$B - \bar{B}$ mixing on domain-wall lattices

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RBC/UKQCD and JLQCD

CIPANP 2022, Orlando

30/08/2022

THE UNIVERSITY of EDINBURGH

European Research Council
Established by the European Commission

NEUTRAL MESON MIXING

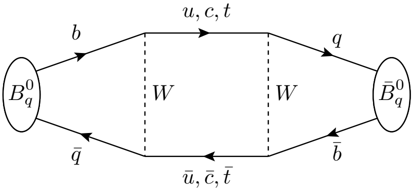
B-mesons B_d, B_s have mass eigenstates

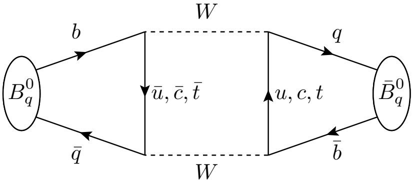
$$|B^0_{qL}\rangle = p_q|B^0_q\rangle + q_q|\bar{B}^0_q\rangle$$
$$|B^0_{qH}\rangle = p_q|B^0_q\rangle - q_q|\bar{B}^0_q\rangle$$

with mass m_{qL} and total decay width Γ_{qL} for the lighter eigenstate. Splittings:

$$\Delta m_q = m_{qH} - m_{qL}$$
$$\Delta\Gamma_q = \Gamma_{qL} - \Gamma_{qH}$$

Experimentally, time dependent probabilities give access to the splittings, e.g.

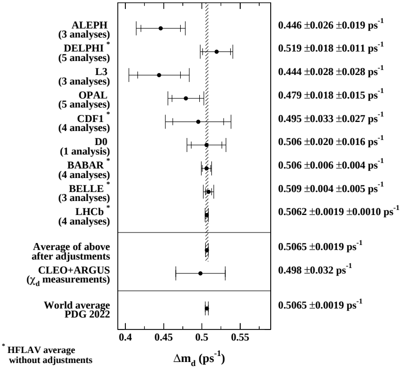


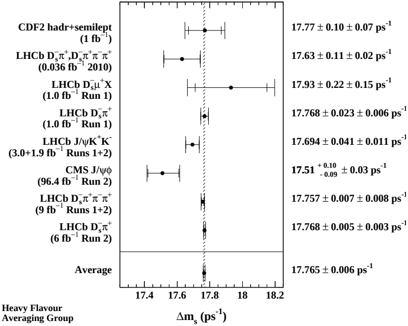


$$\mathcal{P}(B^0_q \to \bar{B}^0_q) = \frac{1}{2}e^{-\Gamma_q t}[\cosh(\frac{1}{2}\Delta\Gamma_q t) - \cos(\Delta m_q t)]|q_q/p_q|^2$$

NEUTRAL MESON MIXING

Experimental results **[HFLAV 2021]**

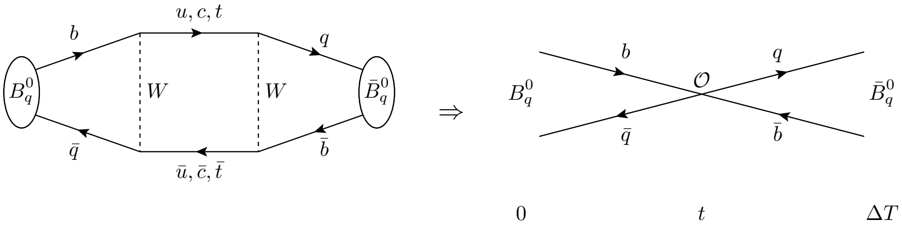




$$\Delta m_s = 17.765(6) ps^{-1}$$

$$\Delta m_d = 0.5065(19) ps^{-1}$$

THEORY



- $\Delta B = 2$ process
- enhanced by top quark $\Rightarrow$ short-distance dominated
- OPE shrinks box diagram to local four-quark operator

$$\langle \bar{B}_q^0 | \mathcal{H}_{\text{eff}}^{\Delta B=2} | B_q^0 \rangle \to \langle \bar{B}_q^0 | \mathcal{O}_i | B_q^0 \rangle$$

- 5 parity-even, dimension 6, $\Delta B = 2$ operators $\mathcal{O}_i$

THEORY

- bag parameters $\mathcal{B}$ give access to mass splittings Δm

$$\mathcal{B}_{B_q}^{[i]} = \frac{\langle \bar{B}_q^0 | \mathcal{O}_i | B_q \rangle}{\langle \bar{B}_q^0 | \mathcal{O}_i | B_q \rangle_{\mathrm{VSA}}}$$

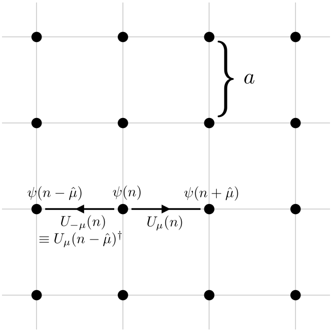
$$\Delta m_q = |V_{td} V_{tq}^*|^2 \mathcal{K} M_{B_q} f_{B_q}^2 \mathcal{B}_{B_q}^{[1]}$$

- V_{tq} from experiment
- $\mathcal{K}$ known (perturbative)
- M_{B_q}, f_{B_q}, $\mathcal{B}_{B_q}^{[i]}$ non-perturbatively from lattice QCD

LATTICE QCD

- Discrete, finite Euclidean space-time grid
    - quark fields ψ on sites n
    - gluons U_μ as gauge links
    - finite lattice spacing a (UV regulator)
    - finite volume L, T (IR regulator)
- Path integral

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int dUd\psi d\bar{\psi}\, \mathcal{O}[U,\psi,\bar{\psi}]e^{-S[U,\psi,\bar{\psi}]}$$



⇒ exponentially decaying & finite estimators

- even relatively small grids have size $\Lambda = (L/a) \times (T/a) = 24^3 \times 48$
    - exact evaluation prohibitively expensive

    ⇒ stochastic sampling of ensembles

LATTICE QCD

Fermionic part of the path integral can be solved explicitly

$$\begin{aligned}\langle \mathcal{O} \rangle &= \frac{1}{Z} \int dU\, e^{-S_G[U]} d\psi d\bar{\psi}\, e^{-S_F[U,\psi,\bar{\psi}]} \mathcal{O}[U, \psi, \bar{\psi}] \\ &= \frac{1}{Z} \int dU\, e^{-S_G[U]} \Big(\prod_f \det[D_f(U)] \Big) \mathcal{O}[U, \psi, \bar{\psi}]\end{aligned}$$

Monte-Carlo simulation: interpret

$$Z^{-1} e^{-S_G[U]} \Big(\prod_f \det[D_f(U)] \Big)$$

as probability weight.

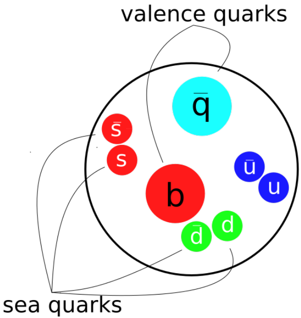
- D_f is matrix with $12 \times \Lambda$ rows and columns

$\Rightarrow$ brute-force inversion prohibitively expensive

$\Rightarrow$ intricate algorithms needed to account for $\det[D_f]$, beyond the scope of this talk

LATTICE QCD

- Fermion determinants describe "sea quarks"
  - quark pairs in fermionic vacuum
  - gives rise to definition of "valence quarks" giving quantum numbers to hadrons
- "quenching": set $\det[D_f] = 1$
  - neglects sea quark contribution
- we use partially quenched setup "$N_f = 2 + 1$"
  - $\det[D_u] = \det[D_d] \neq 1$
  - $\det[D_s] \neq 1$
- sea-quark masses m_l, m_s are inputs
  - typically $M_\pi > M_\pi^{\mathrm{phys}}$



CONTINUUM LIMIT

We need to control on each ensemble

- finite-volume effects $M_\pi L \gtrapprox 4$
- discretisation effects of heavy quark $am_H \ll 1$
    - currently too expensive to control simultaneously

We therefore simulate on multiple ensembles, and take the limits

- lattice spacing $a \to 0$
- box size $L \to \infty$
- sea quark masses $m_q \to m_q^{\mathrm{phys}}$ ($M_\pi \to M_\pi^{\mathrm{phys}}$)
- bottom quark mass $m_b \to m_b^{\mathrm{phys}}$

HEAVY-QUARK STRATEGY

- We simulate at multiple heavy-quark masses $m_h \sim m_c \to m_h \lesssim m_b$
  - $\Rightarrow$ Extrapolation in $m_h \to m_b$ as part of global fit
  - systematically improvable
  - expensive, and analysis of more data
- all other lattice calculations use effective actions for heavy quarks
  - $\Rightarrow$ no extrapolation needed
  - systematic bias
  - not systematically improvable

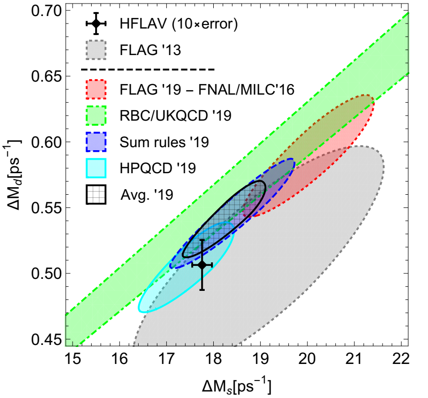
DOMAIN-WALL FERMIONS

- multiple different lattice action discretisations available
- all lattice actions must recover the same physics in the continuum
- we use ”Domain-Wall Fermions”
    - automatic $O(a)$ improvement in absence of odd powers in a
    - $\Rightarrow$ reduced discretisation effects
    - improved chiral symmetry
    - $\Rightarrow$ leads to simple mixing pattern of operators $\mathcal{O}_i$

$B_q - \bar{B}_q$ MIXING ON THE LATTICE

- three independent lattice groups working on $B_q - \bar{B}_q$ mixing
- current tension between Δm_d, Δm_s lattice determinations
    - FNAL/MILC '16 is in tension with experiment
    - HPQCD '19 is compatible with experiment
    - RBC/UKQCD '19 result still missing renormalization factors

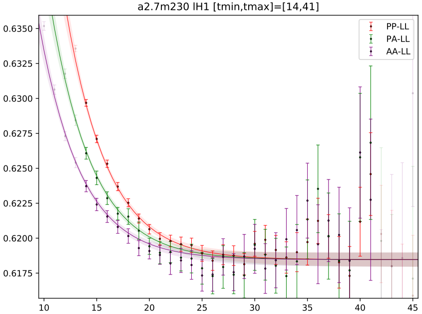
        $\Rightarrow$ we are working on that!



[Di Luzio et al. arxiv 1909.11087]

TWO-POINT FUNCTIONS

- fit to heavy-light / heavy-strange 2pt functions: M_{B_q}, f_{B_q}
- also need pion/kaon 2pt functions: M_π, M_K
- shown here: lightest heavy-light meson on "a2.7m230" ensemble



$$C_2^{\Gamma_1\Gamma_2} = \sum_{\boldsymbol{x}} \langle O_{\Gamma_2}(\boldsymbol{x}, t) O_{\Gamma_1}(\boldsymbol{0}, t)^\dagger \rangle = \sum_{n=0}^{\infty} \frac{M_{\Gamma_2 n} M_{\Gamma_1 n}}{2E_n} \left(e^{-E_n t} + e^{-E_n (T-t)}\right)$$

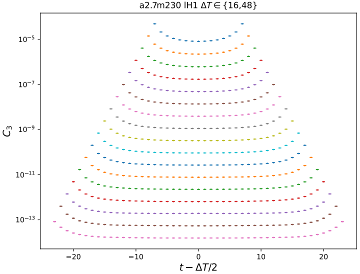
with $\Gamma_i \in \{\gamma_5, \gamma_0\gamma_5\} \equiv \{P, A\}$

$$E_{\text{eff}}^{\Gamma_1\Gamma_2}(t) = \ln\left(C_2^{\Gamma_1\Gamma_2}(t) / C_2^{\Gamma_1\Gamma_2}(t+1)\right)$$

THREE-POINT FUNCTIONS

FORMULA

- four-quark operators $O_q^{\Gamma\Gamma} = (\bar{b}_a \Gamma q_a)(\bar{b}_b \Gamma q_b)$
- different values for $B - \bar{B}$ separation ΔT
- large separations noisier
- small separations excited-state dominated



$$C_3^{\mathcal{O}_q^{\Gamma\Gamma}}(t, \Delta T) = \langle P(\Delta T) \mathcal{O}_q^{\Gamma\Gamma} \bar{P}^\dagger(0) \rangle$$
$$= M_{P0}^2/(4E_0^2) \langle 0 | \mathcal{O}_q^{\Gamma\Gamma} | 0 \rangle e^{-E_0 \Delta T} \times \left[1 + C^{(0|1)} \cosh[\Delta E(t - \Delta T/2)] \right]$$

with 5 linear combinations of $\mathcal{O}_q^{\Gamma\Gamma}$:

$$[VV + AA], [VV - AA], [SS + PP], [SS - PP], [TT]$$

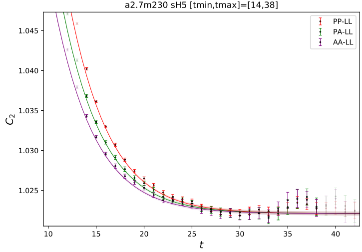
RATIOS

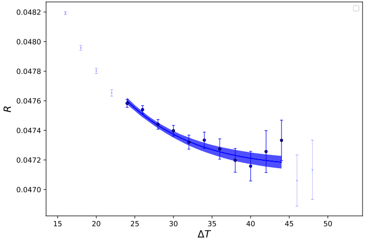
- We look at ratios with similar time behaviour in denominator and numerator

$\Rightarrow$ cancellation of correlations

$$R(\Delta T) = \left. \frac{C_3^{\mathcal{O}_q^{\Gamma\Gamma}}(t, \Delta T)}{C_{PA}(t) C_{AP}(\Delta T - t)} \right|_{t=\Delta T/2}$$

- Example of combined correlated fit to heaviest heavy-strange meson on ”a2.7m230” ensemble





FIT STRATEGY

- We have studied a number of different strategies to fit all these parameters and settled on a **simultaneous**, **fully correlated** fit to:

  $C_2^{PP}(t)$, $C_2^{PA}(t)$, $C_2^{AA}(t)$, $R^{\mathcal{O}}(\Delta T)$

- We define a vector with all data points entering the fit

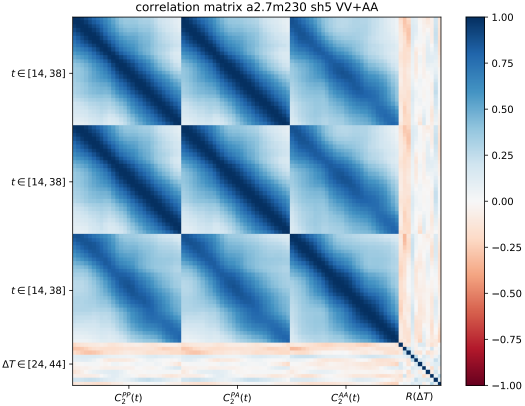
$$C = (C_2^{PP}(t_{\rm min}^{PP}), \ldots, C_2^{PA}(t_{\rm min}^{PA}), \ldots, C_2^{AA}(t_{\rm min}^{AA}), \ldots, R^{\mathcal{O}}(\Delta T_{\rm min}^{\mathcal{O}}), \ldots)$$

$$\Delta = C^{\rm data} - C^{\rm model}$$

- From this we define and minimise a χ^2 function

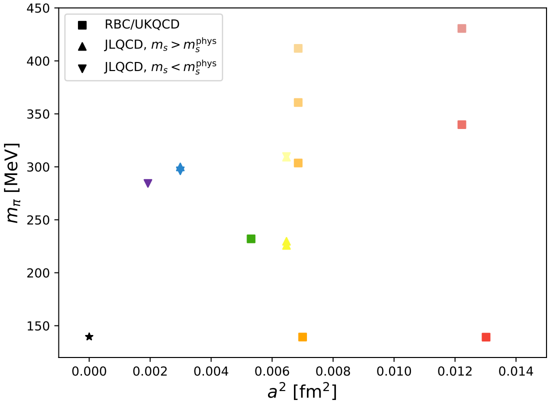
$$\chi^2 = \Delta C_{cov}^{-1} \Delta^T$$

CORRELATION MATRIX



- 2pt functions are highly correlated
- ratios are decorrelated from the 2pt functions
- We improved upon an earlier attempt of fitting 2pt and raw 3pt functions simultaneously, which had high correlations

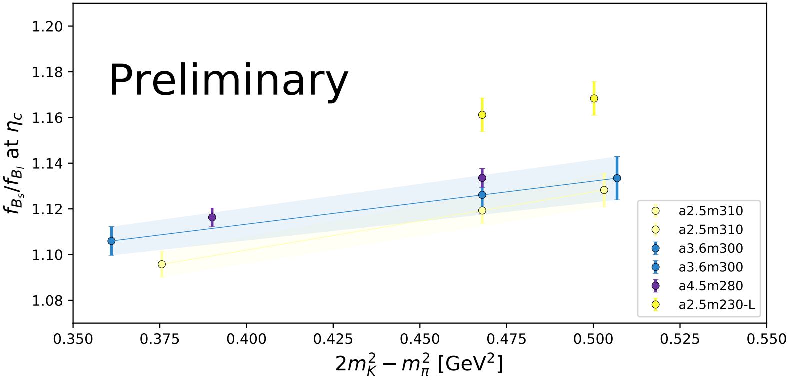
Landscape plot of our ensembles



- 2 ensembles at m_π^{phys}
- JLQCD ensembles are very fine
  - $\Rightarrow$ almost reach m_b^{phys}
- 2 very similar ensembles with $m_\pi L = 3.0$ and $m_\pi L = 4.4$
- 6 different lattice spacings from $a = 0.11\text{fm}$ to $a = 0.044\text{fm}$

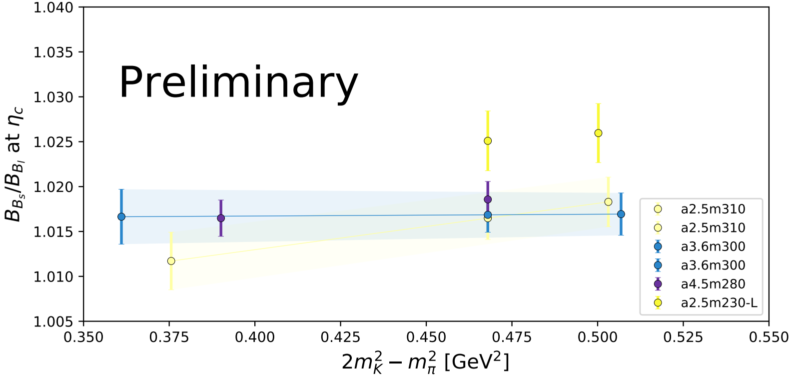
$\Rightarrow$ These strongly constrain the relevant limits we will take in a final global fit to data on all ensembles.

Ratio of decay constants - JLQCD ensembles



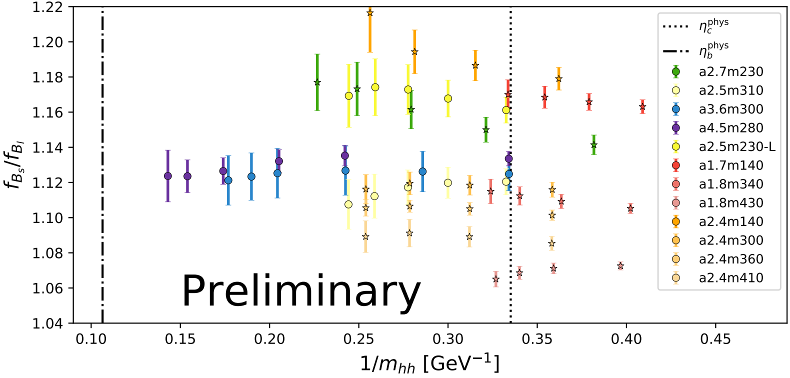
- RBC/UKQCD ensembles have sea-quark tuned to physical $m_s^{\rm phys}$
- Two JLQCD ensembles come in pairs, bracketing $m_s^{\rm phys}$
- interpolation slope is then applied to other ensembles
- only mild dependence on $m_s^{\rm sea}$

Ratio of bag parameters - JLQCD ensembles



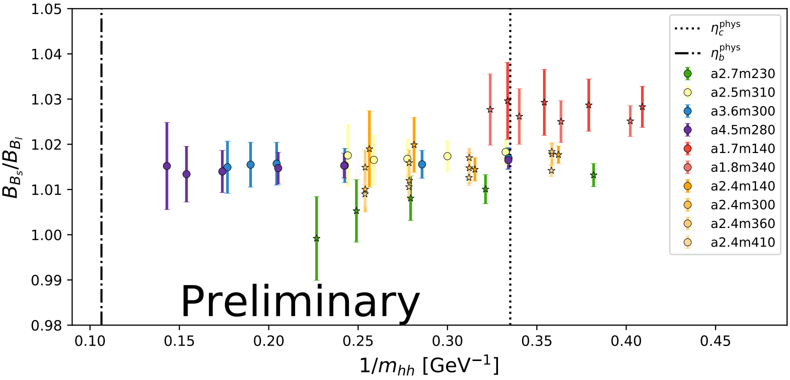
- even milder dependence on $m_s^{\rm sea}$ for bag parameters

Ratio of decay constants



- illustration in the heavy-quark mass reach of the JLQCD ensembles
- dependence on heavy-quark mass is very mild

RATIO OF BAG PARAMETERS - $VV + AA$



- this $SU(3)$-breaking ratio is close to 1
- dependence on heavy-quark mass is very mild

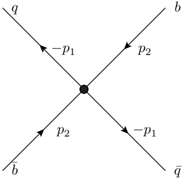
Non-perturbative renormalisation

$$\langle O\rangle_i^{\mathrm{S}}(\mu) = \lim_{a^2\to 0}\sum_{j=1}^{5}[Z_O^{\mathrm{S}}(a,\mu)]_{ij}\langle O\rangle_j^{\mathrm{bare}}(a)$$

for some regularisation independent scheme S at mass scale μ.
Continuum perturbation theory can then match

$$\langle O\rangle_i^{\overline{\mathrm{MS}}}(\mu) = R^{\overline{\mathrm{MS}}\leftarrow \mathrm{S}}\langle O\rangle_i^{\mathrm{S}}(\mu)$$

We use the "RI-SMOM" scheme. Requires computation of four-quark vertices for $(\bar{b}q) \to (\bar{q}b)$. **[Boyle et al. arxiv 1708.03552]**



$\Rightarrow$ analysis of NPR data is underway

Domain-wall operator-mixing matrix

Based on chiral symmetry of our domain-wall fermions, a very simple mixing pattern of the 5 operators arises:

$$\mathcal{O}_1 = \mathcal{O}^{VV+AA}$$
$$\mathcal{O}_2 = \mathcal{O}^{VV-AA}$$
$$\mathcal{O}_3 = \mathcal{O}^{SS-PP}$$
$$\mathcal{O}_4 = \mathcal{O}^{SS+PP}$$
$$\mathcal{O}_5 = \mathcal{O}^{TT}\,.$$

$$\begin{pmatrix} \mathcal{O}_1 & 0 & 0 \\ 0 & \begin{pmatrix} \mathcal{O}_{2/3} & \mathcal{O}_{2/3} \\ \mathcal{O}_{2/3} & \mathcal{O}_{2/3} \end{pmatrix} & 0 \\ 0 & 0 & \begin{pmatrix} \mathcal{O}_{4/5} & \mathcal{O}_{4/5} \\ \mathcal{O}_{4/5} & \mathcal{O}_{4/5} \end{pmatrix} \end{pmatrix}$$

This block-structure means that only $\mathcal{O}_2$, $\mathcal{O}_3$ as well as $\mathcal{O}_4$, $\mathcal{O}_5$ mix, but they are linearly independent from each other and from $\mathcal{O}_1$.
This is a great advantage of chiral domain wall fermions to other lattice discretisations, where a more complicated mixing pattern has to be dealt with.

[Boyle et al. arxiv 1708.03552]

CONCLUSIONS & OUTLOOK

Conclusions:

- We can extract bag parameters and matrix elements $\langle 0|\mathcal{O}|0\rangle$ using a fully correlated fit with a combined χ^2/dof for C_2 and C_3.
- DWF leads to a very simple mixing pattern of the 5 operators due to chiral symmetry
- Set of 15 ensembles
  - 2 ensembles at m_π^{phys}
  - JLQCD ensembles almost reach m_b^{phys}
  - 2 very similar ensembles with $m_\pi L = 3.0$ and $m_\pi L = 4.4$
  - 6 different lattice spacings from $a = 0.044\mathrm{fm}$ to $a = 0.11\mathrm{fm}$

  $\Rightarrow$ These strongly constrain the relevant limits we will take in a final global fit to data on all ensembles.

Next steps

- Non-perturbative renormalisation (NPR) is being worked on
- Extract bag parameters for full 5-operator basis

  $\Rightarrow$ will allow us to constrain $\Delta m_d, \Delta m_s$

This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme under grant agreement No 757646.

BACKUP

Lattice setup

- RBC-UKQCD's 2+1 flavour domain wall fermions **[Blum et al. arxiv 1411.7017]**
  - pion masses from $m_\pi = 139$ MeV to $m_\pi = 430$ MeV
  - several heavy-quark masses from below m_c to $0.5 m_b$, using a stout-smeared action ($\rho = 0.1$, $N = 3$) with $M_5 = 1.0$, $L_s = 12$ and Möbius-scale $= 2$ **[Boyle et al. arxiv:1812.08791]**
  - light and strange quarks: sign function approximated via:
    - Shamir approximation for heavier pion masses
    - Möbius approximation at m_π^{phys} and on the finest ensemble
- JLQCD's 2+1 flavour domain wall fermions **[Kaneko et al. arxiv 1711.11235]**
  - pion masses from $m_\pi = 226$ MeV to $m_\pi = 310$ MeV
  - heavy-quark masses from m_c nearly up to m_b, using the same stout-smeared action.
  - light and strange quarks use the same action as the heavy quarks.

Lattice setup

	L/a	T/a	a^{-1} [GeV]	m_π [MeV]	$m_\pi L$	hits $\times N_{\rm conf}$	collaboration id
a1.7m140	48	96	1.730(4)	139.2	3.9	48×90	R/U C0
a1.8m340	24	64	1.785(5)	339.8	4.6	32×100	R/U C1
a1.8m430	24	64	1.785(5)	430.6	5.8	32×101	R/U C2
a2.4m140	64	128	2.359(7)	139.3	3.8	64×82	R/U M0
a2.4m300	32	64	2.383(9)	303.6	4.1	32×83	R/U M1
a2.4m360	32	64	2.383(9)	360.7	4.8	32×76	R/U M2
a2.4m410	32	64	2.383(9)	411.8	5.5	32×81	R/U M3
a2.5m230-L	48	96	2.453(4)	225.8	4.4	24×100	J C-ud2-sa-L
a2.5m230-S	32	64	2.453(4)	229.7	3.0	16×100	J C-ud2-sa
a2.5m310-a	32	64	2.453(4)	309.1	4.0	16×100	J C-ud3-sa
a2.5m310-b	32	64	2.453(4)	309.7	4.0	16×100	J C-ud3-sb
a2.7m230	48	96	2.708(10)	232.0	4.1	48×72	R/U F1M
a3.6m300-a	48	96	3.610(9)	299.9	3.9	24×50	J M-ud3-sa
a3.6m300-b	48	96	3.610(9)	296.2	3.9	24×50	J M-ud3-sb
a4.5m280	64	128	4.496(9)	284.3	4.0	32×50	J F-ud3-sa

List of ensembles used in this work. For consistency of naming conventions in our set of ensembles from two collaborations, we introduce a shorthand notation in the first column which is used throughout this work. The last column shows names used by RBC/UKQCD ("R/U") and JLQCD ("J").