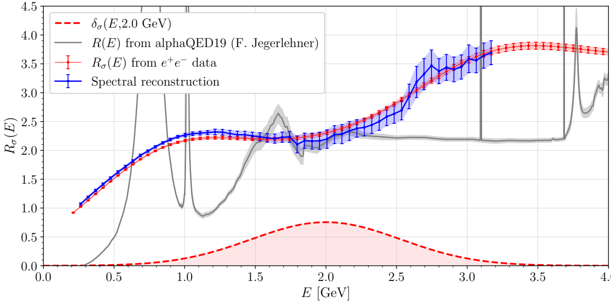
Spectral function reconstruction

[DeSantis et al, Lattice22-Thu]

First "*R(E)*" computation from first principles

$$R_\sigma(E) = \int_{2M_\pi}^{\infty} \mathrm{d}\omega \delta_\sigma(\omega, E) R(\omega) \qquad \delta_\sigma(\omega, E) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(\omega-E)^2}{2\sigma^2}} \qquad \sigma \sim 0.5 \text{ GeV}$$



- How did we get it?

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