Progress on the extraction of generalized parton distributions from experimental data

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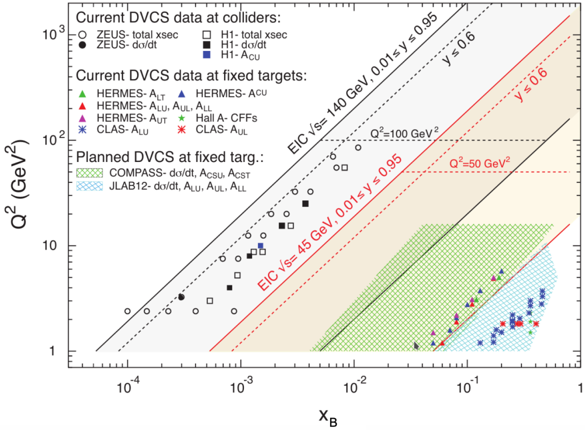
Exciting experimental promises



- The EIC offers to measure exclusive processes sensitive to GPDs at high luminosity on a large kinematic range.
- This triggers interest in unbiased extraction of GPDs from exclusive processes, beyond models parametrized with limited flexibility.
- In this talk, particular emphasis is put on deeply virtual Compton scattering (DVCS) considered as a golden channel of GPD extraction and already thoroughly studied.

Overview

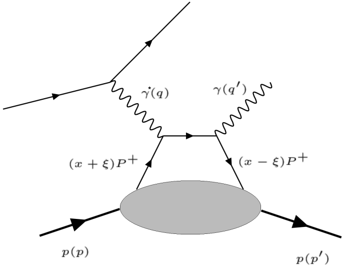
1. Deeply virtual Compton scattering and the structure of hadrons
2. Warming-up: extraction of gravitational form factors
3. Deconvoluting a Compton form factor: shadow GPDs
4. New models of GPDs
5. Perspectives

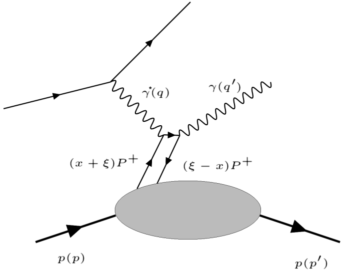
1. Deeply virtual Compton scattering and the structure of hadrons

Deeply virtual Compton scattering and the structure of hadrons

DVCS: lepton hadron scattering via a photon of large virtuality, producing a real photon in the final state. It is an **exclusive process** with an intact recoil proton.

- x is the average light-front plus-momentum (longitudinal momentum in a fast moving hadron) fraction of the struck parton
- ξ describes the light-front plus-momentum transfer, linked to Björken's variable x_B
- $t = \Delta^2$ is the total four-momentum transfer squared





GPDs were introduced more than two decades ago in [Müller *et al*, 1994], [Radyushkin, 1996] and [Ji, 1997].

Tree-level depiction of DVCS for $x > |\xi|$ (left) and $\xi > |x|$ (right)

Deeply virtual Compton scattering and the structure of hadrons

Similarly to usual PDFs in the study of DIS,

- For a large photon virtuality $Q^2 = -q^2$, finite x_B and small t, **factorisation theorems** describe DVCS observables in terms of a hard scattering part, and a soft part described by **generalized parton distributions** (GPDs).
- DVCS observables can be expressed in terms of **Compton form factors** (CFFs) $\mathcal{F}$, which write as convolutions of perturbative **coefficient functions** T_F^a and the **GPDs** F^a:

CFF convolution (leading twist) [Radyushkin, 1997], [Ji, Osborne, 1998], [Collins, Freund, 1999]

$$\mathcal{F}(\xi, t, Q^2) = \sum_{\text{parton type } a} \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} \, T_F^a\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) \frac{F^a(x, \xi, t, \mu^2)}{x^{p_a}} \quad (1)$$

μ is the factorisation / renormalisation scale, α_s the strong coupling, $p_a = 0$ for $a = q$ and $p_a = 1$ for $a = g$.

Properties of GPDs

- The **forward limit** $t \to 0$, $\xi \to 0$ gives back the usual **PDF**

$$H^a(x, \xi = 0, t = 0, \mu^2) = x^{p_a} f^a(x, \mu^2) \tag{2}$$

where $p_a = 0$ if $a = q$ and $p_a = 1$ if $a = g$.

- The evolution of GPDs with scale μ^2 generalizes the evolution kernels of PDFs (DGLAP) and distribution amplitudes (ERBL) [Müller, 1994]

$$\frac{1}{x^{p_a}} \frac{\partial}{\partial \log(\mu^2)} H^a(x, \xi, t, \mu^2) = \sum_b \int_{-1}^{1} \frac{\mathrm{d}y}{\xi} K^{ab}\left(\frac{y}{\xi}, \frac{\xi}{x}, \alpha_s(\mu^2)\right) \frac{H^b(y, \xi, t, \mu^2)}{y^{p_b}} \tag{3}$$

- Because of the parity of the process, DVCS only involves the C-even – or singlet – GPDs, given *e.g.* for H^q by

$$H^{q(+)}(x, \xi, t, \mu^2) = H^q(x, \xi, t, \mu^2) - H^q(-x, \xi, t, \mu^2) \tag{4}$$

Deeply virtual Compton scattering and the structure of hadrons

Polynomiality of Mellin moments: [Ji, 1998], [Radyushkin, 1999]
Due to Lorentz covariance,

$$\int_{-1}^{1} \mathrm{d}x\, x^n H^q(x, \xi, t, \mu^2) = \sum_{k=0 \text{ even}}^{n+1} H^q_{n,k}(t, \mu^2)\xi^k \tag{5}$$

This property implies that the GPD is the Radon transform of a **double distribution** F^q (DD) with an added ***D*-term** on the support $\Omega = \{(\beta, \alpha) \mid |\beta| + |\alpha| < 1\}$:

Double distribution formalism [Radyushkin, 1997], [Polyakov, Weiss, 1999]

$$H^q(x, \xi, t, \mu^2) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha\, \delta(x - \beta - \alpha\xi) \left[F^q(\beta, \alpha, t, \mu^2) + \xi\delta(\beta) D^q(\alpha, t, \mu^2)\right] \tag{6}$$

Deeply virtual Compton scattering and the structure of hadrons

- Remarkably, GPDs allow access to gravitational form factors (GFFs) of the **energy-momentum tensor (EMT)** [Ji, 1997] defined for parton of type a

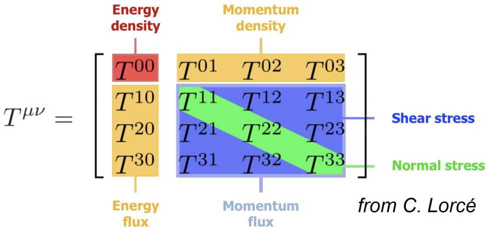
Gravitational form factors [Lorcé *et al*, 2017]

$$\langle p', s' | T_a^{\mu\nu} | p, s \rangle = \bar{u}(p', s') \Bigg\{ \frac{P^\mu P^\nu}{M} A_a(t, \mu^2) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2}{M} C_a(t, \mu^2) + M \eta^{\mu\nu} \bar{C}_a(t, \mu^2) + \frac{P^{\{\mu} i\sigma^{\nu\}\rho} \Delta_\rho}{4M} \left[A_a(t, \mu^2) + B_a(t, \mu^2) \right] + \frac{P^{[\mu} i\sigma^{\nu]\rho} \Delta_\rho}{4M} D_a^{GFF}(t, \mu^2) \Bigg\} u(p, s) \tag{7}$$

where

$$\Delta = p' - p, \ \ t = \Delta^2, \ \ P = \frac{p + p'}{2} \tag{8}$$

Deeply virtual Compton scattering and the structure of hadrons



from C. Lorcé

In the Breit frame ($\vec{P} = 0,\ t = -\vec{\Delta}^2$), radial distributions of energy and momentum in the proton are described by Fourier transforms of the **GFFs** w.r.t. variable $\vec{\Delta}$ [Polyakov, 2003].

- Example of such distribution: radial pressure anisotropy profile

$$s_a(r,\mu^2) = -\frac{4M}{r^2}\int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}}\,\frac{t^{-1/2}}{M^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left[t^{5/2}\,C_a(t,\mu^2)\right] \tag{9}$$

- This pressure profile can be extracted from **GPDs** thanks to *e.g.* for quarks

$$\int_{-1}^{1} \mathrm{d}x\, x\, H^q(x,\xi,t,\mu^2) = A_q(t,\mu^2) + 4\xi^2 C_q(t,\mu^2) \tag{10}$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, E^q(x,\xi,t,\mu^2) = B_q(t,\mu^2) - 4\xi^2 C_q(t,\mu^2) \tag{11}$$

Extraction of GFFs

2. Warming-up: extraction of gravitational form factors from experimental data

Extraction of GFFs

- No need to fully extract the GPDs H or E to conveniently access the GFF $C_q(t,\mu^2)$. Thanks to the **polynomiality property**, $C_q(t,\mu^2)$ depends only on the D-term via

$$\int_{-1}^{1} \mathrm{d}z \, z D^q(z,t,\mu^2) = 4 C_q(t,\mu^2) \tag{12}$$

- Experimental data is sensitive to the D-term through the **subtraction constant** defined by the **dispersion relation** (see *e.g.* [Diehl, Ivanov, 2007])

DVCS dispersion relation

$$\mathcal{C}_H(t,Q^2) = \mathrm{Re}\,\mathcal{H}(\xi,t,Q^2) - \frac{1}{\pi}\int_0^1 \mathrm{d}\xi' \,\mathrm{Im}\,\mathcal{H}(\xi',t,Q^2)\left(\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right) \tag{13}$$

The subtraction constant $\mathcal{C}_H(t,Q^2)$ is a function of the D-term given at LO by

$$\mathcal{C}_H(t,Q^2) = 2\sum_q e_q^2 \int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,Q^2)}{1-z} \tag{14}$$

Extraction of GFFs

- How do we get from

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} \quad \text{to} \quad \int_{-1}^{1} \mathrm{d}z \, z D^q(z,t,\mu^2) \ ? \tag{15}$$

- This is a prototype of the more complicated GPD extraction problem we will face later on. The known solution is through evolution.
- Let's expand the D-term on a basis of Gegenbauer polynomials

$$D^q(z,t,\mu^2) = (1-z^2) \sum_{\text{odd } n} d_n^q(t,\mu^2) \, C_n^{3/2}(z) \tag{16}$$

Then

GFF C_a extraction

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} = 2 \sum_{\text{odd } n} d_n^q(t,\mu^2) \quad \text{and} \quad \int_{-1}^{1} \mathrm{d}z \, z D^q(z,t,\mu^2) = \frac{4}{5} \, d_1(t,\mu^2) \tag{17}$$

Extraction of GFFs

- Because Gegenbauer polynomials diagonalize the LO ERBL [Lepage, Brodsky, 1979], [Efremov, Radyushkin, 1979] evolution kernel, each term $d_n^q(t,\mu^2)^*$ evolves multiplicatively with a different anomalous dimension. Since exponentials are a free family on any non-vanishing interval, the decomposition

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} = 2 \sum_{\text{odd } n} d_n^q(t,\mu^2) \tag{18}$$

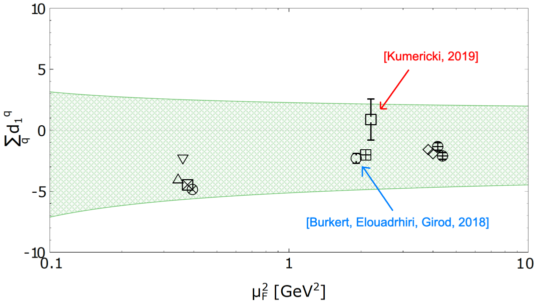
  is **unique, non-ambiguous and theoretically allows to entirely retrieve the D-term from the knowledge of the subtraction constant on any non-vanishing interval in $Q^2 = \mu^2$.**

- All is well on paper, but what about in real life?

*actually, due to the mixing of quarks and gluons under evolution, it is actually a linear combination of d_n^q and d_n^g which evolves multiplicatively, but it does not change the argument

Extraction of GFFs

- Details found in **[Dutrieux et al, Eur.Phys.J.C 81 (2021) 4, 300]**. We perform a neural network fit of CFFs over world DVCS data, which gives a **subtraction constant compatible with 0** $\rightarrow$ also found in [Kumericki, 2019]. Then fixing the t-dependence with an Ansatz and assuming all d_n for $n > 1$ to be 0 gives



In green, 68% confidence interval found for $\sum_q d_1^q(t = 0, \mu^2)$. Results obtained by the two other data-driven extractions highlighted. **But this is essentially a fit with one free parameter d_1^q whose uncertainty reflects the experimental uncertainty on the subtraction constant. What happens in case of a more flexible parametrization?**

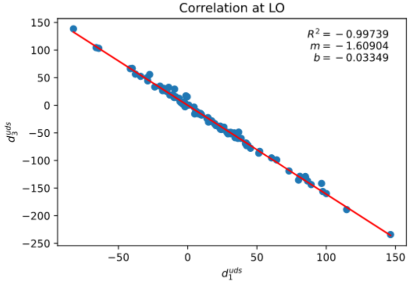
Extraction of GFFs

- To reduce bias, let us also allow d_3^q to be fitted jointly with d_1^q $(\mu_F^2 = 2\ \text{GeV}^2)$

$d_1^{uds}(\mu_{\rm F}^2)$	-0.5 ± 1.2

$\longrightarrow$

$d_1^{uds}(\mu_{\rm F}^2)$	11 ± 25
$d_3^{uds}(\mu_{\rm F}^2)$	-11 ± 26



- Uncertainty explodes, and $d_1^q(\mu_F^2) \approx -d_3^q(\mu_F^2)$! What is going on?

Extraction of GFFs

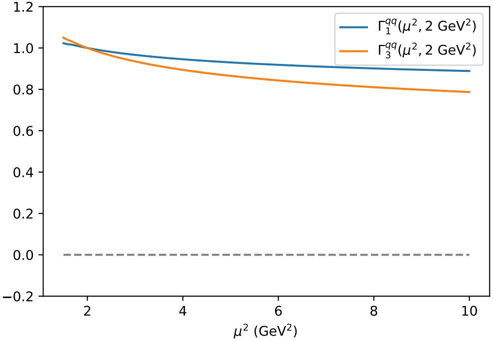
- The LO subtraction constant reads

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z, t, \mu^2)}{1 - z} = 2 \sum_{\text{odd } n} d_n^q(t, \mu^2) \tag{19}$$

  so at fixed scale μ_0^2, $d_1^q(\mu_0^2) = -d_3^q(\mu_0^2)$ does not bring any contribution on the subtraction constant. **Arbitrary large values of $d_1^q(\mu_0^2) = -d_3^q(\mu_0^2)$ are unconstrained by experimental data at fixed scale.**
- Under evolution, the equality $d_1^q(\mu^2) = -d_3^q(\mu^2)$ cannot remain valid, and evolution provides the practical way to make the problem invertible. But if experimental data are only available on a small range on Q^2, we still may have $d_1^q(\mu^2) \approx -d_3^q(\mu^2)$. **Strong difference in the rate of evolution of different d_n on the probed Q^2 range is crucial for the practical possibility of performing an unbiased extraction.**

Extraction of GFFs

Preliminary result



Simplified evolution in the qq sector

$$d_n^q(\mu^2) = \Gamma_n^{qq}(\mu^2, 2\ \mathrm{GeV}^2) d_n^q(2\ \mathrm{GeV}^2) \quad (20)$$

- current range of most DVCS data : [1.5, 4] GeV^2
- Over this range, Γ_1^{qq} and Γ_3^{qq} are numerically very close $\rightarrow$ little actual leverage in evolution to separate the two
- Estimate of the inflation on uncertainty when fitting jointly d_1 and d_3 compared to the sole d_1 :

$$\propto \left(1 - \frac{\Gamma_3^{qq}(Q^2_{\max}, Q^2_{\min})}{\Gamma_1^{qq}(Q^2_{\max}, Q^2_{\min})}\right)^{-1} \quad (21)$$

- **An increase thanks to EIC from [1.5, 4] GeV^2 to [1.5, 50] GeV^2 could yield a decrease by 3 times of the uncertainty on (d_1, d_3) due to the sole effect of increase in Q^2 range, without taking account a better experimental precision.**

3. Deconvoluting a Compton form factor: shadow GPDs

Deconvoluting a Compton form factor

Position of the problem
Assuming a CFF has been extracted from experimental data with excellent precision – and the different gluon and flavor contributions have been separated –, we are left with the convolution:

$$\int_{-1}^{1} \frac{\mathrm{d}x}{\xi} T^q\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) H^q(x, \xi, t, \mu^2) = T^q(Q^2, \mu^2) \otimes H^q(\mu^2) \tag{22}$$

where T^q is a coefficient function computed in pQCD. **Can we then "de-convolute" Eq.** (22) **to recover** $H^{q(+)}(x, \xi, t, \mu^2)$ **from** $T^q(Q^2, \mu^2) \otimes H^q(\mu^2)$**?**

Deconvoluting a Compton form factor

- Question was raised 20 years ago. Evolution was again proposed as a crucial element in [Freund, 1999], but no full-fledged proof of feasibility at NLO was provided.
- Following the same idea as previously, we show that GPDs exist which bring exactly no contributions to the LO and NLO CFF at fixed scale, and study the effect of evolution on such objects. We call them **LO and NLO shadow GPDs**.

Definition of an NLO shadow GPD

For a given scale μ_0^2,

$$\forall \xi, \forall t, \; T^q_{NLO}(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0 \quad \text{and} \quad H^q(x, \xi = 0, t = 0, \mu_0^2) = 0 \tag{23}$$

$$\text{so for } Q^2 \text{ and } \mu^2 \text{ close enough to } \mu_0^2, \; T^q_{NLO}(Q^2, \mu^2) \otimes H^q(\mu^2) = \mathcal{O}(\alpha_s^2(\mu^2)) \tag{24}$$

- Let H^q be an NLO shadow GPD, and G^q be any GPD. Then G^q and $G^q + H^q$ have the same forward limit, and the same NLO CFF up to a numerically small and theoretically subleading contribution.

Shadow GPDs at leading order

- Complete details in **[Bertone et al, Phys.Rev.D 103 (2021) 11, 114019]**
- We search for our shadow GPDs as simple **double distributions (DD)** $F(\beta, \alpha, \mu^2)$ to respect polynomiality, with a zero D-term. Then, thanks to dispersion relations, we can restrict ourselves to the imaginary part only $\mathrm{Im}\ T^q(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0$.
- We search our DD as a polynomial of order N in (β, α), characterized by $\sim N^2$ coefficients c_{mn}:

$$F(\beta, \alpha, \mu_0^2) = \sum_{m+n \leq N} c_{mn}\, \alpha^m \beta^n \qquad (25)$$

- **Leading order** At LO, the imaginary part of the CFF gives

$$\mathrm{Im}\ T^q_{LO}(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) \propto H^{q(+)}(\xi, \xi, \mu_0^2) \qquad (26)$$

and it is easy to build a system of $\sim N$ equations on the $\sim N^2$ coefficients c_{mn} of the polynomial DD and exhibit an infinite number of solutions cancelling the LO CFF.

Shadow GPDs at next-to-leading order

- **First study beyond leading order:** Apart from the **LO** part, the NLO CFF is composed of a **collinear part** (compensating the α_s^1 term resulting from the convolution of the LO coefficient function with the LO evolution of the GPD) and a genuine **1-loop NLO** part.

$$\mathcal{H}^q(\xi, Q^2) = C_0^q \otimes H^{q(+)}(\mu_0^2) + \alpha_s(\mu^2)\, C_1^q \otimes H^{q(+)}(\mu_0^2) + \alpha_s(\mu^2)\, C_{coll}^q \otimes H^{q(+)}(\mu_0^2) \log\left(\frac{\mu^2}{Q^2}\right) \quad (27)$$

An explicit calculation of each term for our polynomial double distribution gives that

$$\mathrm{Im}\ T_{coll}^q(Q^2, \mu^2) \otimes H^q(\mu^2) \propto$$

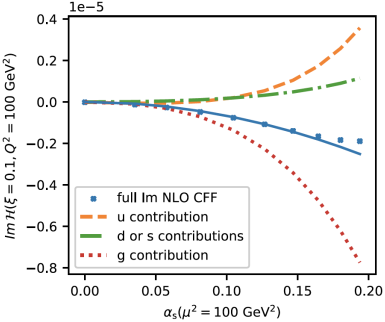
$$\alpha_s(\mu^2) \log\left(\frac{\mu^2}{Q^2}\right)\left[\left(\frac{3}{2} + \log\left(\frac{1-\xi}{2\xi}\right)\right) \mathrm{Im}\ T_{LO}^q \otimes H^q(\mu^2) + \sum_{w=1}^{N+1} \frac{k_w^{(coll)}}{(1+\xi)^w}\right] \quad (28)$$

and assuming $\mathrm{Im}\ T_{LO}^q \otimes H^q(\mu^2) = 0$,

$$\mathrm{Im}\ T_1^q(Q^2, \mu^2) \otimes H^q(\mu^2) \propto \alpha_s(\mu^2)\left[\log\left(\frac{1-\xi}{2\xi}\right) \mathrm{Im}\ T_{coll}^q \otimes H^q(\mu^2) + \sum_{w=1}^{N-1} \frac{k_w^{(1)}}{(1+\xi)^w}\right]$$

Shadow GPDs at next-to-leading order

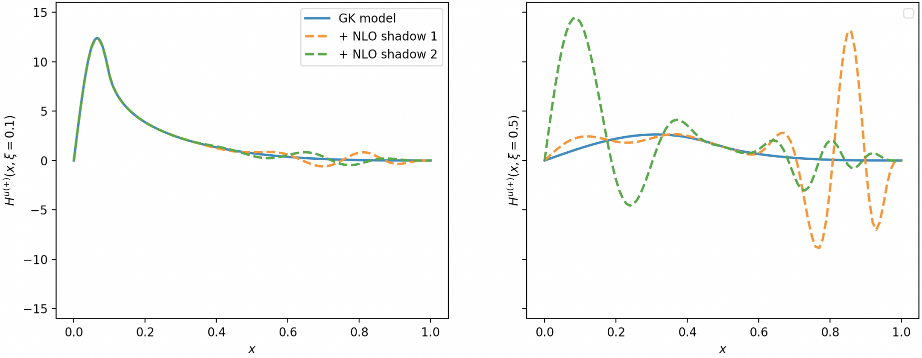
- By linearity of both the CFF convolution and the evolution equation, we can evaluate separately the contribution to the CFF of a quark shadow NLO GPD under evolution.
- Having cancelled all term of order $\mathcal{O}(\alpha_s)$, the contribution of NLO shadow GPDs to the CFF is expected to behave as $\mathcal{O}(\alpha_s^2(\mu^2))$. We probe this prediction on a lever-arm in Q^2 of $[1, 100]$ GeV^2 (typical collider kinematics) using the APFEL++ code.



- The fit by $\alpha_s^2(\mu^2)$ is very good up to values of α_s of the order of its $\overline{MS}$ values. For larger values, large logs and higher orders slightly change the picture.
- The numerical effect of evolution remains very small. For a GPD of order 1, the NLO CFF is only of order 10^{-5}.

Shadow GPDs at next-to-leading order

The orange and brown models are **Goloskokov-Kroll model + NLO shadow GPDs**. For ξ close to 0 and x close to ξ, by design, they are very close, but vastly different otherwise. They give rise to NLO CFFs which are exactly identical at this scale, and different by a negligible amount for expected Q^2 lever arm.

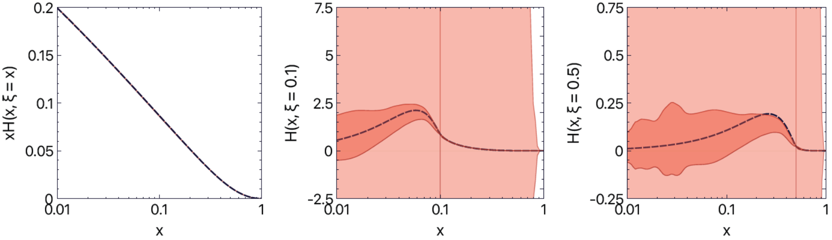


$\xi = 0.1$ (left) and $\xi = 0.5$ (right)

4. New models of GPDs

New models of GPDs

To go beyond models with limited flexibility and take into account the uncertainty stemming from shadow GPDs, we built a **neural network (NN) parametrization of DDs** in **[Dutrieux et al, Eur.Phys.J.C 82 (2022) 3, 252]**, with emphasis on reproducing polynomiality, shadow components and **positivity constraints**. Without positivity,

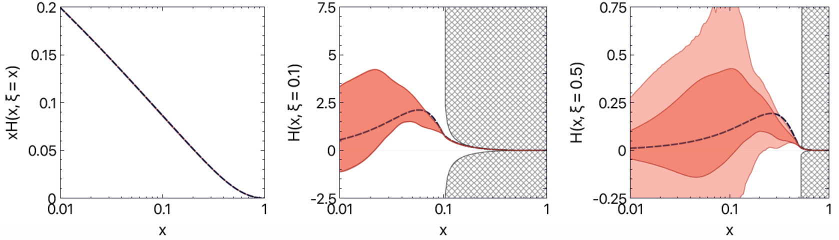


We give ourselves a GPD diagonal $x = \xi$ (left) and a PDF that we fit with our model. The obtained GPDs at $\xi = 0.1$ (middle) and 0.5 (right) are made of the sum of two NNs, one built on the Radyushkin DD Ansatz (dark red) and one specifically emulating the shadow term (light red).

New models of GPDs

Introducing the simplified positivity constraint [Radyushkin, 1999], [Pire *et al*, 1999], [Diehl *et al*, 2001], [Pobylitsa, 2002]

$$|H^q(x,\xi)| \leq \sqrt{\frac{1}{1-\xi^2} f^q\left(\frac{x-\xi}{1-\xi}\right) f^q\left(\frac{x+\xi}{1+\xi}\right)} \tag{30}$$



The grey area depicts the positivity exclusion region.

Perspectives

5. Perspectives

Perspectives

- Other exclusive processes are sensitive to GPDs, like **time-like Compton scattering** (TCS) [Berger *et al*, 2002]. However, the similar nature of its convolution (see [Müller *et al*, 2012]) makes it subject to the similar shadow GPDs issue.
- **Deeply virtual meson production** (DVMP) [Collins *et al*, 1997] is also an important source of knowledge on GPDs, with currently a larger lever arm in Q^2. The process involves form factors of the general form

$$\mathcal{F}(\xi, t) = \int_0^1 \mathrm{d}u \int_{-1}^1 \frac{\mathrm{d}x}{\xi}\, \phi(u) T\left(\frac{x}{\xi}, u\right) F(x, \xi, t) \tag{31}$$

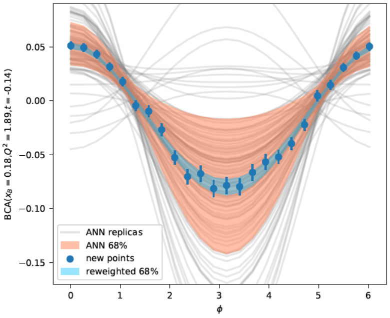
  where $\phi(u)$ is the leading-twist meson distribution amplitude (DA).
- At LO, the GPD and DA parts of the integral factorize and shadow GPDs cancel the form factor.
- Situation at NLO remains to be clarified, it is foreseeable new shadow GPDs (dependent on the DA) could be generated also for this process.

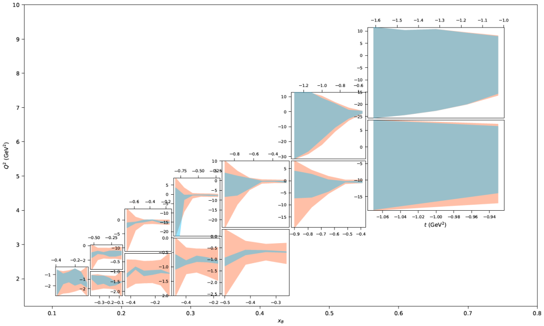
Perspectives

- **New experimental channels**: more experimentally challenging processes offer a richer access to GPDs thanks to more handles with kinematic variables.
    - Double deeply virtual Compton scattering (DDVCS) – proposed at JLab with SOLID (LOI12-15-005) and CLAS12 (LOI12-16-004) – which gives access directly to the (x, ξ) value of GPDs in the ERBL region at LO.
    - Multiparticle production: diphoton [Pedrak *et al*, 2017], photon-rho [Boussarie *et al*, 2017]
- **Lattice QCD**: low order Mellin moments of GPDs do not change significantly the previously exposed picture, as it is easy to produce shadow GPDs with also a given number of vanishing Mellin moments.
- Extraction of the x-dependence of parton distributions is an interesting prospects, whose impact on the current discussion we aim to assess.

Perspectives

- Reducing uncertainties on CFFs itself is a very useful task. *e.g.* proton pressure anisotropy is compatible with 0 largely because of the uncertainty on $\mathrm{Re}\,\mathcal{H}$.
- The proposal to install a positron beam at JLab [Afanasev *et al*, 2019] can help on this task. We have performed in **[Dutrieux et al, Eur.Phys.J.A 57 (2021) 8, 250]** a reweighting of our neural network replicas of CFFs against simulated new experimental points.





Conclusion

- Explicit demonstration of NLO shadow GPDs of considerable size with a very small and subleading contribution to CFFs. **Such shadow GPDs will be hidden in typical statistical and systematic uncertainties of DVCS**. TCS or LO DVMP face similar issues. We foresee that our discussion can be extended to higher order DVCS. Other exclusive processes will help discriminate the DVCS shadow GPDs. Especially DDVCS or Lattice QCD for instance should escape the dimensionality of data problem.
- Potential impact on **hadron tomography** due to the $\xi \to 0$ extrapolation, impact on the extraction of hadron mechanical properties.
- An extraction of GPDs with lesser systematic uncertainty requires a **multi-channel analysis**, and the development of public integrated analysis tools, like **PARTONS** (`https://partons.cea.fr`) and **GeParD** (`https://gepard.phy.hr`).
- More precise data over a much larger Q^2 range promised by future colliders will be very welcome here and for the extraction of mechanical properties as well.
- More theoretical constraints, like **positivity** will play a significant role in reducing the uncertainty.

Conclusion

Thank you for your attention !

Shadow GPDs at leading order

- For our LO shadow GPD, we first want $H^{q(+)}(\xi, \xi, \mu_0^2) = 0$, so we notice that

$$H^{q(+)}(\xi, \xi, \mu_0^2) = \sum_{w=1}^{N+1} \frac{k_w}{(1+\xi)^w} \quad \text{where} \quad k_w = \sum_{u,v} C_w^{uv} \, q_{uv} \, , \quad C_w^{uv} = (-1)^{u+v+w} \binom{v}{u-w}$$

Cancelling the LO CFF

$$H^{q(+)}(\xi, \xi, \mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(C.R) \tag{32}$$

Shadow GPDs at leading order

Cancelling the LO CFF

$$H^{q(+)}(\xi, \xi, \mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(C.R) \tag{32}$$

- We then want $H^{q(+)}(x, \xi = 0, \mu_0^2) = 0$, so we notice that

$$H^{q(+)}(x, 0, \mu_0^2) = \sum_{w=0}^{N+1} q_w x^w \quad \text{where} \quad q_w = \sum_{u,v} Q_w^{uv} \, q_{uv} \, , \quad Q_w^{uv} = 2\delta_w^v$$

Cancelling the forward limit

$$H^{q(+)}(x, \xi = 0, \mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(Q.R) \tag{33}$$

Shadow GPDs at leading order

Cancelling the LO CFF

$$H^{q(+)}(\xi, \xi, \mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(C.R) \tag{32}$$

Cancelling the forward limit

$$H^{q(+)}(x, \xi = 0, \mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(Q.R) \tag{33}$$

- Both linear systems $C.R$ and $Q.R$ are systems of $\sim N$ equations for $\sim N^2$ variables, so the number of solutions grows quadratically with N, order of the polynomial DD.

Shadow GPDs at leading order

Cancelling the LO CFF

$$H^{q(+)}(\xi,\xi,\mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(C.R) \quad (32)$$

Cancelling the forward limit

$$H^{q(+)}(x,\xi=0,\mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(Q.R) \quad (33)$$

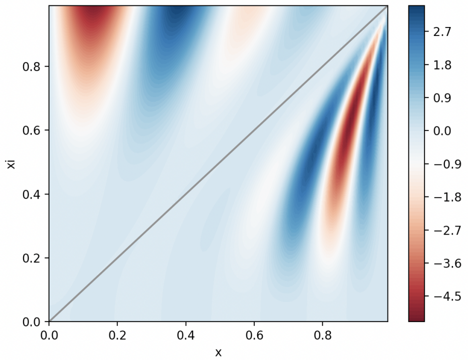
LO shadow GPDs

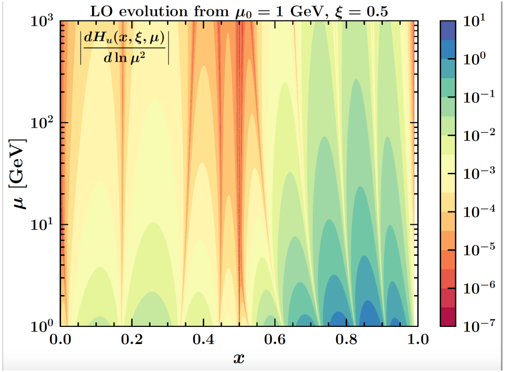
Here is an example of an infinite family of LO shadow DDs, each being of degree $N \geq 9$ odd

$$F_N(\beta,\alpha,\mu_0^2)=\beta^{N-8}\Big[\alpha^8-\frac{28}{9}\alpha^6\left(\frac{N^2-3N+20}{(N+1)N}+\beta^2\right)+\frac{10}{3}\alpha^4\left(\frac{N^2-7N+40}{(N+1)N}+\frac{2(N^2-3N+44)}{3(N+1)N}\beta^2+\beta^4\right)$$
$$-\frac{4}{3}\alpha^2\left(\frac{N^2-11N+60}{(N+1)N}-\frac{N-8}{N}\beta^2-\frac{N^2-3N-28}{(N+1)N}\beta^4+\beta^6\right)+\frac{1}{9}(1-\beta^2)^2\left(\frac{N^2-15N+80}{(N+1)N}-\frac{2(N-8)}{N}\beta^2+\beta^4\right)\Big] \quad (34)$$

Shadow GPDs at next-to-leading order

- Cancelling both terms gives rise to two additional systems with a linear number of equations. The first NLO shadow GPD is found for $N = 21$, and adding the condition that the DD vanishes at the edges of its support gives a first solution for $N = 25$ (see below).





Color plot of an NLO shadow GPD at initial scale 1 GeV^2, and its evolution for $\xi = 0.5$ up to 10^6 GeV^2 via APFEL++ and PARTONS [Bertone].

Neural network modelling of double distributions

Our neural network model for singlet DDs consists of three parts

$$f^{q(+)}(\beta,\alpha) = (1-x^2) f_C^{q(+)}(\beta,\alpha) + (x^2-\xi^2) f_S^{q(+)}(\beta,\alpha) + \xi f_D^{q(+)}(\beta,\alpha) \tag{35}$$

$$f_C^{q(+)} = \frac{q^{(+)}(\beta)}{1-\beta^2} \frac{\mathrm{ANN}_C(|\beta|,\alpha)}{\int_{|\beta|-1}^{1-|\beta|} \mathrm{ANN}_C(|\beta|,\alpha)} \tag{36}$$

$$f_S^{q(+)}(\beta,\alpha) = q^{(+)}(\beta) N_S \left(\frac{\mathrm{ANN}_S^{(1)}(|\beta|,\alpha)}{\int_{|\beta|-1}^{1-|\beta|} \mathrm{ANN}_S^{(1)}(|\beta|,\alpha)} - \frac{\mathrm{ANN}_S^{(2)}(|\beta|,\alpha)}{\int_{|\beta|-1}^{1-|\beta|} \mathrm{ANN}_S^{(2)}(|\beta|,\alpha)} \right) \tag{37}$$