

Observables for scattering on targets with any spin

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Current matrix elements for composite particles with arbitrary spin

- Decompose matrix element in independent non-perturbative objects

$$\langle p', s' | j^\mu | p, s \rangle = \left(G_1(Q^2) [\varepsilon'^* \cdot \varepsilon] + G_3(Q^2) \frac{(q \cdot \varepsilon'^*)(q \cdot \varepsilon)}{2m^2} \right) (p + p')^\mu + G_M(Q^2) ((q \cdot \varepsilon'^*) \varepsilon^\mu - (q \cdot \varepsilon) (\varepsilon'^*)^\mu)$$

- Spin- j fields embedded in objects with $> 2j + 1$ components

- Polarization four vector (ε) for spin 1 $\rightarrow p_\mu \varepsilon^\mu(p, s) = 0$
- Rarita Schwinger for spin 3/2 $\rightarrow \gamma^\mu \psi_\mu(p, s) = 0$

(need for constraints, subsidiary conditions)

- Use $(2j + 1)$ -component spinors

- Via $SL(2, \mathbb{C})$ fundamental rep tensor products [Zwanziger 60s, Polyzou '18]
- Weinberg's construction [64-65] (not yet applied in this context)

Advantages of Weinberg's construction

- Use only exact degrees of freedom (**chiral reps**), no need for constraints
- Easy to switch between forms of dynamics (**instant form, light front**)
- Straightforward **Physical interpretation**
- For parity conserving interactions a **generalized Dirac algebra** is obtained
- “Basic” construction and implementation based on **$\text{su}(2)$ algebra**
- No kinematic singularities (**improved analyticity properties of operators**)

Weinberg's "Feynman rules for Any Spin" [1964]

- Algebra for Generators of the Lorentz group

$$[\mathbb{J}_l, \mathbb{J}_m] = i\epsilon_{lmn}\mathbb{J}_n, \quad [\mathbb{J}_l, \mathbb{K}_m] = i\epsilon_{lmn}\mathbb{K}_n, \quad [\mathbb{K}_l, \mathbb{K}_m] = -i\epsilon_{lmn}\mathbb{J}_n$$

- Two independent $\mathfrak{su}(2)$ subalgebras \rightarrow irreps (j_A, j_B)

$$\mathbb{A}_m = \frac{1}{2}(\mathbb{J}_m + i\mathbb{K}_m), \quad \mathbb{B}_m = \frac{1}{2}(\mathbb{J}_m - i\mathbb{K}_m)$$

$$[\mathbb{A}_l, \mathbb{A}_m] = i\epsilon_{lmn}\mathbb{A}_n, \quad [\mathbb{B}_l, \mathbb{B}_m] = i\epsilon_{lmn}\mathbb{B}_n, \quad [\mathbb{A}_l, \mathbb{B}_m] = 0$$

- Simplest irreps that contain spin- $j \rightarrow (2j + 1 \text{ components})$

- Right-handed $(j, 0)$: $\mathbb{K}_m \rightarrow -i\mathbb{J}_m$

- Left-handed $(0, j)$: $\mathbb{K}_m \rightarrow +i\mathbb{J}_m$

[Wigner(1939)]

Some Representations constructed out of the Chiral ones

- $(0, 0) \rightarrow$ Scalar
- $(1/2, 0) \rightarrow$ Right Weyl spinors & $(0, 1/2) \rightarrow$ Left Weyl spinors
- $(1/2, 0) \oplus (0, 1/2) \rightarrow$ Dirac (spin 1/2) spinors (direct sum)
- $(1/2, 1/2) \rightarrow$ Vector (Defining representation)
- $(1, 0) \rightarrow$ Right Chiral (spin 1) spinors & $(0, 1) \rightarrow$ Left Chiral (spin 1) spinors
- $(1, 0) \oplus (0, 1) \rightarrow$ Dirac (spin 1) spinors (direct sum)
- $(1, 1) \rightarrow$ Tensor

Canonical Space-Time Parameterization

Parameterizations (Foliations) of space-time \rightarrow Specify equal time surfaces

Canonical or Instant time: $x^0 = t$

[Wigner(1939)]

- Defined by rotationless boosts from rest: $\overset{\circ}{p}^\mu = (m, 0, 0, 0)$

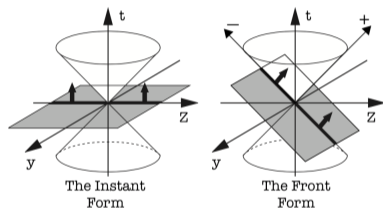
to final momentum: $p^\mu = (E_p, \vec{p}) = (\sqrt{m^2 + \vec{p}^2}, \vec{p})$

$$\Lambda^{\text{IF}} = \exp(i\vec{\mathbb{K}} \cdot \vec{\phi}) = \exp(i\phi \hat{\mathbb{K}} \cdot \hat{\phi})$$

- Then, $p^\mu = (E, \vec{p}) = (\Lambda^{\text{IF}})^\mu{}_\nu \overset{\circ}{p}^\nu$

implies, $\cosh(\phi) = \frac{E}{m}$, $\hat{\phi}_j \sinh(\phi) = \frac{p_j}{m}$

Leading to the well known result: $(\Lambda^{\text{IF}})^\mu{}_\nu = \begin{pmatrix} \frac{E}{m} & & & \\ \frac{\vec{p}}{m} & & & \\ \frac{\vec{p}}{m} & \delta_{ij} + \frac{p_i p_j}{(E+m)m} & & \end{pmatrix}$



Light-Front Space-Time Parameterization

Light Front time: $x^+ = t + z$

$$p^+ = E_p + p_z, \quad p^- = E_p - p_z$$

- Defined by a longitudinal boost followed by a transverse boost

$$\Lambda_{\text{def.}}^{\text{LF}} = \exp [i\vec{\mathbb{G}} \cdot \vec{v}_T] \cdot \exp [i\mathbb{K}_3\eta]$$

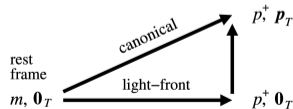
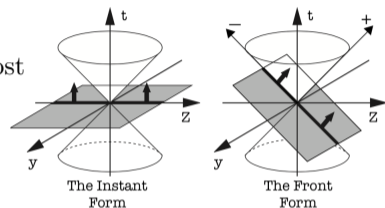
- LF Boost Generators (light front along z -axis),

$$\mathbb{G}_1 = \mathbb{G}_x = \mathbb{K}_x - \mathbb{J}_y, \quad \mathbb{G}_2 = \mathbb{G}_y = \mathbb{K}_y + \mathbb{J}_x, \quad \mathbb{K}_3 = \mathbb{K}_z$$

- Comparing the action of both boosts on the same rest momentum we find the LF boost parameters

$$e^\eta = \frac{p^+}{m}, \quad \vec{v}_T = \frac{\vec{p}_T}{p^+} \rightarrow \Lambda^{\text{LF}} = \exp \left[i \frac{\eta}{p^+ - m} \vec{p}_T \cdot \vec{\mathbb{G}} + i\eta\mathbb{K}_3 \right]$$

Dirac(1949)



Propagator of chiral fields

- Numerator (invariant)

$$\Pi_{\sigma\sigma'}^{(j)}(\vec{p}, \omega) = m^{2j} D_{\sigma\sigma'}^{(j)}[L(\vec{p})] \left(D_{\sigma'\sigma''}^{(j)}[L(\vec{p})] \right)^\dagger = m^{2j} \left(e^{-2\hat{p} \cdot \vec{J}^{(j)} \theta} \right)_{\sigma\sigma'}$$

$$\bar{\Pi}_{\sigma\sigma'}^{(j)}(\vec{p}, \omega) = m^{2j} \bar{D}_{\sigma\sigma'}^{(j)}[L(\vec{p})] \left(\bar{D}_{\sigma'\sigma''}^{(j)}[L(\vec{p})] \right)^\dagger = m^{2j} \left(e^{2\hat{p} \cdot \vec{J}^{(j)} \theta} \right)_{\sigma\sigma'}$$

- Introduction of the t-tensors

$$\Pi_{\sigma\sigma'}^{(j)}(\vec{p}, \omega) = t_{\sigma\sigma'}^{\mu_1\mu_2\cdots\mu_{2j}} p_{\mu_1} p_{\mu_2} \cdots p_{\mu_{2j}}$$

$$\bar{\Pi}_{\sigma\sigma'}^{(j)}(\vec{p}, \omega) = \bar{t}_{\sigma\sigma'}^{\mu_1\mu_2\cdots\mu_{2j}} p_{\mu_1} p_{\mu_2} \cdots p_{\mu_{2j}}$$

- These can also be used to write the boosts/spinors

$$D_{[L(p)]}^{(j)} = t^{\mu_1\mu_2\cdots\mu_{2j}} \tilde{p}_{\mu_1} \tilde{p}_{\mu_2} \cdots \tilde{p}_{\mu_{2j}}$$

$$\bar{D}_{[L(p)]}^{(j)} = \bar{t}^{\mu_1\mu_2\cdots\mu_{2j}} \tilde{p}_{\mu_1} \tilde{p}_{\mu_2} \cdots \tilde{p}_{\mu_{2j}}$$

Instant form dynamics
(Canonical)

$$\tilde{p}_C^\mu = \sqrt{\frac{m}{2(m+p^0)}} (p^0 + m, \vec{p})$$

Light-Front dynamics
(Light Cone time)

$$\tilde{p}_{LF}^\mu = \sqrt{\frac{m}{4p^+}} (p^+ + m, p_\ell, ip_\ell, p^+ - m)$$

Properties of the t-tensors

- **Symmetric** and (covariantly) **traceless**

$$g_{\mu_k \mu_l} t_{\sigma \sigma'}^{\mu_1 \dots \mu_k \dots \mu_l \dots \mu_{2j}} = 0$$

- Transform **covariantly**

$$\left(D_{[\Lambda]}^{(j)} \right)_{\sigma \delta} t_{\delta \delta'}^{\mu_1 \dots \mu_{2j}} \left(D_{[\Lambda]}^{(j)\dagger} \right)_{\delta' \sigma'} = \Lambda_{\nu_1}^{\mu_1} \dots \Lambda_{\nu_{2j}}^{\mu_{2j}} t_{\sigma \sigma'}^{\nu_1 \dots \nu_{2j}}$$

- Right chiral (t) and left chiral (\bar{t})
are related by charge conjugation

$$\bar{t}_{\sigma \sigma'}^{\mu_1 \mu_2 \dots \mu_{2j}} = (\pm) t_{\sigma \sigma'}^{\mu'_1 \mu'_2 \dots \mu'_{2j}}$$

(+ for even (− for odd) spacelike indices)

Algorithm for construction of t -tensors

Construction for t -tensor more insightful than Weinberg's expressions

- The 0-th degree polynomial in the J 's is always $t^{0\dots 0} = \mathbf{1}$

- The linear polynomials
are the Rotation Group Generators $t^{0\dots i\dots 0} = \frac{2}{2j} J_i = \frac{1}{j} J_i$

- From pairwise symmetrizations of the rotation generators

$$\begin{aligned} t^{0\dots m\dots 0\dots n\dots 0} = t^{mn0\dots 0} &= \frac{1}{\frac{(2j)!}{2!(2j-2)!}} \left(\{J_m, J_n\} - \frac{1}{3} \delta_{mn} \sum_{r=1}^3 \{J_r, J_r\} \right) + \frac{1}{3} t^{0\dots 0} \delta_{mn} \\ &= \frac{j}{(2j-1)} \left(\{t^{m0\dots 0}, t^{n0\dots 0}\} - \frac{1}{j} \delta_{mn} t^{0\dots 0} \right) \end{aligned}$$

Algorithm for construction of t-tensors

- Continues for higher orders
 - Matrices have more and more off-diagonal elements

$$t^{lmn0\dots0} = t^{0\dots0l0\dots0m0\dots0n0\dots0} = \frac{j}{(2j-2)} \frac{1}{3} \left(\{t^{l0\dots0}, t^{mn0\dots0}\} + \{t^{m0\dots0}, t^{nl0\dots0}\} + \{t^{n0\dots0}, t^{lm0\dots0}\} \right. \\ \left. - \frac{2}{j} \{ \delta_{lm} t^{n0\dots0} + \delta_{ln} t^{m0\dots0} + \delta_{mn} t^{l0\dots0} \} \right)$$

- Construction stops after j steps (Cayley-Hamilton) $(J-s)(J-s-1)\dots(J+s) = 0$
- The t-tensors contain a **basis for the su(N) algebra** $(N=2j+1)$
- A basis to decompose operators with physical interpretation for each term
(**multipole expansion** \rightarrow **mono-, di-, quadrupole, ...**)

$$\hat{O} = Tr [O] \mathbf{1} + Tr [OJ_i] J_i + Tr [OJ_{ij}] J_{ij} + \dots = \langle O \rangle \mathbf{1} + O_i J_i + O_{ij} J_{ij} + \dots$$

Spin 1/2 Example - $[\mathfrak{su}(2)]$

0-th order terms in $J_i^{(1/2)}$: $t^0 = \mathbf{1}$

Linear terms in $J_i^{(1/2)}$: $t^i = \frac{1}{1/2} J_i^{(1/2)} = \sigma_i$

(Pauli matrices)

$$J_1^{(1/2)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad J_2^{(1/2)} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad J_3^{(1/2)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Quadratic terms in $J_i^{(1/2)}$

$$(J^{(1/2)} - \frac{1}{2}\mathbf{1})(J^{(1/2)} + \frac{1}{2}\mathbf{1}) = 0 \implies (J^{(1/2)})^2 = c_0\mathbf{1} + c_2 J^{(1/2)}$$

Right Chiral Rep

- $t^0 = \mathbf{1}$, $t^i = \sigma_i$
- t^μ Transform Covariantly: $D_{[\Lambda]}^{(1/2)} t^\mu D_{[\Lambda]}^{(1/2)\dagger} = \Lambda_\rho{}^\mu t^\rho$
- Propagator (Lorentz invariant): $\Pi^{(1/2)}(p) = t^\mu p_\mu = \begin{pmatrix} E - p_z & -(p_x - ip_y) \\ -(p_x + ip_y) & E + p_z \end{pmatrix}$
 $p_\mu = (E_p, \vec{p})$
- Boost/spinors (Canonical): $D_{\text{IF}}^{(1/2)} = t^\mu \tilde{p}_\mu^{\text{C}} = \frac{1}{\sqrt{2m(m+p_0)}} \begin{pmatrix} m + p^- & -p_\ell \\ -p_r & m + p^+ \end{pmatrix}$
 $\tilde{p}_\text{C}^\mu = \sqrt{\frac{m}{2(m+p^0)}}(p^0 + m, \vec{p})$

Similarly for the Left Chiral Rep, only change is: $J_i^{(1/2)} \rightarrow \bar{J}^\mu = (1, -\vec{J}^{(1/2)})$

Spin 1 Example - $[\text{su}(3)]$

0-th order terms in $J_i^{(1)}$: $t^{00} = \mathbf{1}$

Linear terms in $J_i^{(1)}$: $t^{0i} = J_i^{(1)}$

$$t^{01} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad t^{02} = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad t^{03} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Quadratic terms in $J_i^{(1)}$: $t^{ij} = \{J_i^{(1)}, J_j^{(1)}\} - \mathbf{1}\delta_{ij}$

$$t^{11} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad t^{22} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad t^{33} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$t^{12} = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad t^{13} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}, \quad t^{23} = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

Cubic terms in $J_i^{(1)}$: $(J^{(1)} - \mathbf{1})(J^{(1)})(J^{(1)} + \mathbf{1}) = 0 \implies (J^{(1)})^3 = c_0\mathbf{1} + c_2J^{(1)} + c_3(J^{(1)})^2$

Right Chiral Rep

- $t^{00} = \mathbf{1}$, $t^{0i} = t^{i0} = J_i^{(1)}$, $t^{ij} = \{J_1^{(1)}, J_1^{(1)}\} - \mathbf{1}\delta_{ij}$

- $t^{\mu\nu}$ Transform covariantly $D_{[\Lambda]}^{(1)} t^{\mu\nu} D_{[\Lambda]}^{(1)\dagger} = \Lambda_{\rho}^{\mu} \Lambda_{\sigma}^{\nu} t^{\rho\sigma}$

- Propagator ($p_{\mu} = (E_p, \vec{p})$): $\Pi^{(1)}(p) = t^{\mu\nu} p_{\mu} p_{\nu} = \begin{pmatrix} (p^{-})^2 & -\sqrt{2}p_{\ell}p^{-} & p_{\ell}^2 \\ \sqrt{2}p_r p^{+} & p^{+}p^{-} + p_{\text{T}}^2 & \sqrt{2}p_{\ell}p^{-} \\ p_r^2 & \sqrt{2}p_r p^{-} & (p^{+})^2 \end{pmatrix}$

- Boost/spinors ($t^{\mu\nu} \tilde{p}_{\mu} \tilde{p}_{\nu}$)

Canonical: $D_{\text{IF}}^{(1)} = \frac{1}{2m(m+p_0)} \begin{pmatrix} (m+p^{-})^2 & -\sqrt{2}p_{\ell}(m+p^{-}) & p_{\ell}^2 \\ -\sqrt{2}p_r(m+p^{-}) & 2(m^2 + mp_0 + p_{\text{T}}^2) & -\sqrt{2}p_{\ell}(m+p^{+}) \\ p_r^2 & -\sqrt{2}p_r(m+p^{+}) & (m+p^{+})^2 \end{pmatrix}$

$$\tilde{p}_{\text{C}}^{\mu} = \sqrt{\frac{m}{2(m+p^0)}}(p^0 + m, \vec{p})$$

Similarly for the **Left Chiral Rep**, **only change is:** $J_i^{(1)} \rightarrow \bar{J}^{\mu} = (1, -\vec{J}^{(1)})$

Generalized Dirac algebra

- Parity conserving reactions are simpler in the direct sum of both chiral representations (like the spin 1/2 case)
- This leads to generalized gamma matrices $\rightarrow \gamma^{\mu_1 \dots \mu_{2j}} = \begin{pmatrix} 0 & t^{\mu_1 \dots \mu_{2j}} \\ \bar{t}^{\mu_1 \dots \mu_{2j}} & 0 \end{pmatrix}$
- Basis for spin-1/2 $\rightarrow \mathbf{1}, \gamma_5, (4)\gamma^\mu, (4)\gamma^\mu \gamma_5, (6)[\gamma^\mu, \gamma^\nu]$
- Basis for spin-1 $\rightarrow \mathbf{1}, \gamma_5, (9)\gamma^{\mu\nu}, (9)\gamma^{\mu\nu} \gamma_5, (6)[\gamma^{\mu_1 \mu_2}, \gamma^{\mu_3 \mu_4}], (10)\{\gamma^{\mu_1 \mu_2}, \gamma^{\mu_3 \mu_4}\}$
- Amplitudes can be evaluated by
 - Constructing expressions for the generalized bilinears
 - Using trace algebra
- Similarly expressions for covariant density matrices can be constructed

Generalized Dirac equation and Gordon identities

- Dirac Equation (constraint on bispinors)

$$(\gamma^{\mu_1 \dots \mu_{2j}} p_{\mu_1 \dots \mu_{2j}} - m^{2j}) u_p^s = (\not{p}^{(j)} - m^{2j}) u_p^s = 0$$

with, $u_p^s = \gamma^{\mu_1 \dots \mu_{2j}} \tilde{p}_{\mu_1 \dots \mu_{2j}}$, $p_{\mu_1 \dots \mu_{2j}} = p_{\mu_1} \dots p_{\mu_{2j}}$

- Gordon identity separates general bilinears into convection and magnetization currents [For spin $\frac{1}{2}$: Lorcé(2017)]

$$\bar{u}_{p'}^{s'} (\Gamma) u_p^s = \frac{1}{2m^{2j}} u_{p'}^{s'} \left(\left\{ \not{P}^{(j)}, \Gamma \right\} + \frac{1}{2} \left[\not{A}^{(j)}, \Gamma \right] \right) u_p^s$$

with,

$$\not{P}^{(j)} = \gamma^{\mu_1 \dots \mu_{2j}} P_{\mu_1 \dots \mu_{2j}} \quad , \quad P_{\mu_1 \dots \mu_{2j}} = \frac{1}{2} (p'_{\mu_1 \dots \mu_{2j}} + p_{\mu_1 \dots \mu_{2j}})$$

$$\not{A}^{(j)} = \gamma^{\mu_1 \dots \mu_{2j}} \Delta_{\mu_1 \dots \mu_{2j}} \quad , \quad \Delta_{\mu_1 \dots \mu_{2j}} = p'_{\mu_1 \dots \mu_{2j}} - p_{\mu_1 \dots \mu_{2j}}$$

EM Current Parameterized by Sachs Form Factors

From spinor

Representation: $\langle p', s' | j^\mu(0) | p, s \rangle = 2P^\mu \left(\mathbf{1} G_C(Q^2) - \frac{\Delta^\rho \Delta^\sigma (t_{\rho\sigma} - \frac{1}{3} g_{\rho\sigma} \mathbf{1})}{2M^2} \frac{P^2}{M^2} G_Q(Q^2) \right)_{s's}$

$$P = \frac{1}{2}(p' + p)$$

$$\Delta = p' - p \quad (\Delta^2 = -Q^2)$$

$$n_t^\nu = (1, 0, 0, 0)$$

$$-i\epsilon^{\mu\rho\sigma\lambda} \left(\frac{\Delta_\rho P_\sigma (t_{\lambda\nu} - \frac{1}{3} g_{\lambda\nu} \mathbf{1}) n_t^\nu}{\sqrt{P^2}} G_M(Q^2) \right)_{s's}$$

Using polarization vectors
[Wang & Lorcé (2022)]

$$\Gamma^{\mu\alpha\beta} = 2P^\mu \left(\Pi^{\alpha\beta} G_C(Q^2) - \frac{\Delta^\rho \Delta^\sigma (\Sigma_{\rho\sigma})^{\alpha\beta}}{2M^2} \frac{P^2}{M^2} G_Q(Q^2) \right)_{s's}$$

$$-i\epsilon^{\mu\rho\sigma\lambda} \left(\frac{\Delta_\rho P_\sigma (\Sigma_\lambda)^{\alpha\beta}}{\sqrt{P^2}} G_M(Q^2) \right)_{s's}$$

Current conservation is guaranteed: $\Gamma^\mu \Delta_\mu = 0$

(on-shell condition $\rightarrow P^\mu \Delta_\mu = 0$)

From Spinor Representation (Parameterized by Sachs Form Factors):

$$j_{\text{Ch}}^\mu(P, \Delta) = 2P^\mu \left(\mathbf{1}G_C - \frac{\Delta^\rho \Delta^\sigma (t_{\rho\sigma} - \frac{1}{3}g_{\rho\sigma} \mathbf{1})}{2M^2} \frac{P^2}{M^2} G_Q \right) - \frac{i\epsilon^{\mu\rho\sigma\lambda} \Delta_\rho P_\sigma (t_{\lambda\nu} - \frac{1}{3}g_{\lambda\nu} \mathbf{1}) n_t^\nu}{\sqrt{P^2}} G_M$$

Textbook Representation
(using Polarization vectors)

$$\langle p', s' | j^\mu(0) | p, s \rangle = \epsilon_{s'}^{*\alpha}(p') j_{\alpha\beta}^\mu(P, \Delta) \epsilon_s^\beta(p)$$

Parameterized by
Covariant Form Factors

$$j_{\alpha\beta}^\mu(P, \Delta) = 2P^\mu \left[g_{\alpha\beta} G_1(Q^2) - \frac{\Delta_\alpha \Delta_\beta}{M^2} G_2(Q^2) \right]$$

$$\epsilon_s^\mu(p) = \left(\frac{\mathbf{p} \cdot \boldsymbol{\epsilon}_s}{M}, \boldsymbol{\epsilon}_s + \frac{\mathbf{p}(\mathbf{p} \cdot \boldsymbol{\epsilon}_s)}{M(p^0 + M)} \right)$$

$$+ \left[\Delta^\alpha g_\beta^\mu - \Delta^\beta g_\alpha^\mu \right] G_3(Q^2)$$

$$\boldsymbol{\epsilon}_\pm = \frac{1}{\sqrt{2}}(\mp 1, -i, 0), \quad \boldsymbol{\epsilon}_0 = (0, 0, 1)$$

The two sets of form factors are related by:

$$\tau = Q^2/(4M^2)$$

$$G_C(Q^2) = G_1(Q^2) + \frac{2}{3}\tau G_Q(Q^2)$$

$$G_M(Q^2) = G_2(Q^2)$$

$$G_Q(Q^2) = G_1(Q^2) - G_2(Q^2) + (1 + \tau)G_3(Q^2)$$

- Weinberg's construction allows for an efficient and manifestly covariant calculation of currents for any spin
- Central (and multifaceted) role for the covariant t-tensors
- Simple algorithm. Only need to know the matrices for the Generators of rotations in the representation of interest.
- Many applications and extensions possible (Parameterization for SIDIS and DVCS)