

# The BOSS bispectrum analysis at one loop from the EFT of LSS

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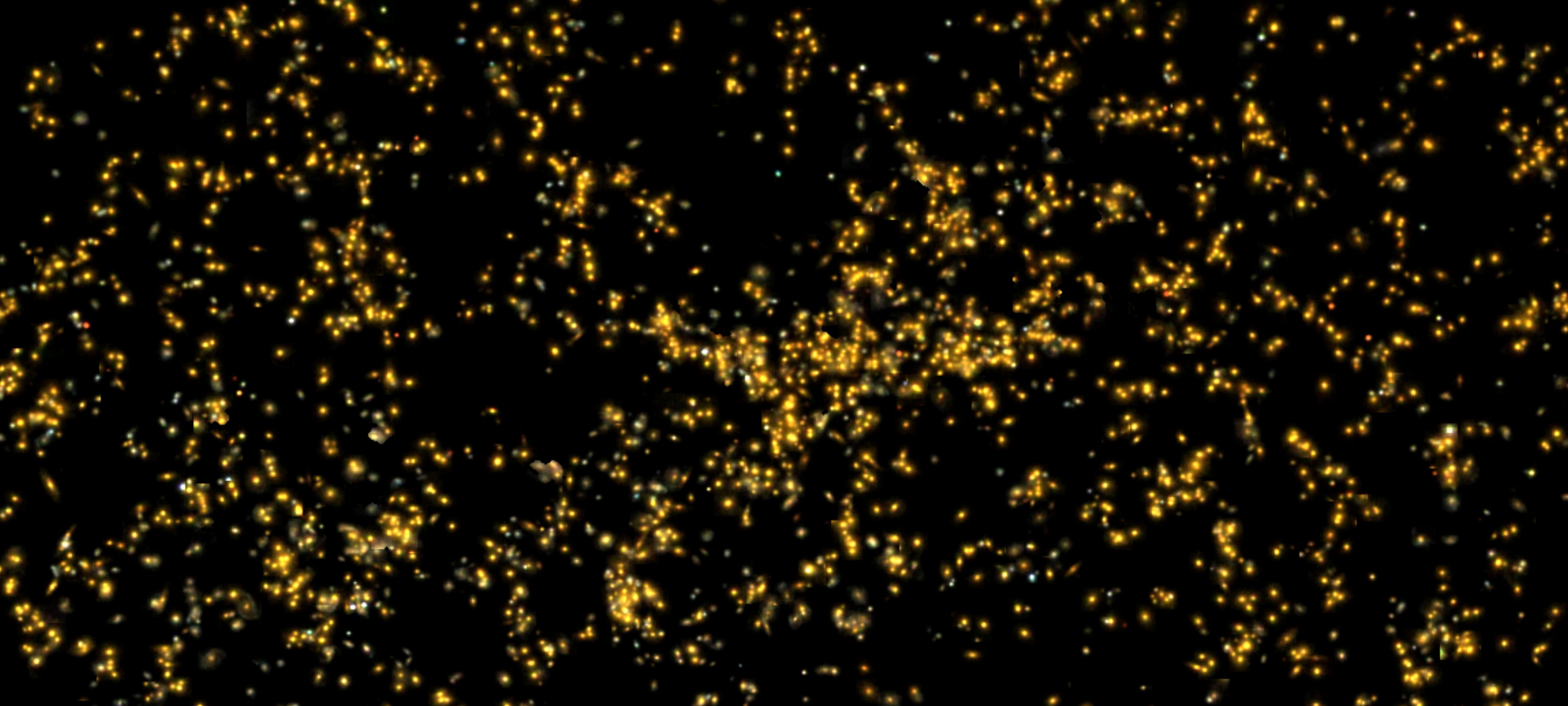
CIPANP, 08/31/22

# outline

- review of EFT of LSS
- recent progress in BOSS analysis from the EFT of LSS
- the road ahead

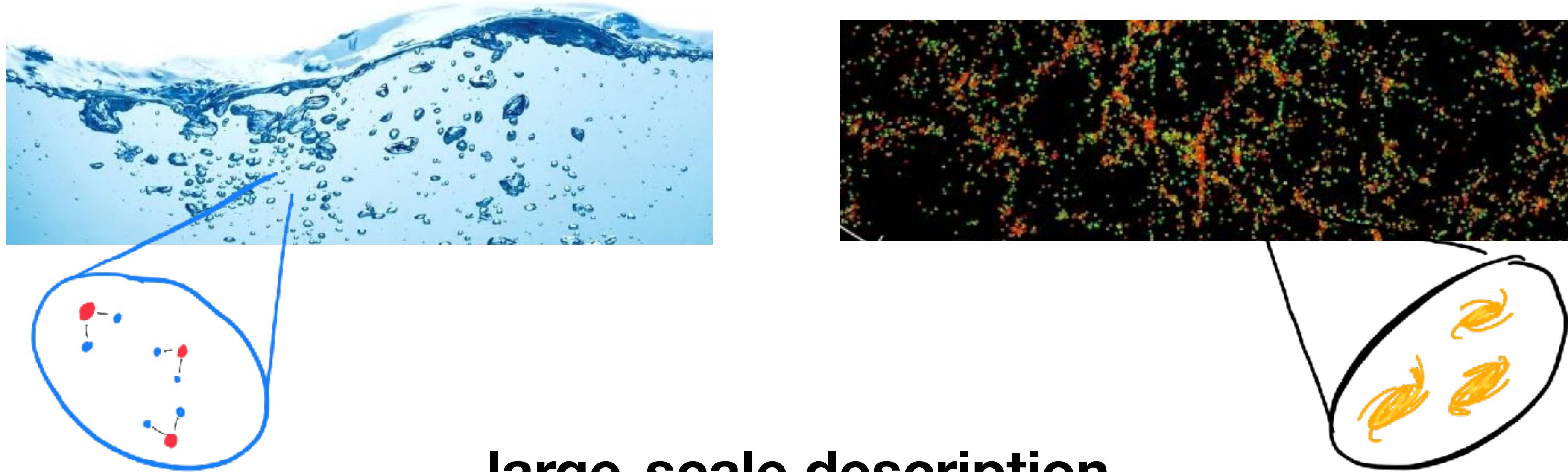
in collaboration with

G. D'Amico, Y. Donath, L.  
Senatore, P. Zhang, D.  
Bragança, D. Sekera, R.  
Sgier, ...



EFT of LSS review

# main idea



viscosity  $\nu$ , ...

EFT parameters  $c_s^2$ , ...

**expansion parameter**

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$$\frac{\lambda_{\text{mfp}}}{L_{\text{obs}}}$$

$$\frac{L_{\text{NL}}}{L_{\text{obs}}} \sim \frac{k_{\text{obs}}}{k_{\text{NL}}}$$

# low energy DOF, scales

non-relativistic, sub-horizon, fluid-like system in an expanding universe

$$ds^2 = -(1 + 2\Phi)dt^2 + a(t)^2(1 - 2\Psi)d\vec{x}^2 \quad \frac{\dot{a}}{a} \equiv H$$

$$\delta(t, \vec{x}) \equiv \frac{\delta\rho(t, \vec{x})}{\bar{\rho}(t)} \quad \delta\rho(t, \vec{x}) \equiv \rho(t, \vec{x}) - \bar{\rho}(t)$$

$$v^i(t, \vec{x}) \quad v \ll 1 \quad \text{non-relativistic}$$

$$\frac{k}{aH} \gg 1$$

sub-horizon  
(Newtonian limit)

$$k_{\text{NL}}^{-1} \sim \frac{v}{aH}$$

finite age of universe  
(like mean free path)

$$k/k_{\text{NL}} \lesssim 1$$

EFT expansion param.

# dark matter EOM

$$\{\rho, v^i, \Phi\} \quad a^{-2}\partial^2\Phi = \frac{3}{2}H^2\Omega_m\delta$$

- mass conservation
- momentum conservation
- Galilean invariance

$$x^i \rightarrow x^i + n^i(t)$$
$$t \rightarrow t + a^2 n^i(t) x^i$$

$$\partial_i \rightarrow \partial_i \quad \partial_t \rightarrow \partial_t - \dot{n}^i(t)\partial_i \quad \rho \rightarrow \rho$$

$$\Phi \rightarrow \Phi - a^2(\ddot{n}^i(t) + 2H\dot{n}^i(t))x^i \quad v^i \rightarrow v^i + a\dot{n}^i(t)$$

$$\dot{\delta} + a^{-1}\partial_i((1+\delta)v^i) = 0$$

EFT of LSS

$$\partial_i \dot{v}^i + H\partial_i v^i + a^{-1}\partial^2\Phi + a^{-1}\partial_i(v^j\partial_j v^i) = -a^{-1}\partial_i(\partial_j \tau^{ij}/\rho)$$

Baumann, Nicolis,  
Senatore, Zaldarriaga 12  
Carrasco, Hertzberg,  
Senatore 12

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$$\tau^{ij} \sim c_s^2 \delta^{ij} \delta + c_2 \partial_j v^j + c_3 \partial_i \delta \partial_j \delta + c_4 \partial_i \partial_j \delta + \dots$$

$$x^i \rightarrow x^i + n^i(t) \\ t \rightarrow t + a^2 n^i(t) x^i$$

$$\dot{\delta} + a^{-1}\partial_i((1+\delta)v^i) = 0$$

$$\partial_i \dot{v}^i + H\partial_i v^i + a^{-1}\partial^2\Phi + a^{-1}\partial_i(v^j\partial_j v^i) = -a^{-1}\partial_i(\partial_j \tau^{ij}/\rho)$$

EFT of LSS

Baumann, Nicolis,  
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Senatore 12

# observables

observables - correlation functions

---

$$\langle \delta(\vec{k}_1) \delta(\vec{k}_2) \rangle = (2\pi)^3 \delta_D(\vec{k}_1 + \vec{k}_2) P(k_1) \quad \text{power spectrum}$$

$$\langle \delta(\vec{k}_1) \delta(\vec{k}_2) \delta(\vec{k}_3) \rangle = (2\pi)^3 \delta_D(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B(k_1, k_2, k_3) \quad \text{bispectrum}$$

perturbative expansion

---

$$P = P_{\text{linear}} + P_{\text{1-loop}} + P_{\text{2-loop}} + \dots$$

$$B = B_{\text{linear}} + B_{\text{1-loop}} + B_{\text{2-loop}} + \dots$$

# perturbative expansion

ansatz:  $\delta_{\vec{k}}(a) = \sum_n \left(\frac{a}{a_0}\right)^n \delta_{\vec{k}}^{(n)}$

general form of the solutions:

$$\delta_{\vec{k}}^{(n)} = \int_{\vec{k}_1, \dots, \vec{k}_n} (2\pi)^3 \delta_D(\vec{k} - \sum_i \vec{k}_i) F_n(\vec{k}_1, \dots, \vec{k}_n) \delta_{\vec{k}_1}^{(1)} \dots \delta_{\vec{k}_n}^{(1)}$$

power spectra:

$$\langle \delta_{\vec{k}}^{(1)} \delta_{\vec{k}'}^{(1)} \rangle = (2\pi)^3 \delta_D(\vec{k} + \vec{k}') P_{11}(k)$$

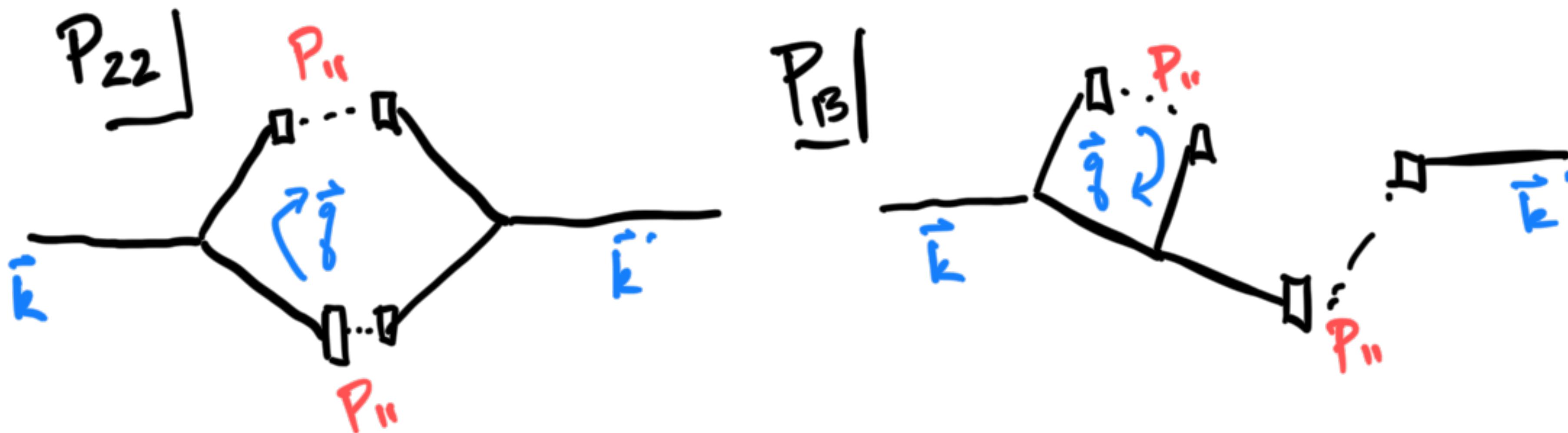
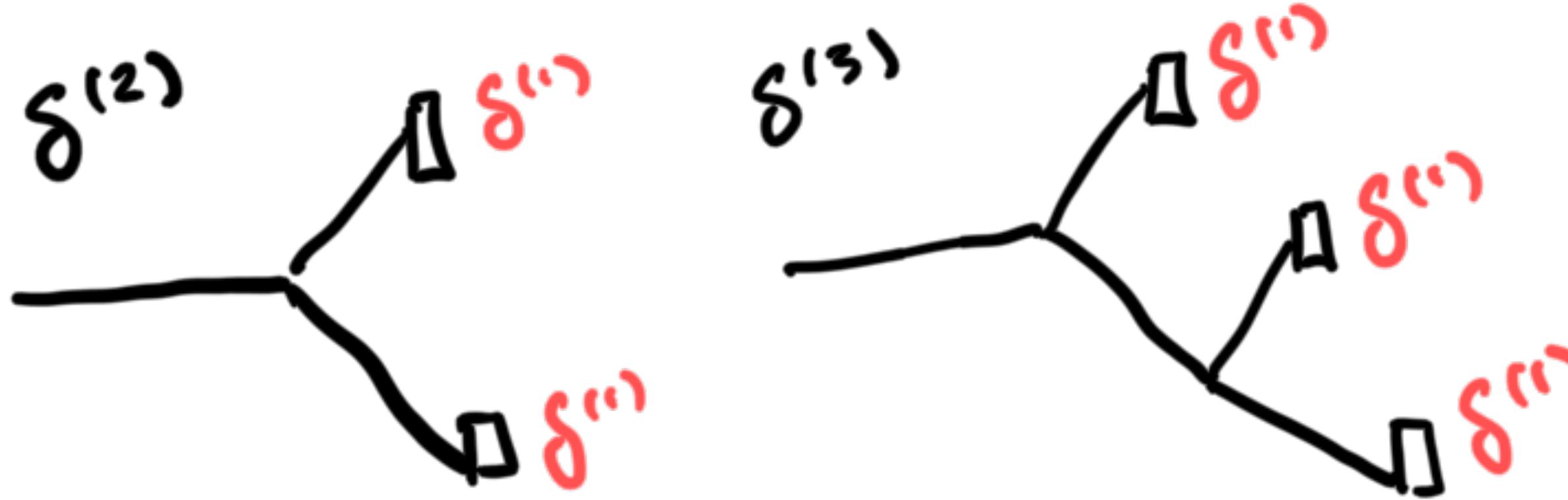
linear

$$\langle \delta_{\vec{k}}^{(2)} \delta_{\vec{k}'}^{(2)} \rangle = (2\pi)^3 \delta_D(\vec{k} + \vec{k}') P_{22}(k)$$

first corrections,  
one-loop terms

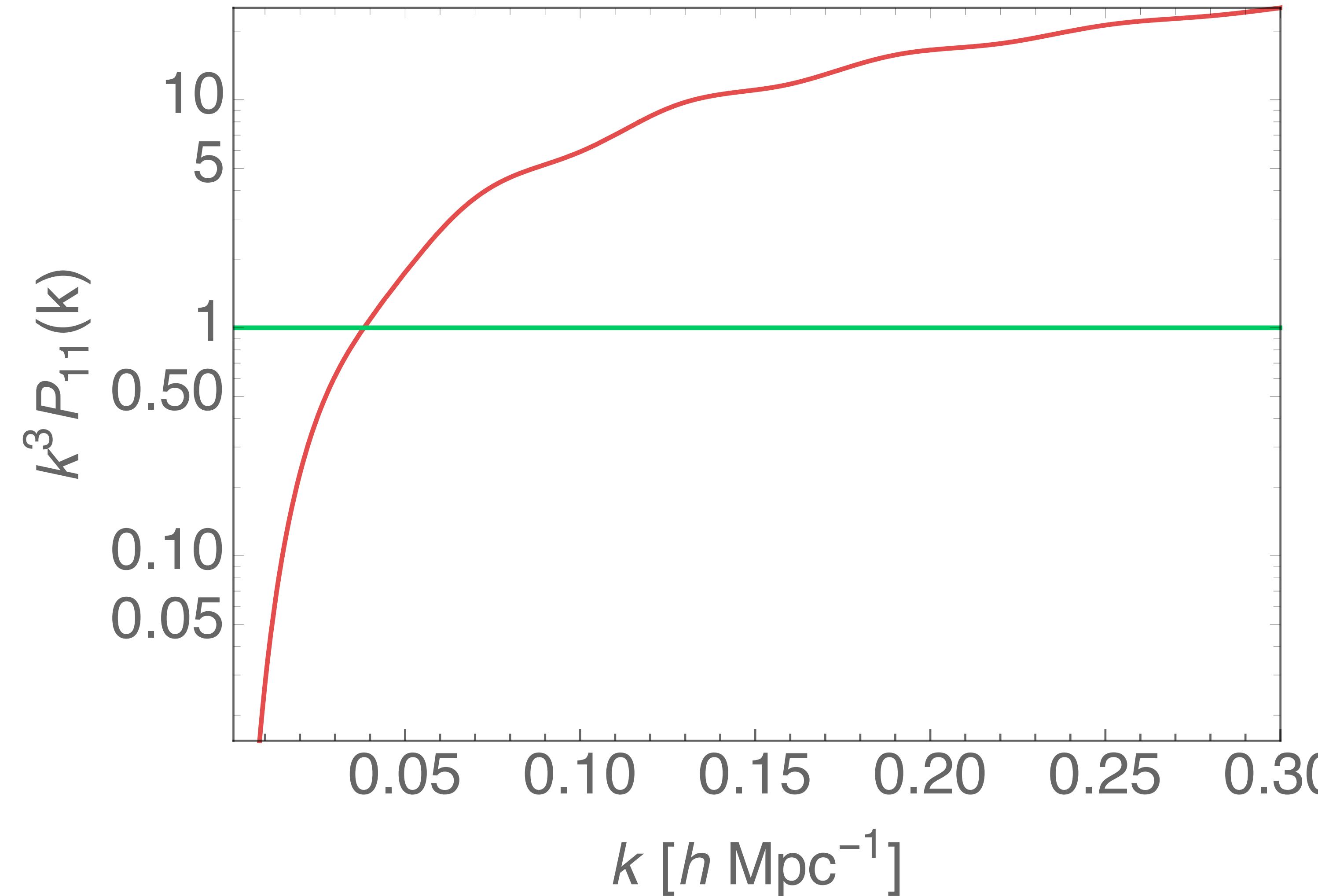
$$2 \langle \delta_{\vec{k}}^{(1)} \delta_{\vec{k}'}^{(3)} \rangle = (2\pi)^3 \delta_D(\vec{k} + \vec{k}') P_{13}(k)$$

# diagrams



# UV structure

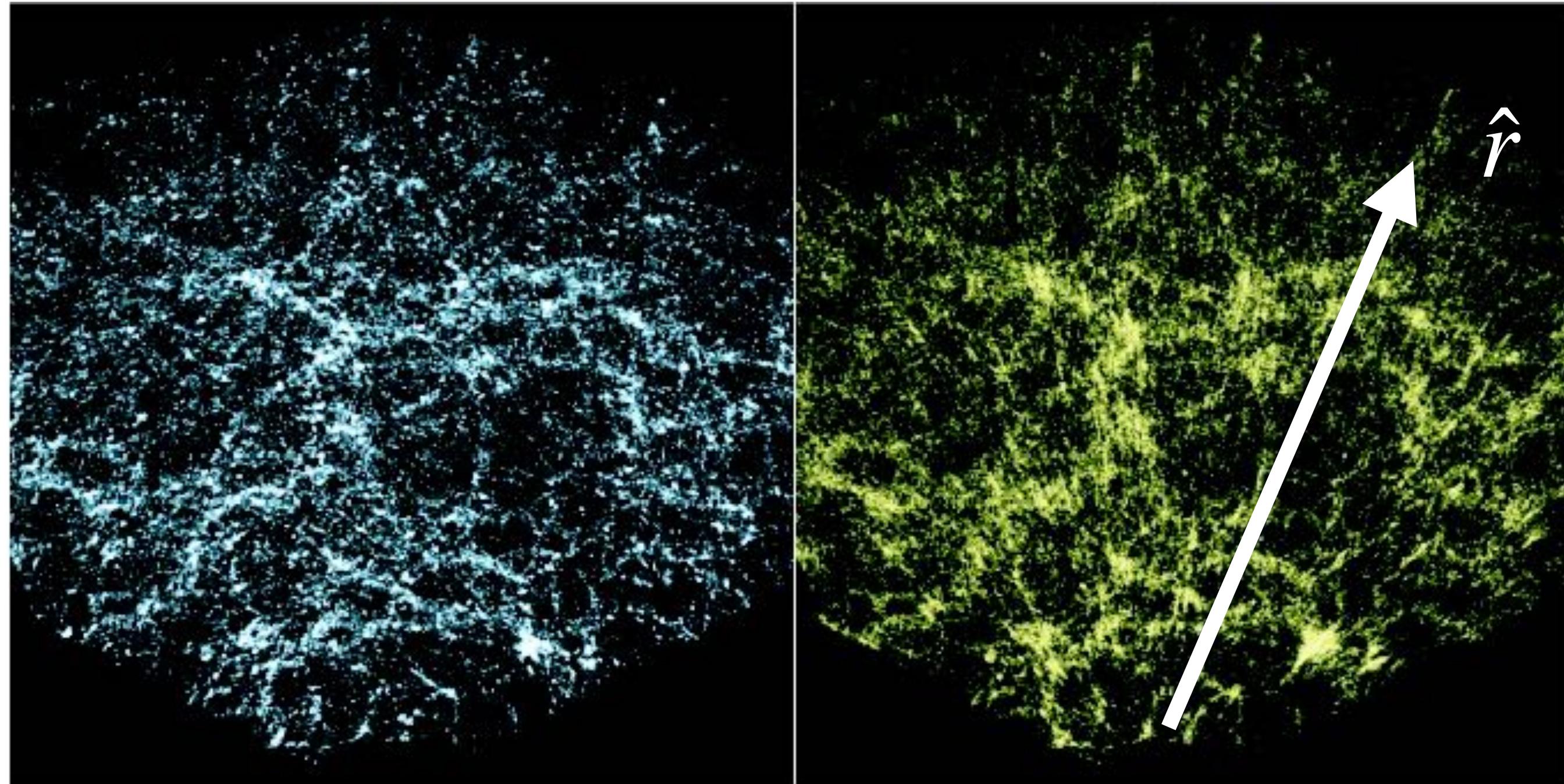
$$P_{13}(k) \rightarrow -\frac{61}{630\pi^2} k^2 P_{11}(k) \int^{\Lambda_{\text{UV}}} dq P_{11}(q)$$



# redshift-space distortions in the EFT

Senatore, Zaldarriaga 14

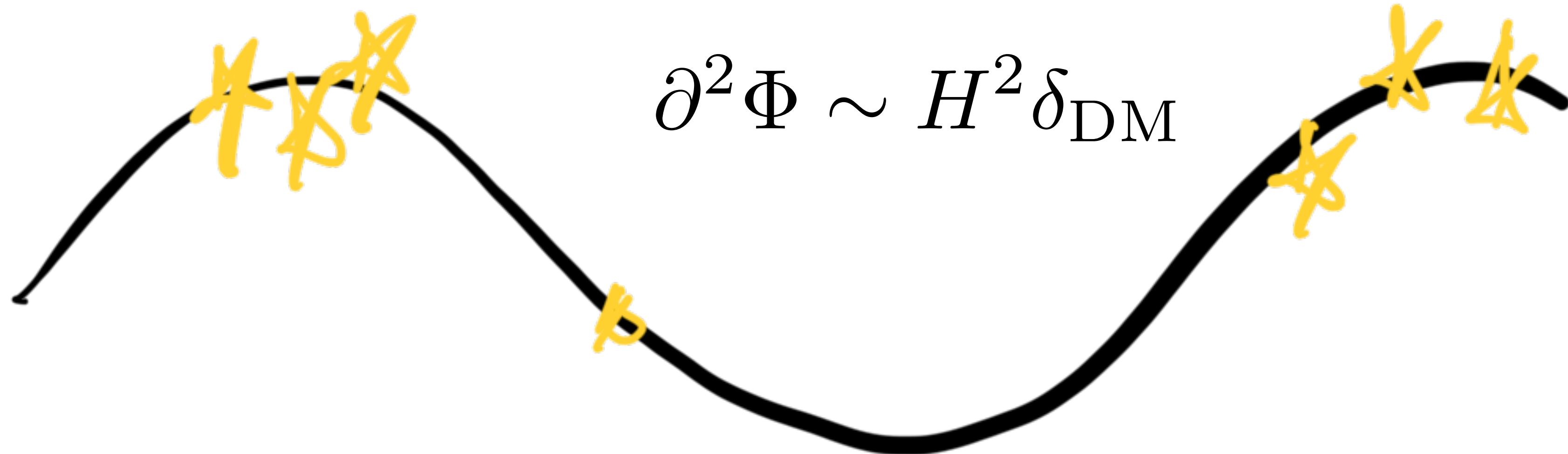
ML, Senatore, et. al. 18



isotropic in  $\vec{x}(z)$   
 $z$  of the Hubble flow

anisotropic in  
 $\vec{x}_r(z) \equiv \vec{x}(z_{obs}(z)) \approx \vec{x}(z) + \frac{\hat{r} \cdot \vec{v}}{aH} \hat{r}$

# galaxy bias



$$\delta_{\text{galaxy}}(\vec{x}, t) = \int^t dt' H(t') f_{\text{galaxy}} (\partial_i \partial_j \Phi(\vec{x}_{\text{fl}}, t'), \partial_i v_j(\vec{x}_{\text{fl}}, t'), \epsilon(\vec{x}_{\text{fl}}, t'), \dots; t')$$

some very complicated dependence, integrated  
over past light cone

perturbation  
theory



$$\delta_{\text{galaxy}} \sim b_1 \delta_{\text{DM}} + b_2 \delta_{\text{DM}}^2 + b_3 \partial_i \delta_{\text{DM}} \partial_i \delta_{\text{DM}}$$

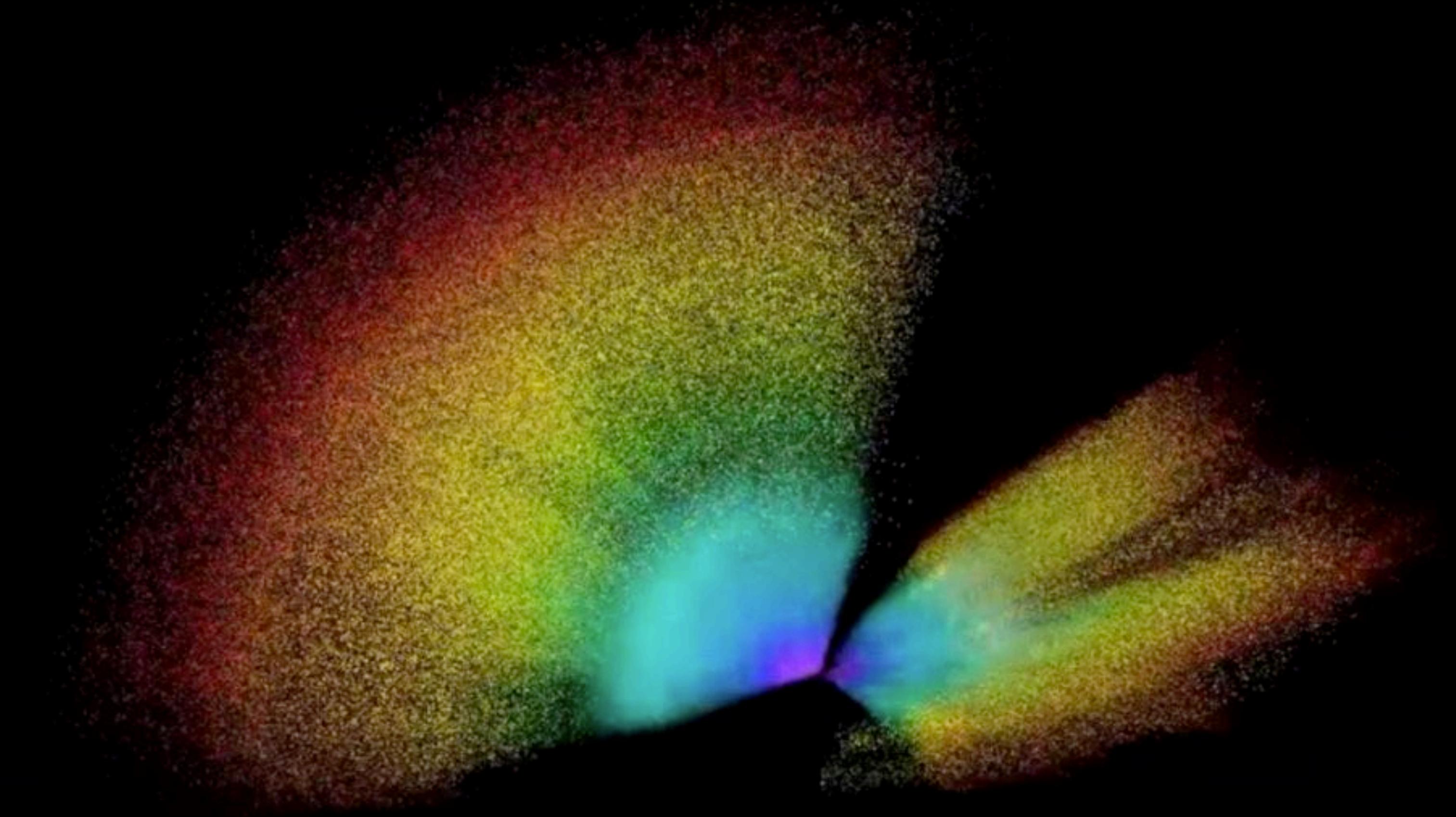
McDonald 06

McDonald, Roy 10

Senatore 15

D'Amico, Donath, **ML**,  
Senatore, Zhang 22

D'Amico, Donath, **ML**,  
Senatore, Zhang [to appear]



the BOSS analysis

# the BOSS analysis

## original power spectrum

- D'Amico, Gleyzes, Kokron, Markovic, Senatore, Zhang, Beutler, Gil-Marín 19
- Ivanov, Simonović, Zaldarriaga 19

## tree-level bispectrum

- D'Amico, Gleyzes, Kokron, Markovic, Senatore, Zhang, Beutler, Gil-Marín 19
- Philcox, Ivanov 21

## time for one-loop bispectrum

- data analysis    D'Amico, Donath, **ML**, Senatore, Zhang 22  
                      Philcox, Ivanov, Cabass, Simonović, Zaldarriaga, Nishimichi 22
- theory model    D'Amico, Donath, **ML**, Senatore, Zhang 22  
                      Philcox, Ivanov, Cabass, Simonović, Zaldarriaga, Nishimichi 22  
                      D'Amico, Donath, **ML**, Senatore, Zhang [to appear]
- theory integration    Philcox, Ivanov, Cabass, Simonović, Zaldarriaga, Nishimichi 22  
                      Anastasiou, Bragança, Senatore, Zheng [to appear]

# the BOSS analysis

one-loop power spectrum and bispectrum

one-loop power spectrum of halos in redshift space  
function of  $(k, \hat{k} \cdot \hat{z})$

$$P_{\text{1-loop tot.}}^{r,h} = P_{11}^{r,h} + (P_{13}^{r,h} + P_{13}^{r,h,ct}) + (P_{22}^{r,h} + P_{22}^{r,h,\epsilon})$$

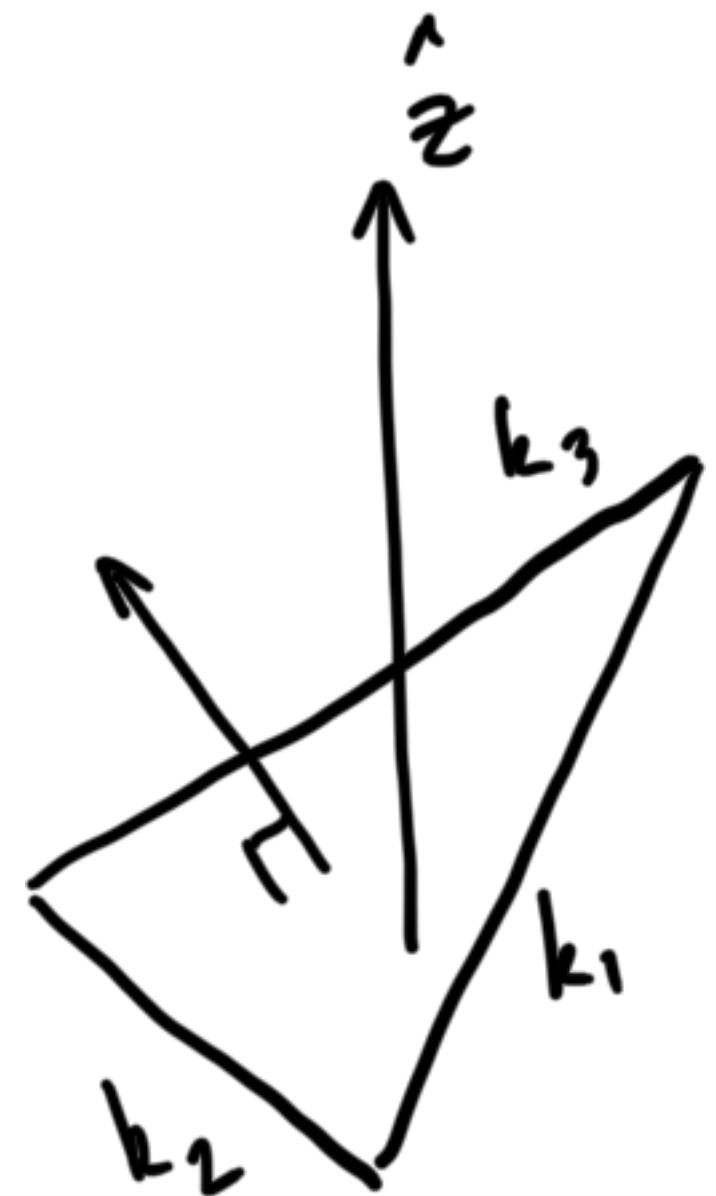
one-loop bispectrum of halos in redshift space  
function of  $(k_1, k_2, k_3, \hat{k}_1 \cdot \hat{z}, \hat{k}_2 \cdot \hat{z})$

$$\begin{aligned} B_{\text{1-loop tot.}}^{r,h} = & B_{211}^{r,h} + (B_{321}^{r,h,(II)} + B_{321}^{r,h,(II),ct}) + (B_{411}^{r,h} + B_{411}^{r,h,ct}) \\ & + (B_{222}^{r,h} + B_{222}^{r,h,\epsilon}) + (B_{321}^{r,h,(I)} + B_{321}^{r,h,(I),\epsilon}) \end{aligned}$$

Philcox, Ivanov, Cabass, Simonović,  
Zaldarriaga, Nishimichi 22

D'Amico, Donath, **ML**,  
Senatore, Zhang 22

$\hat{z}$  is line-of-sight direction



# the BOSS analysis

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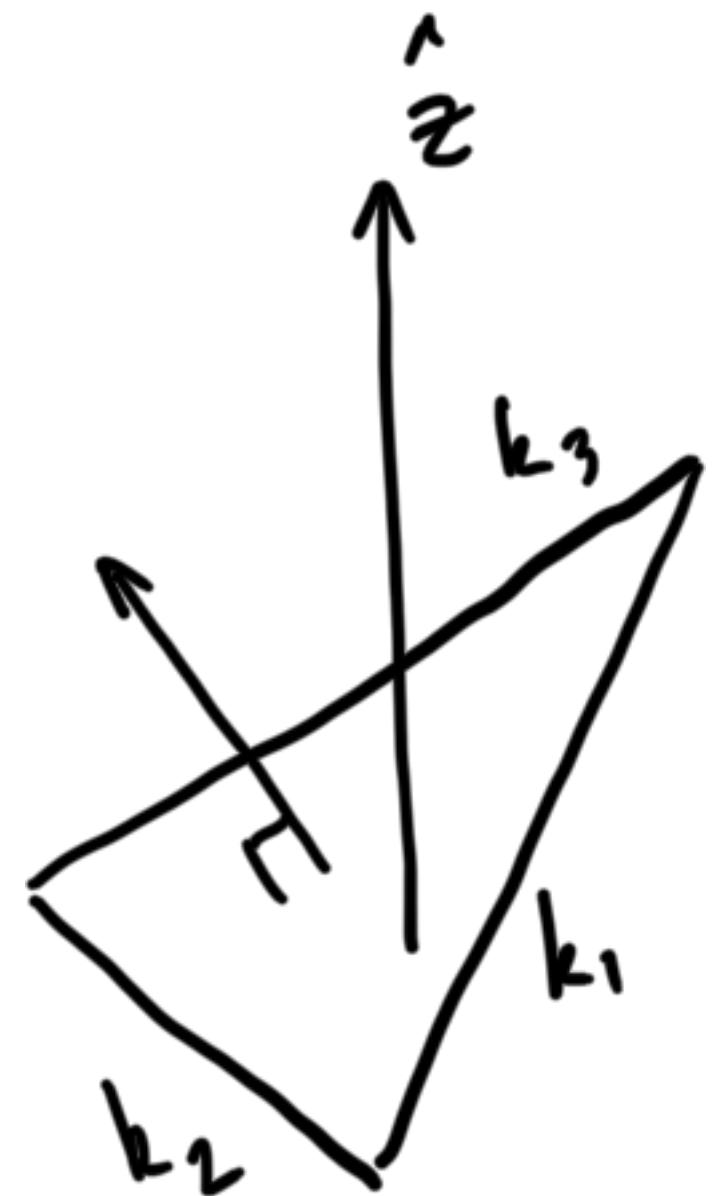
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linear

Philcox, Ivanov, Cabass, Simonović,  
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D'Amico, Donath, **ML**,  
Senatore, Zhang 22

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# the BOSS analysis

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$$\underline{\quad + (B_{222}^{r,h} + B_{222}^{r,h,\epsilon}) + (B_{321}^{r,h,(I)} + B_{321}^{r,h,(I),\epsilon})}$$

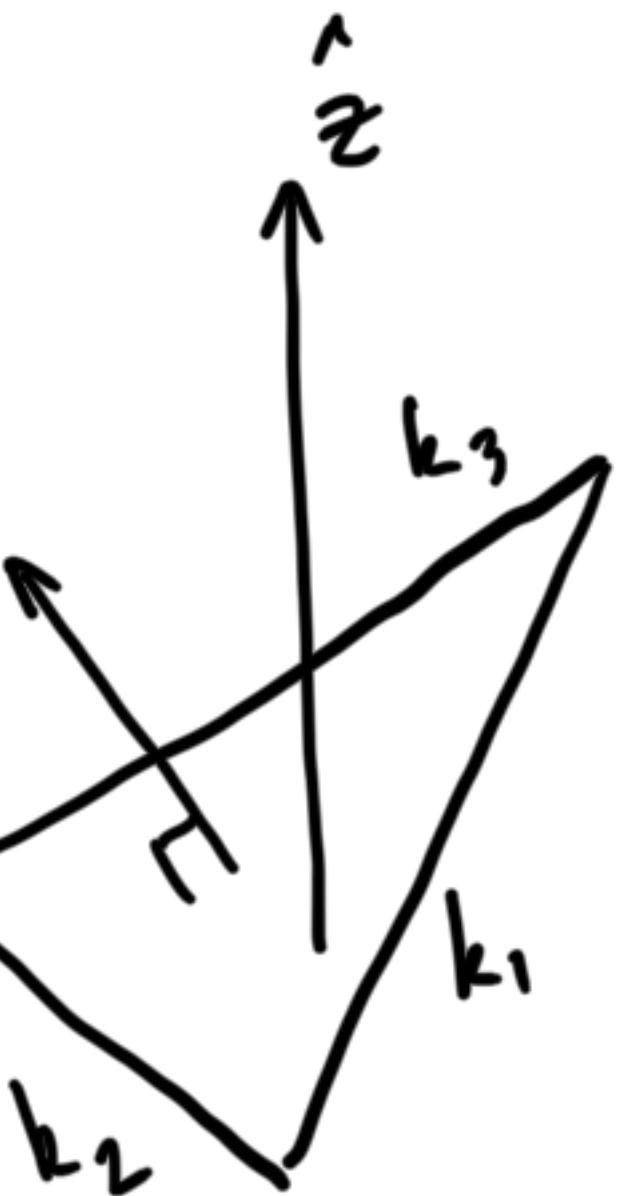
linear

one loop

Philcox, Ivanov, Cabass, Simonović,  
Zaldarriaga, Nishimichi 22

D'Amico, Donath, **ML**,  
Senatore, Zhang 22

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# the BOSS analysis

one-loop power spectrum and bispectrum

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linear

one loop

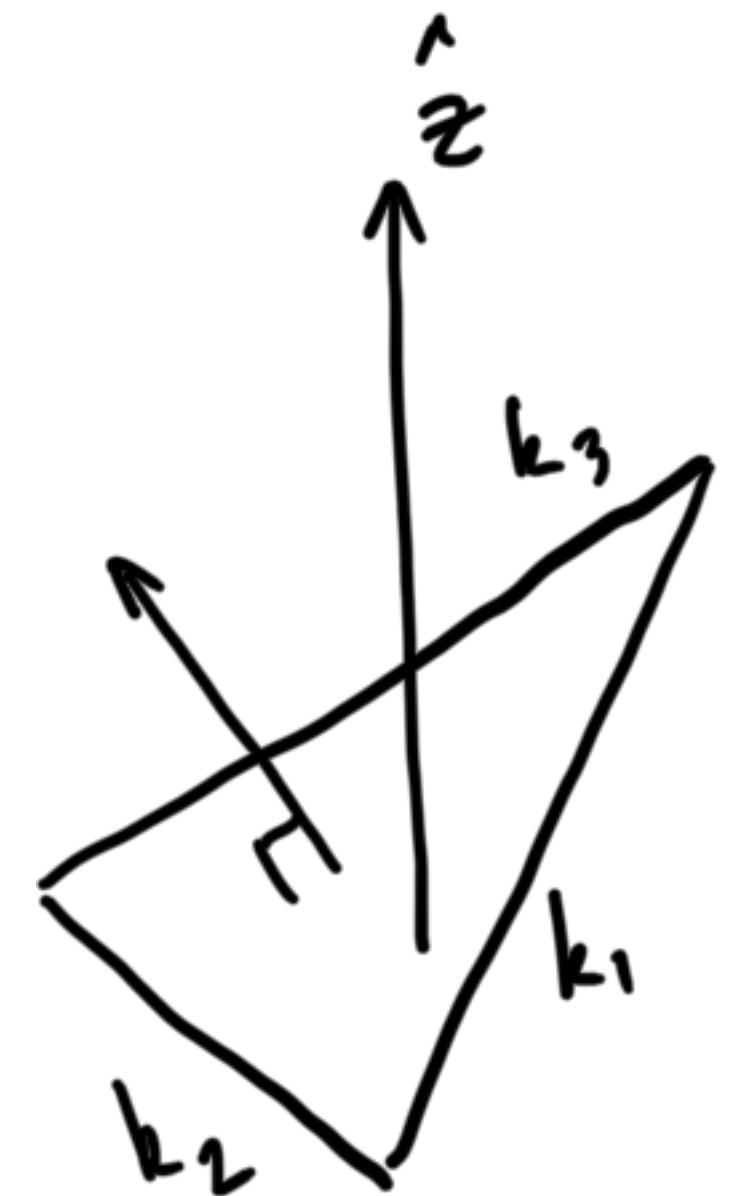
Philcox, Ivanov, Cabass, Simonović,  
Zaldarriaga, Nishimichi 22

D'Amico, Donath, **ML**,  
Senatore, Zhang 22

$\hat{z}$  is line-of-sight direction



**EFT of LSS  
counterterms**



# the BOSS analysis

## multipoles

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power spectrum -  $\ell = \{0,2\}$

$$P_\ell^{r,h}(k) = \frac{2\ell+1}{2} \int_{-1}^1 d\mu \mathcal{P}_\ell(\mu) P^{r,h}(k, \mu)$$

bispectrum monopole

$$B_0^{r,h}(k_1, k_2, k_3) = \frac{1}{4\pi} \int_{-1}^1 d\mu_1 \int_0^{2\pi} d\phi B^{r,h}(k_1, k_2, k_3, \mu_1, \mu_2(\mu_1, \phi))$$

bispectrum quadrupoles (tree level)

$$B_{(2,1)}^{r,h}(k_1, k_2, k_3) \equiv \frac{5}{4\pi} \int_{-1}^1 d\mu_1 \int_0^{2\pi} d\phi \mathcal{P}_2(\mu_1) B^{r,h}(k_1, k_2, k_3, \mu_1, \mu_2(\mu_1, \phi)) ,$$

$$B_{(2,2)}^{r,h}(k_1, k_2, k_3) \equiv \frac{5}{4\pi} \int_{-1}^1 d\mu_1 \int_0^{2\pi} d\phi \mathcal{P}_2(\mu_2(\mu_1, \phi)) B^{r,h}(k_1, k_2, k_3, \mu_1, \mu_2(\mu_1, \phi)) ,$$

$$B_{(2,3)}^{r,h}(k_1, k_2, k_3) \equiv \frac{5}{4\pi} \int_{-1}^1 d\mu_1 \int_0^{2\pi} d\phi \mathcal{P}_2(\mu_3(\mu_1, \phi)) B^{r,h}(k_1, k_2, k_3, \mu_1, \mu_2(\mu_1, \phi)) .$$

$$\mu_2(\mu_1, \phi) = \mu_1 \hat{k}_1 \cdot \hat{k}_2 + \sqrt{1 - \mu_1^2} \sqrt{1 - (\hat{k}_1 \cdot \hat{k}_2)^2} \sin \phi$$

$$\mu_3(\mu_1, \phi) = -k_3^{-1} (k_1 \mu_1 + k_2 \mu_2(\mu_1, \phi))$$

# the BOSS analysis

## parameters

$$P_{11}^{r,h}[b_1], \quad P_{13}^{r,h}[b_1, b_3, b_8], \quad P_{22}^{r,h}[b_1, b_2, b_5],$$

$$B_{211}^{r,h}[b_1, b_2, b_5], \quad B_{321}^{r,h,(II)}[b_1, b_2, b_3, b_5, b_8], \quad B_{411}^{r,h}[b_1, \dots, b_{11}],$$

$$B_{222}^{r,h}[b_1, b_2, b_5], \quad B_{321}^{r,h,(I)}[b_1, b_2, b_3, b_5, b_6, b_8, b_{10}],$$

$$P_{13}^{r,h,ct}[b_1, c_{h,1}, c_{\pi,1}, c_{\pi v,1}, c_{\pi v,3}], \quad P_{22}^{r,h,\epsilon}[c_1^{\text{St}}, c_2^{\text{St}}, c_3^{\text{St}}],$$

$$B_{321}^{r,h,(II),ct}[b_1, b_2, b_5, c_{h,1}, c_{\pi,1}, c_{\pi v,1}, c_{\pi v,3}],$$

$$B_{411}^{r,h,ct}[b_1, \{c_{h,i}\}_{i=1,\dots,5}, c_{\pi,1}, c_{\pi,5}, \{c_{\pi v,j}\}_{j=1,\dots,7}],$$

$$B_{321}^{r,h,(I),\epsilon}[b_1, c_1^{\text{St}}, c_2^{\text{St}}, \{c_i^{\text{St}}\}_{i=4,\dots,13}], \quad B_{222}^{r,h,\epsilon}[c_1^{(222)}, c_2^{(222)}, c_5^{(222)}]$$

scanning  $\{\Omega_m, H_0, \sigma_8\}$

BBN prior on  $\Omega_b h^2$ , Planck fixed  $n_s$

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Senatore, Zhang 22

11 bias parameters

14 response counterterms  
16 stochastic counterterms

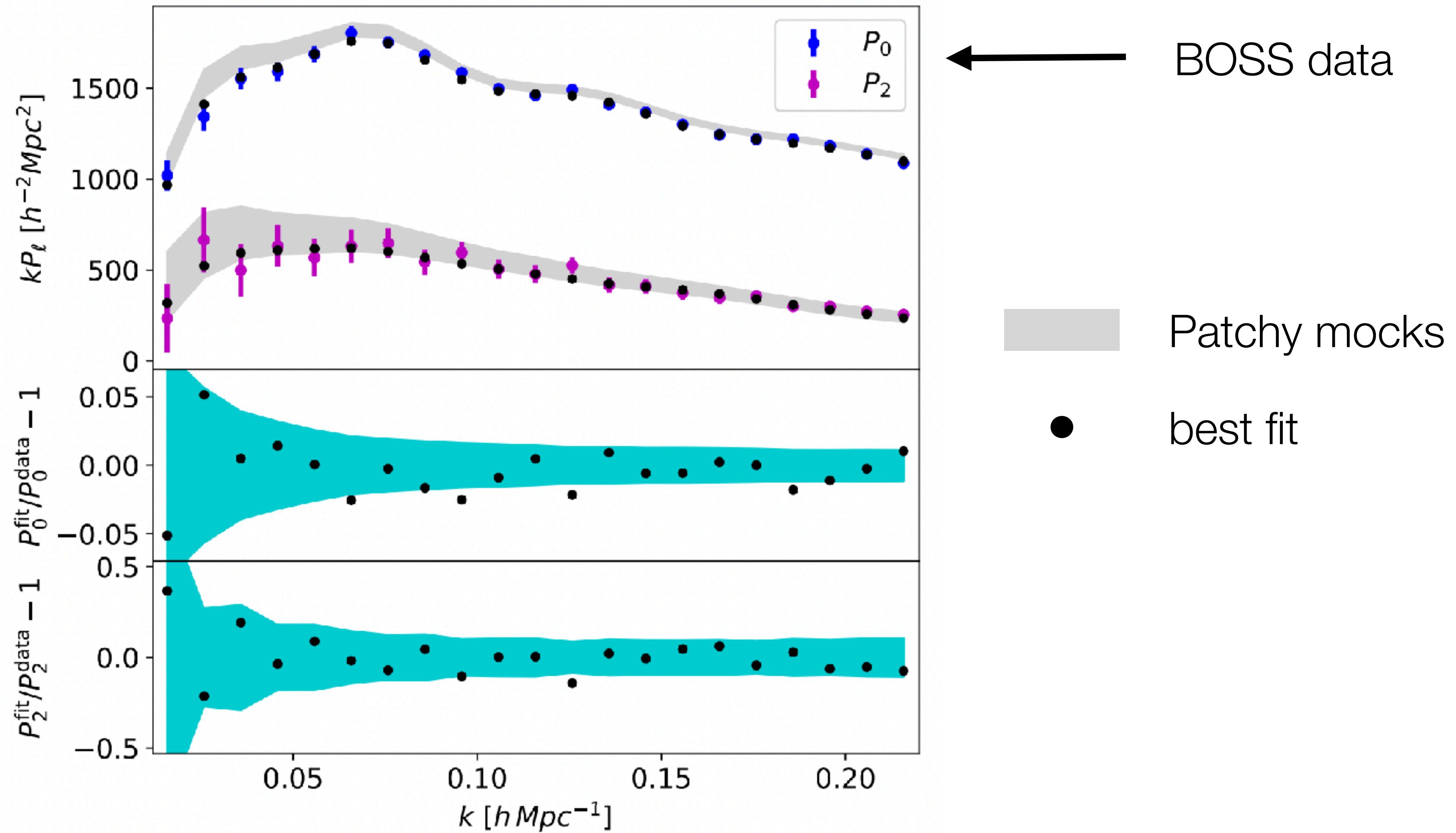
cosmological parameters

# the BOSS analysis

data - power spectrum

BOSS DR12 LRG sample

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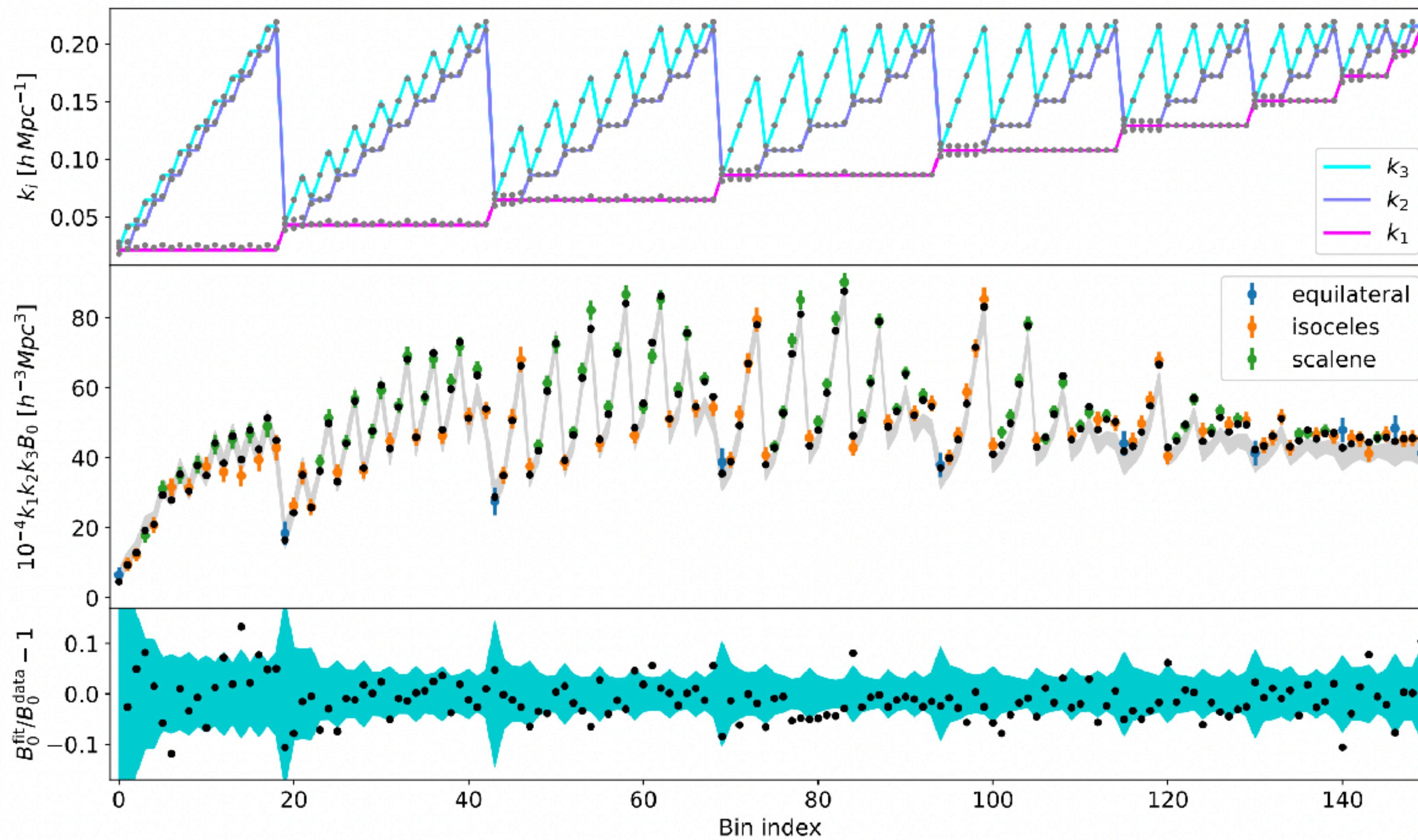


# the BOSS analysis

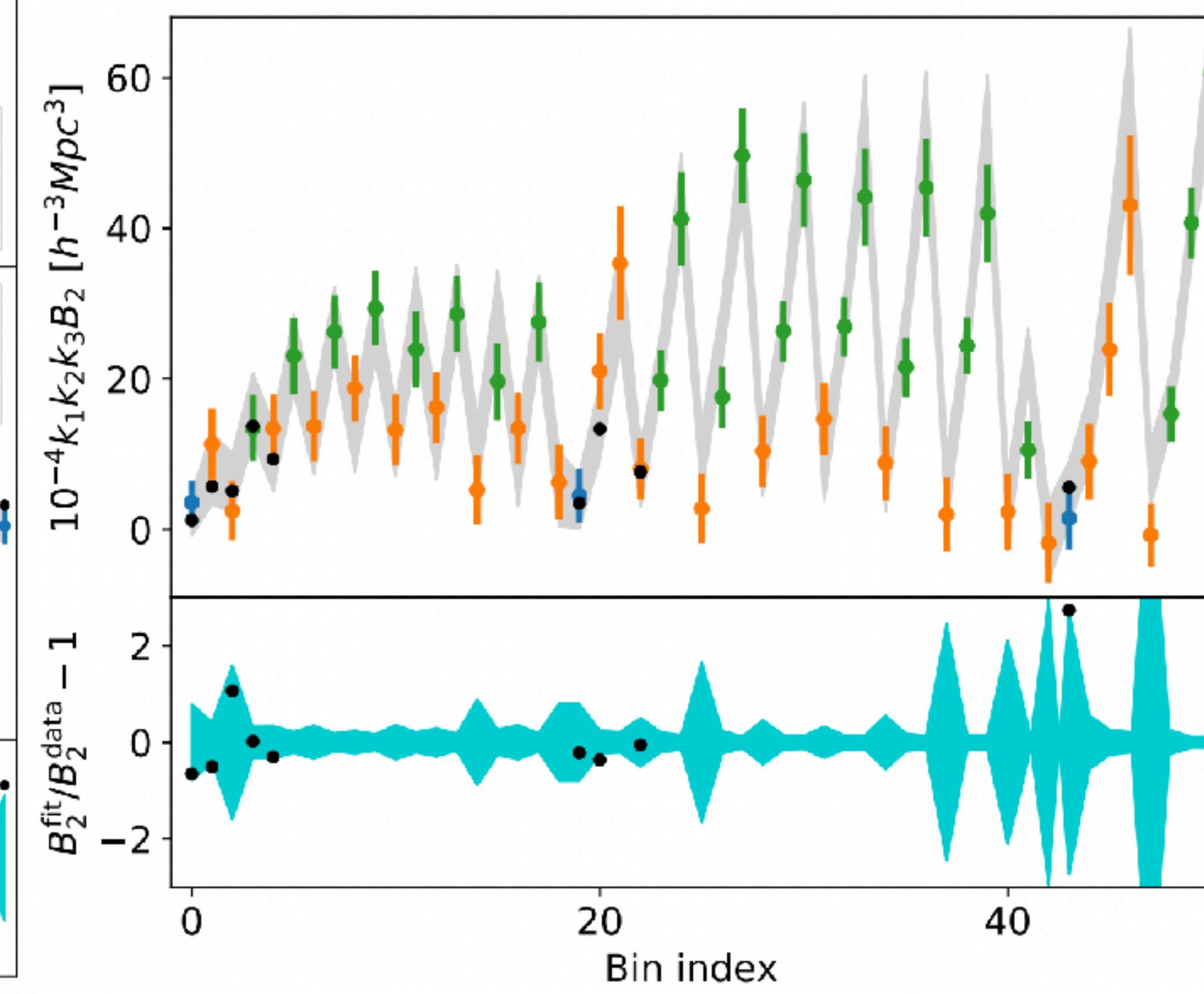
data - bispectrum

BOSS DR12 LRG sample

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Senatore, Zhang 22



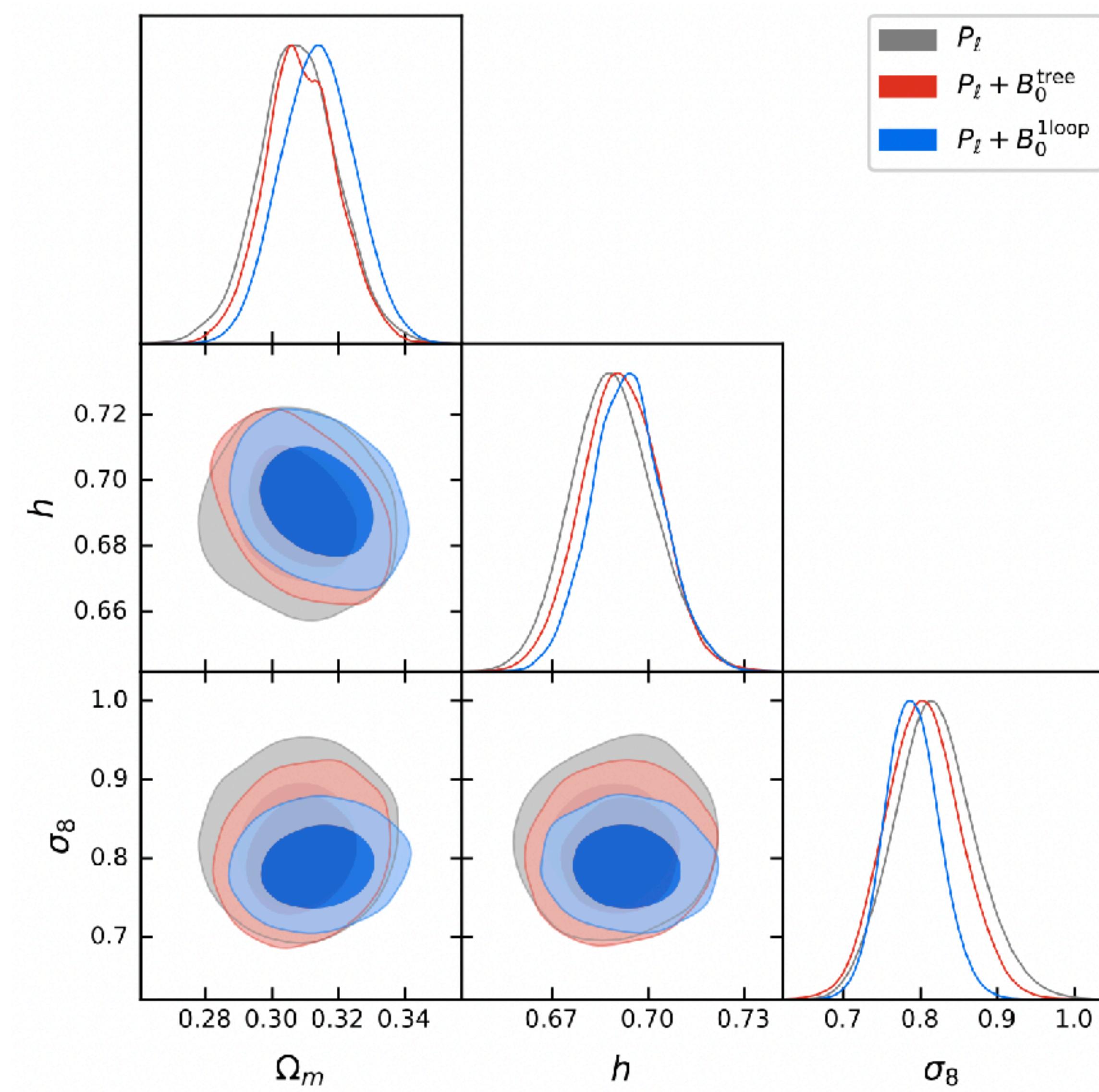
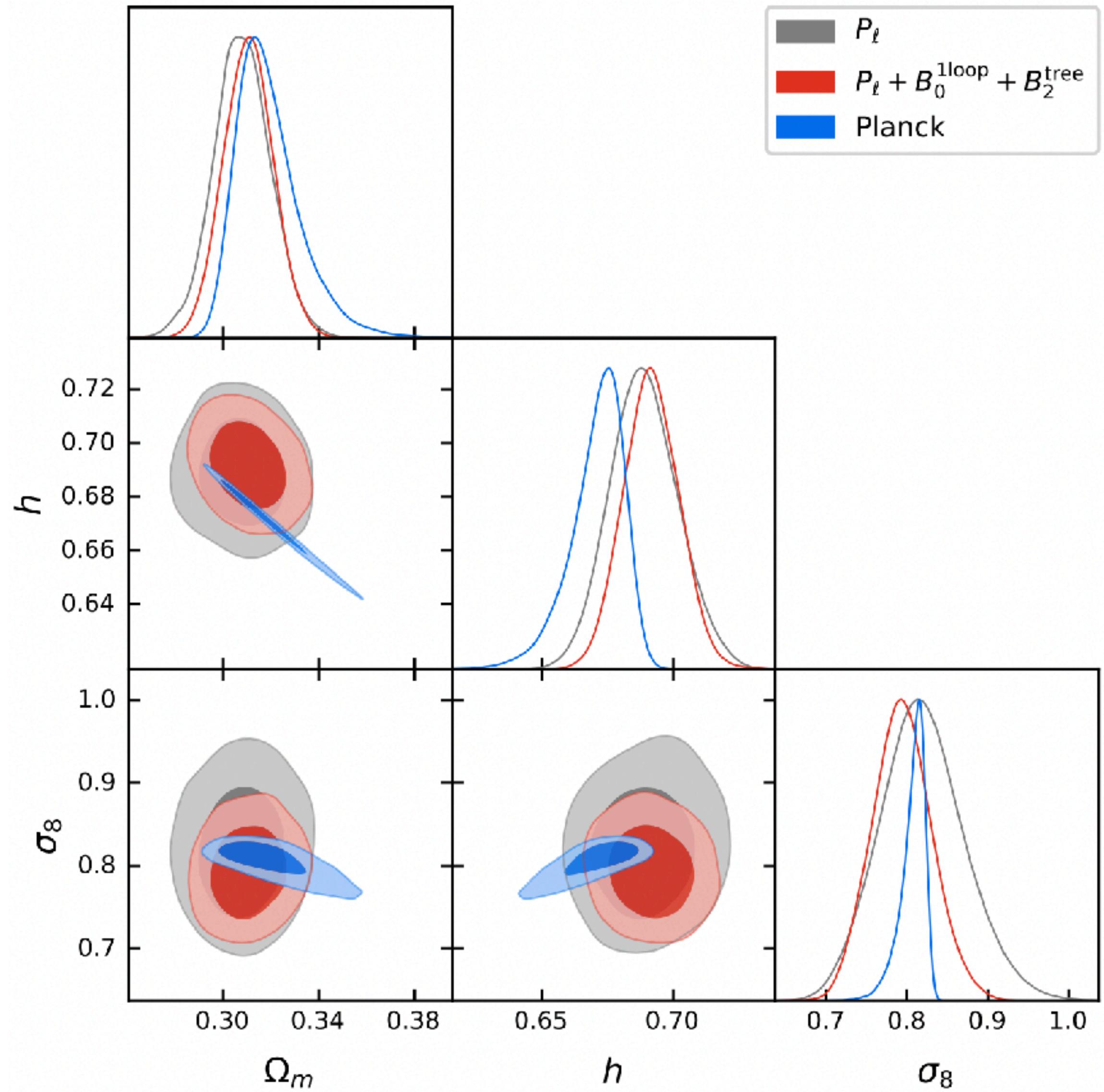
monopole



quadrupole

# the BOSS analysis

results



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# the BOSS analysis

## final takeaway

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percent size of error bars

	<b>OG BOSS, no sound horizon prior</b>	<b>PS only</b>	<b>B_tree</b>	<b>B_1-loop/ B_quad</b>	<b>Planck</b>
$\Omega_m$	few percent, but only on two parameters, and in a degenerate way [ $H(z)$ $r_s(z_d)$ and $D_A(z)/r_s(z_d)$ ]	0.039	0.0356	0.0321	0.0384
$h$		0.0189	0.0174	0.0159	0.0139
$\sigma_8$		0.0635	0.0609	0.0466	0.0160

**no tension with Planck!**

# the BOSS analysis

## analysis details

### - scale cut and kmax values

- $k_{\max}$  can be determined by validation with simulations
- make sure we recover known cosmological parameters

$$k_{\max} = 0.23 h \text{ Mpc}^{-1} \quad \text{one-loop terms}$$

$$k_{\max} = 0.08 h \text{ Mpc}^{-1} \quad \text{tree-level terms}$$

- also via perturbation theory: add NNLO terms and demand parameters shift  $< \sigma/3$

$$P_{\text{NNLO}}(k, \mu) = \frac{1}{4} c_{r,4} b_1^2 \mu^4 \frac{k^4}{k_{\text{NL,R}}^4} P_{11}(k) + \frac{1}{4} c_{r,6} b_1 \mu^6 \frac{k^4}{k_{\text{NL,R}}^4} P_{11}(k)$$

$$B_{\text{NNLO}}(k_1, k_2, k_3, \mu, \phi) = 2c_{\text{NNLO},1} K_2^{r,h}(\vec{k}_1, \vec{k}_2; \hat{z}) K_1^{r,h}(\vec{k}_2; \hat{z}) f \mu_1^2 \frac{k_1^4}{k_{\text{NL,R}}^4} P_{11}(k_1) P_{11}(k_2)$$

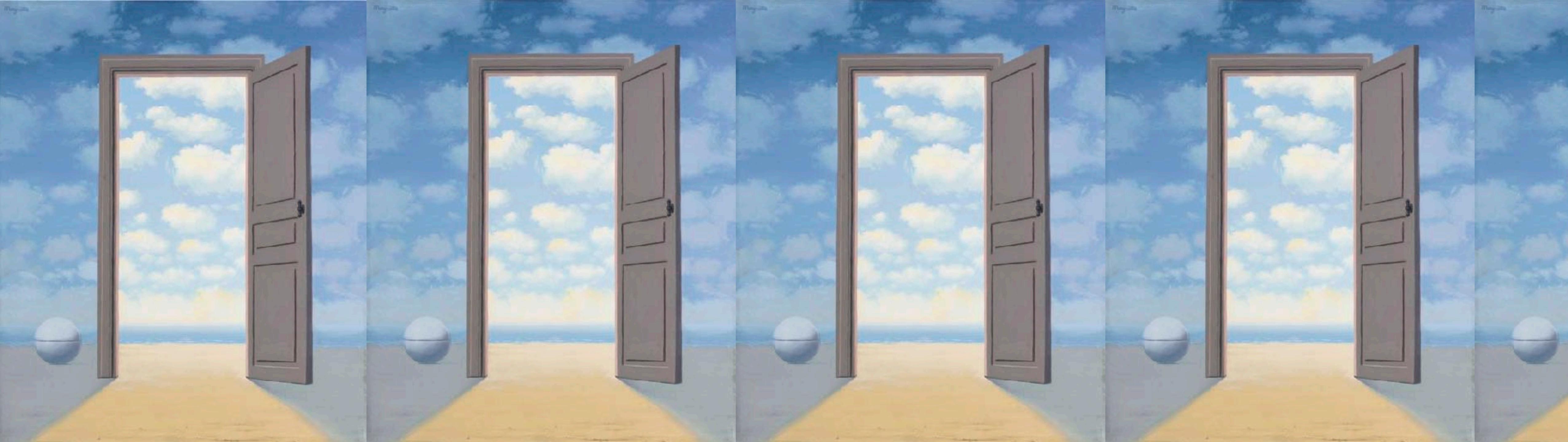
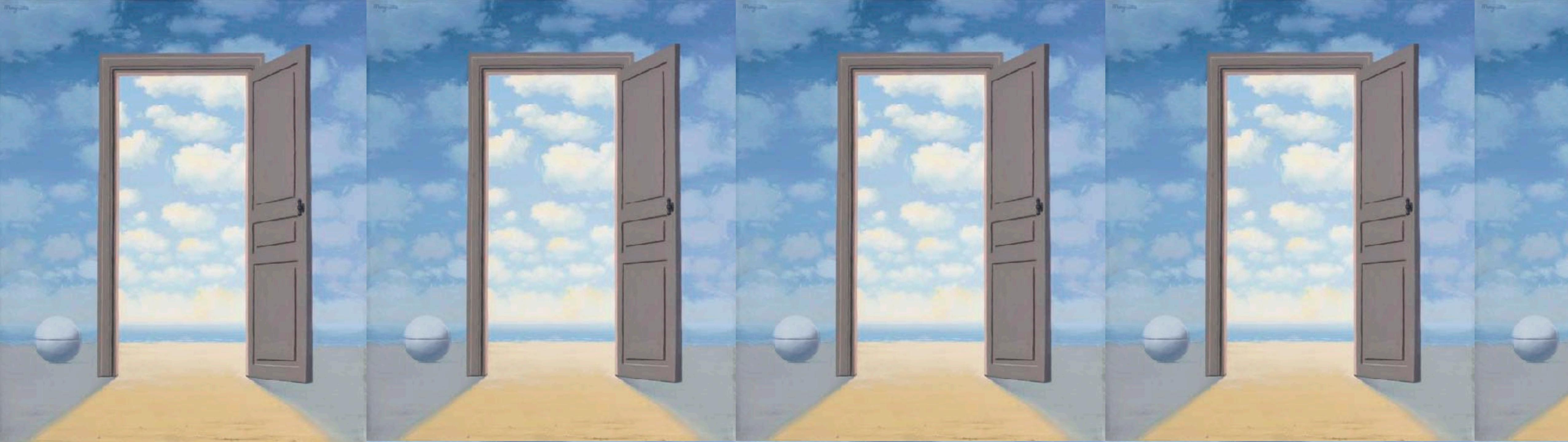
$$\begin{aligned} &+ c_{\text{NNLO},2} K_1^{r,h}(\vec{k}_1; \hat{z}) K_1^{r,h}(\vec{k}_2; \hat{z}) P_{11}(k_1) P_{11}(k_2) f \mu_3 k_3 \frac{(k_1^2 + k_2^2)}{4k_1^2 k_2^2 k_{\text{NL,R}}^4} \left[ -2\vec{k}_1 \cdot \vec{k}_2 (k_1^3 \mu_1 + k_2^3 \mu_2) \right. \\ &\left. + 2f \mu_1 \mu_2 \mu_3 k_1 k_2 k_3 (k_1^2 + k_2^2) \right] + \text{perms.} \end{aligned}$$

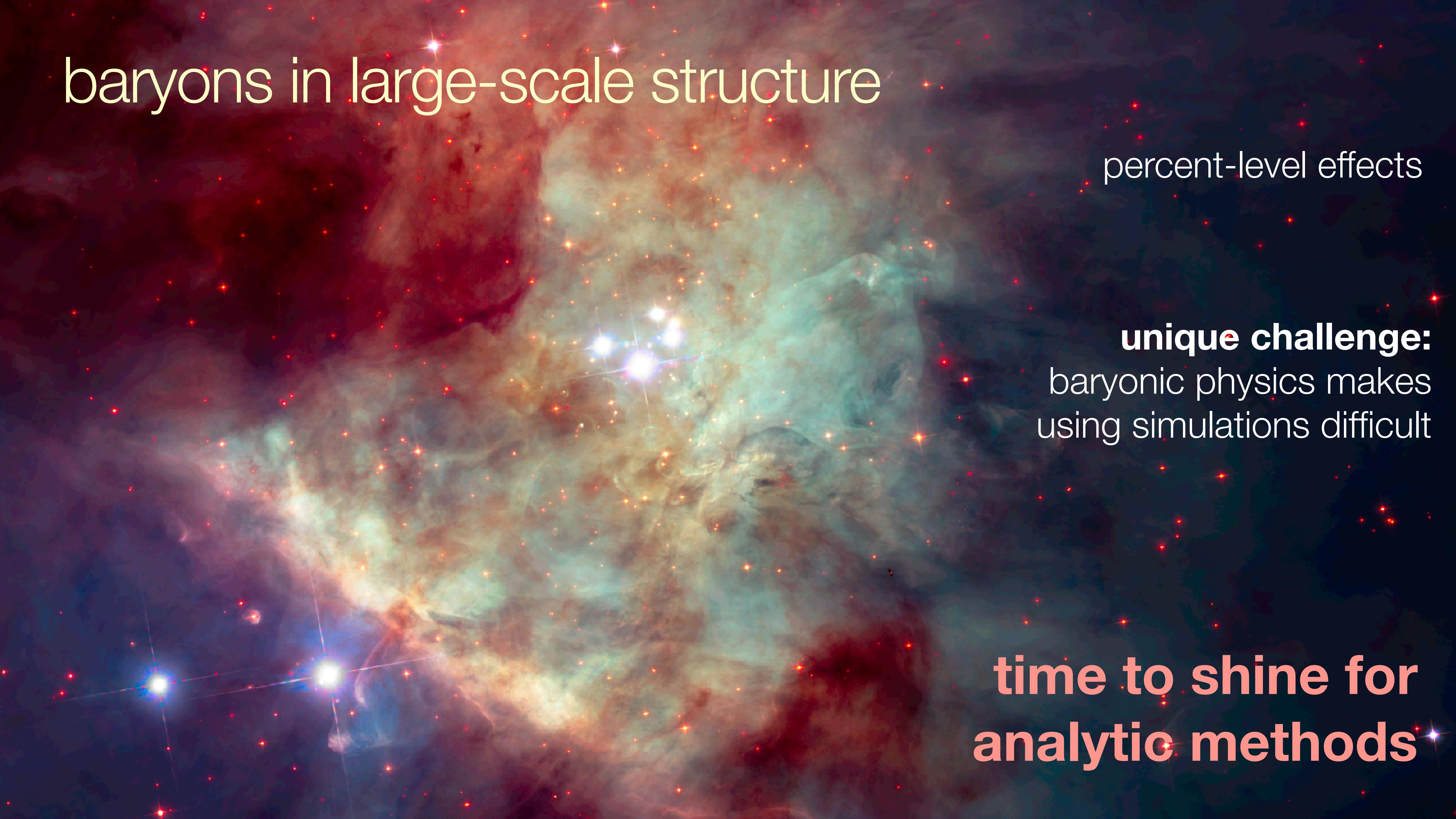
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	$\Omega_m$	$h$	$\sigma_8$
$\Delta_{\text{shift}}/\sigma_{\text{stat}}$	-0.03	-0.09	-0.03



the road ahead



A vibrant nebula with intricate, swirling patterns of red, orange, yellow, and blue. A dense cluster of young stars is visible in the center-left, with several bright, multi-colored stars emitting radial light rays. The background is a dark, star-filled space.

baryons in large-scale structure

percent-level effects

**unique challenge:**  
baryonic physics makes  
using simulations difficult

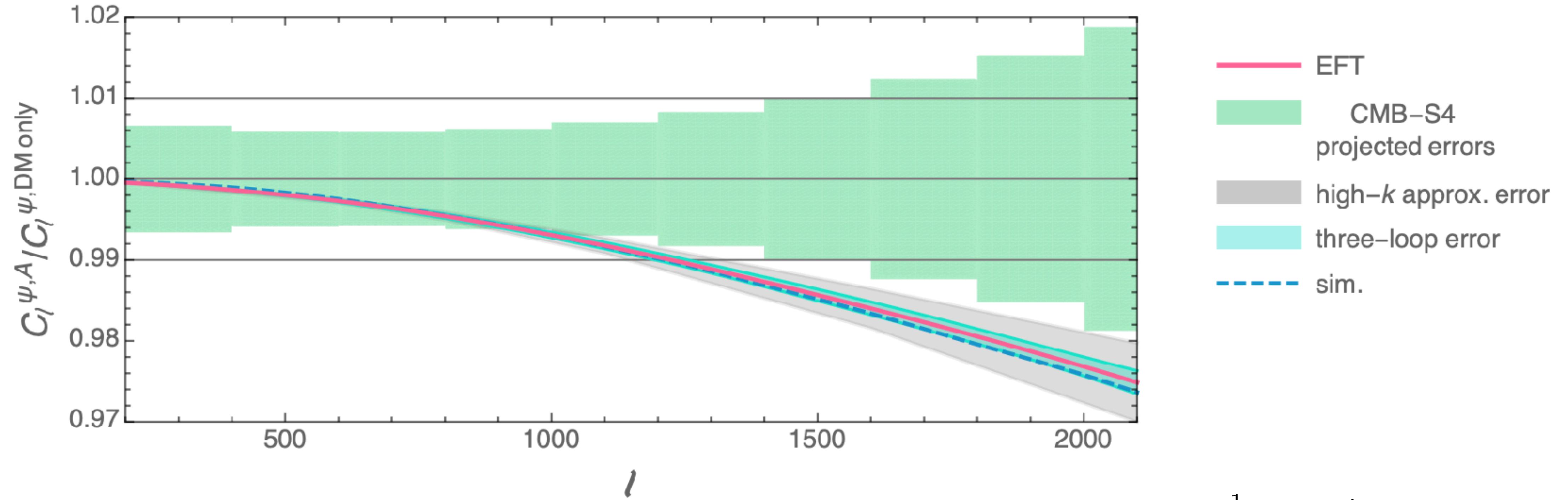
time to shine for  
analytic methods

# baryons in large-scale structure

## lensing potential

Bragança, **ML**, Sekera  
Senatore, Sgier 20

**ML**, Perko, Senatore 14



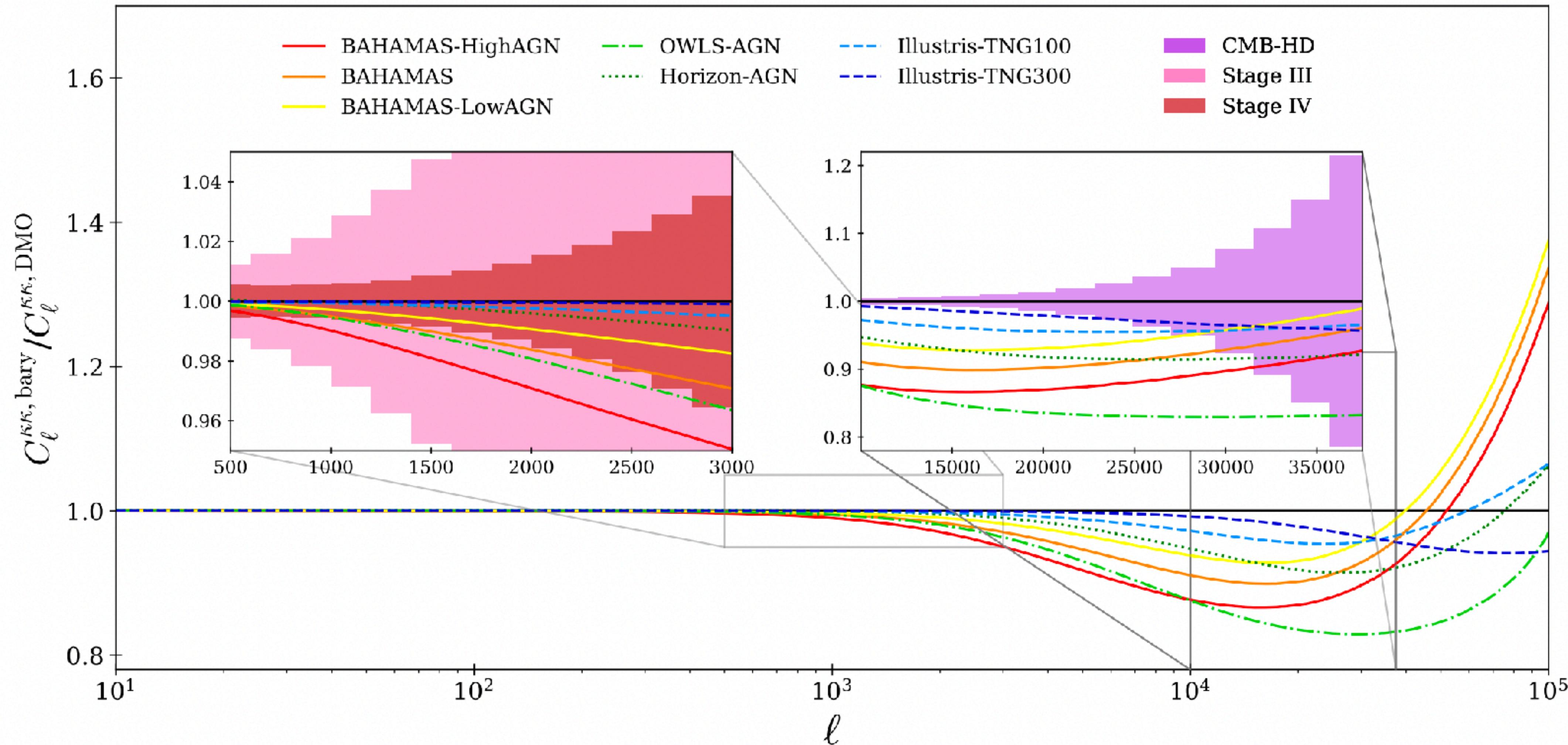
$$C_\ell^\psi = \frac{8\pi^2}{\ell^3} \int_0^{\chi_*} d\chi \chi P_\Phi \left( a(\chi), k = \frac{\ell}{\chi} \right) \left( \frac{\chi_* - \chi}{\chi_* \chi} \right)^2$$

$$\chi(a) = \int_a^1 \frac{da'}{(a')^2 H(a')}$$

$$P_\Phi(a, k) = \frac{9 \Omega(a)^2 \mathcal{H}(a)^4}{8\pi^2} \frac{P^A(a, k)}{k}$$

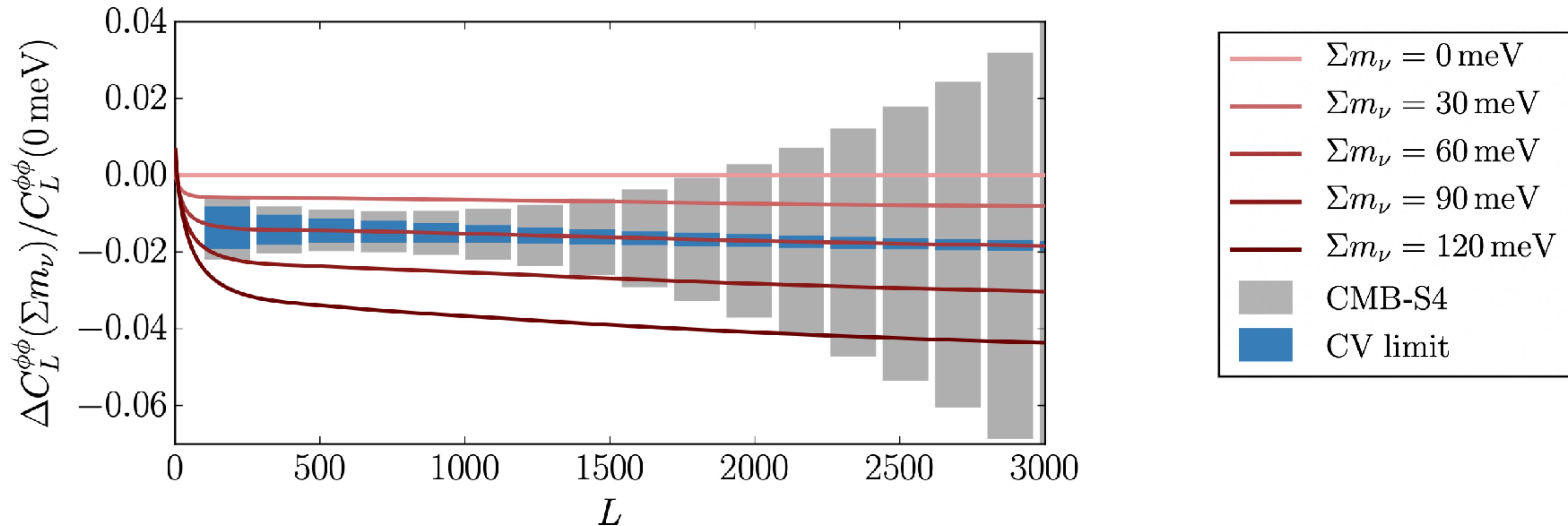
# baryons in large-scale structure

## baryon simulations



# baryons in large-scale structure

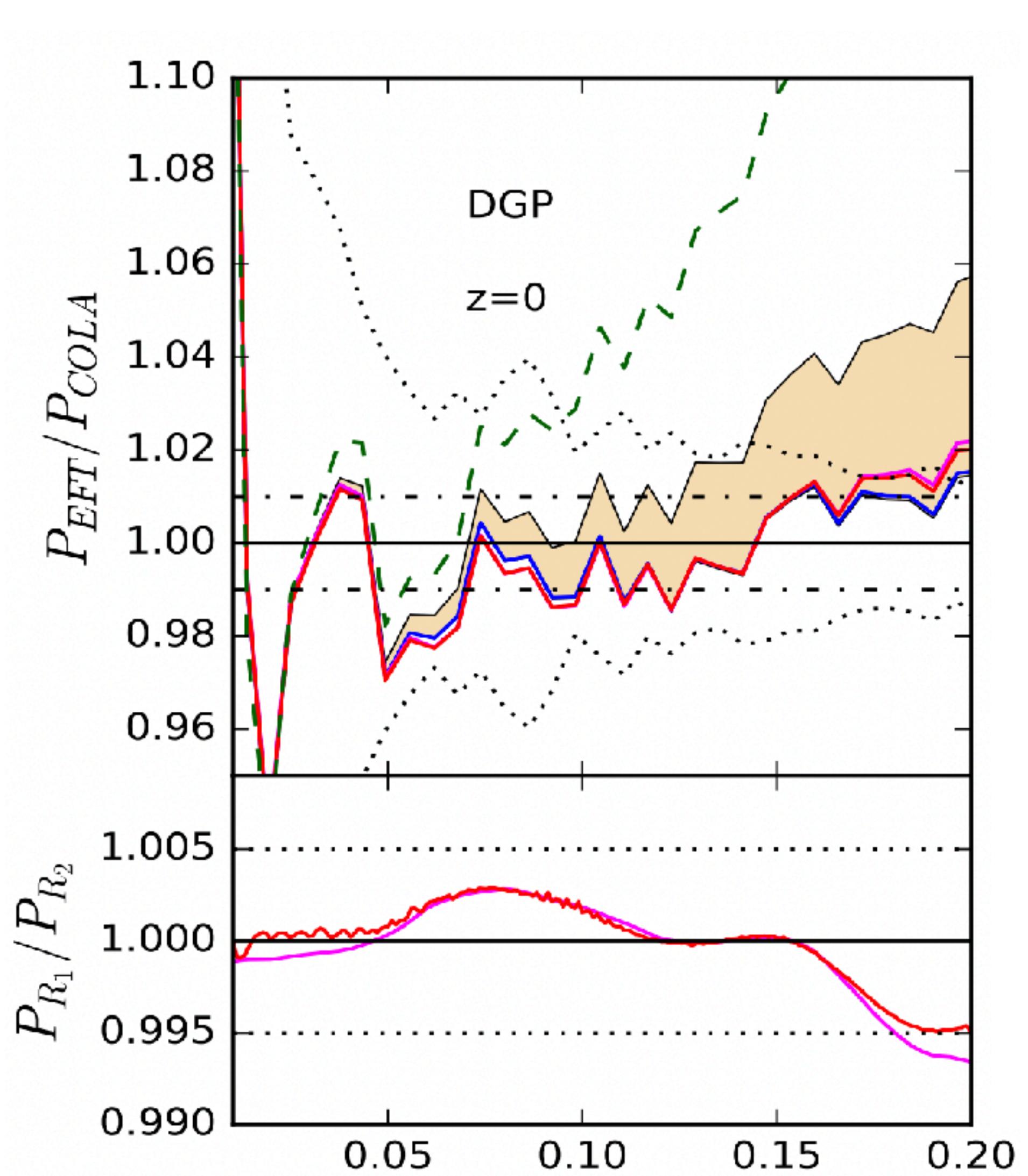
massive neutrinos



# beyond $\Lambda$ CDM/dark energy

## EFT of DE for EFT of LSS

- clustering quintessence in the EFT of LSS  
**ML**, Maleknejad, Senatore 17
- general EFT of DE in EFT of LSS  
Cusin, **ML**, Vernizzi 18  
Cusin, **ML**, Vernizzi 18
- comparisons to DE simulations  
Bose, Koyama, **ML**, Vernizzi, Winther 18
- unique LSS signatures from DE  
Crisostomi, **ML**, Vernizzi 19  
**ML** 20

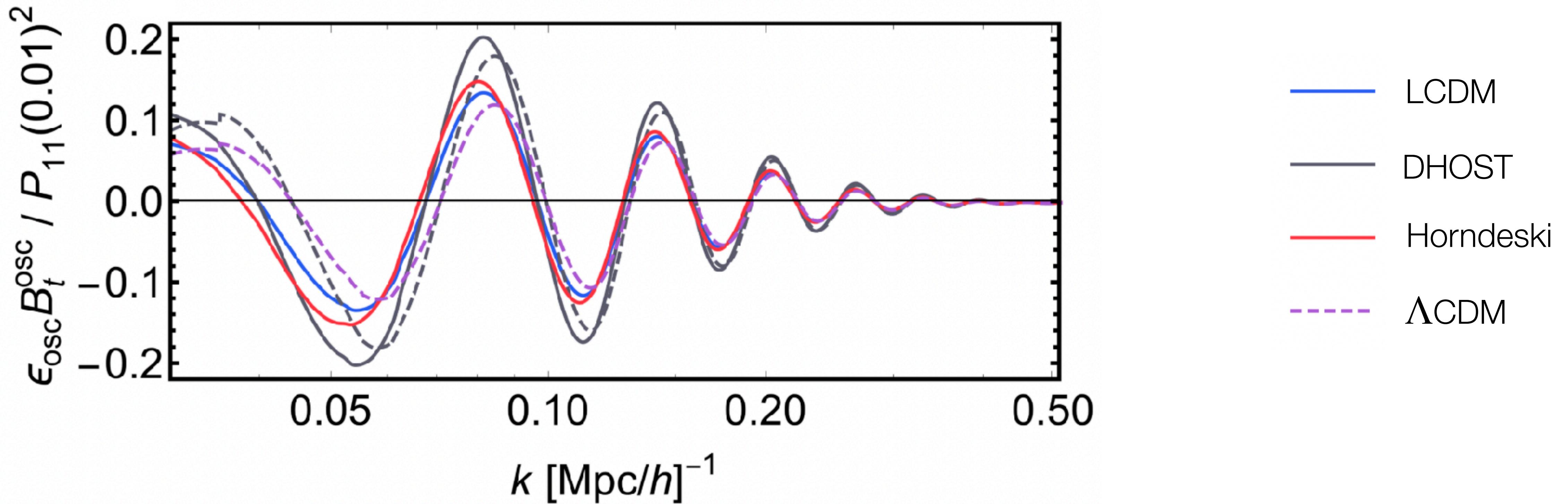


Bose, Koyama, **ML**, Vernizzi, Winther 18

# beyond $\Lambda$ CDM/dark energy violation of LSS consistency conditions

ML 20

Crisostomi, **ML**, Vernizzi 19



# conclusions

- after many years of theoretical development, the EFT of LSS has been successfully applied to real data, by multiple groups
- even with old data, the theory is powerful enough to place strong constraints (and much better than the original analysis)
- we are getting ready for new data, where EFT techniques will be indispensable
- there are many directions to go in the future - baryons, dark energy, massive neutrinos, fuzzy DM/ ALPs, more observables, higher precision, numerical insights, computational techniques, ...
- a lot of great physics opportunity!

for more info on EFTs in cosmology,  
see our Snowmass paper:

Cabass, Ivanov, **ML**, Mirbabayi,  
Simonović 22

# conclusions

- after many years of theoretical development, the EFT of LSS has been successfully applied to real data, by multiple groups
- even with old data, the theory is powerful enough to place strong constraints (and much better than the original analysis)
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Simonović 22

**Thank  
You!**



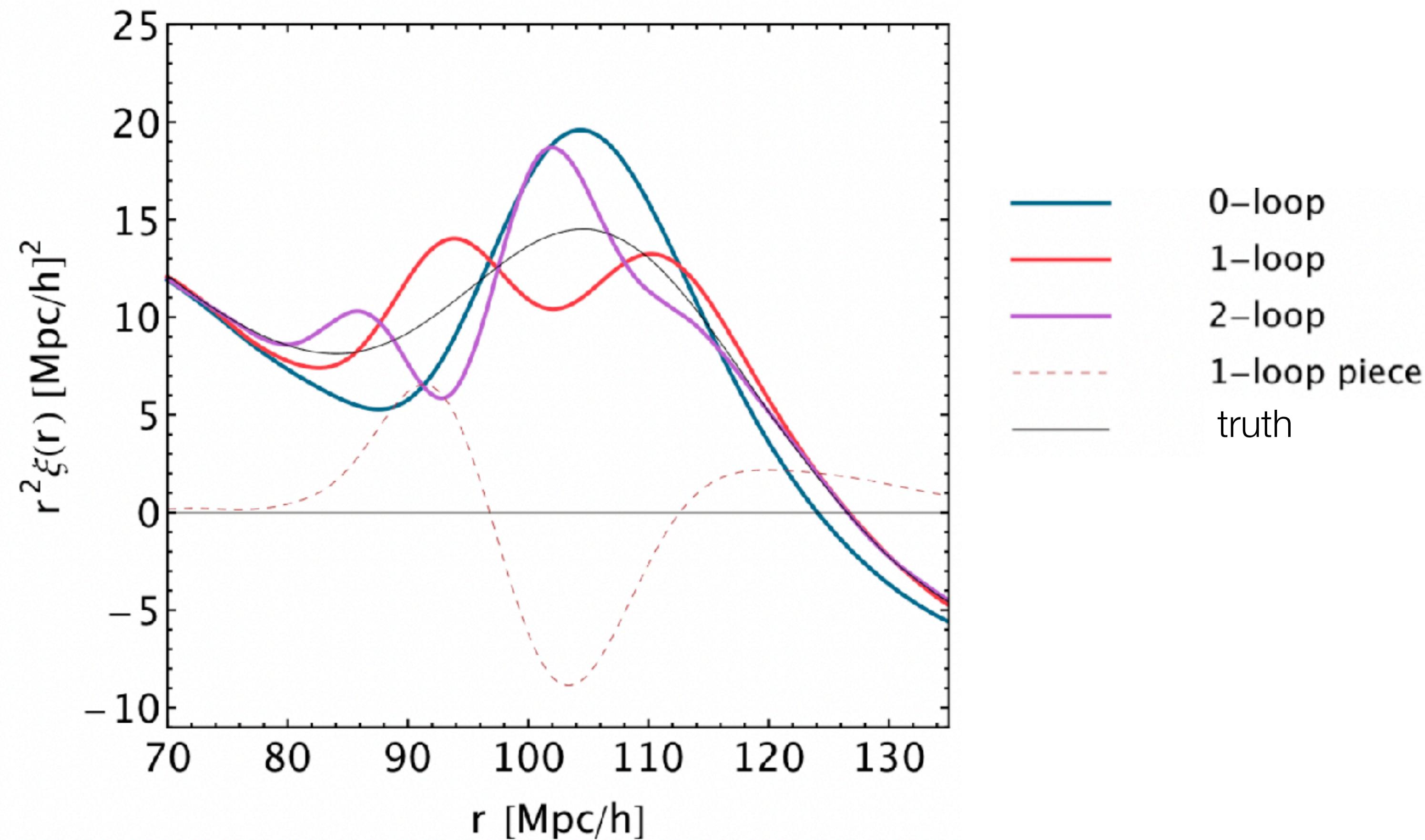
**extra slides**

# analytic resummation of BAO

ML, Senatore 18

non-resummed baryon acoustic oscillations

$\xi(r)$  is Fourier  
transform  
of  $P(k)$

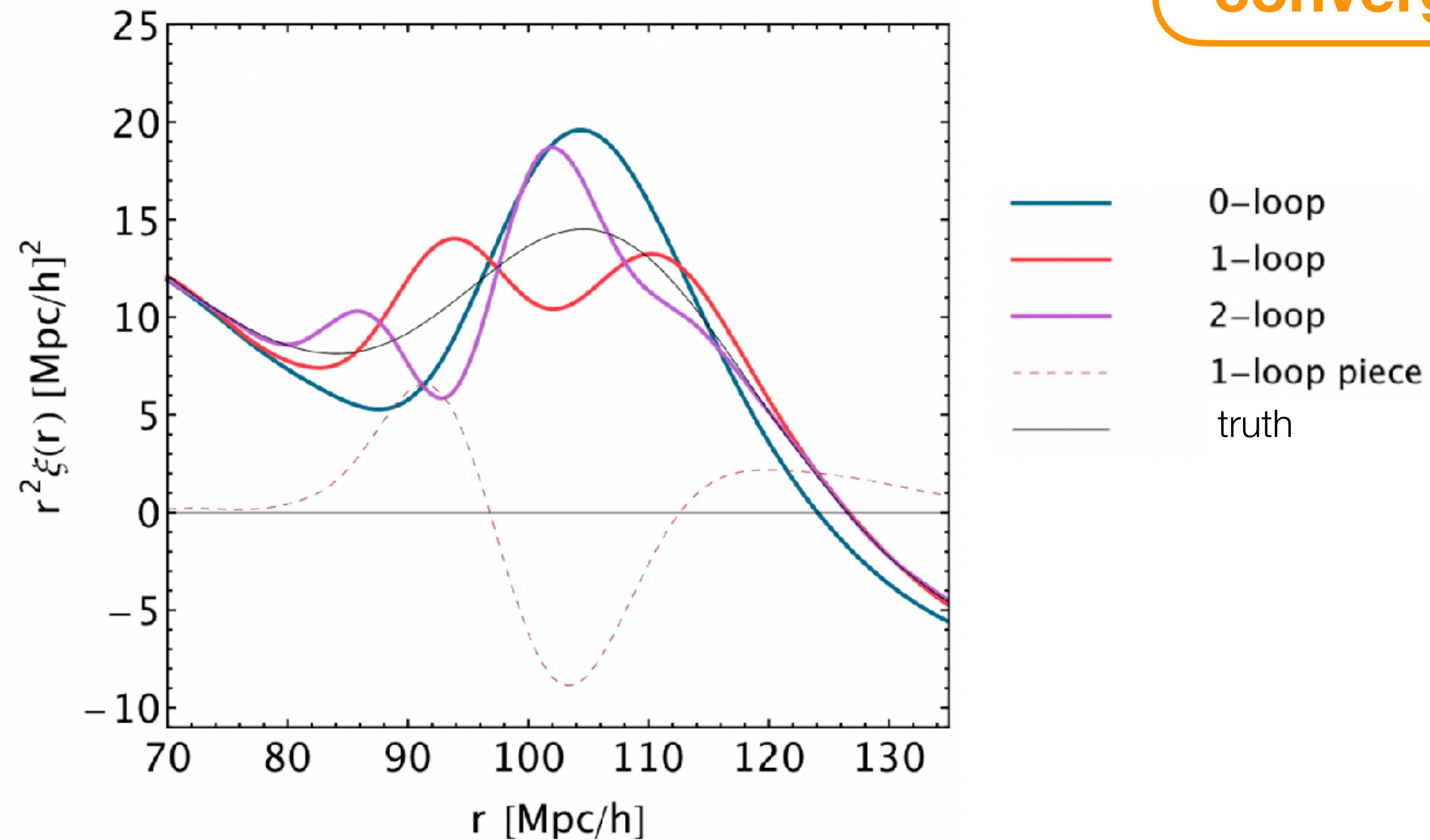


# analytic resummation of BAO

ML, Senatore 18

non-resummed baryon acoustic oscillations

$\xi(r)$  is Fourier transform of  $P(k)$



not  
converging!

# analytic resummation of BAO

ML, Senatore 18

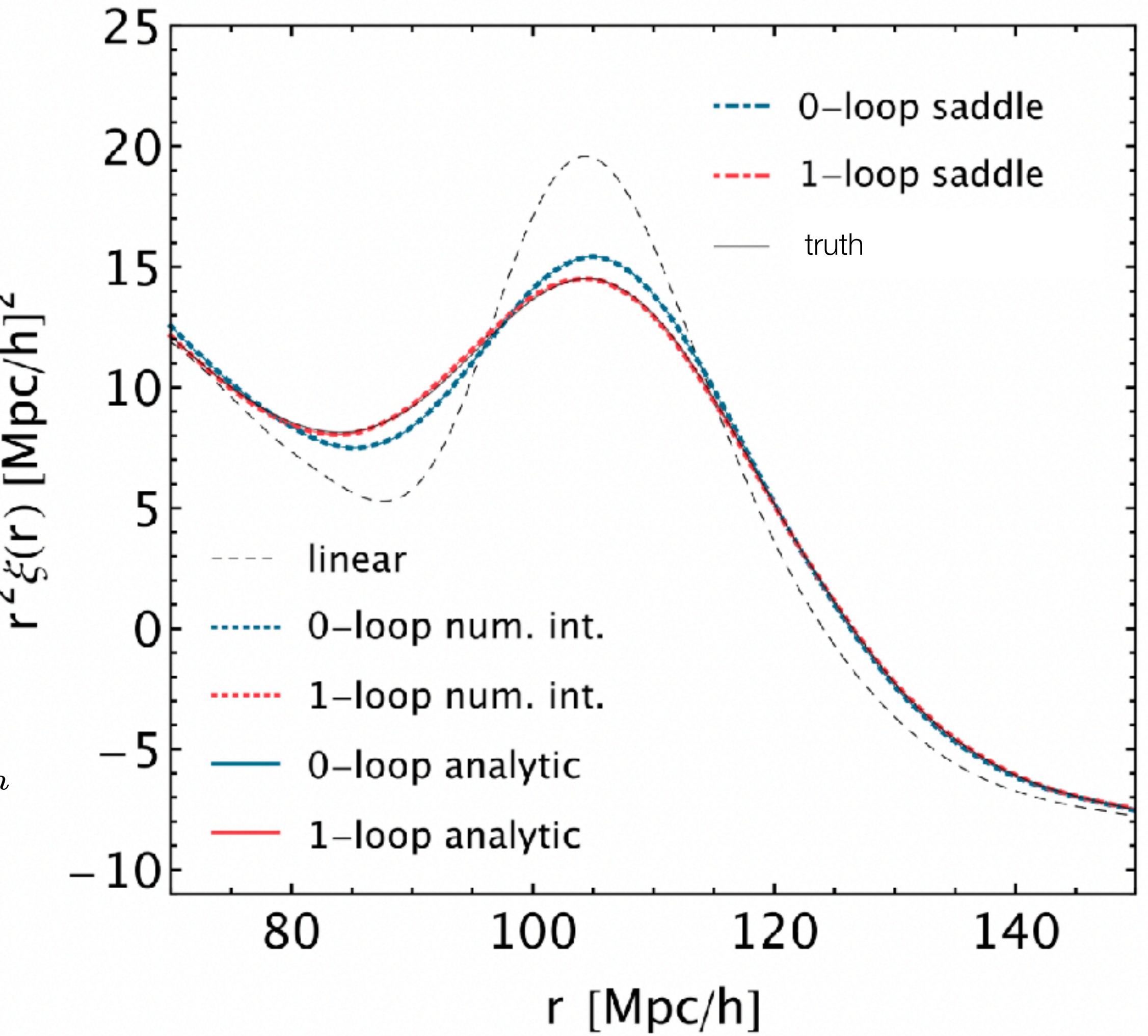
- large IR effects controlled by the equivalence principle
- soft mode effect on hard modes can be resummed

$$\xi_{\text{resum}}(r) \sim \int d^3q e^{S(\vec{r}, \vec{q})} \xi(q)$$

- morally similar to resumming soft emissions in QED and non-abelian gauge theories

complex power- laws:  $\xi_{\text{in}}(q) = q^{-3} \sum_m c_m q^{2\nu_m}$

loops have same  
integrals as massless  
3D QFT



# analytic resummation of BAO

ML, Senatore 18

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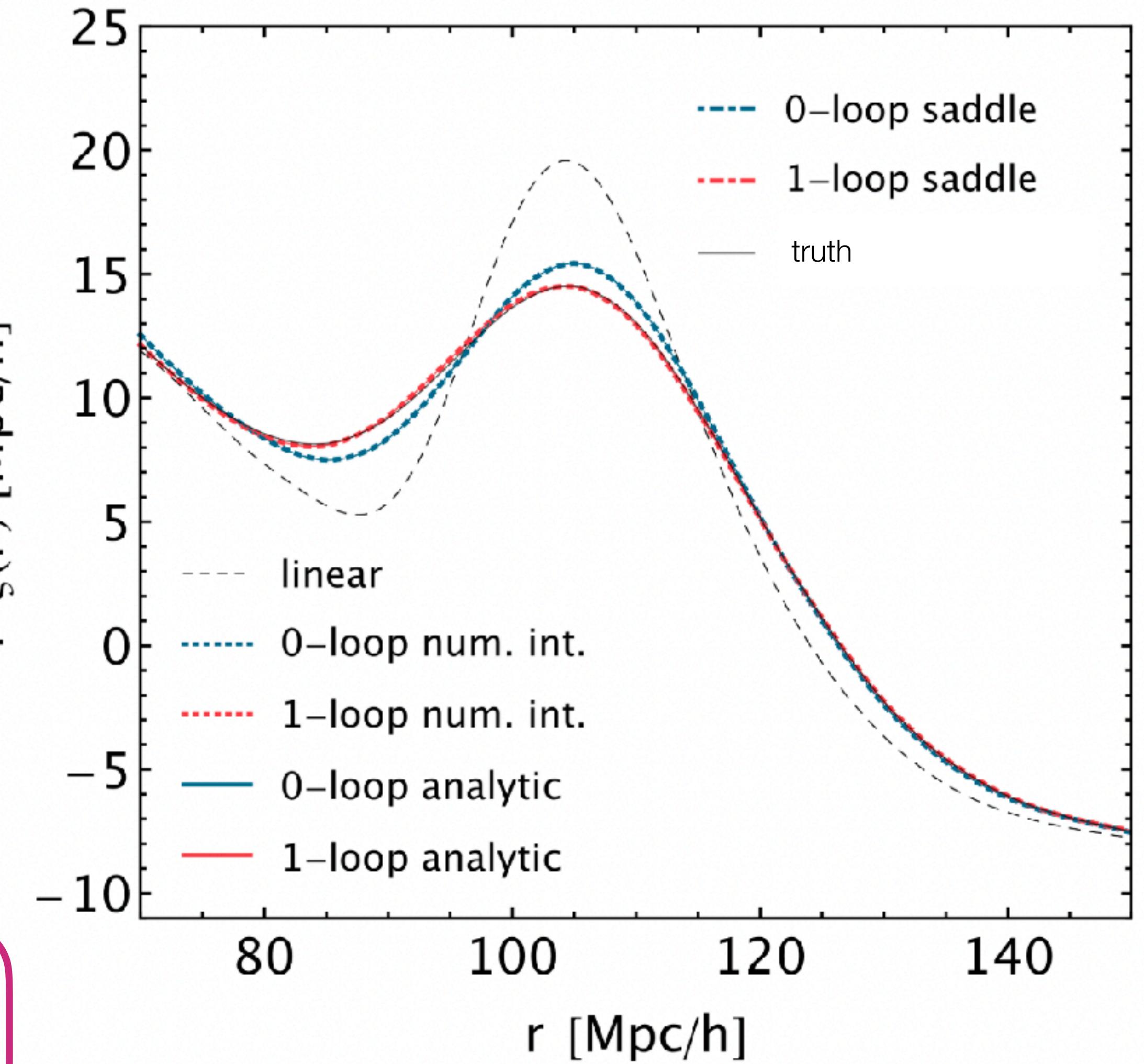
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complex power- laws:  $\xi_{\text{in}}(q) = q^{-3} \sum_m c_m q^{2\nu_m}$

loops have same integrals as massless 3D QFT

integrals over  $e^S$  can be done analytically



# the BOSS analysis

## analysis details

D'Amico, Donath, **ML**,  
Senatore, Zhang 22

### - IR resummation

- exact for one-loop power spectrum (described above)
- tree-level bispectrum, wiggle/no-wiggle approximation

$$B_{211}^{r,h} = 2K_1^{r,h}(\vec{k}_1; \hat{z})K_1^{r,h}(\vec{k}_2; \hat{z})K_2^{r,h}(\vec{k}_1, \vec{k}_2; \hat{z})P_{\text{LO}}(k_1)P_{\text{LO}}(k_2) + \text{ 2 perms.}$$

$$P_{\text{LO}}(k) = P_{\text{nw}}(k) + (1 + k^2 \Sigma_{\text{tot}}^2) e^{-k^2 \Sigma_{\text{tot}}^2} P_{\text{w}}(k)$$

Ivanov, Sibiryakov 18

- bispectrum loops we use

$$P_{\text{NLO}}(k) = P_{\text{nw}}(k) + e^{-k^2 \Sigma_{\text{tot}}^2} P_{\text{w}}(k)$$

in non-integrated linear spectra

# the BOSS analysis

## analysis details

### - Alcock-Paczynski effect

- cartesian coords obtained from reference cosmology
- AP effect converts to true cosmology

$$q_{\perp} = \frac{D_A(z)H_0}{D_A^{\text{ref}}(z)H_0^{\text{ref}}} , \quad q_{\parallel} = \frac{H^{\text{ref}}(z)/H_0^{\text{ref}}}{H(z)/H_0}$$

$$k = \frac{k^{\text{ref}}}{q_{\perp}} \left[ 1 + (\mu^{\text{ref}})^2 \left( \frac{1}{F^2} - 1 \right) \right]^{1/2} , \quad \mu = \frac{\mu^{\text{ref}}}{F} \left[ 1 + (\mu^{\text{ref}})^2 \left( \frac{1}{F^2} - 1 \right) \right]^{-1/2} \quad F = q_{\parallel}/q_{\perp}$$

- exact on tree-level, loop is small

### - window function

- exact for power spectrum, approx. for tree-level bispectrum

Gil-Marín et. al. 14

$$B_{211}^{r,h} = 2K_1^{r,h}(\vec{k}_1; \hat{z})K_1^{r,h}(\vec{k}_2; \hat{z})K_2^{r,h}(\vec{k}_1, \vec{k}_2; \hat{z})[W * P_{11}](\vec{k}_1)[W * P_{11}](\vec{k}_2) + \text{ 2 perms.}$$

$$[W * P_{11}](\vec{k}) = \int \frac{d^3 k'}{(2\pi)^3} W(\vec{k} - \vec{k}') P_{11}(\vec{k}')$$

D'Amico, Donath, **ML**,  
Senatore, Zhang 22

# the BOSS analysis

## analysis details

### - binning

- exact for power spectrum and tree-level bispectrum

$$B_{(\ell,i),\text{bin}}^{r,h}(k_1, k_2, k_3) = \frac{2\ell + 1}{N_T} \sum_{\vec{q}_1 \in k_1} \sum_{\vec{q}_2 \in k_2} \sum_{\vec{q}_2 \in k_2} \delta_K(\vec{q}_1 + \vec{q}_2 + \vec{q}_3) B^{r,h}(\vec{q}_1, \vec{q}_2, \vec{q}_3) \mathcal{P}_\ell(\mu_i)$$

$$\rightarrow B_{(\ell,i),\text{bin}}^{r,h}(k_1, k_2, k_3) = \frac{1}{V_T} \int_{k_1} dq_1 \int_{k_2} dq_2 \int_{k_3} dq_3 q_1 q_2 q_3 \frac{\beta(\Delta_q)}{8\pi^4} B_{(\ell,i)}^{r,h}(q_1, q_2, q_3)$$

- effective wavenumbers for one-loop bispectrum

### - likelihood

$$-2 \ln \mathcal{P} = (T_i - D_i) C_{ij}^{-1} (T_j - D_j) - 2 \ln \mathcal{P}_{\text{pr}}$$

some EFT parameters  
appear linearly

$$T_i = g_\alpha T_{G,i}^\alpha + T_{NG,i}$$

$$-2 \ln \mathcal{P} = g_\alpha F_{2,\alpha\beta} g_\beta - 2g_\alpha F_{1,\alpha} + F_0$$

Gaussian integrals can be done  
analytically

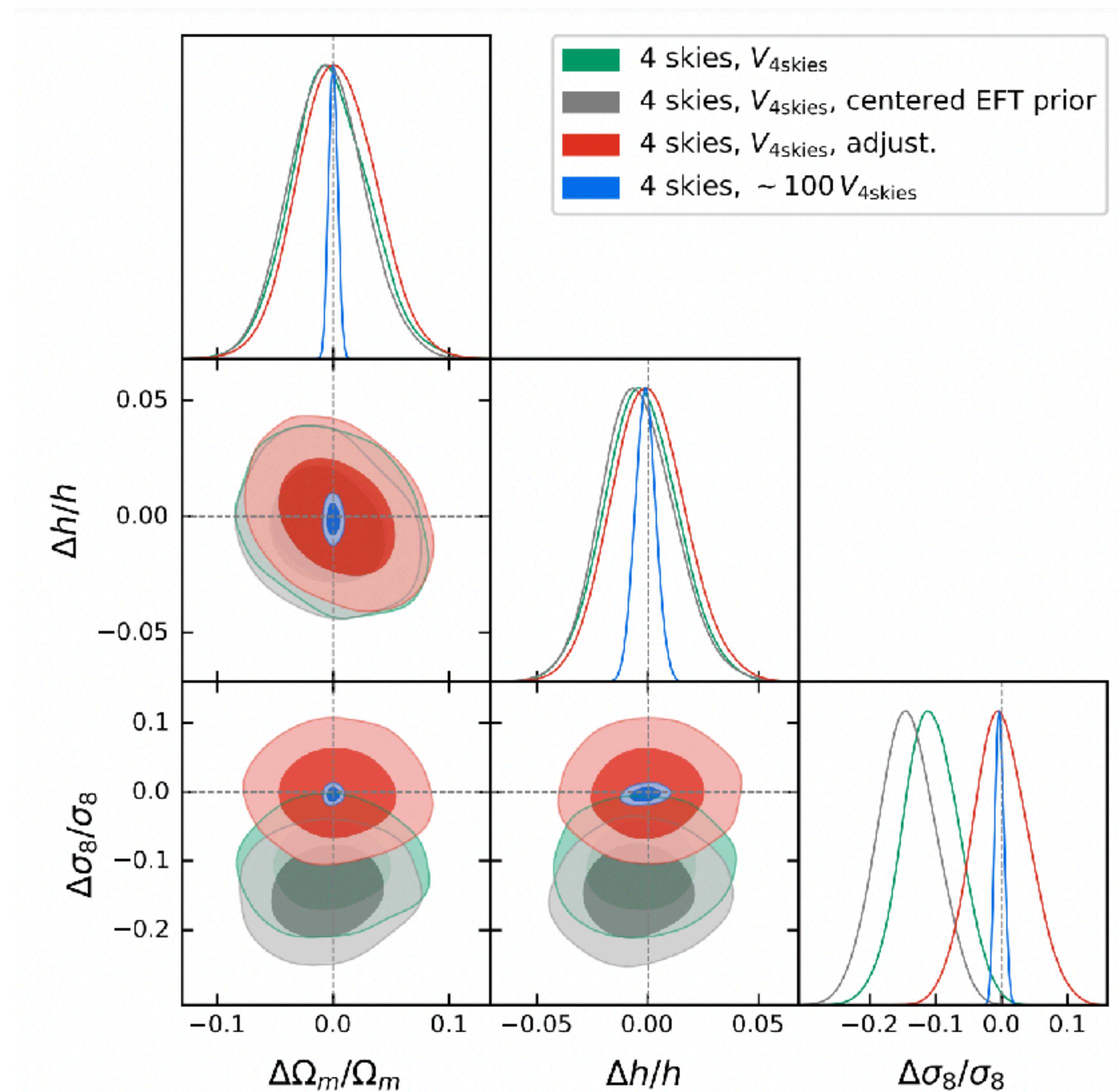
D'Amico, Donath, **ML**,  
Senatore, Zhang 22

# the BOSS analysis

## analysis details

### - pipeline validation - phase space effects

- useful test: generate “data” with your model, then run your pipeline to recover input parameters
  - we find a bias (**green curves**)
  - error in pipeline? no, because drastically reducing covariance we find agreement (**blue curves**)
  - need to use EFT priors centered on true values instead of zero? doesn’t change much (**gray curves**)
  - phase space effects, i.e. integrating out Gaussian parameters in non-Gaussian posterior
  - fix with linear shift in log-posterior
- $$\ln \mathcal{P}_{\text{pr}}^{\text{ph. sp. 4sky}} = -48 \left( \frac{b_1}{2} \right) + 32 \left( \frac{\Omega_m}{0.31} \right) + 48 \left( \frac{h}{0.68} \right)$$
- unbiased fit (**red curves**)



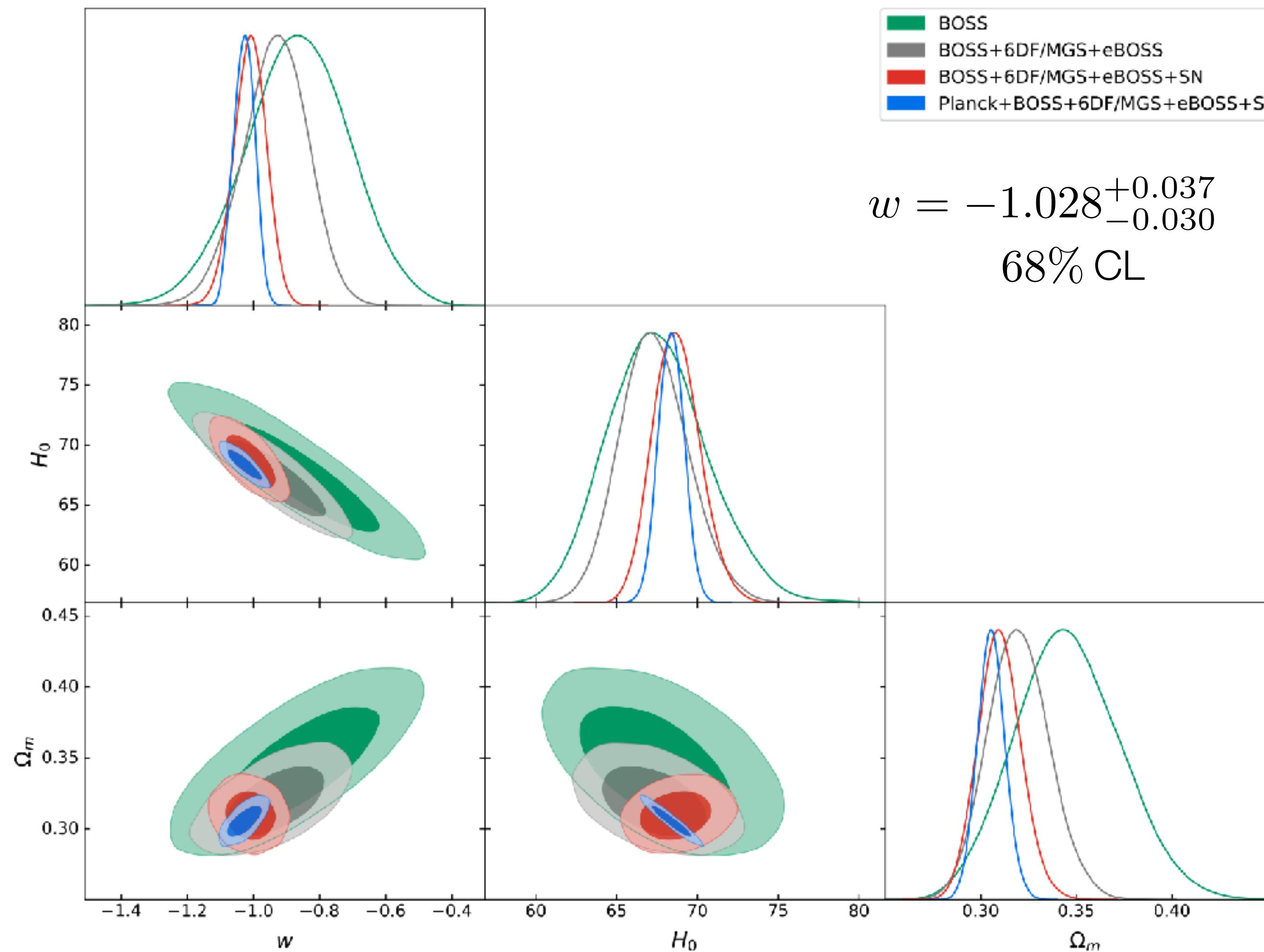
# the BOSS analysis

beyond  $\Lambda$ CDM

quintessence

D'Amico, Donath, Senatore, Zhang 21

(theory paper: **ML**, Maleknejad, Senatore 17)



# EFT for two fluids

## notation

fluid variables	$\rho_\sigma \equiv \bar{\rho}_\sigma(1 + \delta_\sigma)$	$\sigma \in \{c, b\}$
	$v_\sigma^i$	
adiabatic or 'total'	$\rho_A \equiv \rho_c + \rho_b$	$\bar{\rho}_\sigma = w_\sigma \bar{\rho}_A$
	$\delta_A \equiv w_c \delta_c + w_b \delta_b$	
	$v_A^i \equiv w_c v_c^i + w_b v_b^i$	
isocurvature or 'relative'	$\delta_I \equiv \delta_c - \delta_b$	
	$v_I^i \equiv v_c^i - v_b^i$	
momentum density	$\pi_\sigma^i \equiv \rho_\sigma v_\sigma^i$	

# EFT for two fluids

constructing the EOM

Braganca, ML, Sekera,  
Senatore, Sgier, 2020

individual mass conservation

$$\partial_t \int d^3x a^3 \rho_\sigma = 0$$

total momentum conservation

$$\partial_t \int d^3x a^4 (\pi_c^i + \pi_b^i) = 0$$

Galilean invariance

$$\begin{aligned} \partial_i &\rightarrow \partial_i , \quad \partial_t \rightarrow \partial_t - \dot{n}^i(t) \partial_i , \\ \rho_\sigma &\rightarrow \rho_\sigma , \quad \pi_\sigma^i \rightarrow \pi_\sigma^i + \rho_\sigma a \dot{n}^i(t) , \\ \Phi &\rightarrow \Phi - a^2 (\ddot{n}^i(t) + 2H \dot{n}^i(t)) x^i \end{aligned}$$

Continuity:  $\dot{\rho}_\sigma + 3H\rho_\sigma + a^{-1}\partial_i\pi_\sigma^i = 0 ,$

Momentum:  $\dot{\pi}_c^i + 4H\pi_c^i + a^{-1}\partial_j \left( \frac{\pi_c^i \pi_c^j}{\rho_c} \right) + a^{-1}\rho_c \partial_i \Phi = +a^{-1}\gamma^i - a^{-1}\partial_j \tau_c^{ij} ,$

$\dot{\pi}_b^i + 4H\pi_b^i + a^{-1}\partial_j \left( \frac{\pi_b^i \pi_b^j}{\rho_b} \right) + a^{-1}\rho_b \partial_i \Phi = -a^{-1}\gamma^i - a^{-1}\partial_j \tau_b^{ij} .$

# EFT for two fluids

constructing the EOM

Braganca, ML, Sekera,  
Senatore, Sgier, 2020

Continuity:  $\dot{\rho}_\sigma + 3H\rho_\sigma + a^{-1}\partial_i\pi_\sigma^i = 0 ,$

total momentum conservation

$$\partial_t \int d^3x a^4 (\pi_c^i + \pi_b^i) = 0$$

Momentum:  $\dot{\pi}_c^i + 4H\pi_c^i + a^{-1}\partial_j \left( \frac{\pi_c^i \pi_c^j}{\rho_c} \right) + a^{-1}\rho_c \partial_i \Phi = +a^{-1}\gamma^i - a^{-1}\partial_j \tau_c^{ij} ,$

$$\dot{\pi}_b^i + 4H\pi_b^i + a^{-1}\partial_j \left( \frac{\pi_b^i \pi_b^j}{\rho_b} \right) + a^{-1}\rho_b \partial_i \Phi = -a^{-1}\gamma^i - a^{-1}\partial_j \tau_b^{ij} .$$

1)  $(\rho_c + \rho_b)\partial_i \Phi = \bar{\rho}_A(1 + \delta_A)\partial_i \Phi$

total derivative

$$a^{-2}\partial^2 \Phi = \frac{3}{2}\Omega(t)H(t)^2\delta_A \quad \downarrow \quad \delta_A \partial_i \Phi = \frac{2a^{-2}}{3\Omega(t)H(t)^2} \partial_j \left( \partial_i \Phi \partial_j \Phi - \frac{1}{2}\delta_{ij}(\partial \Phi)^2 \right)$$

2) effective force  $\gamma^i \sim v_I^i \sim \pi_I^i$  is new possible counterterm!

# EFT for two fluids

effective force and stress tensors

$$\begin{aligned}\partial_i(\partial\tau_\rho)_c^i - \partial_i(\gamma)_c^i = & -g w_b aH \partial_i v_I^i + 9(2\pi)H^2 \left\{ \frac{c_{c,g}^2}{k_{NL}^2} (w_c \partial^2 \delta_c + w_b \partial^2 \delta_b) + \frac{c_{c,v}^2}{k_{NL}^2} \partial^2 \delta_c \right. \\ & + \frac{1}{k_{NL}^2} (c_{1c}^{cc} \partial^2 \delta_c^2 + c_{1c}^{cb} \partial^2 (\delta_c \delta_b) + c_{1c}^{bb} \partial^2 \delta_b^2) \\ & \left. + \frac{c_{4c,g}^2}{a^2 k_{NL}^4} (w_c \partial^4 \delta_c + w_b \partial^4 \delta_b) + \frac{c_{4c,v}^2}{a^2 k_{NL}^4} \partial^4 \delta_c \right\} + \dots\end{aligned}$$

$$\partial_i(\partial\tau_\rho)_b^i - \partial_i(\gamma)_b^i = +g w_c aH \partial_i v_I^i + \dots$$



3 independent EFT parameters per fluid  
+ linear counterterm  $g(a)$

# EFT for two fluids

effective force and stress tensors

$$\partial_i(\partial\tau_\rho)_c^i - \partial_i(\gamma)_c^i = -g w_b aH \partial_i v_I^i + 9(2\pi)H^2 \left\{ \frac{c_{c,g}^2}{k_{NL}^2} (w_c \partial^2 \delta_c + w_b \partial^2 \delta_b) + \frac{c_{c,v}^2}{k_{NL}^2} \partial^2 \delta_c \right. \\ \left. + \frac{1}{k_{NL}^2} (c_{1c}^{cc} \partial^2 \delta_c^2 + c_{1c}^{cb} \partial^2 (\delta_c \delta_b) + c_{1c}^{bb} \partial^2 \delta_b^2) \right. \\ \left. + \frac{c_{4c,g}^2}{a^2 k_{NL}^4} (w_c \partial^4 \delta_c + w_b \partial^4 \delta_b) + \frac{c_{4c,v}^2}{a^2 k_{NL}^4} \partial^4 \delta_c \right\} + \dots$$

new linear counterterm!

$$\partial_i(\partial\tau_\rho)_b^i - \partial_i(\gamma)_b^i = +g w_c aH \partial_i v_I^i - \dots$$

→ 3 independent EFT parameters per fluid  
+ linear counterterm  $g(a)$

# baryon fits

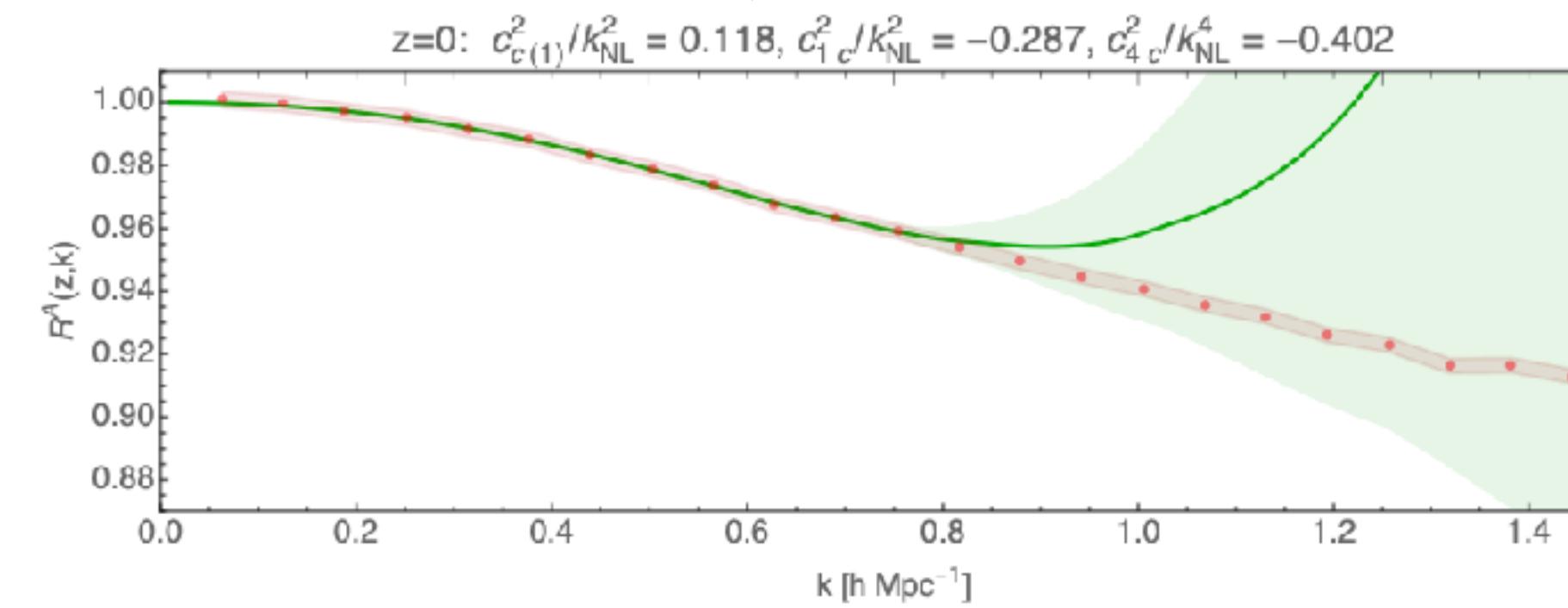
- two-loop ratio of power spectrum with baryons to DM-only power spectrum
- very small cosmic variance in the ratio
- simple perturbative expansion
- linear counterterm is negligible

$$\frac{P_{\text{EFT}}^\sigma}{P_{\text{DM-only}}} \Big|_2 = 1 - 4\pi \Delta c_{\sigma(1)}^2 \frac{k^2}{k_{\text{NL}}^2} + \frac{2\pi}{P_{11}^A} \left( \Delta c_{\sigma(1)}^2 \left( 2 \frac{k^2}{k_{\text{NL}}^2} P_{\text{1-loop}}^A + P_{\text{1-loop}}^{A,(c_s)} \right) + \Delta c_{1\sigma}^2 P_{\text{1-loop}}^{A,\text{(quad,1)}} \right) + \frac{8\pi^2}{17} \frac{k^4}{k_{\text{NL}}^4} \left( 14[\Delta c_{\sigma(1)}^2]^2 - 6c_{s(1)}^2 \Delta c_{\sigma(1)}^2 + 17\Delta c_{4\sigma}^2 \right) ,$$

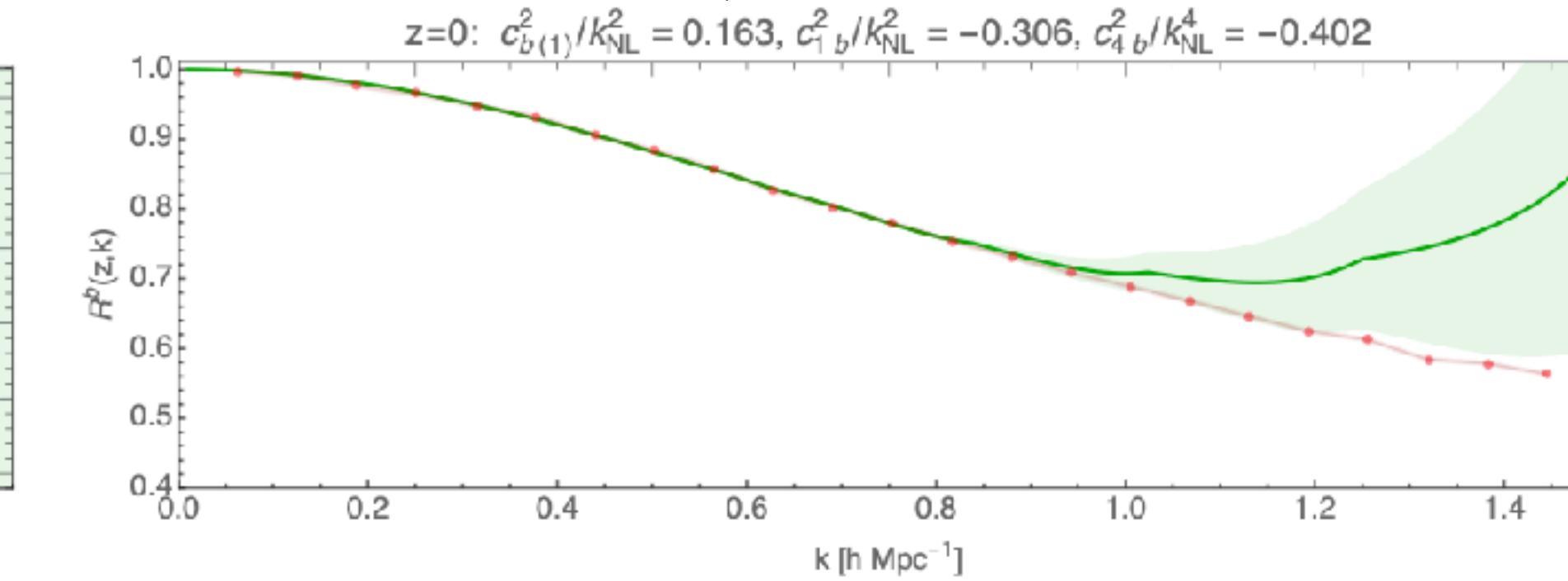
$\sigma = (\text{CDM, baryons, total})$

# baryon fits

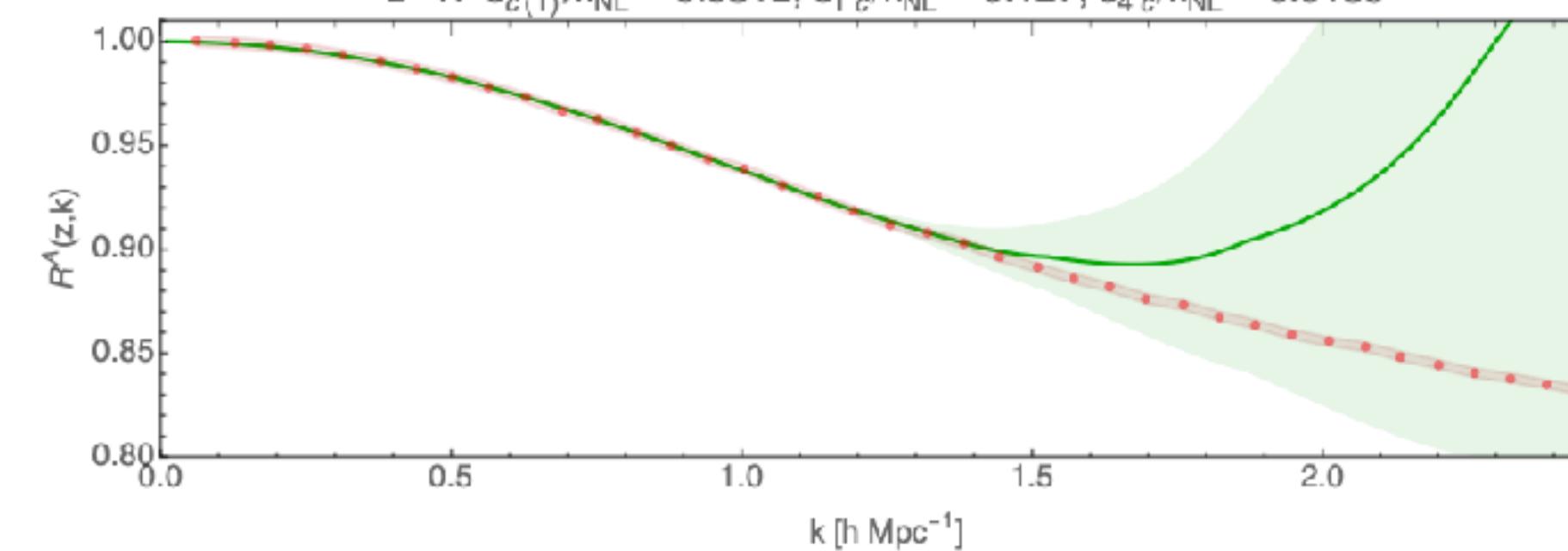
$P^A / P^{\text{DM only}}$



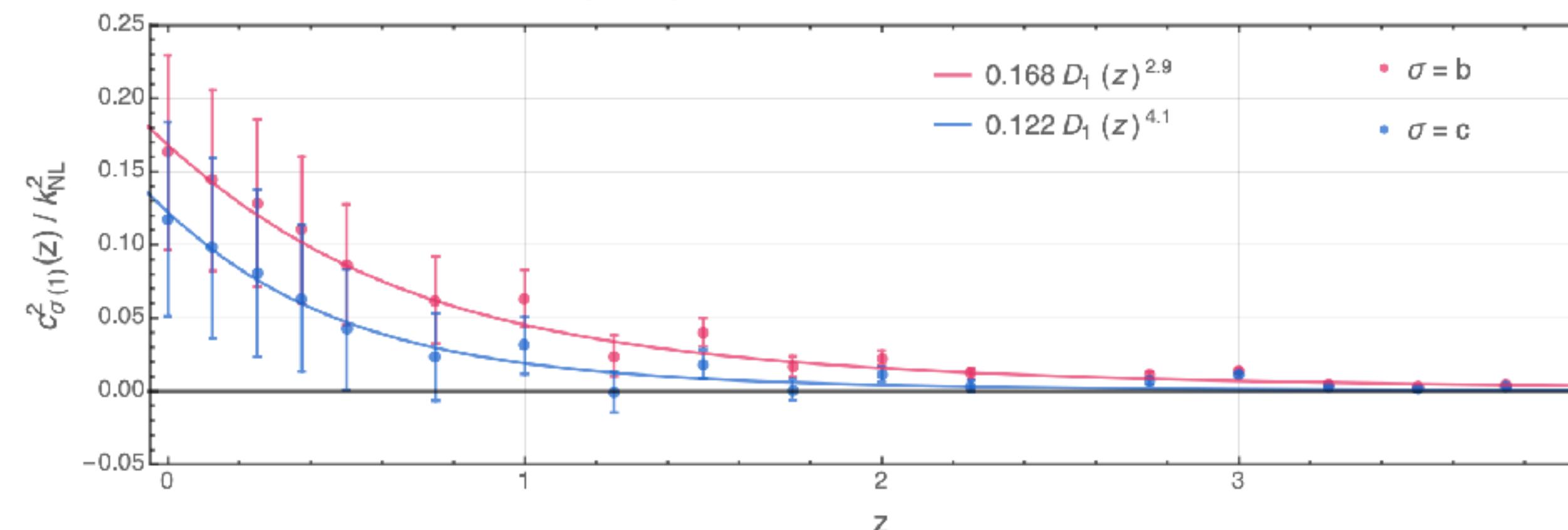
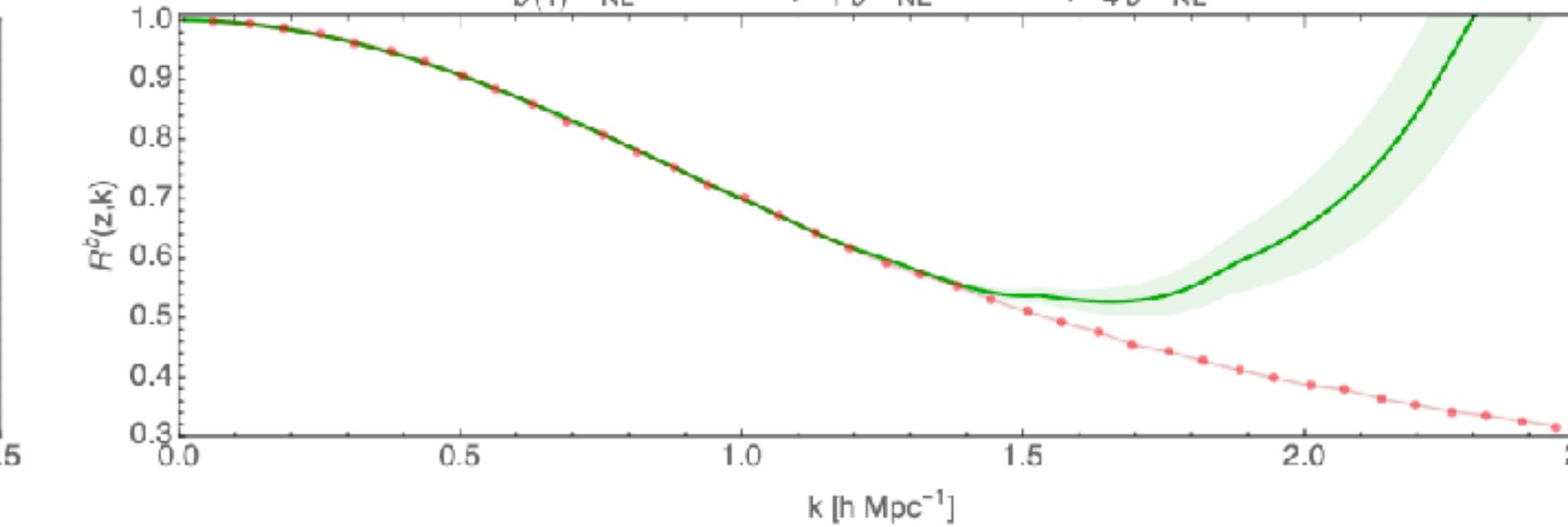
$P^b / P^{\text{DM only}}$



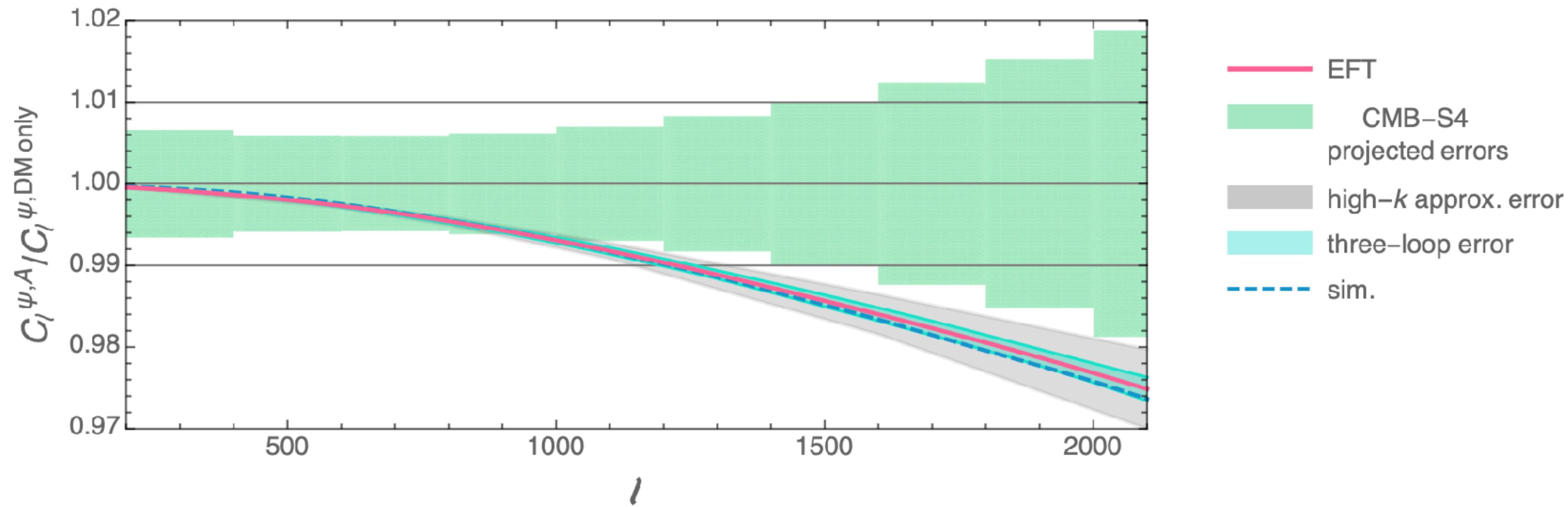
$z=1: c_{c(1)}^2/k_{\text{NL}}^2 = 0.0312, c_{1c}^2/k_{\text{NL}}^2 = 0.127, c_{4c}^2/k_{\text{NL}}^4 = 0.0139$



$z=1: c_{b(1)}^2/k_{\text{NL}}^2 = 0.0633, c_{1b}^2/k_{\text{NL}}^2 = 0.114, c_{4b}^2/k_{\text{NL}}^4 = 0.0138$



# lensing potential with baryons



$$C_\ell^\psi = \frac{8\pi^2}{\ell^3} \int_0^{\chi_*} d\chi \chi P_\Phi \left( a(\chi), k = \frac{\ell}{\chi} \right) \left( \frac{\chi_* - \chi}{\chi_* \chi} \right)^2$$

$$\chi(a) = \int_a^1 \frac{da'}{(a')^2 H(a')}$$

$$P_\Phi(a, k) = \frac{9 \Omega(a)^2 \mathcal{H}(a)^4}{8\pi^2} \frac{P^A(a, k)}{k}$$

# EFT for two fluids

linear relative velocity counterterm

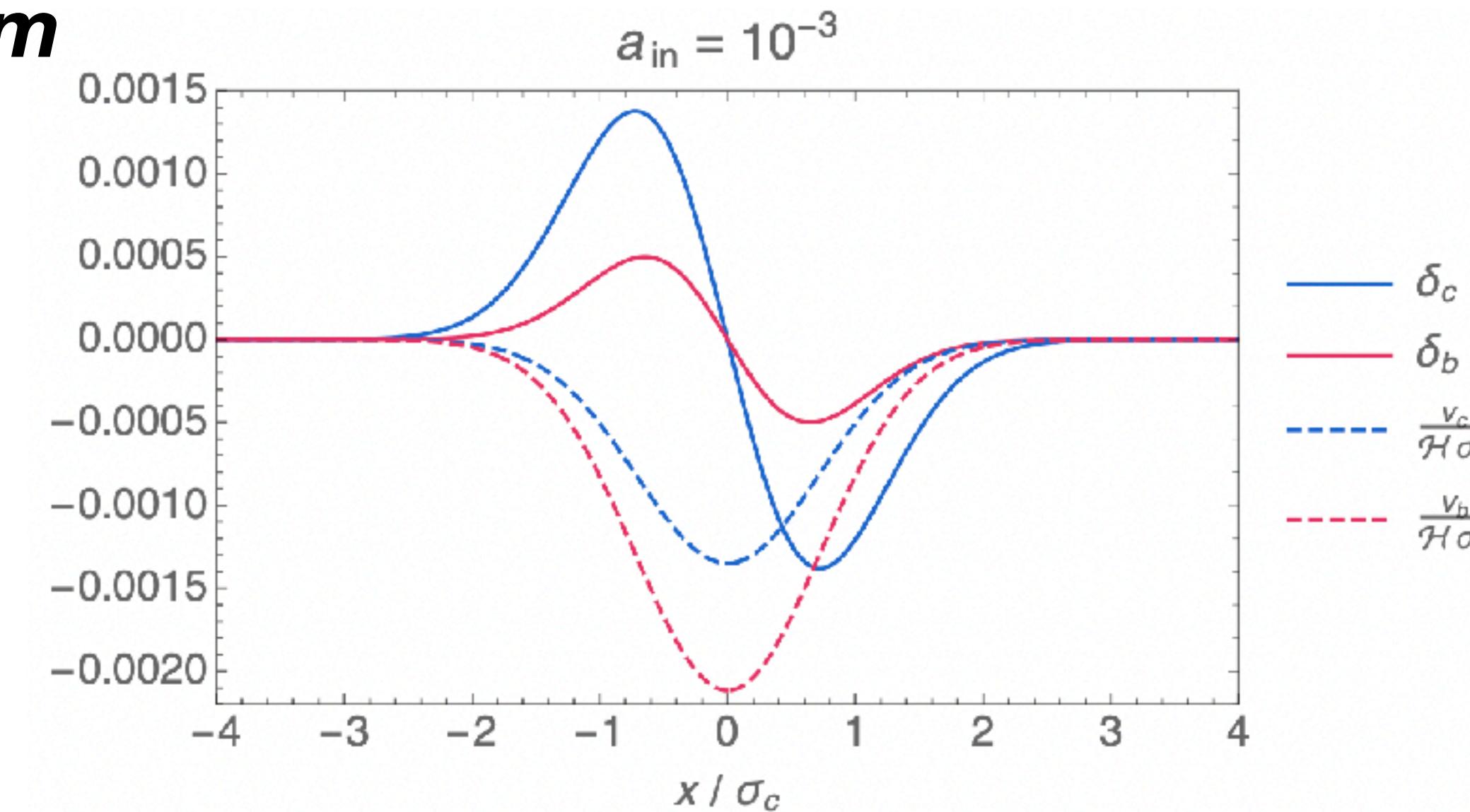
size estimation - UV model

we solve numerically the fully non-linear one-dimensional equations.

we then smooth the numerical solutions with a top hat on a large enough scale so that they are perturbative. we can then show that these smoothed fields satisfy the linear equations ***with the linear relative velocity counterterm***

$$a\mathcal{H}\frac{\partial\delta_\sigma}{\partial a} + \frac{\partial}{\partial x}((1+\delta_\sigma)v_\sigma) = 0 ,$$

$$a\mathcal{H}\frac{\partial}{\partial a}\frac{\partial v_\sigma}{\partial x} + \mathcal{H}\frac{\partial v_\sigma}{\partial x} + \frac{3}{2}\Omega\mathcal{H}^2\delta_A + \frac{\partial}{\partial x}\left(v_\sigma\frac{\partial v_\sigma}{\partial x}\right) = 0$$



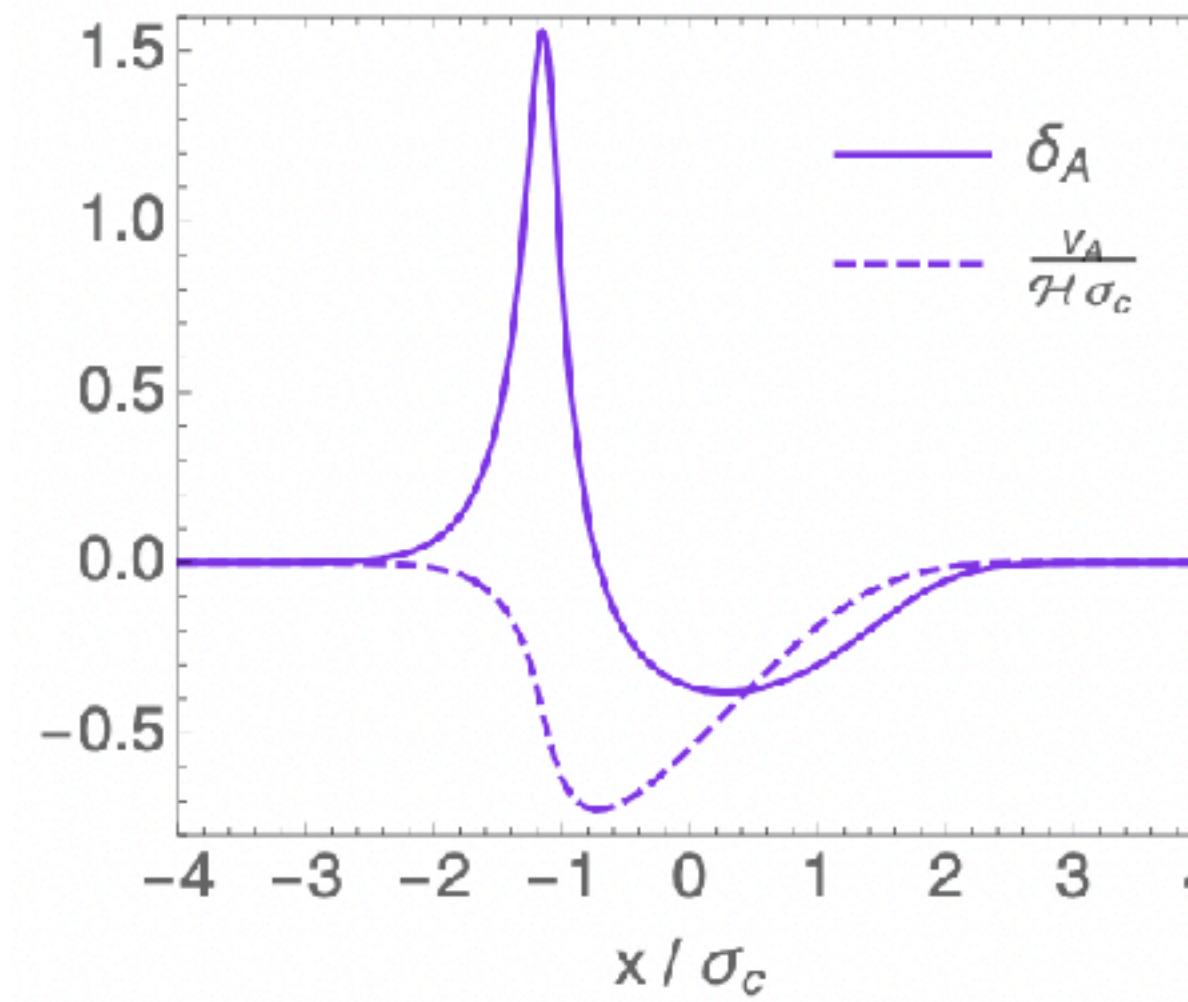
Braganca, ML, Sekera,  
Senatore, Sgier, 2020

# EFT for two fluids

Braganca, ML, Sekera,  
Senatore, Sgier, 2020

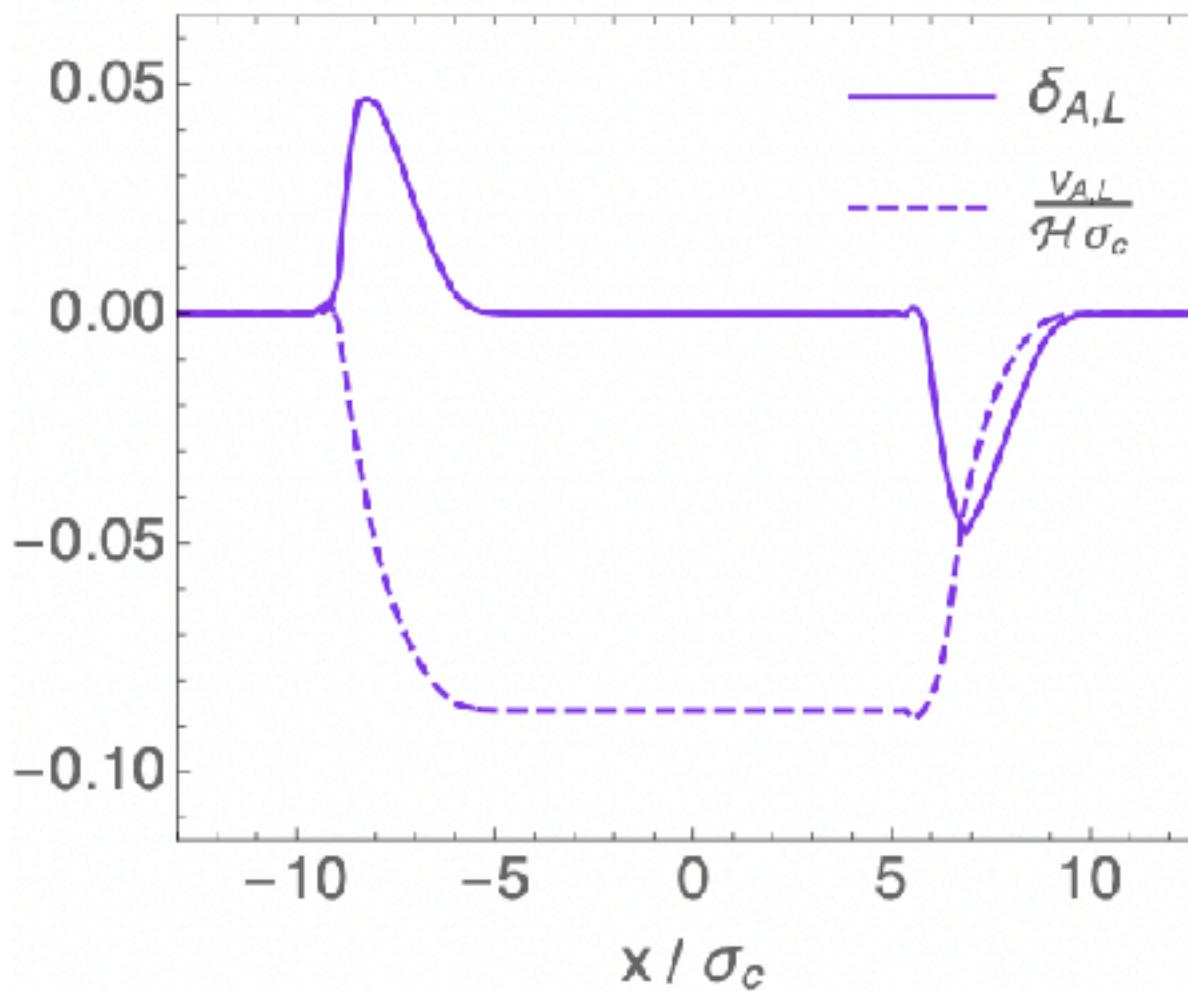
linear relative velocity counterterm  
size estimation - UV model

nonlinear  
solutions

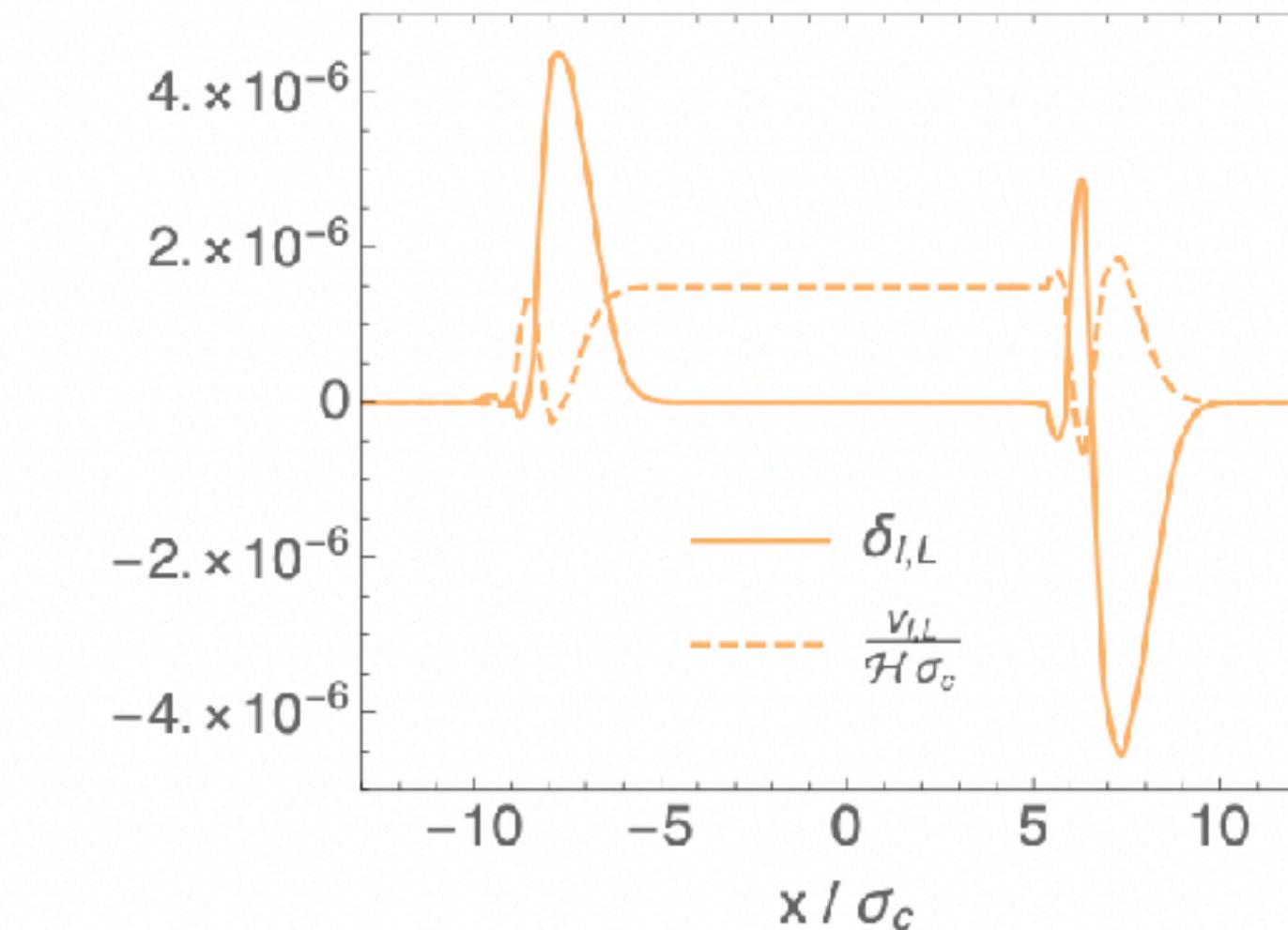
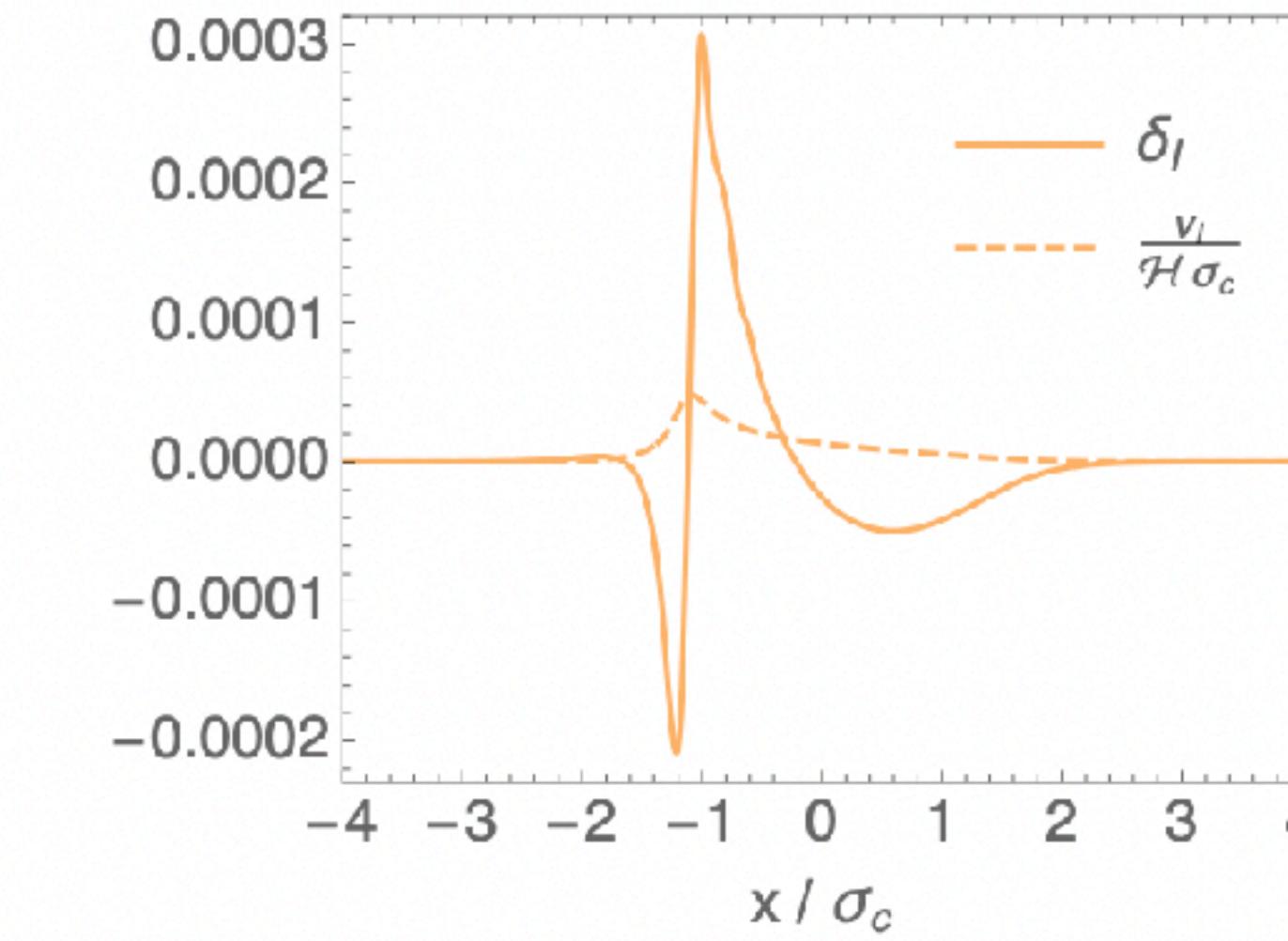


smoothed  
solutions

$L = 15\sigma_c$



$a = 0.5$



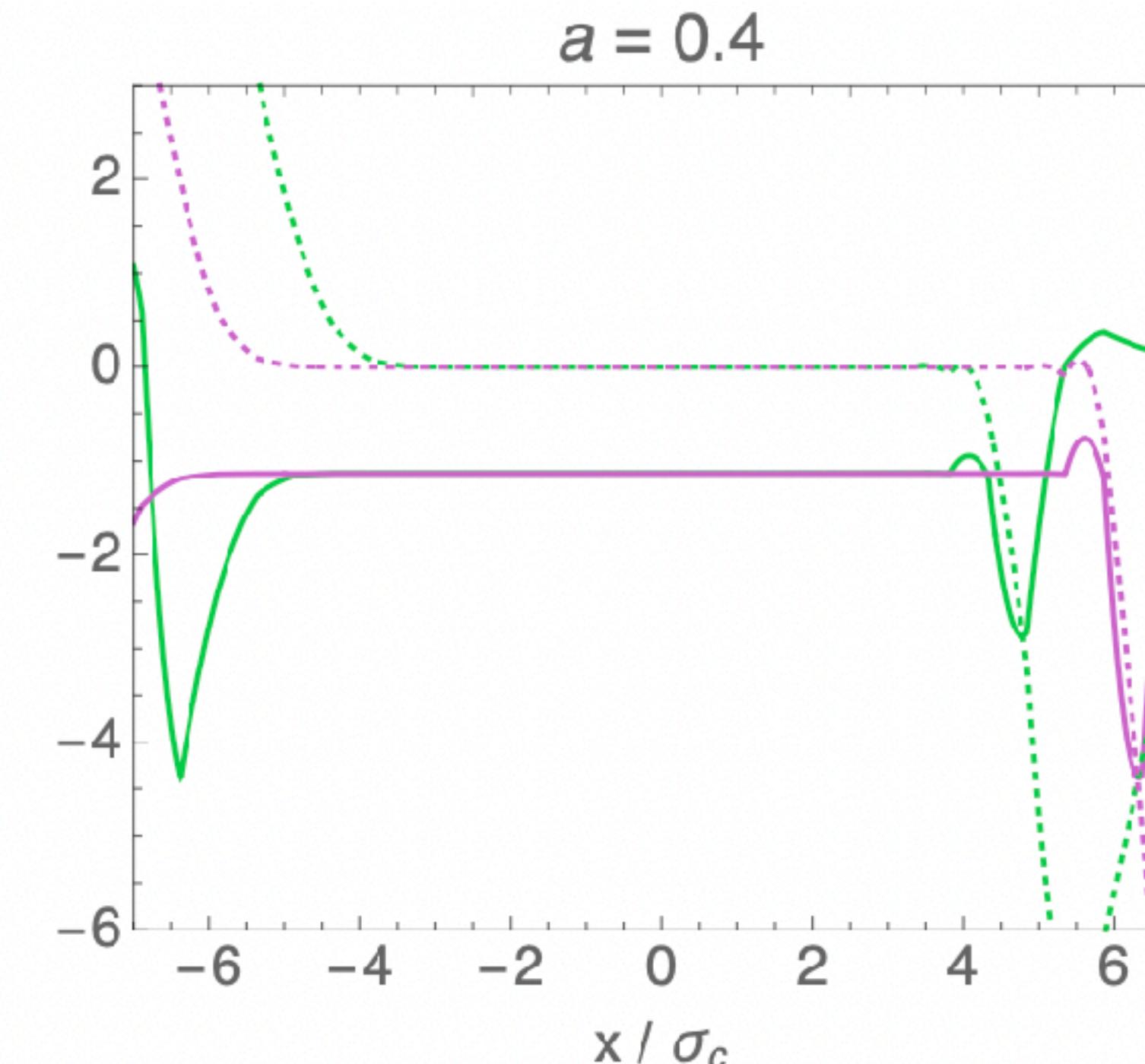
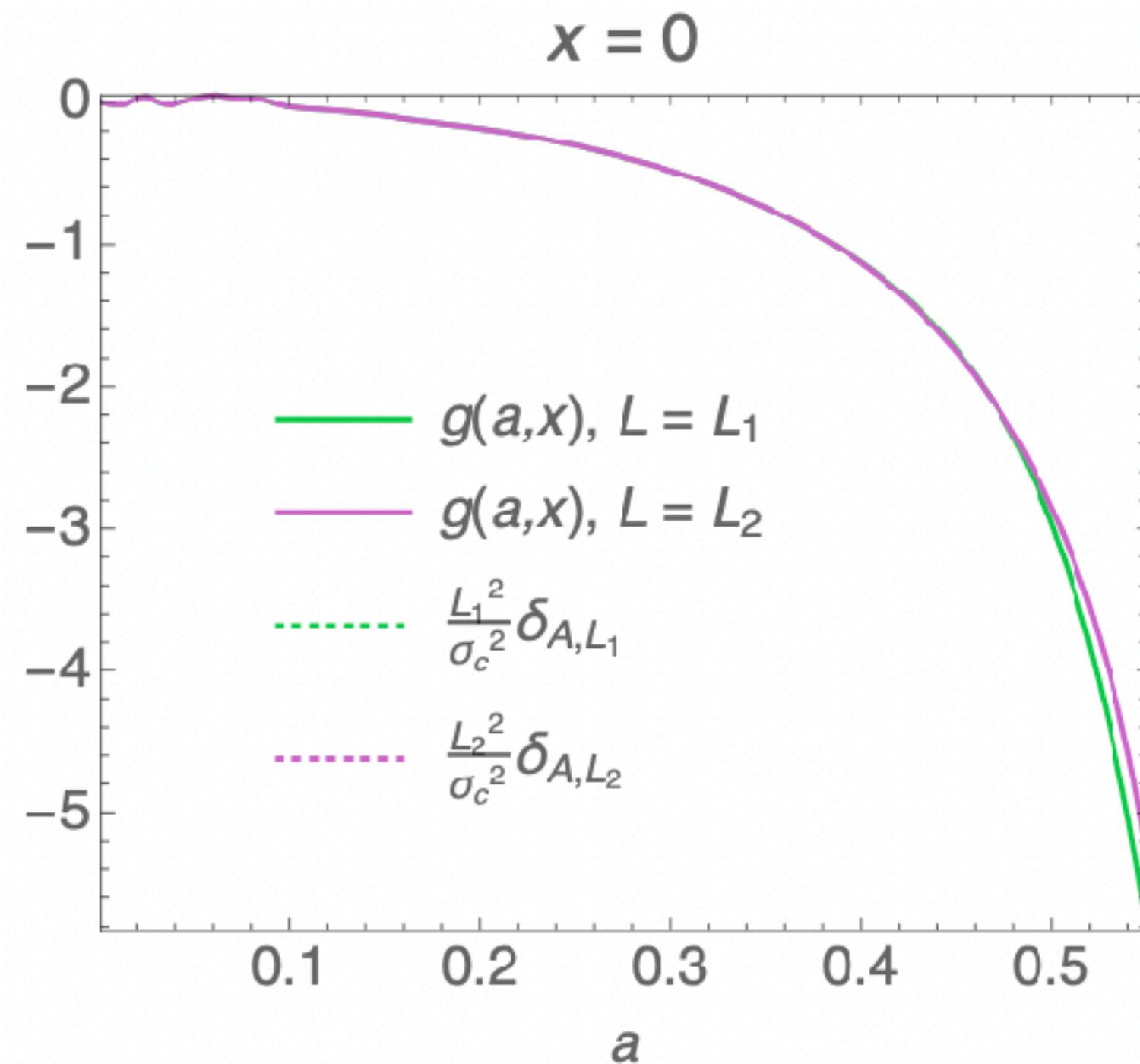
# EFT for two fluids

Braganca, ML, Sekera,  
Senatore, Sgier, 2020

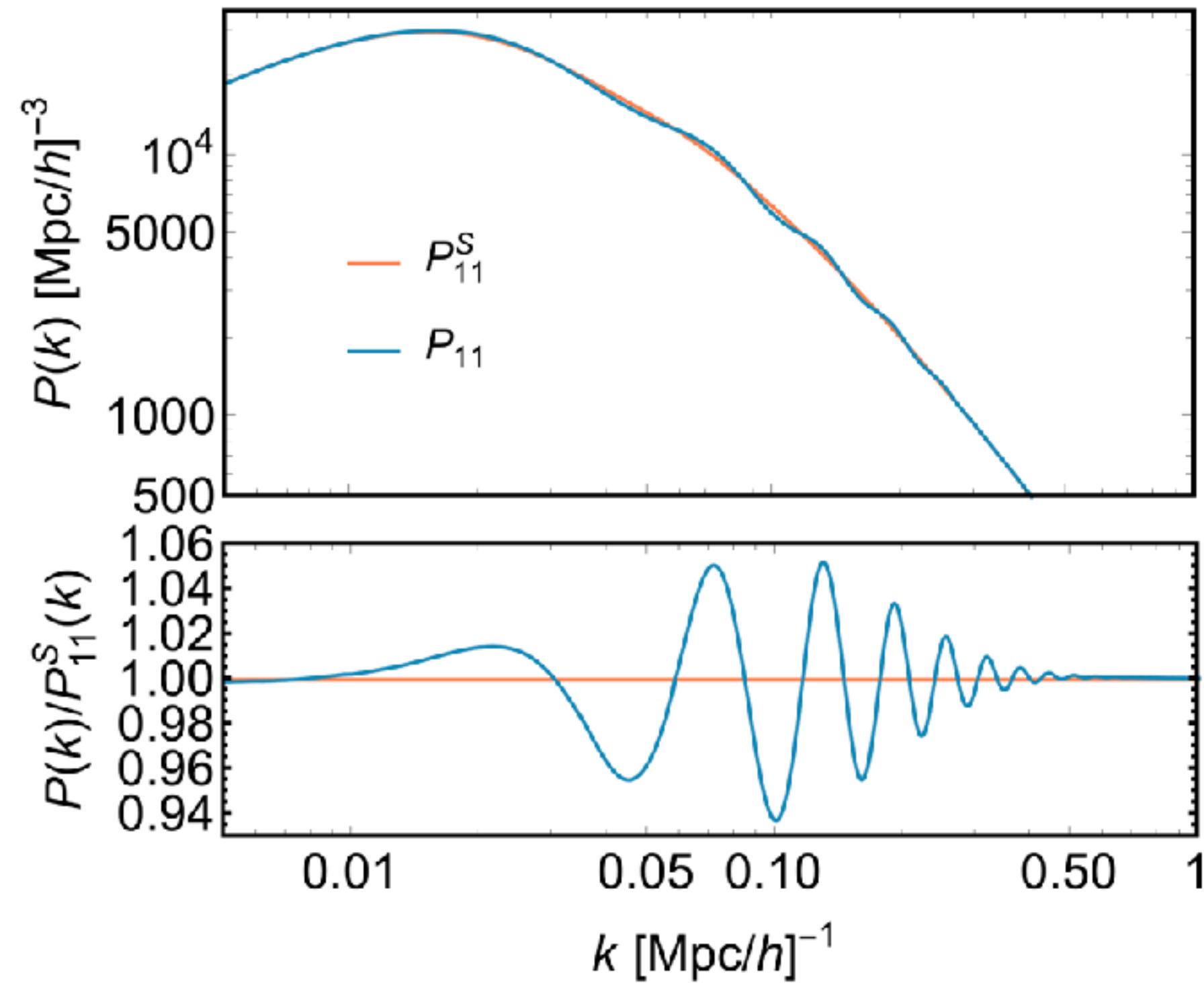
linear relative velocity counterterm  
size estimation - UV model

equation for smoothed field

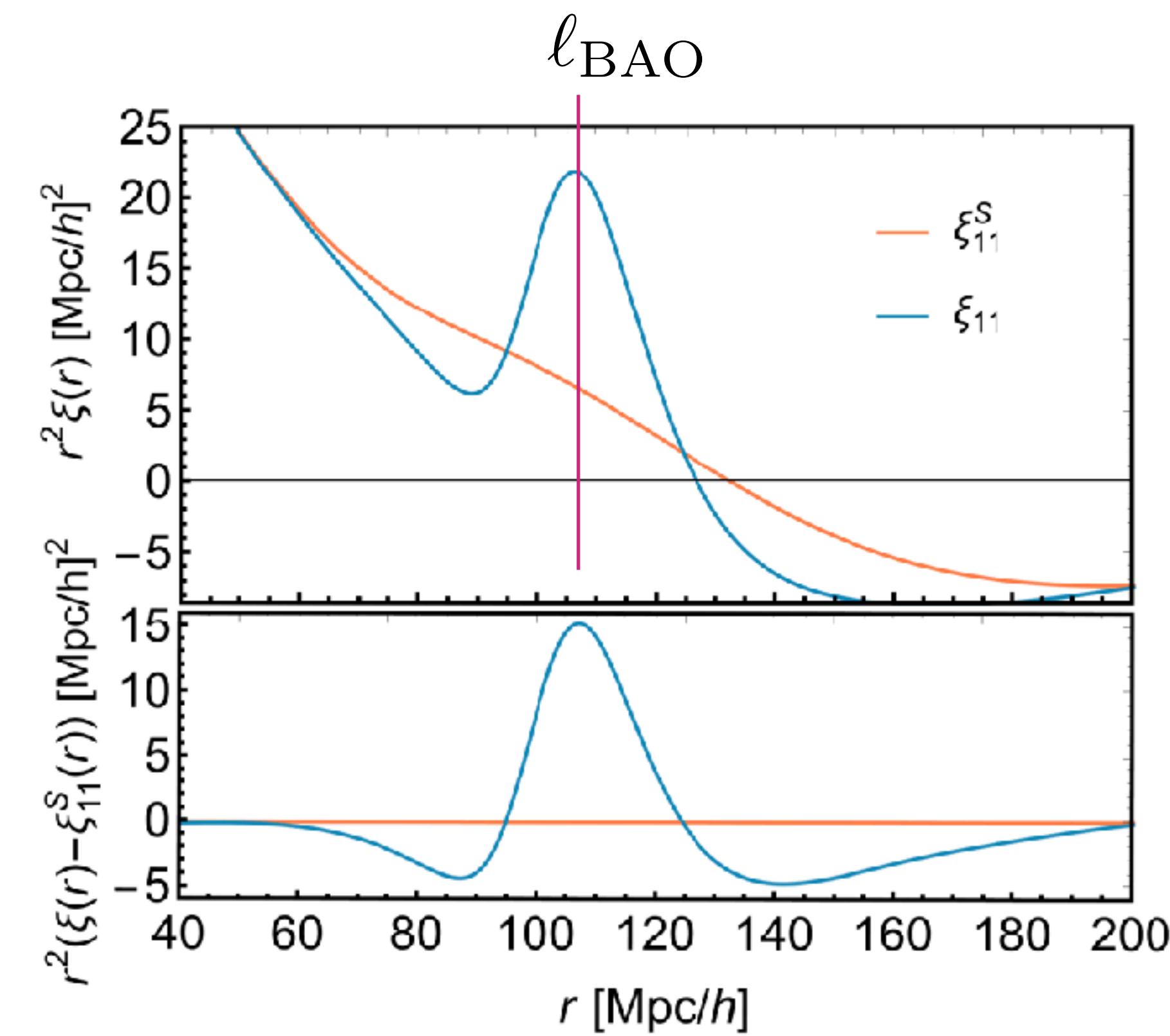
$$a^2 \delta_{I,L}''(a, x) + \left( 2 + \frac{a\mathcal{H}'(a)}{\mathcal{H}(a)} \right) a \delta_{I,L}'(a, x) = g(a, x) a \delta_{I,L}'(a, x) + \dots$$



# reminder



$$\frac{2\pi}{\ell_{\text{BAO}}}$$



$$\xi(r) = \int_{\vec{k}} P(k) e^{i\vec{k}\cdot\vec{r}} = \int_0^\infty dk \frac{k^2}{2\pi^2} \frac{\sin kr}{kr} P(k)$$

# BAO

from the consistency conditions above

$$\lim_{q \rightarrow 0} \frac{B_t(\vec{q}, \vec{k}_1, \vec{k}_2)}{P_{11}(q)} \approx -\frac{\vec{q} \cdot (\vec{k}_1 + \vec{k}_2)}{q^2} P_{11}(k_1) \approx 0$$

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and expanding to next order in  $q/k$

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the coefficient above is fixed, but the next one is not.  
is there a regime where this can be the leading term?

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YES, for the BAO

BAO

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$$\textcolor{blue}{\mathrm{BAO}}$$

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$$k \, \ell_{\text{BAO}} \gg 1$$

$$k \, \ell_{\text{BAO}} \sim 10$$

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coefficient fixed by  
equivalence principle

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# BAO

## galaxies

$$\delta_g = b_1 \delta + \dots$$

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clean measurement of bias  
coefficient

# BAO

## power spectrum

$$\begin{aligned} P_{\text{1-loop}}(k) &\approx \int_{q \lesssim k} \frac{d^3 q}{(2\pi)^3} \left( \frac{\vec{q} \cdot \vec{k}}{q^2} \right)^2 P_{11}(q) \left( P_{11}(|\vec{k} - \vec{q}|) - P_{11}(k) \right) \\ &\approx 0 - \int_{q \lesssim k} \frac{d^3 q}{(2\pi)^3} \left( \frac{\vec{q} \cdot \vec{k}}{q^2} \right)^2 P_{11}(q) \left( \frac{q^2(1 - \mu^2)}{2k} \frac{\partial P_{11}(k)}{\partial k} + \frac{q^2 \mu^2}{2} \frac{\partial^2 P_{11}(k)}{\partial k^2} \right) \end{aligned}$$

$$\mu \equiv \hat{k} \cdot \hat{q}$$

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normally this term cancels, but again, it can have a large effect on the BAO, especially because it is enhanced in the IR limit by  $k^2/q^2$

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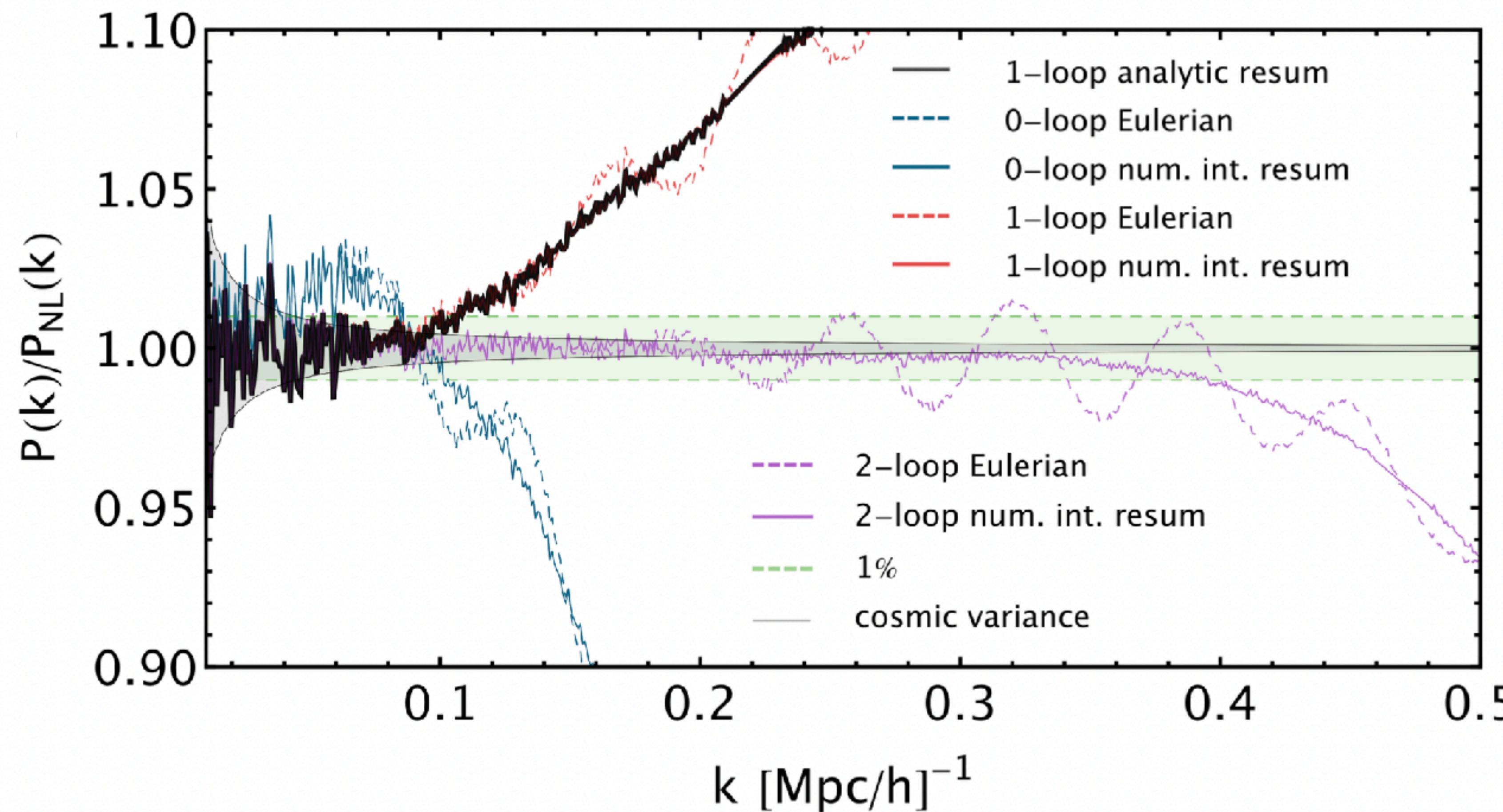
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but importantly, we know this coefficient because of the consistency conditions

# IR resummation

power spectrum



# the BOSS analysis beyond $\Lambda$ CDM

quintessence

D'Amico, Donath, Senatore, Zhang 21

(theory paper: **ML**, Maleknejad, Senatore 17)

Hubble tension

D'Amico, Senatore, Zhang, Zheng 21

- many models try to ease tension by making  $H$  larger before recombination
- Planck has precision constraint on angular acoustic scale

$$\theta_s(z_{\text{CMB}}) \sim r_s(\Omega_m h^2)^{0.4} H_0^{0.2}$$

- amplitude of acoustic peaks gives

$$A \sim r_s^{-0.26} (\Omega_m h^2)^{-0.25}$$

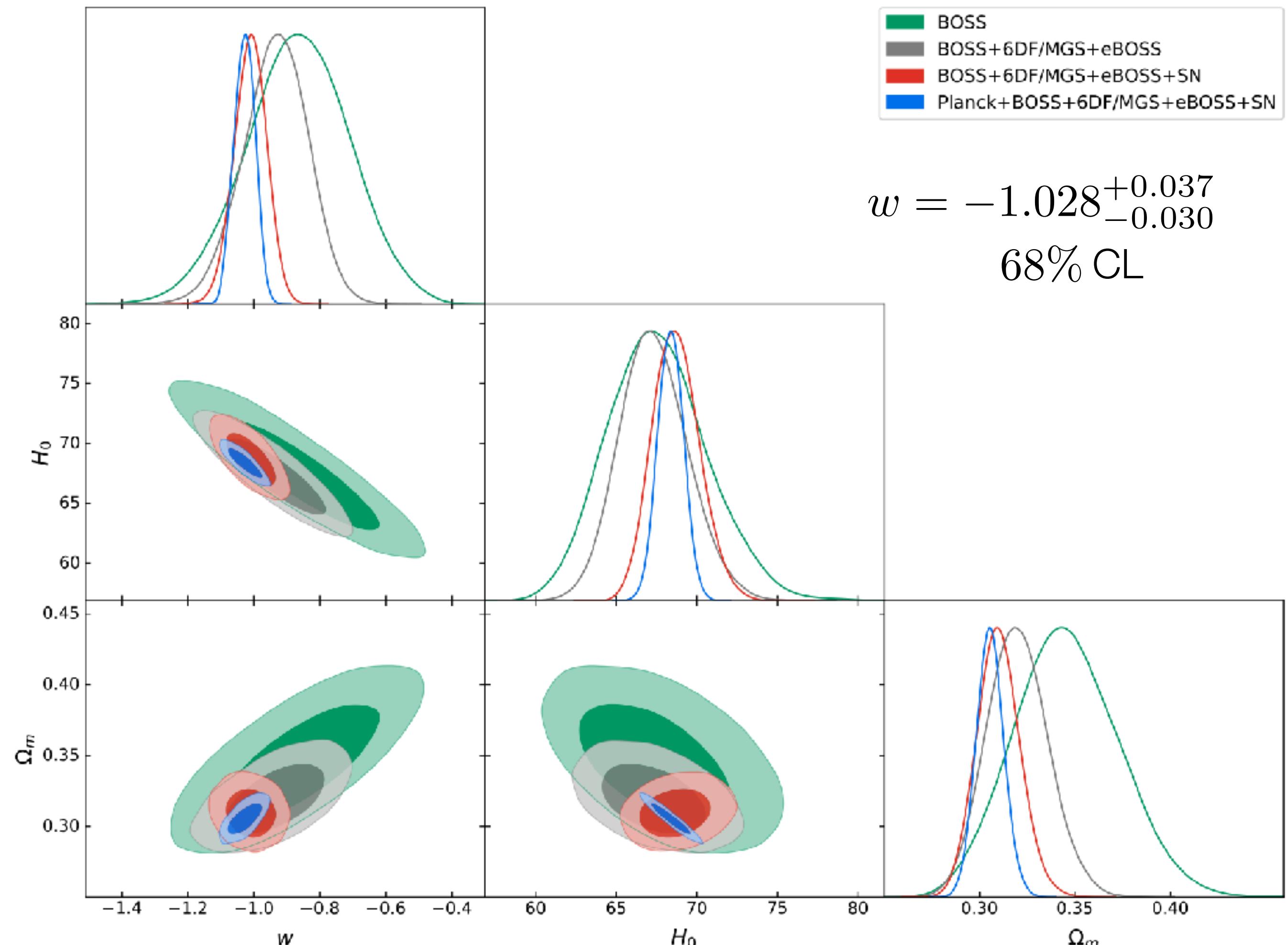
- so there is an  $r_s$ - $H_0$  degeneracy

- but LSS constrains

$$\theta_s(z_{\text{LSS}}) \sim r_s(\Omega_m h^2)^{0.1} H_0^{0.8}$$

and breaks degeneracy

- these models do *not* reduce  $H_0$  tension



# equations in Fourier space

convert to time variable  $a$  using  $\frac{da}{dt} = aH(a)$

$$\delta' \equiv \partial_a \delta$$

$$\mathcal{H} \equiv aH$$

$$\Theta \equiv -\partial_i v^i / \mathcal{H}$$

$$a\delta'_{\vec{k}}(a) - \Theta_{\vec{k}}(a) = \int_{\vec{q}} \alpha(\vec{k} - \vec{q}, \vec{q}) \delta_{\vec{k}-\vec{q}}(a) \Theta_{\vec{q}}(a)$$

$$a\Theta'_{\vec{k}}(a) + \left(1 + \frac{a\mathcal{H}'}{\mathcal{H}}\right) \Theta_{\vec{k}}(a) - \frac{3\Omega_m(a)}{2} \delta_{\vec{k}}(a) = \int_{\vec{q}} \beta(\vec{k} - \vec{q}, \vec{q}) \Theta_{\vec{k}-\vec{q}}(a) \Theta_{\vec{q}}(a)$$

$$- (2\pi)c_s^2(a) \frac{k^2}{k_{\text{NL}}^2} \delta_{\vec{k}}(a)$$

EFT of LSS

“interaction vertices” from the non-linear EOM

$$\alpha(\vec{k}_1, \vec{k}_2) = 1 + \frac{\vec{k}_1 \cdot \vec{k}_2}{k_1^2} , \quad \beta(\vec{k}_1, \vec{k}_2) = \frac{|\vec{k}_1 + \vec{k}_2|^2 \vec{k}_1 \cdot \vec{k}_2}{2k_1^2 k_2^2}$$