Fermilab E989 Muon g-2 Experiment

The Magnetic Field Measurement and Systematics

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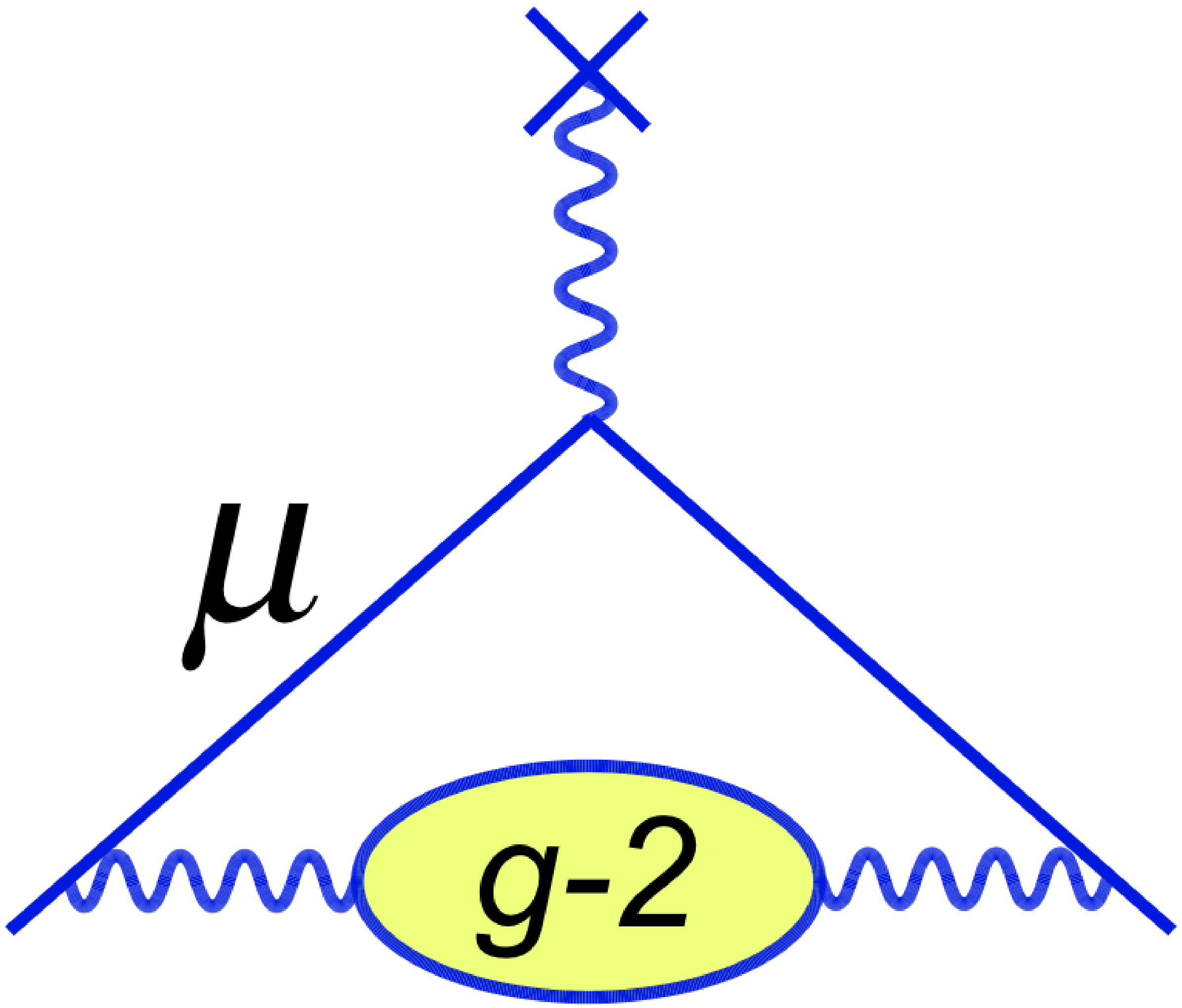
Table of Contents



Overview of E989

Anomalous Magnetic Dipole of Muon

$$a_\mu = \frac{g-2}{2}$$

g = 2 in Dirac Equation for spin ½ particle

Anomalous magnetic moment is the value deviating from g-factor in Dirac equation

Part 1

Overview of E989

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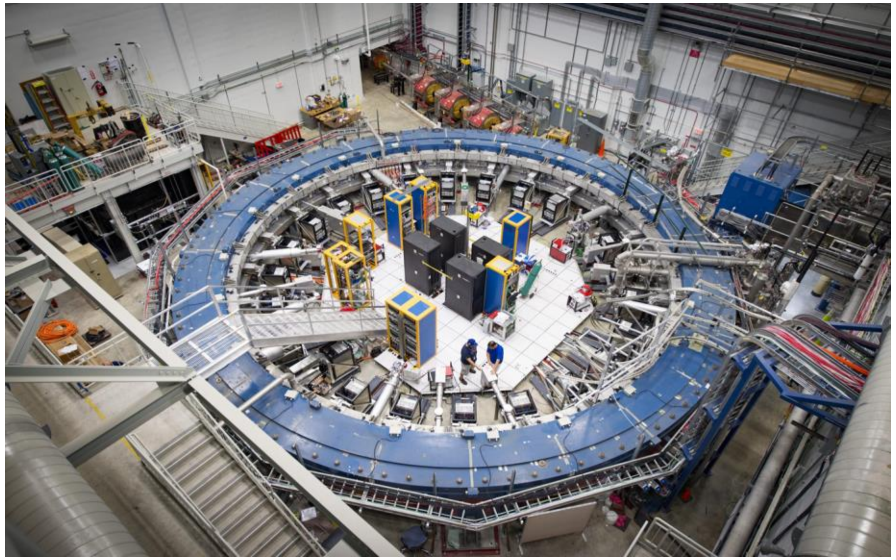


- Fermilab E989, the Muon g-2 experiments, goal is to measure the anomalous magnetic dipole of the muon, a_μ to a precision of **0.14 ppm.**

- The main goal is to test the Standard Model's predictions and to search the evidence of **Beyond Standard Model (BSM) and New Physics (NP).**

Part 1

Overview of E989

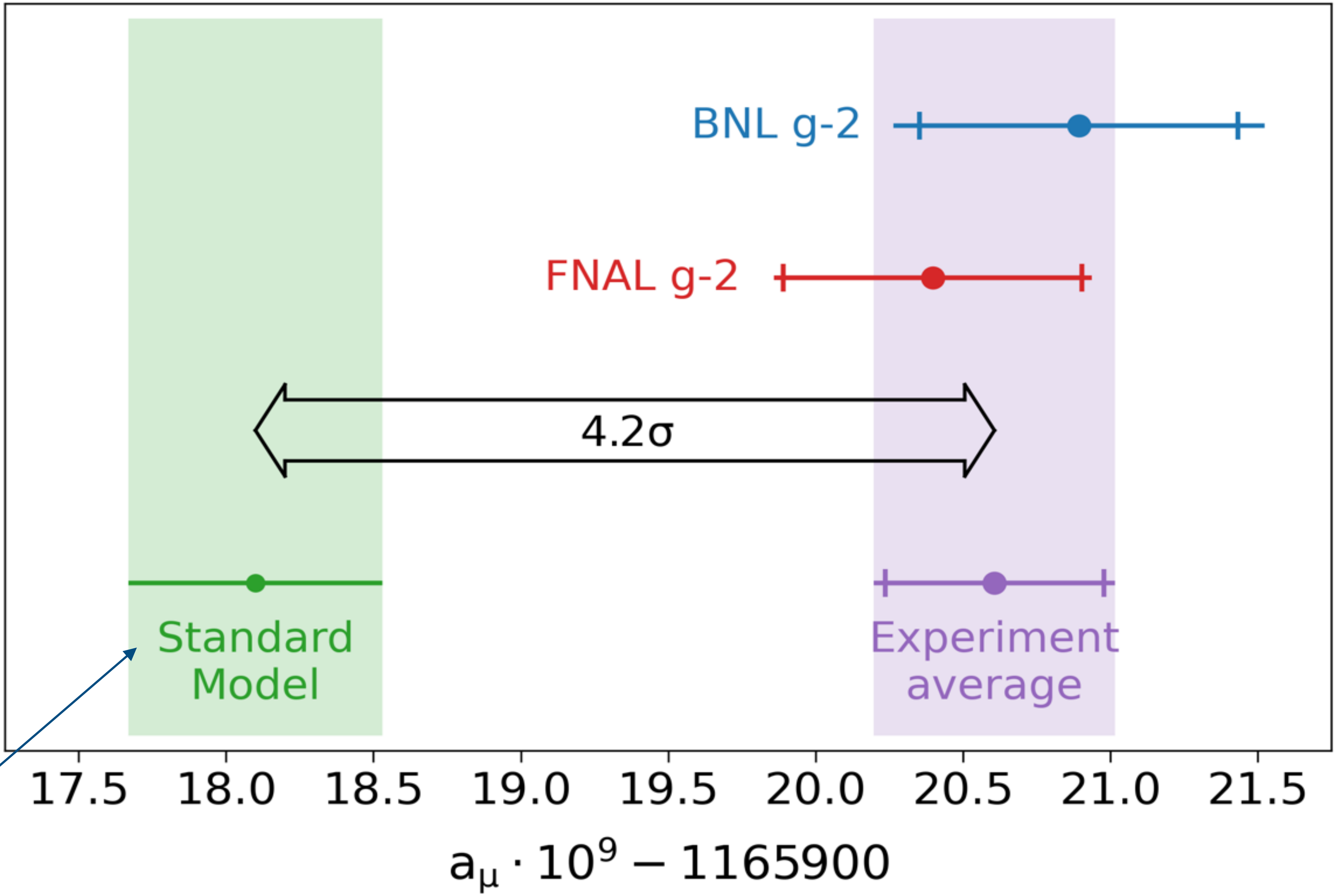
Run 1 Result (2021)

- Result a_μ to 460 ppb
- Confirmed BNL experimental results (20 years later)

a_μ^{SM} = 116 591 810 (43)
a_μ^{EXP}= 116 592 061 (41)

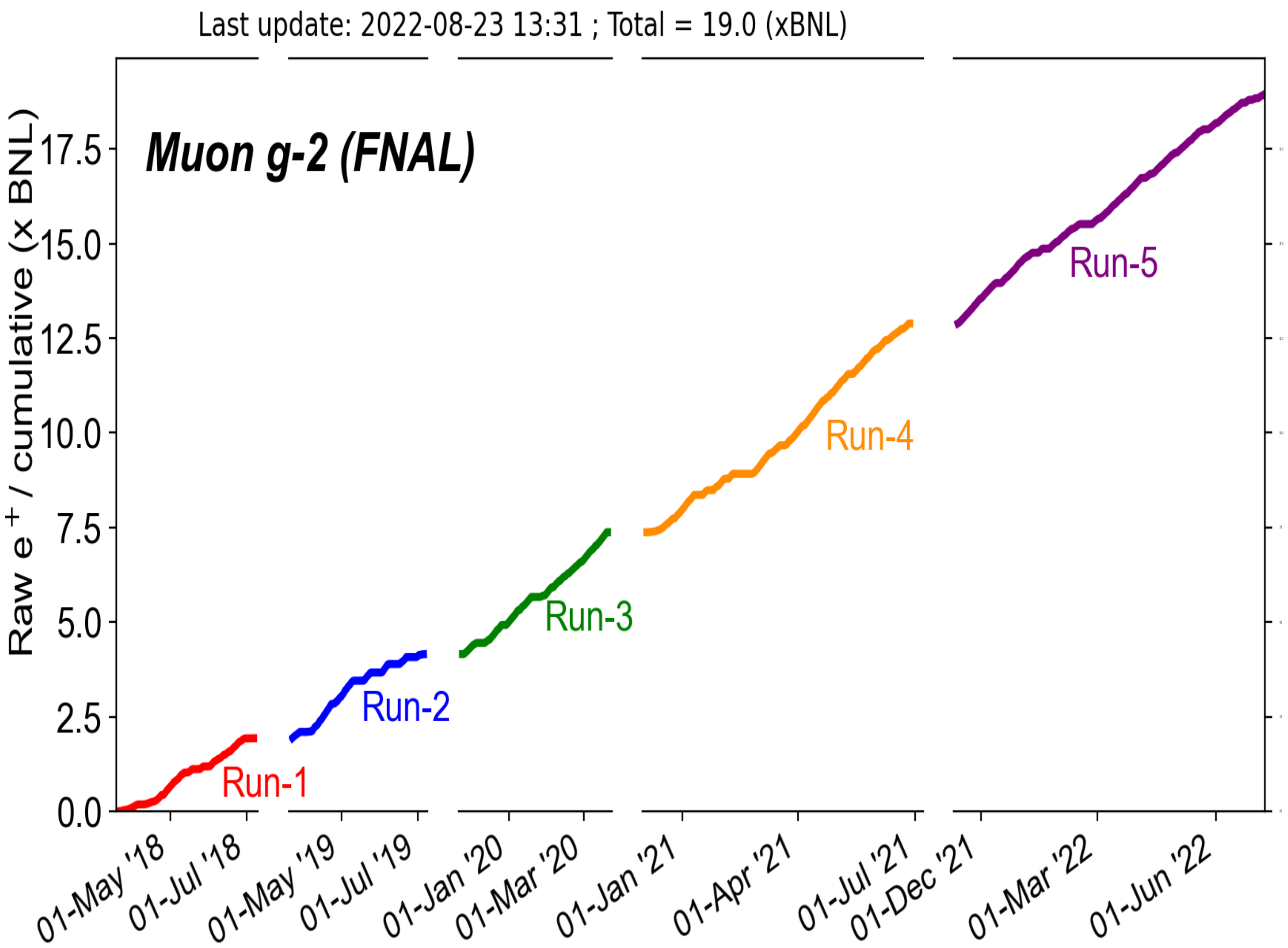
$a_\mu^{EXP} - a_\mu^{SM}$ = (251±59) = 4.2 σ
[unit: 10^{-11}]

The white paper from the theory invitation [https://muon-gm2-theory.illinois.edu/white-paper/]



Overview of E989

Run 1 and Beyond



- Run 1 : 6 % of total expected data
- Much more data to analysis
- Projected to publish Run 2/3 at the spring of 2023
- Run 5 just finished few months ago (2022)
- Run 6 will begin after summer shutdown with sharing the beam with mu2e experiment

Overview of E989

Overview

$$a_\mu = \frac{\omega_a}{\overline{\omega_p}'(T_r)} \frac{\mu_p'(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g}{2} \qquad (T_r = 34.7\ ^\circ\mathrm{C})$$

Overview of E989

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Collaboration Measurement

$$R'_\mu \equiv \frac{\omega_a}{\bar{\omega}'_p(T_r)} \approx \frac{f_{clock}\,\omega_a^m (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{calib} \langle \omega_p(x,y,\phi) \times M(x,y,\phi) \rangle (1 + B_k + B_q)}$$

f_{clock} : the master clock, unblinding factor

ω_a^m: the measured precession frequency

C_i: four beam-dynamic corrections

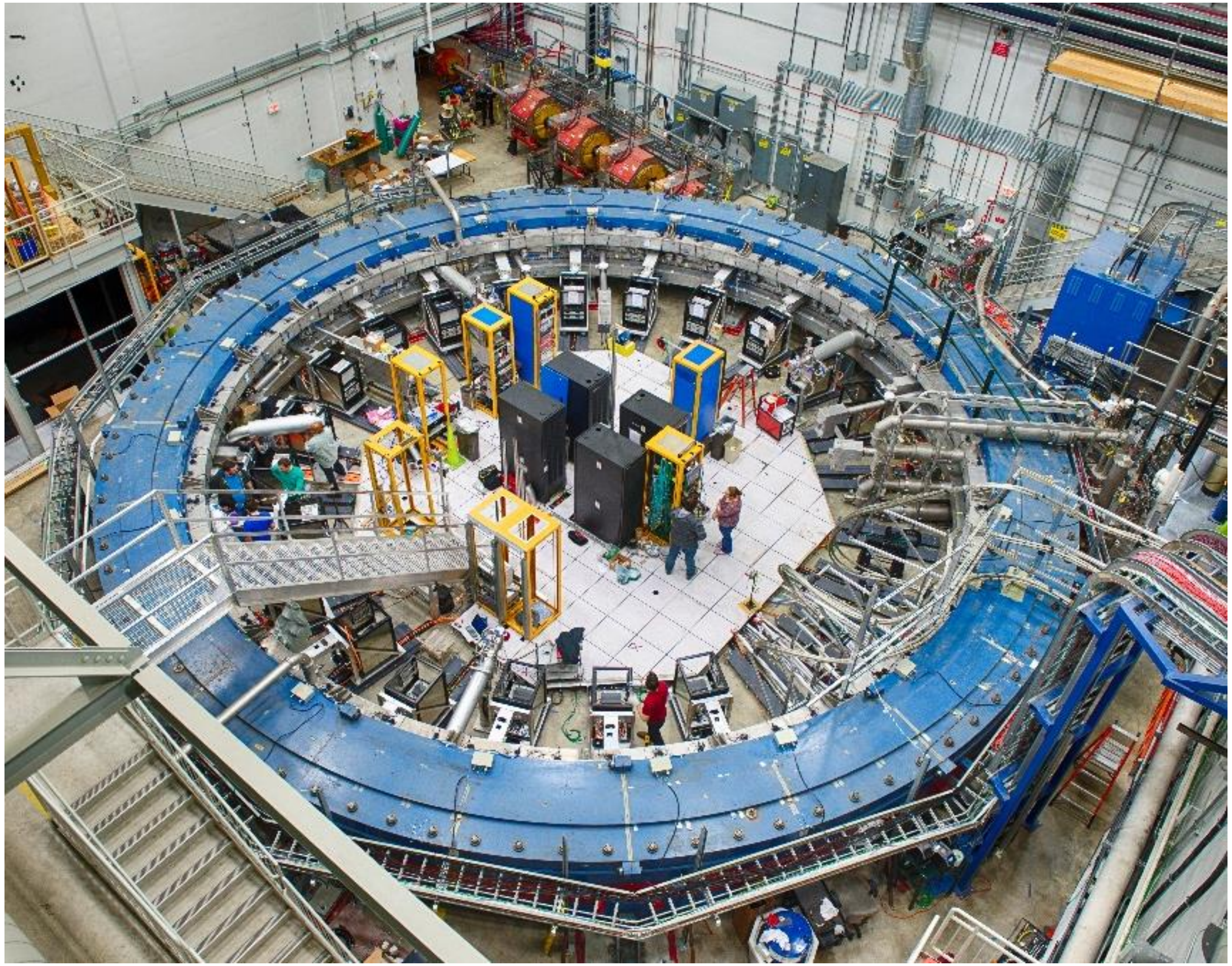
f_{calib}: the probe calibration factor

$\langle \omega_p(x,y,\phi) \times M(x,y,\phi) \rangle$: the field map, weighted by detected positron and muon distribution averaged over space and time.

B_i: Two fast magnetic transient correction

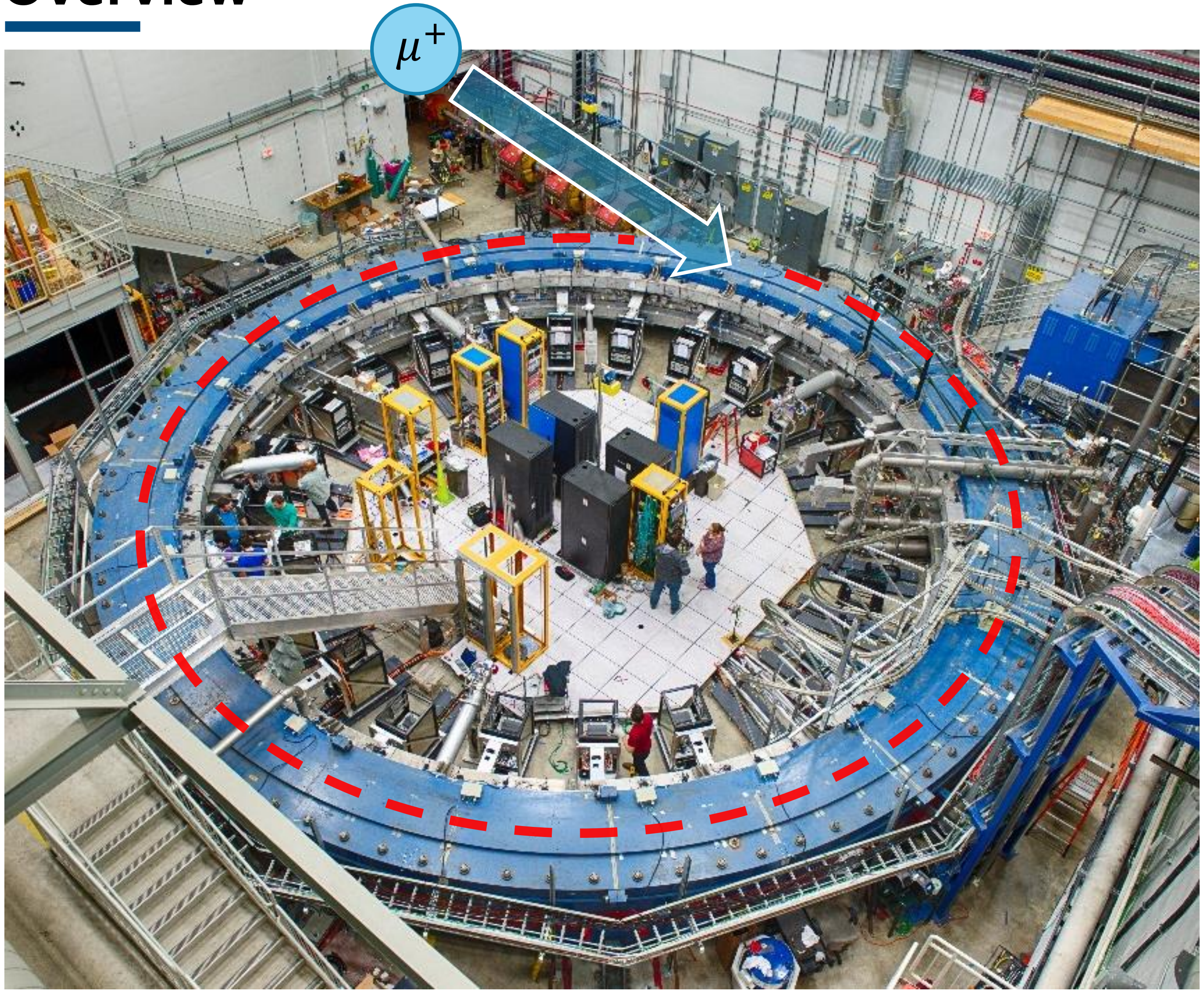
E989 Muon g-2 Storage Ring

Overview



E989 Muon g-2 Storage Ring

Overview



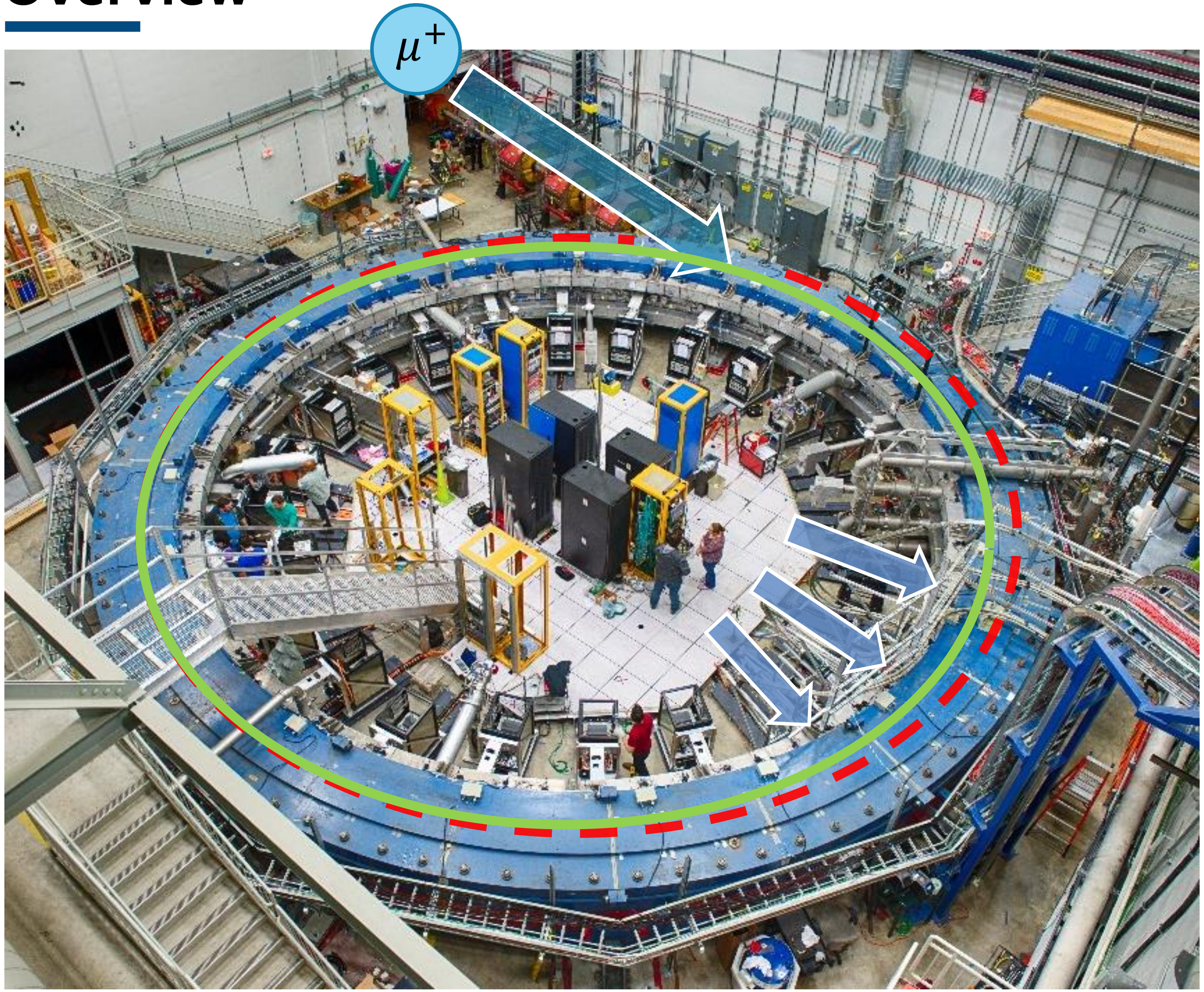
▶ Injection

- Polarized anti-muon bunch (3.1 GeV) is injected into the storage ring (~14 m diameter).
- Superconducting **inflector magnet** cancels the main focusing magnetic field (1.45 T) to inject the bunch into the storage ring tangentially.

Part 2

E989 Muon g-2 Storage Ring

Overview



▶ Injection

- Polarized anti-muon bunch (3.1 GeV) is injected into the storage ring (~14 m diameter).
- Superconducting **inflector magnet** cancels the main focusing magnetic field (1.5 T) to inject the bunch into the storage ring tangentially.

▶ Kickers

- Muons are kicked onto the design orbit by the fast non-ferric **kicker magnet** system.

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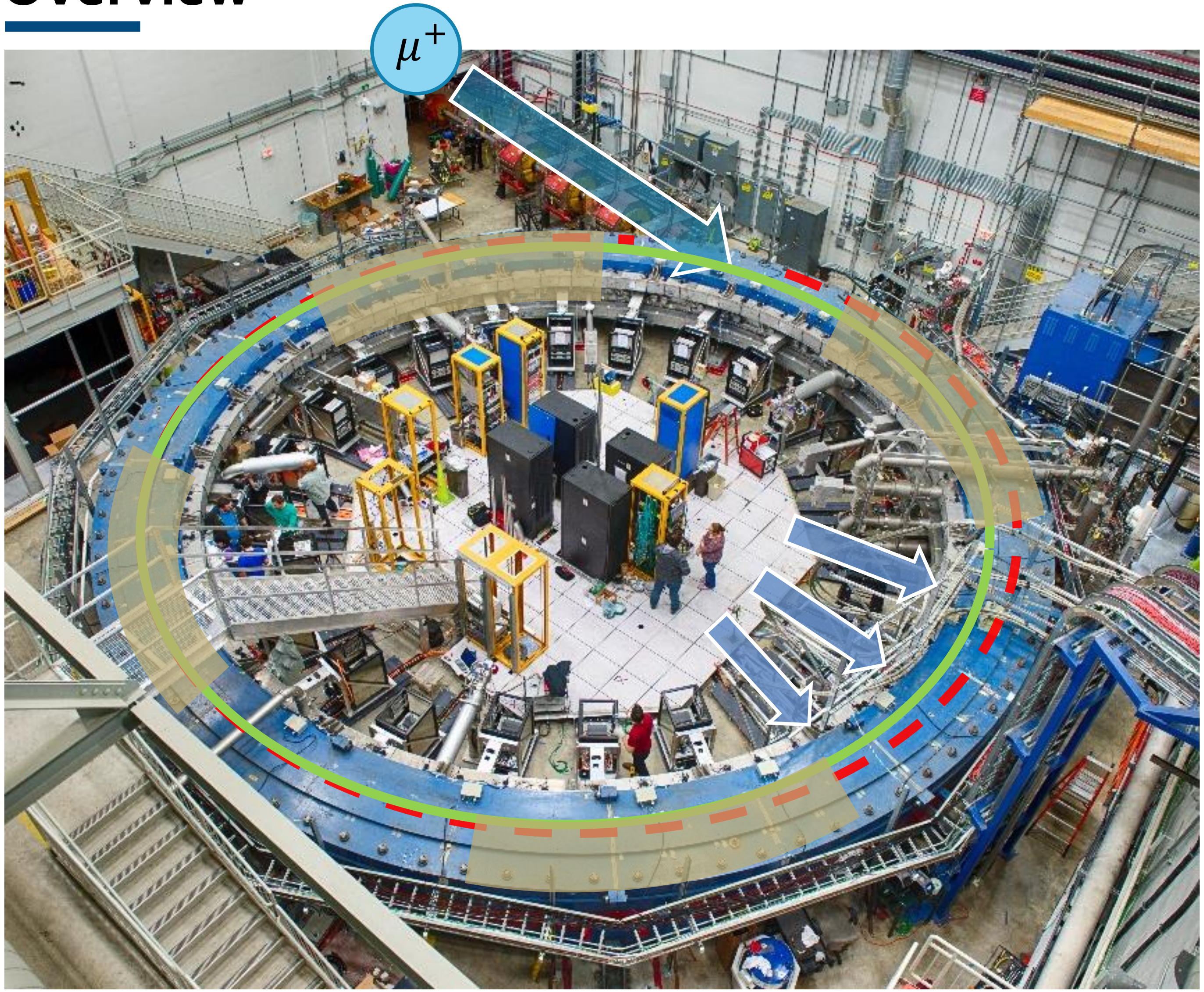

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▶ Vertical focusing

- **Electrostatic Quadrupoles** (ESQ) focuses the beam vertically.
- 4 Quadrupole sections (long and short for each) cover 43% of the circumference

E989 Muon g-2 Storage Ring

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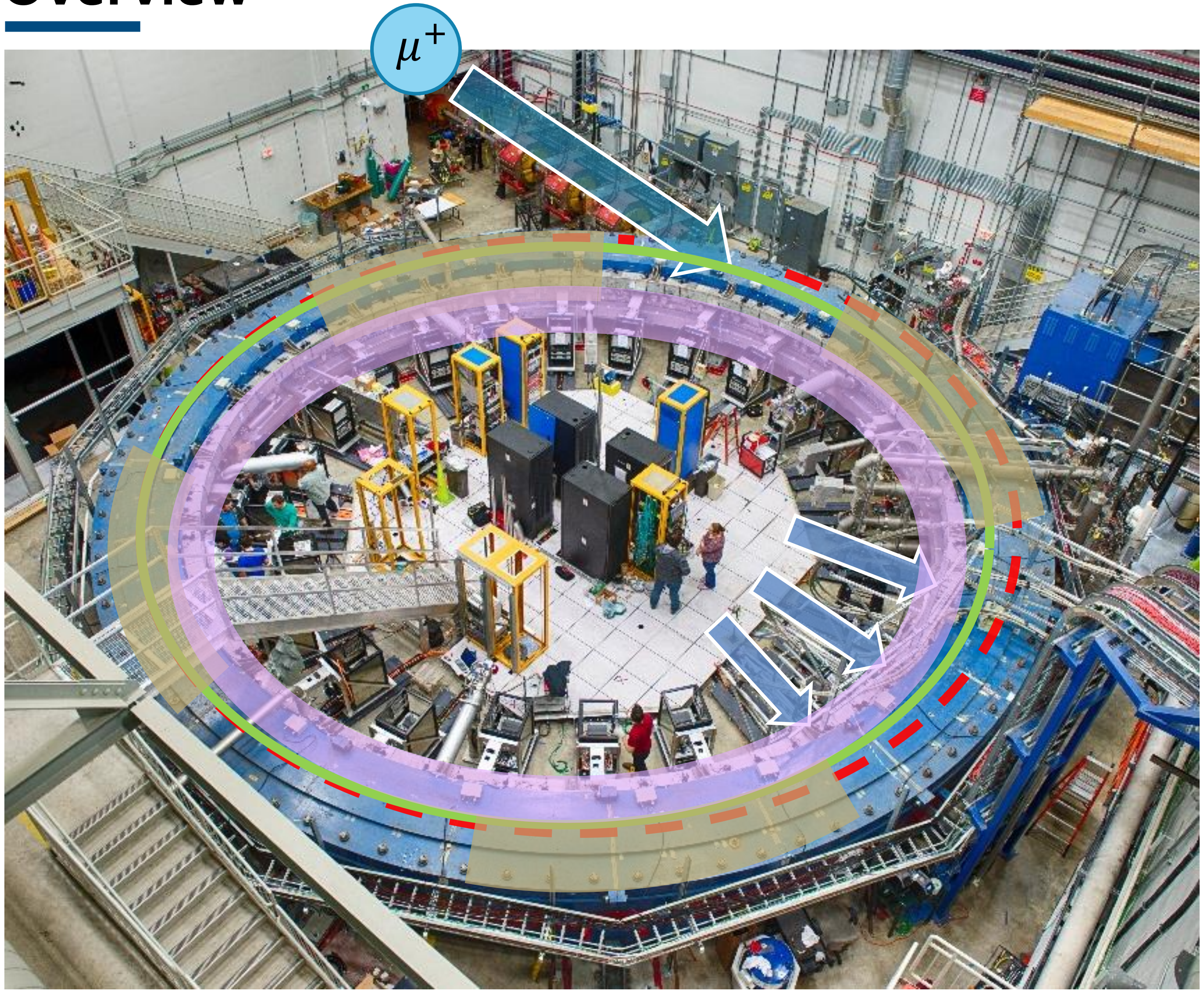

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- **Electrostatic Quadrupoles** (ESQ) focuses the beam vertically.
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▶ Detection

- Muons decay into positrons which curl into 24 electromagnetic **calorimeters** surrounding the storage ring.

Part 3

NMR Technique and Field Measurement

NMR Probes

Trolley Probe

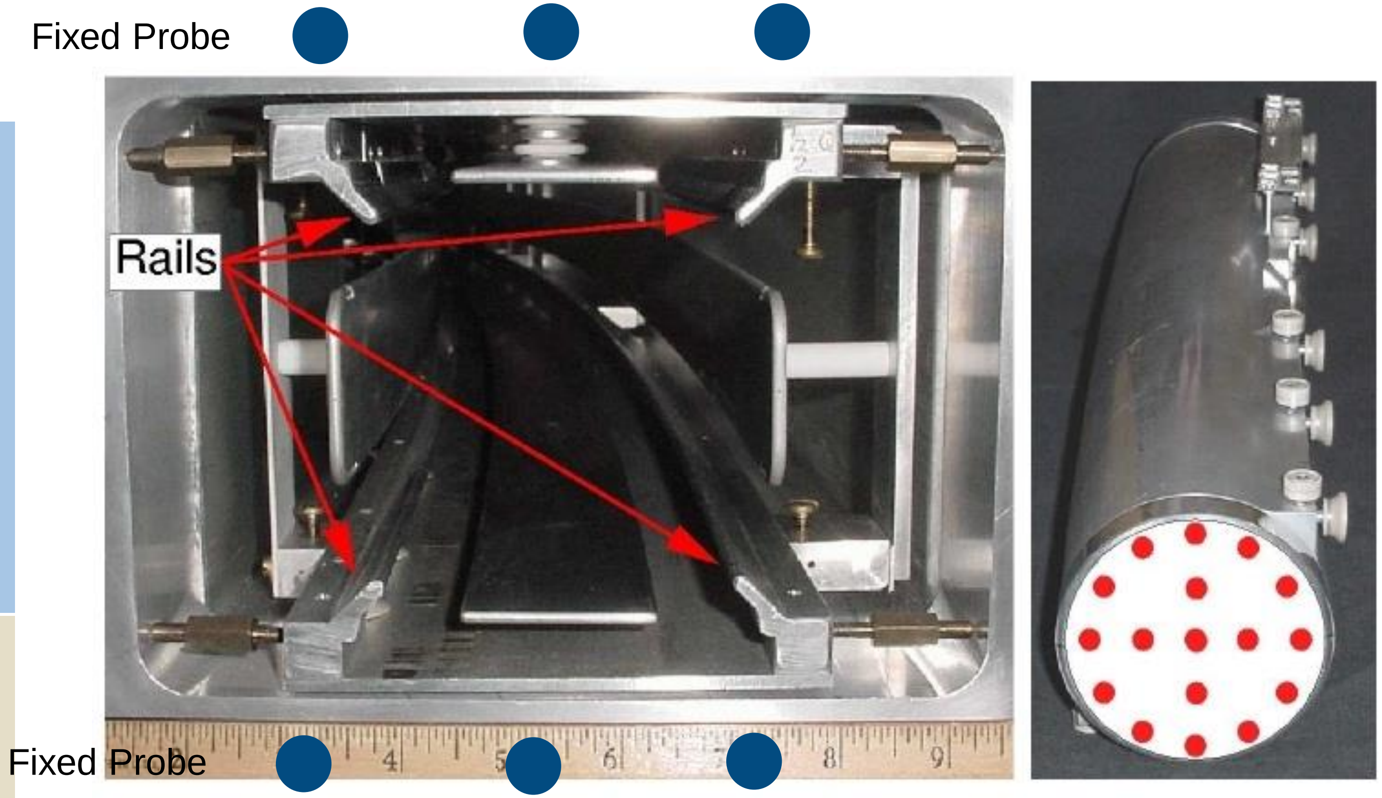
Location:
Total 17 probes carried by trolley

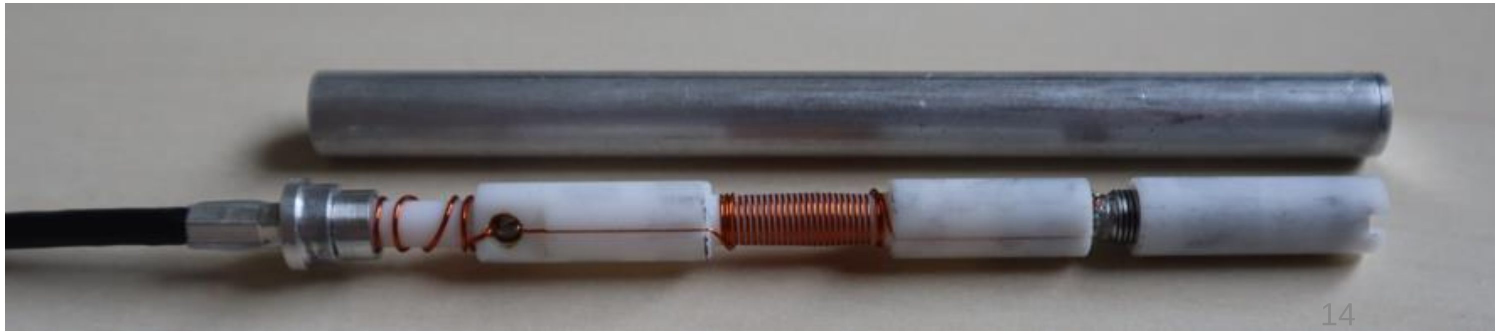
Purpose:
Periodic field maps inside of the ring every 3-5 days

Fixed Probe

Location:
Total 378 probes mounted at fixed locations outside (top and bottom) of the muon orbit

Purpose:
Continuously monitor field drift while muons are present in the ring

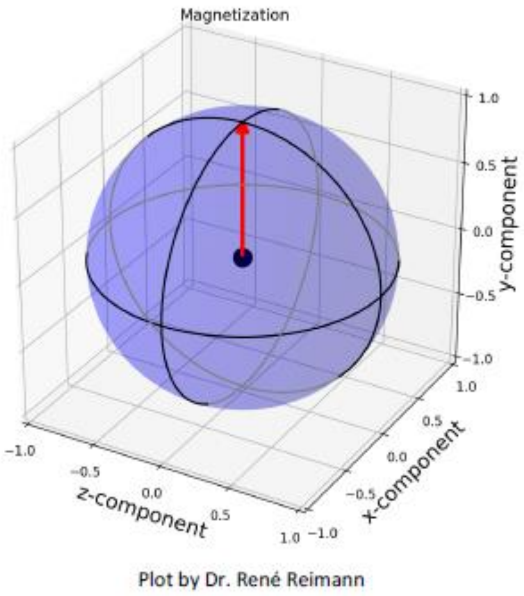




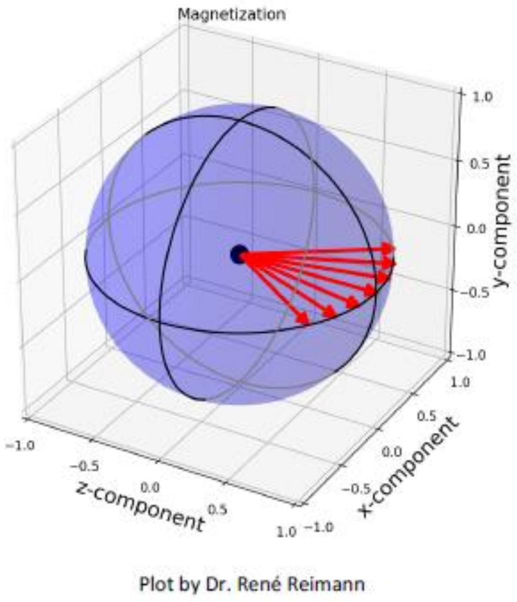
NMR Technique and Field Measurement

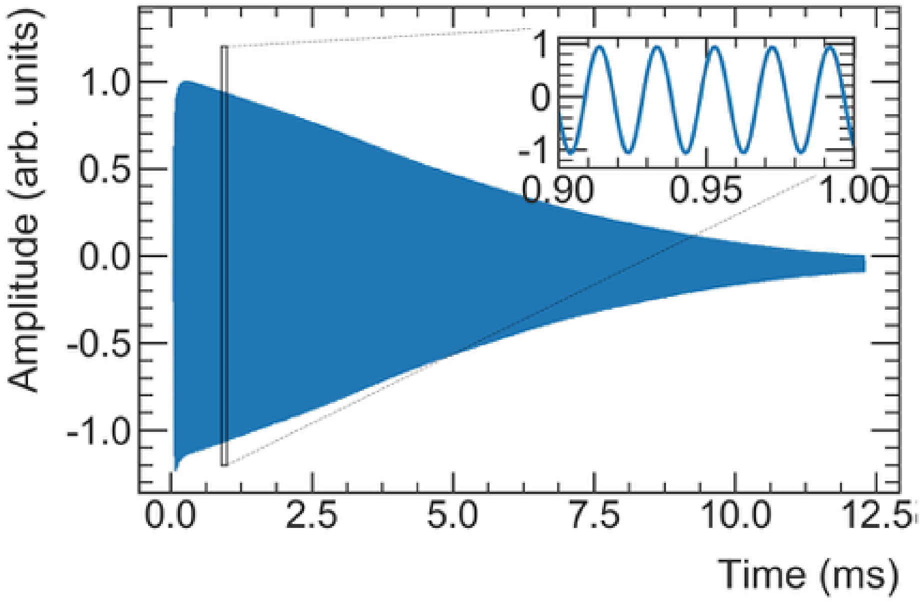
Frequency Extraction Method

1. RF ($\pi/2$) pulse : proton spins flip from $\hat{z}$ - direction to x-y plane (90 degree)
2. Free induction decay (FID) NMR signal digitization



RF Pulse





Typical FID from a trolley probe

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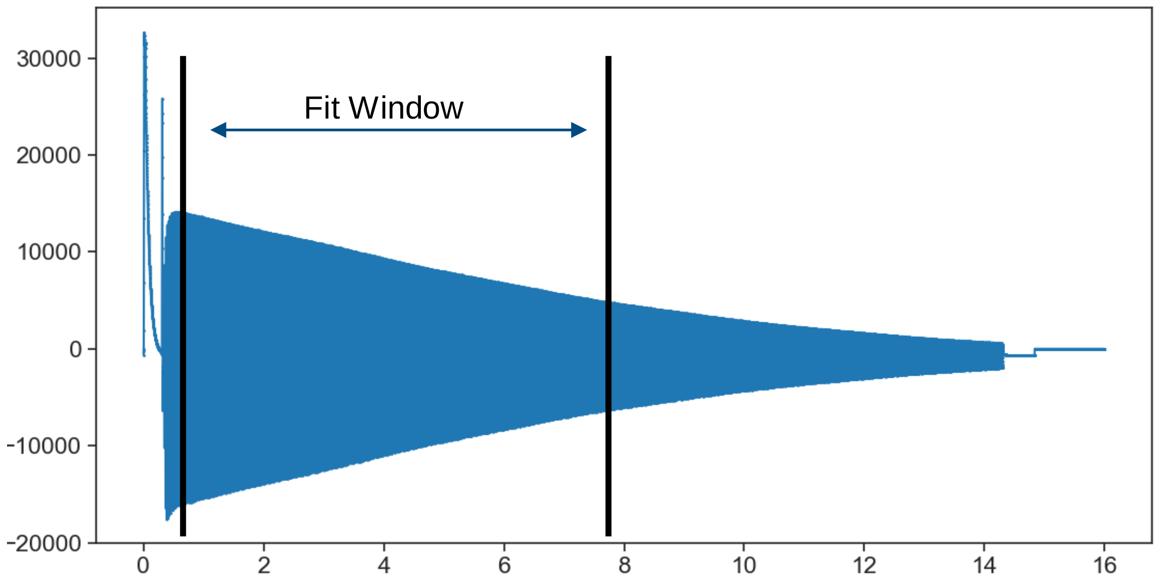
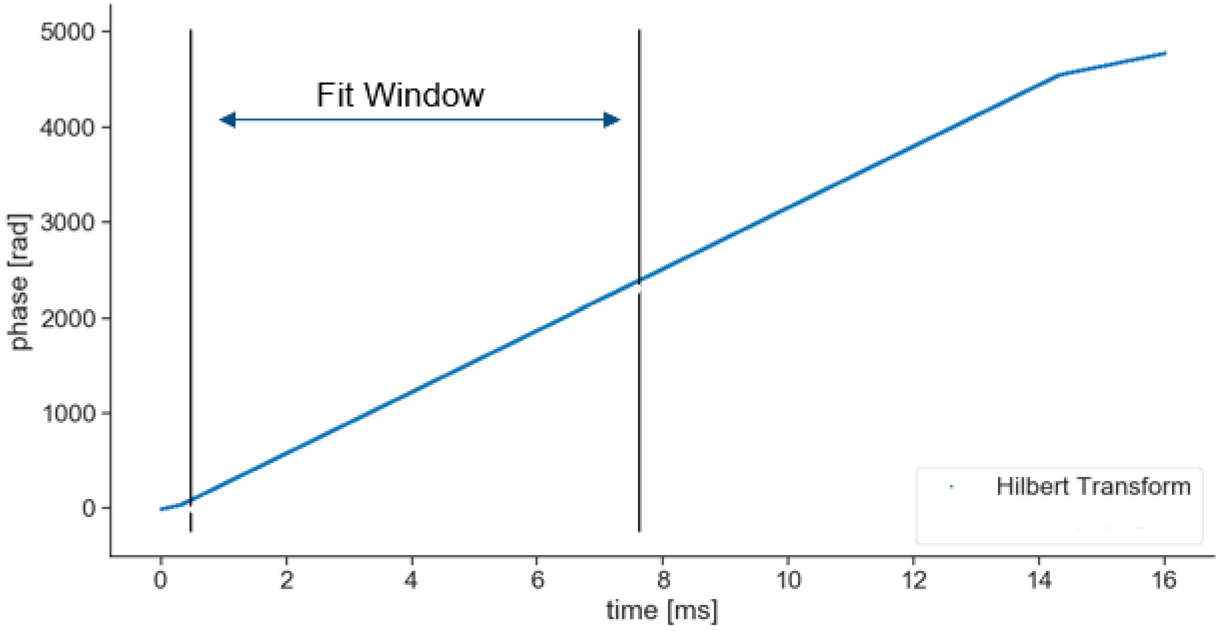
3. Fit window optimization and phase function extraction by using Hilbert Transform



Trolley Probe FID



Trolley Probe Phase Function

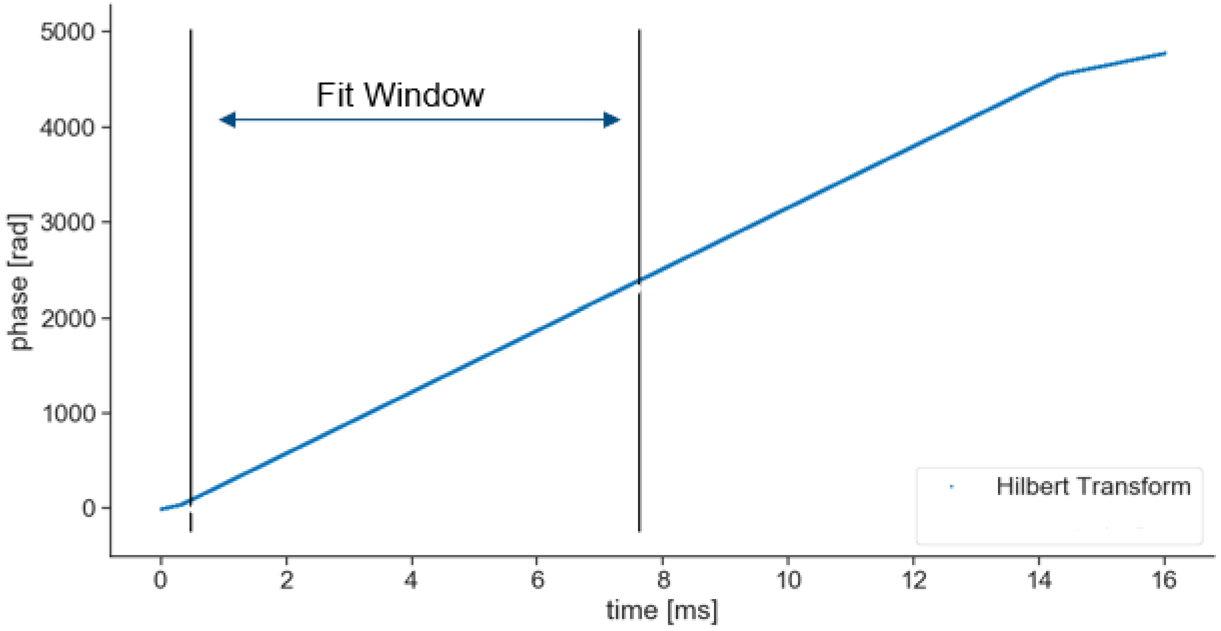
NMR Technique and Field Measurement

Frequency Extraction Method

1. RF ($\pi/2$) pulse : proton spins flip from $\hat{z}$ - direction to x-y plane (90 degree)
2. Free induction decay (FID) NMR signal digitization
3. Fit window optimization and phase function extraction by using Hilbert Transform
4. Linear Fit to extract the frequency

$$\omega = \frac{d\Phi(t)}{dt}\Big|_{t=0}$$

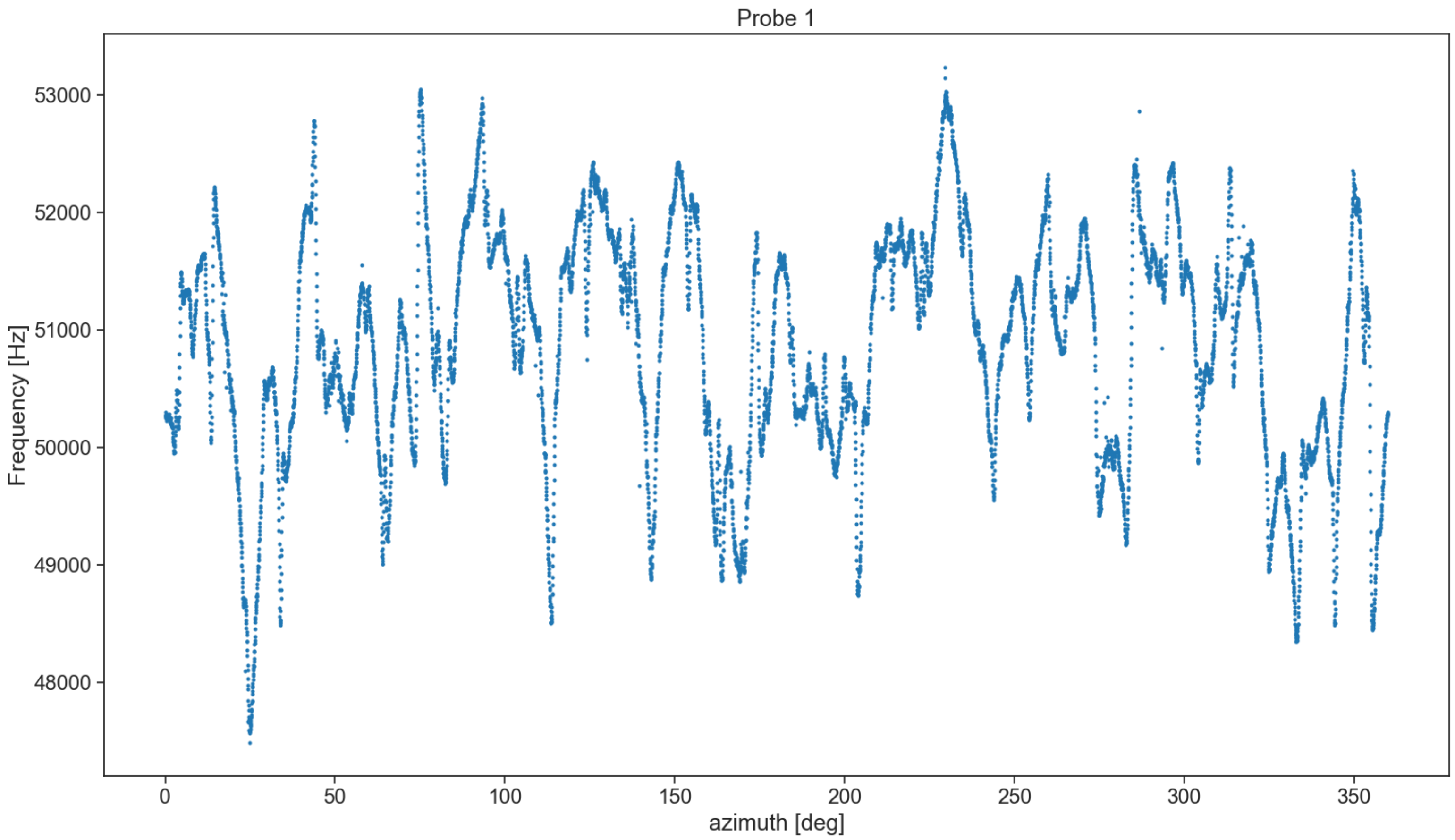
Φ(t) = Phase Function

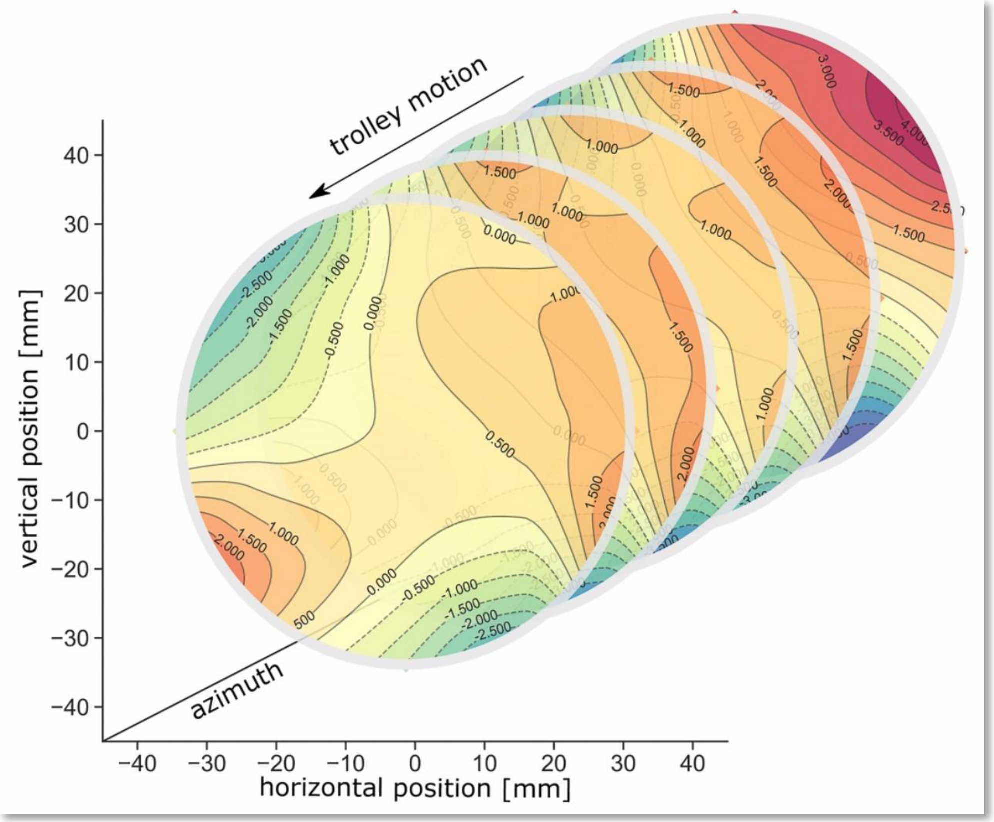


Trolley Probe Phase Function

NMR Technique and Field Measurement

Frequency vs Azimuth





Calibration

Motivation

- Because 17 trolley probes are in an aluminum shell, the materials perturb the magnetic field.
- The correction (calibration) is needed to measure the true field.

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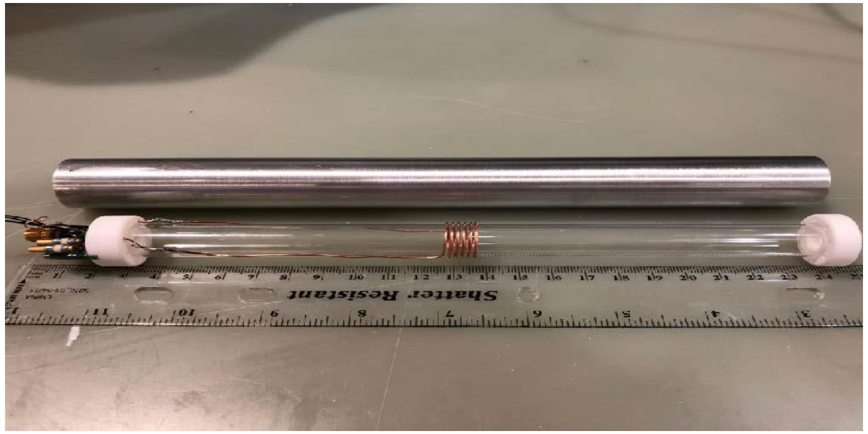
Plunging Probe

- Only one special probe made at UMASS
- Used as the calibration probe.
- Built with well-known geometry and well-measured perturbation for absolute calibration

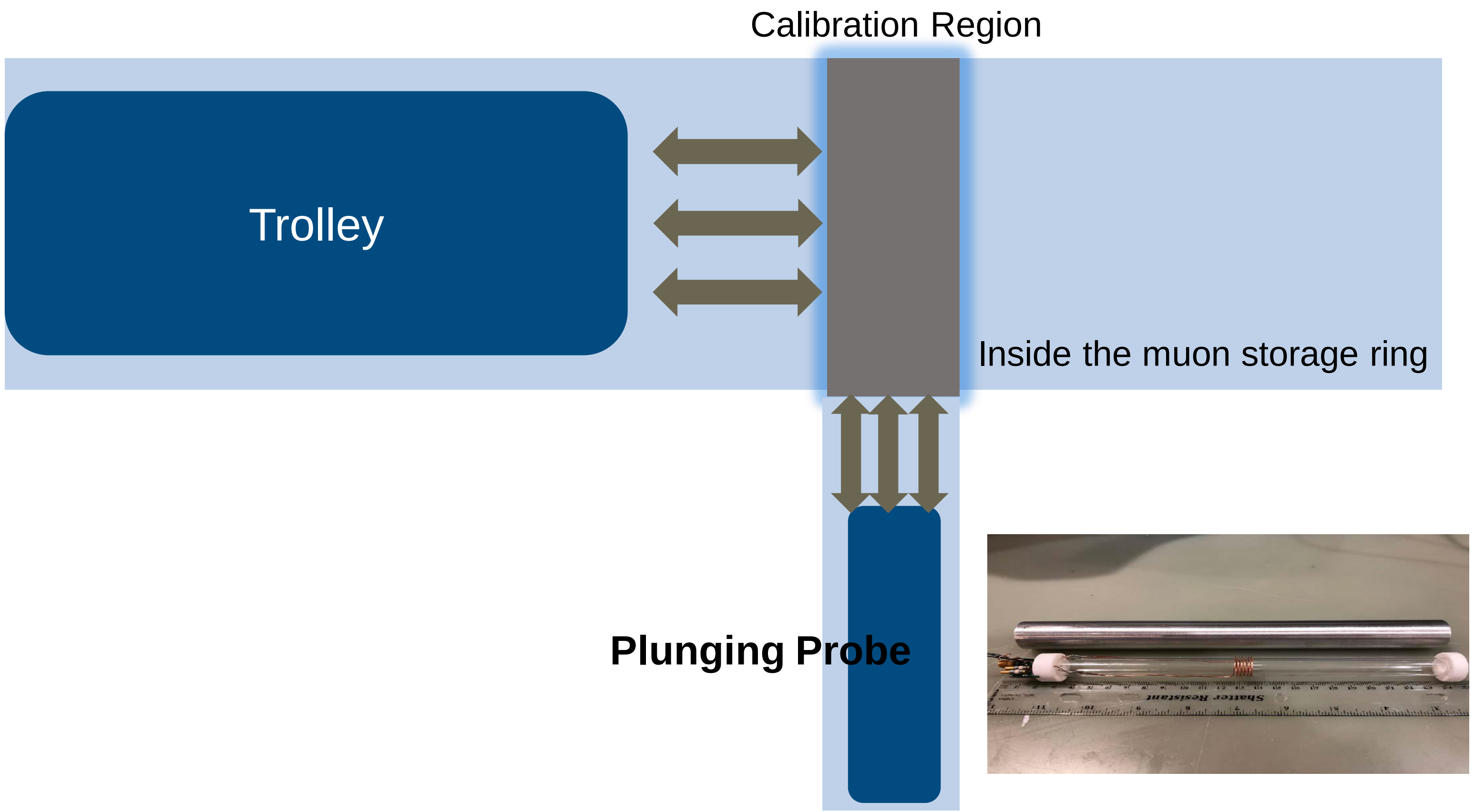


Part 4

Calibration

Procedure

- Swap the trolley and plunging probe in the calibration region inside of the ring
- Find the difference between trolley probe and plunging probe field measurement

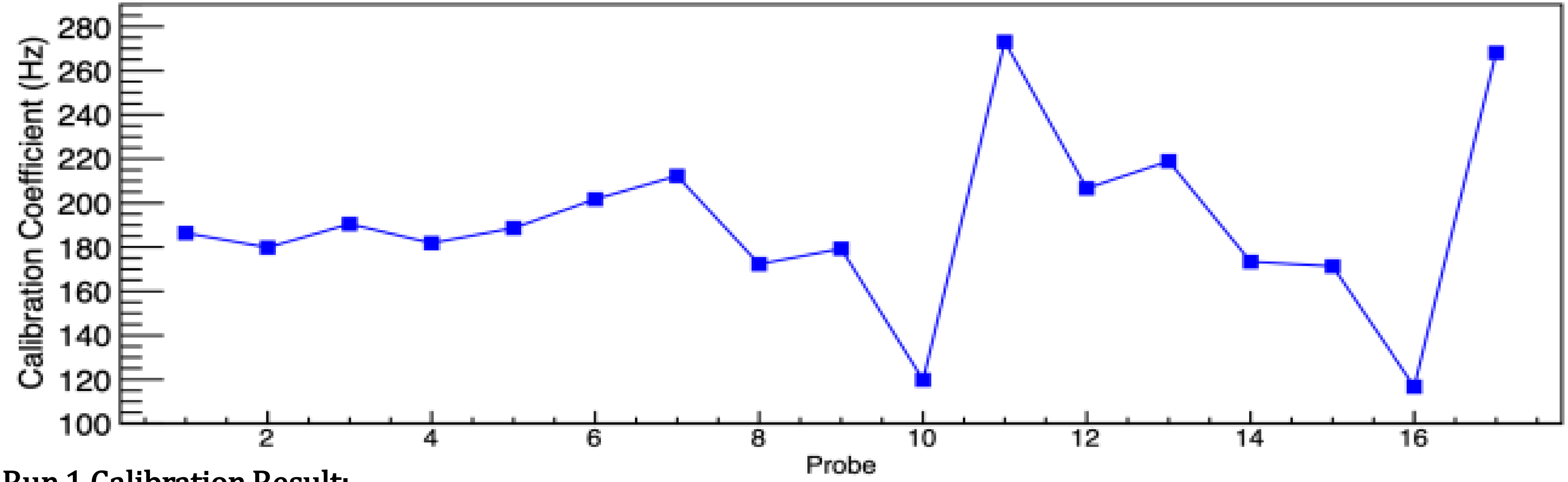


Few things to consider:

- Need to shim the calibration region to make the local field uniform
- Make sure the trolley and plunging probe position is aligned
- Dependence on temperature

Calibration

Result



Run 1 Calibration Result:

- Avg statistical uncertainty : 11.3 ppb (0.7 Hz)
- Avg systematic uncertainty: 19.4 ppb (1.2 Hz)

- Blinded studies from 3 analyzers with 3 different analysis methods
- The mean difference about 3 ppb (0.185 Hz) and standard deviations about 8 ppb (0.494 Hz)

Systematics

Run 1 Systematics

$$R'_\mu \equiv \frac{\omega_a}{\overline{\omega}'_p(T_r)} \approx \frac{f_{clock}\,\omega_a^m(1 + C_e + C_p + C_{ml} + C_{pa})}{f_{calib}\langle\omega_p(x,y,\phi) \times M(x,y,\phi)\rangle(1 + B_k + B_q)}$$

Data set	$\tilde{\omega}'_p(T_r)/2\pi$ (Hz)	Uncertainty (ppb)
Run-1a	61,791,871.2	115
Run-1b	61,791,937.8	127
Run-1c	61,791,845.4	125
Run-1d	61,792,003.4	108
	Average over all data sets	
Field Measurements		56
ESQ Transient		92
Kicker Transient		37
Total		114

The goal is 70 ppb

Systematics

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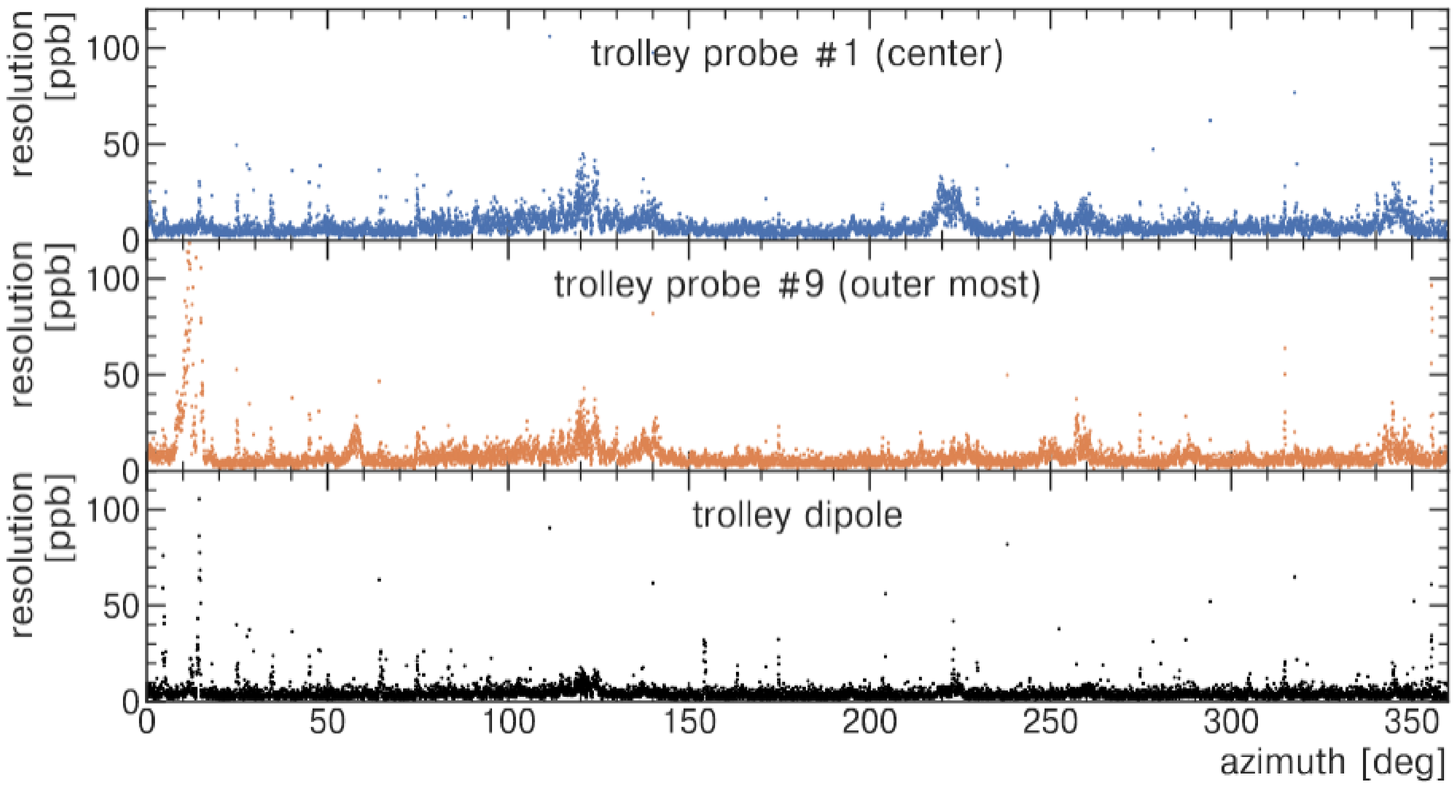
Calibration

Field Tracking

Muon weightening

Systematics

Field Measurement: Statistical Uncertainty of Frequency Extraction

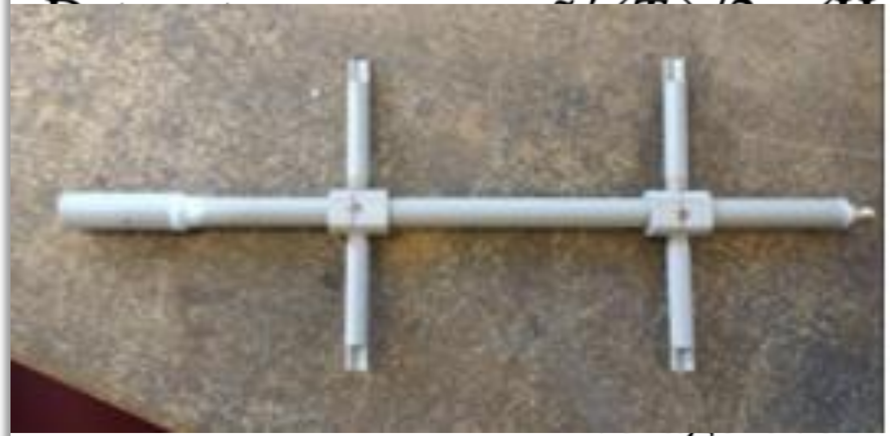


- Depends on predominantly the length of the fit range
- Use ***stepper run***
  - Move trolley 0.05 degree and stop for a moment and measure the field
  - Get rid of any motion effect
- Beside the inflector region, dipole resolution are < 50 ppb around the ring and average resolution about 10 ppb level

Systematics

Run 1 Systematics

$$R'_\mu \equiv \frac{\omega_a}{\tilde{\omega}'_p(T_r)} \approx \frac{f_{clock}\,\omega_a^m(1 + C_e + C_p + C_{ml} + C_{pa})}{f_{calib}\langle\omega_p(x,y,\phi) \times M(x,y,\phi)\rangle(1 + B_k + \boldsymbol{B_q})}$$



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Largest uncertainty

Improvements to the determination of the ESQ transient from additional measurements

Systematics

Run 1 Systematics

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Data se	(Hz)	Uncertainty (ppb)
Run-1a	1.2	115
Run-1b	7.8	127
Run-1c	5.4	125
Run-1d	3.4	108
	er all data sets	
		56
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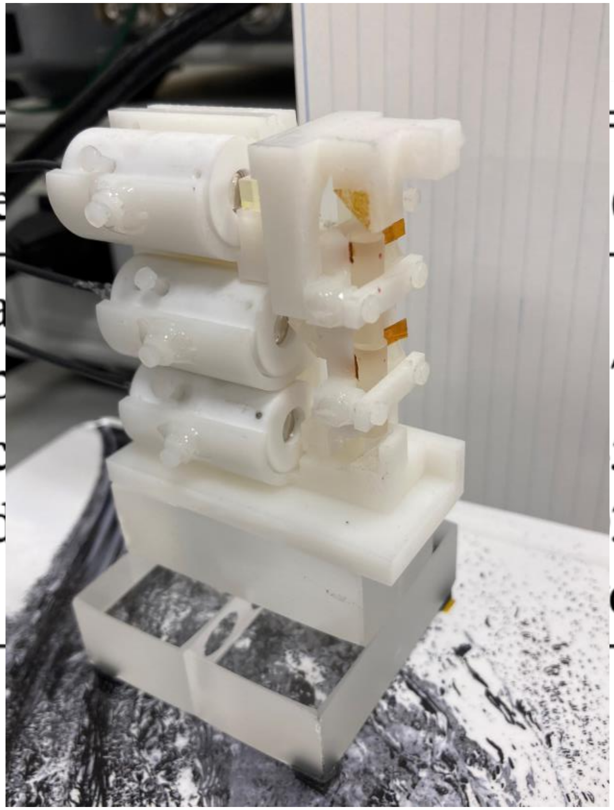
Total		114



The goal is 70 ppb

After the pulse, eddy currents are induced and perturbs the magnetic field during the measurement

Correction is measured using a Faraday magnetometer designed using no metal

Part 6

Conclusion

Summary

Collaborator measurement:

$$\boldsymbol{R'_\mu} \equiv \frac{\boldsymbol{\omega_a}}{\boldsymbol{\bar{\omega}'_p(T_r)}} \approx \frac{f_{clock}\,\omega_a^m(1 + C_e + C_p + C_{ml} + C_{pa})}{f_{calib}\langle\omega_p(x,y,\phi)\ \times M(x,y,\phi)\rangle(1 + B_k + B_q)}$$

- Precise Field Measurement is needed to calculate anomalous muon dipole moment, a_μ
- ω_p is measured by trolley, running around the ring every 3-5 days, and by fixed probe, tracking the field drift
- f_{calib} is from the calibration of trolley probes to plunging probe, well-known geometry and well-known perturbation
- Systematic studies for field measurement and ESQ and Kicker Transients

Conclusion

What we improved beyond Run 1: NMR Technique

Calibration

- Automatic script implemented
- Better shimming measurements.
- Better temperature controls
- Better alignment

Systematic	Correction (ppb)	Uncertainty (ppb)
Absolute calibration	0	15
Trolley calibration	0	28
Configuration	−1	23
Trolley baseline $m^{tr}(0)$	−13	25
Fixed probe baseline $m^{fp}(0)$	0	8
Fixed probe runs $m^{fp}(t)$	0	1
Total	−14	48

Systematic correction and uncertainty from Run 1

NMR Technique

- New Fit Window Optimization method
  - implemented to the production
  - Improved bad probes (short FID probes) resolutions
- Upgrade Data Quality Cut (DQC) / Production campaign

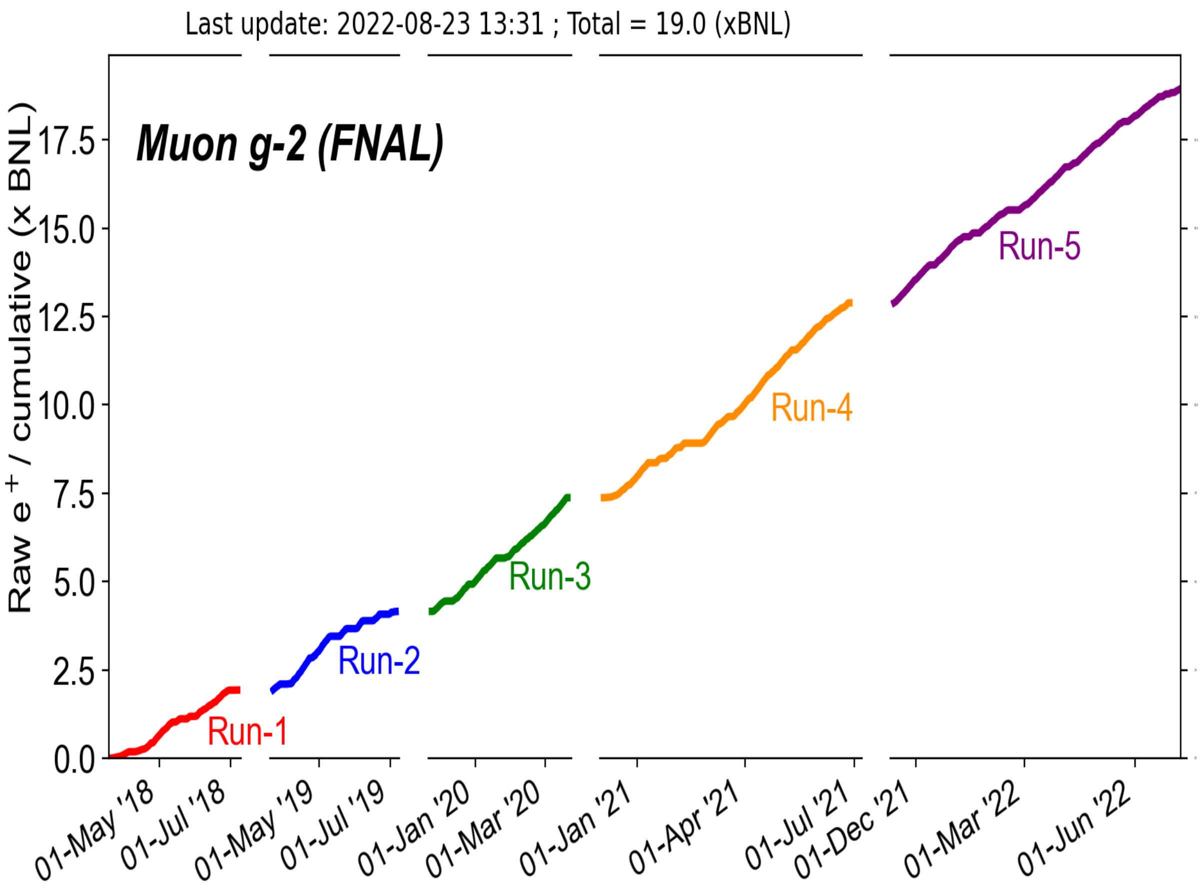
Interpolation

- Two separate interpolation team (blinded study)
- Improve and upgrade their framework for better precision

Part 6

Conclusion

What we improved beyond Run 1: Field Measurement



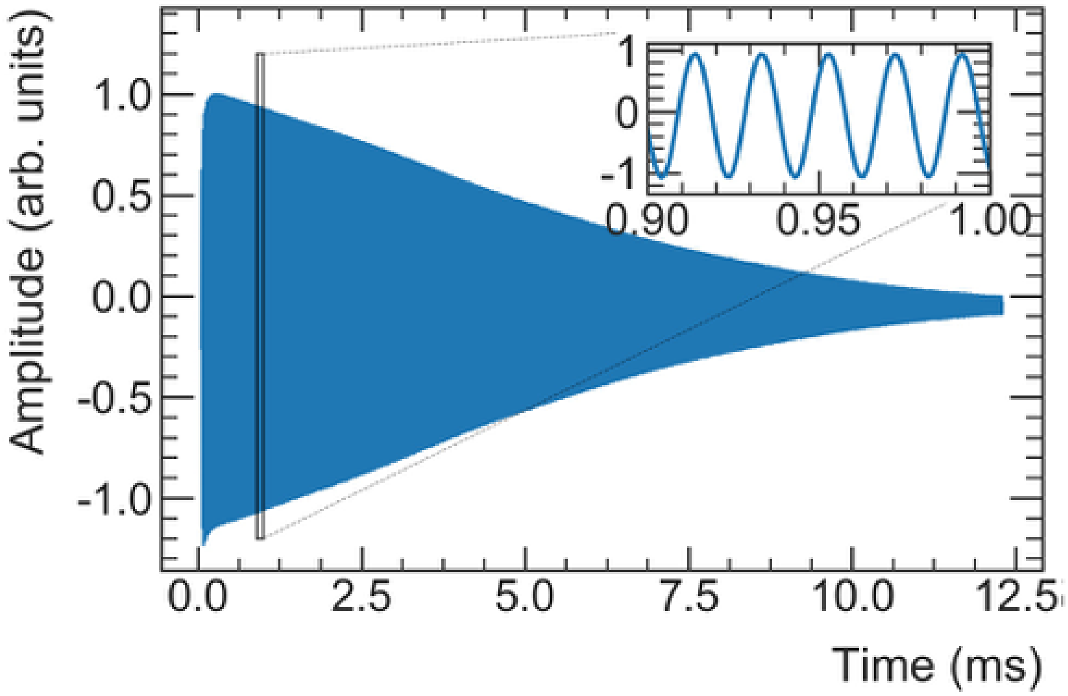
- Replaced the damaged resistors after Run 1
- Upgraded the experimental hall cooling system (Beyond Run 2) - *Temperature*
- Upgraded the kickers to deflect beam on the ideal orbit (Beyond Run 2) - *Kicker*
- Improved mapping the perturbed magnetic field resulting from quad transient field - *ESQ*

The goal is 70 ppb

Back-up

NMR Technique

FID (Free Induction Decay)



Typical FID from a trolley probe

- $\pi/2$ pulse : proton spins from $\hat{z}$ - direction to x-y plane
- FID signal: Proton spins precessing at angular frequency ω and rotating magnetic field induced EMF in the pick-up coils. Signal read out by electronics
- Amplitude decays due to energy loss (T_1) and decoherence (T_2).

NMR Technique

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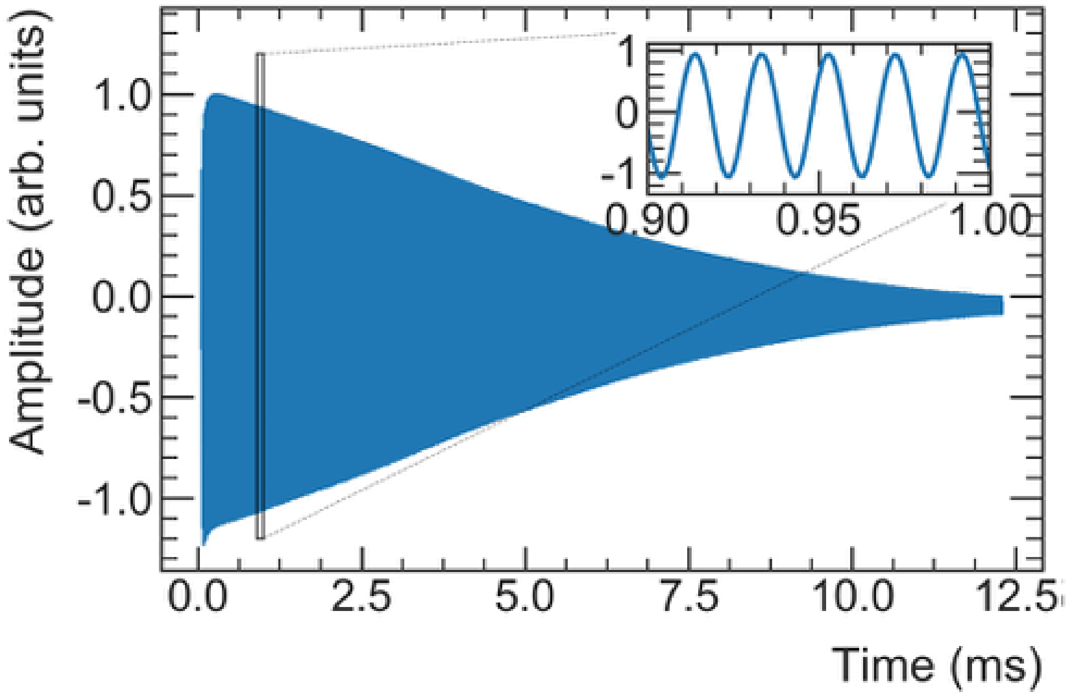

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Uniform Field Model

$$f(t) = A \exp\left(-\frac{t}{\tau}\right) \exp\big(i(\omega t + \phi_0)\big),$$

Constant initial phase

$$\frac{1}{\tau} = \frac{1}{T_1} + \frac{1}{T_2}$$

Back up

Non-uniform FID model

$\Omega(x, y, z)$: Precession angular frequencies as a function of position
$A(x, y, z)$: Initial amplitude of the spin precession signal as a function of position
$\eta(x, y, z)$: Response function of the coil as a function of position

Assumption:

- All spins precess independently with the same time constant τ
- The precession phase (ϕ_0) at the end of the $\pi/2$ pulse is the same for all spins

With these two assumptions, one can integrate the contribution of each spin and obtain the total signal:

$$f(t) = \exp\left(-\frac{t}{\tau}\right) \iiint_{-\infty}^{+\infty} \exp(i\Omega(x, y, z)t + \phi_0)\, A(x, y, z)\, \eta(x, y, z)\, dxdydz$$

where the precession angular frequency Ω becomes space-dependent. Insert the identity operator $\int \delta(\omega - \Omega)\, d\omega$ for rewrite the equation above for final equation on slide 17

Back up

Non-uniform Field

- In a ***non-uniform field***…
  - Protons at different positions precess at different angular frequency, $\boldsymbol{\Omega}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$
  - Initial amplitude of spin precession signal, $\boldsymbol{A}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$
  - The response function of the coil, $\boldsymbol{\eta}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$

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  - The response function of the coil, $\boldsymbol{\eta}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$

Non-uniform Field Model

$$f(t) = N \exp\left(-\frac{t}{\tau}\right) \int_{-\infty}^{\infty} g(\omega) \exp\left(i\,(\omega t + \phi_0)\right) d\omega$$

$$g(\omega) = \frac{1}{N} \iiint_{-\infty}^{+\infty} \delta\left(\omega - \Omega(x, y, z)\right) A(x, y, z)\, \eta(x, y, z)\ dxdydz$$

$g(\omega)$: signal distribution function. Usually Peak at ω_0, the frequency at the probe center.

Back up

Hilbert Transform

FID model in a non-uniform field:

$$f(t) = N \exp\left(-\frac{t}{\tau}\right) \int_{-\infty}^{\infty} g(\omega) \exp\big(i\,(\omega t + \phi_0)\big)\, d\omega$$

Back up

Hilbert Transform

FID model in a non-uniform field:

$$f(t) = N \exp\left(-\frac{t}{\tau}\right) \int_{-\infty}^{\infty} g(\omega) \exp\big(i\,(\omega t + \phi_0)\big)\, d\omega$$

Let $\omega = \omega_0 + \Delta\omega$, the FID function:

$$f(t) = N \exp\left(-\frac{t}{\tau}\right) \exp(i\,(\omega_0 t + \phi_0)) \int_{-\infty}^{\infty} g(\Delta\omega) \exp\big(i\,(\Delta\omega t)\big)\, d\Delta\omega$$

Back up

Hilbert Transform

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Let $\omega = \omega_0 + \Delta\omega$, the FID function:

$$f(t) = N \exp\left(-\frac{t}{\tau}\right) \exp(i\,(\omega_0 t + \phi_0)) \underline{\int_{-\infty}^{\infty} g(\Delta\omega) \exp\big(i\,(\Delta\omega t)\big)\, d\Delta\omega}$$

$\Updownarrow$

Additional factor for non-uniform field

Uniform Field FID Model at $\omega = \omega_0$*:*

$$f(t) = A \exp\left(-\frac{t}{\tau}\right) \exp\big(i(\omega_0 t + \phi_0)\big)$$

Back up

Hilbert Transform

FID model in a non-uniform field:

$$f(t) = N \exp\left(-\frac{t}{\tau}\right) \int_{-\infty}^{\infty} g(\omega) \exp\left(i\ (\omega t + \phi_0)\right) d\omega$$

Let $\omega = \ \omega_0 + \ \Delta\omega$, the FID function:

$$f(t) = N \exp\left(-\frac{t}{\tau}\right) \exp(i\ (\omega_0 t + \phi_0)) \underline{\int_{-\infty}^{\infty} g(\Delta\omega) \exp\left(i\ (\Delta\omega t)\right) d\Delta\omega}$$

$\Updownarrow$

Uniform Field FID Model at $\omega = \ \omega_0$:

$$f(t) = A \exp\left(-\frac{t}{\tau}\right) \exp\left(i(\omega_0 t + \ \phi_0)\right)$$

Additional factor for non-uniform field

$\Updownarrow$

Exactly same as the inverse Fourier Transform of $g(\Delta\omega)$.

Ran Hong's Note: DocDB [18347]

Hilbert Transform

FID model in a non-uniform field at $\omega = \omega_0 + \Delta\omega$:

$$f(t) = N \exp\left(-\frac{t}{\tau}\right) \exp(i\,(\omega_0 t + \phi_0)) \underline{\int_{-\infty}^{\infty} g(\Delta\omega) \exp\big(i\,(\Delta\omega t)\big)\, d\Delta\omega}$$

The inverse Fourier Transform of $g(\Delta\omega)$.

Therefore, we can write it as combination of sine and cosine:

$$\int_{-\infty}^{+\infty} g(\Delta\omega) \exp\big(i(\Delta\omega t)\big)\, d\Delta\omega = C(t) + i\,S(t) = \sqrt{C^2(t) + S^2(t)} \cdot \exp(i \tan^{-1}(S(t)/C(t)))$$

where,

$$S(t) = \int_{-\infty}^{\infty} g(\Delta\omega)\sin(\Delta\omega t) d\Delta\omega$$

$$C(t) = \int_{-\infty}^{\infty} g(\Delta\omega)\cos(\Delta\omega t) d\Delta\omega$$

Back up

Hilbert Transform

FID model in a non-uniform field at $\omega = \omega_0 + \Delta\omega$:

$$f(t) = N \exp\left(-\frac{t}{\tau}\right) \exp(i\,(\omega_0 t + \phi_0)) \int_{-\infty}^{\infty} g(\Delta\omega) \exp\big(i\,(\Delta\omega t)\big)\, d\Delta\omega$$

$$\int_{-\infty}^{+\infty} g(\Delta\omega) \exp\big(i(\Delta\omega t)\big)\, d\Delta\omega = C(t) + i\,S(t) = \underline{\sqrt{C^2(t) + S^2(t)}} \cdot \exp(i\, \underline{\tan^{-1}(S(t)/C(t))})$$

$$E(t) = \sqrt{C^2(t) + S^2(t)}$$

Signal Envelope

$$\phi(t) = \tan^{-1}\frac{S(t)}{C(t)}$$

Final FID model :

$$f(t) = N \exp\left(-\frac{t}{\tau}\right) E(t) \exp\big(i\,(\omega_0 t + \phi(t) + \phi_0)\big),$$

$$\Phi(t) = \omega_0 t + \phi(t) + \phi_0$$

Phase Function

Back up

Zero-Crossing

Zero-cross counting:

$$f = \frac{N_c}{\Delta t} = \frac{N_{zc} - 1}{2\left(t_{last} - t_{first}\right)}$$

N_{ZC} : the number of zero-counting within the user-defined fit window
$t_{first}(t_{last})$:the first (last) zero-crossing the time of the data in the analysis window

Back up

Run 1 Calibration Analysis Methods

Field Oscillation:

Bingzhi: fixed probes from 90 to 180 and 200 to 290 degrees.
David: 164 fixed probes from 90 to 270 degree that are under 30 ppb resolution.
Ran: Select power supply feedback probes and reading of probe 209 and 224 (drift tracking reference data)

Field Drift:

Bingzhi: ABA method
David: ABA method
Ran: Average a data point with an error bar and a time stamp equal to the middle of the measurement period for PP. Use linear interpolation or extrapolation using nearest drift correction data point for trolley

Position Correction:

Bingzhi: Barcode Method

Part 6

Back up

Run 2/3 Calibration Analysis

- Two analyzers per each Run
  - Run 2 : Bingzhi and David
  - Run 3 : Bingzhi and Chris
  - Run 4 and beyond: Chris and David

Run	Bingzhi	Chris H	David K
Run 2	V		V
Run 3	V	V	
Run 4 (Pre)		V	V
Run 4 (Post)		V	V
Run 5 (Pre)		V	V
Run 5 (Post) – Future		V	V

	Bingzhi	Chris H	David K (David Flay)
Frequency Extraction	Hilbert Transform	Hilbert Transform	Advanced zero-crossing algorithm for plunging probe
Field Oscillation	Fixed probes from 90 to 180 and 200 to 290 deg	Fixed probes from 90 to 180 and 190 to 270 deg	164 fixed probes from 90 to 270 deg that are under 30 ppb resolution
Field Drift	ABA	ABA	ABA
Framework	Root/C++ macro	Python/gm2-package	Umass framework

Back up

Simulation Tool

- The precessing magnetic moments (spins) generates an oscillating flux in the pick-up coil, and thus an oscillating voltage in it.

- **In the simulation…**
    - the flux through the pick-up coil generated by a spin needs to be calculated.
    - Then the motion (precession) of the spin is modeled with some approximations and assumptions to simplify the calculation.
    - Finally, the signals from all spins in the NMR sample are added together to form the FID output.

Simulation

Simulated FID Generation

1. Define the geometry of the probe
2. Calculate the static magnetic field generated by the coil $\vec{B}$
3. Define external field map
4. Generate spins randomly in the sample volume
5. Create an histogram of ω
6. For each spin, fill the bin that its ω_i belongs to by *S*
7. For each frequency bin *j*, generate FID vector at the sampling frequency
8. Sum over all frequencies in the frequency histogram to obtain the generated FID
9. Scale the FID to the amplitude, add baseline, add noise, and etc.

Back up

Simulation Tool

- Magnetic Field at position $\vec{r_s}$: Single-turn coil, XY plane, with current *I*

$$\vec{B}(\vec{r_s}) = \frac{\mu_0}{4\pi} \oint_C \frac{I d\vec{r_c} \times \vec{r_{cs}}}{r_{cs}^3}$$

where *C* is the contour along the coil, and $\vec{r_{cs}} = \vec{r_s} + \vec{r_c}$

- Flux : with magnetic momentum, μ

$$\Phi_{\mu C} = \vec{\mu} \cdot \vec{B}(\vec{r_s})/I$$

Back up

Systematic Uncertainty

Azimuthal Average:

$$\langle f \rangle = \frac{1}{2\pi}\int_0^{2\pi} \Omega(\varphi)\ d\varphi$$

$\Omega(\varphi)$: NMR frequency of external field

True Field measurement:

$$\omega_{meas,i} = \int_0^{2\pi} \Omega(\varphi)\, S_i(\varphi)\ d\varphi \text{ , for event } i$$

$S(\varphi)$: The sensitivity of the probe

Uncertainty:

$$\eta = \langle f \rangle - \sum_i \Psi_i \cdot \omega_{meas,i} = \frac{1}{2\pi}\int_0^{2\pi} \Omega(\varphi)\ d\varphi - \sum_i \Psi_i \cdot \int_0^{2\pi} \Omega(\varphi)\, S_i(\varphi)\ d\varphi$$

Ψ_i : Weight for each measurements due to oversampling (= $\Delta\varphi_i$: that’s what we use)

Back up

Run 1 Systematic Uncertainty

$$\boldsymbol{R'_\mu} \equiv \frac{\boldsymbol{\omega_a}}{\boldsymbol{\overline{\omega}'_p(T_r)}} \approx \frac{f_{clock}\,\omega_a^m(1 + C_e + C_p + C_{ml} + C_{pa})}{f_{calib}\langle\omega_p(x,y,\phi)\ \times M(x,y,\phi)\rangle(1 + B_k + B_q)}$$

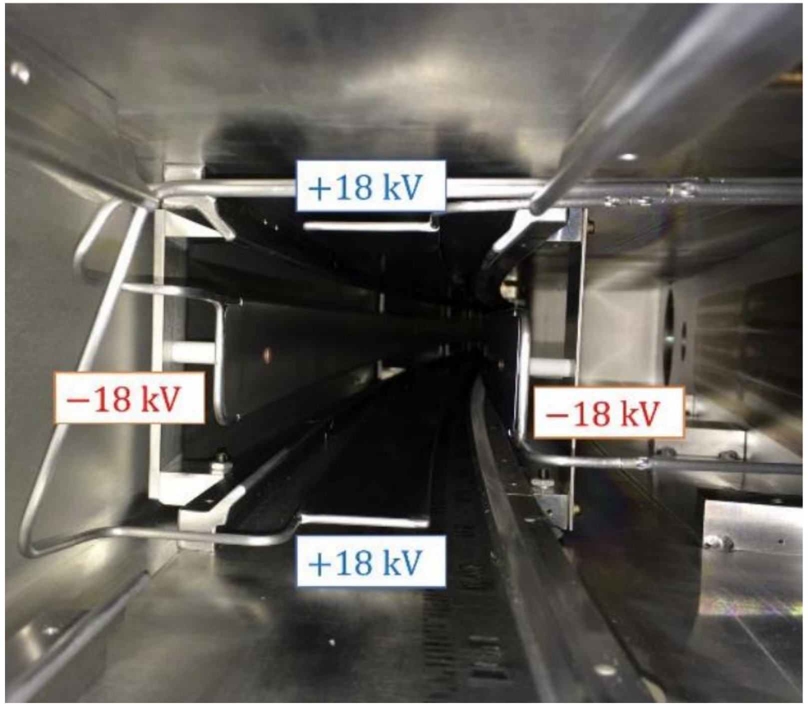
Quantity	Correction terms (ppb)	Uncertainty (ppb)
ω_a^m (statistical)	$\cdots$	434
ω_a^m (systematic)	$\cdots$	56
C_e	489	53
C_p	180	13
C_{ml}	−11	5
C_{pa}	−158	75
$f_{\text{calib}}\langle\omega_p(x,y,\phi) \times M(x,y,\phi)\rangle$	$\cdots$	56
B_k	−27	37
B_q	−17	92
$\mu'_p(34.7°)/\mu_e$	$\cdots$	10
m_μ/m_e	$\cdots$	22
$g_e/2$	$\cdots$	0
Total systematic	$\cdots$	157
Total fundamental factors	$\cdots$	25
Totals	544	462

Part 6

Back up

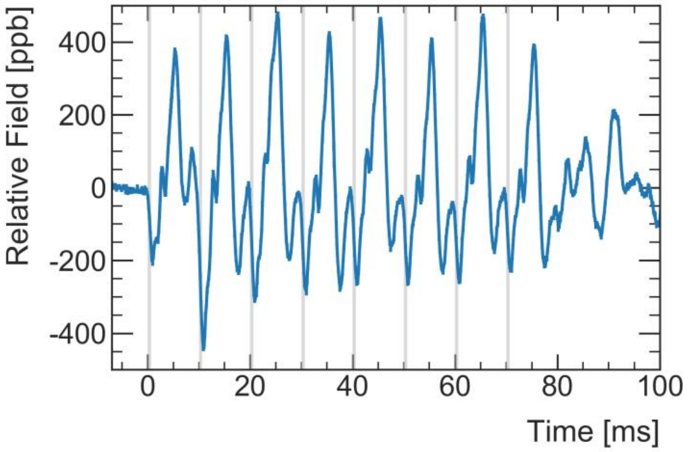
$$R'_\mu \equiv \frac{\omega_a}{\overline{\omega}'_p(T_r)} \approx \frac{f_{clock}\,\omega_a^m(1 + C_e + C_p + C_{ml} + C_{pa})}{f_{calib}\langle\omega_p(x,y,\phi)\ \times M(x,y,\phi)\rangle(1 + B_k + B_q)}$$

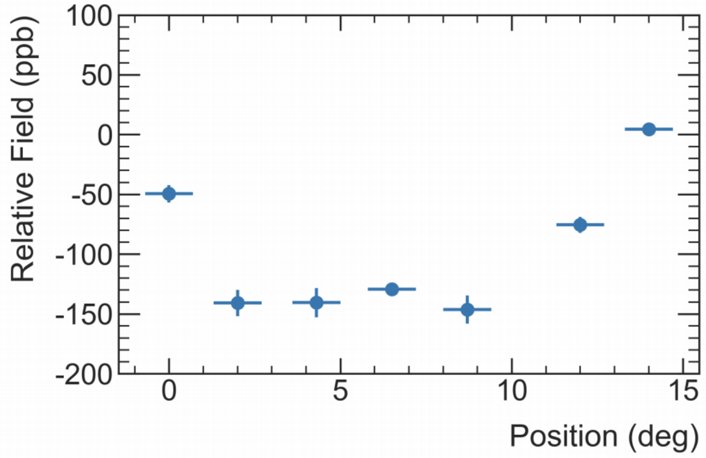
Run 1 ESQ Transient Uncertainty



Dataset	Run-1a	Run-1b	Run-1c	Run-1d
Correcti on [ppb]	-15 ± 8 3	-19 ± 1 03	-19 ± 1 03	-15 ± 83

- The ESQs are pulsed approximately every 10 ms
- Cover about 43% of the storage ring azimuth
- Mechanical vibrations of the charge plates result in a transient field
- Special NMR probe was built to measure the effect
- Measurements are mapped extensively in one quad section of the storage ring

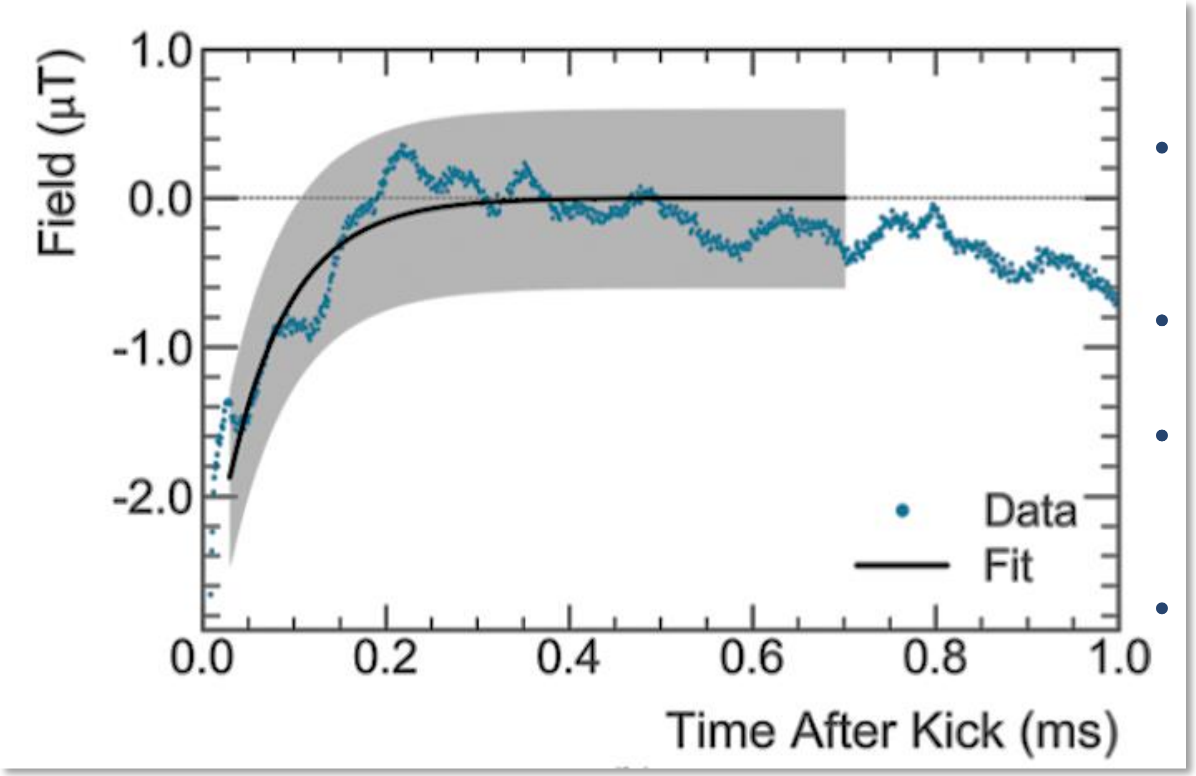




Back up

$$R'_\mu \equiv \frac{\omega_a}{\overline{\omega}'_p(T_r)} \approx \frac{f_{clock}\,\omega_a^m (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{calib}\langle \omega_p(x,y,\phi) \times M(x,y,\phi)\rangle (1 + B_k + B_q)}$$

Run 1 Kicker Uncertainty



- Kickers are operated at ~220 G and creates a 150 ns pulse that rings
- Cover about 8.5% of the storage ring azimuth
- After the pulse, eddy currents are induced and perturbs the magnetic field during the measurement period
- Correction is measured using a Faraday magnetometer designed using no metal

Dataset	Run-1
Correction [ppb]	-27 ± 37

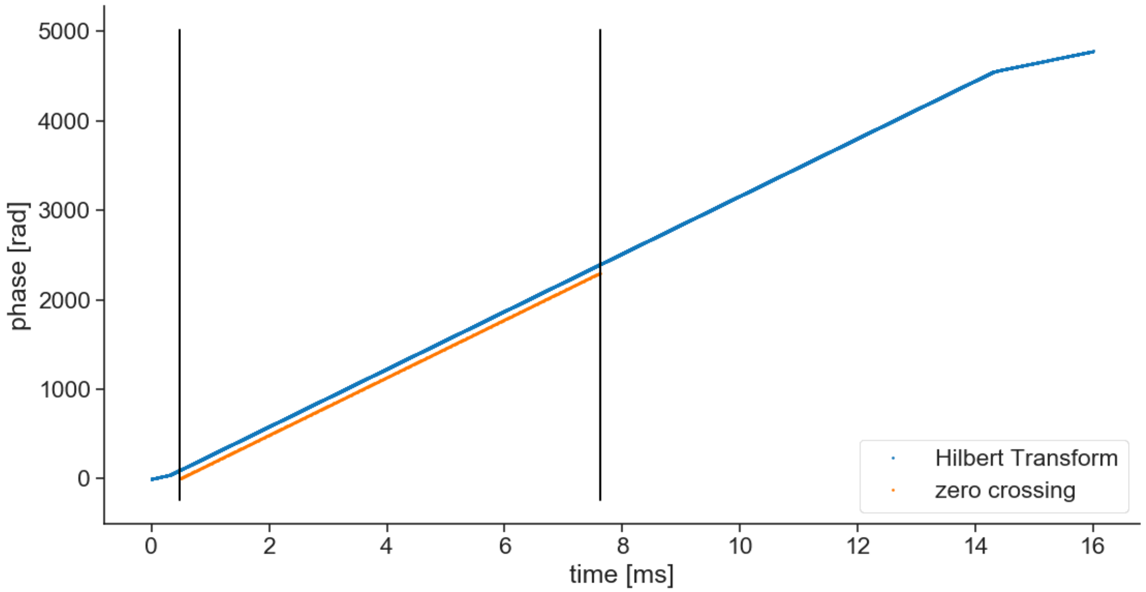
Uncertainty Analysis

Major Concern (Hilbert Transform)

- The statistical uncertainty of $\overline{\omega}$ extracting using $d\Phi/dt$ at $t = 0$.
- The accuracy of $\overline{\omega}$ extraction using using $d\Phi/dt$ at $t = 0$.
- The systematic uncertainty for azimuthally averaged field

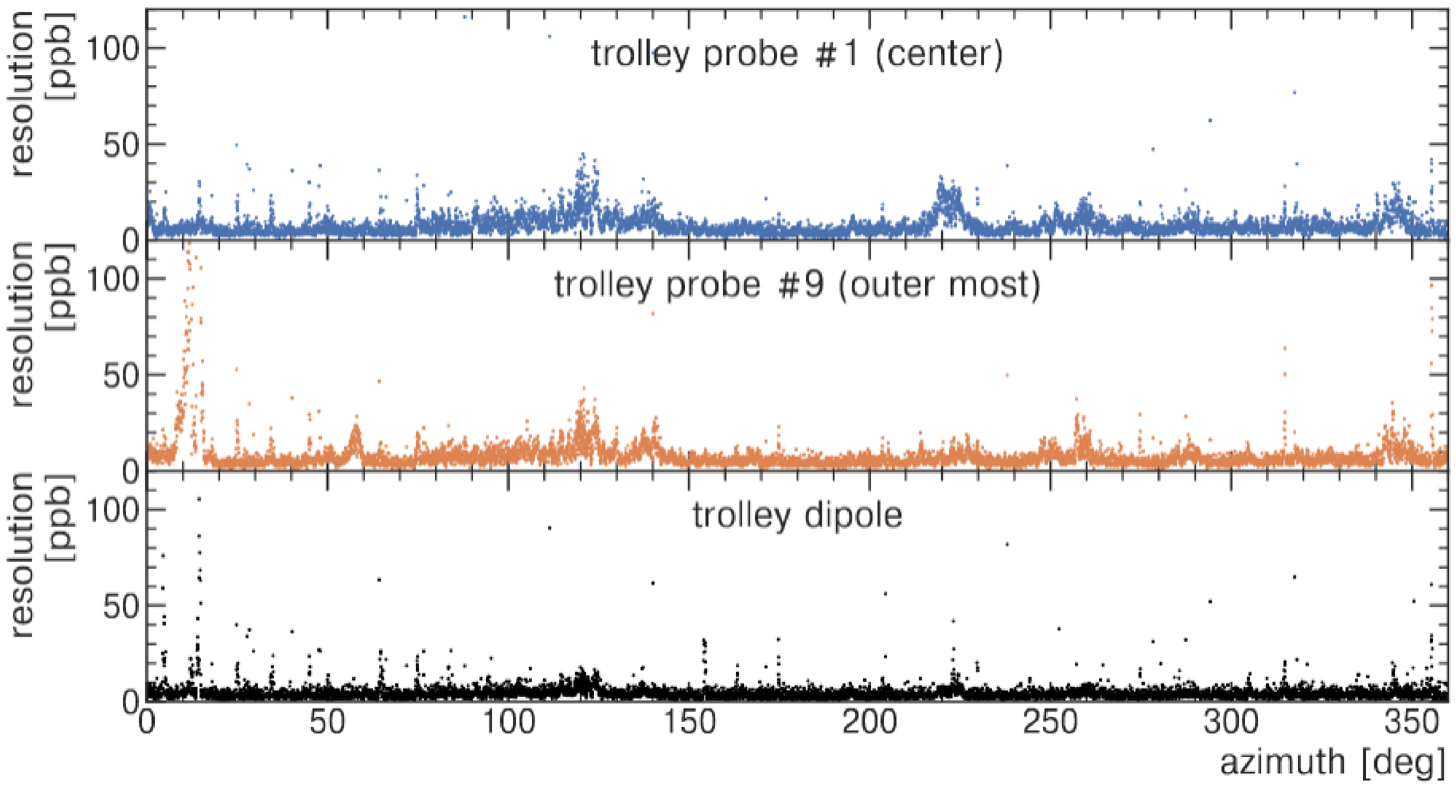
$$\omega = \left.\frac{d\Phi(t)}{dt}\right|_{t=0}$$

Φ(t) = Phase Function



Uncertainty Analysis

Statistical Uncertainty



- Depends on predominantly the length of the fit range
- Use ***stepper run***
  - Move trolley 0.05 degree and stop for a moment and measure the field
  - Get rid of any motion effect
- Beside the inflator region, dipole resolution are < 50 ppb around the ring and average resolution about 10 ppb level

Uncertainty Analysis

The Fit Accuracy

The fit accuracy : the difference between the extracted ω and true frequency ω_{truth}

- True frequency, ω_{truth} : evaluate from the **simulation tool**

$$\omega_{truth} = \int \Omega(\varphi)\; S(\varphi)\, d\varphi$$

where $\Omega(\varphi) = external\ field\ (true\ field)$ and $S(\varphi) = probe\ sensitivity\ function$

- Trolley takes about 9000 measurements around the storage ring
- Take average of 9000 events to evaluate the fit accuracy for the azimuthally averaged field

Probe Id	Fit δ (Hz)
1	0.033
2	0.048
3	0.034
4	0.029
5	0.020
6	0.029
7	0.079
8	0.039
9	0.098
10	0.036
11	0.043
12	0.051
13	0.037
14	0.025
15	0.021
16	0.048
17	0.031

Fit Accuracy for the azimuthally averaged

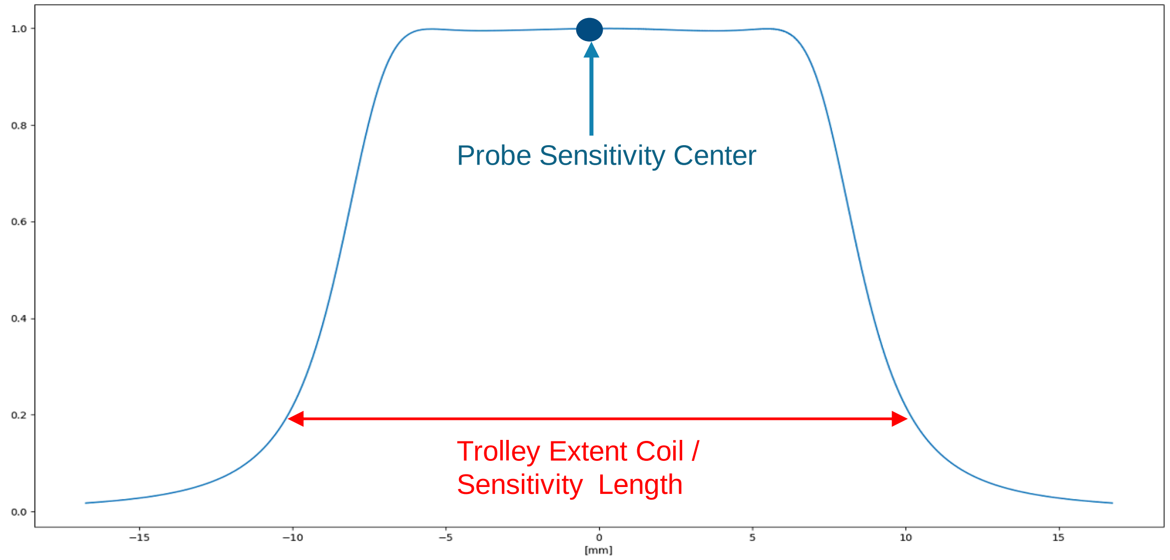
Uncertainty Analysis

The systematic uncertainty for azimuthally averaged field

- Trolley probe extent: ~ 15 mm

- Probe sensitivity exists (shown on the left).

Sensitivity function $S(\varphi)$ of Trolley Long Probe



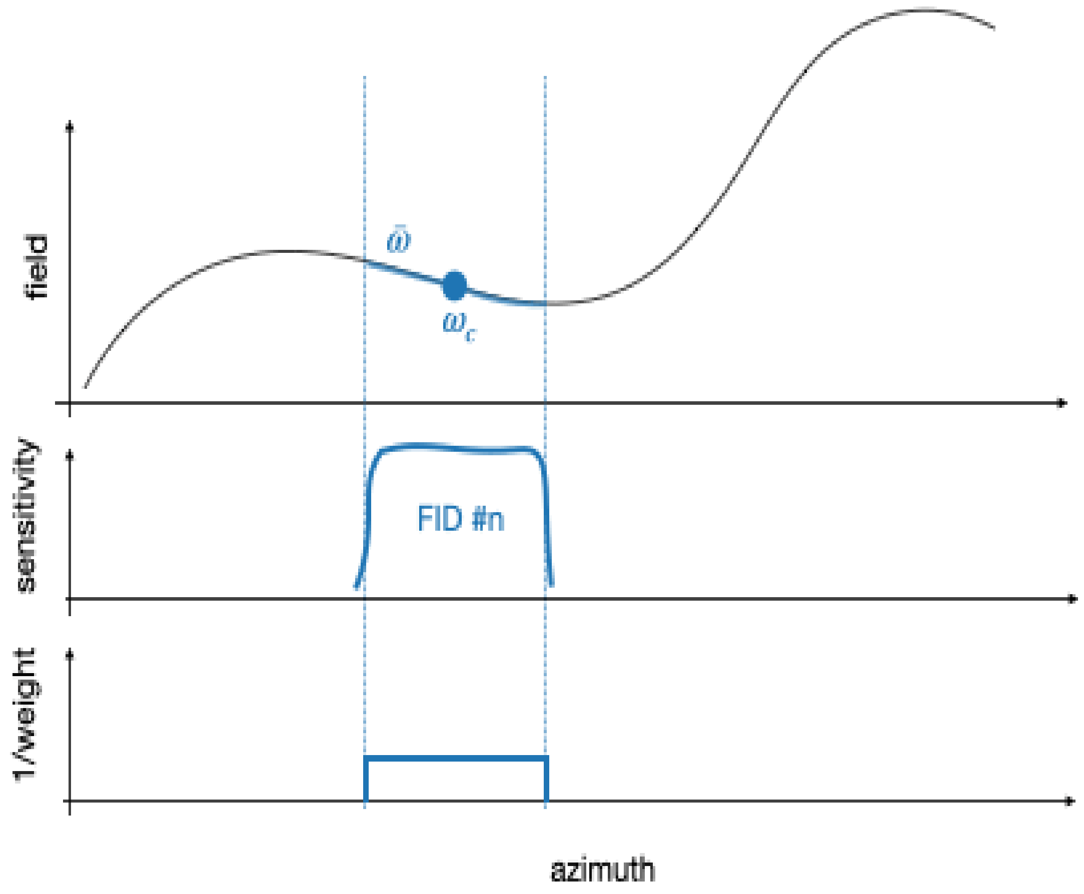
Unit: mm

Part 5

Uncertainty Analysis

The systematic uncertainty for azimuthally averaged field

- Trolley probe extent: ~ 15 mm
- Probe sensitivity exists (shown on the left)
- While trolley runs along the ring, there are some regions where trolley measures multiple times.



Animation for trolley measurements around the ring

Part 5

Uncertainty Analysis

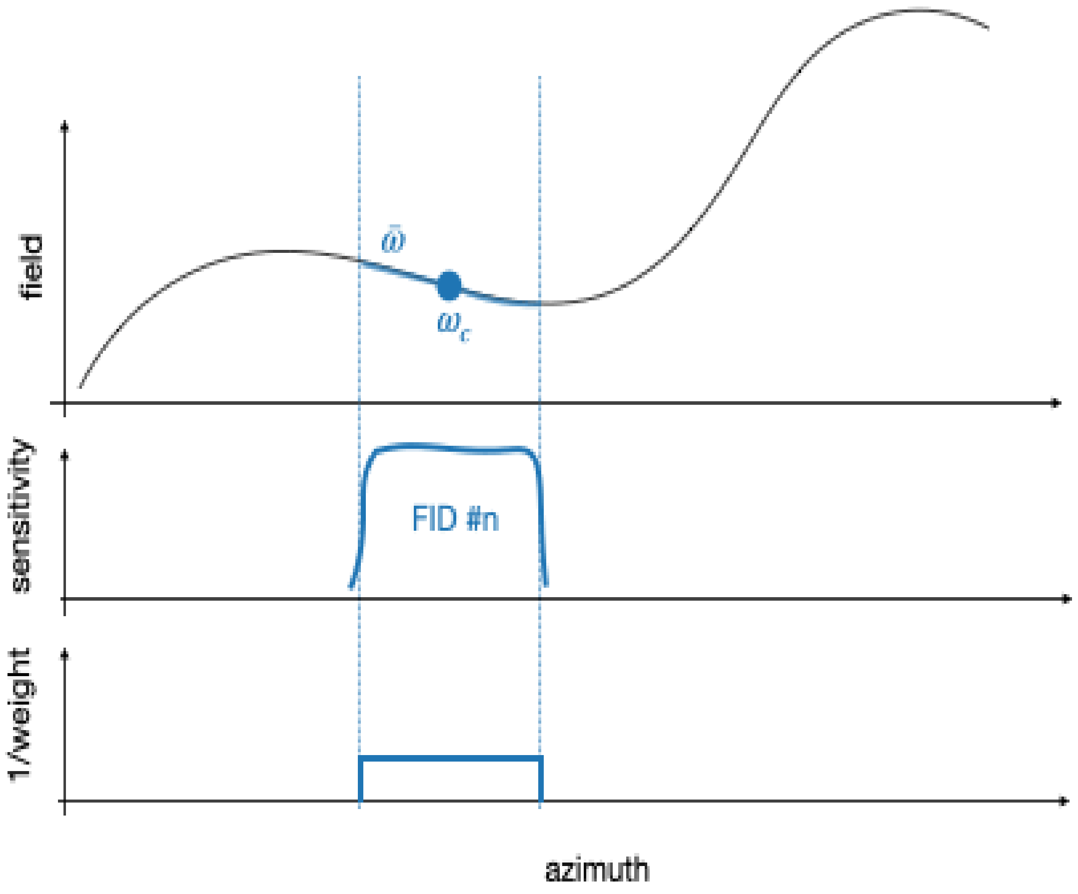
The systematic uncertainty for azimuthally averaged field

- Trolley probe extent: ~ 15 mm
- Probe sensitivity exists (shown on the left)
- While trolley runs along the ring, there are some regions where trolley measures multiple times.

The systematic uncertainty:
the systematic difference from oversampling.

Result

-0.0032 Hz $\pm$ 0.0092 Hz (0.052 ppb)



Animation for trolley measurements around the ring