Universität Zürich UZH

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Inclusive semileptonic B decays on the lattice

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Based on P. Gambino, S. Hashimoto, SM, M. Panero, F. Sanfilippo, S. Simula, A. Smecca and N. Tantalo, arXiv:2203.11762

Long-standing tension in V_{cb}

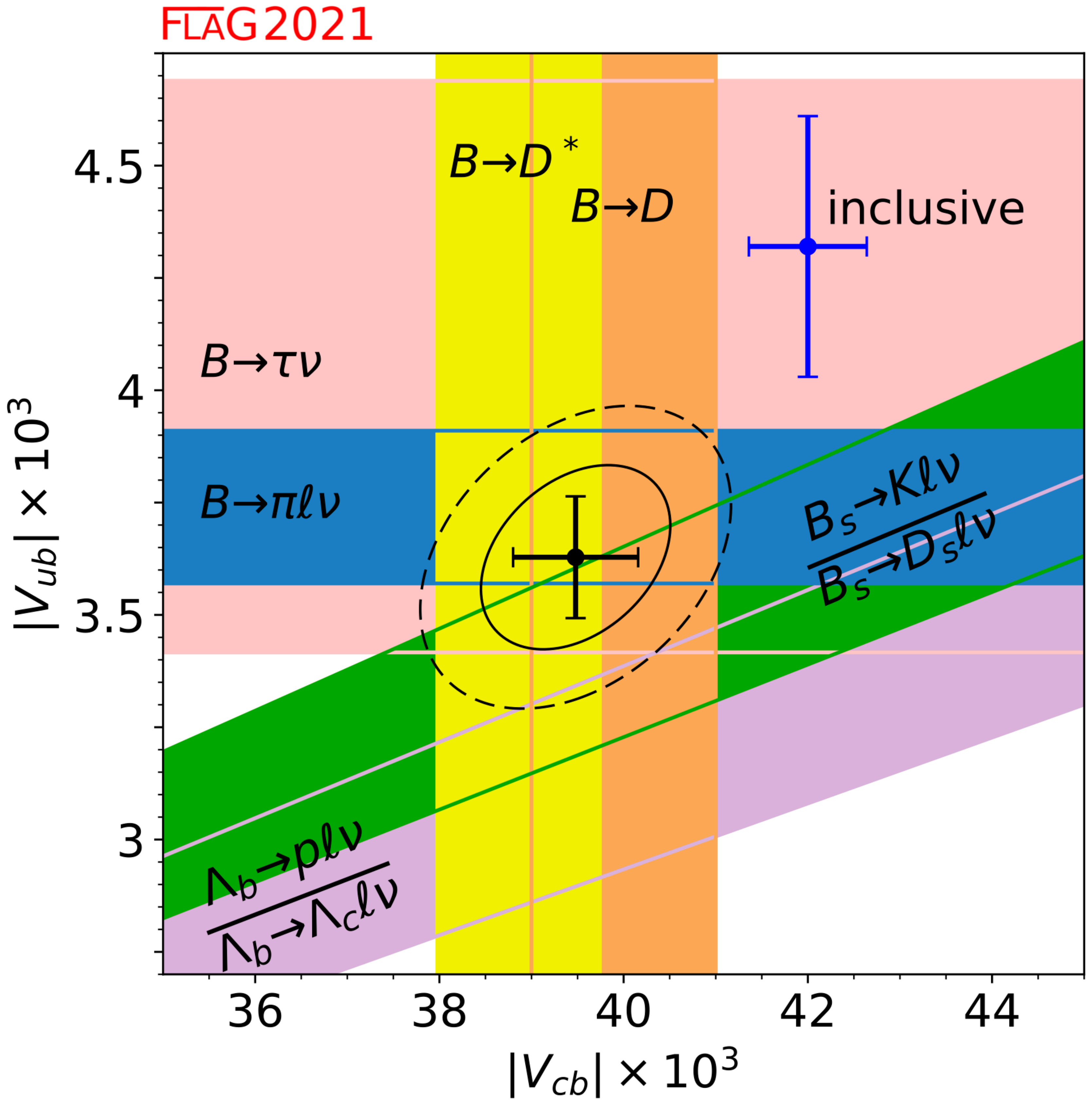
There is a long-standing discrepancy between inclusive and exclusive determinations of V_{cb}

$$\left|V_{cb}^{\text{incl}}\right| = (42.00 \pm 0.64)\ 10^{-3}$$

$\sim 3\sigma$

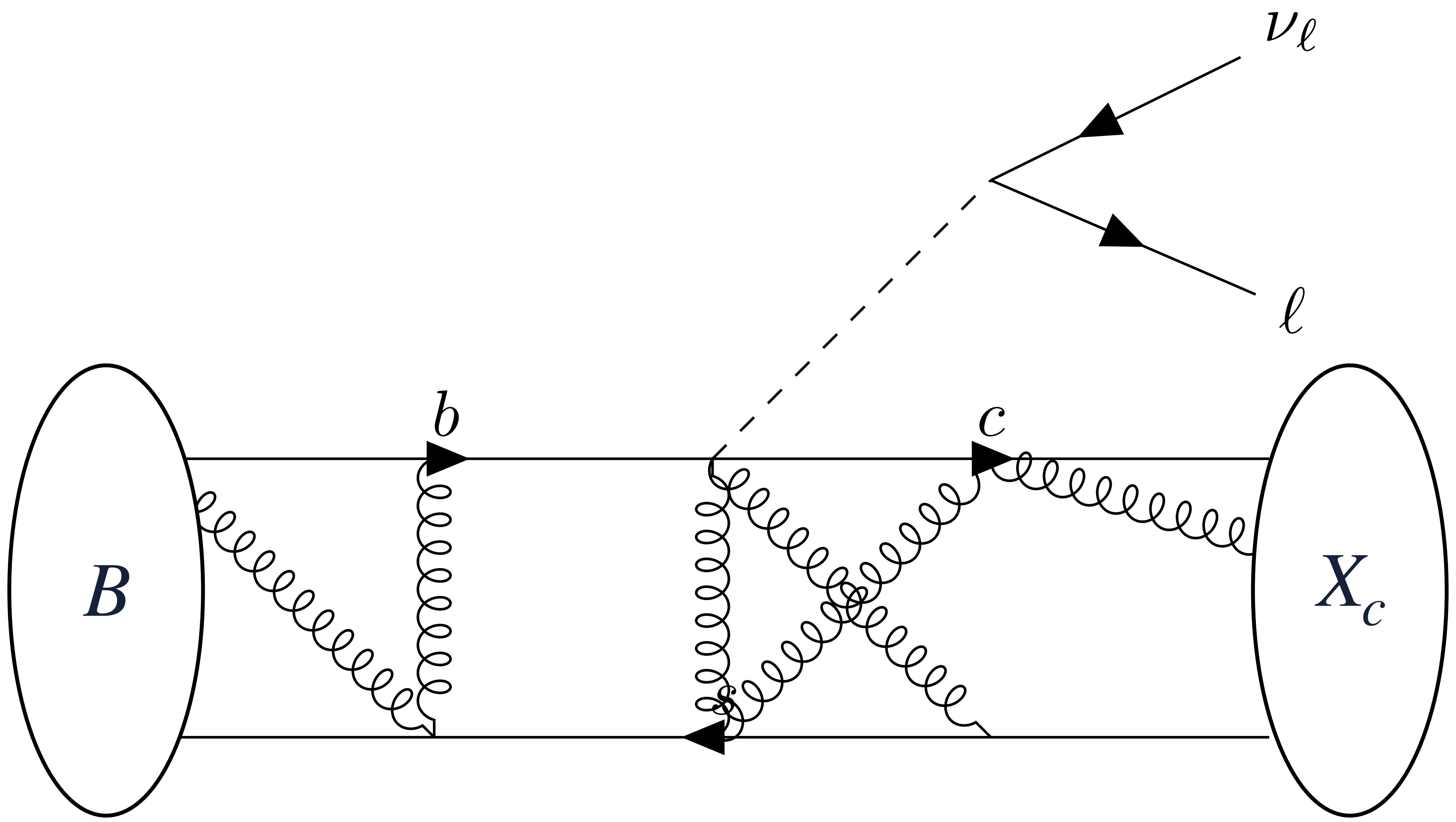
$$\left|V_{cb}^{\text{excl}}\right| = (39.16 \pm 0.67) \times 10^{-3}$$

[FLAG 2021]



Inclusive B decays

We consider the inclusive decay $B \to X_c \ell \nu_{\ell'}$



Inclusive decays are independent of the final state.
Through an OPE short and long distance QCD effects are separated.

OPE for inclusive B decays

The triple differential decay rate can be factorized as into a leptonic and hadronic tensor as

$$\frac{\mathrm{d}^3\Gamma}{\mathrm{d}\mathbf{q}^2\mathrm{d}\omega\mathrm{d}E_\ell} = \frac{G_\mathrm{F}^2 \left|V_{cb}\right|^2}{8\pi^3} L_{\mu\nu} W^{\mu\nu}$$

which contain the leptonic and the **non-perturbative** hadronic dynamics, respectively.

After integrating over E_ℓ we can write the total decay rate as

$$\Gamma = \frac{G_\mathrm{F}^2 \left|V_{cb}\right|^2}{24\pi^3} \int_0^{\mathbf{q}^2_\mathrm{max}} \mathrm{d}\mathbf{q}^2 \sqrt{\mathbf{q}^2} \int_{\omega_\mathrm{min}}^{\omega_\mathrm{max}} \mathrm{d}\omega X\left(\omega, \mathbf{q}^2\right)$$

OPE for inclusive B decays

The hadronic tensor is given by

$$W^{\mu\nu} = \sum_{X_c} (2\pi)^3 \, \delta^{(4)} \left(p - q - r\right) \frac{1}{2E_B(\mathbf{p})} \left\langle \overline{B}(\mathbf{p}) \middle| J^{\mu\dagger} \middle| X_C(\mathbf{r}) \right\rangle \left\langle X_C(\mathbf{r}) \middle| J^{\nu} \middle| \overline{B}(\mathbf{p}) \right\rangle$$

with

$$J^{\mu} = \bar{c}\gamma^{\mu} \left(1 - \gamma_5\right) b = V^{\mu} - A^{\mu}$$

Through an OPE we can express it in terms of B meson matrix elements of local operators.

OPE for inclusive B decays

Then general inclusive observables are smeared differential distributions, given by a double series in λ_{QCD}/m_b and α_s

$$M_i = M_i^{(0,0)} + \frac{\alpha_s}{\pi} M_i^1 + \frac{\mu_\pi^2}{m_b^2}\left(M_i^{\pi,0} + \frac{\alpha_s}{\pi} M_i^{(\pi,1)}\right) + \frac{\mu_G^2}{m_b^2}\left(M_i^{G,0} + \frac{\alpha_s}{\pi} M_i^{(G,1)}\right)$$
$$+ \frac{\rho_D^3}{m_b^3} M_i^{(D,0)} + \frac{\rho_{LS}^3}{m_b^3} M_i^{(LS,0)} + \cdots,$$

expressed in terms of perturbative Wilson coefficients and non-perturbative matrix elements,

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expressed in terms of perturbative Wilson coefficients and non-perturbative matrix elements, for instance

$$\mu_\pi^2 = \frac{1}{m_B}\left\langle B \left| \bar{b}_v (i\vec{D})^2 b_v \right| B \right\rangle, \quad \mu_G^2 = \frac{1}{m_B}\left\langle B \left| \bar{b}_v \frac{i}{2}\sigma_{\mu\nu} G^{\mu\nu} b_v \right| B \right\rangle.$$

OPE for inclusive B decays

- The non-perturbative parameters $\mu_\pi, \mu_G, \rho_D, \rho_{LS}$ and quark masses m_b, m_c are **crucial** input parameters and extracted from experimental data.

→ A fully non-perturbative computation of inclusive observables as proposed in 2005.13730 would provide a check of the OPE.

Idea: Treat lattice simulations as a virtual laboratory to probe the non-perturbative dynamics.

Let us go back to the total decay width

$$\Gamma = \frac{G_F^2 \left| V_{cb} \right|^2}{24\pi^3} \int_0^{\mathbf{q}^2_{\max}} \mathrm{d}\mathbf{q}^2 \sqrt{\mathbf{q}^2} \int_{\omega_{\min}}^{\omega_{\max}} \mathrm{d}\omega X\left(\omega, \mathbf{q}^2\right)$$

The ω-integral, $\bar{X}$, is our gateway to compute Γ on the lattice. It is of the form

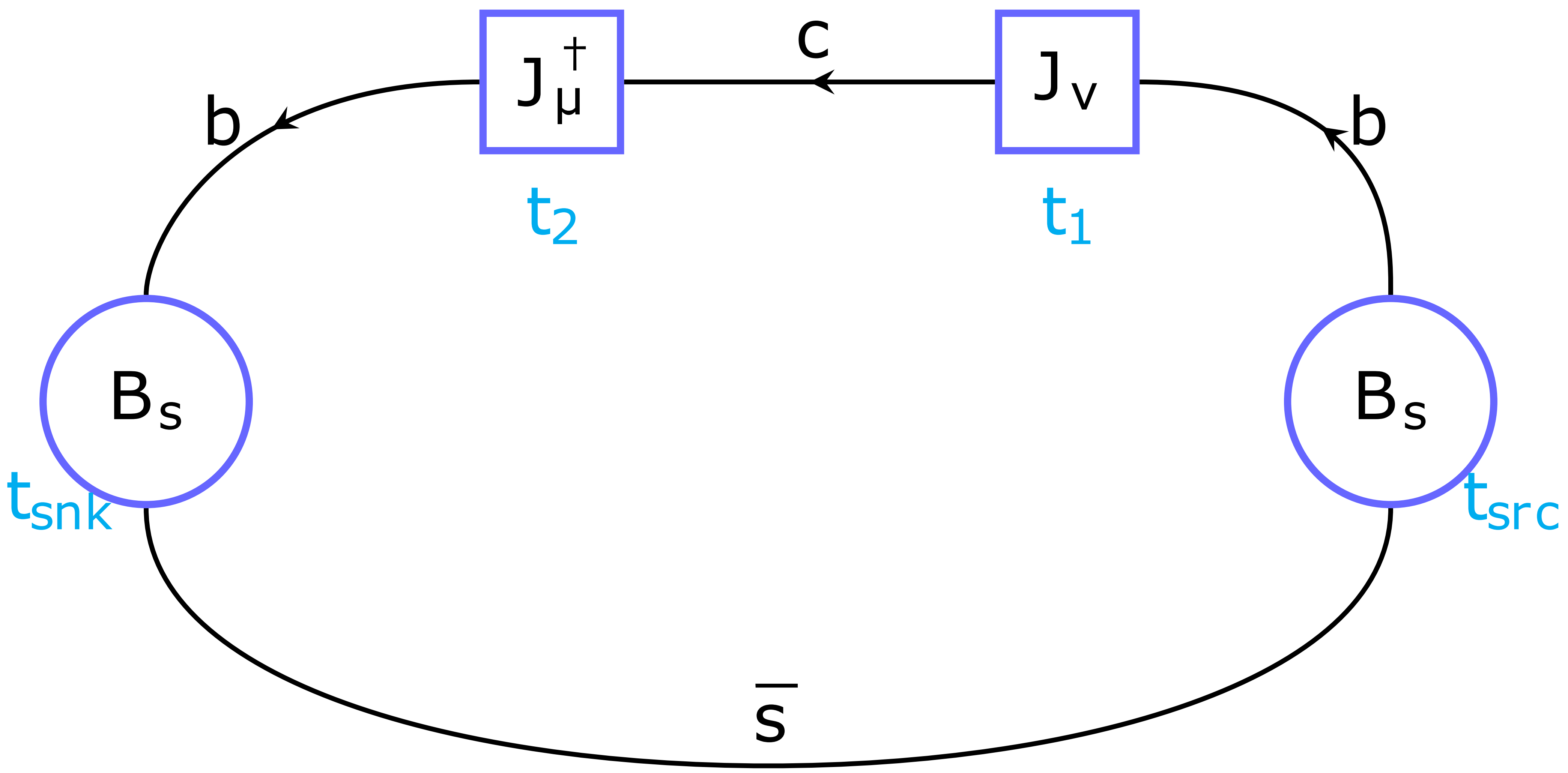
$$\bar{X} = \int_{\omega_{\min}}^{\omega_{\max}} \mathrm{d}\omega \underbrace{k_{\mu\nu}}_{\text{kinematics}} \times \underbrace{W^{\mu\nu}}_{\text{lattice}}$$

As expected we need a non-perturbative computation of the hadronic tensor.

Inclusive B decays on the lattice

However, $W_{\mu\nu}$ cannot be computed directly. Instead it is **linked** to the 4-point correlator $C_{\mu\nu}$ below:

$$C_{\mu\nu}\left(T_{\text{snk}}, t_2, t_1, t_{\text{src}}; \mathbf{q}\right) =$$



[Gambino, Hashimoto, 2005.13730]

Inclusive B decays on the lattice

It turns out that in a finite volume the 4-point function $C_{\mu\nu}$ is related to the hadronic tensor $W_{\mu\nu}$ by the simple relation

$$C_{\mu\nu}(t;\mathbf{q}) = \int_0^\infty \mathrm{d}\omega W_{L,\mu\nu}(\omega,\mathbf{q}) e^{-\omega t}$$

We remember that $\bar{X}$ is given by

$$\bar{X} = \int_{\omega_{\min}}^{\omega_{\max}} \mathrm{d}\omega \underbrace{k_{\mu\nu}(\mathbf{q},\omega)}_{\text{kinematics}} \times W^{\mu\nu} = \int_{\omega_{\min}}^{\infty} \mathrm{d}\omega W^{\mu\nu} \underbrace{k_{\mu\nu}(\mathbf{q},\omega)\theta(\omega_{\max}-\omega)}_{\text{kernel operator}}$$

$$= \int_{\omega_{\min}}^{\infty} \mathrm{d}\omega W^{\mu\nu} K_{\mu\nu}(\mathbf{q},\omega)$$

Inclusive B decays on the lattice

The remaining problem is to find a relation between

$$C_{\mu\nu}\left(t;\mathbf{q}\right) = \int_0^\infty \mathrm{d}\omega W_{L,\mu\nu}\left(\omega,\mathbf{q}\right)e^{-\omega t} \overset{?}{\longleftrightarrow} \int_{\omega_{\min}}^\infty \mathrm{d}\omega W^{\mu\nu} K_{\mu\nu}\left(\mathbf{q},\omega\right) = \bar{X}$$

Idea: We can approximate any smooth function $K_{\mu\nu}(\omega)$ as

$$K^{\mu\nu}(\omega) = \sum_\tau g_\tau^{\mu\nu} e^{-\omega\tau}\,.$$

Inclusive B decays on the lattice

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$$K^{\mu\nu}(\omega) = \sum_\tau g_\tau^{\mu\nu} e^{-\omega\tau} .$$

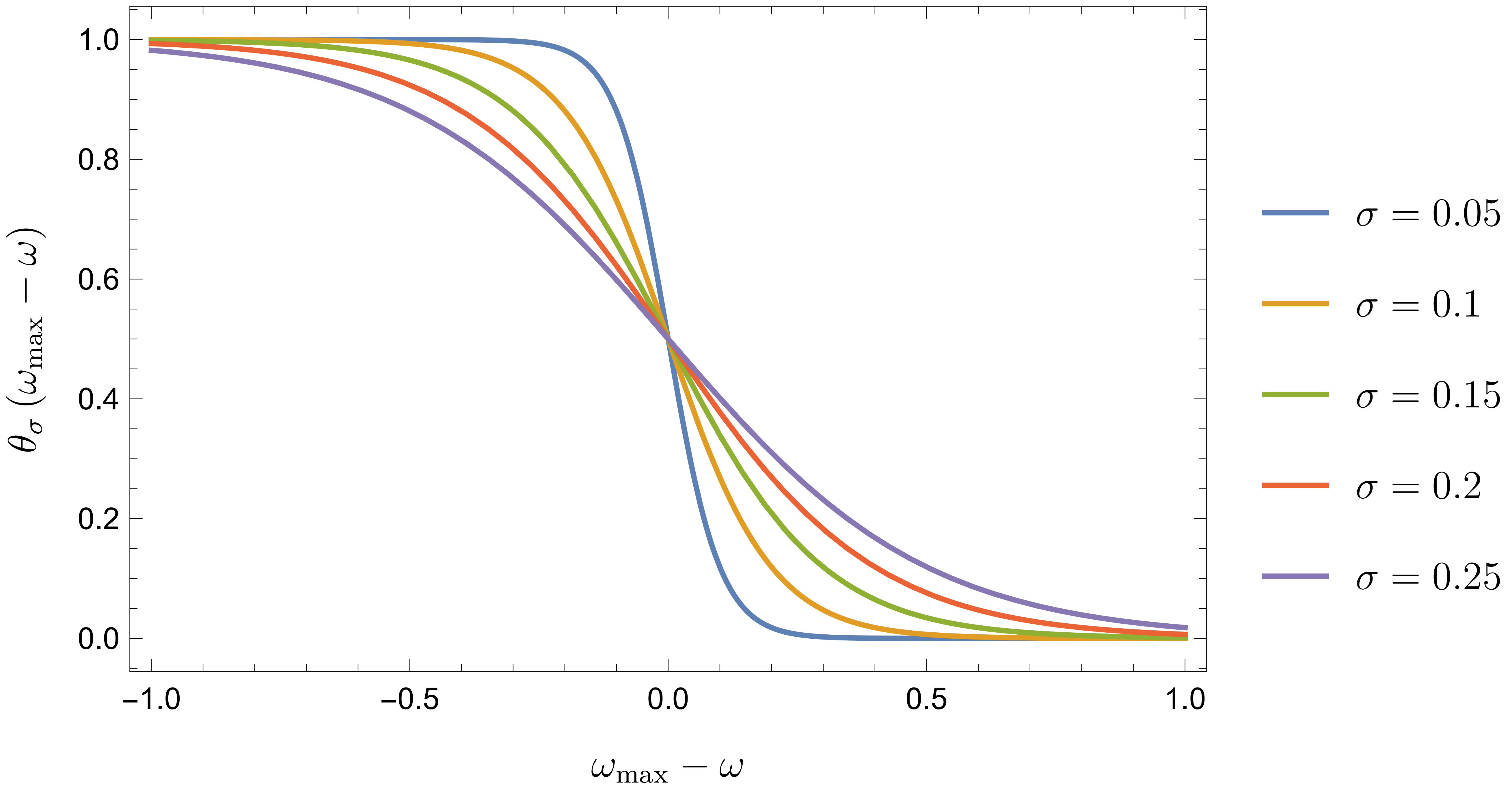
But the kernel above is **not smooth** at all!

→ We need to introduce a **smeared** kernel!

Inclusive B decays on the lattice

We replace the θ-function with a smeared version:

$$\theta(x) \to \theta_\sigma(x) = \frac{1}{1 + e^{\frac{x}{\sigma}}}$$



Inclusive B decays on the lattice

Finally we have all the ingredients to compute inclusive observables on the lattice!

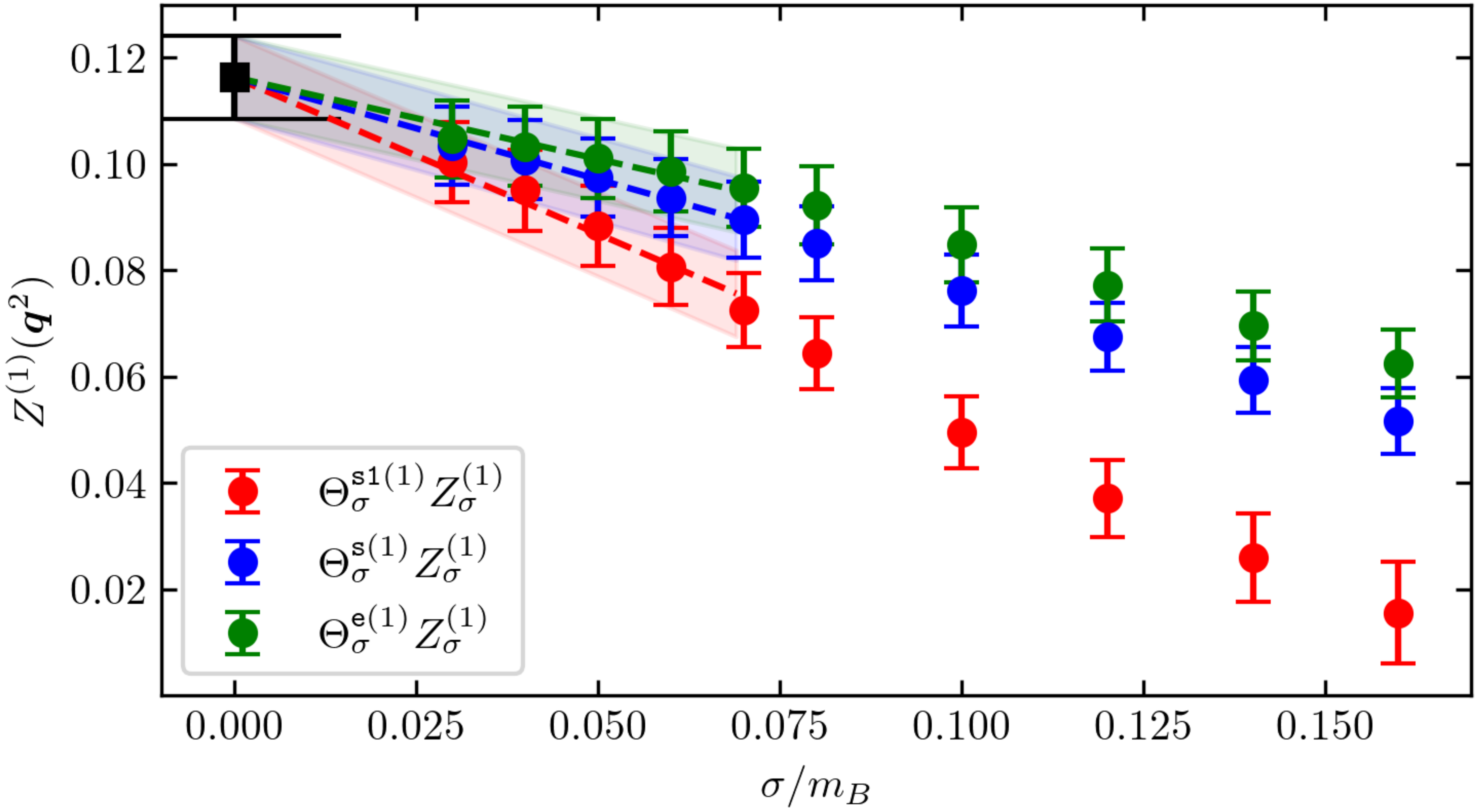
$$\lim_{\sigma\to 0}\lim_{V\to 0}\sum_{\tau} g_{\tau,\mu\nu} C^{\mu\nu}(t;\mathbf{q}) = \lim_{\sigma\to 0}\int_0^\infty \mathrm{d}\omega W^{\mu\nu}(\omega,\mathbf{q}) k_{\mu\nu}(\mathbf{q},\omega)\,\theta_\sigma(\omega_{\max}-\omega)$$

So in order to compute the total decay rate on the lattice, one has to

- Evaluate the 4-point correlator on the lattice
- Replace the kernel $K_{\mu\nu}$ with a smooth function
- Approximate the smooth kernel
- Take the infinite volume limit
- Extrapolate to the original kernel

Inclusive B decays on the lattice

Extrapolation to $\sigma = 0$



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Lattice calculation

JLQCD

Uses DWF QCD action with $N_f = 2 + 1$ sea quarks

Single (!) Lattice spacing: $a = 0.055\ \mathrm{fm}$

Quark masses: $m_c = m_c^{\mathrm{phys}}$

$m_b = 2.44 m_c$

Unphysically light B_s meson mass: $M_{B_s} \approx 3.45\ \mathrm{GeV}$

ETMC

Uses Twisted Mass QCD action with $N_f = 2 + 1 + 1$ sea quarks

Single (!) Lattice spacing: $a = 0.0815(30)\ \mathrm{fm}$

Quark masses: $m_c^{\mathrm{phys}} = 1176(39)\ \mathrm{MeV}$

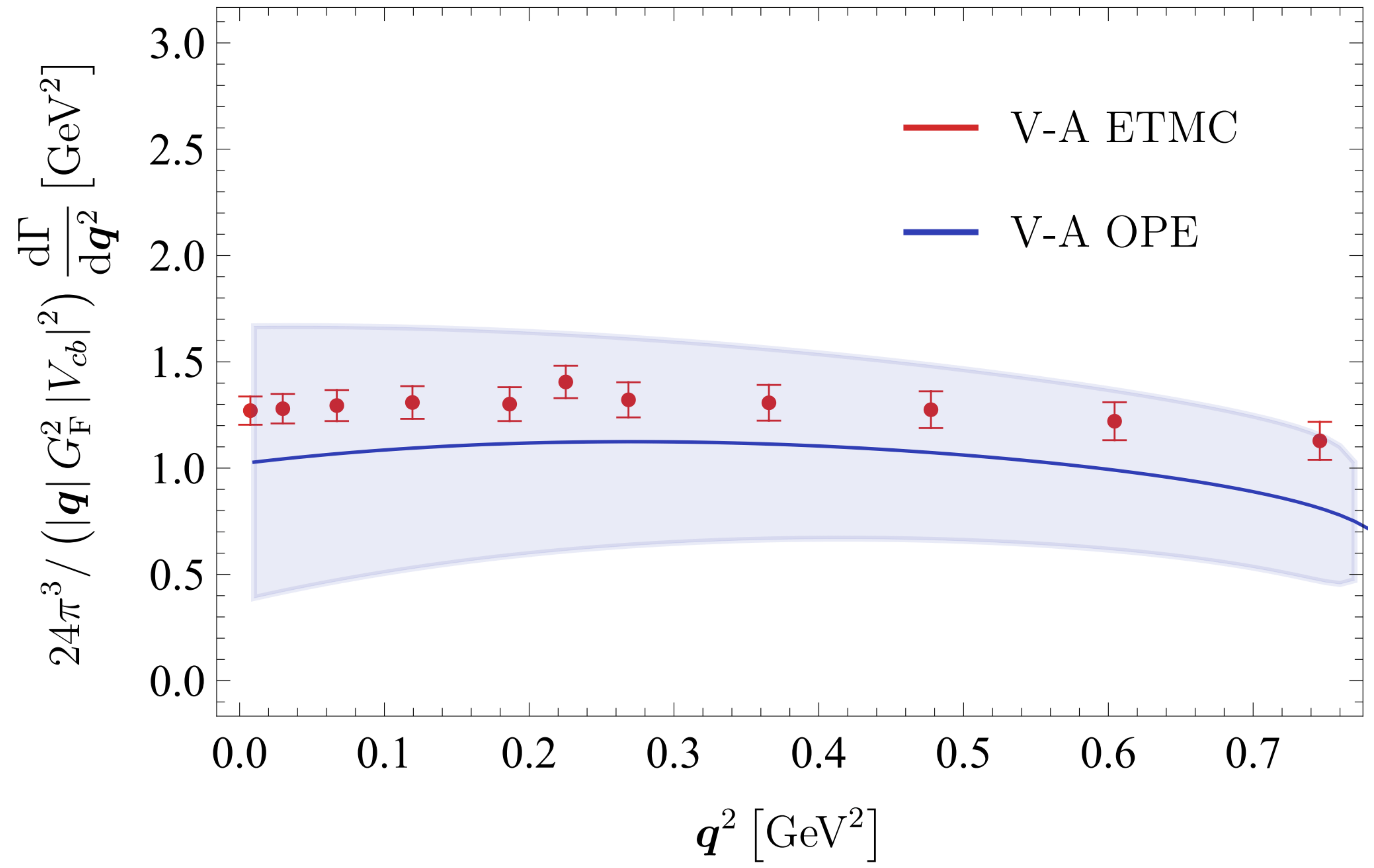
$m_b = 2 m_c$

Unphysically light B_s meson mass: $M_{B_s} \approx 3.08(11)\ \mathrm{GeV}$

Results

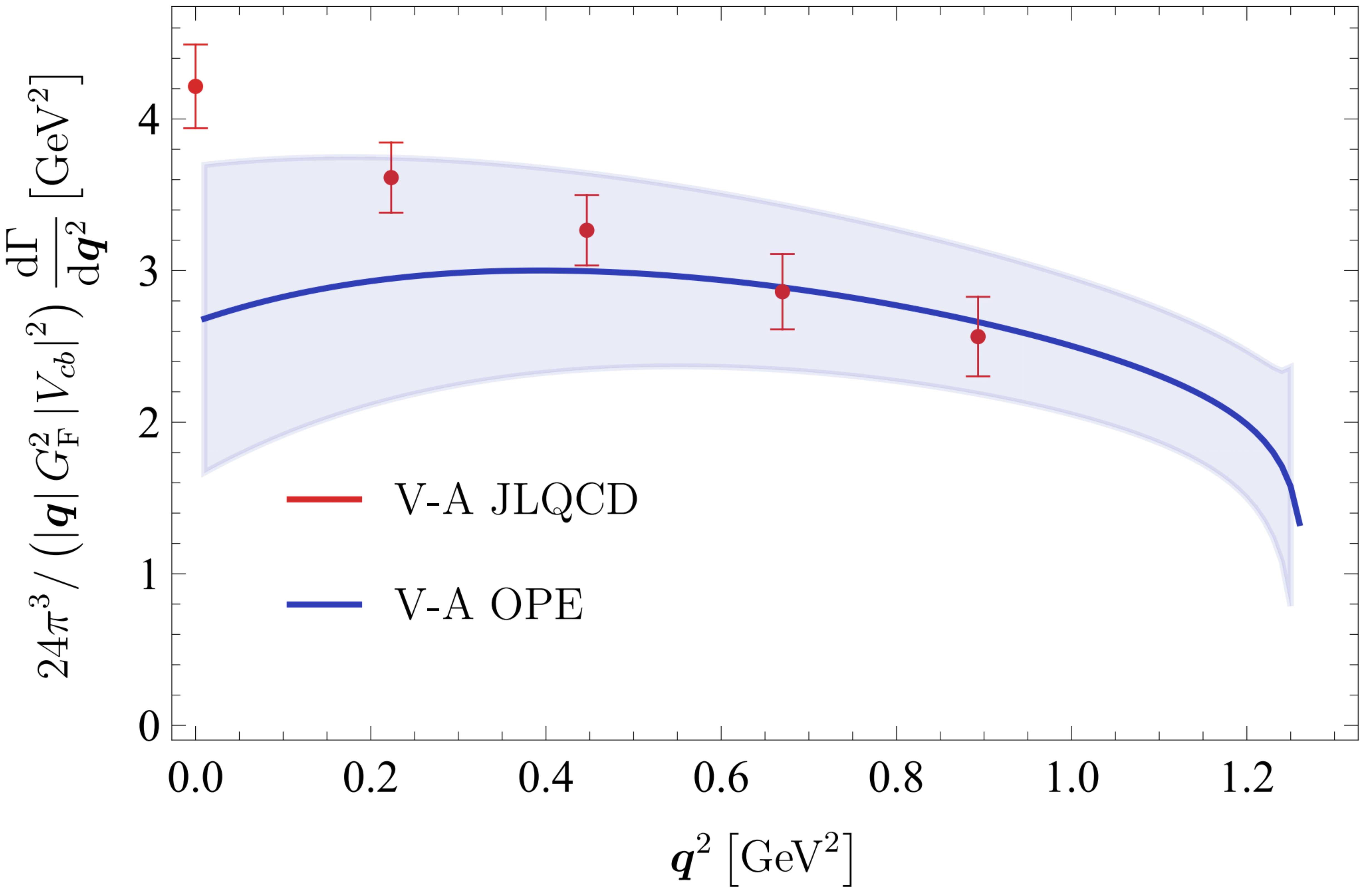
We performed two OPE calculations, for JLQCD and ETMC kinematics.

Including corrections up to $\mathcal{O}\left(\Lambda_{\mathrm{QCD}}/m_b^3\right)$ and $\mathcal{O}\left(\alpha_s\right)$ we find an excellent agreement between lattice and OPE:



Results

$\mathbf{q}^2$-spectrum for JLQCD kinematics:



Results

OPE uncertainties cancel in ratios of observables → define differential moments

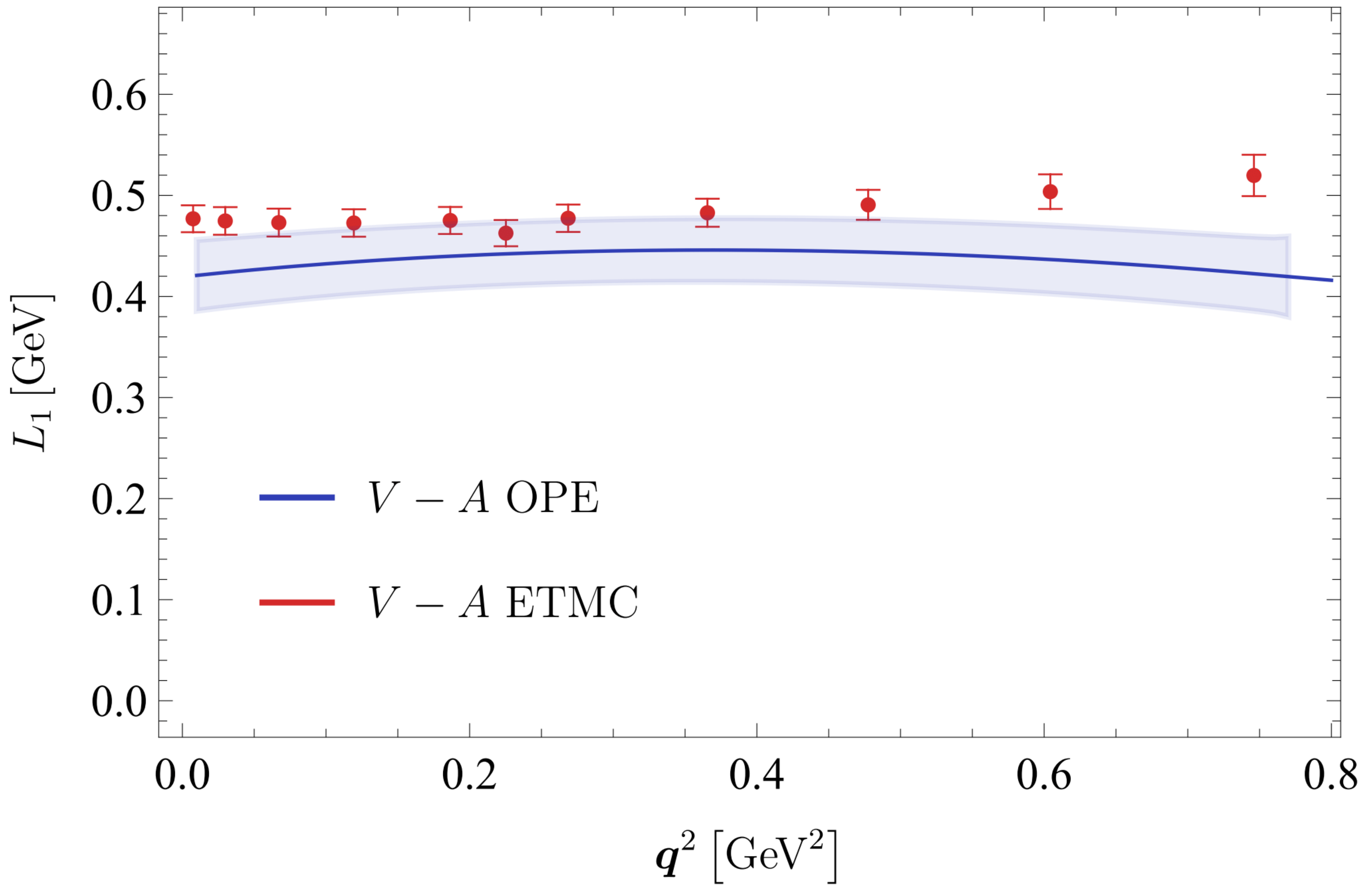
Hadronic invariant mass moments

$$H_n\left(\mathbf{q}^2\right) = \frac{\int \mathrm{d}\omega \mathrm{d}E_\ell \left(\omega^2 - \mathbf{q}^2\right)^n \left[\frac{\mathrm{d}\Gamma}{\mathrm{d}\mathbf{q}^2 \mathrm{d}\omega \mathrm{d}E_\ell}\right]}{\int \mathrm{d}\omega \mathrm{d}E_\ell \left[\frac{\mathrm{d}\Gamma}{\mathrm{d}\mathbf{q}^2 \mathrm{d}\omega \mathrm{d}E_\ell}\right]}$$

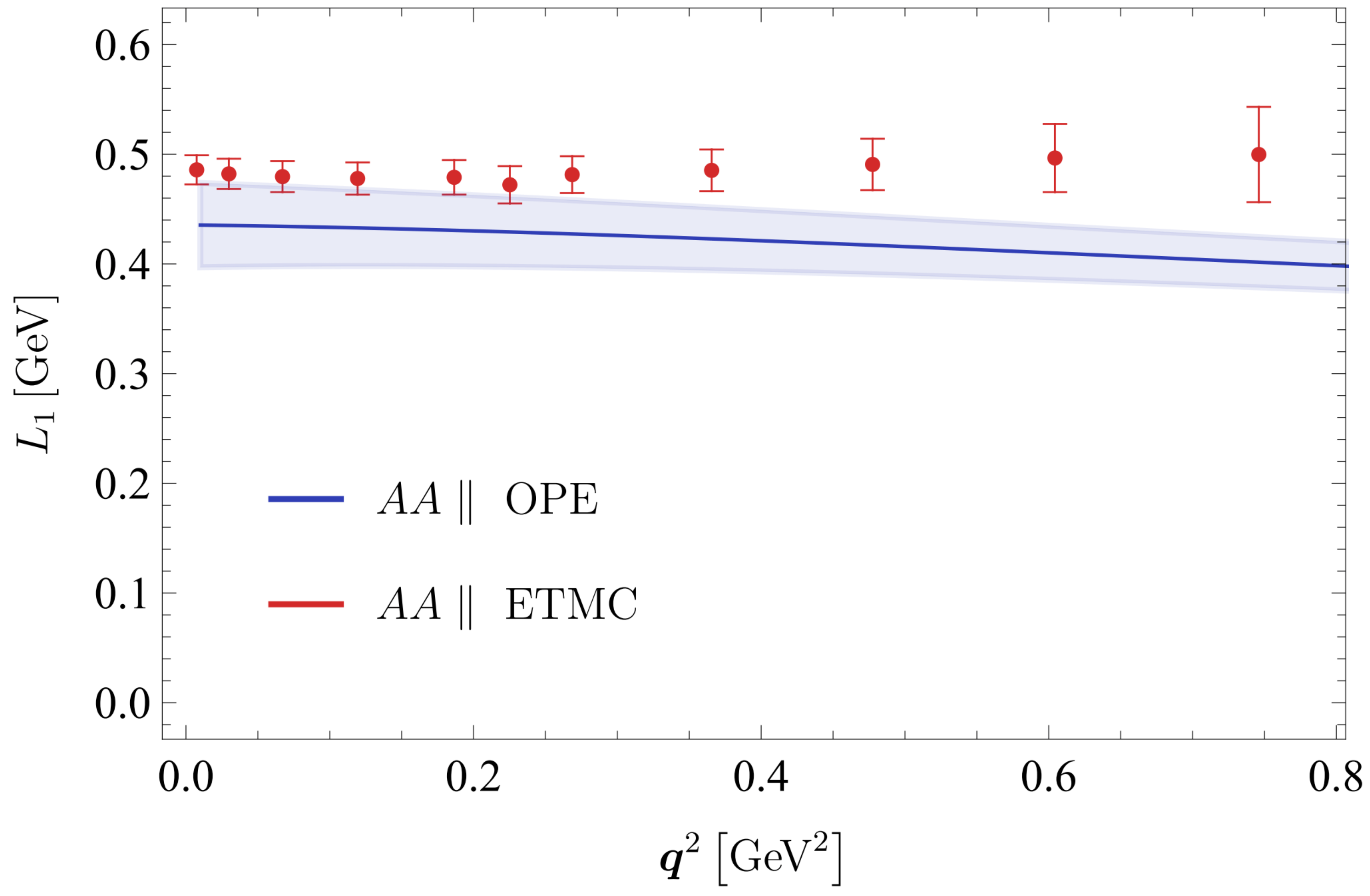
Lepton energy moments

$$L_{n_\ell}\left(\mathbf{q}^2\right) = \frac{\int \mathrm{d}\omega \mathrm{d}E_\ell E_\ell^{n_\ell} \left[\frac{\mathrm{d}\Gamma}{\mathrm{d}\mathbf{q}^2 \mathrm{d}\omega \mathrm{d}E_\ell}\right]}{\int \mathrm{d}\omega \mathrm{d}E_\ell \left[\frac{\mathrm{d}\Gamma}{\mathrm{d}\mathbf{q}^2 \mathrm{d}\omega \mathrm{d}E_\ell}\right]}$$

Results



0.6
0.5
0.4
0.3
0.2
0.1
0.0
L_1 [GeV]
$V - A$ OPE
$V - A$ ETMC
0.0
0.2
0.4
0.6
0.8
q^2 [GeV^2]



0.6
0.5
0.4
0.3
0.2
0.1
0.0
L_1 [GeV]
$AA \parallel$ OPE
$AA \parallel$ ETMC
0.0
0.2
0.4
0.6
0.8
q^2 [GeV^2]

Results

The last thing left to do is to compute the fully integrated moments:

	ETMC	OPE
$\Gamma/\lvert V_{cb}^2\rvert \times 10^{13}$ (GeV)	$0.987(60)$	$1.20(46)$
$\langle E_\ell \rangle$ (GeV)	$0.491(15)$	$0.441(43)$
$\langle E_\ell^2 \rangle$ (GeV ^{2})	$0.263(16)$	$0.207(49)$
$\langle E_\ell^2 \rangle - \langle E_\ell \rangle^2 (\text{GeV}^2)$	$0.022(16)$	$0.020(8)$
$\langle M_X^2 \rangle$ (GeV^2)	$3.77(9)$	$4.32(56)$

Summary & Outlook

- We have calculated the semileptonic decay rate and moments at unphysical m_b
- Good agreement with the OPE.
- Next steps:
  - Perform a lattice calculation at different values of lattice spacing, volumes and b-quark masses to perform extrapolations $a \to 0, V \to \infty, m_b \to m_b^{\text{phys}}$.
  - Calculation by Southampton group underway, using RBC/UKQCD ensembles
  - Apply this method to inclusive D-meson decays for direct comparison to experiment.
  - Extend the method to decays like $B \to X_u \ell \nu_\ell$ to extract V_{ub}.

Thanks for your attention!

Backup

Inputs of the OPE calculation

m_b^{kin} (JLQCD)	2.70 ± 0.04
$\overline{m}_c(2\ \text{GeV})$ (JLQCD)	1.10 ± 0.02
m_b^{kin} (ETMC)	2.39 ± 0.08
$\overline{m}_c(2\ \text{GeV})$ (ETMC)	1.19 ± 0.04
μ_π^2	0.57 ± 0.15
ρ_D^3	0.22 ± 0.06
$\mu_G^2(m_b)$	0.37 ± 0.10
ρ_{LS}^3	-0.13 ± 0.10
$\alpha_s^{(4)}(2\ \text{GeV})$	0.301 ± 0.006

Backup

Comparison with smooth kernel (ETMC: $\sigma = 0.12$, JLQCD: $\sigma = 0.1$)