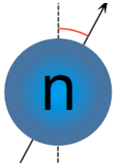


Neutron Measurements to Probe the Hadronic Weak Interaction

Murad Sarsour
for the NSR Collaboration

Georgia State University

CIPANP 2022
14th Conference on the Intersections of Particle and Nuclear Physics

U.S. DEPARTMENT OF ENERGY | Office of Science

NN WEAK INTERACTION

- Hadronic Weak Interaction (HWI) between nucleons is not well constrained. Low energy, non-perturbative regime makes calculations and experiments difficult.
  - Short range (~0.01 fm << size of nucleon at ~1 fm)
    - First order sensitive to short range quark-quark correlations in hadrons ⇒ gives insight into QCD in the non-perturbative strongly interacting limit.
  - Dominated by the strong of interaction
    - Use parity violation (PV) to isolate Weak contribution ⇒ PV effects would probe the degree of Weak-Strong interference at short distance scales without exciting the nucleons

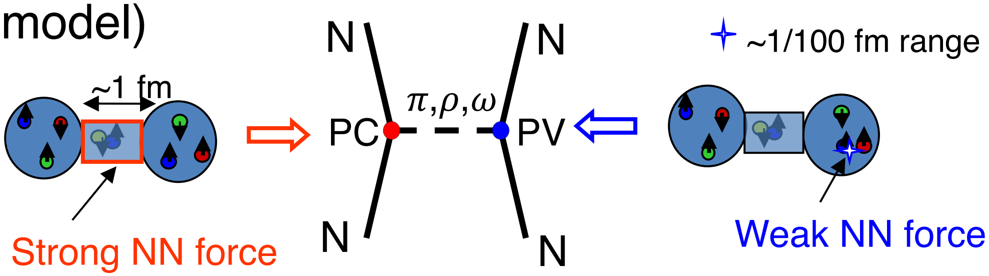
- Effective Theories
  - Meson exchange pictures, $\not{\pi}^{\text{EFT}}$, χ^{EFT}, Large-N_c

- Lattice QCD

NN WEAK INTERACTION

- “Traditional picture” (DDH model)

Meson exchange!



  - Dominated by low mass mesons, leads to 6 weak meson coupling constants with ΔI = 0,1,2: $h_\pi^1, h_\omega^{0,1}, h_\rho^{0,1,2}$

- “Modern picture”
  - Effective (EFT) operators with built-in QCD symmetries and weak & strong potentials
  - Pion-less EFT with 5 low energy contact interaction terms:

$$\Lambda_0^{^1S_0-^3P_0}, \Lambda_0^{^3S_1-^1P_1}, \Lambda_1^{^1S_0-^3P_0}, \Lambda_1^{^3S_1-^3P_1}, \Lambda_2^{^1S_0-^3P_0}$$

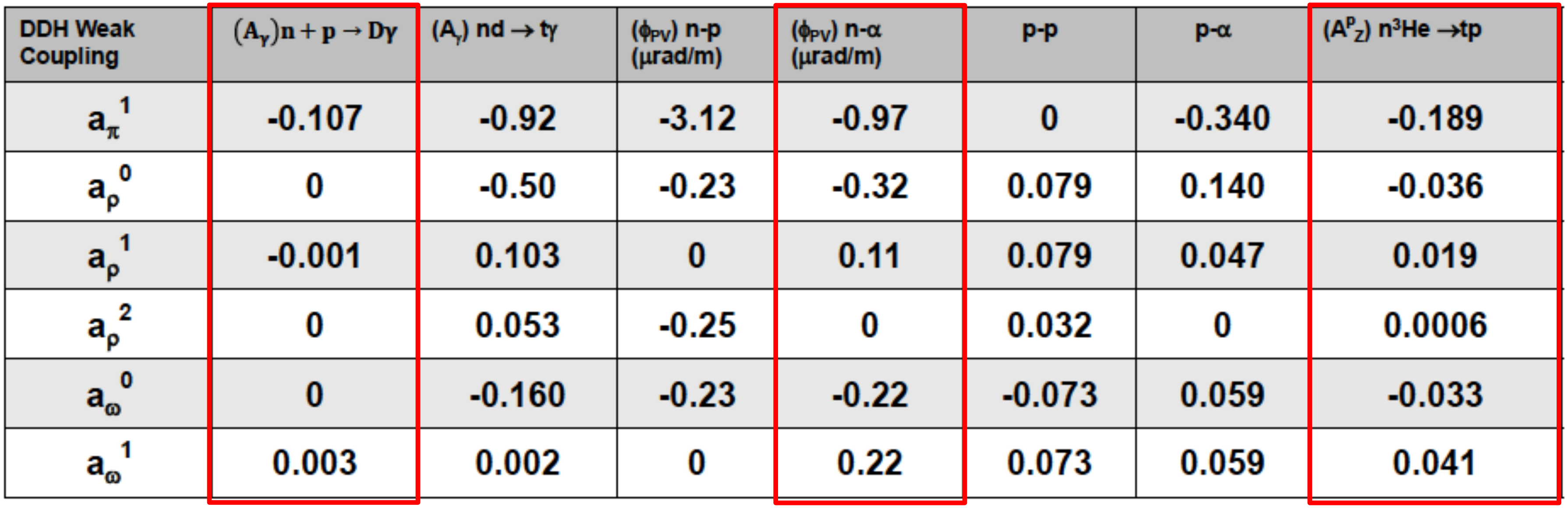
  - Chiral EFT, again with 5 constants + pion and two-pion exchange
  - Large-N_c framework, attempt to reduce to two dominant couplings

- Different experiments sensitive to different linear combinations

DDH MODEL: Observables

- Large nuclei: Large PV effects but hard to relate to the underlying observables
- Simple NN systems (np, pp, pD, n^{3}He, n^{4}He, pα): Small PV effects but predictions with minimum theoretical uncertainty
- Generally, observables are expressed in terms of theoretically calculated coefficients and coupling constants,

$$O_{PV} = a_\pi^1 h_\pi^1 + a_\omega^0 h_\omega^0 + a_\omega^1 h_\omega^1 + a_\rho^0 h_\rho^0 + a_\rho^1 h_\rho^1 + a_\rho^2 h_\rho^2$$



DDH Weak Coupling	(A_γ) n + p → Dγ	(A_γ) nd → tγ	(ϕ_{PV}) n-p (μrad/m)	(ϕ_{PV}) n-α (μrad/m)	p-p	p-α	(A^P_Z) n^{3}He → tp
a_π^1	-0.107	-0.92	-3.12	-0.97	0	-0.340	-0.189
a_ρ^0	0	-0.50	-0.23	-0.32	0.079	0.140	-0.036
a_ρ^1	-0.001	0.103	0	0.11	0.079	0.047	0.019
a_ρ^2	0	0.053	-0.25	0	0.032	0	0.0006
a_ω^0	0	-0.160	-0.23	-0.22	-0.073	0.059	-0.033
a_ω^1	0.003	0.002	0	0.22	0.073	0.059	0.041

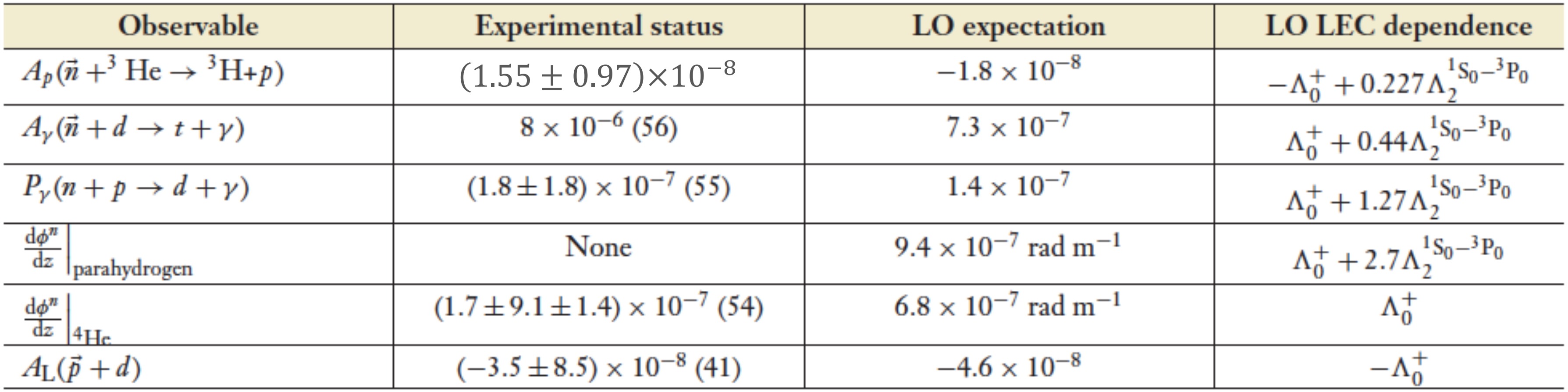
MORE RECENT THEORETICAL DEVELOPMENTS

- EFT approach leads 5 S-P amplitudes (involving five degrees of freedom), it's model independent and has quantifiable errors and a direct connection to QCD
- Combined EFT/large-N_c leads to lowest-order (LO) 2D characterization of PNC

Danilov parameter	N_c Trend	Exp't	LO+LQCD
$\Lambda_0^+ \equiv \frac{3}{4}\Lambda_0^{^3S_1-^1P_1} + \frac{1}{4}\Lambda_0^{^1S_0-^3P_0}$	$\sim \; N_c$	~300	301
$\Lambda_2^{^1S_0-^3P_0}$	$\sim \; N_c \sin^2\theta_w$	~160	-137
$\Lambda_0^- \equiv \frac{1}{4}\Lambda_0^{^3S_1-^1P_1} - \frac{3}{4}\Lambda_0^{^1S_0-^3P_0}$	$\sim \; 1/N_c$		-46
$\Lambda_1^{^1S_0-^3P_0}$	$\sim \; \sin^2\theta_w$		4.67
$\Lambda_1^{^3S_1-^3P_1}$	$\sim \; \sin^2\theta_w$	~740	859

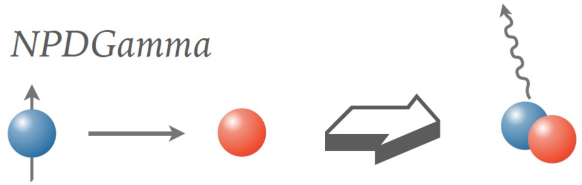
Girish Muralidhara (INT 2022), Phys. Lett. B **833**, 137372 (2022)

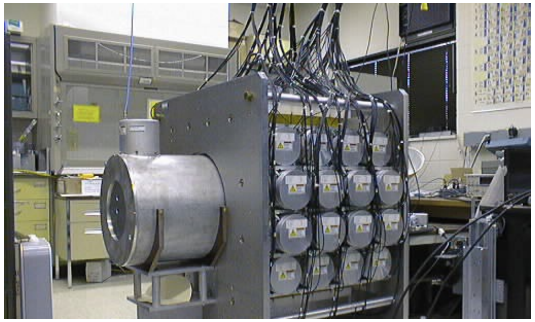
- S. Gardner, W.C. Haxton, B.R. Holstein, Annu. Rev. Nucl. Part. Sci. **67**, 69 (2017)



Observable	Experimental status	LO expectation	LO LEC dependence
$A_p(\vec{n} + {}^3\mathrm{He} \rightarrow {}^3\mathrm{H} + p)$	$(1.55 \pm 0.97) \times 10^{-8}$	-1.8×10^{-8}	$-\Lambda_0^+ + 0.227\Lambda_2^{^1S_0-^3P_0}$
$A_\gamma(\vec{n} + d \rightarrow t + \gamma)$	8×10^{-6} (56)	7.3×10^{-7}	$\Lambda_0^+ + 0.44\Lambda_2^{^1S_0-^3P_0}$
$P_\gamma(n + p \rightarrow d + \gamma)$	$(1.8 \pm 1.8) \times 10^{-7}$ (55)	1.4×10^{-7}	$\Lambda_0^+ + 1.27\Lambda_2^{^1S_0-^3P_0}$
$\left.\frac{d\phi^n}{dz}\right\rvert_{\mathrm{parahydrogen}}$	None	9.4×10^{-7} rad m^{-1}	$\Lambda_0^+ + 2.7\Lambda_2^{^1S_0-^3P_0}$
$\left.\frac{d\phi^n}{dz}\right\rvert_{^4\mathrm{He}}$	$(1.7 \pm 9.1 \pm 1.4) \times 10^{-7}$ (54)	6.8×10^{-7} rad m^{-1}	Λ_0^+
$A_L(\vec{p} + d)$	$(-3.5 \pm 8.5) \times 10^{-8}$ (41)	-4.6×10^{-8}	$-\Lambda_0^+$

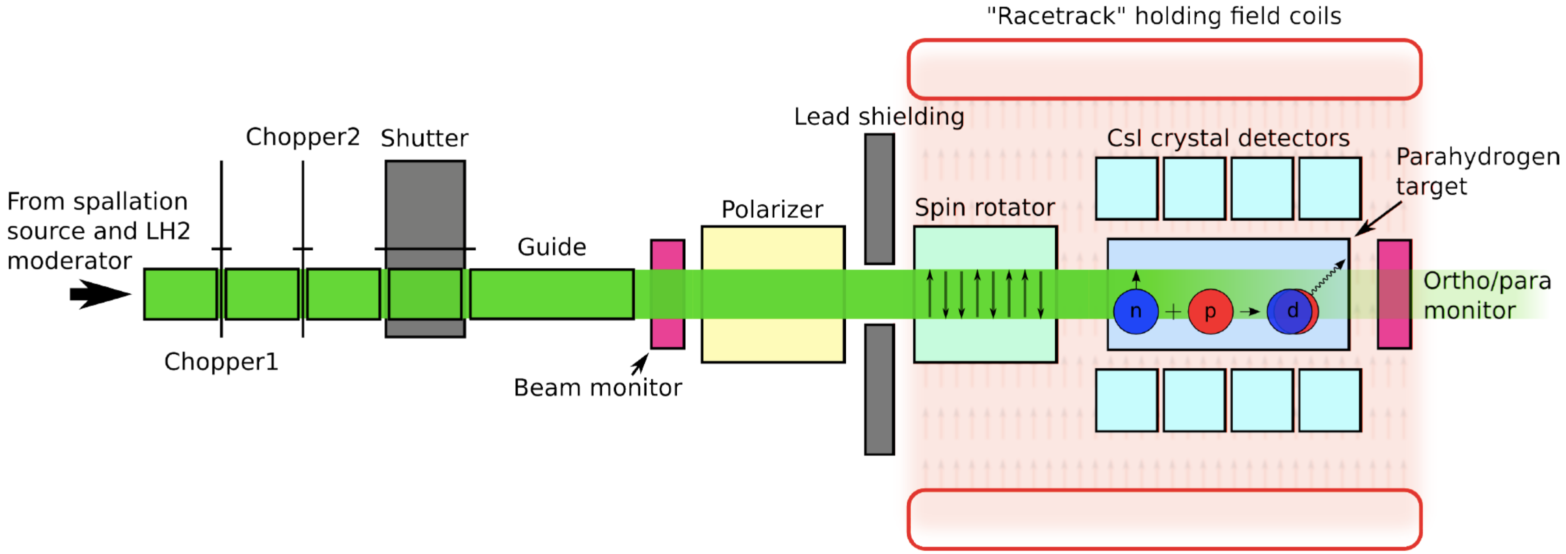
The NPDGamma EXPERIMENT





$$\boxed{\frac{d\sigma}{d\Omega} \propto \frac{1}{4\pi}\left(1 + A_\gamma \cos\theta\right)}$$

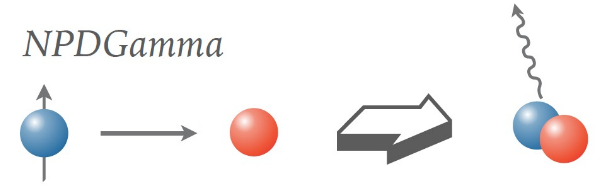
A_γ – P-odd asymmetry in the gammas emitted from polarized slow neutron capture on protons.



- No nuclear structure uncertainty in the interpretation of A_γ^{np}
- Small contribution from heavy mesons allowing determination of h_π^1

$$A_\gamma^{np} = -0.114\mathrm{h}_\pi^1 - 0.001\mathrm{h}_\rho^1 + 0.002\mathrm{h}_w^1$$

The NPDGamma EXPERIMENT



- LANSCE (LANL) : 2006 – 2007

Gericke *et al.*, PRC **83**, 015505 (2011)

$$A_{\gamma}^{np} = [-1.2 \pm 2.1(\text{stat}) \pm 0.2(\text{syst})]\times 10^{-7}$$

- SNS (ORNL) : 2008 - 2012

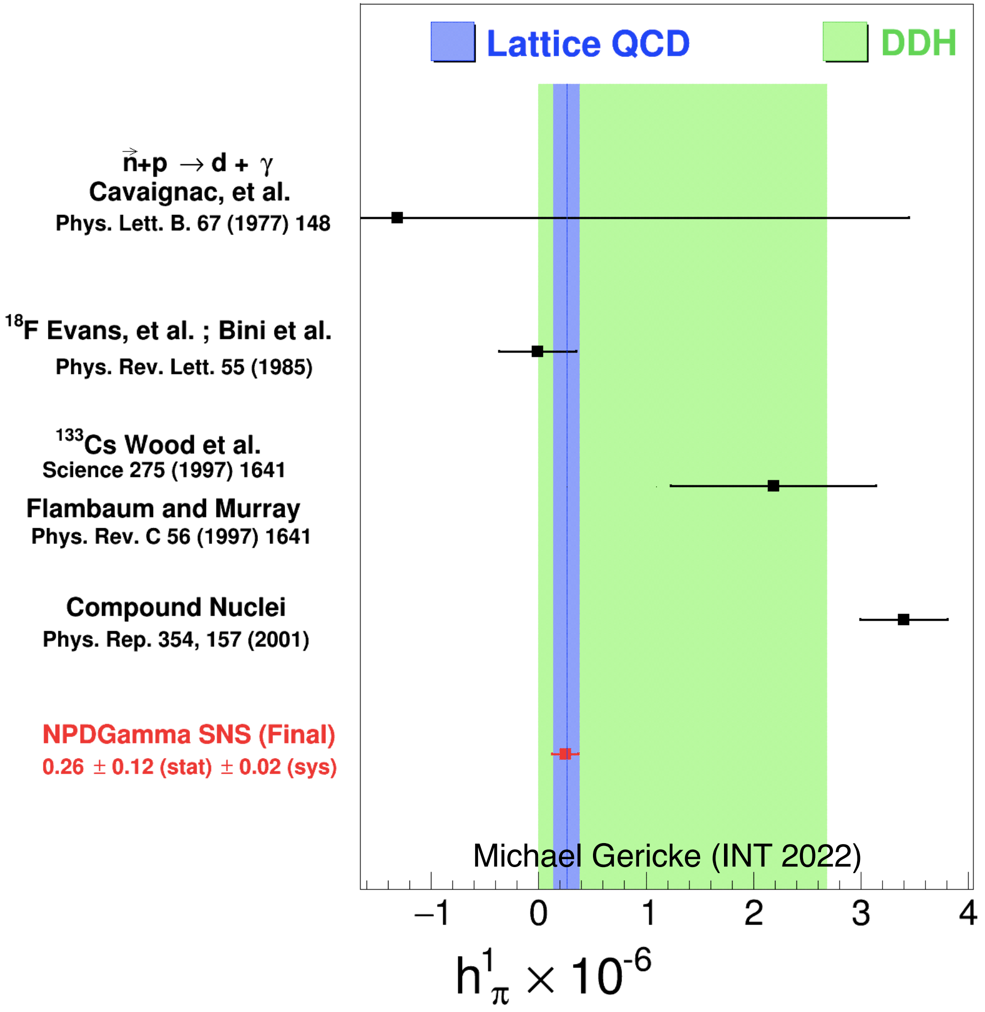
D. Blyth *et al.*, PRL **121**, 242002 (2018)

$$A_{\gamma}^{np} = [-3.0 \pm 1.4(\text{stat}) \pm 0.2\ (\text{syst})]\times 10^{-8}$$

- DDH Model

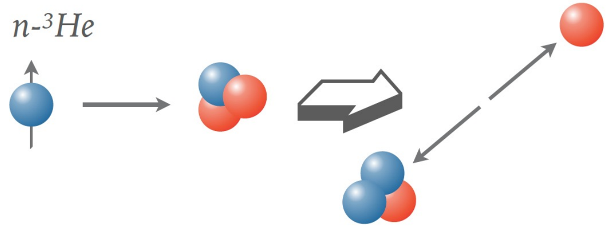
$$A_{\gamma}^{np} \approx -0.11 h_{\pi}^{1}$$

$$h_{\pi}^{1} = [2.6 \pm 1.2(\text{stat}) \pm 0.2\ (\text{syst})]\times 10^{-7}$$

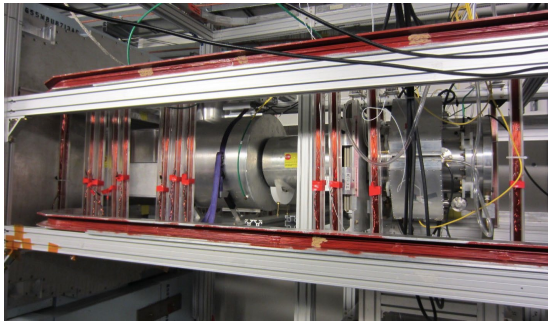


ΔS = 0 π-N weak coupling measurements

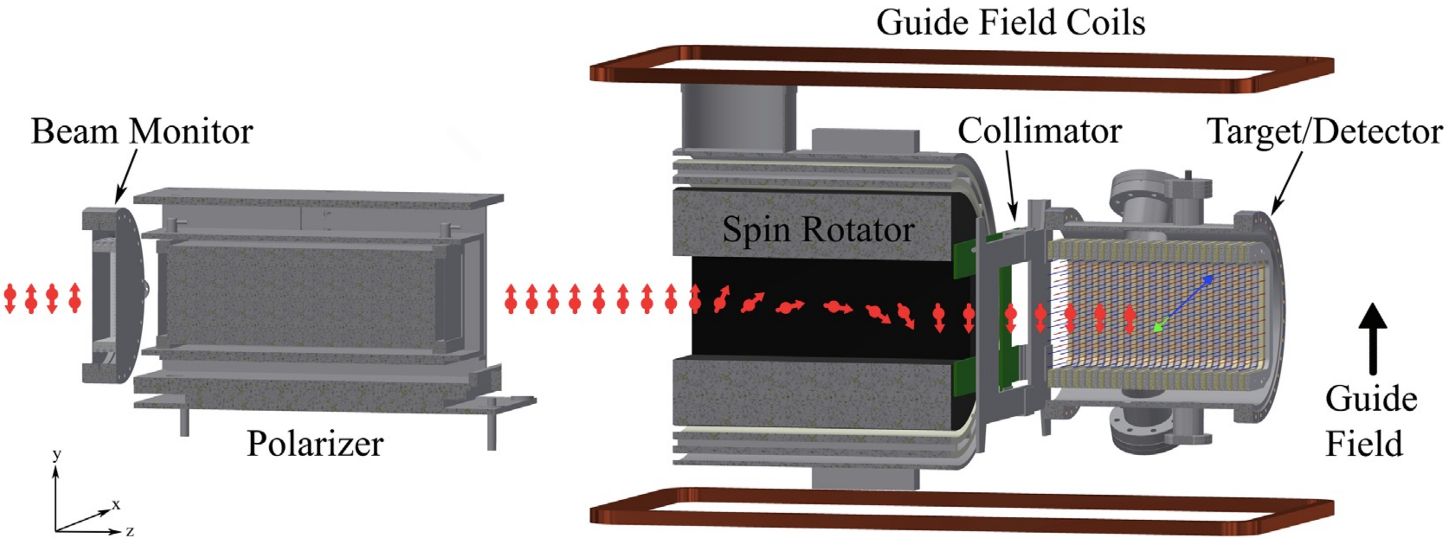
The n-^{3}He EXPERIMENT



$$A_{meas} = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow} = P\varepsilon\left(A_{PV}\cos(\theta_p) + A_{PC}\sin(\theta_p)\right)$$

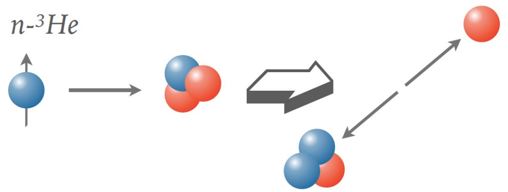


A_{pv} - P-odd up-down asymmetry in the angular distribution of protons with respect to the neutron spin direction.



$$A_{PV} = -0.185h^1_\pi - 0.038h^0_\rho - 0.023h^0_\omega + 0.023h^1_\rho + 0.05h^1_\omega - 0.001h^2_\rho$$

The n-^{3}He EXPERIMENT



- SNS (ORNL) : 2014 - 2015

$$A_{PV} = [1.55 \pm 0.97(\text{stat}) \pm 0.24(\text{sys})] \times 10^{-8}$$

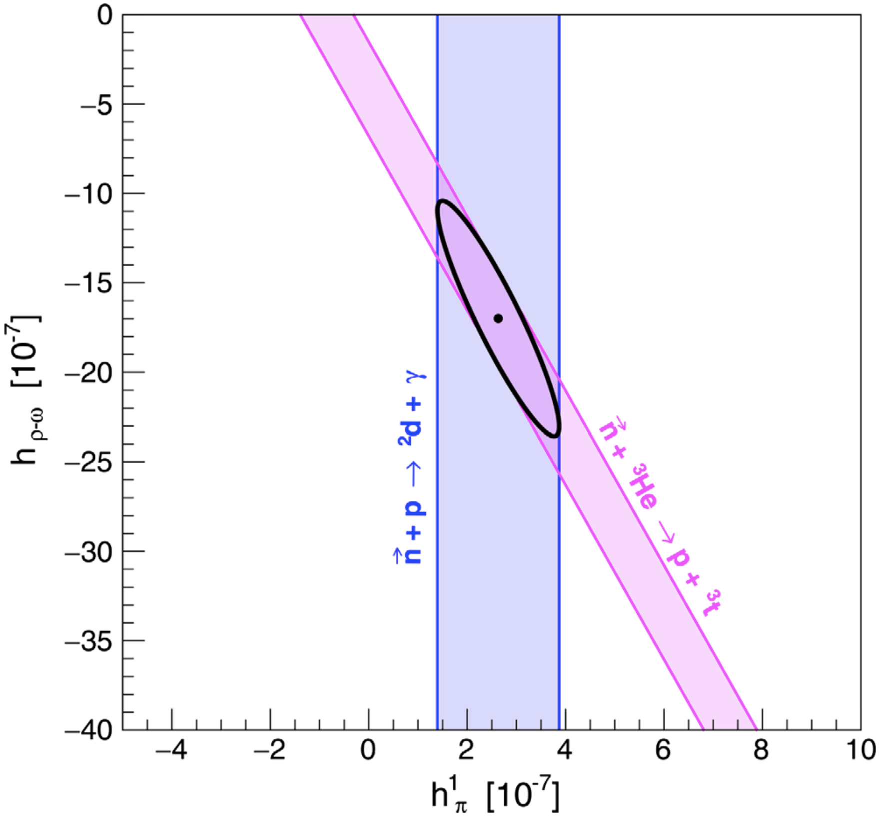
Blyth et al., Phys. Rev. Lett. **125**, 131803 (2020)

- DDH Model

$$A_{PV} = -0.185h_\pi^1 - 0.038h_\rho^0 - 0.023h_\omega^0 + 0.023h_\rho^1 + 0.050h_\omega^1 - 0.001h_\rho^2$$

- Pionless EFT + Large N_c

$$A_{PV} = -\Lambda_0^+ + 0.227\,\Lambda_2^{{}^1S_0-{}^3P_0}$$



○ Including pp, pα and ^{19}F, we have (ARNPS **67**, 69 (2017))

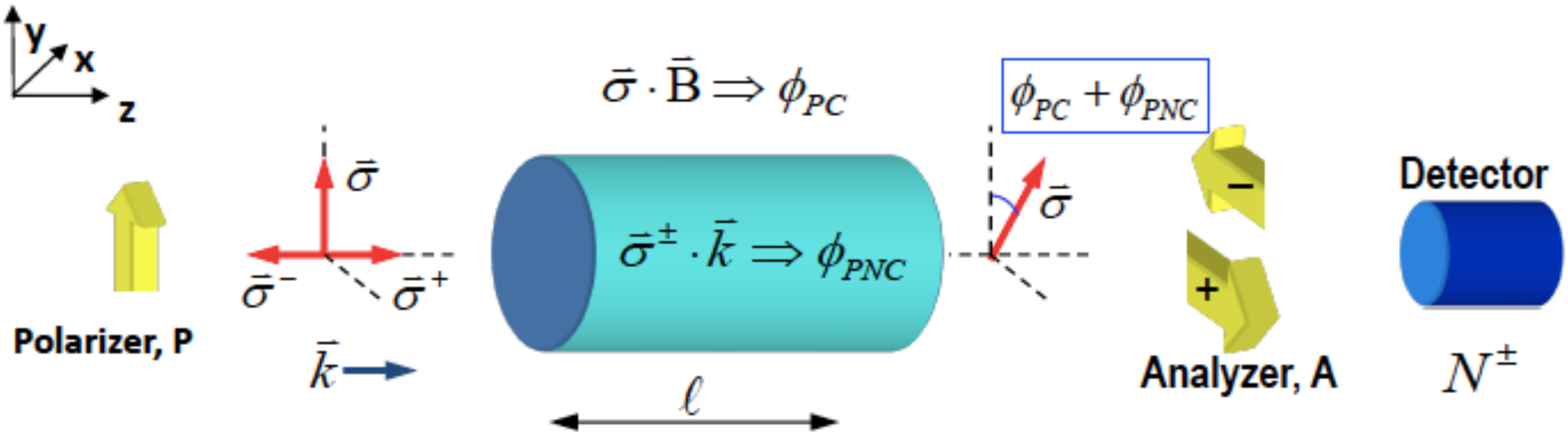
$$\Lambda_2 = 324 \quad \& \quad \boxed{\Lambda_0^+ = 717}$$

○ Including pp, n^{3}He, we have (Michael Gericke (INT 2022))

$$\Lambda_2 = 183 \quad \& \quad \boxed{\Lambda_0^+ = 264}$$

The n-^{4}He EXPERIMENT

Neutron Spin Rotation (NSR)



$$\boxed{sin(\phi_{PNC}) = \frac{1}{PA}\frac{N^+ - N^-}{N^+ + N^-}}$$

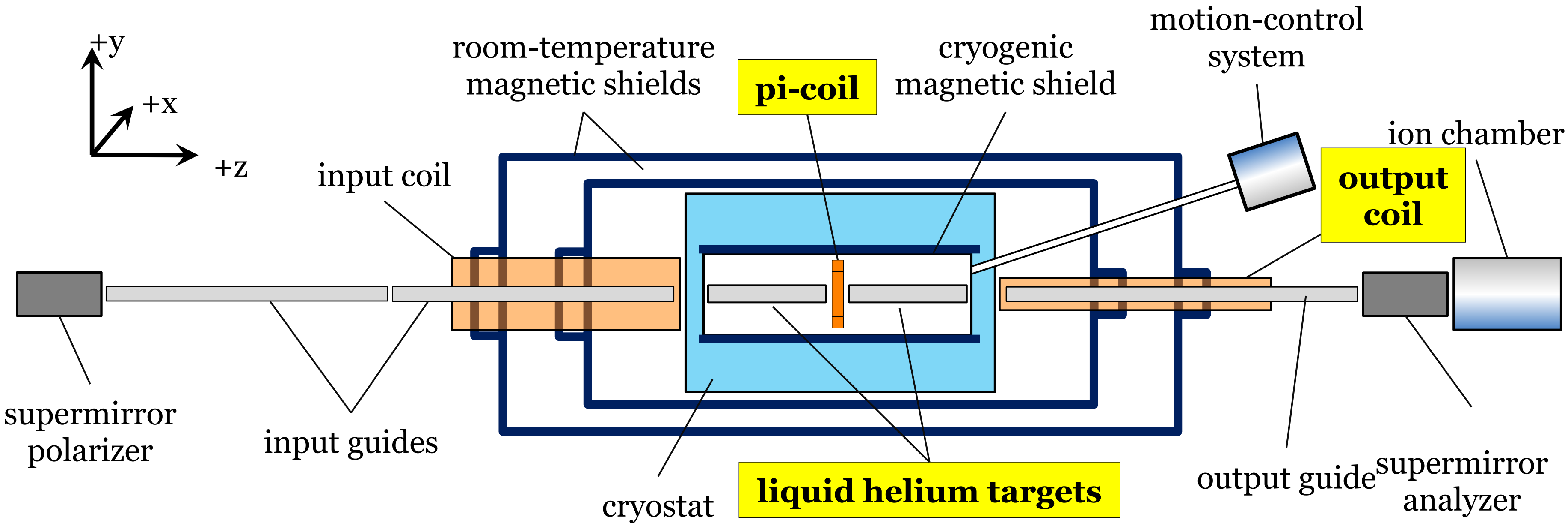
Expected Size:

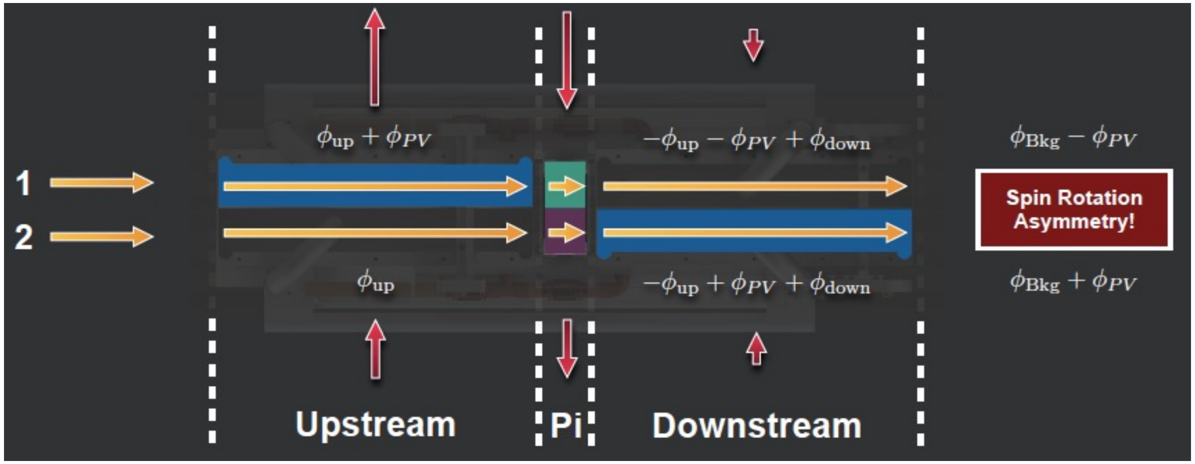
$$\frac{d\phi_{PNC}}{dz} \sim 10^{-7}(rad/m)$$

Experimental challenges

- Reducing $\vec{\sigma}_n \cdot \vec{B} \Rightarrow \phi_{PC}$
- Effectively canceling what is left
- Controlling noise
- Controlling other systematics

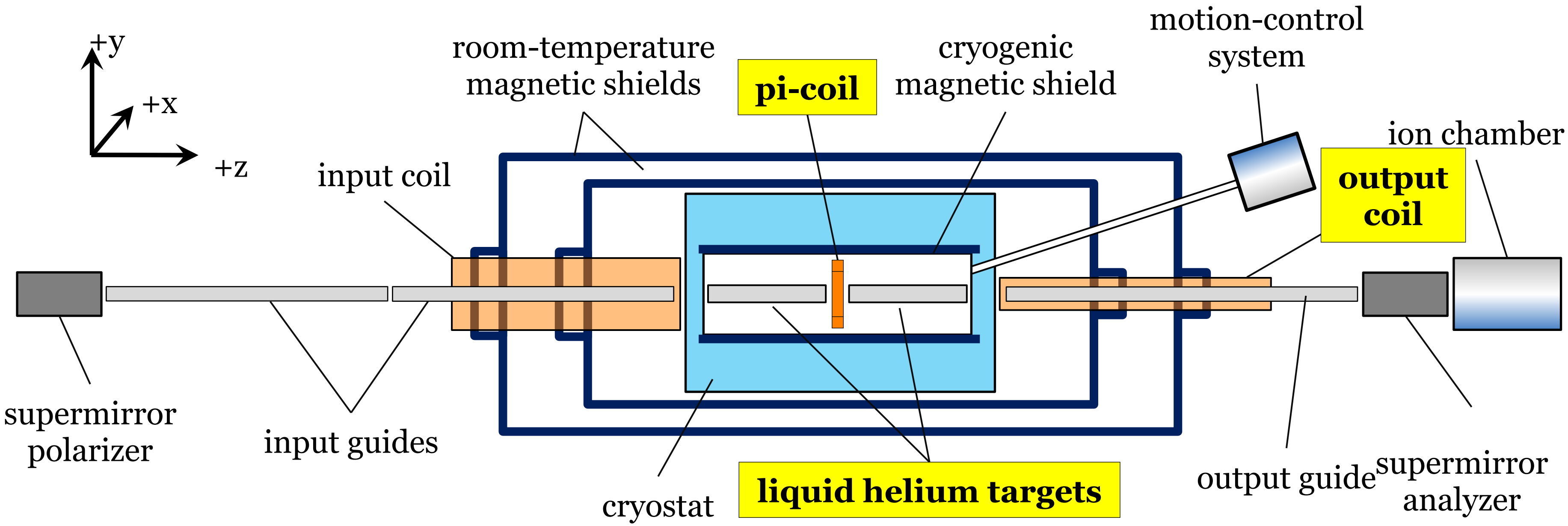
n-^{4}He: NSR II





$$\Delta\phi_{Bkg} = 2.9\times10^{-7}\ \mathrm{rad\ m^{-1}}$$

n-^{4}He: NSR II



$$\boxed{\sin(\phi_{PNC}) = \frac{1}{PA}\frac{N^+ - N^-}{N^+ + N^-}}$$

Measures the horizontal component of neutron spin for a vertically-polarized beam

NIST-NG6 2008

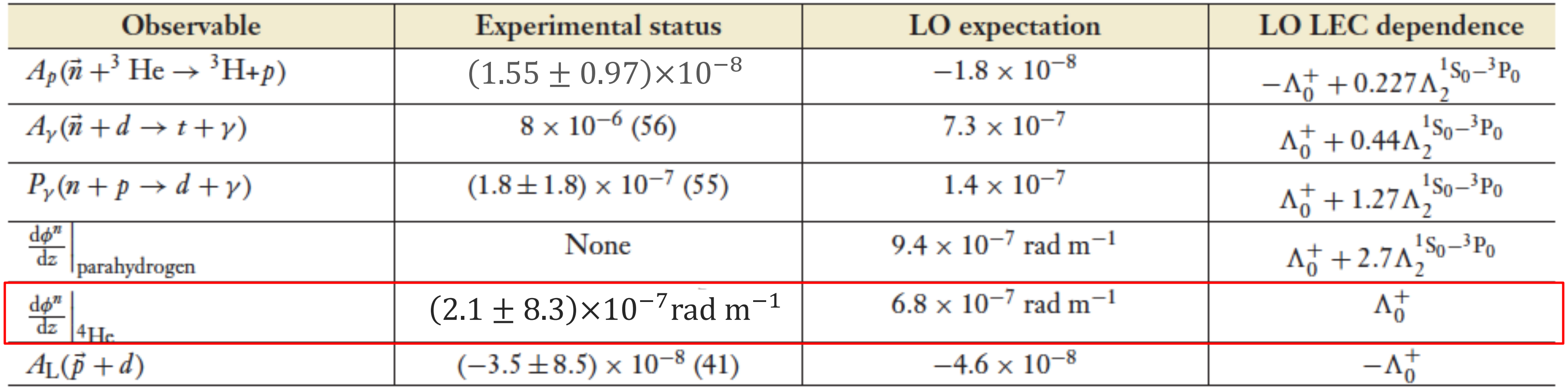
$$\boxed{\frac{d\phi_{PV}}{dz} = [+2.1 \pm 8.3(\text{stat.})^{+2.9}_{-0.2}(\text{sys.})]\times 10^{-7}\,\text{rad/m}}$$

W.M. Snow *et al.*, PRC **83**, 022501(R) (2011)
W.M. Snow *et al.*, RSI **86**, 055101 (2015)
H. E. Swanson,*et al.*, PRC **100**, 015204 (2019)

NN Weak Amplitudes in EFT+ 1/N_c & NSR

- Impact of new/potential experiments on the current status!

S. Gardner, W.C. Haxton, B.R. Holstein, Annu. Rev. Nucl. Part. Sci. **67**, 69 (2017)



Observable	Experimental status	LO expectation	LO LEC dependence
$A_p(\vec{n}+{}^3\mathrm{He}\to{}^3\mathrm{H}+p)$	$(1.55\pm0.97)\times10^{-8}$	-1.8×10^{-8}	$-\Lambda_0^+ + 0.227\Lambda_2^{^1S_0-^3P_0}$
$A_\gamma(\vec{n}+d\to t+\gamma)$	8×10^{-6} (56)	7.3×10^{-7}	$\Lambda_0^+ + 0.44\Lambda_2^{^1S_0-^3P_0}$
$P_\gamma(n+p\to d+\gamma)$	$(1.8\pm1.8)\times10^{-7}$ (55)	1.4×10^{-7}	$\Lambda_0^+ + 1.27\Lambda_2^{^1S_0-^3P_0}$
$\left.\frac{d\phi^n}{dz}\right\|_{\mathrm{parahydrogen}}$	None	9.4×10^{-7} rad m^{-1}	$\Lambda_0^+ + 2.7\Lambda_2^{^1S_0-^3P_0}$
$\left.\frac{d\phi^n}{dz}\right\|_{^4\mathrm{He}}$	$(2.1\pm8.3)\times10^{-7}$ rad m^{-1}	6.8×10^{-7} rad m^{-1}	Λ_0^+
$A_L(\vec{p}+d)$	$(-3.5\pm8.5)\times10^{-8}$ (41)	-4.6×10^{-8}	$-\Lambda_0^+$

$$\boxed{\Lambda_0^+ = 717\times10^{-7} \to 264\times10^{-7}}$$

Michael Gericke (INT 2022)

Spin Rotation Theoretical Predictions

$$\left.\frac{d\phi_{PV}}{dz}\right|_{^4\mathrm{He}} = 6.8\times10^{-7}\mathrm{rad/m} \longrightarrow \left.\frac{d\phi_{PV}}{dz}\right|_{^4\mathrm{He}} = 2.5\times10^{-7}\mathrm{rad/m}$$

S. Gardner, W. C. Haxton, B. R. Holstein
Ann. Rev. Nucl. Part. Sci. ***67**, 69 (2017)*

$$\left.\frac{d\phi_{PV}}{dz}\right|_{^4\mathrm{He}} = 1.2\times10^{-6}\mathrm{rad/m} \longrightarrow \left.\frac{d\phi_{PV}}{dz}\right|_{^4\mathrm{He}} = 4.4\times10^{-7}\mathrm{rad/m}$$

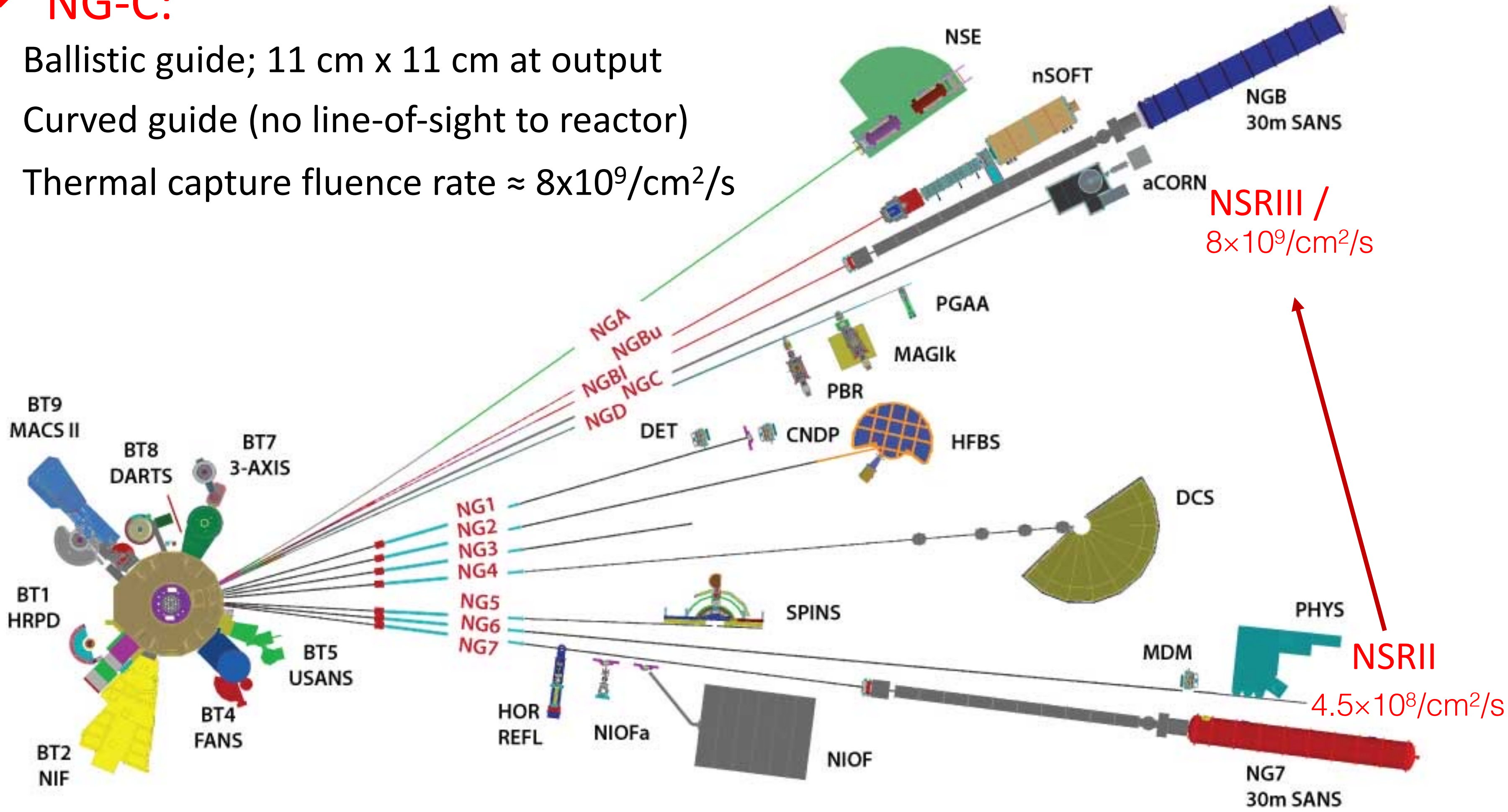
R. Lazauskas and Y.-H. Song,
Phys. Rev. C 99, 054002 (2019)

A ≤10^{-7} rad/m precision measurement of n-^{4}He spin rotation would provide a useful non-zero result in a field with rich theoretical parameter space and with not many experiments.

NIST Center for Neutron Research

❖ NG-C:

- Ballistic guide; 11 cm x 11 cm at output
- Curved guide (no line-of-sight to reactor)
- Thermal capture fluence rate ≈ 8x10^9/cm^2/s



NSRIII /
8×10^9/cm^2/s

NSRII
4.5×10^8/cm^2/s

Reactor **20 MW**
(fission neutrons)

Moderation D_2O →

Thermal neutrons

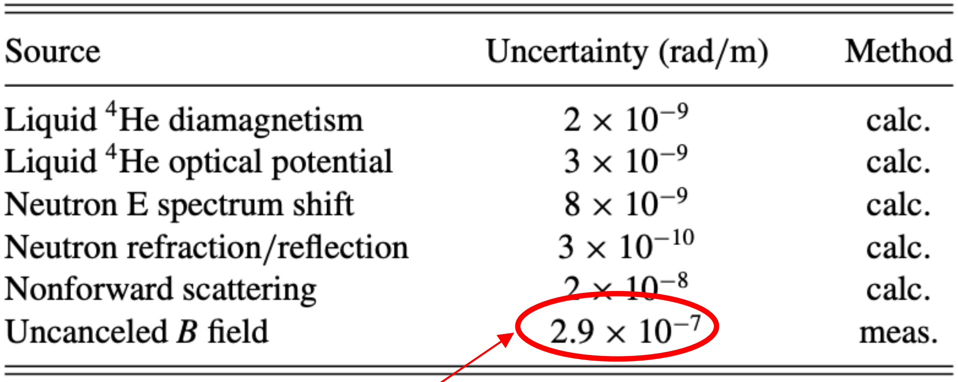
Moderation LH →

Cold neutrons
T=20K

NSR-III: Cryogenics & Target Improvements

NSR-II Potential Systematic Effects

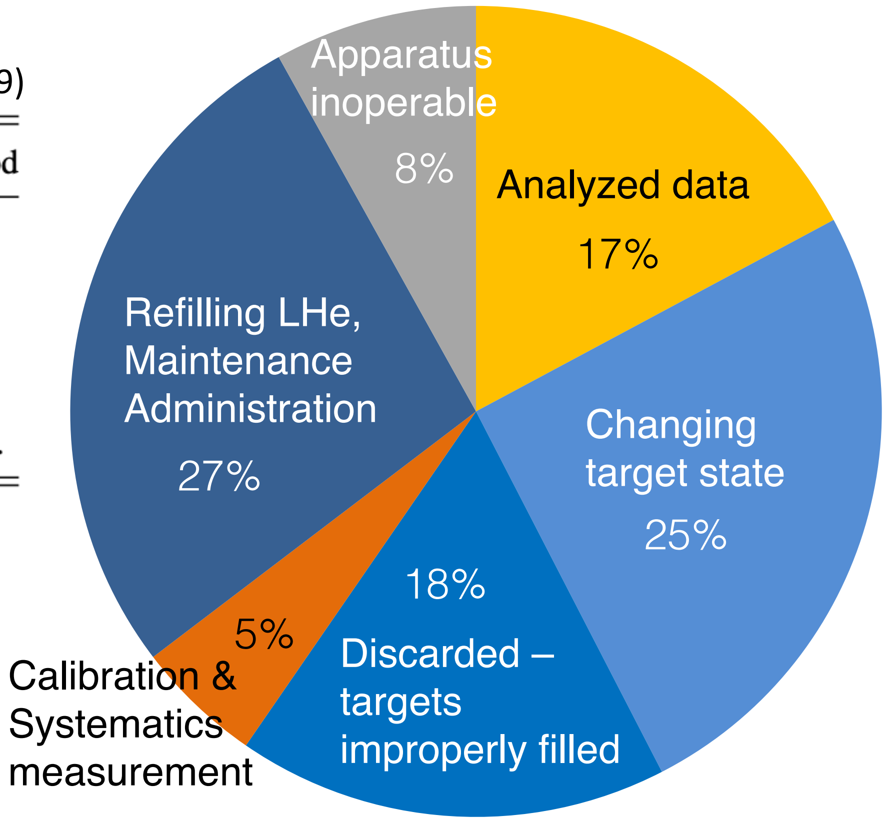
H. E. Swanson *et al.*, PRC **100**, 015204 (2019)



Source	Uncertainty (rad/m)	Method
Liquid ^{4}He diamagnetism	2×10^{-9}	calc.
Liquid ^{4}He optical potential	3×10^{-9}	calc.
Neutron E spectrum shift	8×10^{-9}	calc.
Neutron refraction/reflection	3×10^{-10}	calc.
Nonforward scattering	2×10^{-8}	calc.
Uncanceled B field	2.9×10^{-7}	meas.

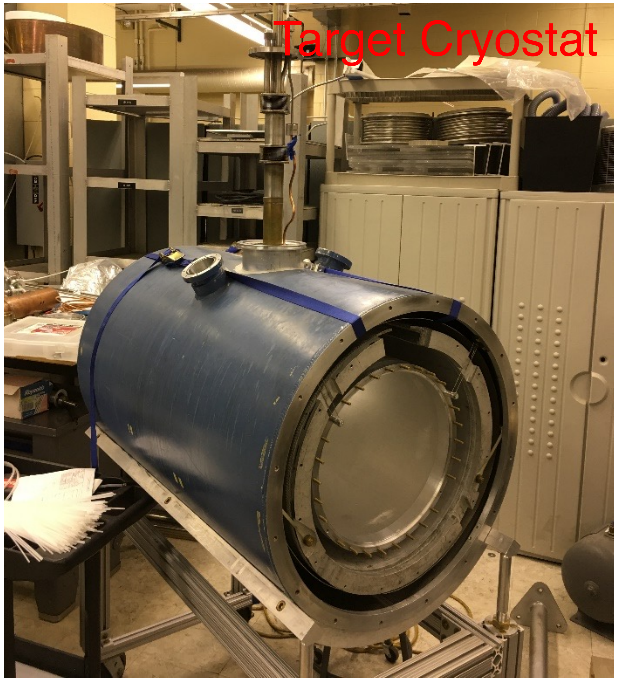
Dominated by background fields ~100 μG

120 NSR-II "Reactor On" days



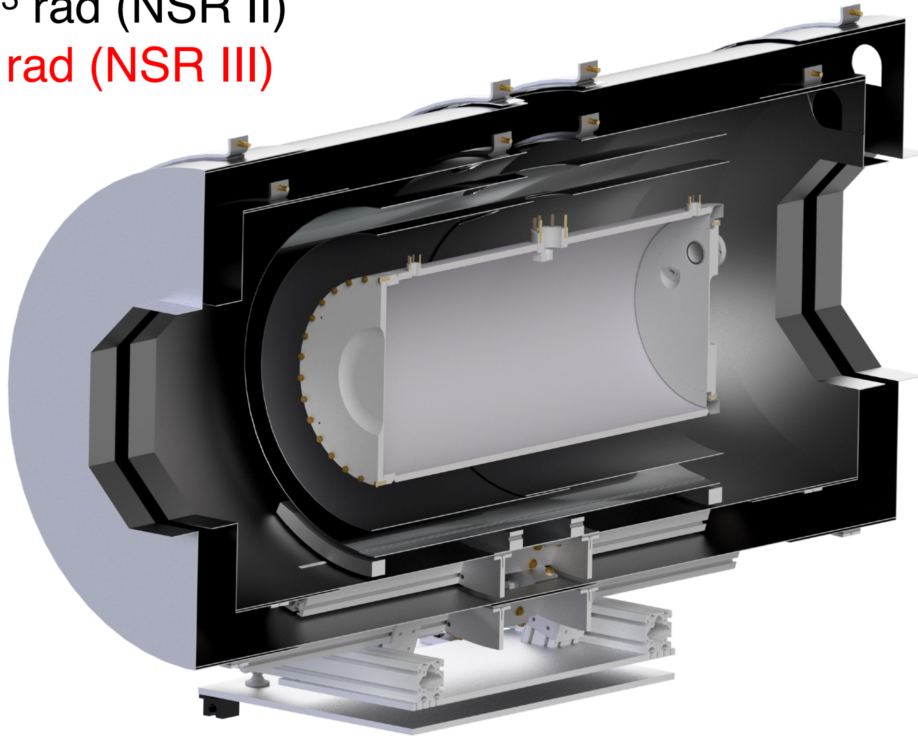
Magnetic Shielding

- Background fields dominate systematic



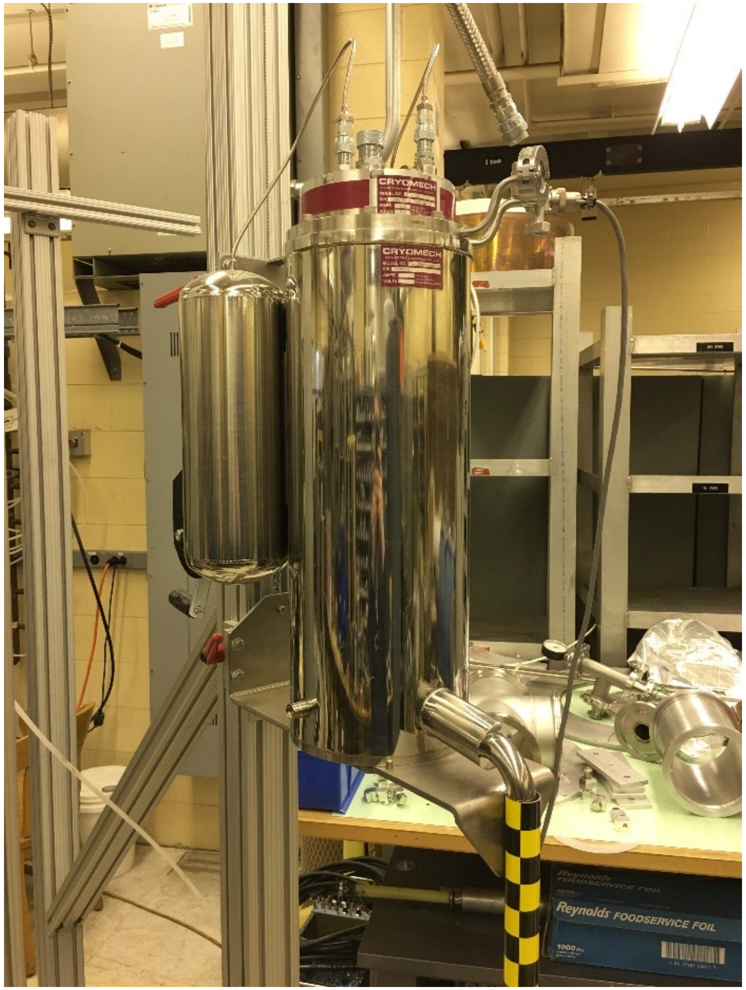
Rotations:
100 μG ➔ 10^{-3} rad (NSR II)
10 μG ➔ 10^{-4} rad (NSR III)

Initial measurement of 2 outer layers gives 10-100 μG



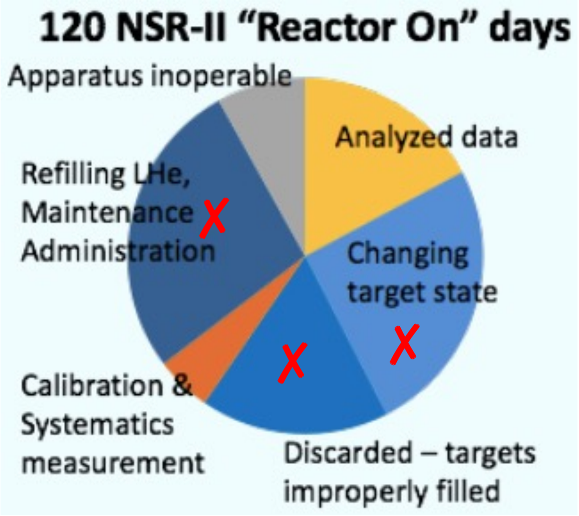
- 4 concentric cylindrical MuMetal shields (<10 μG)
- End caps for longitudinal field reduction
- Non-magnetic support structure
- Semi-active field control from trim coils around target — fine control over longitudinal field magnitude and gradient

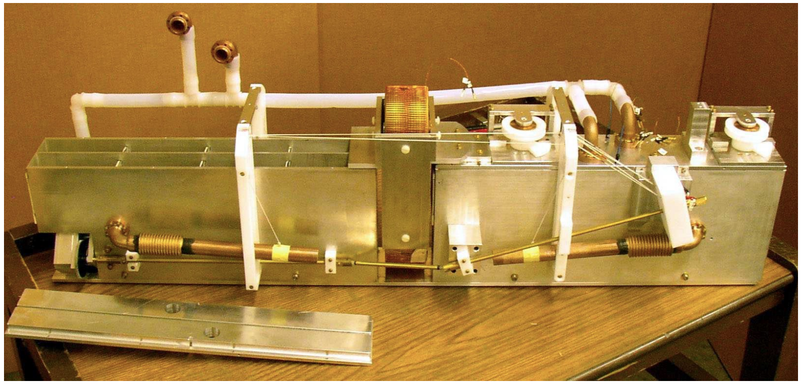
n-^{4}He Spin Rotation Target



- Cryomech pulse-tube reliquefier: tested for 3 months of continuous operation

- 5x5 cm^2 ⇒ 10x10 cm^2

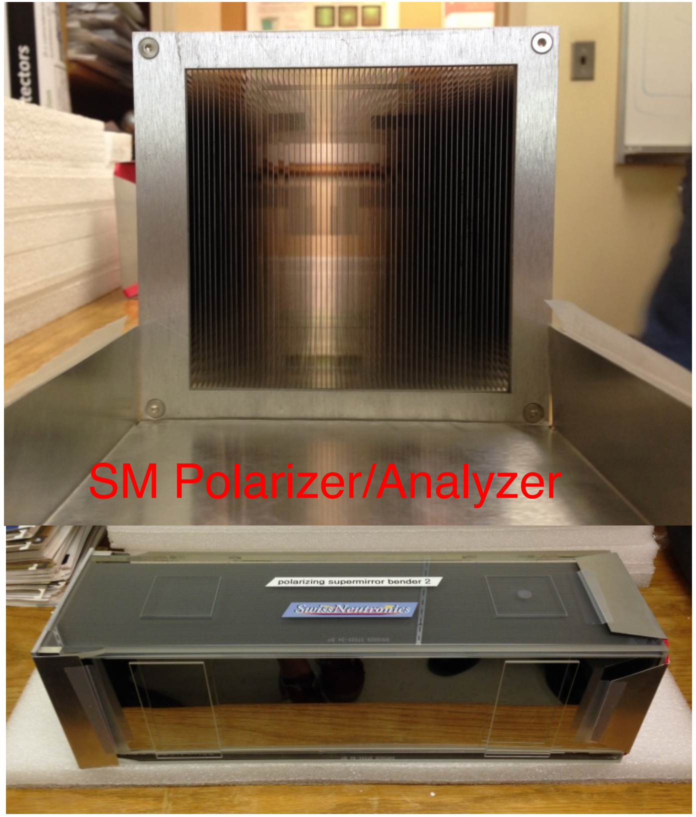




- Mechanical drive motor and drain actuation done externally at room temp away from sensitive field region
- Centrifugal pump and drains mounted to 4 helium target chambers at 4K

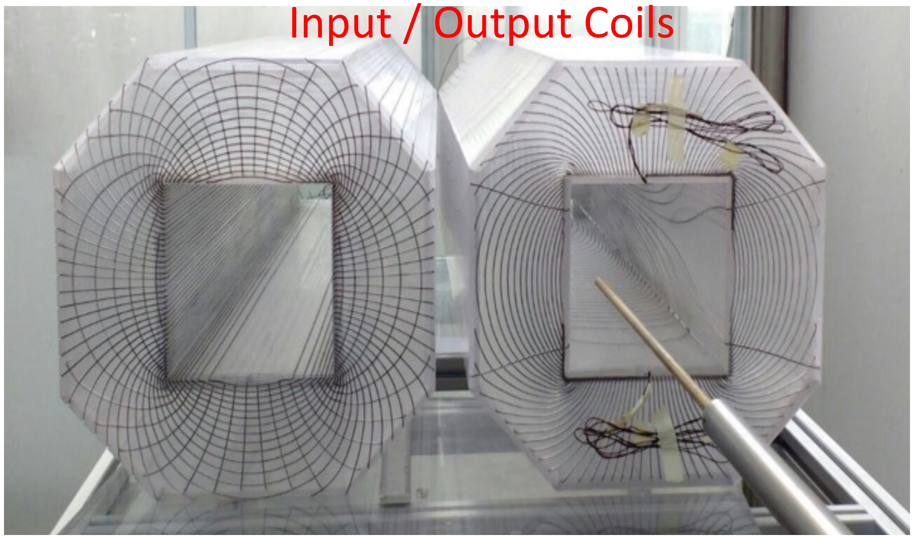
Other Beam-Line Components

Procured by NIST

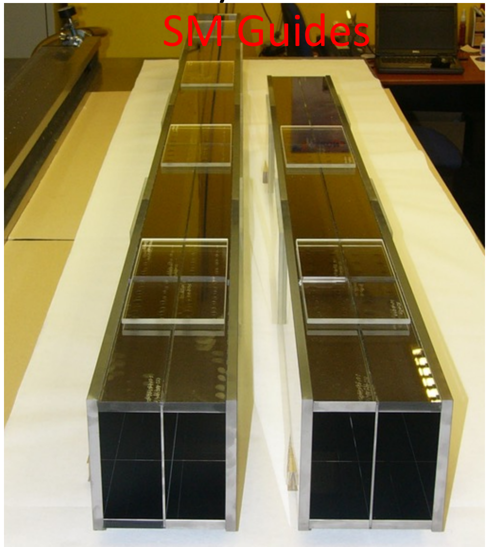


- Matched Pair of SwissNeutronics Polarizing Benders Courtesy of NIST (m=2.5)
- Attenuates and bends beam by ~15 mRad
- Achieves p > 95% polarization

Built at UNAM



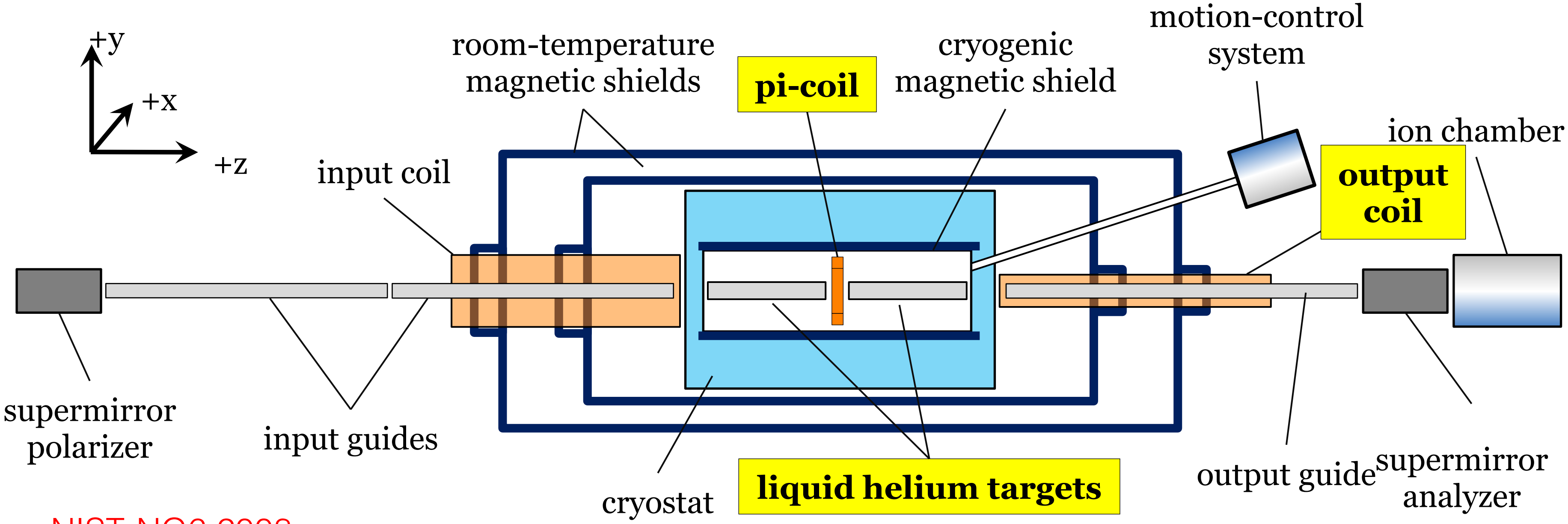
Procured by BARC in India



- 10cm×10cm, 1.25m and 2.0m non-magnetic supermirror neutron guides (NiMo-Ti)
- m = 2.0, R>90%, matching NGC phase space
- depolarization probability / bounce <1%

Spin Rotation III

Measures the horizontal component of neutron spin for a vertically-polarized beam



NIST-NG6 2008

$$\boxed{\frac{d\phi_{PV}}{dz} = [+2.1 \pm 8.3(\text{stat.})^{+2.9}_{-0.2}(\text{sys.})]\times 10^{-7}\,\text{rad/m}}$$

W.M. Snow *et al.*, PRC **83**, 022501(R) (2011)
W.M. Snow *et al.*, RSI **86**, 055101 (2015)
H. E. Swanson,*et al.*, PRC **100**, 015204 (2019)

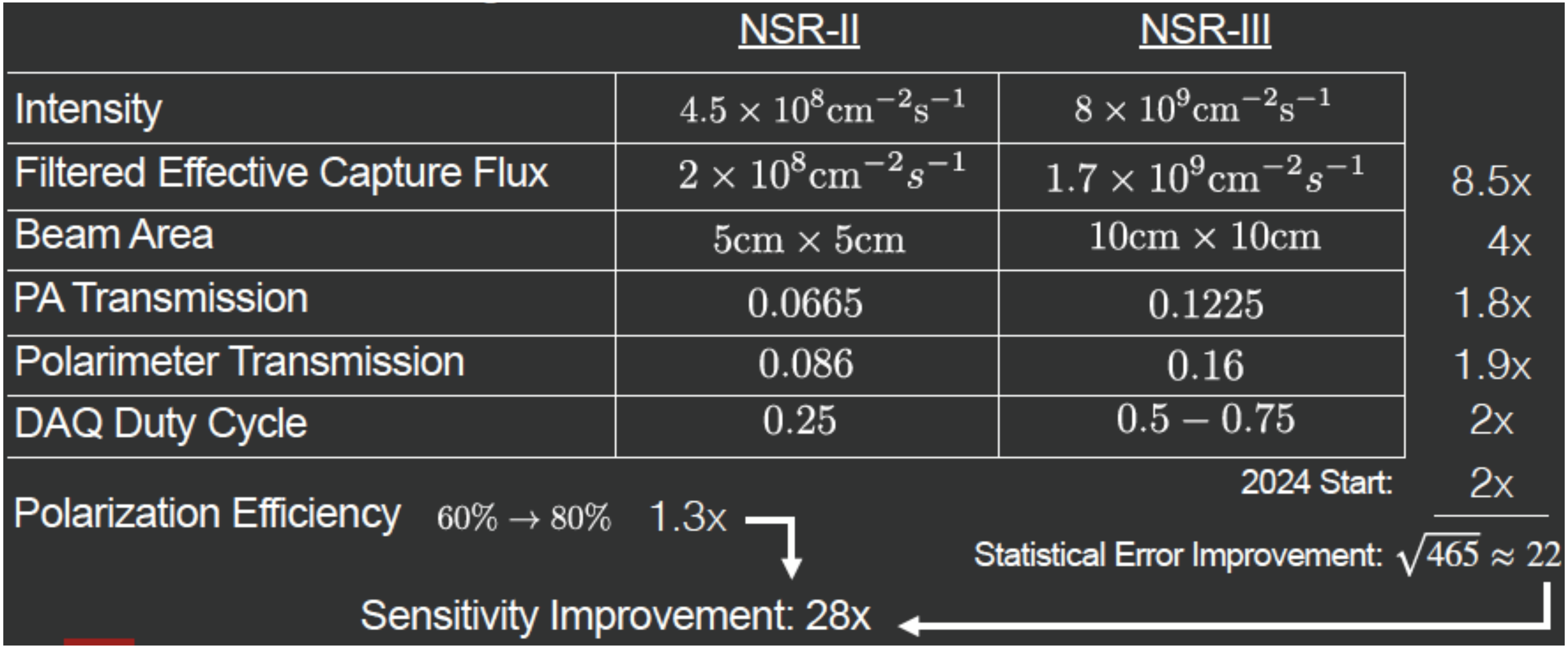
- ~~4.5×10^{8}/cm^{2}/s NG6~~ → 8×10^{9}/cm^{2}/s NGC
- ~~5cm×5cm NG6~~ → 10cm×10cm NGC
- ~~float glass guides (m=0.62)~~ → super-mirror guides (m=2)
- New SM Polarizer/Analyzer
- ~~100 μG~~ → 10 μG in target region
- Be filter cuts spectrum <4Å to limit under rotation by pi-coil

NSR III Status

- ✓ Supermirror Waveguides (new / BARC)
  - ○ Tested (October 2014 at LENS)
- ✓ Input/output coils (new / UNAM)
- ✓ New supermirror polarizer and analyzer (new / NIST)
  - ○ Tested at LENS
- ✓ Pi-coil (new / IU)
- ✓ Ion chamber (new / IU)
  - ○ Tested and functioning as expected
- ✓ Data Acquisition – Ready

- ○ Liquid helium target
  - ✓ Cryostat
  - ✓ Helium re-liquefier commissions with equivalent heat load
  - ○ Target and He pump construction and testing in progress

Statistical Projections



	NSR-II	NSR-III	
Intensity	$4.5 \times 10^8 \mathrm{cm}^{-2}\mathrm{s}^{-1}$	$8 \times 10^9 \mathrm{cm}^{-2}\mathrm{s}^{-1}$	
Filtered Effective Capture Flux	$2 \times 10^8 \mathrm{cm}^{-2} s^{-1}$	$1.7 \times 10^9 \mathrm{cm}^{-2} s^{-1}$	8.5x
Beam Area	$5\mathrm{cm} \times 5\mathrm{cm}$	$10\mathrm{cm} \times 10\mathrm{cm}$	4x
PA Transmission	0.0665	0.1225	1.8x
Polarimeter Transmission	0.086	0.16	1.9x
DAQ Duty Cycle	0.25	$0.5 - 0.75$	2x

2024 Start: 2x

Polarization Efficiency $60\% \to 80\%$ 1.3x

Statistical Error Improvement: $\sqrt{465} \approx 22$

Sensitivity Improvement: 28x

Projected Sensitivity Improvement: 28x

- ~10^{-7} rad/m within ~1 week
- ~10^{-8} rad/m with 3 reactor cycles

Theoretical predications:

$$\left\{ \left. \frac{d\phi_{\mathrm{PV}}}{dz} \right|_{^4\mathrm{He}} = 2.5 \times 10^{-7} \mathrm{rad/m} \right.$$

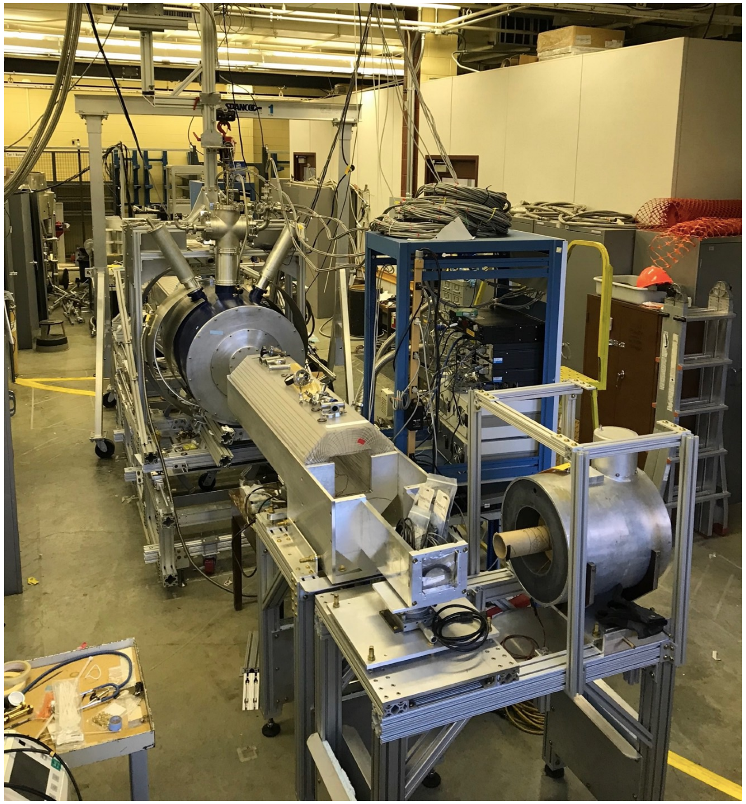
Scheduling Delays

NCNR Status:

- COVID19 shutdown significantly impacted the neutron beam lifetime data collection which is currently occupying the NG-C beam line
- Reactor event has resulted in an unplanned shutdown from early 2021-present
- NSRIII n-4He unlikely to see beam at NCNR until 2024, unclear how scheduled shutdown can be utilized for setup

Summary

- HWI is still one of the least understood aspects of nuclear physics. Significant recent theoretical work but still lack a sufficient number of precision measurements to constrain the set of couplings.
- NPDGamma and n^{3}He are the first two recent statistically significant precision measurements in few nucleon systems.
- NSRIII (n^{4}He) provides additional, much needed, significant precision measurement in the NN weak sector.
- NSRIII collaboration has an apparatus nearing readiness for an n^{4}He spin rotation measurement at the level ~$[\pm 1.0(\mathrm{stat}) \pm 1.0(\mathrm{sys})]\times 10^{-8}$ rad/m.
- The critical path items are the LHe pump, LHe target, and radiation shielding
- NIST will announce the details of their restart plan in the next couple of weeks



NSR-III Collaboration

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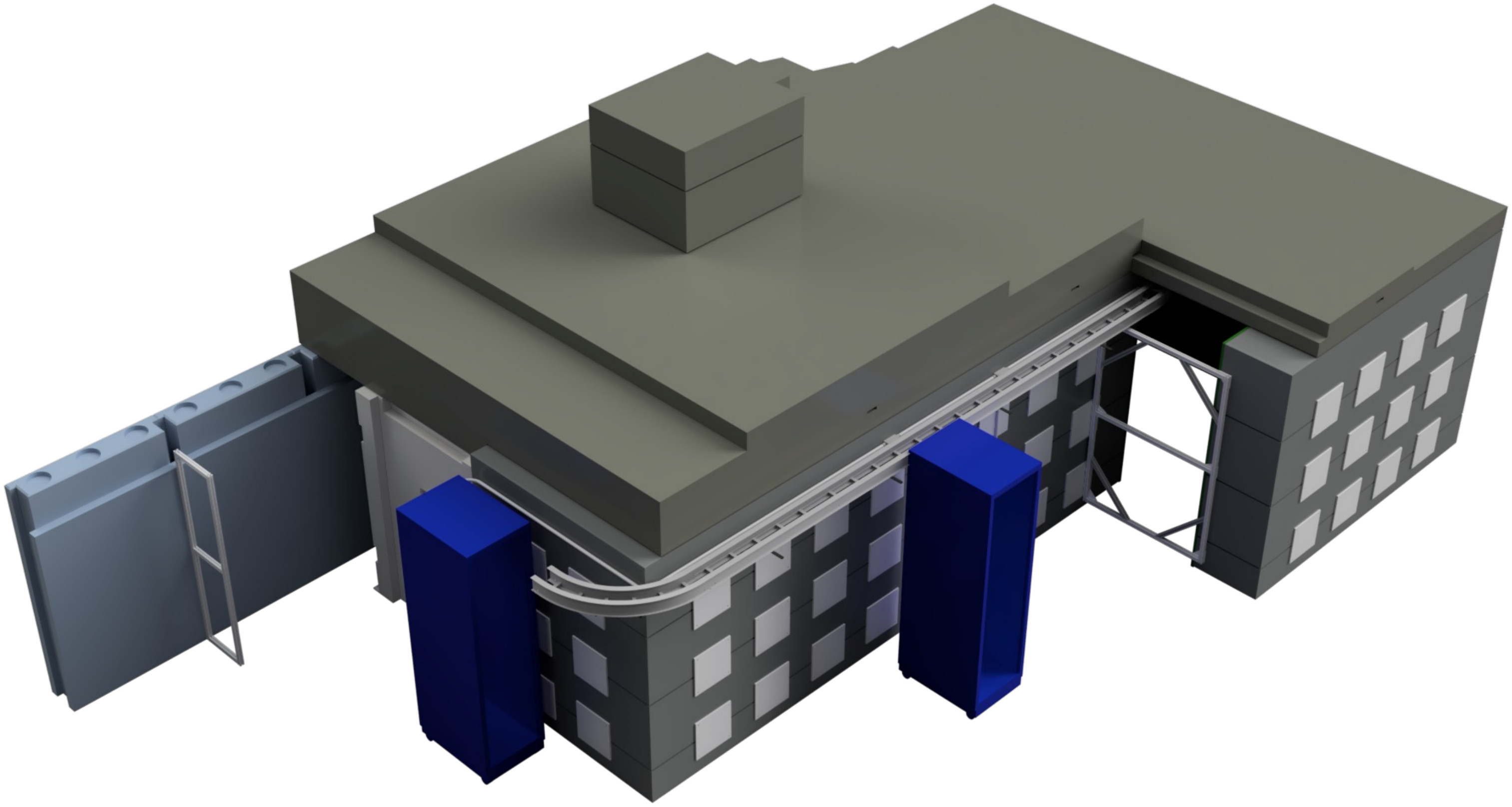
Support From:
NSF, DOE
PAPIIT
NIST BARC

Thank You

Backup

Shielding

100mrem/h backgrounds in vicinity of must be attenuated by factor of 100x
IU providing in-hand lead, borated poly, and concrete shielding
Radiation shielding in engineering safety phase for internal roof reinforcement



PV NSR in Liquid Parahydrogen

- Feasible because strong interaction spin-flip scattering off parahydrogen (S = 0 molecules) is forbidden.
- Largest predicted value by EFT/large-Nc calculations but no experiments to date.

$$\left.\frac{\mathrm{d}\phi_{\mathrm{PV}}}{\mathrm{d}z}\right|_{\mathrm{H}_2} = 9.4 \times 10^{-7}\mathrm{rad/m} \qquad \text{LO LEC Dependence: } \Lambda_0^{+} + 2.7\Lambda_2^{{}^1S_0 - {}^3P_0}$$

- Access combination of the LO LECs and along with NSR in ^{4}He offers independent measurements of LO LECs with different systematics from previous measurements

Experiment

- Use the same beamline components as ^{4}He with 20 cm target (neutron mean free path in para-hydrogen) – NSR collaboration developed extensive para-hydrogen target experience from npdgamma experiment
  - The target will be operated at 16-17 K where the equilibrium ortho to para hydrogen ratio is 0.03 %. If needed, a catalyst of paramagnetic material can be used to convert the ortho-hydrogen to the para configuration.
- Preliminary statistical calculations indicate 10^{-7} rad/m with ~1year of beam
- Higher neutron scattering rate in the hydrogen than liquid helium ➔ extensive simulation studies are ongoing to better understand the associated systematics

General Forward Scattering Amplitude:

$$f(0) = A + B[\sigma_n \cdot \mathbf{S}_N] + \underbrace{C[\sigma_n \cdot \mathbf{k}_n]}_{\boxed{P-\text{odd}}} + \underbrace{D[\mathbf{S}_N \cdot \mathbf{k}_n]}_{\boxed{P-\text{odd}}} + \underbrace{E[\sigma_n \cdot (\mathbf{k}_n \times \mathbf{S}_N)]}_{\boxed{P,T-\text{odd}}}$$

Unpolarized Target $\Longrightarrow$ $f(0) = A + C[\sigma_n \cdot \mathbf{k}_n] = f_{\text{PC}} + f_{\text{PV}}(\sigma_n \cdot \mathbf{k}_n)$

Coherent Wave Equation:

$$\boxed{[\nabla^2 + k_0^2 n^2]\,\langle\psi_c\rangle = 0}$$

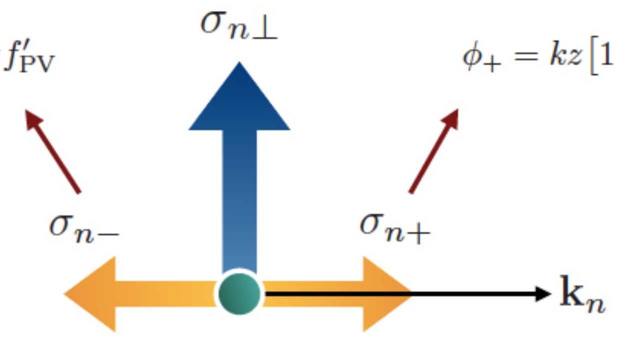
where $n = 1 + \dfrac{2\pi\rho f(0)}{k^2}$

$$\langle\psi_c\rangle \propto e^{i\mathbf{k}\cdot\mathbf{r}} \rightarrow e^{ink_0z} = \underbrace{e^{-\operatorname{Im}[n]k_0z}}_{\text{Attenuation}}\underbrace{e^{i\operatorname{Re}[n]k_0z}}_{\text{Accumulated Phase}}$$

$f_{\text{PV}} \propto k$ ➡ Momentum dep. of PV part of f(0) gives energy-independent phase accumulation in absence of resonances ⬅ $\theta = \operatorname{Re}[n]k_0z$

$\phi_- = kz[1 + \frac{2\pi\rho}{k^2}f_{\text{PC}}] - 2\pi\rho z f'_{\text{PV}}$

$\phi_+ = kz[1 + \frac{2\pi\rho}{k^2}f_{\text{PC}}] + 2\pi\rho z f'_{\text{PV}}$



$f'_{\text{PV}} = \dfrac{f_{\text{PV}}}{k}$

Constant for low-k

$$|\sigma_{n\perp}\rangle_f = \frac{1}{\sqrt{2}}[e^{i\phi_+}|\sigma_{n+}\rangle + e^{i\phi_-}|\sigma_{n-}\rangle] \Longrightarrow \varphi_{\text{PV}} = \phi_+ - \phi_- = 4\pi\rho z f'_{\text{PV}} \Longrightarrow \frac{\mathrm{d}\phi_{\text{PV}}}{\mathrm{d}z}$$