3D Imaging of Nucleons and Nuclei

(Andreas Metz, Temple University)

- Introduction and motivation
- 3D in momentum space:
  transverse momentum dependent parton distributions (TMDs)
- 1D+2D in mixed momentum and position space:
  related to generalized parton distributions (GPDs)
- Going beyond 3D:
  partonic Wigner functions
- Nuclei
- Some prospects at the EIC
- Conclusions

supported by the NSF

Nucleon Structure: the Nobel Prizes

- Protons' anomalous magnetic moment (Estermann, Frisch, Stern, 1933)

$$\mu_p \sim 3 \times \mu_{\text{Dirac}}$$

$\rightarrow$ proton cannot be pointlike

- Elastic electron-proton scattering (Hofstadter, McAllister, 1955)

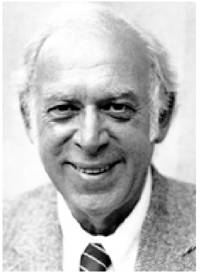
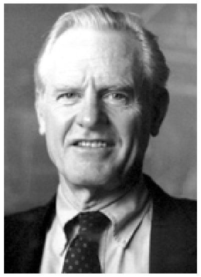
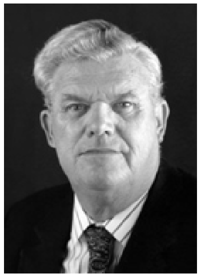
$$r_{p,\text{RMS}}^{\text{charge}} = (0.74 \pm 0.24)\ \text{fm}$$

$\rightarrow$ first rather precise idea about size of the proton

- Deep-inelastic electron-proton scattering (Friedman, Kendall, Taylor et al, 1968)

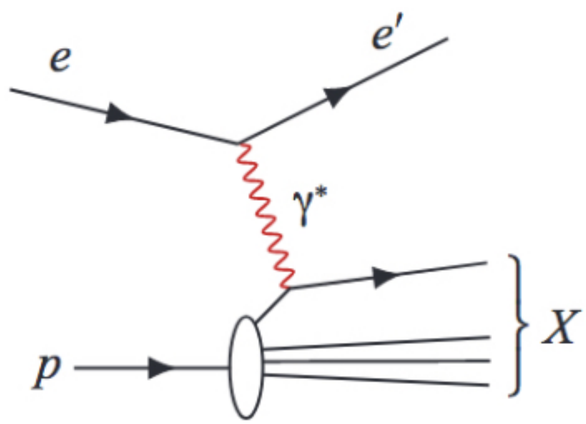
$\rightarrow$ proton has partonic substructure

$\rightarrow$ experiments paved ground for discovery of QCD

Inclusive Deep-Inelastic Scattering (DIS) and 1D Imaging

- Process: $e + p \to e + X$



  - probing of partons (quarks, gluons) with (longitudinal) momentum fraction x $(0 < x < 1)$
  - QCD factorization of $\sigma_{ep\to eX}$ into perturbatively calculable cross section $\hat{\sigma}_{eq\to eq}$ and parton distribution function (PDF) $f_1^q(x)$

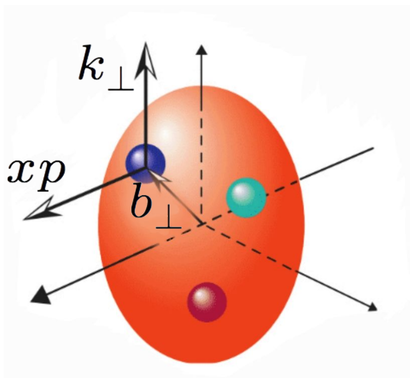
$$\sigma_{ep\to eX} = \sum_q \int_0^1 dx\, f_1^q(x)\, \hat{\sigma}_{eq\to eq} + \mathcal{O}(1/Q^2)$$

  - factorization underlies all parton structure studies
  - gluons enter at higher order in perturbation theory

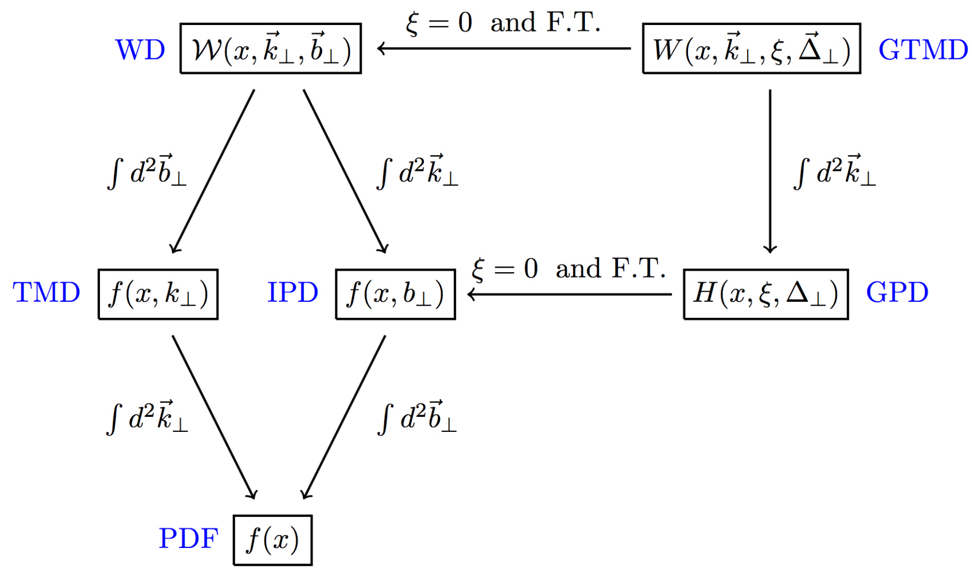
- 1D quark PDFs (including polarization of nucleon and quarks)
  - $f_1^q(x)$ unpolarized PDF (→ Hobbs, HE, Tu, 14:00 / Cocuzza, PDF, Tu, 15:30)
  - $g_1^q(x)$ helicity PDF (→ Sato, PDF, We, 14:30)
  - $h_1^q(x)$ transversity PDF

3D Imaging and Beyond

Adding $\vec{k}_\perp$ or/and $\vec{b}_\perp$



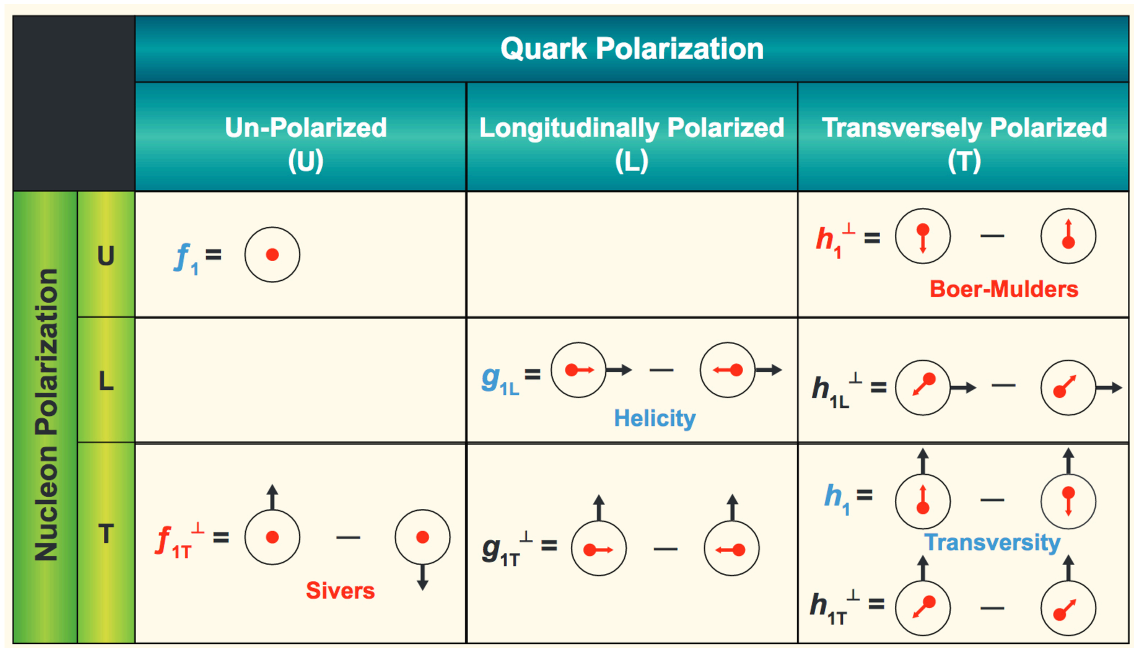
Overview of partonic functions



- Main objects of interest for multi-dimensional imaging
  1. $f(x, k_\perp)$ TMDs
  2. $f(x, b_\perp)$ Impact parameter distributions (Fourier transforms of GPDs)
  3. $\mathcal{W}(x, \vec{k}_\perp, \vec{b}_\perp)$ Wigner distributions (5-D quasi-probability distributions)

3D Imaging in Momentum Space: Overview of TMDs

- Quark TMDs for spin-$\frac{1}{2}$ hadron



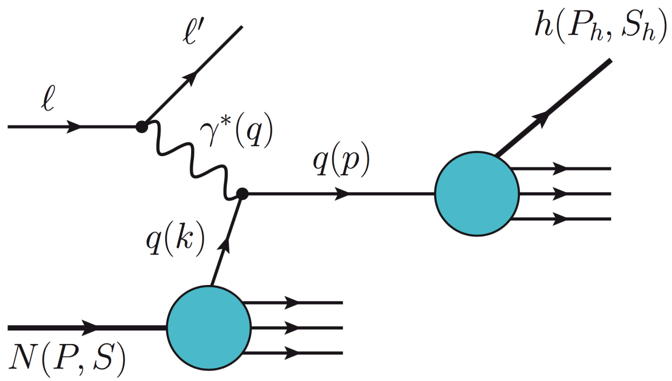
(arXiv:1212.1701)

  - transverse momentum dependent $f_1(x, k_\perp)$ $g_1(x, k_\perp)$ $h_1(x, k_\perp)$
  - 5 additional functions
  - here: main focus on f_1 h_1 $f_{1T}^\perp$ (Sivers function)

- Similarly, 8 gluon TMDs

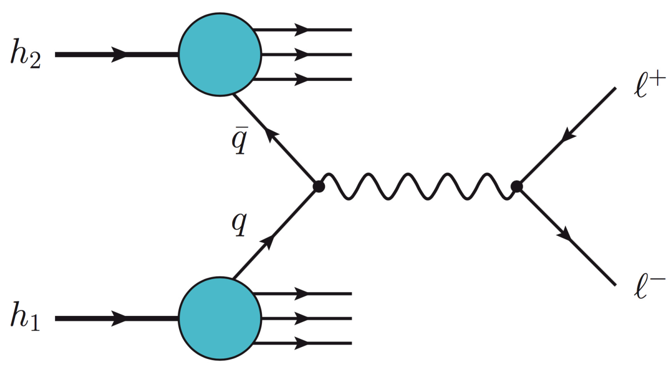
Key Processes for Measuring TMDs

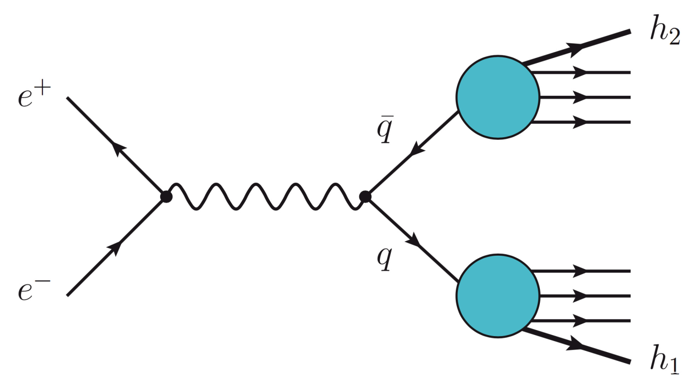
- Semi-inclusive DIS: $\ell + N \to \ell + h + X$



sensitive to TMD PDFs and
TMD fragmentation functions (FFs)
(→ Moffat, PDF, Sa, 15:30)

- Drell-Yan process and electron-positron annihilation

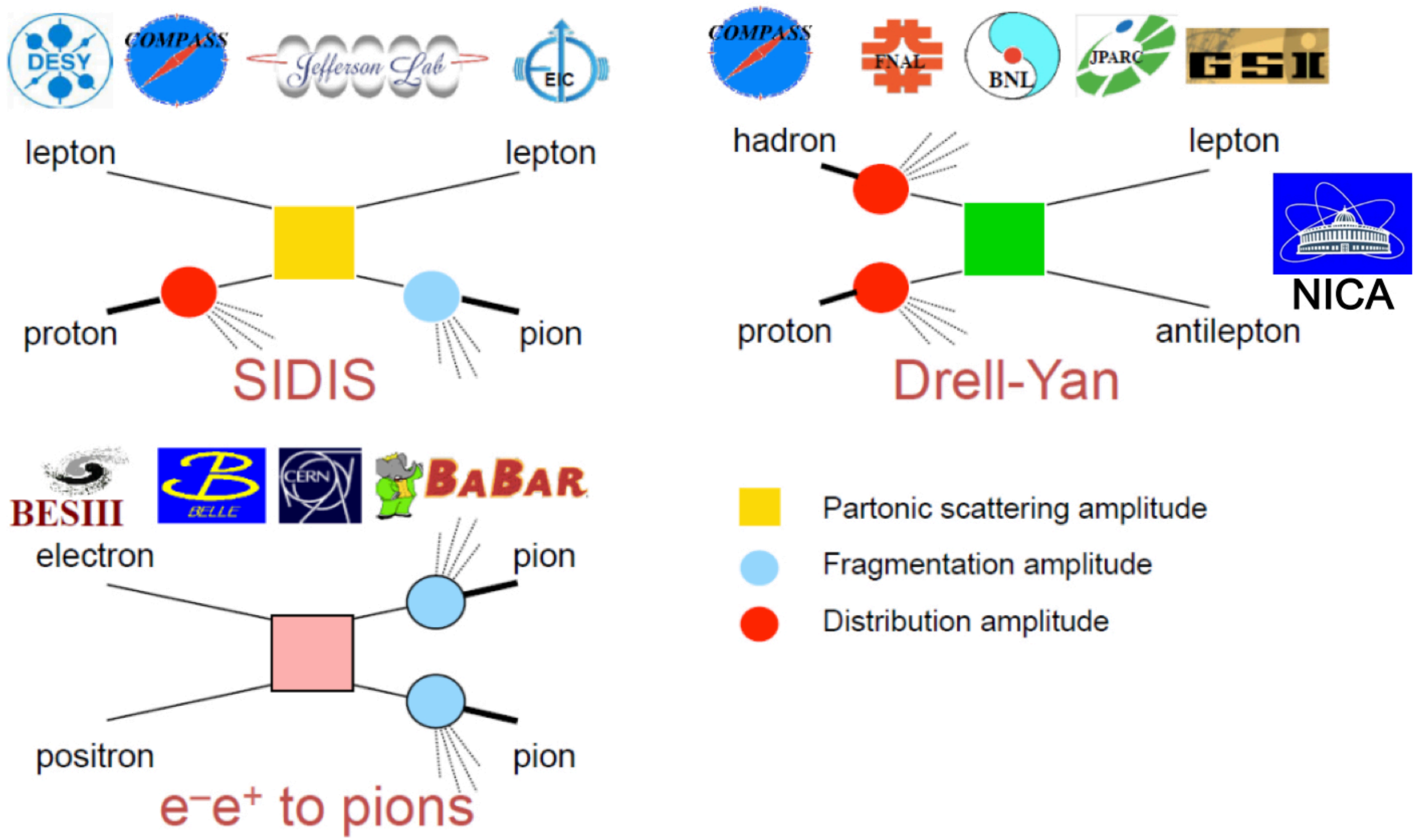




sensitive to TMD PDFs

sensitive to TMD FFs

- Overview of experimental facilities



(Grosse Perdekamp, TMD Summer School, 2017)

Semi-Inclusive DIS: Observables and Sample Data

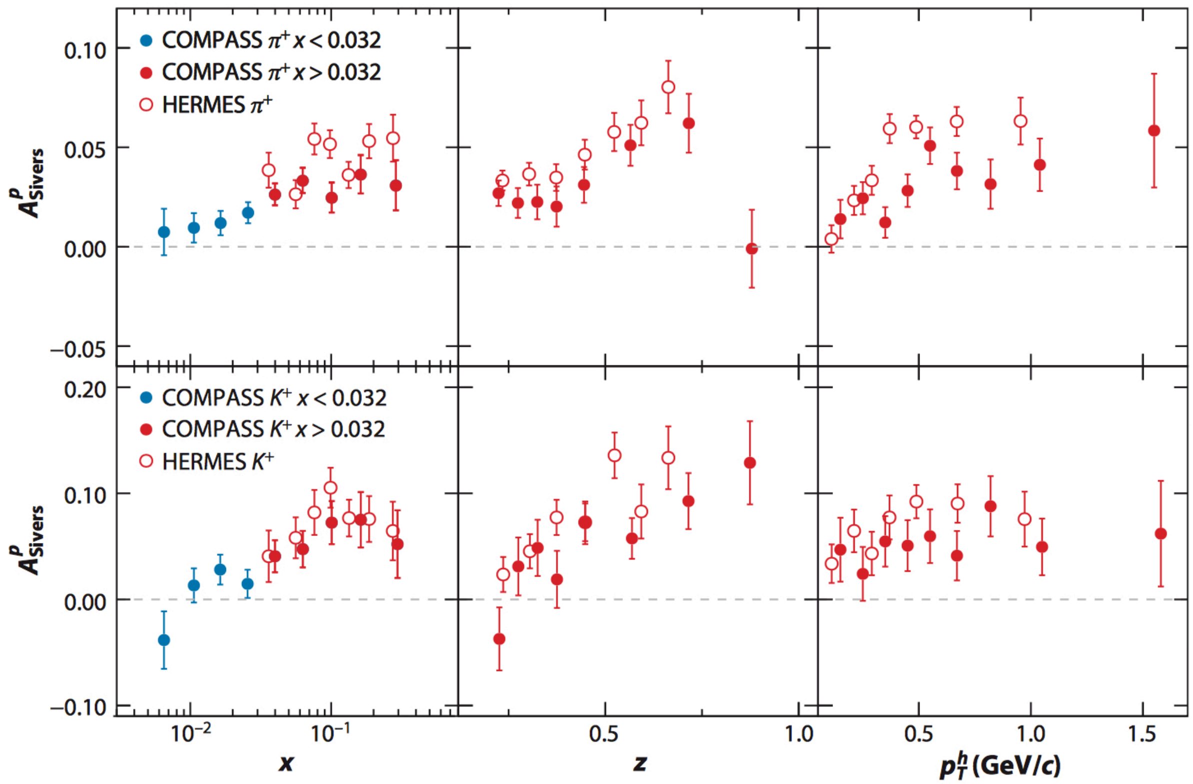
- Structure functions in leading order pQCD factorization (notation: Bacchetta et al, JHEP 2006)

$$
\begin{aligned}
F_{UU,T} &= x \sum_q e_q^2 \int d^2\vec{k}_\perp \, d^2\vec{p}_\perp \, \delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp - \vec{p}_\perp) \, f_1^q(x, k_\perp) \, D_1^q(z, p_\perp) \\
F_{UU}^{\cos 2\phi_h} &\sim h_1^\perp \otimes H_1^\perp \\
F_{UL}^{\sin 2\phi_h} &\sim h_{1L}^\perp \otimes H_1^\perp \\
F_{LL} &\sim g_1 \otimes D_1 \\
F_{UT,T}^{\sin(\phi_h-\phi_S)} &\sim f_{1T}^\perp \otimes D_1 \qquad \text{[Sivers effect]} \\
F_{UT}^{\sin(\phi_h+\phi_S)} &\sim h_1 \otimes H_1^\perp \qquad \text{[Collins effect]} \\
F_{UT}^{\sin(3\phi_h-\phi_S)} &\sim h_{1T}^\perp \otimes H_1^\perp \\
F_{LT}^{\cos(\phi_h-\phi_S)} &\sim g_{1T} \otimes D_1
\end{aligned}
$$

- two TMD FFs for unpolarized final-state hadrons: D_1 $H_1^\perp$ (Collins function)
- all 8 TMD-PDFs can be studied
- all 8 structure functions have been measured

- Sample data for Sivers asymmetry $\sim F_{UT,T}^{\sin(\phi_h - \phi_S)}$

data from COMPASS and HERMES



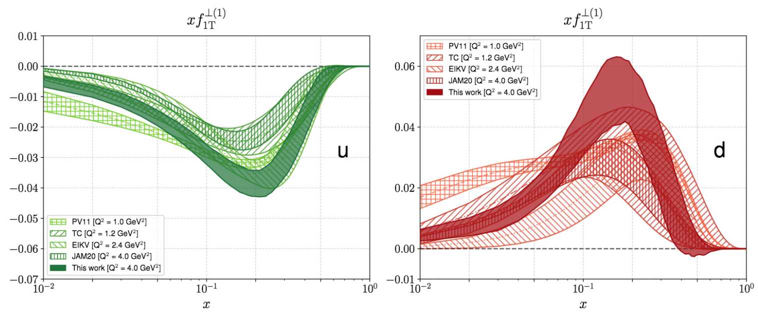
(compilation: Grosse Perdekamp, Yuan, Ann. Rev. Nucl. Part. Sci. 2015)

  - measurements for π^+ and K^+
  - robust nonzero effects

Results for Sivers Function

(Bacchetta et al, PLB 2020)

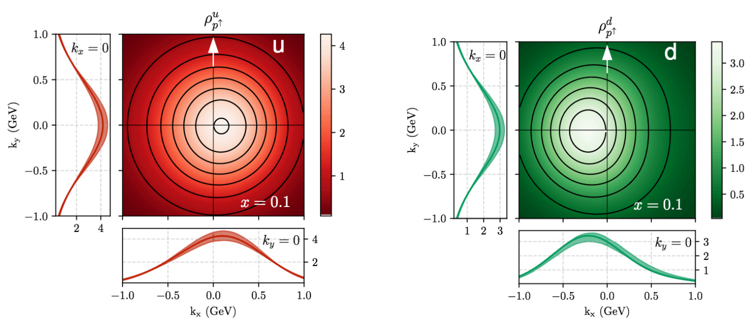
- (Moment of) Sivers function for proton (at scale $\mu = 2\,\mathrm{GeV}$)



$$f_{1T}^{\perp(1)}(x) = \int d^2\vec{k}_\perp \, \frac{k_\perp^2}{2M^2} \, f_{1T}^{\perp}(x, k_\perp)$$

reversed sign for up and down quarks

- Density $\rho_{\vec{S}}(x, \vec{k}_\perp)$ of unpolarized quarks in transversely polarized proton



$$\rho_{\vec{S}}(x, \vec{k}_\perp) = f_1(x, k_\perp) - \frac{(\vec{k}_\perp \times \vec{S}_\perp)_z}{M} \, f_{1T}^{\perp}(x, k_\perp)$$

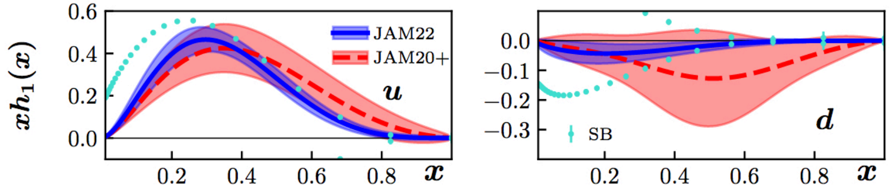
nonzero $f_{1T}^{\perp}$ "distorts" $\rho_{\vec{S}}(x, \vec{k}_\perp)$

- Results from other groups available as well

Results for Transversity Distribution

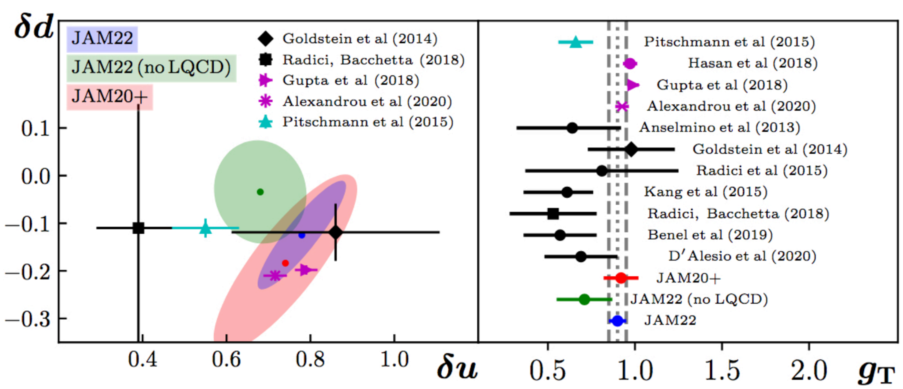
(JAM Collaboration, PRD 2022)

- Transversity ($\mu = 2\,\mathrm{GeV}$) from global analysis of single transverse-spin asymmetries, exploiting the Collins effect in SIDIS, and data from e^+e^- and pp ($\rightarrow$ state of the art)



- Tensor charge

$$\delta q = \int_0^1 dx \left(h_1^q(x) - h_1^{\bar{q}}(x) \right) \qquad\qquad g_T = \delta u - \delta d$$



  - JAM22 fit also uses g_T from lattice QCD

- PS: Generally, lattice QCD can play important role in TMD physics
  ($\rightarrow$ Zhao, QCD-PDF, Fr, 10:55)

1D+2D Imaging: Overview of GPDs

- GPDs appear in hard exclusive reactions

- GPDs depend on 3 variables: $\mathrm{GPD}(x, \xi, t)$

- Quark GPDs for spin-$\frac{1}{2}$ hadron

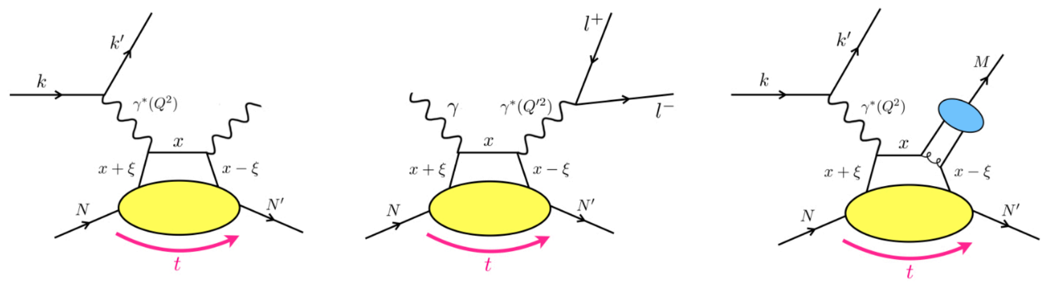
N / q	U	L	T
U	H		E_T
L		$\tilde{H}$	$\tilde{E}_T$
T	E	$\tilde{E}$	$H_T \quad \tilde{H}_T$

  - relation to PDFs: $H^q(x, 0, 0) = f_1^q(x)$ (similarly for $\tilde{H},\ H_T$)
  - relation to form factors: $\int_{-1}^{1} dx\, H^q(x, \xi, t) = F_1^q(t)$ etc.
  - GPDs for transverse quark polarization hard to measure

- Similarly, 8 gluon GPDs

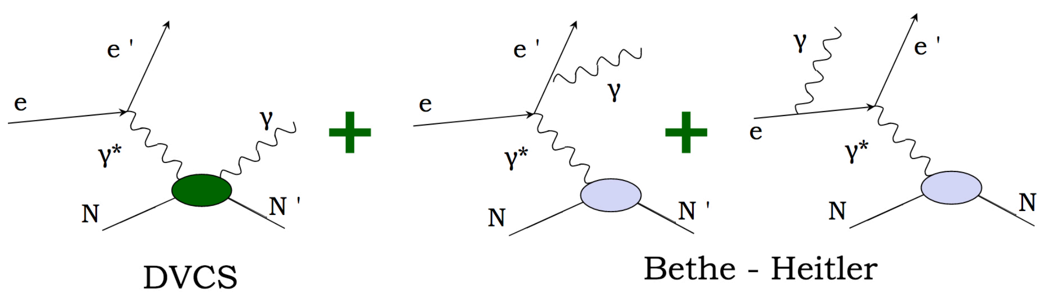
Key Processes for Measuring GPDs

- Deep virtual Compton scattering (DVCS), Time-like Compton scattering (TCS), Hard exclusive meson production (HEMP) (→ Boer, PDF, Th, 13:00)



  - data available for all those processes

- More details on important DVCS process



(Sokhan, HUGS Summer School, 2018)

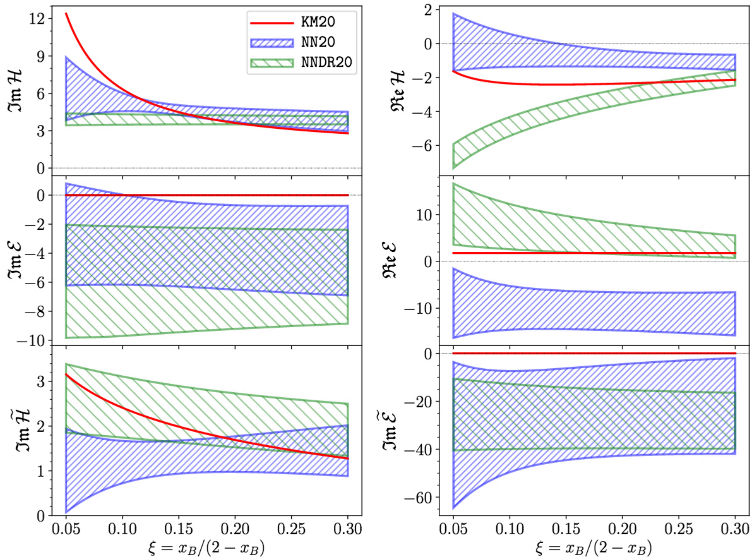
  - interference between DVCS and Bethe-Heitler amplitude plays key role
  - $\sigma_{\mathrm{int}} \sim$ electromagnetic form factor $\times$ Compton form factor

Compton Form Factors (CFFs)

- Example of CFF

$$\mathcal{H}(\xi,t) = \sum_q e_q^2 \int_{-1}^{1} dx\, H^q(x,\xi,t) \left(\frac{1}{\xi - x - i\varepsilon} - \frac{1}{\xi + x - i\varepsilon} \right) + \mathcal{O}(\alpha_s)$$

- Extracted CFFs ($t = -0.2\,\mathrm{GeV}^2$) (Čuić, Kumerički, Schäfer, PRL 2020)



- results also from other groups
- extraction of CFFs difficult
- how to get GPDs from CFFs, that is, how to solve DVCS inverse problem ? (information on x lost in observable CFFs)
  (see, for instance, Bertone et al, PRD 2021)
- existing GPD parameterizations have (significant) model dependence
  ($\rightarrow$ Dutrieux, PDF, We, 13:00)
- in principle, data from other processes could help

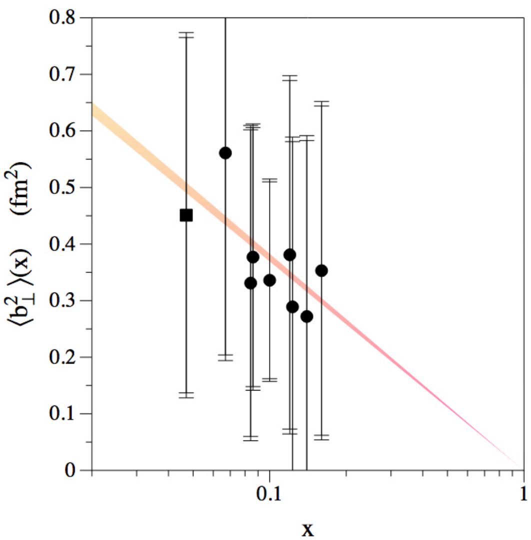
GPDs and Imaging

- Impact parameter distributions ($\xi = 0$): density interpretation (Burkardt, PRD 2000 / ...)

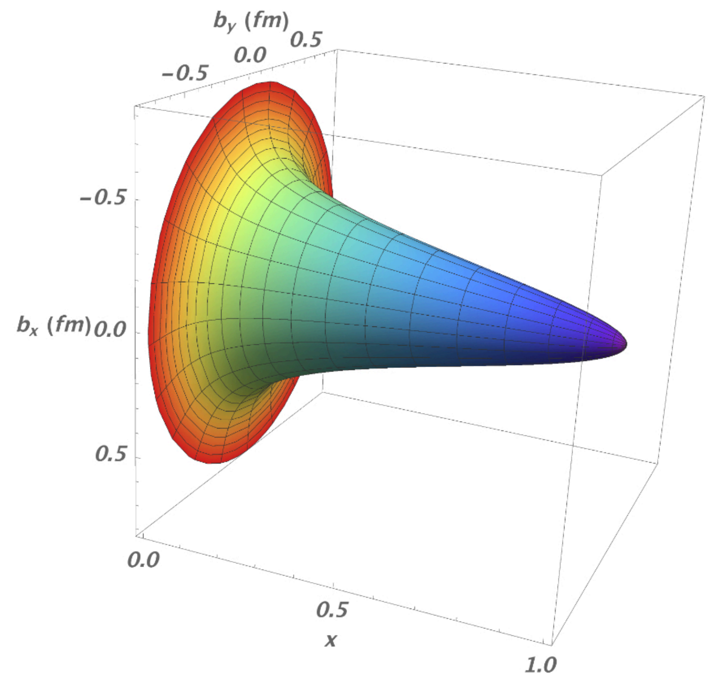
$$f(x, b_\perp) \sim \int d^2\vec{\Delta}_\perp \, e^{-i\,\vec{\Delta}_\perp \cdot \vec{b}_\perp} \, \mathrm{GPD}(x, \xi = 0, \Delta_\perp) \qquad t \overset{\xi=0}{=} -\vec{\Delta}_\perp^{\,2}$$

- Sample results (Dupré, Guidal, Vanderhaeghen, PRD 2017)

$$\langle b_\perp^2 \rangle(x) = \frac{\int d^2\vec{b}_\perp \, \vec{b}_\perp^{\,2} \, H(x, b_\perp)}{\int d^2\vec{b}_\perp \, H(x, b_\perp)}$$



impact parameter distribution



– (model-dependent) extraction of $H(x, b_\perp)$, using data from JLab and HERMES

– important new experimental results from COMPASS (COMPASS Collaboration, PLB 2019)

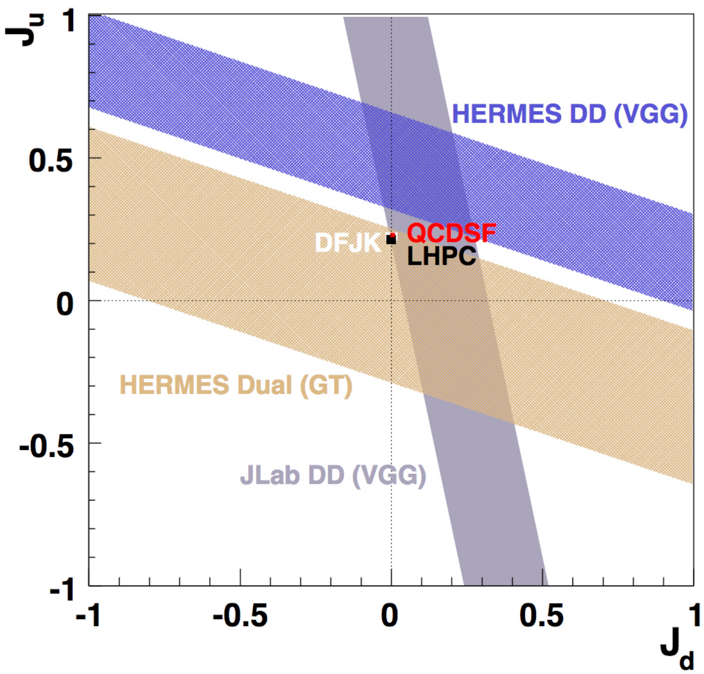
GPDs and Spin Sum Rule

- Ji spin sum rule (Ji, PRL 1996)

$$\tfrac{1}{2} = J^q_{\text{Ji}} + J^g_{\text{Ji}} = \tfrac{1}{2}\,\Delta\Sigma + L^q_{\text{Ji}} + J^g_{\text{Ji}}$$

$$J^q_{\text{Ji}} = \int_{-1}^{1} dx\, x\, \left(H^q + E^q\right)\Big|_{t=0} \qquad \text{similarly for } J^g_{\text{Ji}}$$

- (Model-dependent) results, based on experimental data (HERMES Collaboration, JHEP 2008)



– extraction from HERMES data, using two different models (HERMES DD, HERMES Dual)

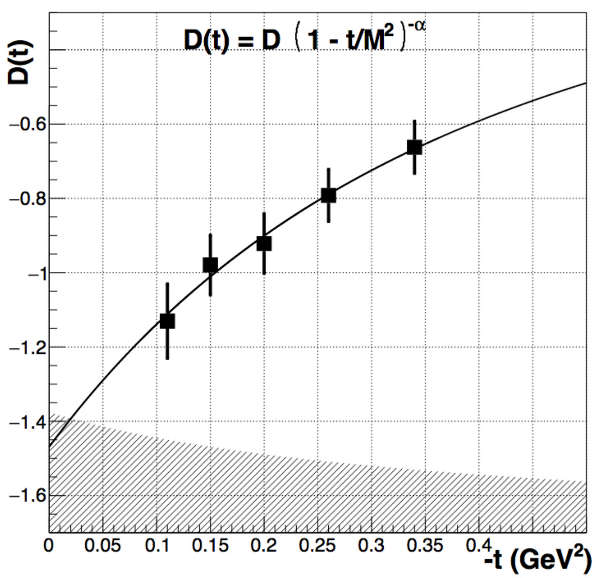
– errors large

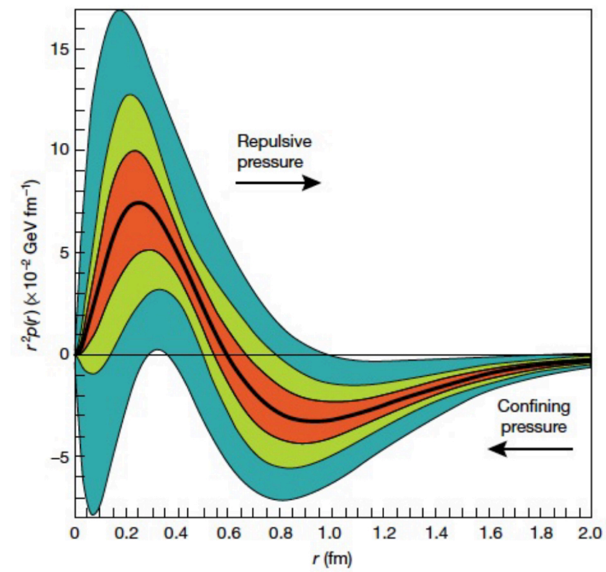
– LQCD calculations (QCDSF, LHPC) have small errors in comparison

GPDs and Distributions of Pressure and Shear

- Gravitational form factor $D(t)$ contains information about pressure and shear distributions of partons (Polyakov, Shuvaev, PLB 2002)
  - $D(t)$ appears in real part of CFF $\mathcal{H}(\xi, t)$

- Results based on data from JLab (Burkert, Elouadrhiri, Girod, Nature 2018, 2021)





  - results for pressure distribution for 3 scenarios:
    (i) prior to JLab data, (ii) with JLab data, (iii) with future JLab data
  - results for distribution of shear forces available as well
  - exciting recent development

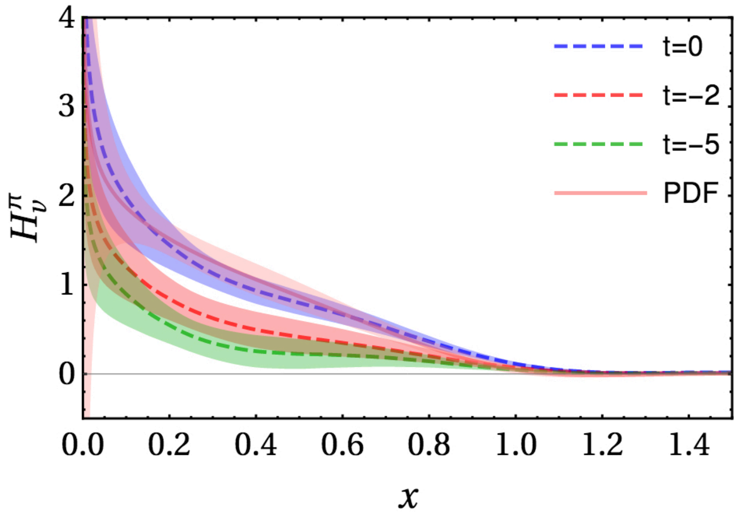
GPDs: Pioneering Results from LQCD

- Access to x-dependence of parton distributions in lattice QCD via Euclidean parton correlators (Ji, PRL 2013 / ...)
  ($\rightarrow$ Cichy, PDF, We, 14:00 / Monahan, PDF, Sa, 16:45)

- Some first results for GPDs

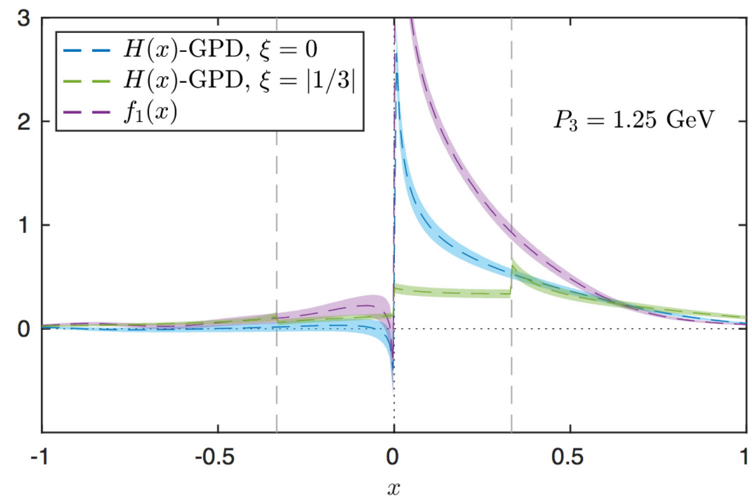
pion GPD $(\xi = 0)$

(Chen, Lin, Zhang, NPB 2019)



GPD H for proton $(t = -0.69\,\mathrm{GeV}^2)$

(ETM Collaboration, PRL 2020)



  - pioneering calculations quite encouraging
  - more results have been published, and further results to appear (Bhattacharya et al, 2022)
    ($\rightarrow$ Cichy, PDF, We, 14:00)

Wigner Functions

- Wigner functions in non-relativistic QM (Wigner, 1932)
  - calculable from wave function (1D)

$$\mathcal{W}(x,k) = \int \frac{dx'}{2\pi}\, e^{-\,i\,k\,x'} \psi^*\left(x - \frac{x'}{2}\right)\psi\left(x + \frac{x'}{2}\right)$$
$$= \int \frac{dk'}{2\pi}\, e^{+\,i\,k'\,x}\, \tilde{\psi}^*\left(k - \frac{k'}{2}\right)\tilde{\psi}\left(k + \frac{k'}{2}\right)$$

  - relation to densities and observables

$$|\psi(x)|^2 = \int dk\, \mathcal{W}(x,k) \qquad |\tilde{\psi}(k)|^2 = \int dx\, \mathcal{W}(x,k)$$
$$\langle O(x,k)\rangle = \int dx\, dk\, O(x,k)\, \mathcal{W}(x,k)$$

- Partonic Wigner functions $\mathcal{W}(x, \vec{k}_\perp, \vec{b}_\perp)$ (Belitsky, Ji, Yuan, PRL, PRD, 2003 / Lorcé, Pasquini, 2011)
  - significant progress in identifying observables for partonic Wigner functions
  - partonic Wigner functions open up unique opportunities (e.g., spin-orbit correlations)

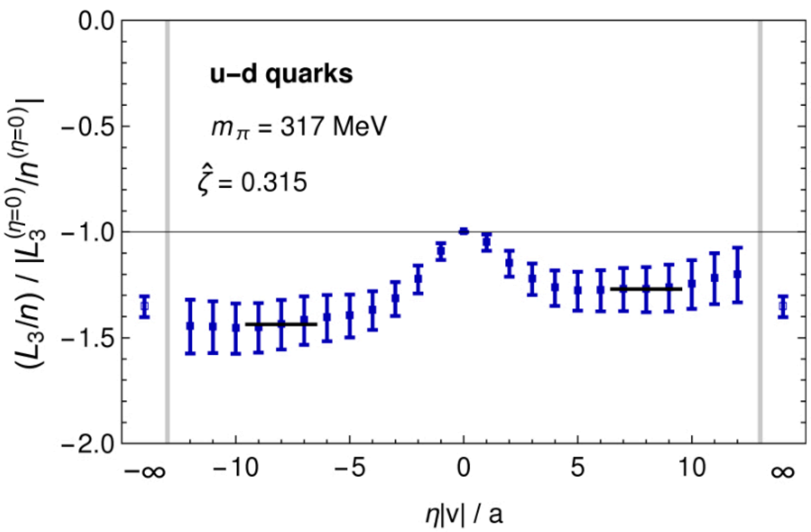
- Orbital Angular Momentum of Partons
  - Jaffe-Manohar spin sum rule (Jaffe, Manohar, NPB, 1989)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{\rm JM} + L^g_{\rm JM}$$

  - parton OAM in longitudinally polarized nucleon (Lorcé, Pasquini, PRD, 2011 / Hatta, PLB, 2011)

$$L^q_z = \int dx\, d^2\vec{k}_\perp\, d^2\vec{b}_\perp\, (\vec{b}_\perp \times \vec{k}_\perp)_z\, \mathcal{W}^{q\,[\rm unp]}_L(x, \vec{k}_\perp, \vec{b}_\perp)$$

  - exploratory calculation of $L^{u-d}_{\rm JM}$ in lattice QCD (Engelhardt et al, PRD, 2017, 2019)



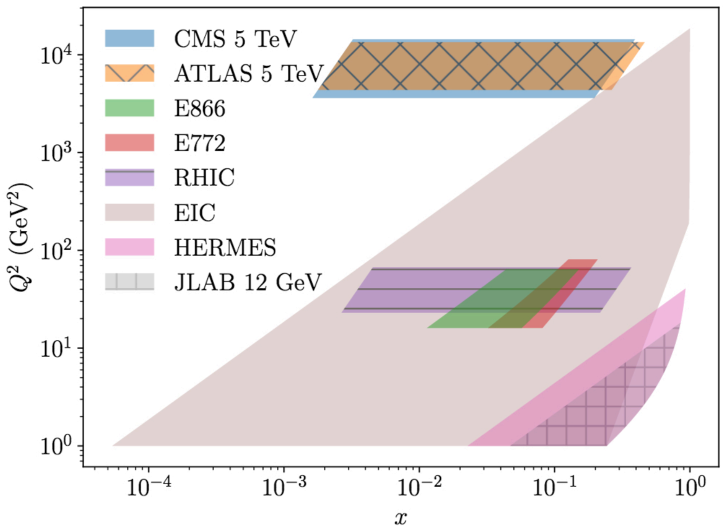
    * figure shows $L^{u-d}_{\rm JM}/|L^{u-d}_{\rm Ji}|$ (for large $\eta|v|/a$)
    * considerable numerical difference btw $L^{u-d}_{\rm JM}$ and $L^{u-d}_{\rm Ji}$

  - developments in this area are milestone in spin physics

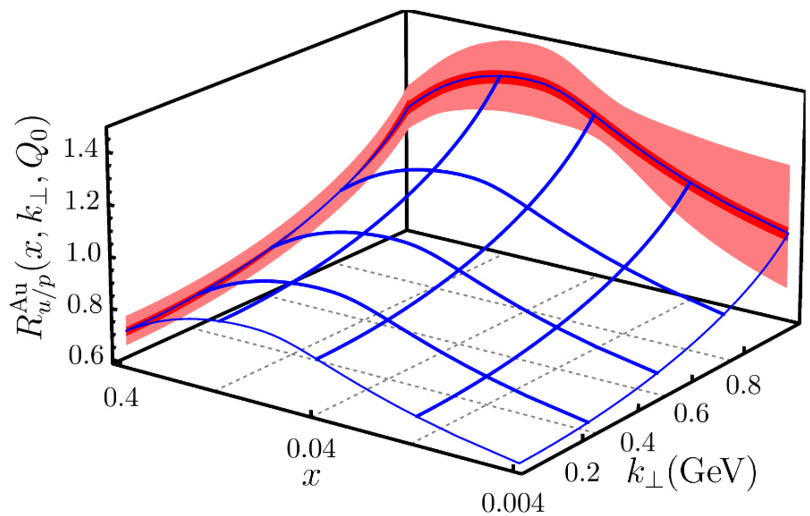
First Global Analysis of TMDs in Nuclei

(Alrashed et al, 2021)

kinematic coverage



result for up quark TMD $f_1^{u/p}$

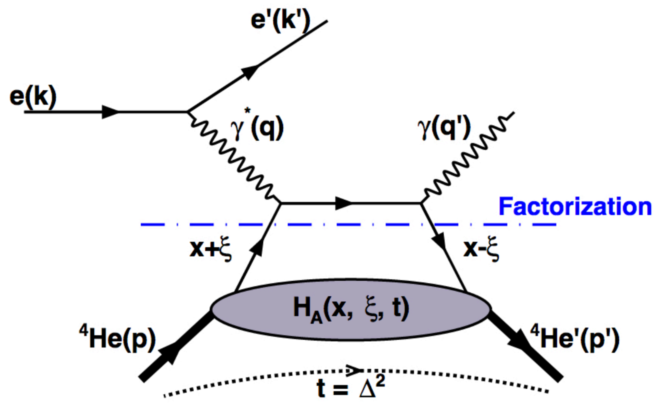


- Nuclear data for semi-inclusive DIS and Drell-Yan (fixed target and collider)
- Ratio of TMD PDFs for bound proton in nucleus over free proton
- Work also includes study of nuclear TMD FFs

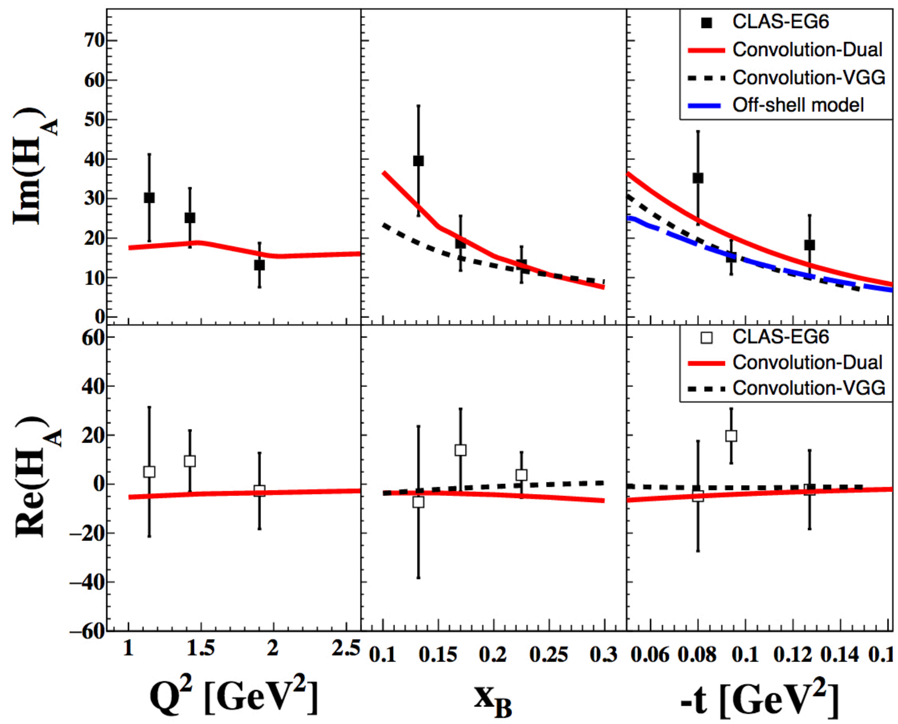
First Exclusive Measurement of DVCS off ^{4}He

(Hattawy et al [CLAS Collaboration], PRL 2017, 2019 / PRC 2021)

handbag diagram



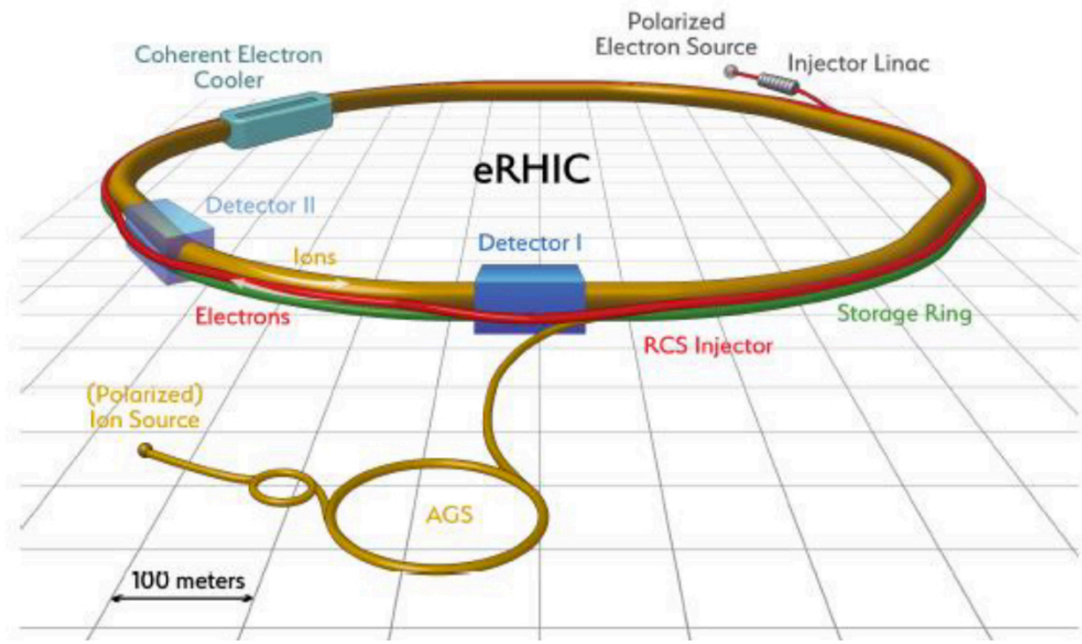
result for Compton form factor



- ^{4}He (spin-0) has one Compton form factor only
- Measurement opens the path for 1D+2D imaging of nuclei
- Coherent (and incoherent) scattering provides new insights into nuclear structure

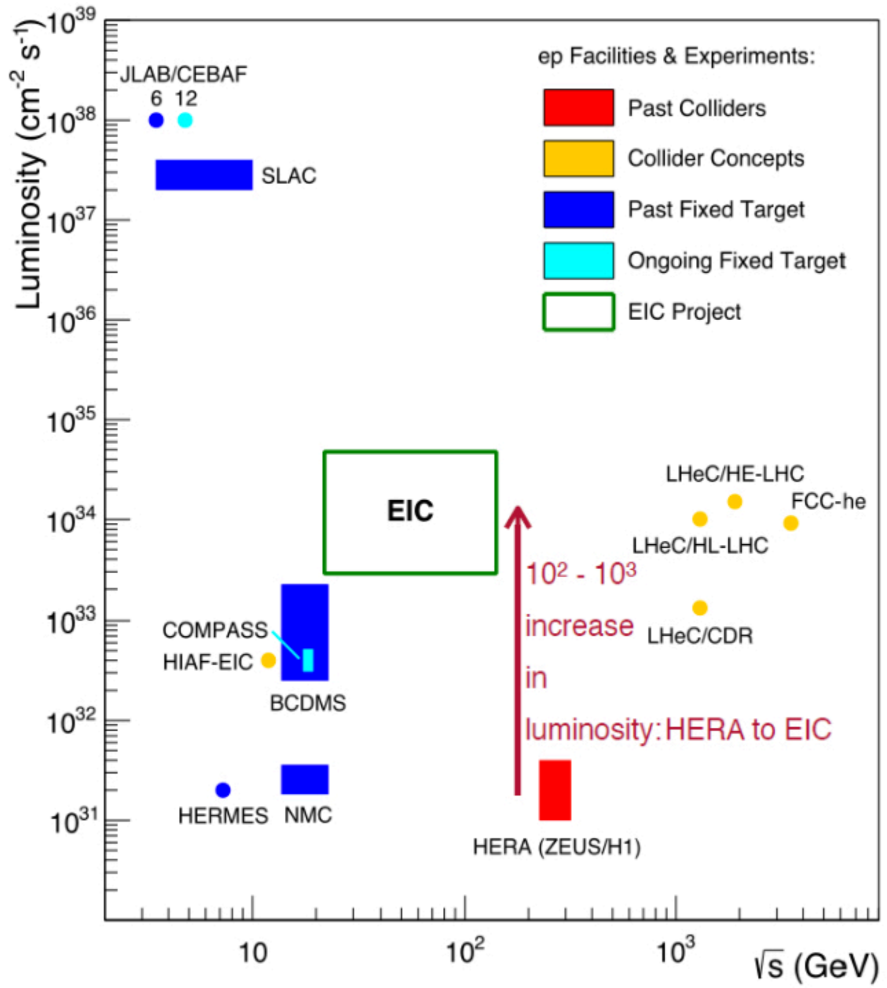
EIC Facility: Overview

- New polarized $e-p/A$ collider at Brookhaven National Lab (largely funded by DOE, operational ~ 2030) (→ Arratia, HE, Tu, 13:00)
- Layout and main parameters



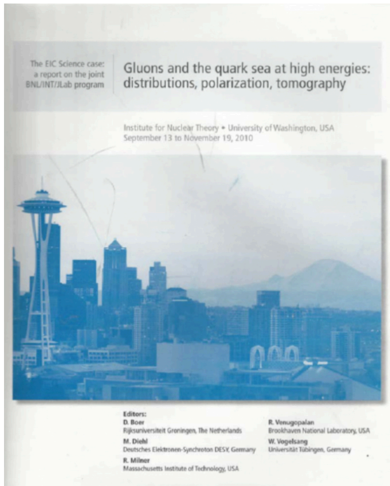
  - one IR/detector part of project; community pushes for second IR/detector
  - (polarized) electrons, protons, light ions; unpolarized heavy ions
  - center-of-mass energy: $\sim 29 - 140\,\mathrm{GeV}$
  - luminosity: $\sim 10^{33} - 10^{34}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$

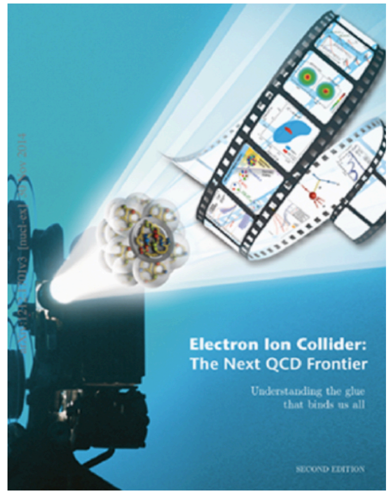
- Comparison with other facilities

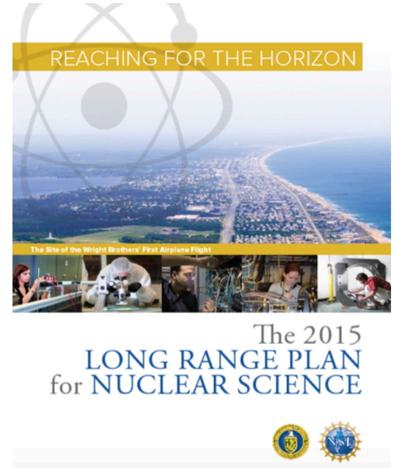


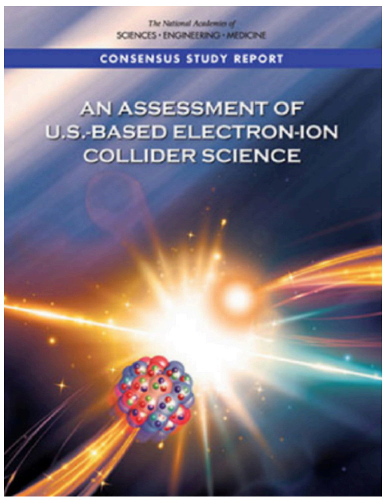
  - advantage over fixed-target machines: cm-energy
  - advantages over HERA ep collider: luminosity, polarization, nuclei

- Some key initiatives and documents





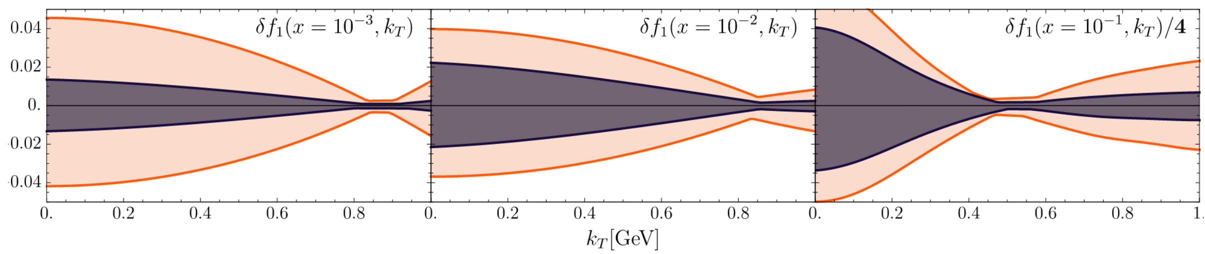




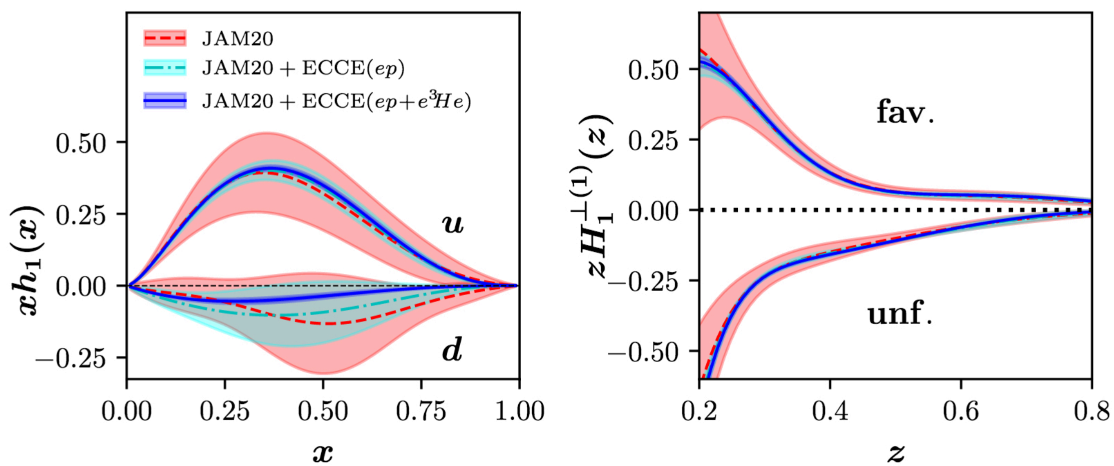
2018 NAS Report: "The committee finds that the science that can be addressed by an EIC is compelling, fundamental and timely."

Selected EIC Impact Studies: TMDs

- Unpolarized TMD f_1 with ECCE detector ($\mathcal{L} = 10\,\text{fb}^{-1}$) (Seidl, Vladimirov et al, 2022)



- Transversity h_1 and Collins function $H_1^{\perp}$ with ECCE detector (Seidl, Pitonyak et al, 2022)

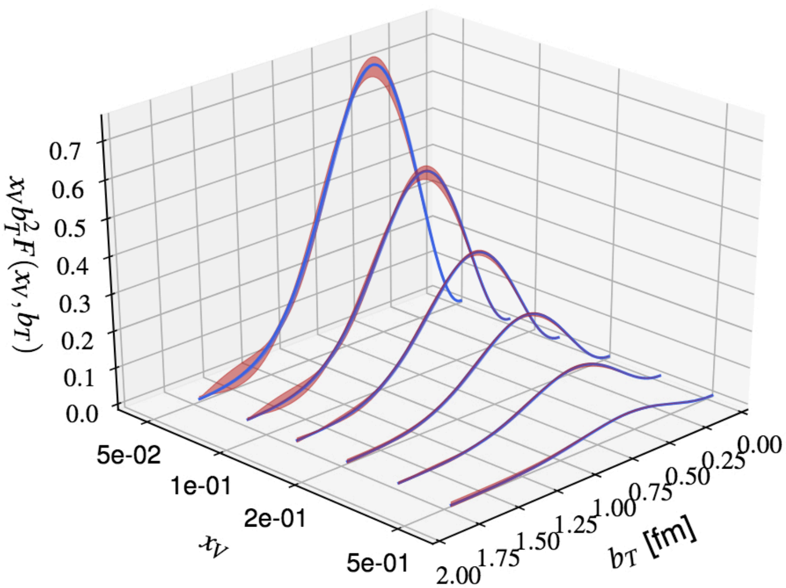


- Similar promising results (expected) for other TMDs

Selected EIC Impact Study: GPDs

(→ Sokhan, PDF, Tu, 14:25)

- Quarkonium production: $\gamma\, p \to \Upsilon\, p$ (theoretically clean) (Joosten, Meziani, PoS, 2018)



$$x_V = \frac{Q^2 + M_V^2}{2P \cdot q}$$

$$M_\Upsilon^2 \approx 89.5\,\mathrm{GeV}^2$$

  - Fourier transform of t distribution of cross section
  - study for $89.5\,\mathrm{GeV}^2 \le Q^2 + M_\Upsilon^2 \le 91\,\mathrm{GeV}^2$ and $\mathcal{L} = 100\,\mathrm{fb}^{-1}$
  - small errors for transverse profiles → strong constraints on 3D gluon distributions

- General advantages for 3D imaging at EIC
  1. (significant) reduction of uncertainties
  2. suppression of contaminations due to power corrections
  3. extension of kinematic range (access to sea quarks and gluons; saturation regime)

Conclusions

- Nucleon structure studies have long and successful history

- Presently, multi-dimensional imaging of hadronic systems (nucleons, nuclei, pions, ...) is very dynamic field (tremendous progress)

- Quantities characterizing the multi-dimensional imaging
  - TMDs: 3D distributions in momentum space
  - GPDs: $\rightarrow$ 1D+2D distributions in mixed momentum and position space
  - Wigner functions: beyond 3D

- Combining experimental data and LQCD results will be crucial to further the field

- Data from the EIC are expected to move field to the next level