

# Properties of nuclei from chiral EFT interactions

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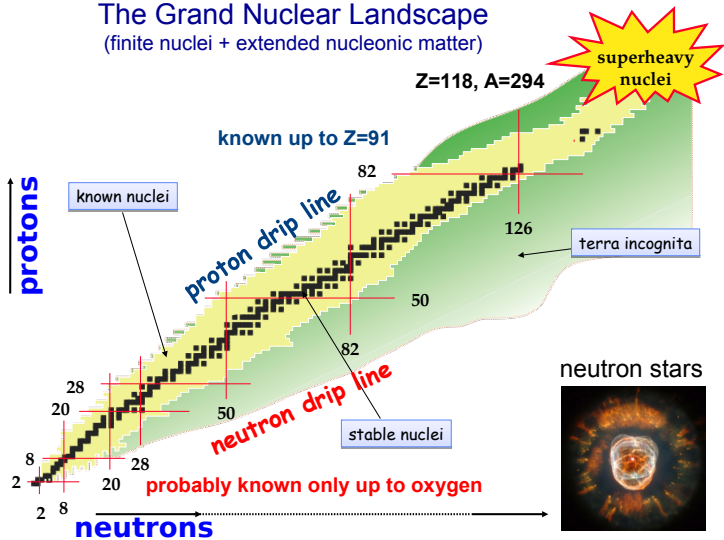
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National Energy Research  
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# The big picture

## The Grand Nuclear Landscape (finite nuclei + extended nucleonic matter)



- The nuclear Hamiltonian and the method
- Nuclear energies and charge radii
- Nuclear and neutron matter EOS
- Electro-magnetic currents and nuclear magnetic moments
- Conclusions

Model: non-relativistic nucleons interacting with an effective nucleon-nucleon force (NN) and three-nucleon interaction (TNI).

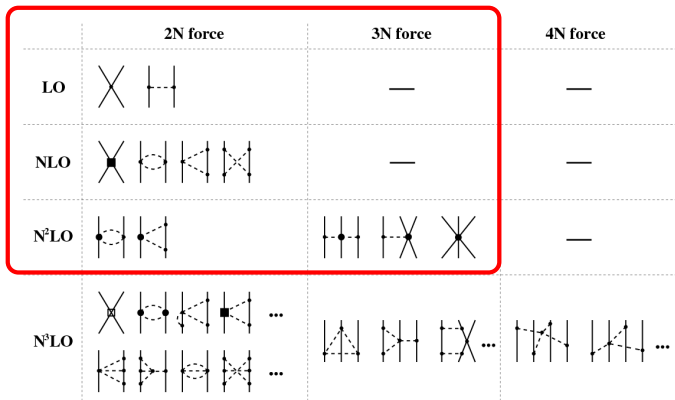
$$H = -\frac{\hbar^2}{2m} \sum_{i=1}^A \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

$v_{ij}$  NN fitted on scattering data.

$V_{ijk}$  typically constrained to reproduce light systems ( $A=3,4$ ).

- “Phenomenological/traditional” interactions (Argonne/Illinois)
- Local chiral forces up to N<sup>2</sup>LO (Gezerlis, et al. PRL 111, 032501 (2013), PRC 90, 054323 (2014), Lynn, et al. PRL 116, 062501 (2016)).

# Nuclear Hamiltonian



Expansion in powers of  $Q/\Lambda$ ,  $Q \sim 100$  MeV,  $\Lambda \sim 1$  GeV.

Long-range physics given by pion-exchanges (no free parameters).

Short-range physics: contact interactions (LECs) to fit.

Operators need to be regulated  $\rightarrow$  **cutoff dependency!**

Order's expansion provides a way to quantify uncertainties!

Error quantification (one possible scheme). Define

$$Q = \max\left(\frac{p}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right),$$

where  $p$  is a typical nucleon's momentum or  $k_F$  for matter,  $\Lambda_b$  is the cutoff, and calculate:

$$\Delta(N2LO) = \max\left(Q^4|\hat{O}_{LO}|, Q^2|\hat{O}_{LO} - \hat{O}_{NLO}|, Q|\hat{O}_{NLO} - \hat{O}_{N2LO}\right)$$

Epelbaum, Krebs, Meissner (2014).

# Quantum Monte Carlo

Propagation in imaginary time:

$$H \psi(\vec{r}_1 \dots \vec{r}_N) = E \psi(\vec{r}_1 \dots \vec{r}_N) \quad \psi(t) = e^{-(H-E_T)t} \psi(0)$$

Ground-state extracted in the limit of  $t \rightarrow \infty$ .

Propagation performed by

$$\psi(R, t) = \langle R | \psi(t) \rangle = \int dR' G(R, R', t) \psi(R', 0)$$

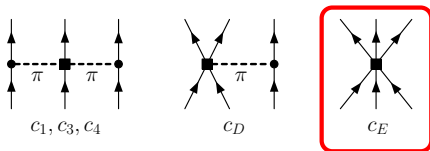
- Importance sampling:  $G(R, R', t) \rightarrow G(R, R', t) \Psi_I(R') / \Psi_I(R)$
- Constrained-path approximation to control the sign problem.  
Unconstrained-path calculation possible in several cases (exact).

GFMC includes all spin-states of nucleons in the w.f., nuclei up to  $A=12$   
AFDMC samples spin states, bigger systems, less accurate than GFMC

Ground-state obtained in a **non-perturbative way**. Systematic uncertainties within 2-3 %.

See Carlson, et al., Rev. Mod. Phys. 87, 1067 (2015)

# Chiral three-body forces, some issue



Equivalent forms of operators entering in  $V_E$  (Fierz-rearrangement):

$$1, \quad \sigma_i \cdot \sigma_j, \quad \tau_i \cdot \tau_j, \quad \sigma_i \cdot \sigma_j \tau_i \cdot \tau_j, \quad \sigma_i \cdot \sigma_j \tau_i \cdot \tau_k, \quad [(\sigma_i \times \sigma_j) \cdot \sigma_k][(\tau_i \times \tau_j) \cdot \tau_k]$$

Epelbaum et al (2002). We investigated the following choices:

$$V_{E\tau} = \frac{c_E}{\Lambda_\chi^4 F_\pi^4} \sum_{i < j < k} \sum_{\text{cyc}} \tau_i \cdot \tau_k \delta(r_{kj}) \delta(r_{ij})$$

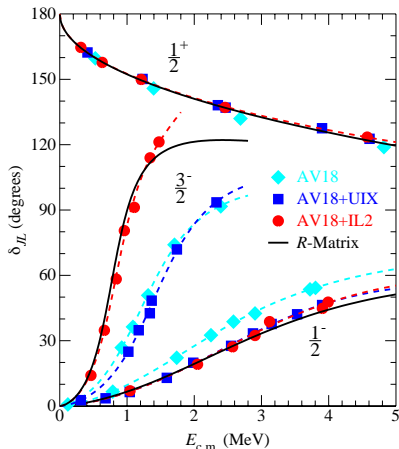
$$V_{E1} = \frac{c_E}{\Lambda_\chi^4 F_\pi^4} \sum_{i < j < k} \sum_{\text{cyc}} \delta(r_{kj}) \delta(r_{ij})$$

Qualitative differences expected, i.e. consider  ${}^4\text{He}$  vs neutron matter!



# Chiral three-body forces

Coefficients  $c_D$  and  $c_E$  fit to reproduce the binding energy of  $^4\text{He}$  and neutron- $^4\text{He}$  scattering.  $\rightarrow$  **more information on  $T=3/2$  part of three-body interaction.** (vs  $A=3, 4$ )



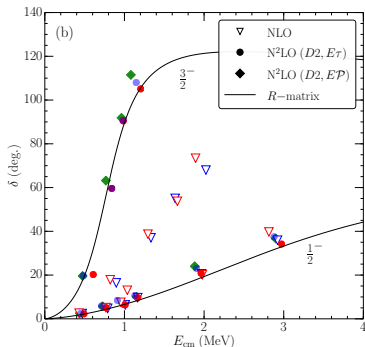
GFMC neutron- $^4\text{He}$  results using Argonne Hamiltonians.

Nollett, Pieper, Wiringa, Carlson, Hale, PRL (2007).

# $^4\text{He}$ binding energy and p-wave n- $^4\text{He}$ scattering

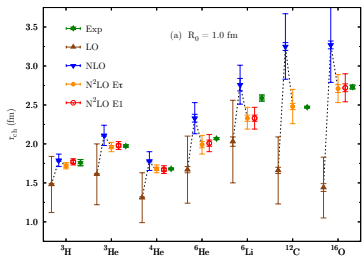
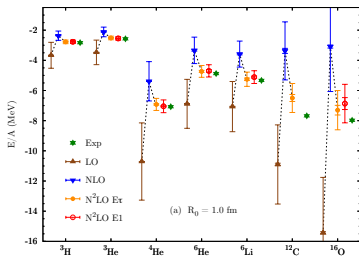
Regulator:  $\delta(r) = \frac{1}{\pi\Gamma(3/4)R_0^3} \exp(-(r/R_0)^4)$

Cutoff  $R_0$  taken consistently with the two-body interaction.



Lynn, Tews, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk PRL (2016).

Energies and charge radii, **cutoff 1.0 fm**:



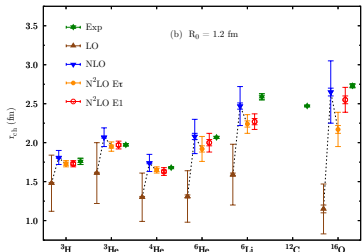
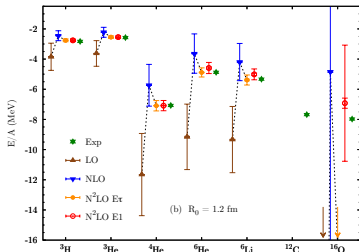
Lonardonì, et al., PRL (2018), PRC (2018).

Qualitative good description of both energies and radii.

Good convergence (although uncertainties still large if LO included).

**Different  $V_E$  operators give similar results.**

Energies and charge radii, **cutoff 1.2 fm**:

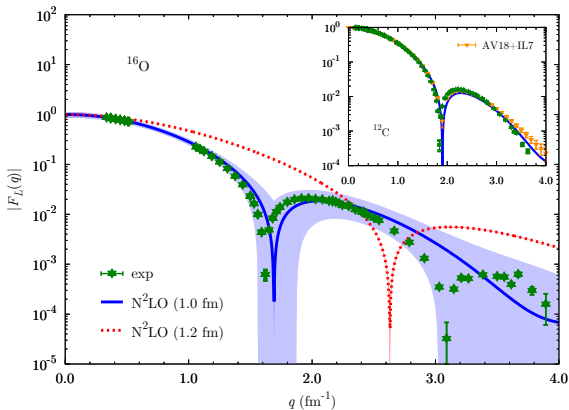


Lonardoni, et al., PRL (2018), PRC (2018).

Qualitative good description up to  $A=6$ .

Different  $V_E$  operators give very different results for  $^{16}\text{O}$ .

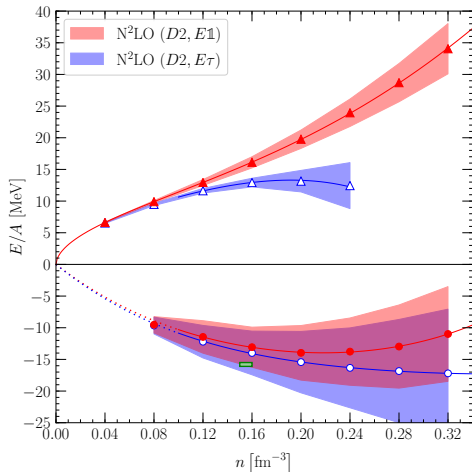
# Charge form factor



Lonardoni, et al., PRL (2018), PRC (2018).

Hard interaction reproduces exp. Similar good results for other nuclei.

# Nuclear and neutron matter EOS

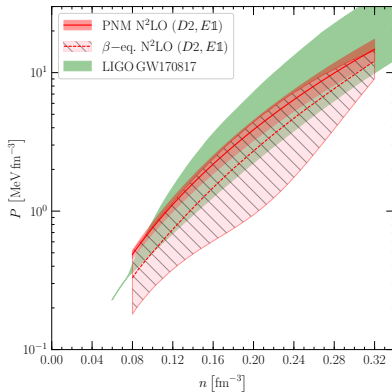


Lonaroni, et al., Phys. Rev. Research 2, 022033(R) (2020).

Symmetric nuclear matter less dependent upon the choice of  $V_E$  (cf. nuclei).  $E_\tau$  is very bad for neutron matter.

# Nuclear and neutron matter EOS

Pressure of pure neutron matter and  $\beta$ -equilibrated matter.



Lonardonì, et al., Phys. Rev. Research 2, 022033(R) (2020).

Very good agreement with LIGO posterior from GW.

# Electro-magnetic currents

	Single-nucleon	Two-nucleon	Three-nucleon
$Q^{-3}$			
$Q^{-1}$			
$Q^0$			
$Q^1$		<p>depend on <math>d_{8,9,18,21,22}</math>    parameter-free</p> <p>parameter-free</p> <p>depend on <math>C_{2,4,5,7}</math> and <math>L_{1,2}</math>    depend on <math>C_T</math> (known)</p>	<p>depend on <math>C_T</math> (known)</p>



# Electro-magnetic currents

But, within chiral EFT, there are different ways to construct the currents.

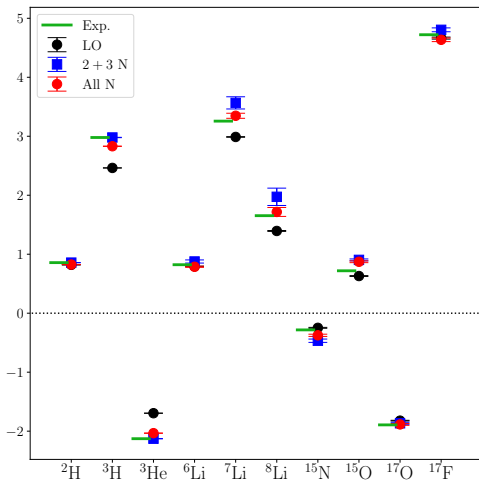
Hamiltonian	Pisa/Norfolk/WUSTL	Bochum
$Q^0$ LO	$Q^{-2}$ LO	$Q^{-3}$ LO
$Q^1$ –	$Q^{-1}$ NLO	$Q^{-2}$ –
$Q^2$ NLO	$Q^0$ N2LO	$Q^{-1}$ NLO
$Q^3$ N2LO	$Q^1$ N3LO	$Q^0$ N2LO
		$Q^1$ N3LO
	2 LECS (contact)	2 LECS (contact)
	1 LEC (OPE)	

Understanding which currents to use, chiral order, regulators, continuity equation, etc., is still work in progress.

The Pisa/etc currents will be used in the next few slides (with some adjustment...).

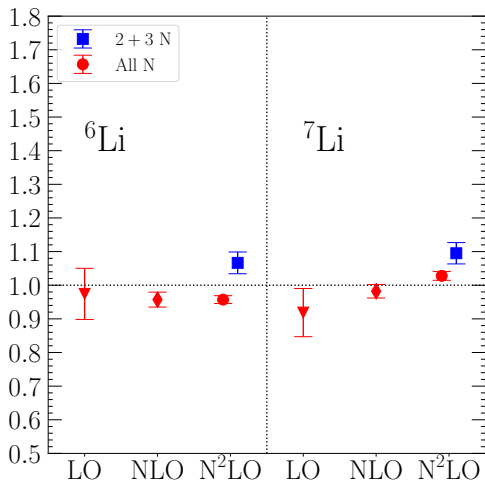
# Nuclear magnetic moments

Fit: up to  $A=3$  or including all nuclei.



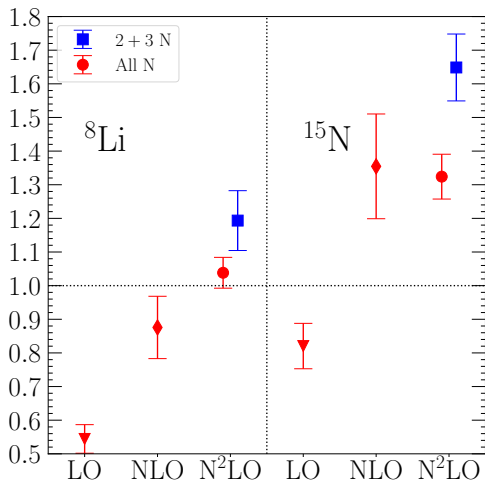
# Nuclear magnetic moments

Order by order calculation. Same order in the Hamiltonian and currents.



Pretty good convergence.

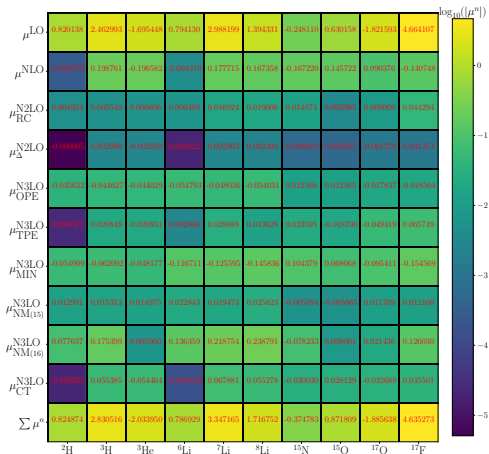
# Nuclear magnetic moments



In these cases the convergence is not clear (LO errors are smaller than NLO).

# Nuclear magnetic moments

Magnetic moments, single contributions.



In several cases, the contribution of N3LO terms is larger than N2LO and NLO.

- Chiral EFT provides a way to constrain nuclear interactions and estimate systematic uncertainties
- Nuclear energies and radii well reproduced by the hard interaction. But several issues with the softer one.
- Role of  $V_E$  critical for neutron matter, but symmetric nuclear matter is qualitatively ok.
- EOS compatible with GW observations.
- Still a lot of work to do to derive EM currents in a consistent way (work in progress).
- Overall good description of nuclear magnetic moments. Best fitting strategy not clear yet.
- Convergence with the chiral expansion not clear.

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