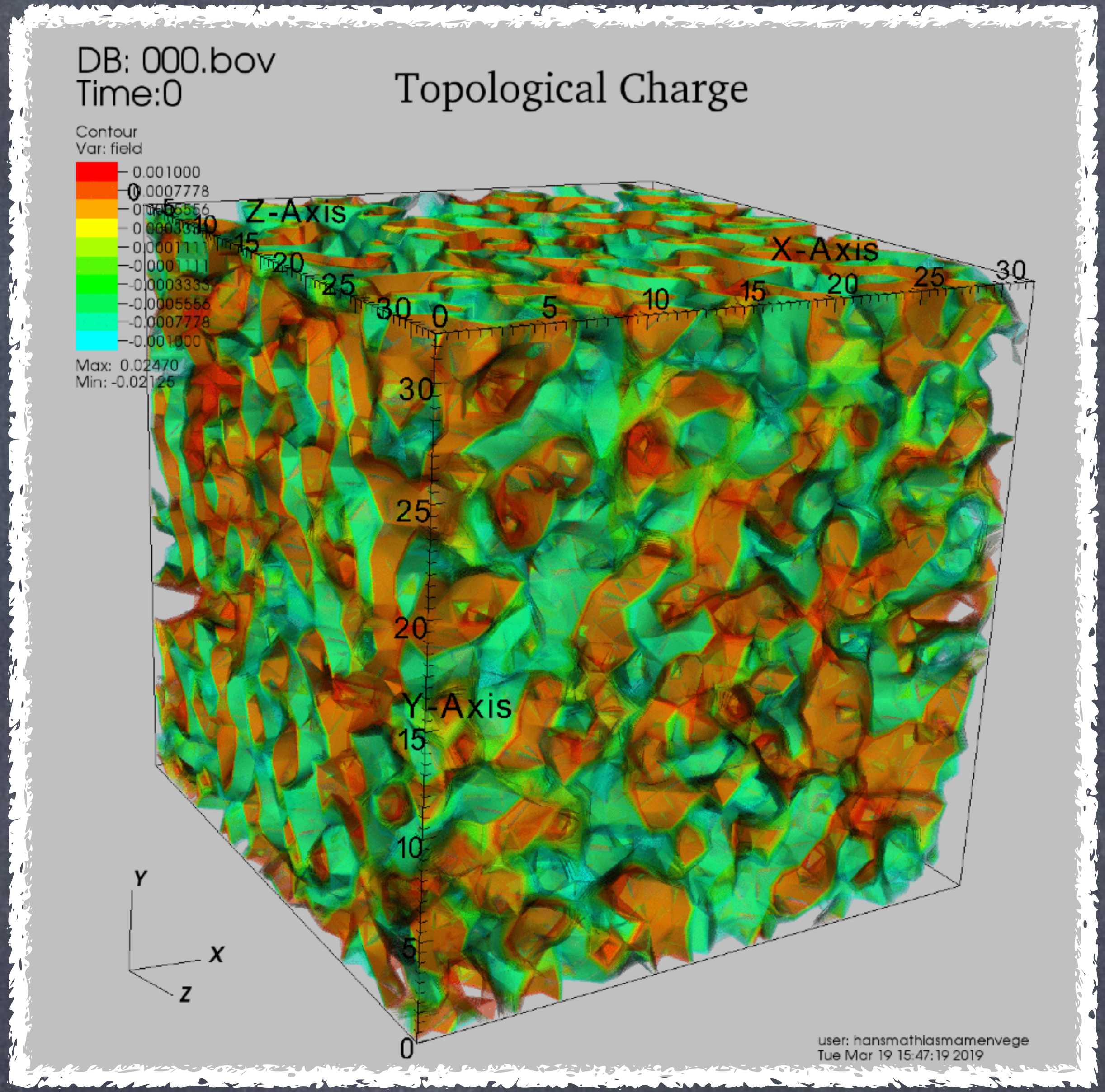
Electric Dipole Moments from Lattice QCD

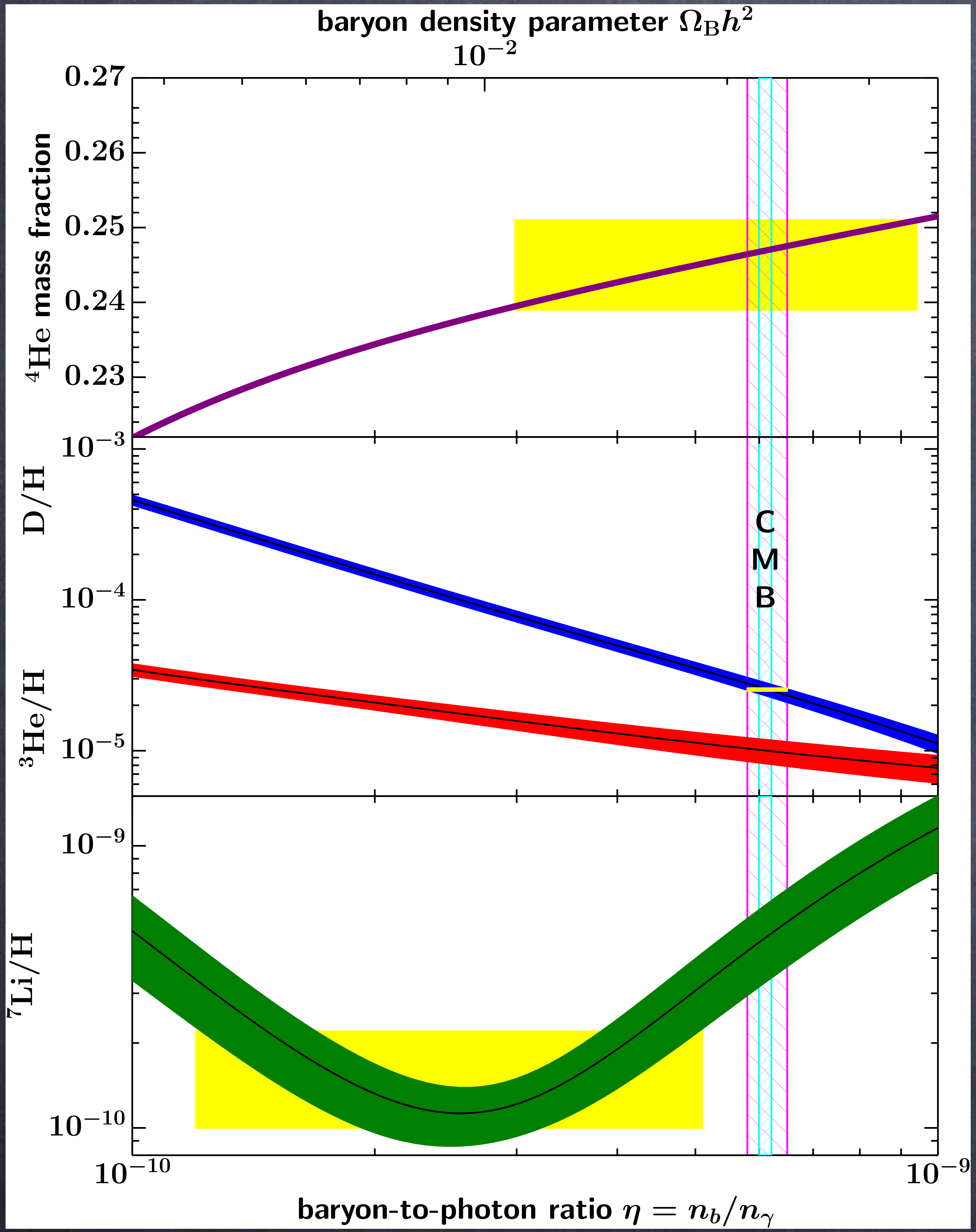
Andrea Shindler



CIPANP 2022

08.31.2022

Matter antimatter asymmetry



PDG 2021

$$\eta = \frac{n_B}{n_\gamma} \qquad n_B = n_b - n_{\bar{b}} \qquad \eta = \frac{\text{(matter) - (antimatter)}}{\text{relic photons}}$$

$$\eta = (6.143 \pm 0.190) \times 10^{-10}$$ Concordance range

$$\Omega_b h^2 = 0.02230 \pm 0.00021 \Rightarrow \eta = (6.104 \pm 0.058) \times 10^{-10}$$

PLANCK

Fields, Olive, Yeh, Young: 2020

Baryon net asymmetry ==>

Sakharov: 1967

$$X \longrightarrow Y + B$$

- Baryon number violation
- C and CP violation $\Gamma(X \longrightarrow Y + B) = \Gamma(\overline{X} \longrightarrow \overline{Y} + \overline{B})$

$$\Gamma(X \longrightarrow q_L q_L) + \Gamma(X \longrightarrow q_R q_R) = \Gamma(\overline{X} \longrightarrow \bar{q}_R \bar{q}_R) + \Gamma(\overline{X} \longrightarrow \bar{q}_L \bar{q}_L)$$

- Interactions out of thermal equilibrium

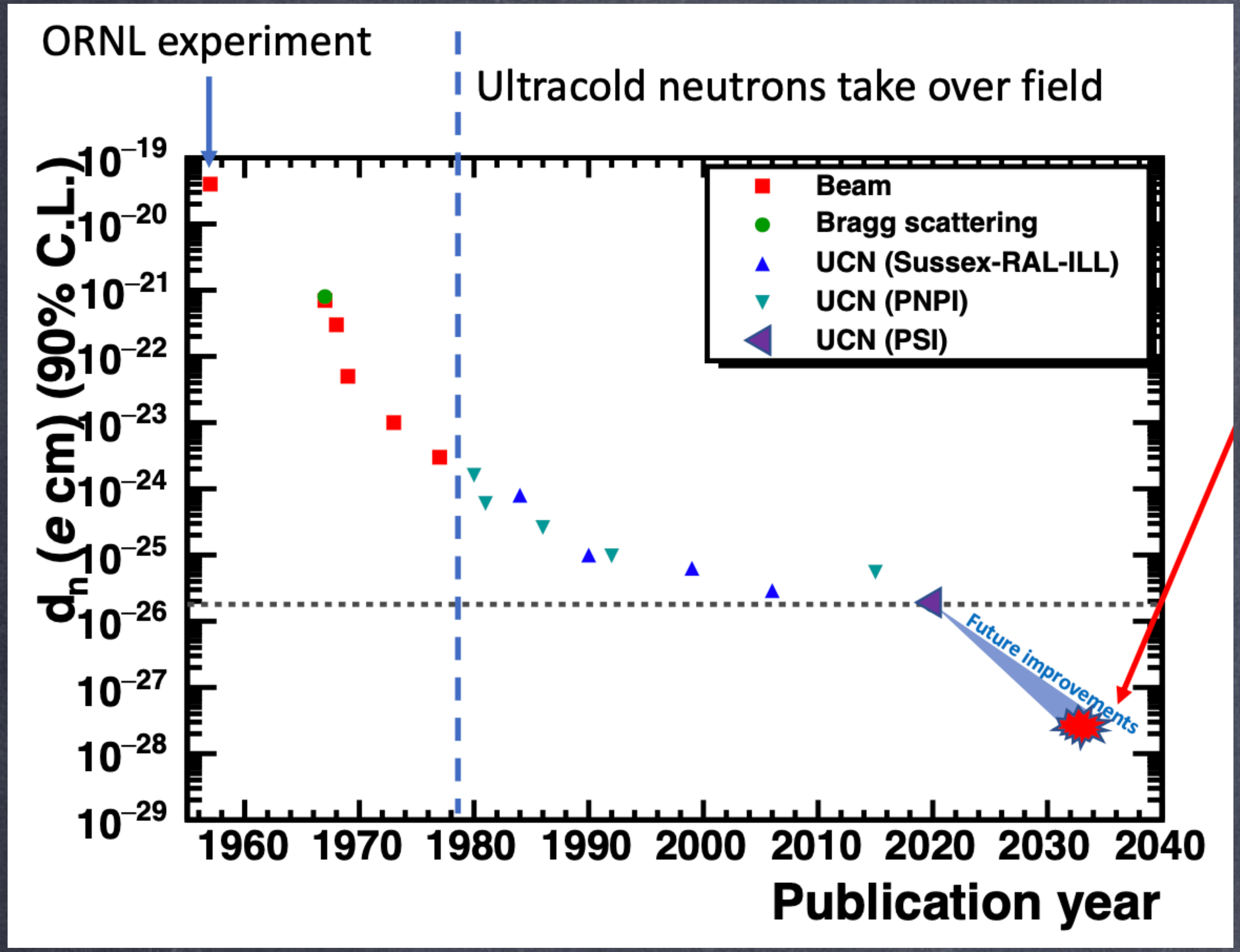
$$\eta_{SM} \sim 10^{-26}$$

Gavela, Hernandez, Orloff, Pene: 1994

Huet, Sather: 1995

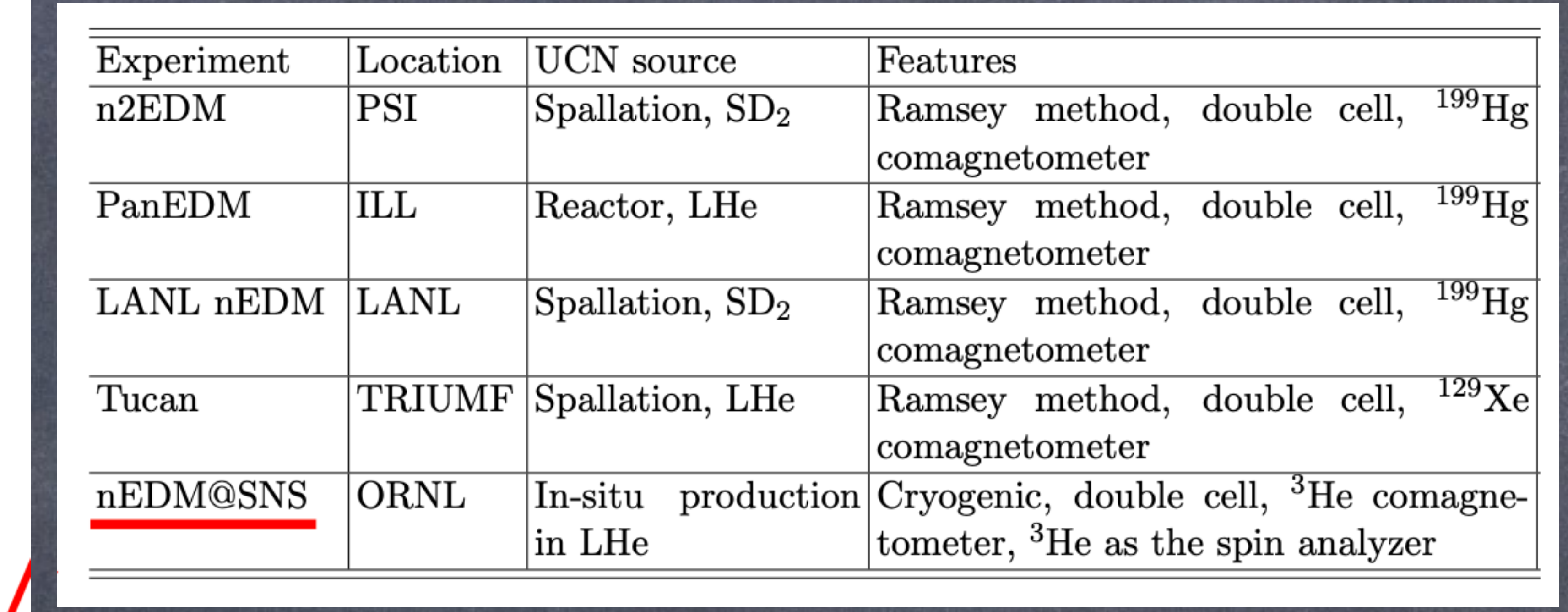
New source of CP violation

Neutron EDM



Alarcon et al.: 2022
Snowmass Summer Study Report

$$|d_n| < 1.8 \times 10^{-26}\ e\ \mathrm{cm}\ (90\%\ \mathrm{C.L.})$$



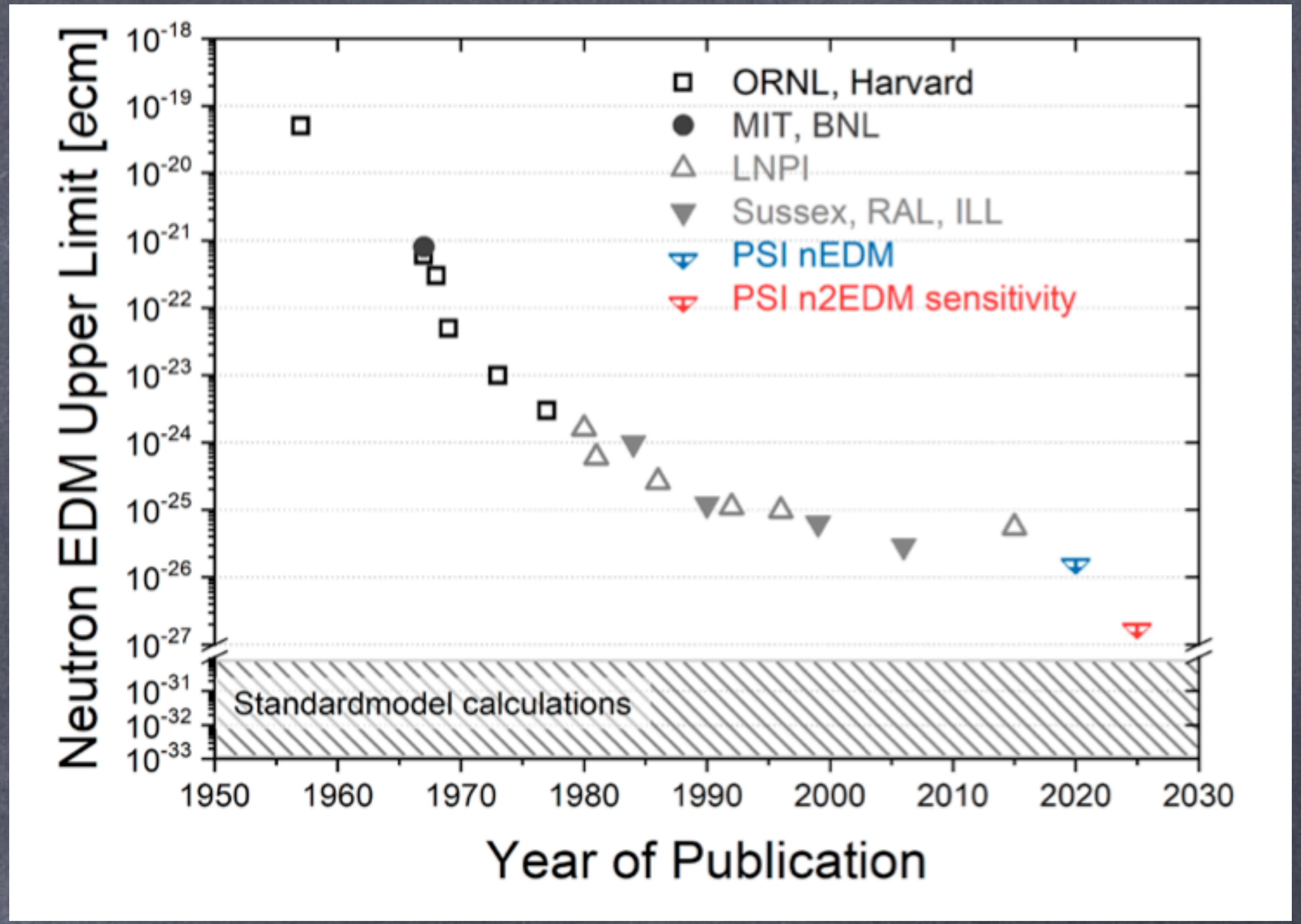
Experiment	Location	UCN source	Features
n2EDM	PSI	Spallation, SD$_2$	Ramsey method, double cell, ^{199}Hg comagnetometer
PanEDM	ILL	Reactor, LHe	Ramsey method, double cell, ^{199}Hg comagnetometer
LANL nEDM	LANL	Spallation, SD$_2$	Ramsey method, double cell, ^{199}Hg comagnetometer
Tucan	TRIUMF	Spallation, LHe	Ramsey method, double cell, ^{129}Xe comagnetometer
nEDM@SNS	ORNL	In-situ production in LHe	Cryogenic, double cell, ^{3}He comagnetometer, ^{3}He as the spin analyzer

$$\nu = -2(\mu \cdot B + d \cdot E)/h$$

$$\sigma_{d_n} \sim \frac{\hbar}{2\alpha E T_{f_p}\sqrt{N}}$$

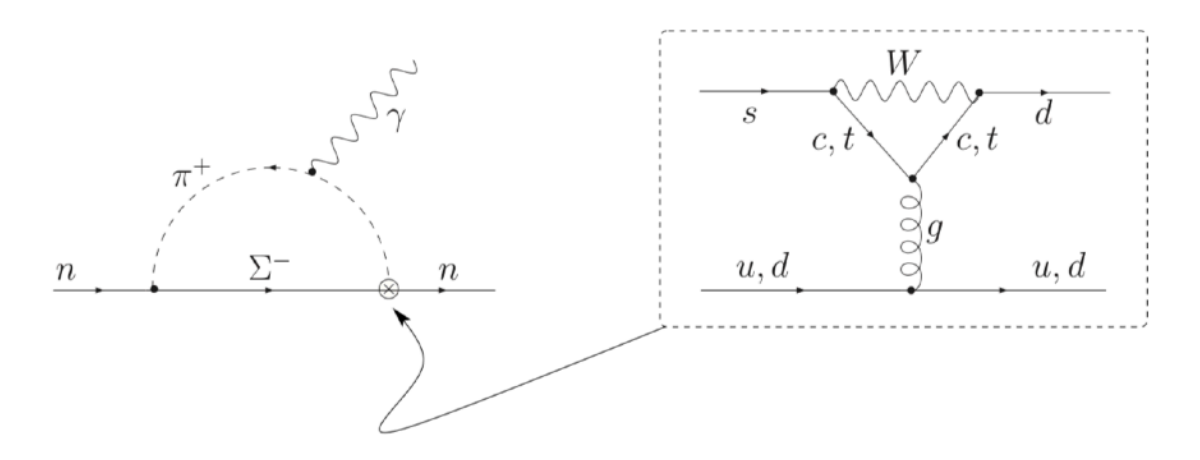
Control over systematics (factor 5)
B non-uniformity

Abel et al.: 2020
PSI

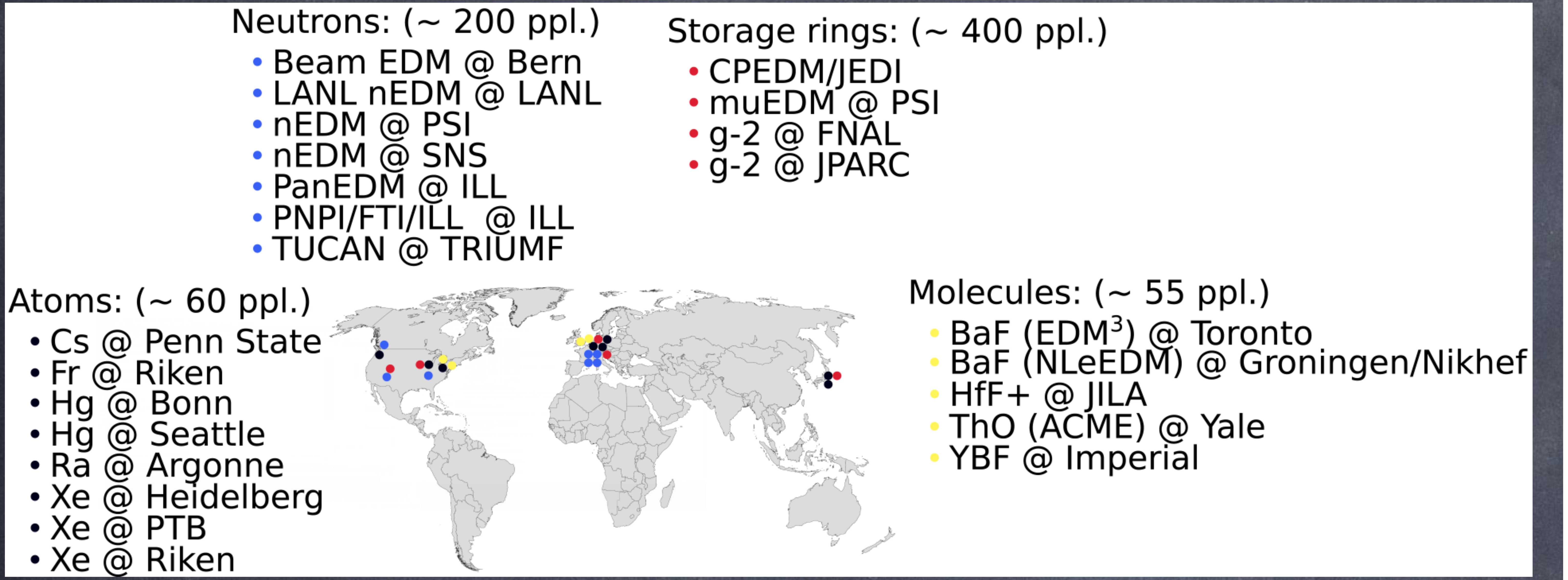


$$(d_n)_{\mathrm{SM}=(1-6)\times 10^{-32}e\quad \mathrm{cm}}$$

Shabalin: 1978-1980
Khriplovich, Zhitnitsky: 1982
Gavela et al. : 1982
Seng: 2015



EDM experiments in the world



Neutrons: (~ 200 ppl.)

- Beam EDM @ Bern
- LANL nEDM @ LANL
- nEDM @ PSI
- nEDM @ SNS
- PanEDM @ ILL
- PNPI/FTI/ILL @ ILL
- TUCAN @ TRIUMF

Storage rings: (~ 400 ppl.)

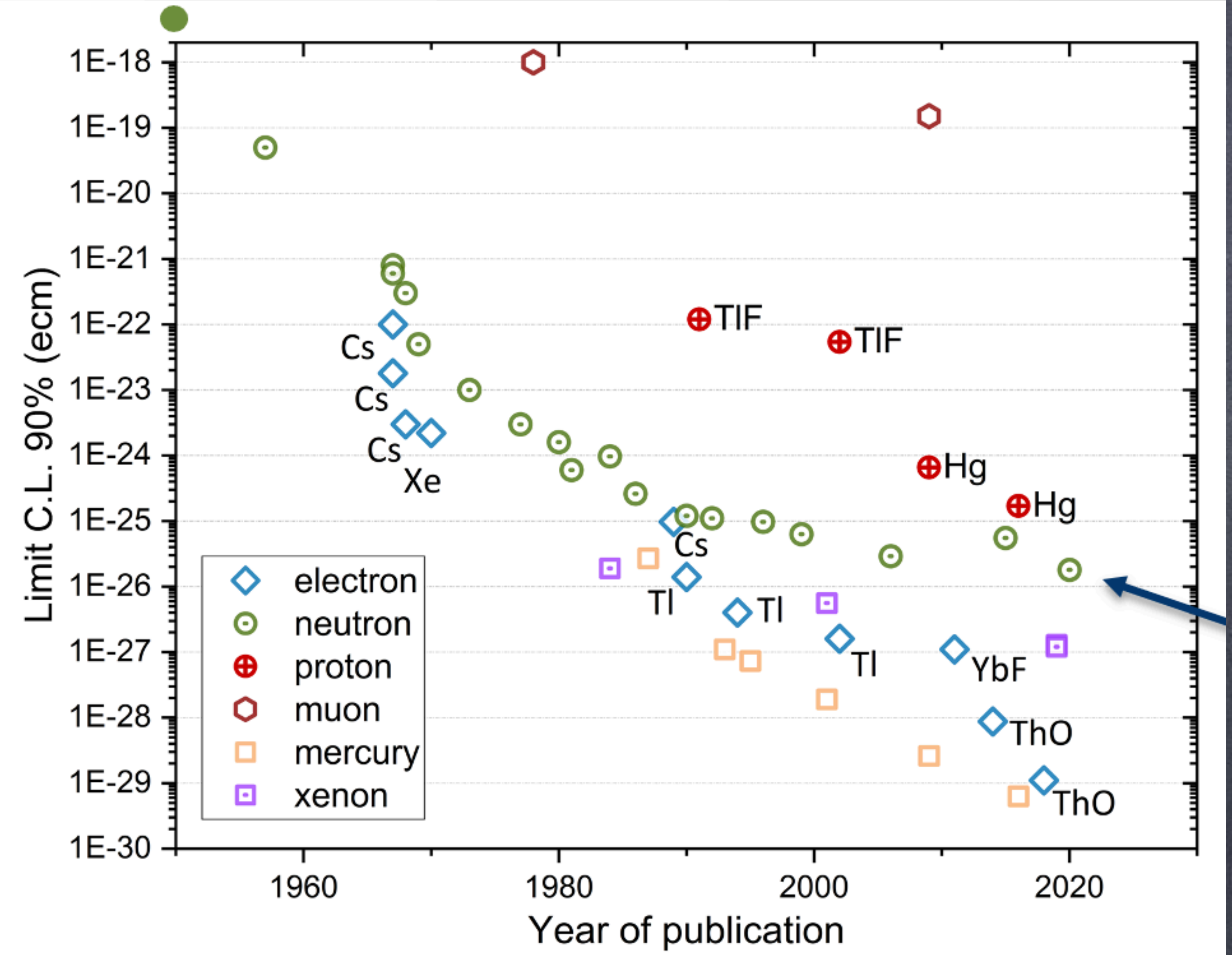
- CPEDM/JEDI
- muEDM @ PSI
- g-2 @ FNAL
- g-2 @ JPARC

Atoms: (~ 60 ppl.)

- Cs @ Penn State
- Fr @ Riken
- Hg @ Bonn
- Hg @ Seattle
- Ra @ Argonne
- Xe @ Heidelberg
- Xe @ PTB
- Xe @ Riken

Molecules: (~ 55 ppl.)

- BaF (EDM^3) @ Toronto
- BaF (NLeEDM) @ Groningen/Nikhef
- HfF+ @ JILA
- ThO (ACME) @ Yale
- YBF @ Imperial



$$|d_n| < 1.8 \times 10^{-26}\ e\ \mathrm{cm}\ (90\%\ \mathrm{C.L.})$$

Abel et al.: 2020
PSI

$$|d_{^{199}Hg}| < 7.4 \times 10^{-30}\ e\ \mathrm{cm}\ (95\%\ \mathrm{C.L.})$$

Griffith et al.: 2009
Graner et al.: 2017

$$|d_{^{225}Ra}| < 1.4 \times 10^{-23}\ e\ \mathrm{cm}\ (95\%\ \mathrm{C.L.})$$

Bishof et al: 2016

5 (10) yrs

$$|d_n| \lesssim 1 - 3 \times 10^{-27}\ e\ \mathrm{cm}\ (2 \times 10^{-28}\ e\ \mathrm{cm})$$

SNS

$$|d_{^{199}Hg}| \lesssim 5 \times 10^{-31}\ e\ \mathrm{cm}\ (5 \times 10^{-32}\ e\ \mathrm{cm})$$

Bonn

$$|d_{^{225}Ra}| \lesssim 3 \times 10^{-27}\ e\ \mathrm{cm}$$

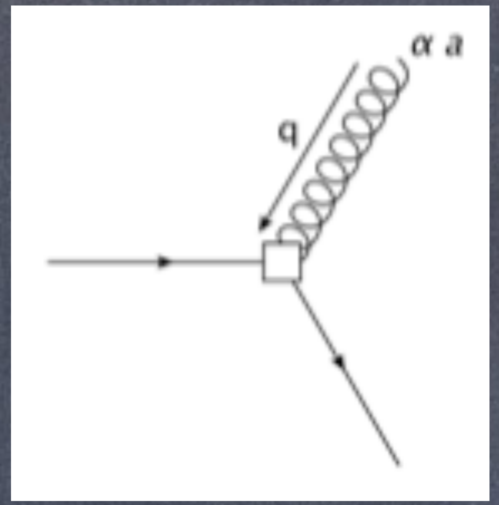
ANL

CP-violating sources

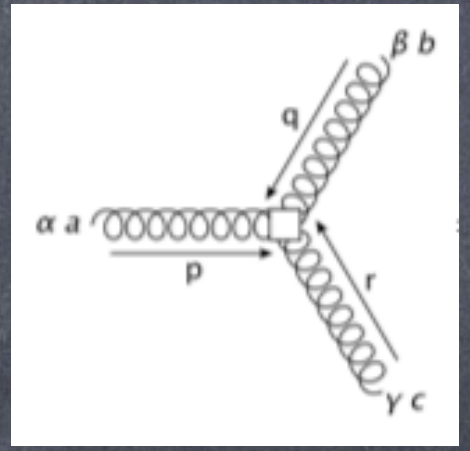
- Full list of dimension 5 and 6 operators is known

Buchmuller, Wyler: 1986
de Rujula et al.: 1991
Grzadkowski et al: 2010

$$\mathcal{O}_{\mathrm{CE}} = \sum_{f=u,d,s,\dots} \overline{\psi}_f(x)\gamma_5\sigma_{\mu\nu}G^a_{\mu\nu}T^a\psi_f(x)$$



$$\mathcal{O}_{\mathrm{gE}}(x) = \frac{1}{6} i f^{abc} G^a_{\mu\rho}(x) G^b_{\nu\rho}(x) G^c_{\lambda\sigma}(x)\epsilon_{\mu\nu\lambda\sigma}$$



Weinberg: 1989

$$\mathcal{O}_{\mathrm{qEDM}} = \sum_{f=u,d,s,\dots} \overline{\psi}_f(x)\gamma_5\sigma_{\mu\nu}\psi_f(x)F_{\mu\nu}$$

$$\mathcal{L}_{\mathrm{QCD}+\theta} = \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \sum_{f=u,d,s,\dots} \overline{\psi}_f(x)\left\{\gamma_\mu\left[\partial_\mu + gA^a_\mu T^a\right] + m_f\right\}\psi_f(x) - i\bar{\theta}q(x)$$

$$q(x) = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}\left\{G_{\mu\nu}(x)G_{\rho\sigma}(x)\right\}$$

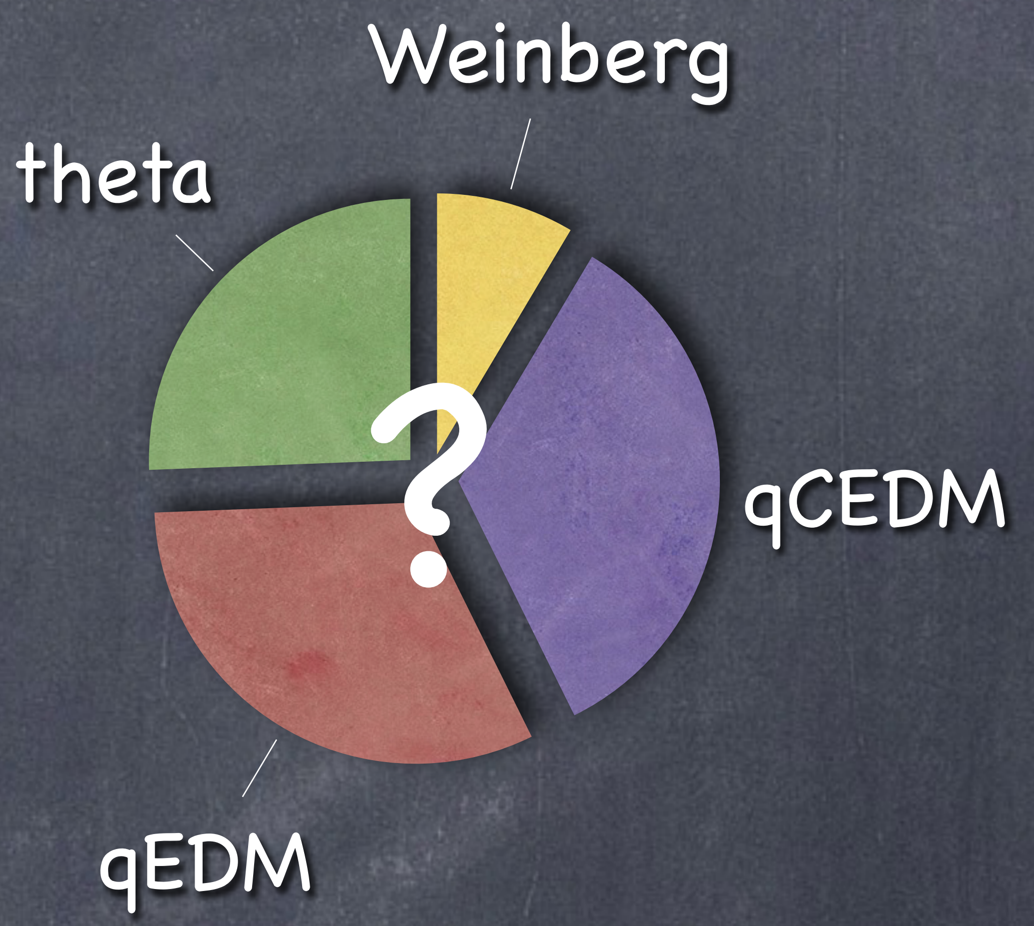
4-fermion operators

The role of lattice QCD

$$d_{\mathrm{N}} = M_{\mathrm{N}}^{\theta}\bar{\theta} + \left(\frac{v}{\Lambda}\right)^2 \sum_i M_{\mathrm{N}}^{(i)}\widetilde{d}_i \qquad \langle N|J_\mu O_{\cancel{CP}}|N\rangle \to d\,\underline{E}\cdot\underline{S}$$

M_{N}^{θ} ⟶ Hadronic matrix element topological charge

$M_{\mathrm{N}}^{(i)}$ ⟶ Hadronic matrix element CP odd operators



The role of LQCD is to provide
the renormalized matrix elements

Only in this way it is possible to
interpret experimental results and
disentangle all CP violating sources

Shintani et al.: 2005
Berruto, Blum, Orginos, Soni 2006
A.S., Luu, de Vries: 2014-2015
Guo, Meißner, et al. : 2010-
Liang, Draper, Liu, Yang
Alexandrou et al. (ETMC): 2015-2020
Abramczyk et al. : 2017-
Dragos, Kim, Luu, Monahan, Rizik, A.S., de Vries, Yousif:
2015-2021
Yoon, Bhattacharya, Cirigliano, Gupta, Mereghetti: 2015-2021

The role of lattice QCD

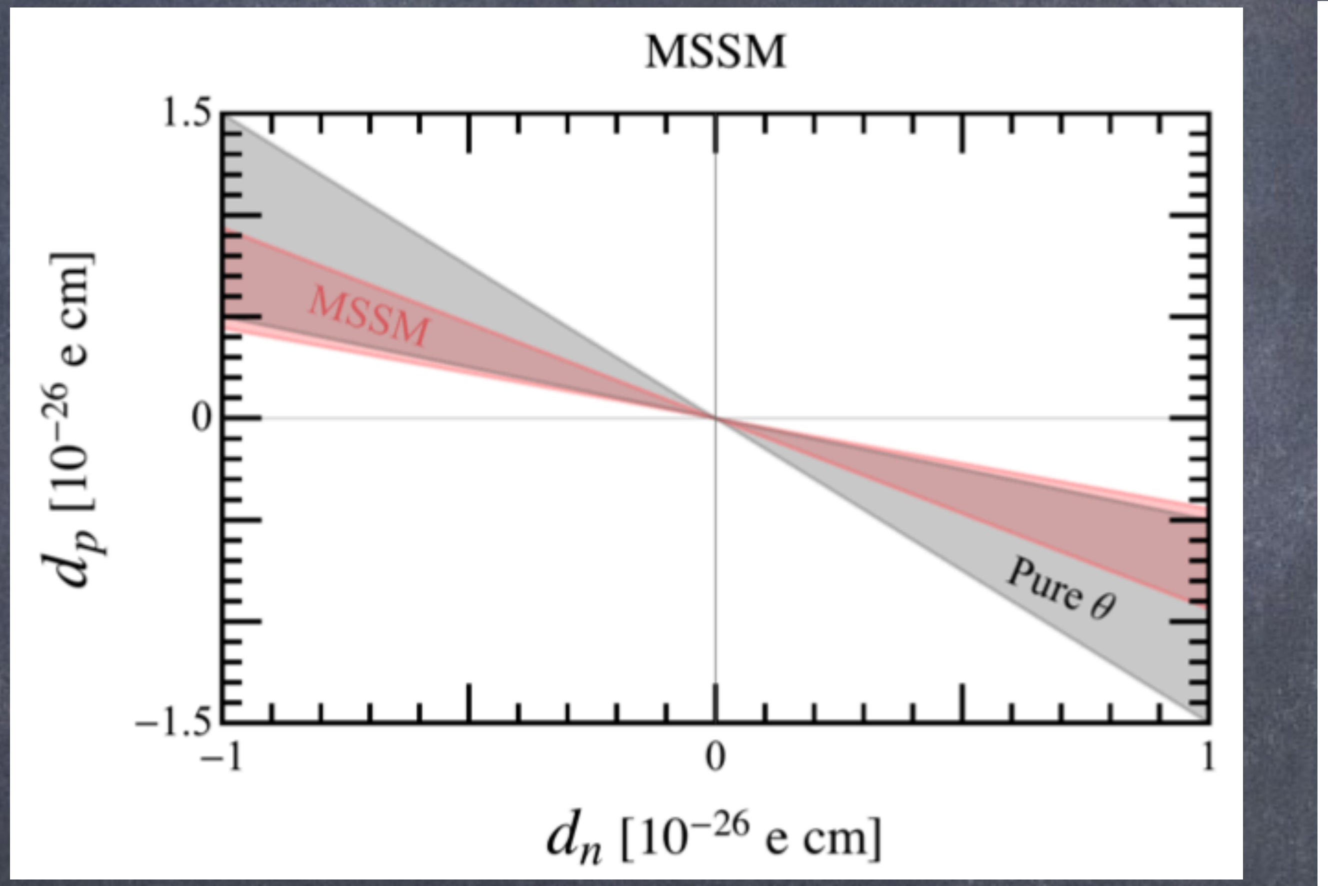
$$d_{\rm N} = M_{\rm N}^{\theta}\bar{\theta} + \left(\frac{v}{\Lambda}\right)^2 \sum_i M_{\rm N}^{(i)} \widetilde{d}_i$$

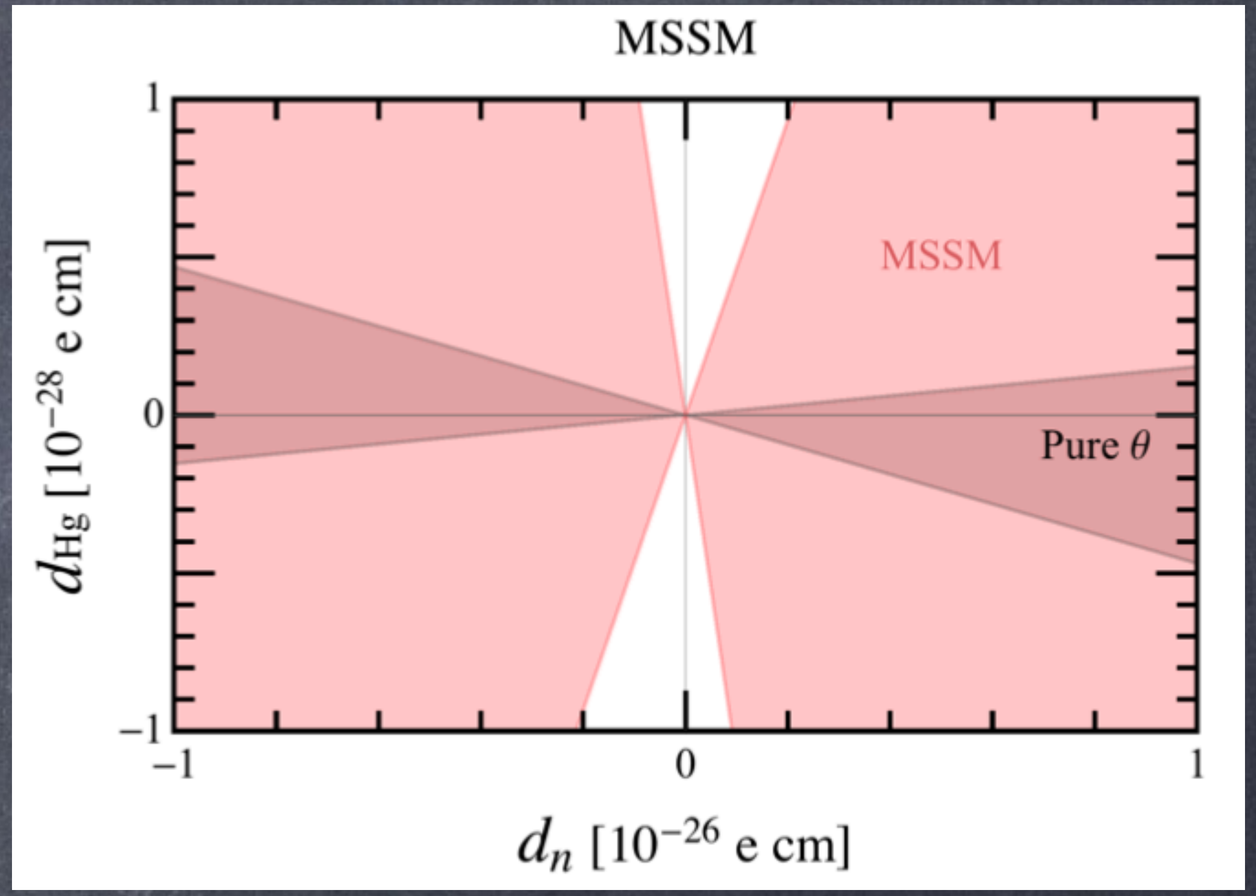
$$\begin{aligned} d_n = & - (1.5 \pm 0.7) \cdot 10^{-3}\bar{\theta} e \text{ fm} \\ & - (0.2 \pm 0.01) d_u + (0.78 \pm 0.03) d_d + (0.0027 \pm 0.016) d_s \\ & - (0.55 \pm 0.28) e\tilde{d}_u - (1.1 \pm 0.55) e\tilde{d}_d + (50 \pm 40) \text{MeV} e\tilde{d}_G \end{aligned}$$

Alarcon et al.: 2022
Snowmass Summer Study Report

- Need a portfolio of EDM experiments. Single EDM experiment not sufficient even if the LEC are correlated in a given model
- Even if only 1 CPV source is active at hadronic scales the large uncertainties dilute the nominal constraining and diagnosing power of EDM searches (excluding the quark EDM)
- To avoid cancellation at hadronic scales (dilution) need precision on matrix elements of 10-25 %

Chien et al.: 2015

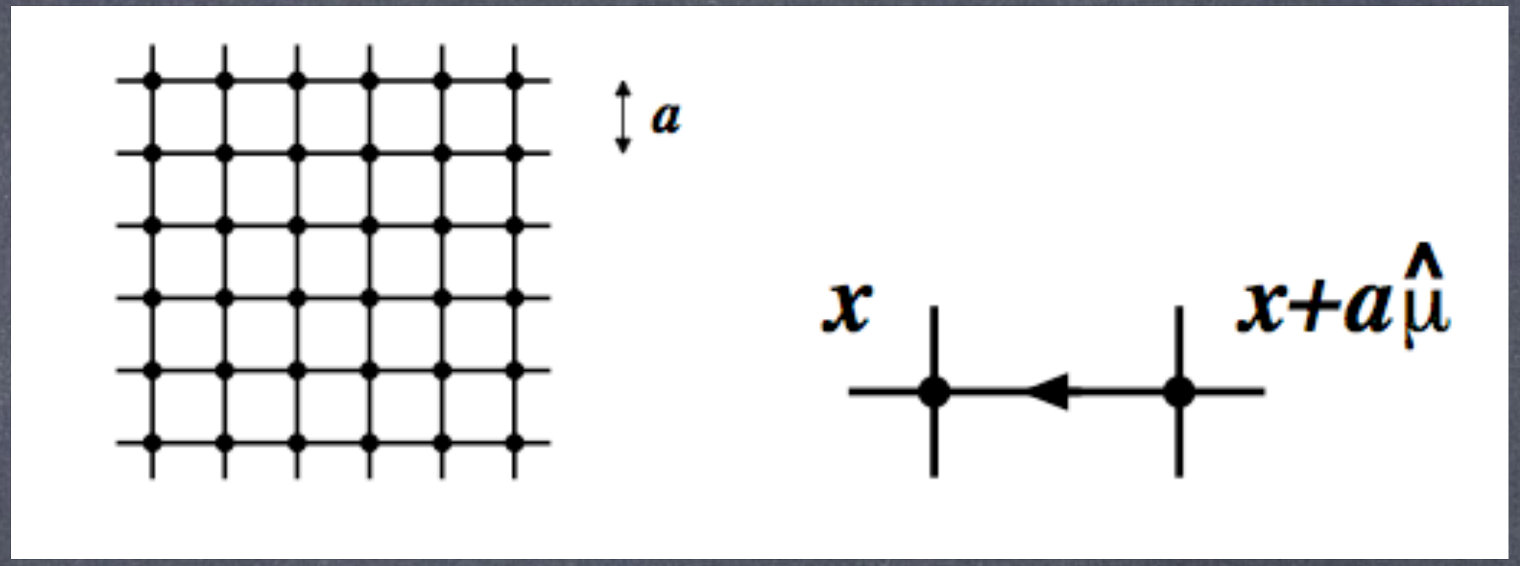


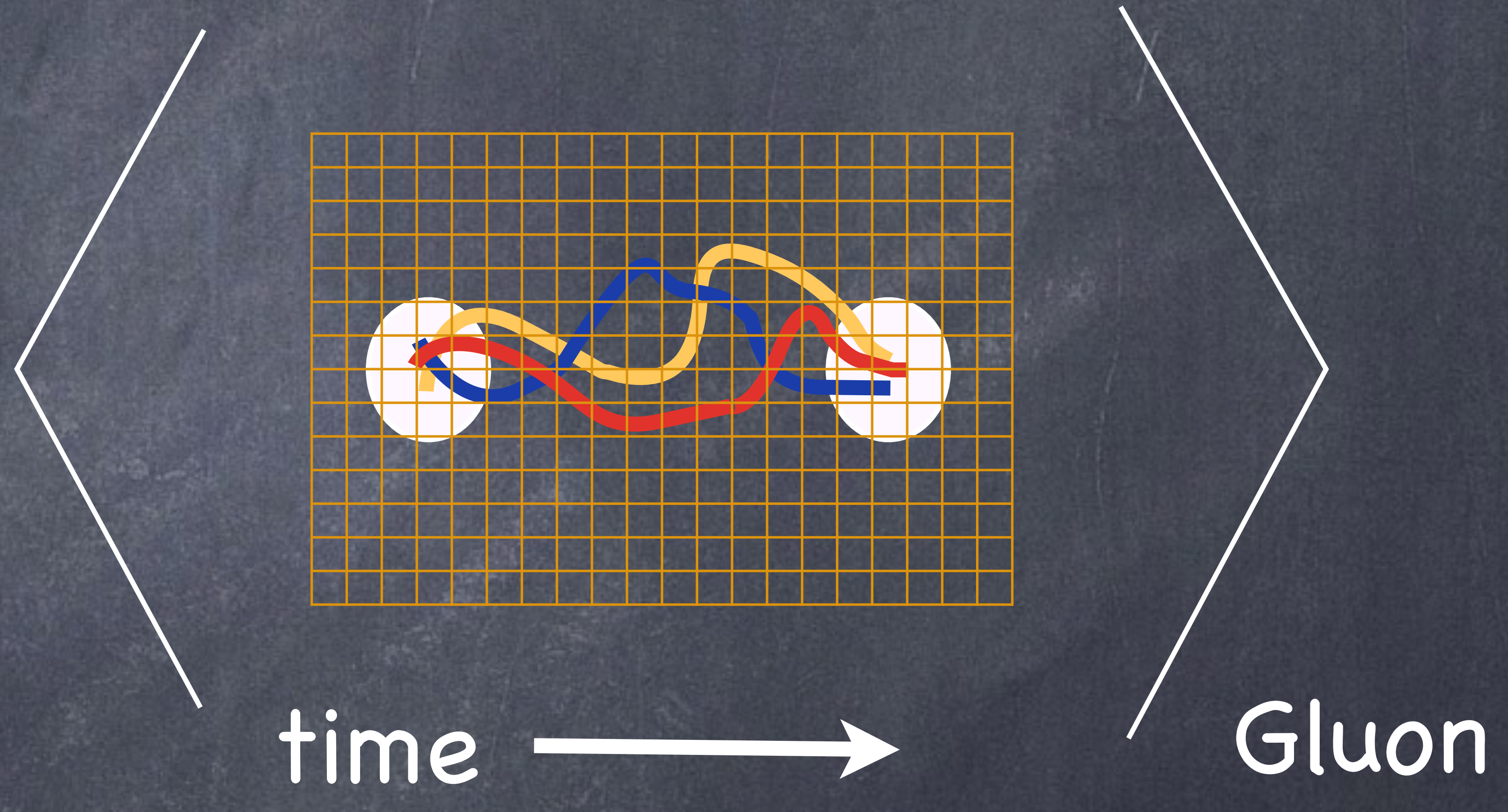


de Vries et al.: 2021
Alarcon et al.: 2022
Snowmass Summer Study Report

Challenges

- Continuum limit a —> 0. Renormalization. Power divergences. Cost $\sim \frac{1}{a^6}$
  - Gradient Flow ==> more applications
- Signal-to-noise
  - Gradient Flow + noise reduction
- Computationally challenging ==> Noise increases exponentially, and in some cases signal vanishes in the chiral limit





SYMLAT

Status

- Theta-term nucleon EDM —> first results 1409.2735 1507.02343 1809.03487 1902.03254
    - Renormalization, S/N
- Quark-chromo EDM —> renormalization
    - Power divergences —> PT 1810.05637 2005.04199 2111.1149
      Non-perturbative 1810.10301 2106.07633
    - Logs/mixing —> 2111.1149
- 3 gluon operator —> PT power divergences 2005.04199
  Preliminary studies for 1711.04730
  renormalization (power divergences) 1810.05637
  —> Logs/mixing

Collaborators:
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E. Mereghetti
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G. Pederiva
M. Rizik
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C. Monahan
(W&M - JLAB)


P. Stoffer
(U. Zurich - PSI)

Lattice QCD Status

A.S: 2021

- Quark EDM —> simplest calculation with Lattice QCD. Precision 3%-5%. No Disc.

Gupta et al.: 2018

- Theta-term nucleon EDM —> few calculations: 2 σ effect

Dragos, Luu, A.S.,
de Vries, Yousif: 2019

Alexandrou et al.
(ETMC): 2021

Yoon, Bhattacharya, Cirigliano,
Gupta, Mereghetti: 2021

- Quark-chromo EDM —> No Lattice QCD calculation, but now new promising approach —> first results on renormalization

Rizik, Monahan, A.S.: 2018-2020
A.S.: 2020
Kim, Luu, Rizik, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer: 2021

- 3 gluon operator —> No Lattice QCD calculation, but now new promising approach

Rizik, Monahan, A.S.: 2018-2020
A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer

Theta-term

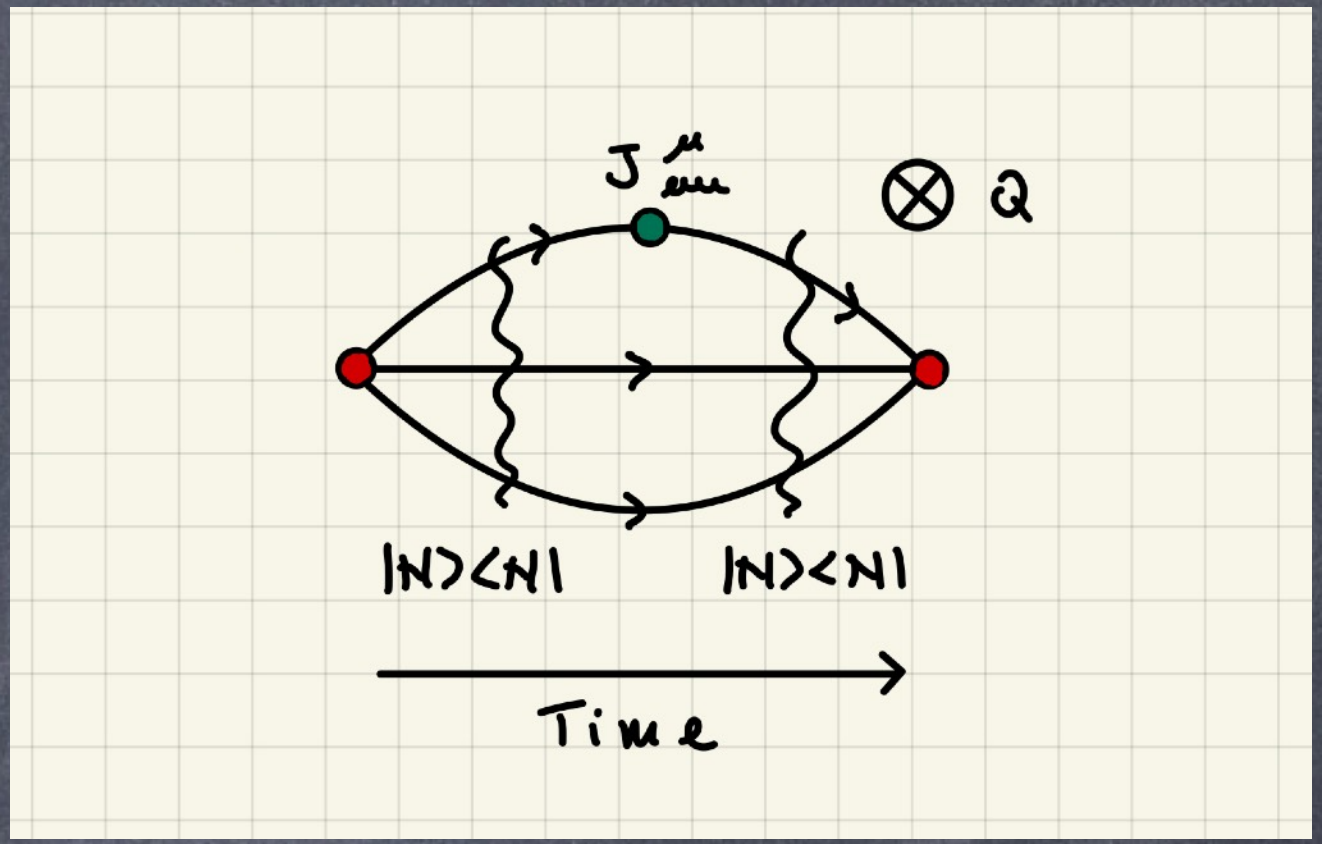
EDM from θ-term

$$\langle N^\theta(\underline{p}', s')|J^{\text{em}}_\mu|N^\theta(\underline{p}, s)\rangle = \overline{u}_N(\underline{p}', s')\Gamma^{\bar\theta}_\mu(q^2)u_N(\underline{p}, s)$$

$$G^\theta_{NJ_\mu N} = \langle N(y_0,\underline{p}_2)J^\mu_{\text{em}}(x_0,\underline{q})N^\dagger(0,\underline{p}_1)\rangle_\theta$$

$$\begin{aligned}\Gamma^{\bar\theta}_\mu(q^2) &= h(\bar\theta^2)\left[F_1(q^2)\gamma_\mu + \frac{i}{2M_N}F_2(q^2)\sigma_{\mu\nu}q_\nu\right] \\ &+ i\bar\theta g(\bar\theta^2)\frac{1}{2M_N}\underline{F_3(q^2)}\sigma_{\mu\nu}\gamma_5 q_\nu\end{aligned}$$

Abramczyk et al. : 2019



$$\boxed{|d_N| = F_3(0)/2M_N}$$

q=p'-p

$$\mathrm{e}^{-S} \simeq \mathrm{e}^{-S_{\text{QCD}}}\,[1 + i\theta Q]$$

Shintani et al.: 2005
Berruto, Blum, Orginos, Soni: 2005

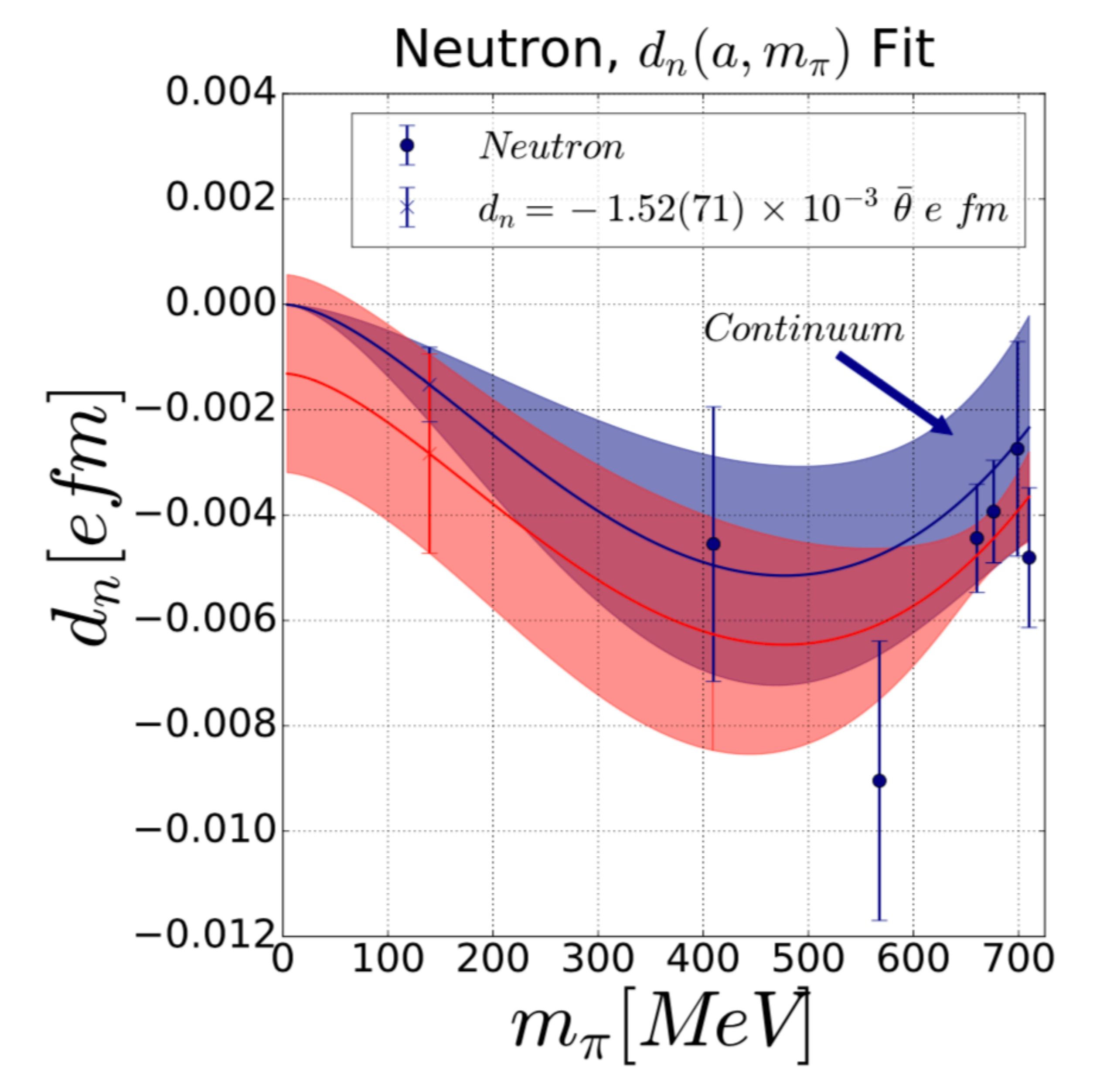
$$\langle\mathcal{O}\rangle_{\bar\theta} \simeq \langle\mathcal{O}\rangle_{\bar\theta=0} + i\bar\theta\,\langle\mathcal{O}\;Q\rangle_{\bar\theta=0} + O(\bar\theta^2) \qquad Q = \int d^4x\; q(x)$$

Problem: definition of Q on the lattice –> gradient flow or count zero modes (chiral symmetry)

Neutron EDM

Dragos, Luu, A.S., de Vries, Yousif: 2019

A.S: 2021

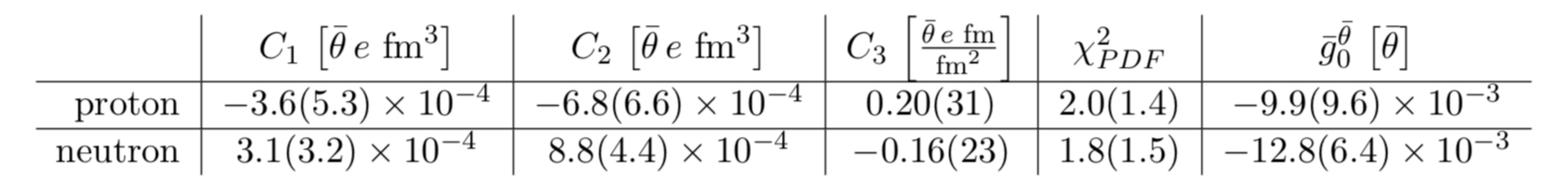


$$d_n(\bar{\theta}) = \bar{d}_n - \frac{e g_A \bar{g}_0^{\bar{\theta}}}{8\pi^2 F_\pi} \ln \frac{m_\pi^2}{M_N^2}$$

Ottnad et al.: 2010

Mereghetti et al.: 2011

$$d_{n/p}(a, m_\pi) = C_1^{n/p} m_\pi^2 + C_2^{n/p} m_\pi^2 \ln \frac{m_\pi^2}{M_N^2} + C_3^{n/p} a^2$$



	$C_1\ [\bar{\theta}\, e\ \mathrm{fm}^3]$	$C_2\ [\bar{\theta}\, e\ \mathrm{fm}^3]$	$C_3\ \left[\frac{\bar{\theta}\, e\ \mathrm{fm}}{\mathrm{fm}^2}\right]$	χ^2_{PDF}	$\bar{g}_0^{\bar{\theta}}\ [\theta]$
proton	$-3.6(5.3) \times 10^{-4}$	$-6.8(6.6) \times 10^{-4}$	$0.20(31)$	$2.0(1.4)$	$-9.9(9.6) \times 10^{-3}$
neutron	$3.1(3.2) \times 10^{-4}$	$8.8(4.4) \times 10^{-4}$	$-0.16(23)$	$1.8(1.5)$	$-12.8(6.4) \times 10^{-3}$

$$\boxed{d_n^{\mathrm{phys}} = -0.00152(71)\ \bar{\theta}\ e\ \mathrm{fm}}$$

$$|\bar{\theta}| < 1.98 \times 10^{-10} (90\%\mathrm{CL})$$

$$\bar{g}_0^{\bar{\theta}} = -1.28(64) \cdot 10^{-2} \bar{\theta}$$

$$\bar{g}_0^{\bar{\theta}} = -1.47(23) \cdot 10^{-2} \bar{\theta}$$

Crewther et al.: 1980

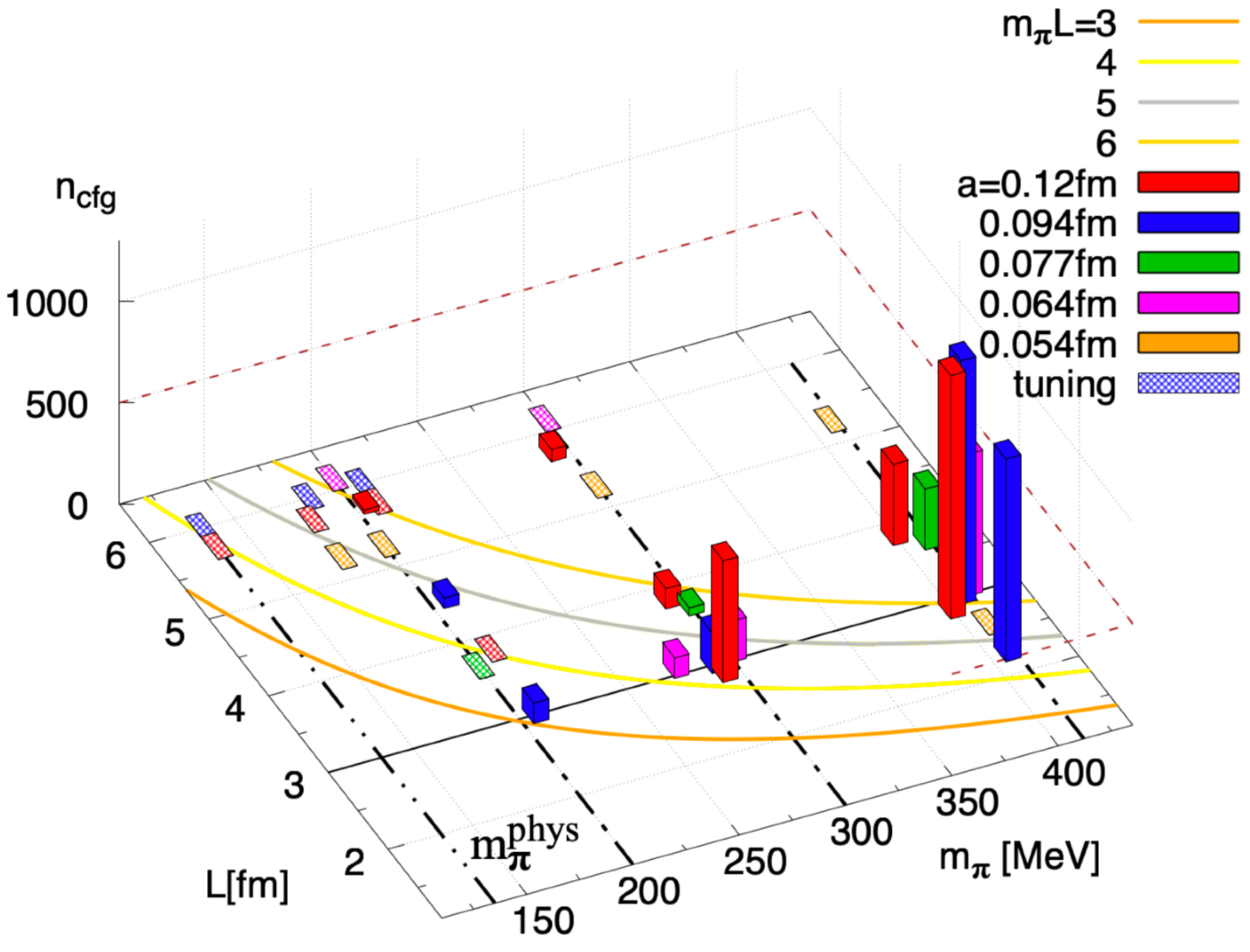
de Vries et al.: 2015

Ab-initio determination of $\bar{g}_0^{\theta}$

Future with Open Science

- OpenLat: open science initiative. Gauges with SWF open to the whole community

Cuteri, Francis, Fritzsch, Orginos, Pederiva, Rago, A.S., Walker-Loud, Zafeiropoulos

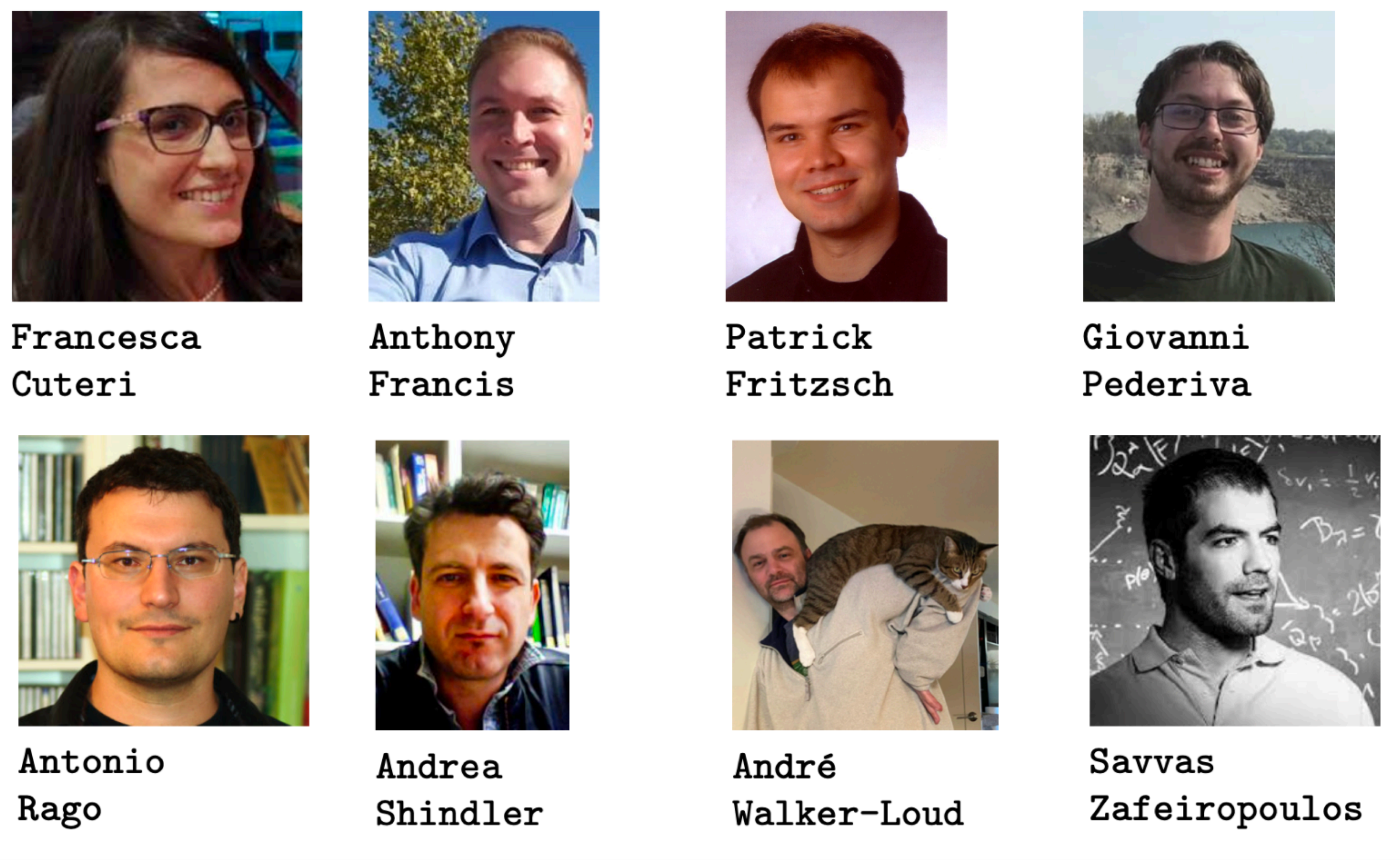


https://openlat1.gitlab.io

PRACE

Neutron EDM with Stabilized Wilson Fermions: the theta term

78.5 M core-h CPU (Irène Joliot Curie)
134 M core-h GPU (Piz Daint)



Francis, Kim, Luu,
Pederiva, A.S.,
Zafeiropoulos

Neutron EDM & Schiff moment

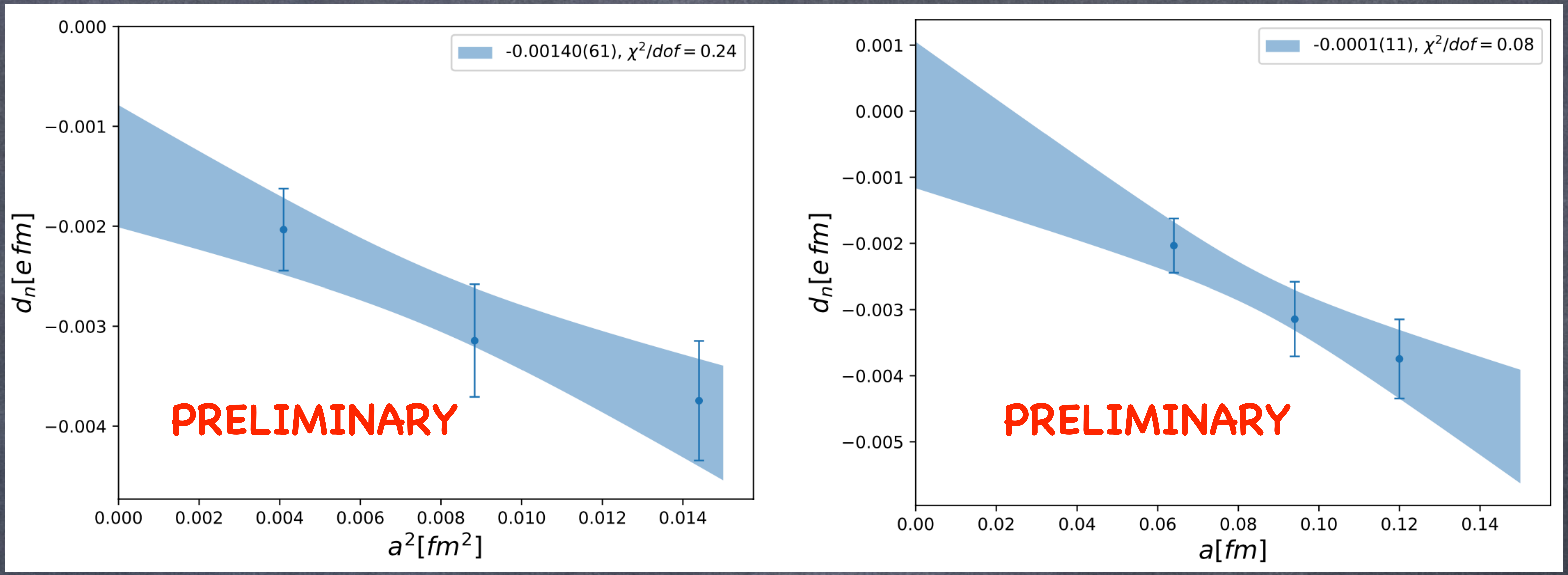
m_π = 400 MeV
t/t_0 = 1.9
a = 0.065-0.12 fm
c_V still missing

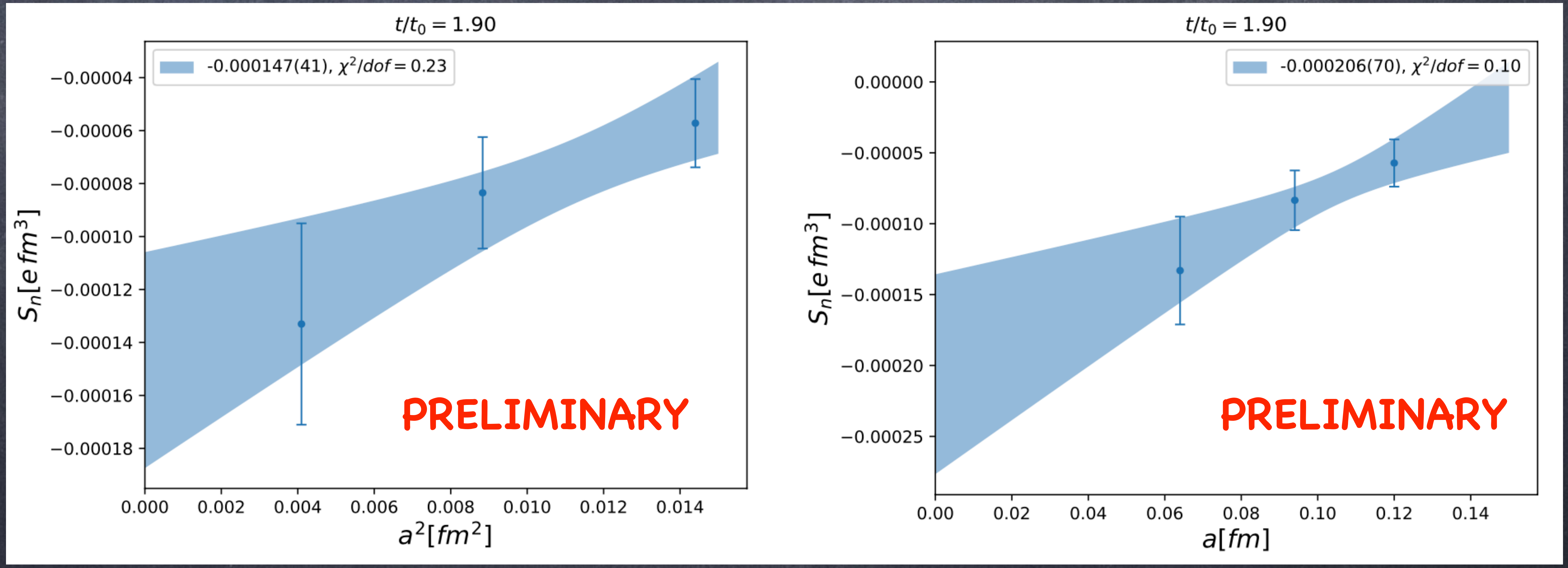
Systematics

- Excited state contamination
- Finite volume effects
- Pion mass dependence
- Different definition of charge

$$|S_N| = 1.7(3) \times 10^{-4} \bar{\theta}\ e\ \mathrm{fm}^3$$

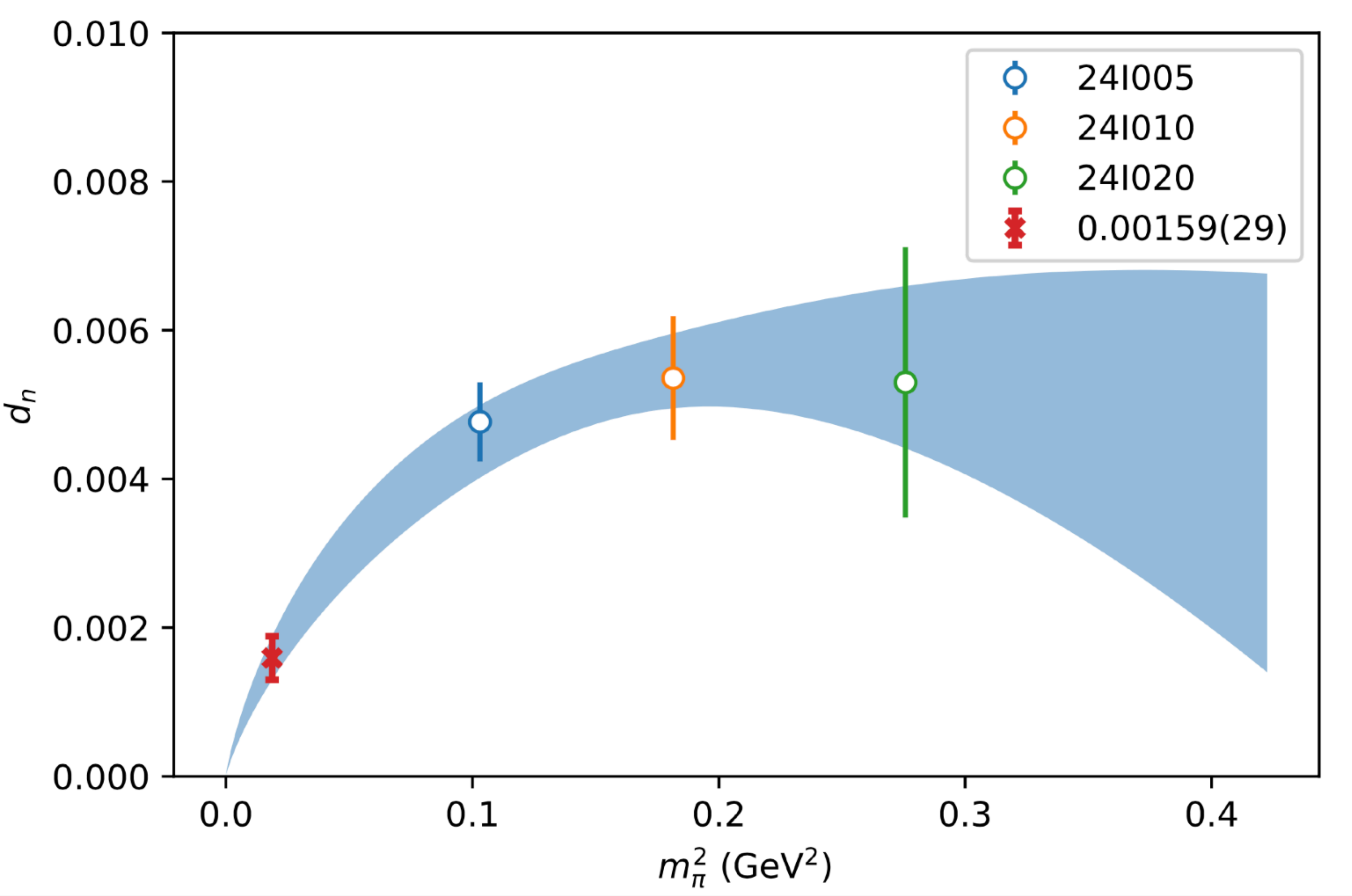
Mereghetti et al.: 2011

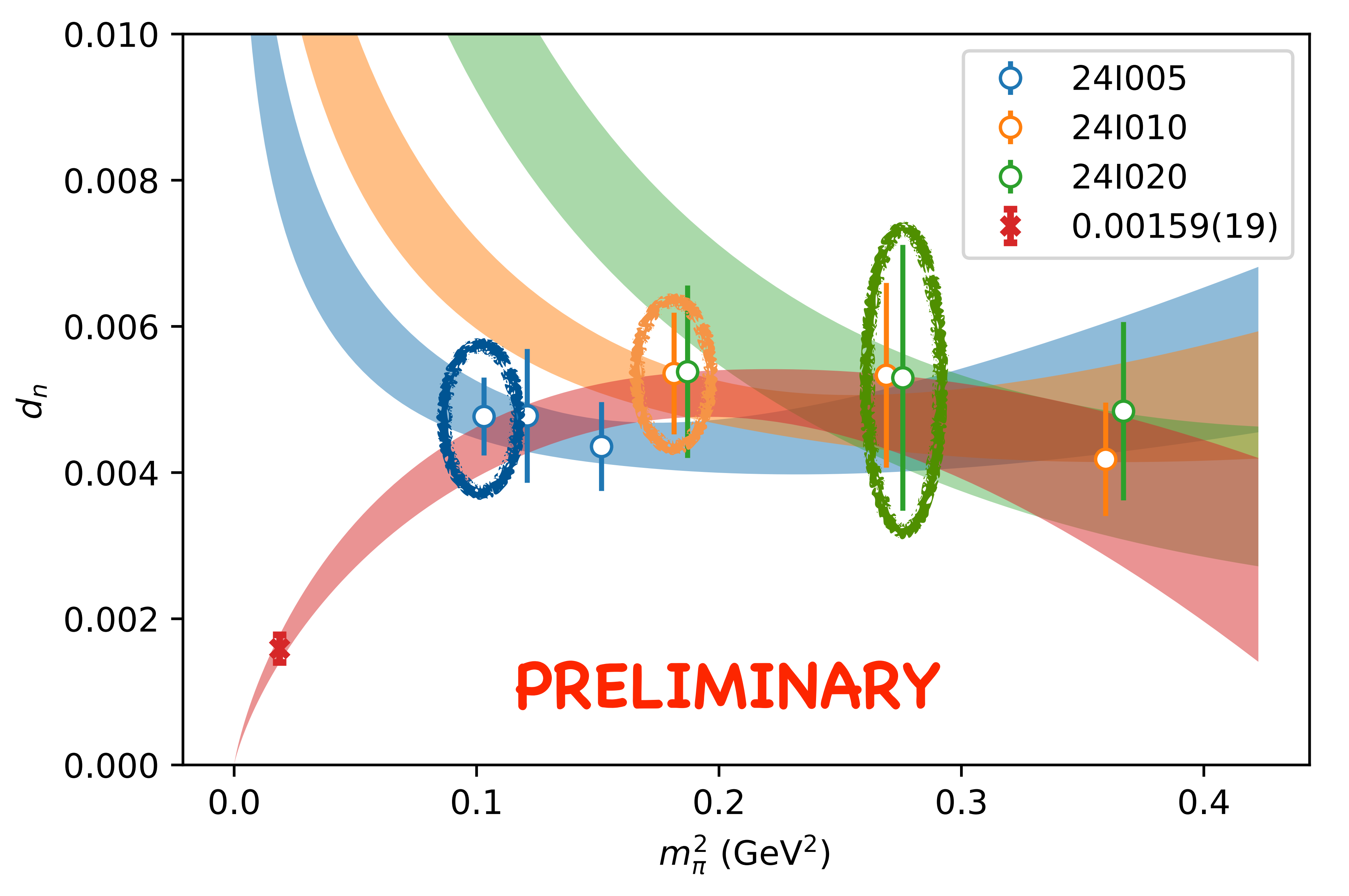




Neutron EDM

Liang, Draper, Liu, Yang: in progress





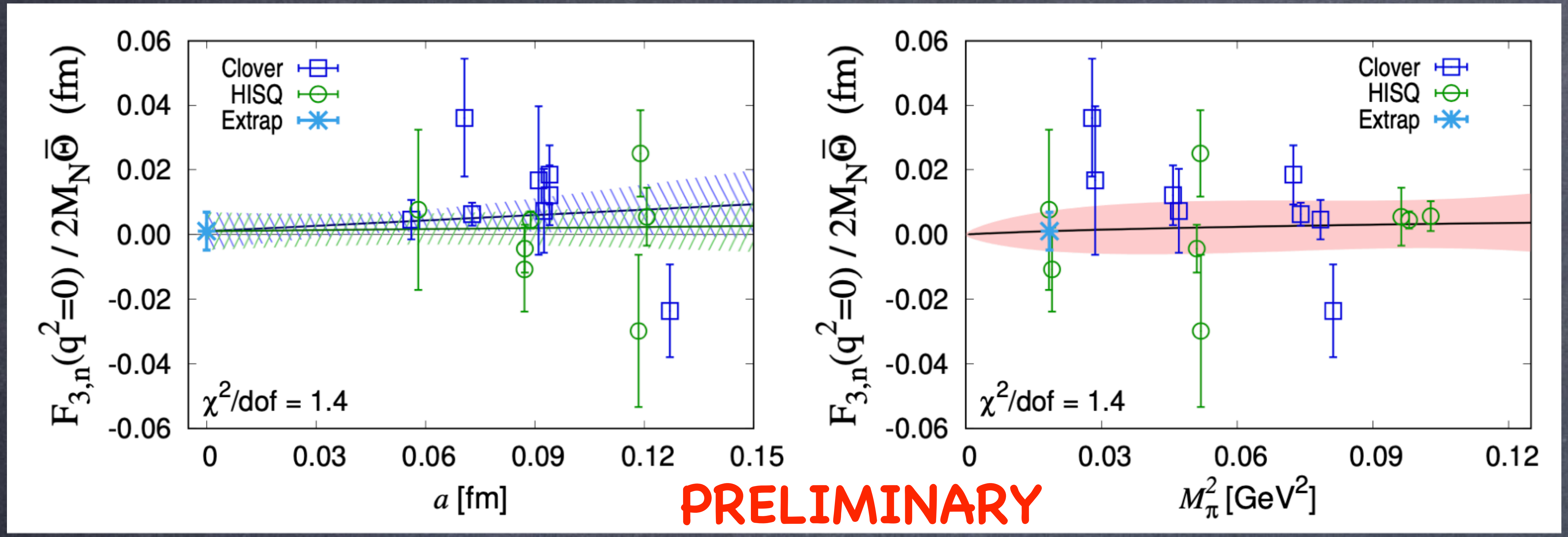
- Overlap fermions
- a = 0.114 fm
- Charge defined with zero modes
- Partially quenched points
  - Unitary points: m_π = 340, 430, 570 MeV

Systematics

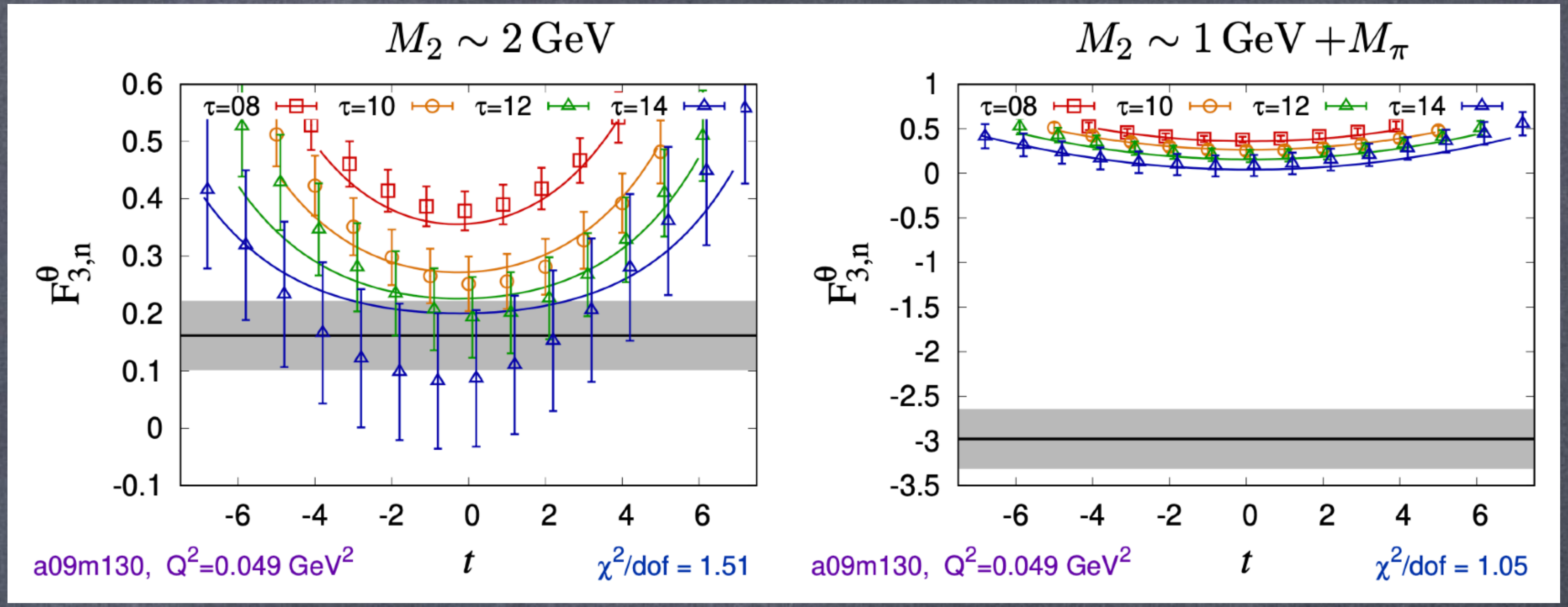
- Excited state contamination
  - t/a = 6, 7, 8
- Sampling topological sectors
- Finite volume and cutoff effects

$$d_n = 0.00159(19)\bar{\theta}\ e\ \mathrm{fm}$$

Neutron EDM

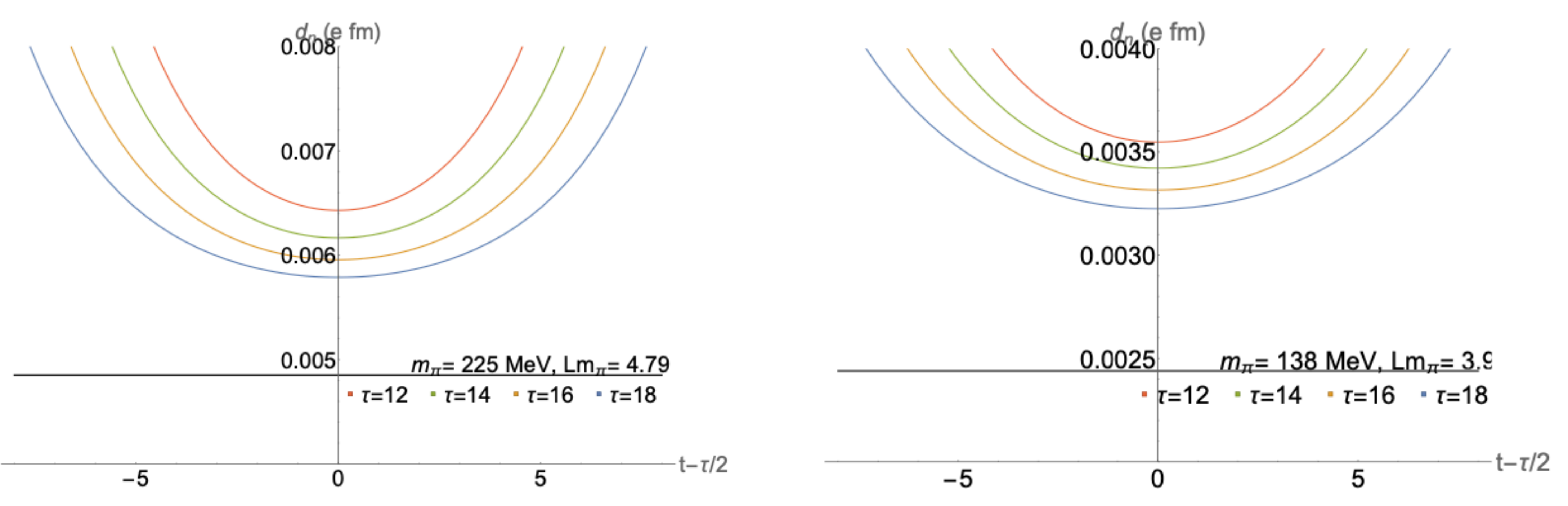


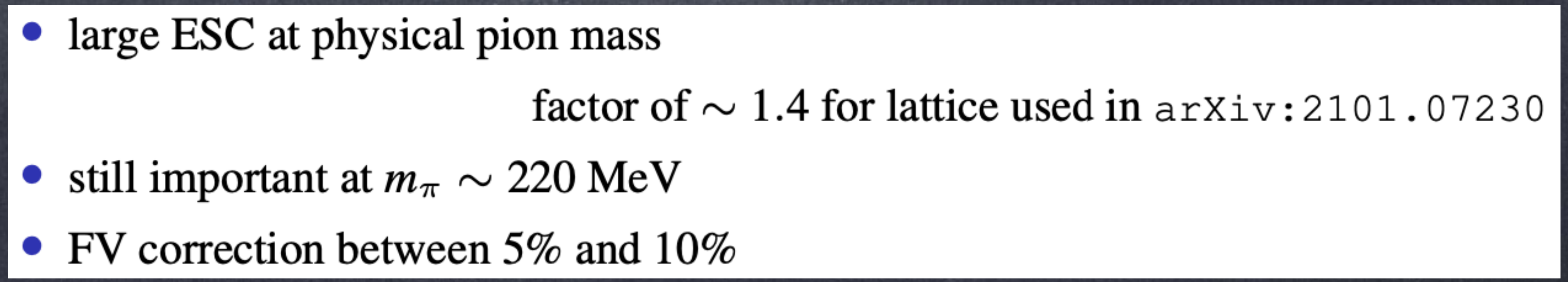
PRELIMINARY



Bhattacharya et al. : 2021, and in progress

- First results with a new discretization
- Long autocorrelation time
- Emphasis in excited states contamination
  - Disentangling contributions from excited states





- large ESC at physical pion mass

  factor of ~ 1.4 for lattice used in `arXiv:2101.07230`
- still important at $m_\pi \sim 220$ MeV
- FV correction between 5% and 10%

Mereghetti

CP-violation strong interactions

Ai, Cruz, Garbrecht, Tamarit: 2021

$$\mathcal{L}_{\text{pion}} = \frac{1}{4} f_\pi^2 \text{Tr} D_\mu U D^\mu U^\dagger + (a f_\pi^3 \text{Tr} M U + |b| e^{-i\xi} f_\pi^4 \det U + \text{h.c.})$$

$$|d_n| \propto (\xi + \alpha_u + \alpha_d + \alpha_s)$$

It is generally thought that $\xi = \theta$ **[Baluni, Crewther et al]**

$\xi = -\sum_i \alpha_i$ Alternative option ➡ **CP conservation**

$$d_n \propto (\xi + \sum_i \alpha_i) = 0$$

Only real computation that we know of is **`t Hooft**'s, using **dilute instanton gas** and yielding $\xi = \theta$ **(➡ CP violation)**

-2 -2

$$\frac{1}{VT}\int d^4x\, \langle \bar{\psi}_i P_L \psi_i \rangle = \lim_{N\to\infty} \lim_{VT\to\infty} \frac{\sum_{\Delta n=-N}^{N} \frac{1}{VT}\int d^4x\, \langle \bar{\psi}_i P_L \psi_i \rangle_{\Delta n}}{\sum\limits_{\Delta m=-N}^{N} Z_{\Delta m}} = 2\mathfrak{m}_i\, \partial_{\mathfrak{m}_i \mathfrak{m}_i^*} \beta(\mathfrak{m}_k \mathfrak{m}_k^*),$$

Topological classification only **enforced in infinite volume**, which **fixes ordering**

$VT \to \infty$ before $\sum_{\Delta n}$	$\sum_{\Delta n}$ before $VT \to \infty$
$\mathcal{L} \to \mathcal{L} - \bar{\psi}(x)\Gamma e^{i\alpha\gamma^5}\psi(x)$	$\mathcal{L} \to \mathcal{L} + \bar{\psi}(x)\Gamma e^{-i\theta\gamma^5}\psi(x)$
Alignment with $\bar{\psi} m \exp(i\alpha\gamma^5)\psi$	Misaligned with $\bar{\psi} m \exp(i\alpha\gamma^5)\psi$
No CP-violating observables	CP-violating observables

- In order to resolve the ambiguity, we must **match effective** $\det U$ **term in** the chiral Lagrangian with **results for correlators in QCD, paying special attention to complex phases**
- Next we proceed to calculate the **phase of QCD correlators** starting from the **path integral** and using **clustering** and the **index theorem**.

Needs to be resolved –>

More precise LQCD calculations

Reanalysis in χPT

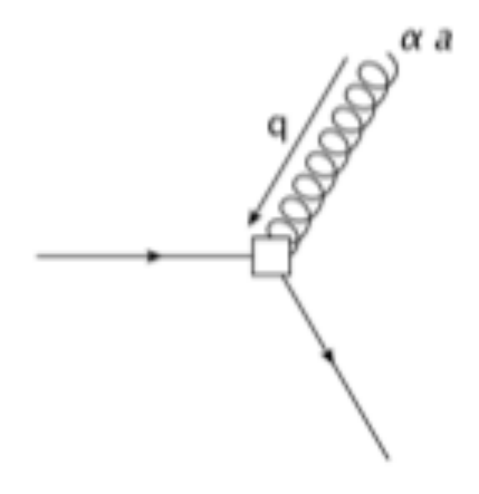
Quark-Chromo EDM

Quark-Chromo EDM

Bhattacharya, Cirigliano, Gupta, Mereghetti, Yoon: 2015

$$\mathcal{O}_{\rm CE}(x) = \sum_{f=u,d,s,\ldots} \overline{\psi}_f(x)\gamma_5\sigma_{\mu\nu}G^a_{\mu\nu}T^a\psi_f(x)$$

$$P(x) = \sum_{f=u,d,s,\ldots} \overline{\psi}_f(x)\gamma_5\psi_f(x)$$



$$[\mathcal{O}_{\rm CE}]_R = Z_{\rm CE}\left[\mathcal{O}_{\rm CE} - \frac{C}{a^2}P\right] + \cdots$$

RI-MOM Off-shell

$\frac{1}{a}$ d=4 —> 2 operators + 3 O(m)

$\log a$ d=5 —> 3 operators + (7 + 5) O(m,m^2) + 4 “nuisance”

Power divergences need to be subtracted non-perturbatively

Maiani, Martinelli, Sachrajda: 1992

Gradient flow

Lüscher: 2013

$$\partial_t \chi(x,t) = \Delta \chi(x,t) \qquad \partial_t \bar{\chi}(x,t) = \bar{\chi}(x,t)\overleftarrow{\Delta}$$

$$\chi(x, t = 0) = \psi(x)$$

$$\bar{\chi}(x, t = 0) = \bar{\psi}(x)$$

$$x_\mu = (x_0, \mathbf{x}) \qquad t = \text{flow} - \text{time} \qquad [t] = -2$$

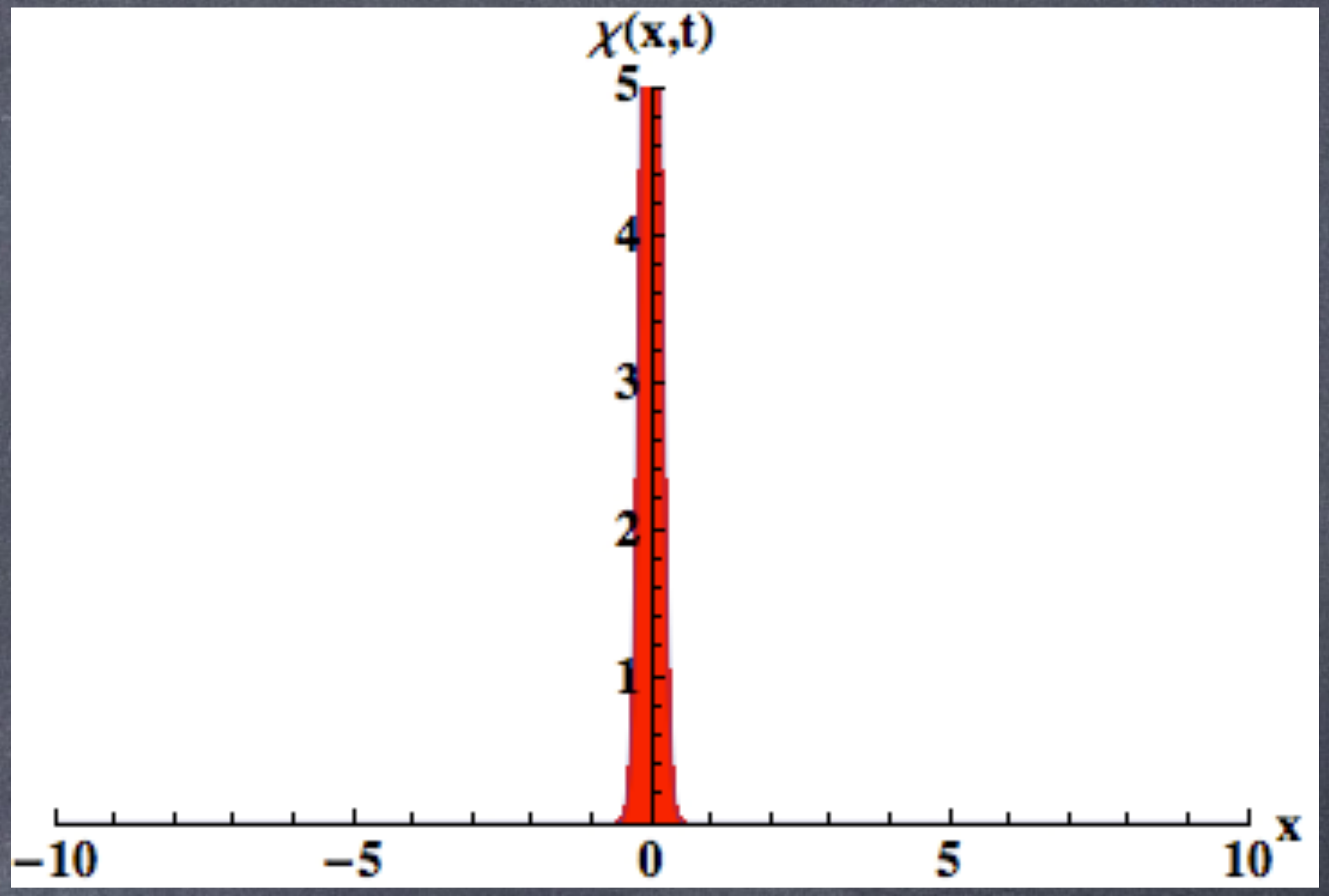
$$\Delta = D_{\mu,t} D_{\mu,t} \qquad D_{\mu,t} = \partial_\mu + B_{t,\mu}$$

Gradient flow

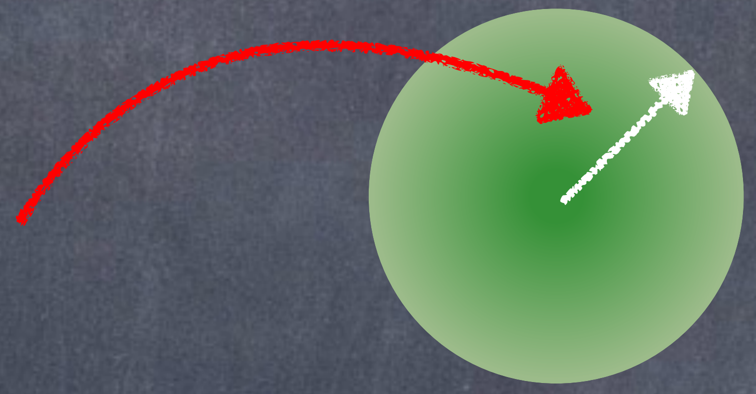
Lüscher: 2013

$$\chi(x,t) = \int d^4y\ K(x-y,t)\psi(y)$$

$$K(x,t) = \frac{\mathrm{e}^{-\frac{x^2}{4t}}}{(4\pi t)^2}$$



- Smoothing over a range $\sqrt{8t}$



- Gaussian damping at large momenta

$$\chi_R(x,t) = Z_\chi^{1/2}\chi(x,t) \qquad \mathcal{O}(x,t) = \overline{\chi}(x,t)\Gamma(x,t)\chi(x,t) \qquad \mathcal{O}_R = Z_\chi\mathcal{O}$$

$$\Sigma_t = \langle\overline{\chi}(x,t)\chi(x,t)\rangle \qquad \Sigma_{t,R} = Z_\chi\Sigma_t$$

Lüscher: 2010, 2013
Lüscher, Weisz: 2011

No additive divergences

All fermion operators renormalize multiplicatively with same factor

Strategy - Short flow-time expansion

A.S., Luu, de Vries: 2014-2015
Dragos, Luu, A.S. de Vries: 2018-2019
Rizik, Monahan, A.S.: 2018-2020
A.S.: 2020
Kim, Luu, Rizik, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer: 2021

$$\underset{\underset{\text{LQCD}}{\uparrow}}{[\mathcal{O}_i(t)]_{\mathrm{R}}} = \sum_i \underset{\underset{\text{PT - LQCD}}{\uparrow}}{c_{ij}(t,\mu)} \boxed{[\mathcal{O}_i(t=0,\mu)]_{\mathrm{R}}} + O(t)$$

$$c_{ij}(t,\mu) = \delta_{ij} + \frac{\alpha_s(\mu)}{4\pi} c_{ij}^{(1)}(t,\mu) + O(\alpha_s^2)$$

- Calculation of matrix elements with flowed fields
  - Easy renormalization (no power divergences)
- Calculation of Wilson coefficients
  - Insert OPE in off-shell amputated 1PI Green's functions
- Determination of the physical renormalized matrix element at zero flow-time

Quark-Chromo EDM

Rizik, Monahan, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer : 2021

$$[\mathcal{O}_i(t)]_{\mathrm{R}} = \sum_i c_{ij}(t,\mu)\,[\mathcal{O}_i(t=0,\mu)]_{\mathrm{R}} + O(t)$$

$$\mathcal{O}_{CE}(x,t) = \bar{\chi}(x,t)\tilde{\sigma}_{\mu\nu}G_{\mu\nu}(x,t)\chi(x,t) \qquad \tilde{\sigma}^{\mathrm{HV}}_{\mu\nu} = -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\sigma_{\alpha\beta} \qquad \tilde{\sigma}^{\mathrm{NDR}}_{\mu\nu} = \sigma_{\mu\nu}\gamma_5$$

$$\mathcal{O}_P(x) = \bar{\psi}(x)\gamma_5\psi(x)$$

$$\mathcal{O}_{m^2P}(x) = m^2\bar{\psi}(x)\gamma_5\psi(x)$$

$$\mathcal{O}_{m\theta}(x) = m\mathrm{tr}[G_{\mu\nu}\widetilde{G}_{\mu\nu}]$$

$$\mathcal{O}_E(x) = \bar{\psi}(x)\tilde{\sigma}_{\mu\nu}F_{\mu\nu}(x)\psi(x)$$

$$\begin{aligned}\mathcal{O}^R_{CE}(x;t) = {} & c_P(t,\mu)\mathcal{O}^{\mathrm{MS}}_P(x;\mu) + c_{m\theta}(t,\mu)\mathcal{O}^{\mathrm{MS}}_{m\theta}(x;\mu) + c_E(t,\mu)\mathcal{O}^{\mathrm{MS}}_E(x;\mu) \\ & + c_{CE}(t,\mu)\mathcal{O}^{\mathrm{MS}}_{CE}(x;\mu) + c_{mP^2}(t,\mu)\mathcal{O}^{\mathrm{MS}}_{m^2P}(x;\mu) + \cdots\end{aligned}$$

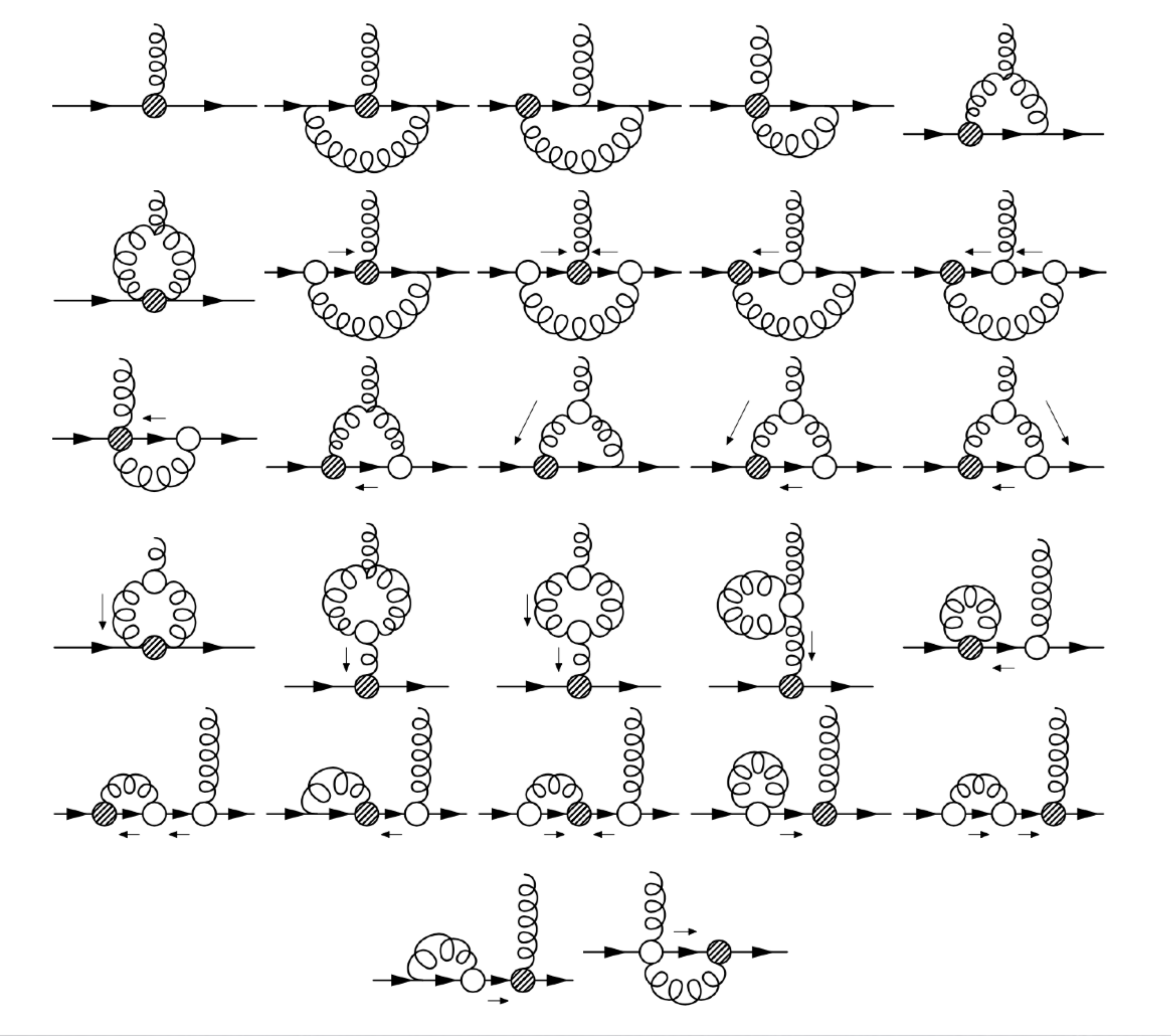
$$Z_\chi^{-n/2}\left\langle(\psi)^{n_\psi}\left(\bar{\psi}\right)^{n_{\bar{\psi}}}(A_\mu)^{n_A}\,\mathcal{O}_i(t)\right\rangle^{\mathrm{amp}} = c_{ij}(t)\left(Z^{\mathrm{MS}}_{jk}\right)^{-1}\left\langle(\psi)^{n_\psi}\left(\bar{\psi}\right)^{n_{\bar{\psi}}}(A_\mu)^{n_A}\,\mathcal{O}_k\right\rangle^{\mathrm{amp}}$$

$$c_{ij}(t,\mu) = \delta_{ij} + \frac{\alpha_s(\mu)}{4\pi}c^{(1)}_{ij}(t,\mu) + O(\alpha_s^2)$$

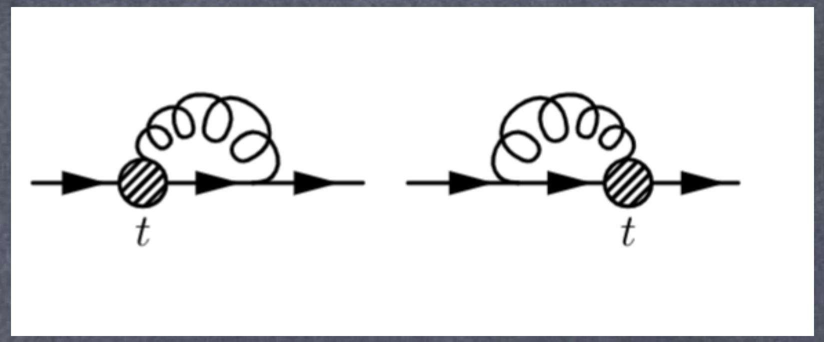
Quark-Chromo EDM

Rizik, Monahan, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S.,
Stoffer : 2021

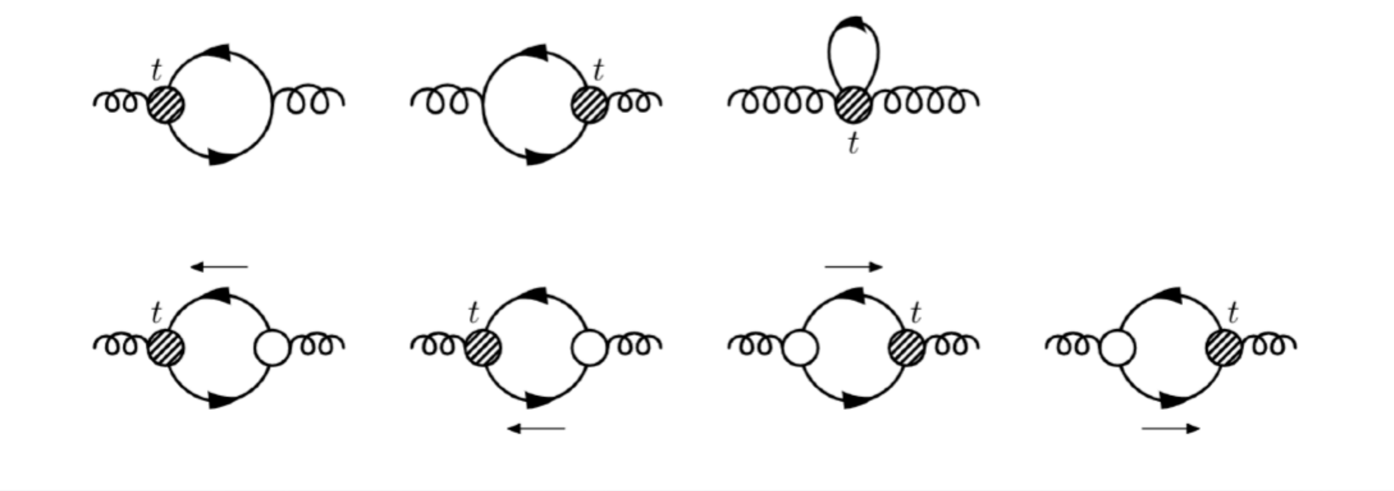
$c_{CE}(t,\mu)$



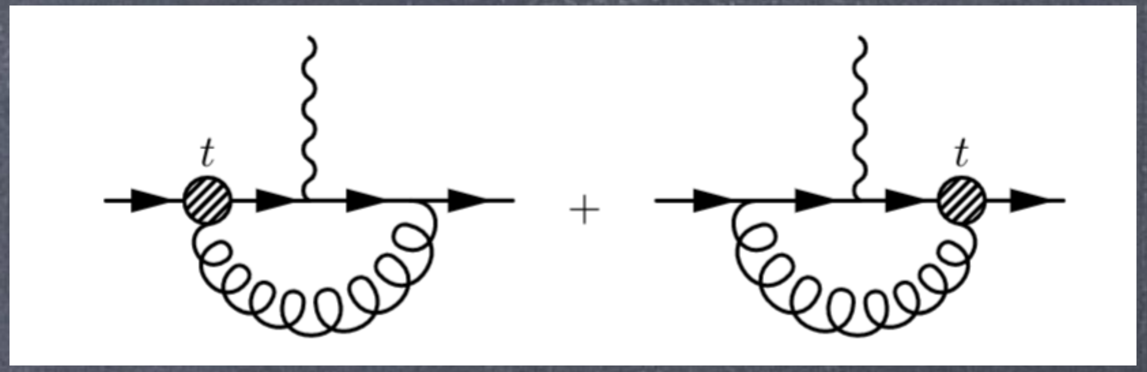
$c_P(t,\mu)$ $c_{m^2P}(t,\mu)$



$c_{m\theta}(t,\mu)$



$c_E(t,\mu)$



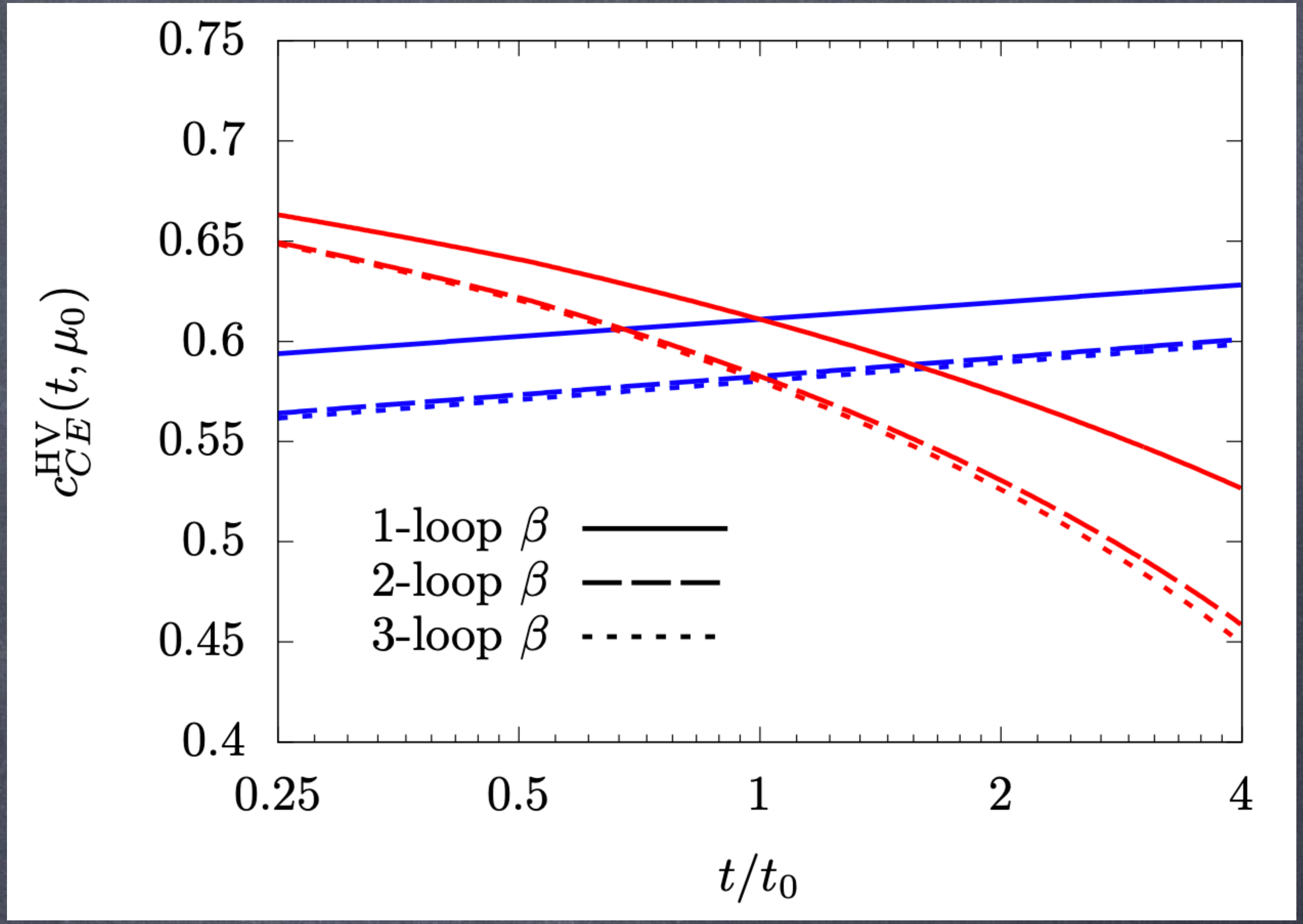
- Expand integrands of loop integrals in all scales excluding t
  - Analytic structure altered —> distortion of IR structure
  - in matching equation the IR modification drops out in the difference
  - Expanding loop integrals in the RHS vanish in DR —> UV and IR are identical
  - The LHS is UV-finite, beside the renormalization of the bare parameters and flowed fermion fields
  - The IR singularities on the LHS exactly match the UV MS counterterms

Scale dependence matching coefficients

$$\bar{\mu}_0 = 3 \text{ GeV} \to \mu_0 = 1.13 \text{ GeV}$$

$$t_0 = \frac{1}{8\pi\mu_0^2}$$

Red - Blue = $A_1\alpha_s^2(\mu_0^2)\log^2(8\pi t\mu_0^2) + A_2\alpha_s^2(\mu_0^2)\log(8\pi t\mu_0^2) + O(\alpha_s^3)$



$t \in [t_0/4, 4t_0]$

10%-20% uncertainties from PT at 1-loop

Rizik, Monahan, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer : 2021

Quark-Chromo EDM: non-perturbative renormalization (power divergences)

Kim, Luu, Rizik, A.S.:2020

- Non-perturbative determination of power divergences
- Continuum limit impossible with other methods. Uncontrolled systematics

$$\Gamma_{CP}(x_4;t) = a^3 \sum_{\mathbf{x}} \left\langle \mathcal{O}_{CE}^{ij}(x_4,\mathbf{x};t) P^{ji}(0,\mathbf{0};0) \right\rangle$$

$$\Gamma_{PP}(x_4;t) = a^3 \sum_{\mathbf{x}} \langle P^{ij}(x_4,\mathbf{x};t) P^{ji}(0,\mathbf{0}) \rangle$$

$$\left[\overline{R}_P(x_4;t)\right]_R = t\,\frac{[\Gamma_{CP}(x_4;t)]_R}{[\Gamma_{PP}(x_4;t)]_R} \longrightarrow$$

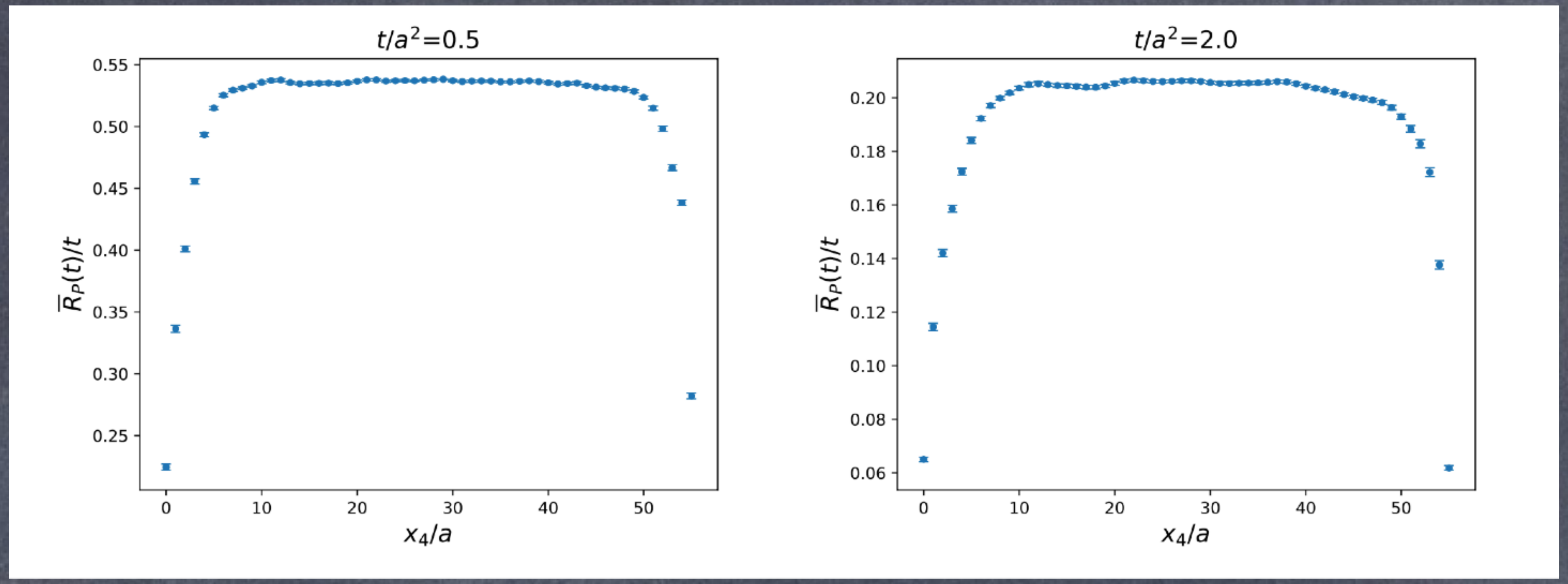
Coefficient linear divergence at 1-loop

- Method is general and can be used for other matrix elements: CP-odd kaon decays, higher moments of PDFs, B-mesons decay rates,...

Non-perturbative renormalization (power divergences)

Kim, Luu, Rizik, A.S.:2020

$$\frac{\left[\overline{R}_P(x_4;t)\right]_R}{t}$$

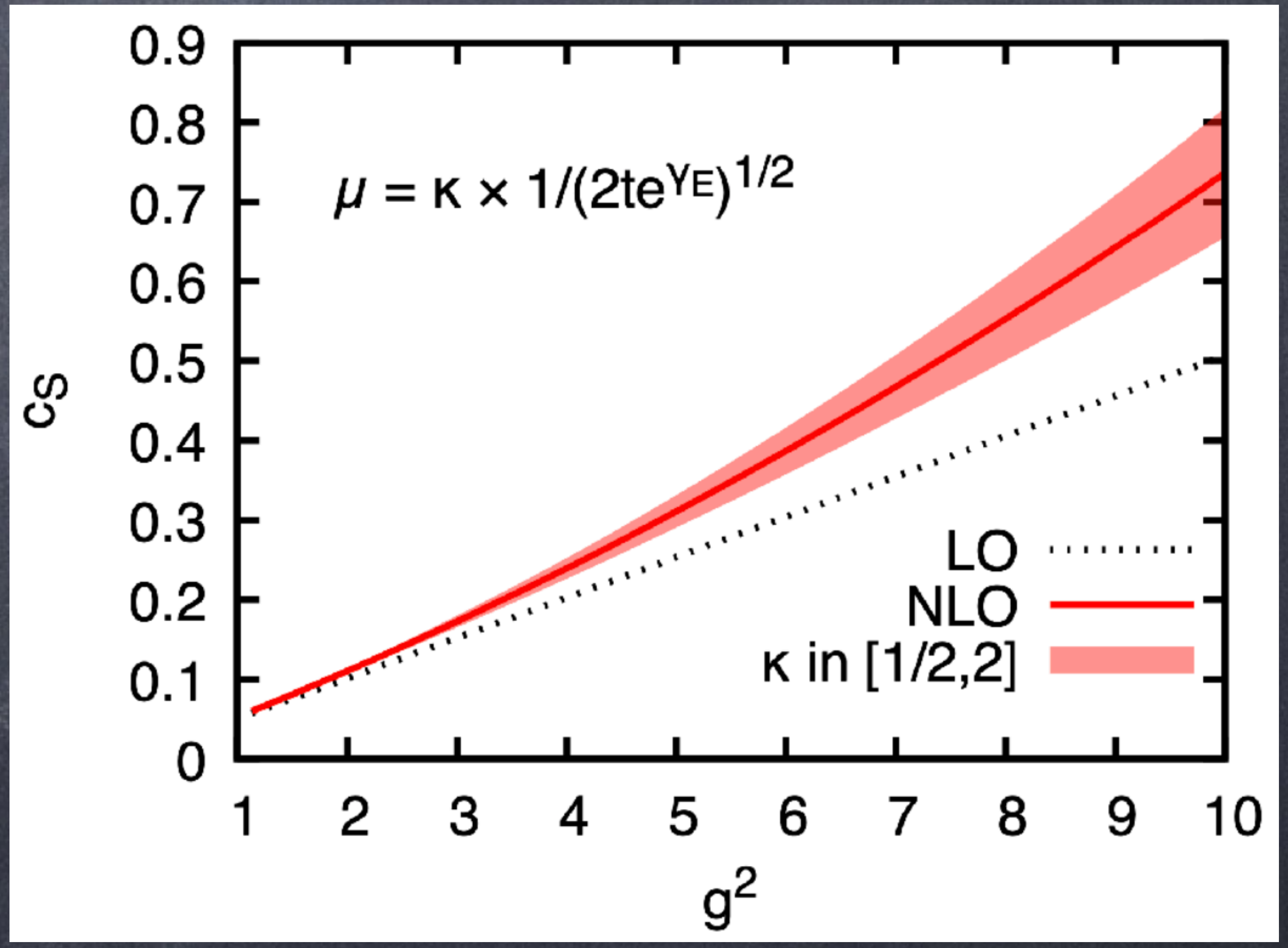


$$c_P(t) = \frac{\bar{g}^2}{2\pi^2} + \left(\frac{\bar{g}^2}{4\pi^2}\right)^2 \left[x_0 + x_1 \log \mu^2 t\right]$$

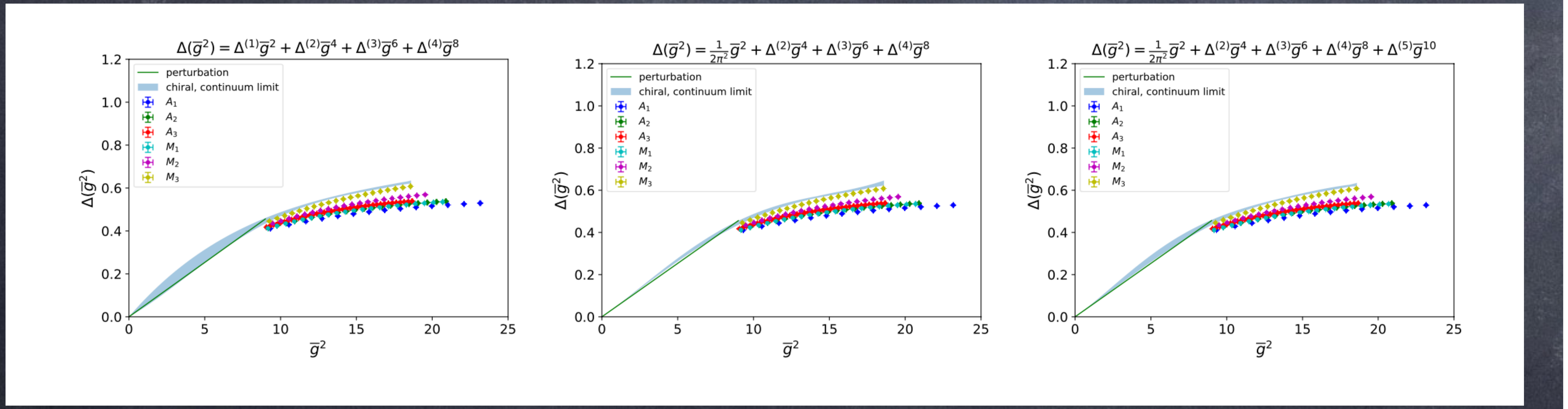
Rizik, Monahan, A.S.: 2020

Borgulat, Harlander, Rizik, A.S.

Warm-up MDM —> 2-loops (226 - 3375 FD)

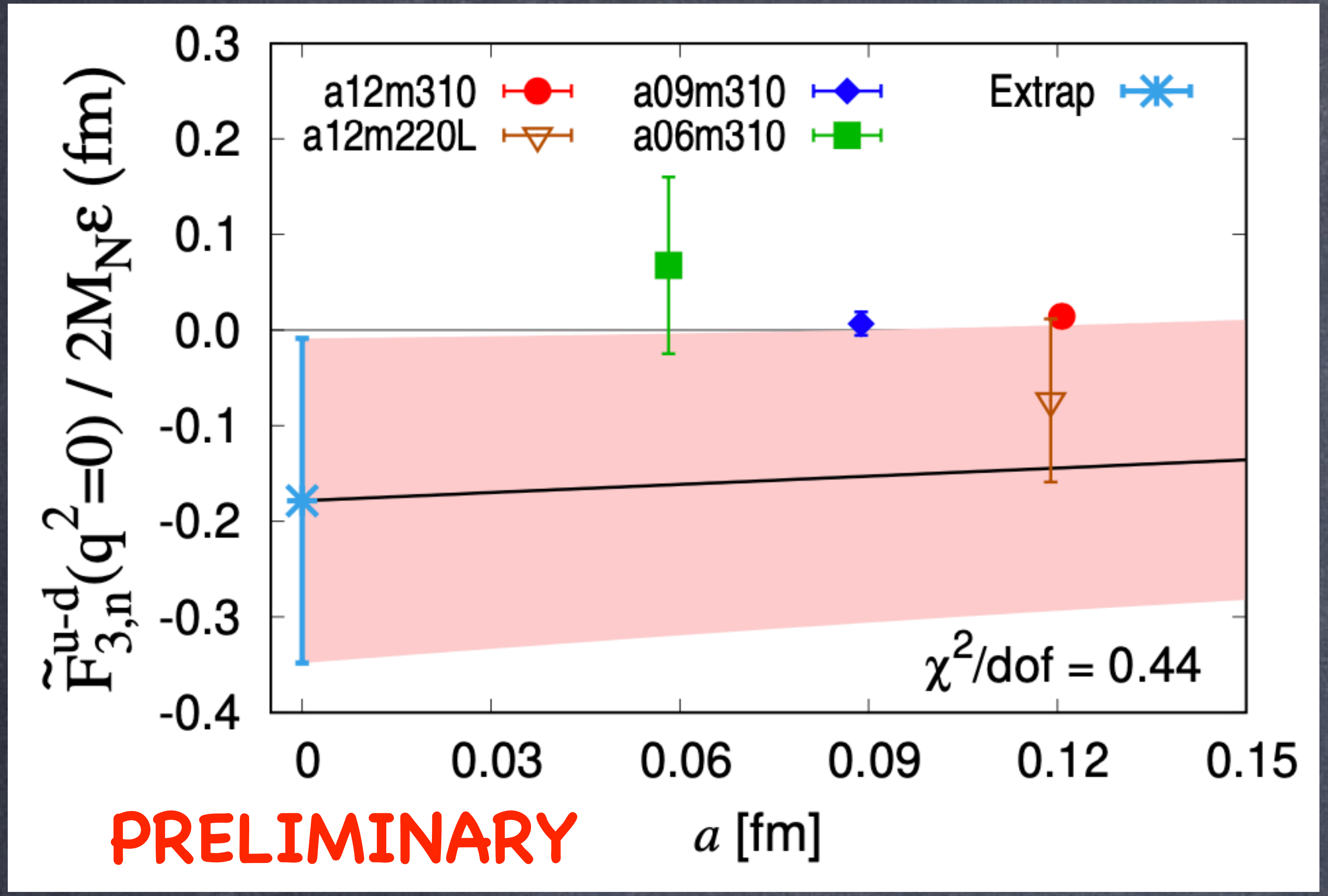


Coefficient linear divergence

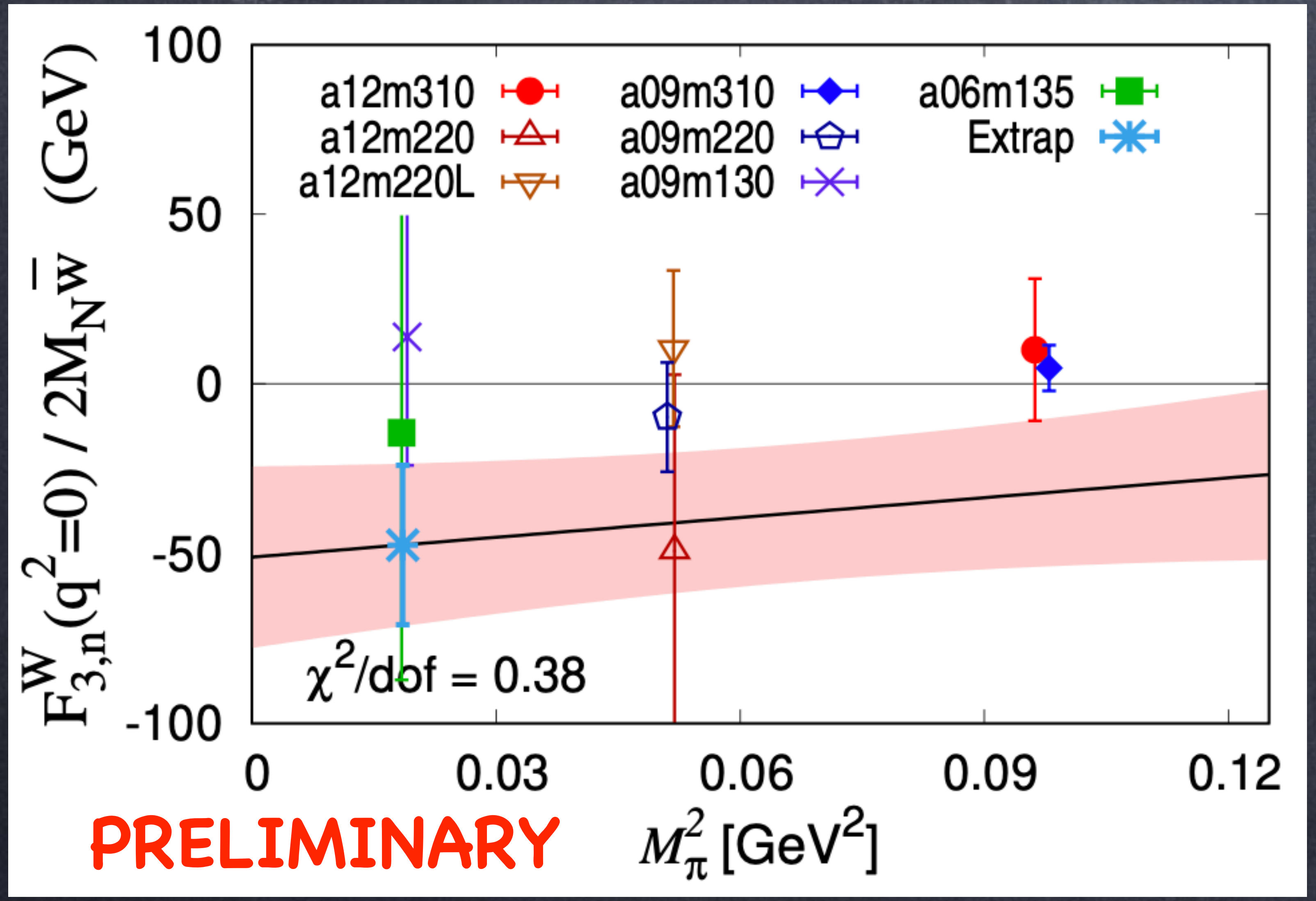


Lattice QCD results

Bhattacharya et al. : in progress



- Bare form factor —> power divergence subtracted
- qCEDM defined in standard manner
- Results have better signal than theta-term at m_π = 300 MeV



- Bare form factor —> no renormalization
- 3g operator defined with gradient flow
- Mixing calculable non-pert (1/t) and pert (log(t))

Dragos, Luu, A.S. de Vries: 2018
Rizik, Monahan, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer : 2021

Work in progress

- Improve determination of nEDM from theta-term

  Francis, Kim, Luu, Pederiva, A.S., Zafeiropoulos

- Matching coefficients of the CP-violating 3-gluon (d=6 op.) and 4-fermion operators at 1-loop

  Mereghetti, Monahan, Rizik, A.S., Stoffer

- Matching coefficients of qCEDM at 2-loops

  Borgulat, Harlander, Rizik, A.S.

- Non-perturbative determination of power divergences with SWF

  Kim, Luu, Pederiva, Rizik, A.S.

- Calculation of the qCEDM in a nucleon

- Non-perturbative renormalization scheme with GF

  Hasenfratz A., Monahan, Rizik, A.S., Witzel

- OpenLat: open science initiative. Gauges with SWF open to the whole community

  Cuteri, Francis, Fritzsch, Pederiva, Rago, A.S., Walker-Loud, Zafeiropoulos

Near term goals

- Several calculations of the theta EDM. ChiPT is consistent with our results
- Calculate theta-term contribution to the nucleon EDM with GF and improved S-to-N ratios.
  O(10-20%) for theta-term in the next 2-3 years ==>
  next level of computer time proposals
- Non-perturbative renormalization and exploratory calculations of qCEDM matrix elements.
  Use of the gradient flow is critical.
- PT for the 3-gluon operator and explore matrix elements
- Determine in PT the flow-time dependence of other higher dimensional operators

Lattice QCD is moving towards a determination of nucleon EDM

Stay tuned

Neutron EDM Workshop

NEUTRON ELECTRIC DIPOLE MOMENT: FROM THEORY TO EXPERIMENT

01 August 2022 — 05 August 2022

ECT* - Villa Tambosi

Strada delle Tabarelle, 286
Trento - Italy

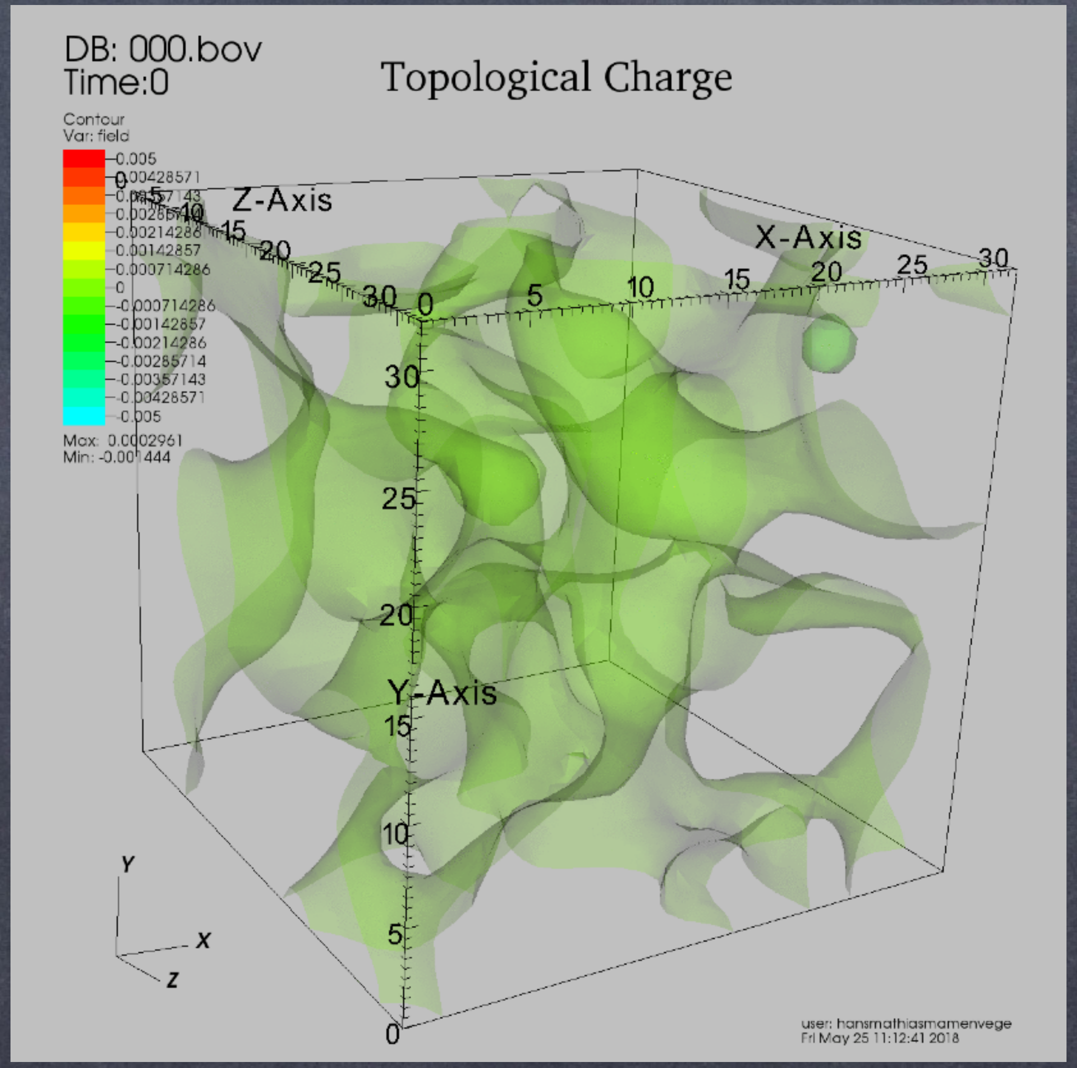
Organizers: A. Athenodorou, A.S., M.P. Lombardo

https://www.ectstar.eu/workshops/neutron-electric-dipole-moment-from-theory-to-experiment/

List of speakers

- Philipp Schmidt-Wellenburg, Peter Fierlinger, Takeyasu Ito, Florian Piegsa, Kent Leung,...
- Ulf Meißner, Michael Ramsey-Musolf, Emanuele Mereghetti, Laura Covi,...
- Costantia Alexandrou, Andre Walker-Loud, Keh-Fei Liu, Boram Yoon,...

Thank you!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Backup Slides

4+1 Local field theory

Lüscher
2010-2013

$$S = S_{\mathrm{G}} + S_{\mathrm{G,fl}} + S_{\mathrm{F,QCD}} + S_{\mathrm{F,fl}}$$

$$S_{\mathrm{F,fl}} = \int_0^\infty dt \int d^4x \,\left[\overline{\lambda}(t,x)\,(\partial_t - \Delta)\,\chi(t,x) + \overline{\chi}(t,x)\left(\overleftarrow{\partial}_t - \overleftarrow{\Delta}\right)\lambda(t,x)\right]$$

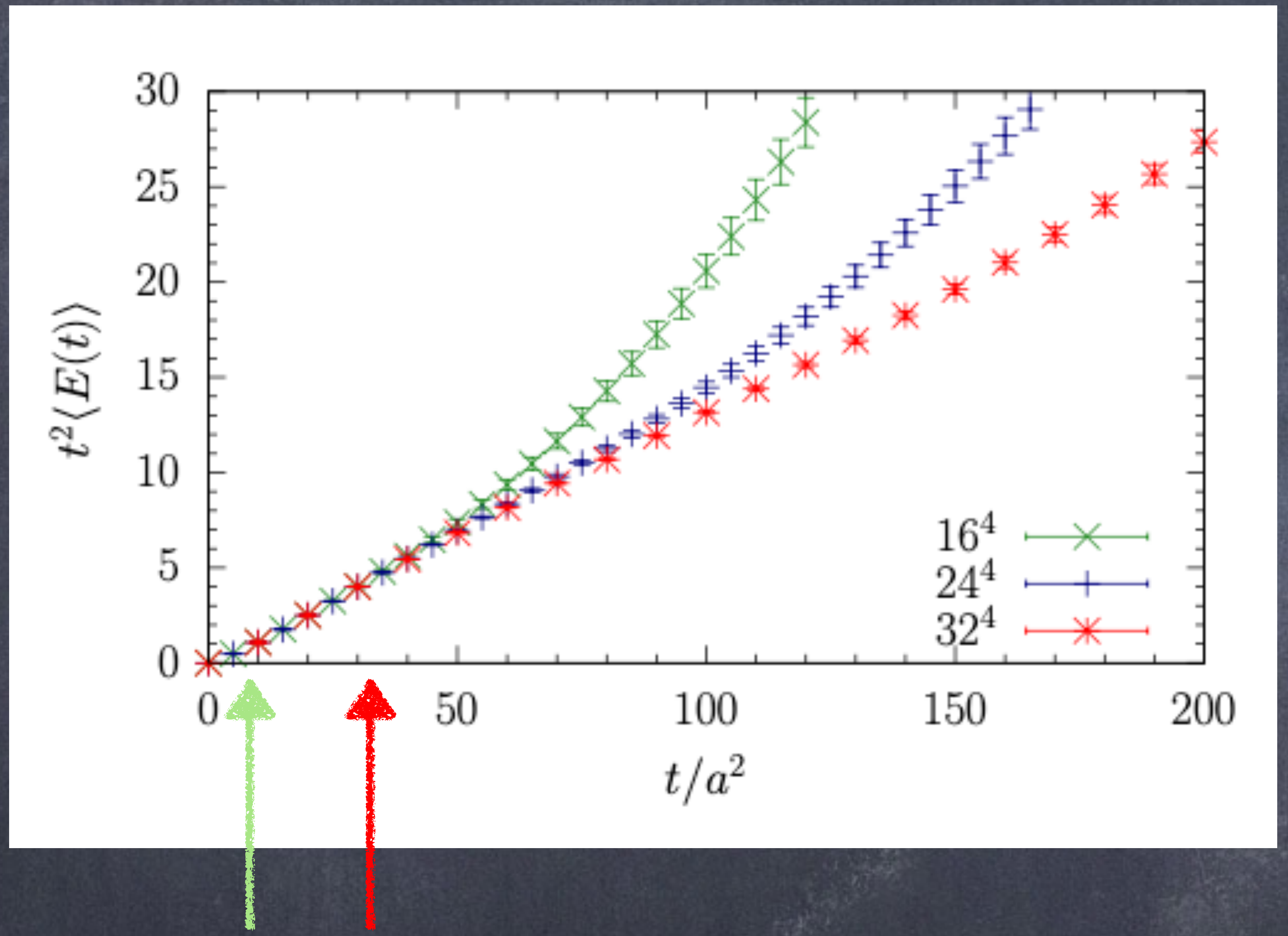
- Wick contractions
- Renormalization. All order proof for gauge sector — Lüscher, Weisz: 2011
- Chiral symmetry and Ward identities — Lüscher: 2013
A.S.:2013
- Wilson twisted mass — A.S.: 2013

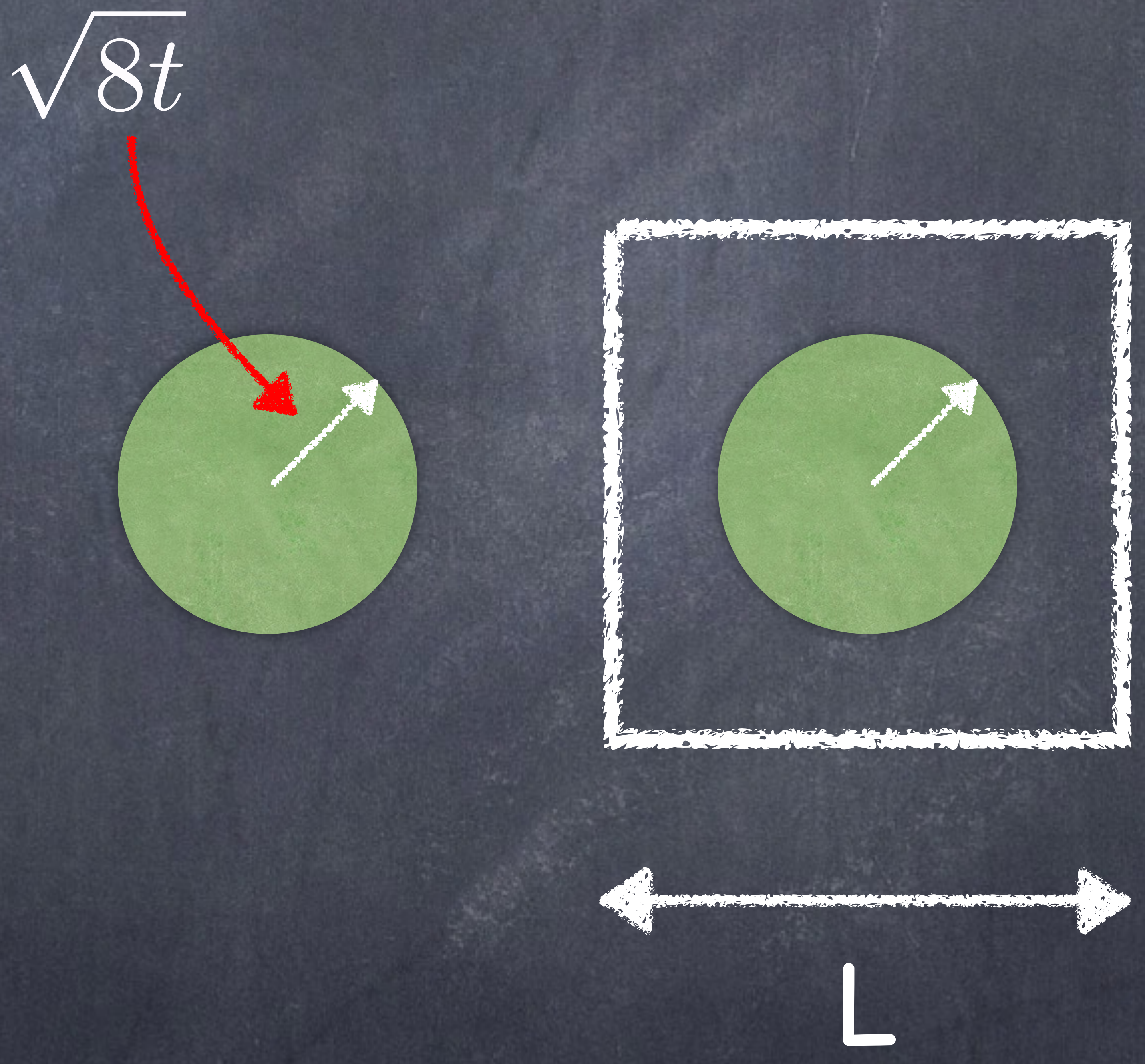
Nakamura, Schierholz 2106.11369

Players in the arena

$$E(t) = \frac{1}{2}\mathrm{Tr}\left[G_{\mu\nu}(x,t)G_{\mu\nu}(x,t)\right] \qquad g^2_{GF}(t) = \frac{16\pi^2}{3}t^2\left\langle E(t)\right\rangle$$

Lüscher:2010





$$32^4 \Rightarrow \frac{L}{2} = 16a \Rightarrow \sqrt{8t} < 16a \Rightarrow \frac{t}{a^2} < 32$$

$$16^4 \Rightarrow \frac{L}{2} = 8a \Rightarrow \sqrt{8t} < 8a \Rightarrow \frac{t}{a^2} < 8$$

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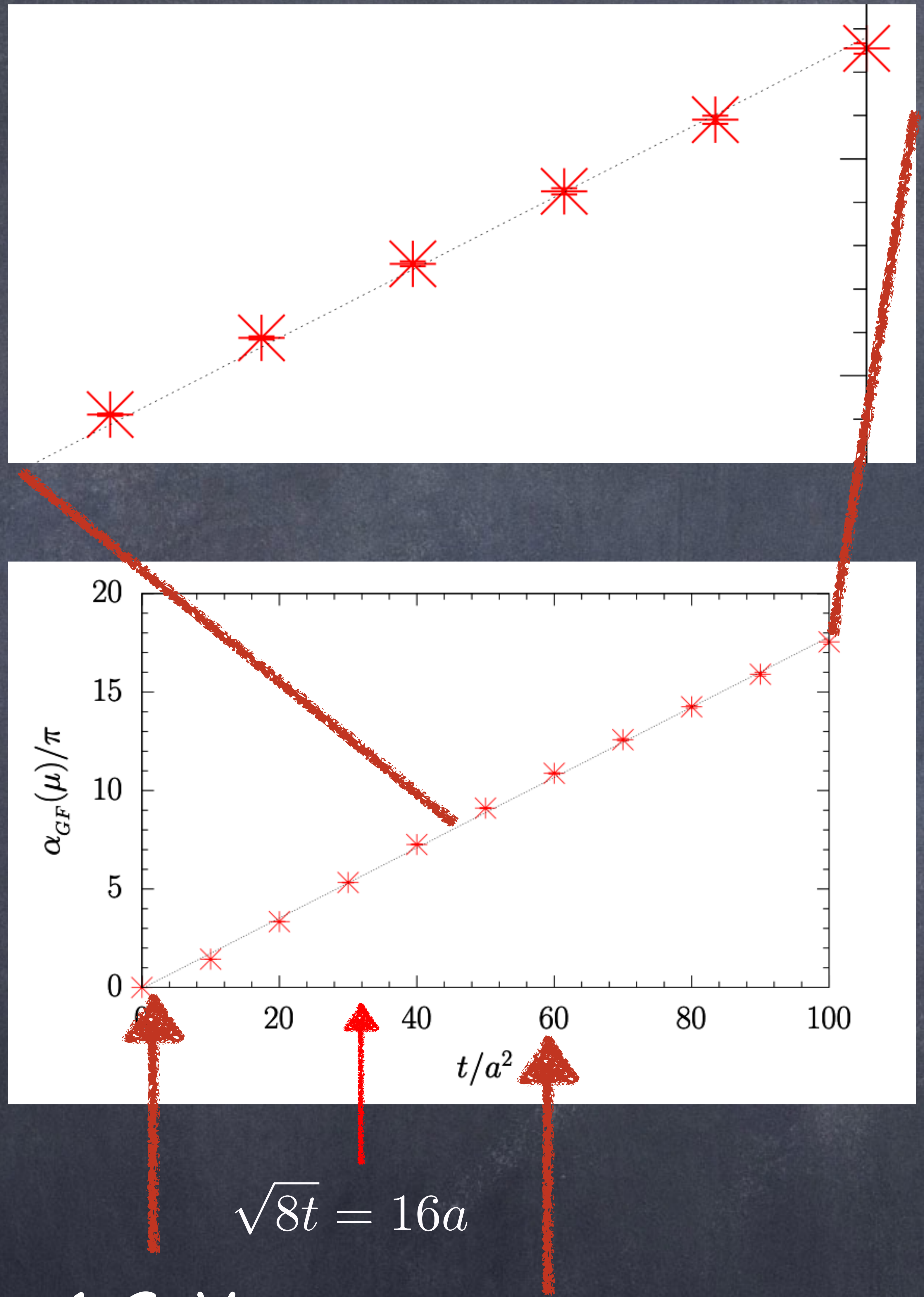

$\sqrt{8t} = 16a$

1 GeV

100 MeV

$$\alpha_{GF}(t) \propto t \quad ? \qquad \mu^2 = \frac{1}{8t}$$

$$\alpha_{GF}(\mu) \propto \frac{1}{\mu^2}$$

$$\beta_{GF} = \frac{\partial \alpha_{GF}(\mu)}{\partial \mathrm{log}\mu} \Rightarrow \beta_{GF} = -2\alpha_{GF}$$

$$\mu \ll 1 \text{ GeV}$$

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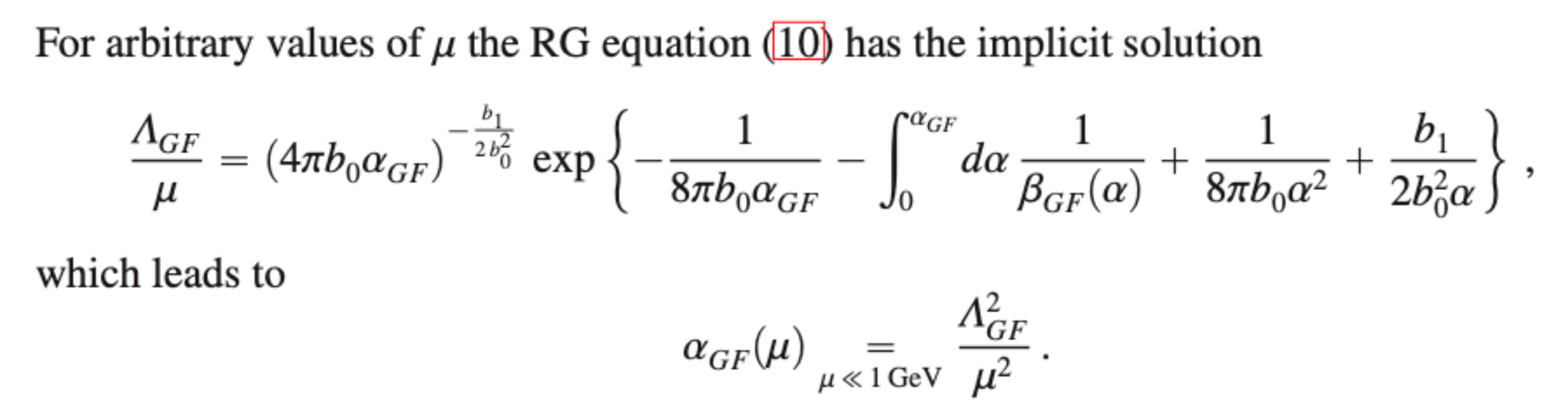

For arbitrary values of μ the RG equation (10) has the implicit solution

$$\frac{\Lambda_{GF}}{\mu} = (4\pi b_0 \alpha_{GF})^{-\frac{b_1}{2b_0^2}} \exp\left\{-\frac{1}{8\pi b_0 \alpha_{GF}} - \int_0^{\alpha_{GF}} d\alpha \, \frac{1}{\beta_{GF}(\alpha)} + \frac{1}{8\pi b_0 \alpha^2} + \frac{b_1}{2b_0^2 \alpha}\right\} ,$$

which leads to

$$\alpha_{GF}(\mu) \underset{\mu \ll 1\,\mathrm{GeV}}{=} \frac{\Lambda_{GF}^2}{\mu^2} .$$

?

$$\frac{\Lambda_{GF}}{\mu} = \exp\left\{-\int_1^{\alpha_{GF}} d\alpha \, \frac{1}{\beta(\alpha)}\right\} \Rightarrow \alpha_{GF} = \frac{\Lambda_{GF}^2}{\mu^2}$$

Incorrect ?

$$\frac{\partial \alpha}{\partial \log \mu} = -2\alpha \Rightarrow \alpha_0 \mu_0^2 = \alpha \mu^2 = \text{constant}$$

Is it the same Λ parameter ?

From now on only the vector scheme. Why?

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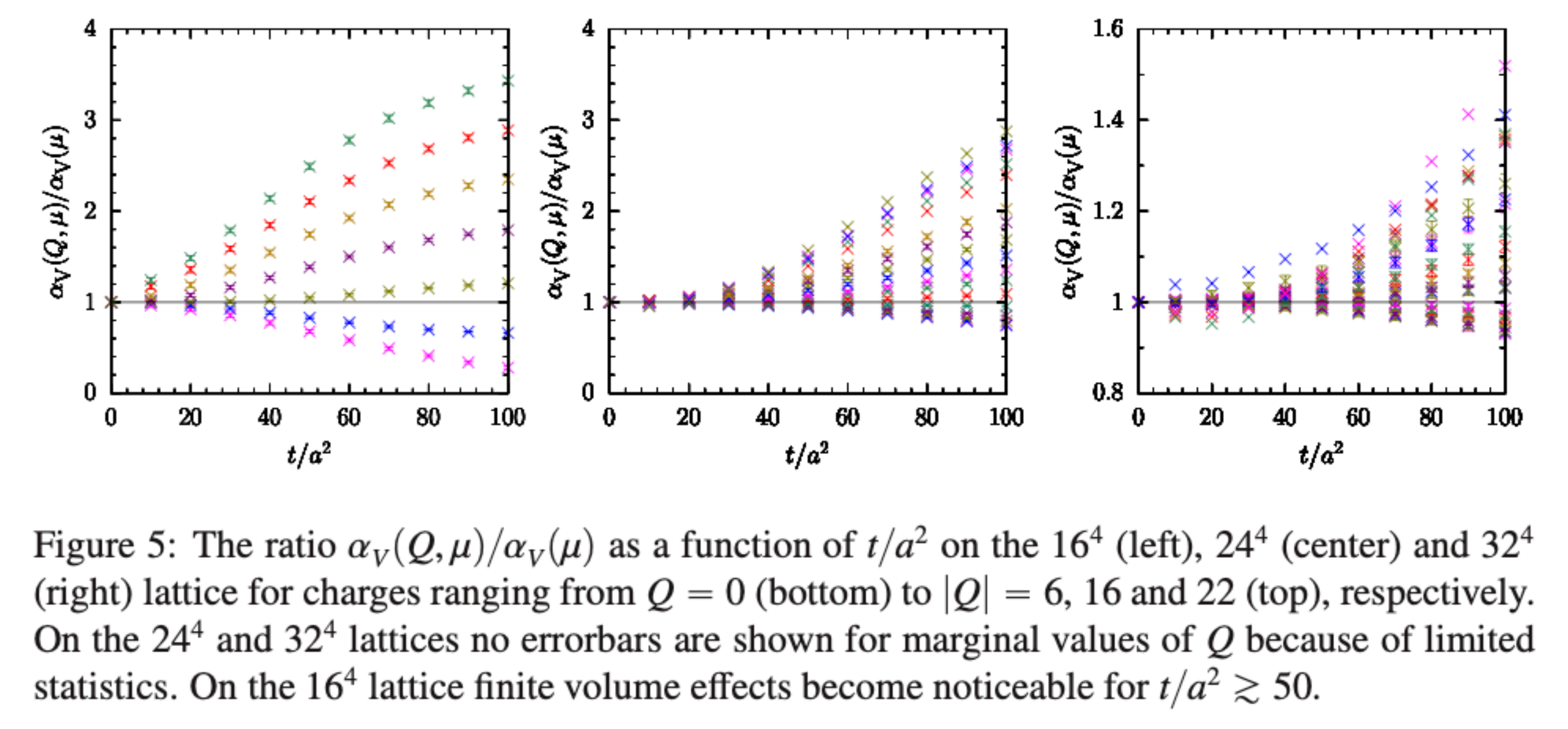
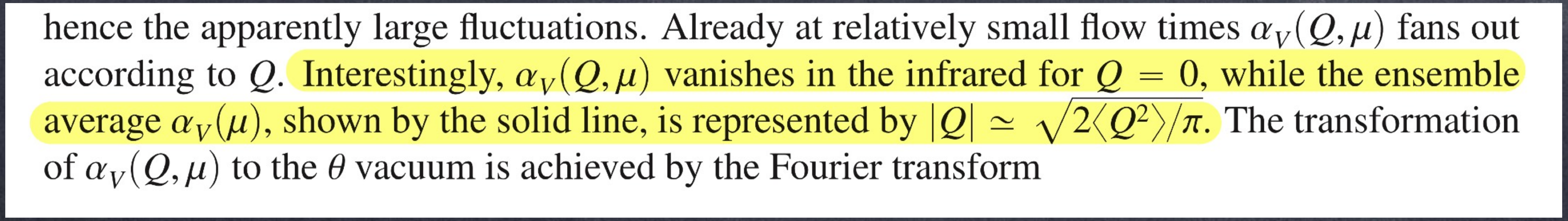


Figure 5: The ratio $\alpha_V(Q,\mu)/\alpha_V(\mu)$ as a function of t/a^2 on the 16^4 (left), 24^4 (center) and 32^4 (right) lattice for charges ranging from $Q = 0$ (bottom) to $|Q| = 6$, 16 and 22 (top), respectively. On the 24^4 and 32^4 lattices no errorbars are shown for marginal values of Q because of limited statistics. On the 16^4 lattice finite volume effects become noticeable for $t/a^2 \gtrsim 50$.

N=5000 configurations

$$K \propto \frac{\langle Q^4 \rangle_c}{\langle Q^2 \rangle_c} < 0$$



hence the apparently large fluctuations. Already at relatively small flow times $\alpha_V(Q,\mu)$ fans out according to Q. Interestingly, $\alpha_V(Q,\mu)$ vanishes in the infrared for $Q = 0$, while the ensemble average $\alpha_V(\mu)$, shown by the solid line, is represented by $|Q| \simeq \sqrt{2\langle Q^2 \rangle/\pi}$. The transformation of $\alpha_V(Q,\mu)$ to the θ vacuum is achieved by the Fourier transform

Nakamura, Schierholz 2106.11369

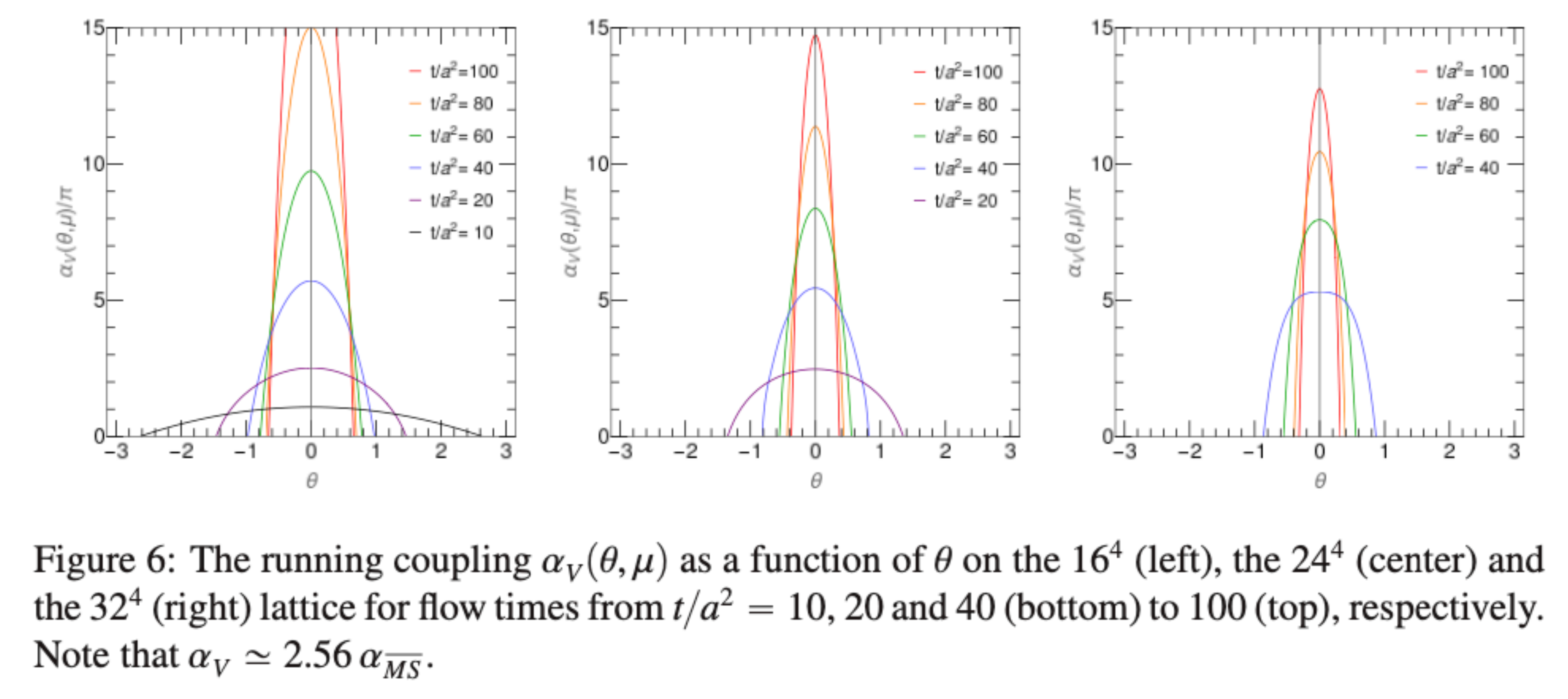
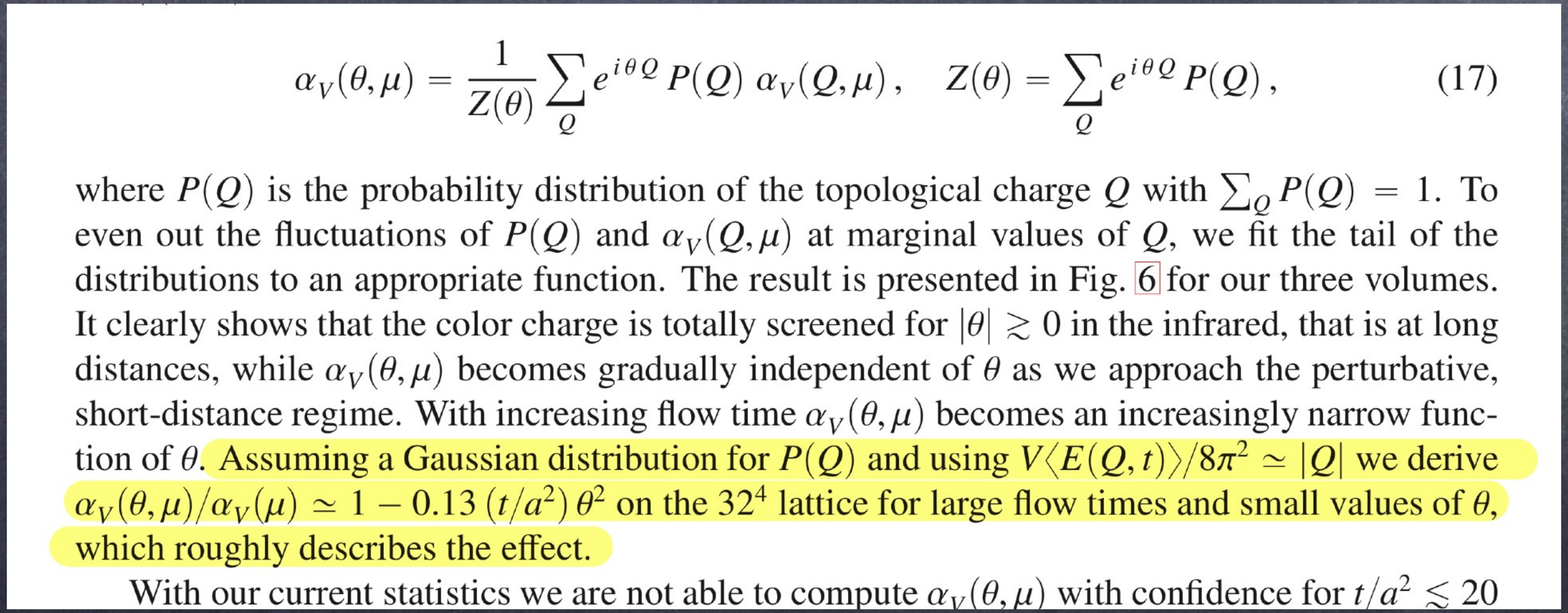


Figure 6: The running coupling $\alpha_V(\theta,\mu)$ as a function of θ on the 16^4 (left), the 24^4 (center) and the 32^4 (right) lattice for flow times from $t/a^2 = 10$, 20 and 40 (bottom) to 100 (top), respectively. Note that $\alpha_V \simeq 2.56\,\alpha_{\overline{MS}}$.

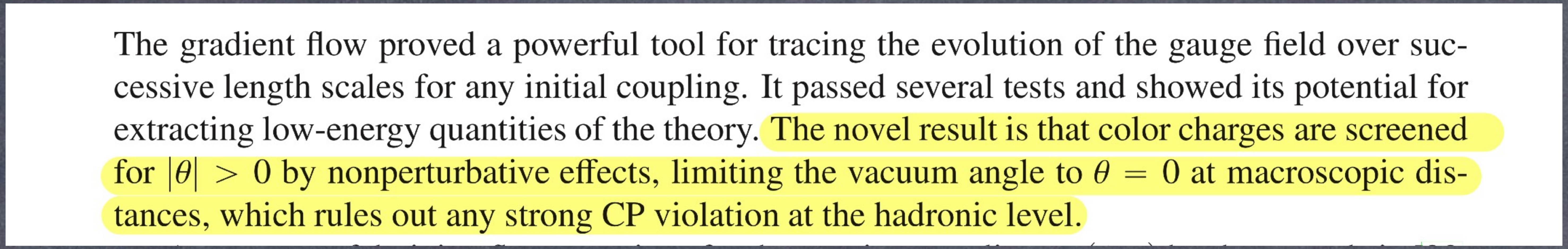


$$\alpha_V(\theta,\mu) = \frac{1}{Z(\theta)}\sum_Q e^{i\theta Q}\,P(Q)\;\alpha_V(Q,\mu)\,,\quad Z(\theta) = \sum_Q e^{i\theta Q}\,P(Q)\,, \tag{17}$$

where $P(Q)$ is the probability distribution of the topological charge Q with $\sum_Q P(Q) = 1$. To even out the fluctuations of $P(Q)$ and $\alpha_V(Q,\mu)$ at marginal values of Q, we fit the tail of the distributions to an appropriate function. The result is presented in Fig. 6 for our three volumes. It clearly shows that the color charge is totally screened for $|\theta| \gtrsim 0$ in the infrared, that is at long distances, while $\alpha_V(\theta,\mu)$ becomes gradually independent of θ as we approach the perturbative, short-distance regime. With increasing flow time $\alpha_V(\theta,\mu)$ becomes an increasingly narrow function of θ. Assuming a Gaussian distribution for $P(Q)$ and using $V\langle E(Q,t)\rangle/8\pi^2 \simeq |Q|$ we derive $\alpha_V(\theta,\mu)/\alpha_V(\mu) \simeq 1 - 0.13\,(t/a^2)\,\theta^2$ on the 32^4 lattice for large flow times and small values of θ, which roughly describes the effect.

With our current statistics we are not able to compute $\alpha_V(\theta,\mu)$ with confidence for $t/a^2 \lesssim 20$

Nakamura, Schierholz 2106.11369



The gradient flow proved a powerful tool for tracing the evolution of the gauge field over successive length scales for any initial coupling. It passed several tests and showed its potential for extracting low-energy quantities of the theory. The novel result is that color charges are screened for $|\theta| > 0$ by nonperturbative effects, limiting the vacuum angle to $\theta = 0$ at macroscopic distances, which rules out any strong CP violation at the hadronic level.

- Smoothing kills confinement properties
- Analysis at large t —> FVE
- Medium-Large Q small statistics
- Mixing NP with Perturbative regime
- Why the need to change scheme
- Negative Kurtosis, but Gaussian P(Q)
- Only 1 coarse lattice spacing quenched

==> strong conclusion needs to be supported by a more robust calculations

Topological charge on the lattice

Discretize ⟶ $$q(x) = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}\left\{G_{\mu\nu}(x)G_{\rho\sigma}(x)\right\}$$

Continuity in space is lost
On the lattice it has no topological significance

Geometrical definition:

extend the lattice gauge field to a continuous one
Field between lattice points -> judicious interpolation
Smooth gauge fields (bound on field tensor)

Lüscher: 1982
Phillips, Stones: 1986

Fermionic definition:

Anomalous Ward Identity

Smit: 1980

$$\partial_\mu A_\mu = 2mP + \text{extra terms}$$

$$\partial_\mu A_\mu = 2mP + 2iN_f q_L$$

$$Q_L \propto m \sum_x \mathrm{Tr}\left(\gamma_5 S\right)$$

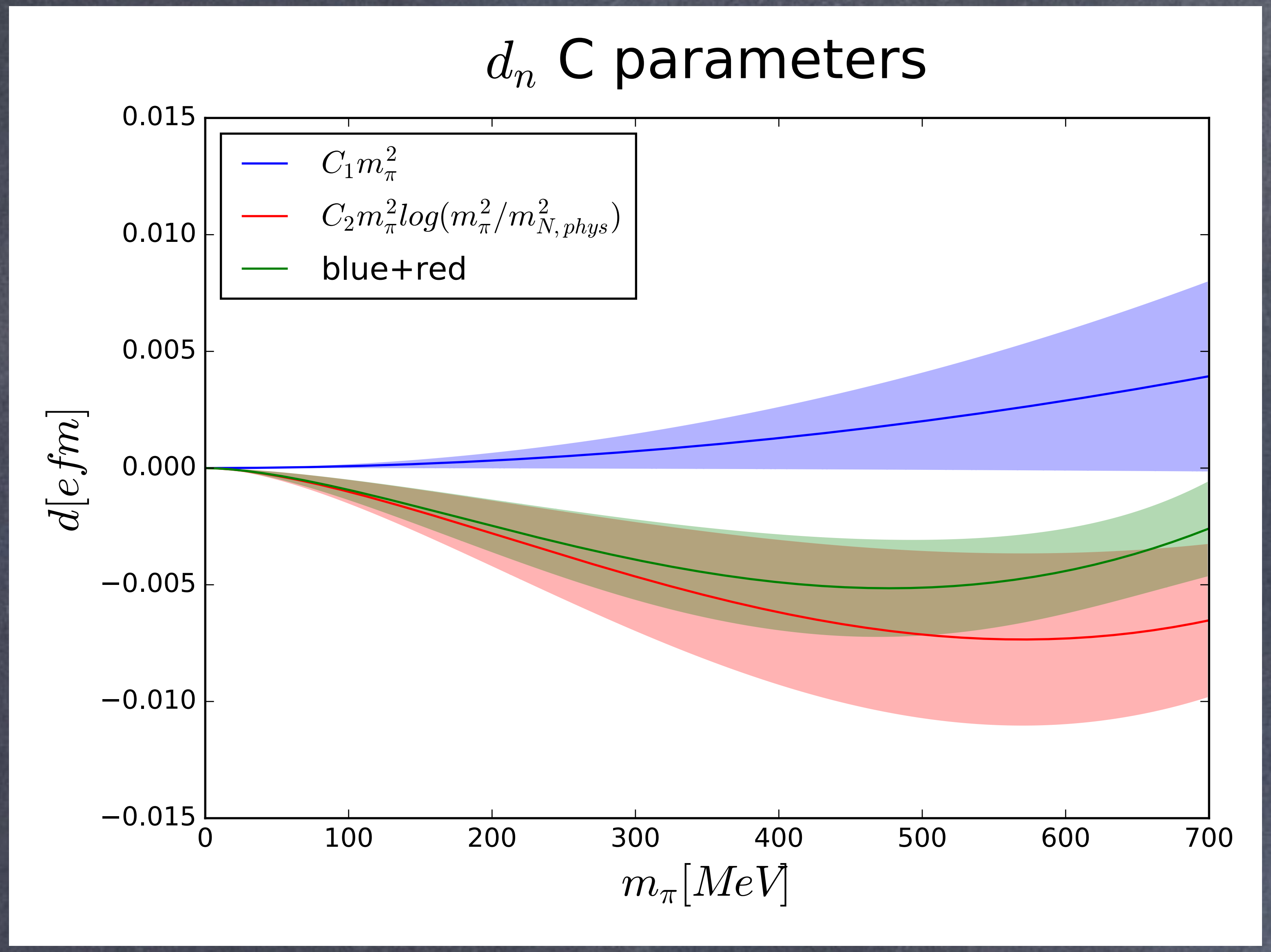
$$Q = n_+ - n_-$$

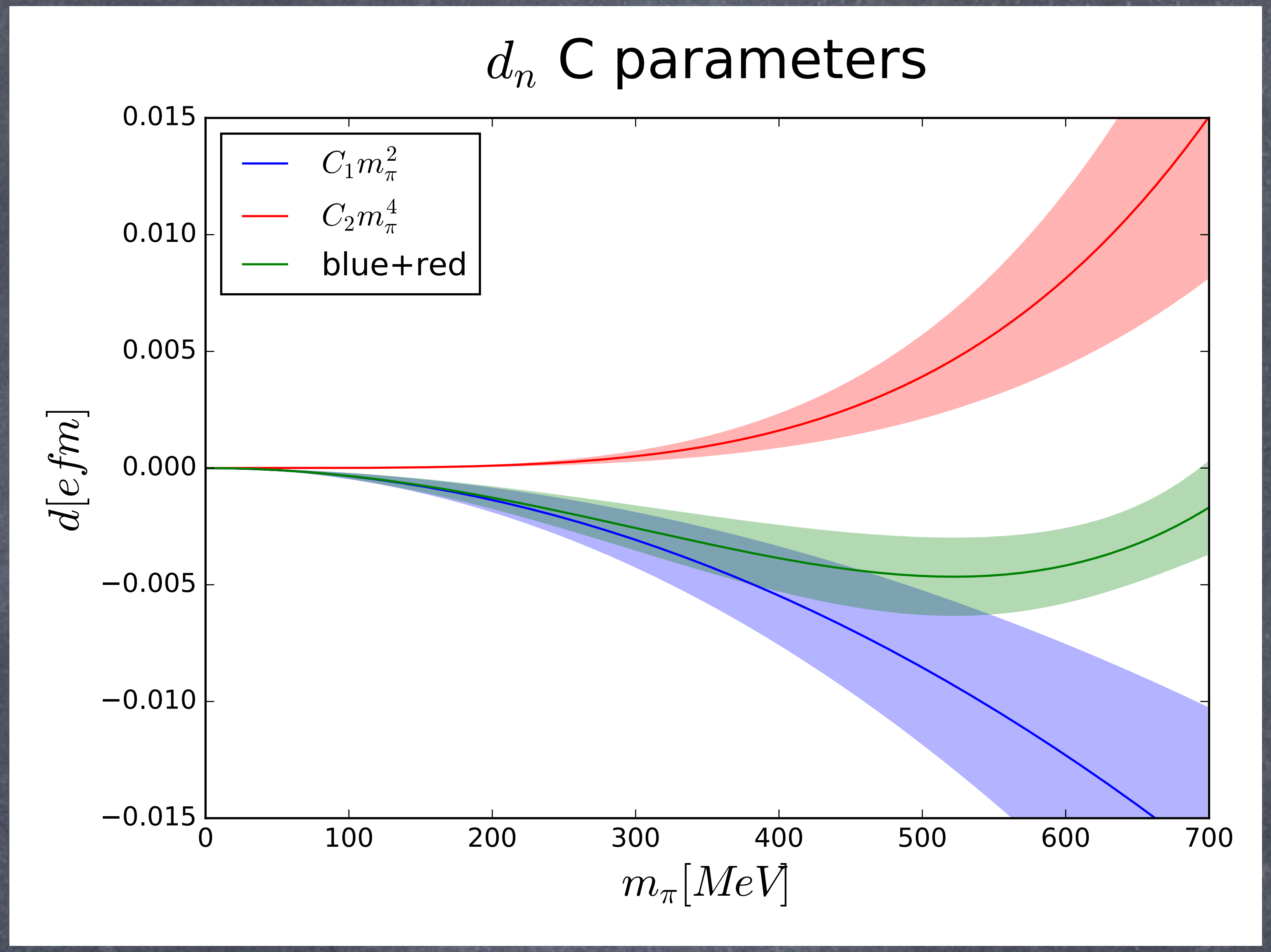
Atiyah, Singer: 1971

P. Hasenfratz: 1998
P. Hasenfratz, Laliena, Niedermeyer: 1998

ChPT-inspired fit

Dragos, Luu, A.S.,
de Vries, Yousif:
2019





$$d_{n/p}(m_\pi) = C_1^{n/p} m_\pi^2 + C_2^{n/p} m_\pi^2 \ln \frac{m_\pi^2}{M_N^2}$$

$$d_{n/p}(m_\pi) = C_1^{n/p} m_\pi^2 + C_2^{n/p} m_\pi^4$$

Data naturally favor the ChiPT-inspired pion mass dependence ==> log dominance

Topological charge

$$Q(t_f) = \int d^4x\ q(x,t_f) \qquad q(x,t_f) = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}\left\{G_{\mu\nu}(x,t_f)G_{\rho\sigma}(x,t_f)\right\}$$

$$\left\langle \cdots [Q(t_f)]_R \cdots \right\rangle = \left\langle \cdots Q(t_f) \cdots \right\rangle$$

Flow time fixed in physical units

$$\partial_{t_f} q(x,t_f) = \partial_\mu w_\mu(x,t_f) \qquad w_\mu(x,t_f) = \frac{1}{8\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}\left[G_{\nu\rho}(x,t_f)D_\alpha(t_f)G_{\alpha\sigma}(x,t_f)\right]$$

⇓

$$\partial_{t_f} Q(t_f) = 0 \qquad Q = \int d^4x\ q(x,t_f)$$

Polyakov: 1987
Lüscher: 2010
Giusti: 2015

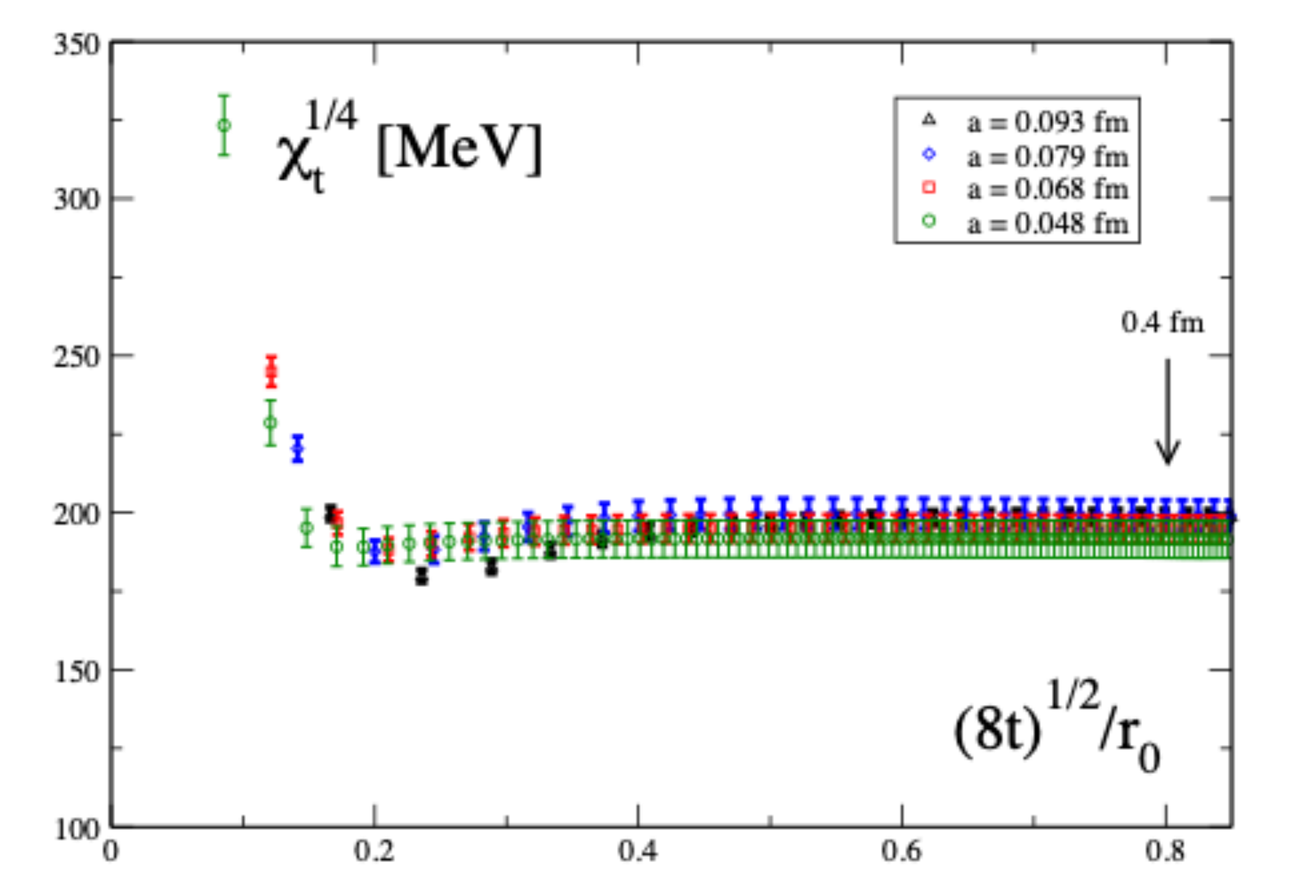
Topological charge

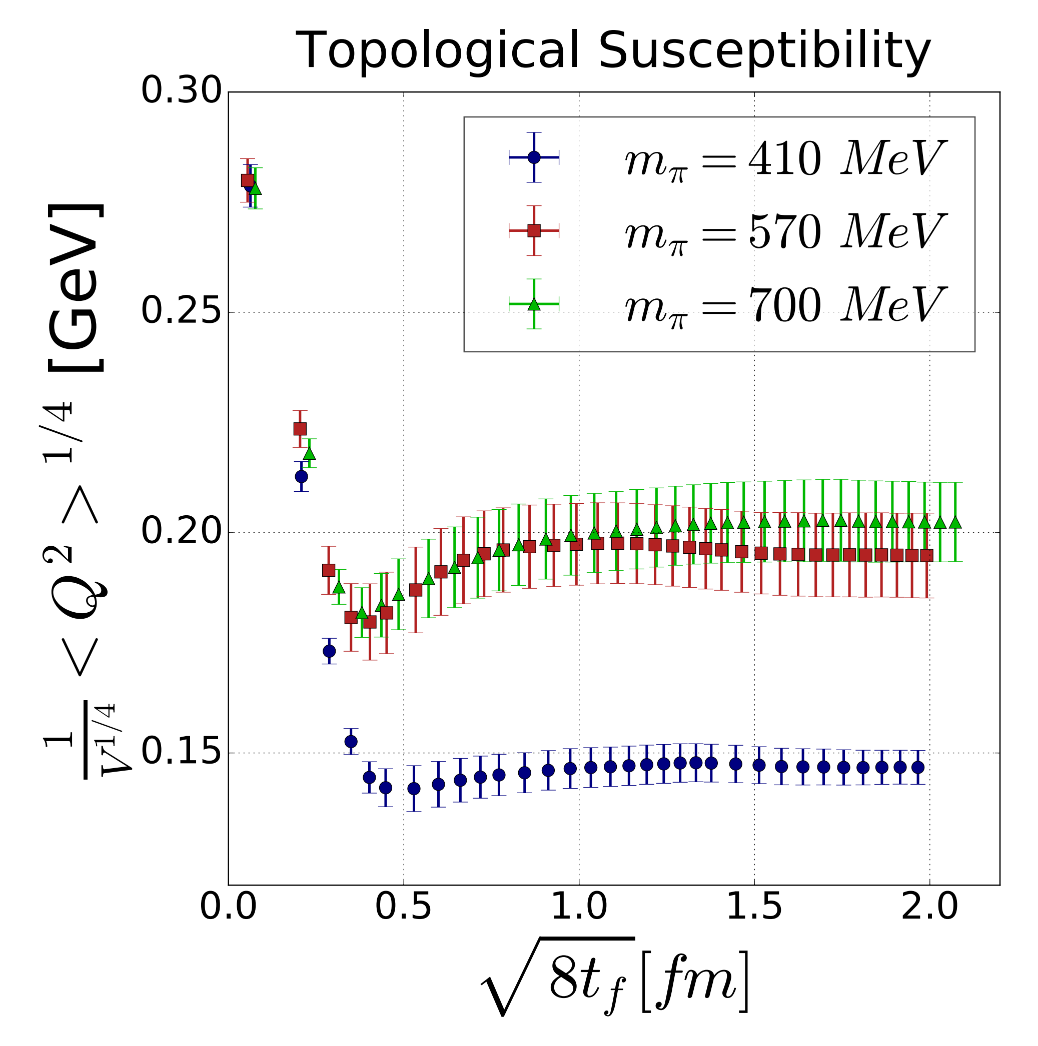
$$\chi_t^{1/4} = 191(7)\ \mathrm{MeV}$$

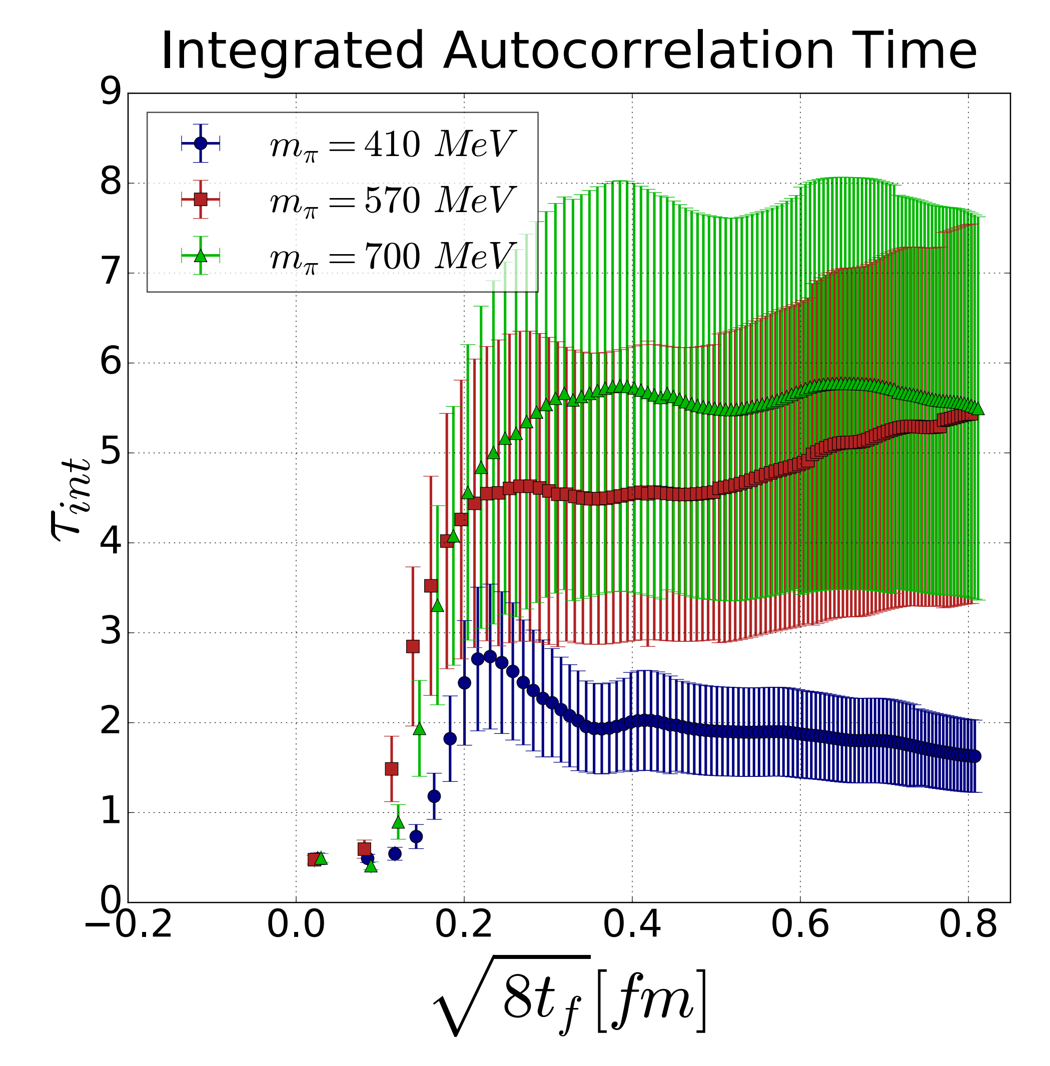
$$q(x,t_f) = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}\left\{G_{\mu\nu}(x,t_f)G_{\rho\sigma}(x,t_f)\right\}$$

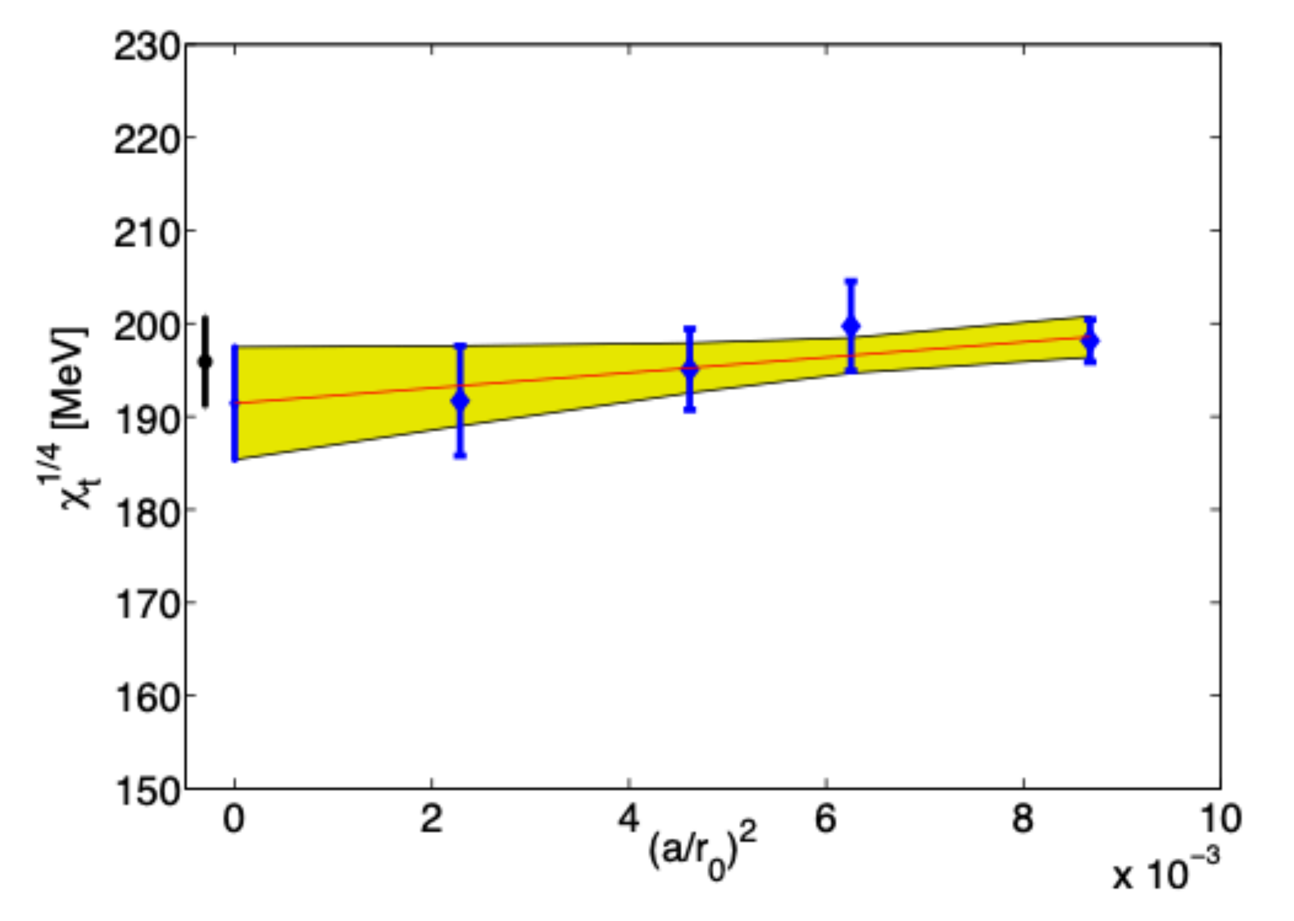
$$Q(t_f) = \int d^4x\ q(x,t_f)$$

$$\chi_t = \frac{1}{V}\int d^4x\ d^4y\ \langle q(x,t_f)q(y,t_f)\rangle$$



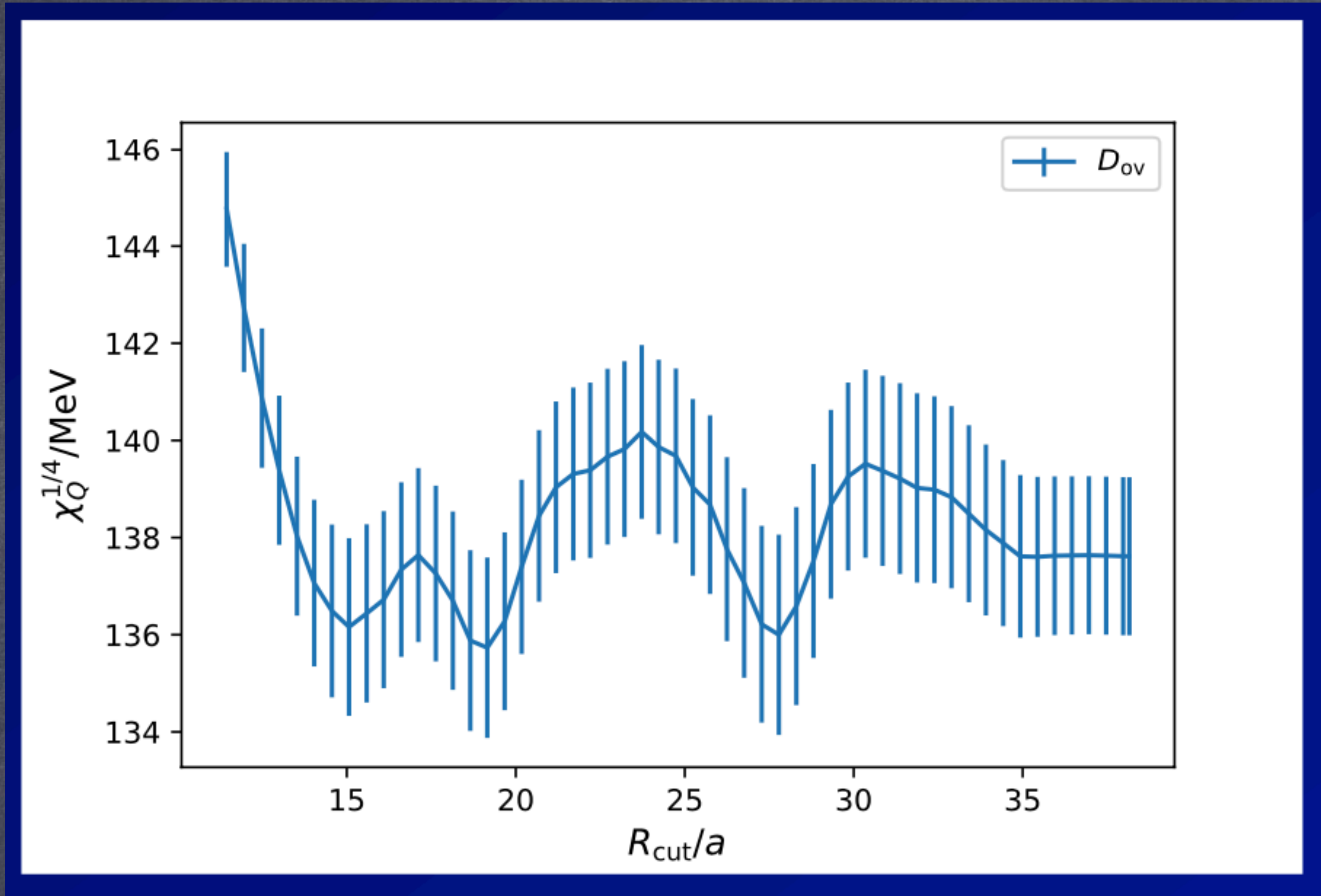


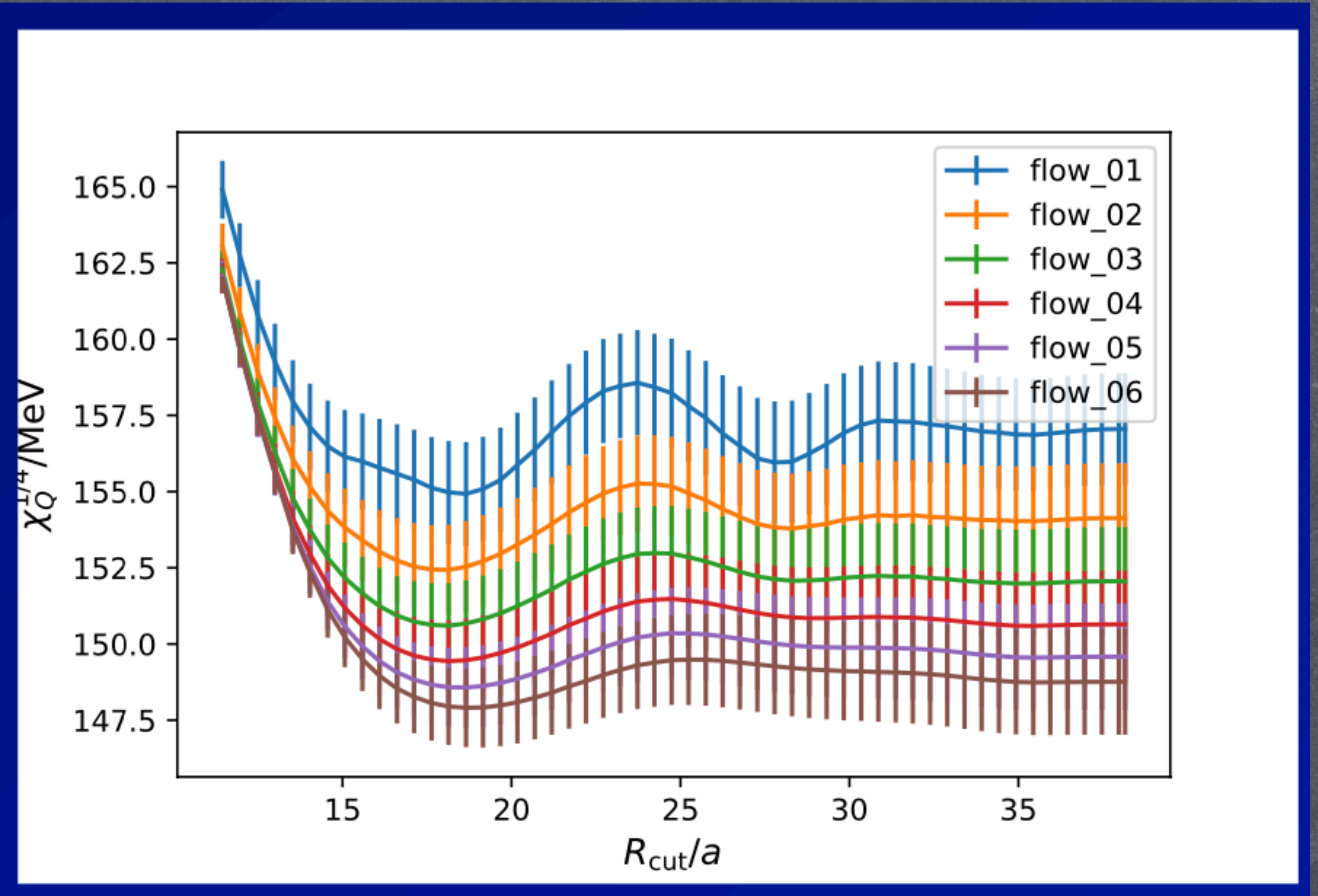


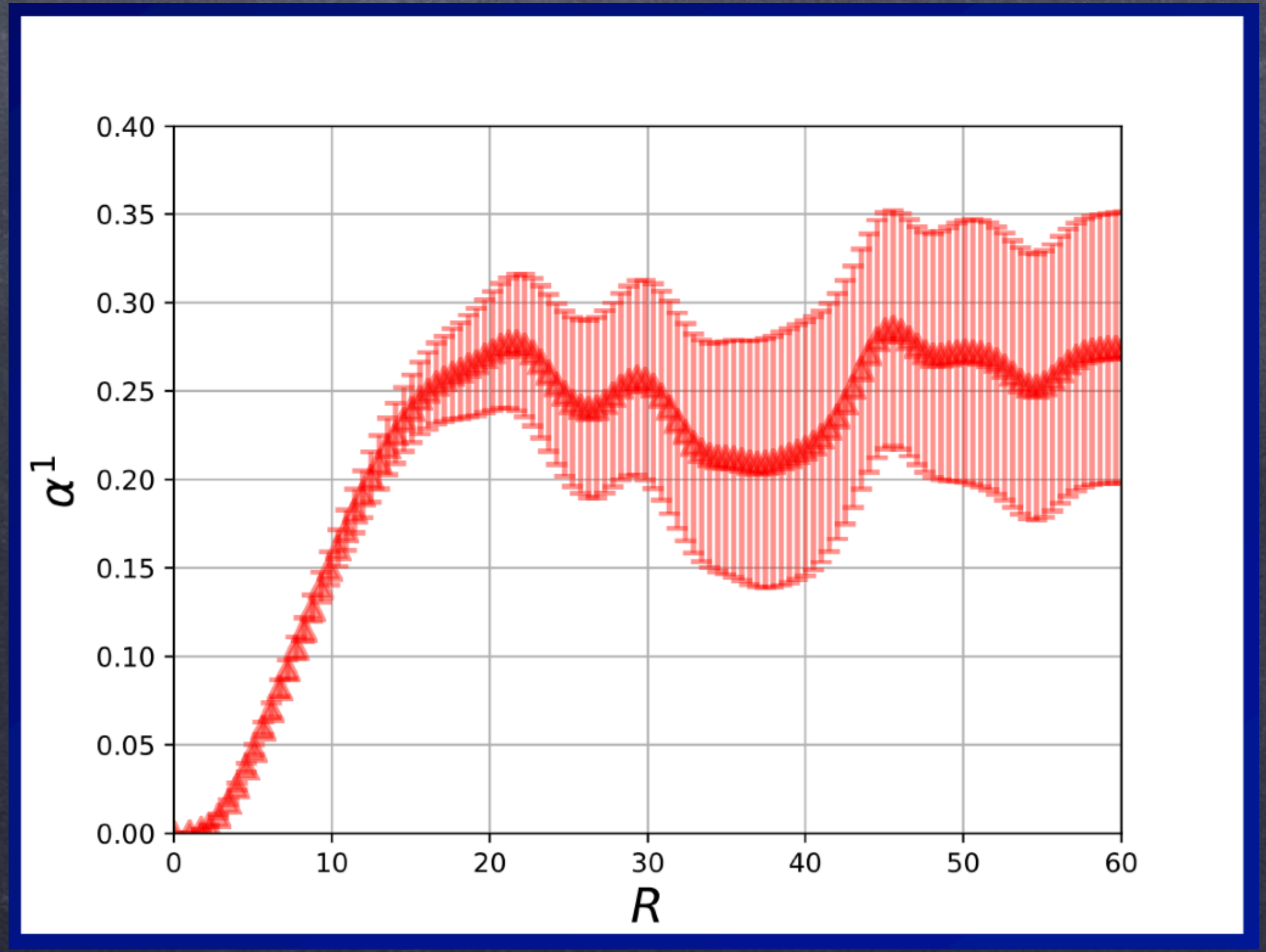


A.S., de Vries, Luu: 2014, 2015

Lattice QCD results



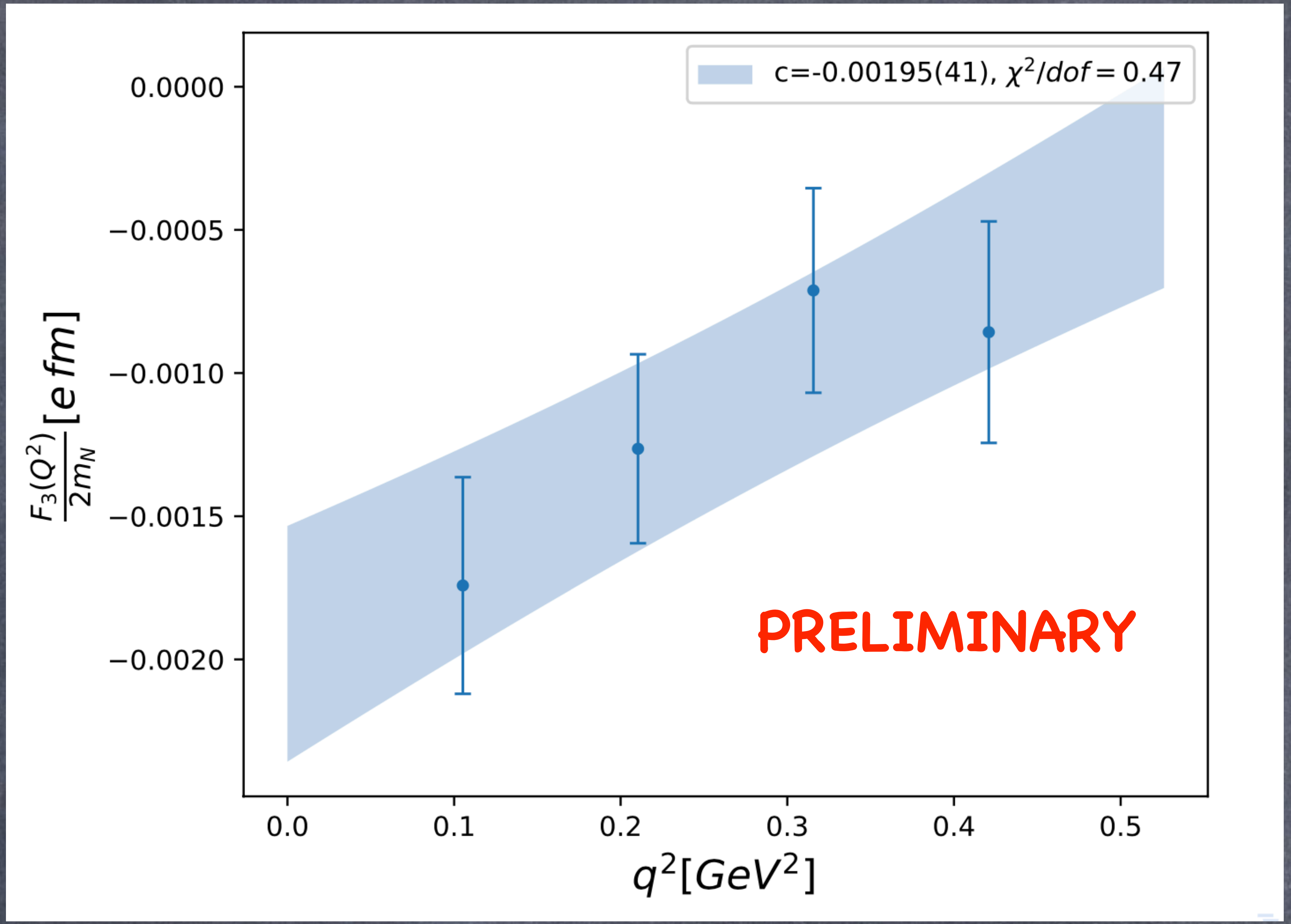




CP-odd form factor

Francis, Kim, Luu,
Pederiva, A.S.,
Zafeiropoulos

m_π = 400 MeV
t/t_0 = 1.9



$$\frac{F_3^{P/N}(Q^2)}{2M_N} = d_{P/N} - S_{P/N}Q^2 + H_{P/N}(Q^2)$$

$$\frac{d_P}{d_N} < 0 \qquad \frac{S_P}{S_N} < 0$$

Mereghetti et al.: 2011

Narayanan,
Neuberger:2006

Gradient flow

Lüscher 2010-2013
Lüscher, Weisz 2011

$$\boxed{\begin{aligned}&\partial_t B_\mu(x,t) = D_\nu(t) G_{\nu\mu}(x,t)\\[2em]&B_\mu(x,t)\big|_{t=0} = A_\mu(x)\end{aligned}}$$

$$D_\nu(t) = \partial_\nu + [B_\nu(x,t), \cdot]$$

$$x_\mu = (\underline{x}, x_4) \qquad t = \text{flow-time} \qquad [t] = -2$$

$$G_{\mu\nu}(x,t) = \partial_\mu B_\nu(x,t) - \partial_\nu B_\mu(x,t) + [B_\mu, B_\nu]$$

Continuous form of stout-smearing

Morningstar, Peardon: 2004

Scale setting

Lüscher: 2010
BMW:2012

Quasi-PDF

Monahan, Orginos: 2015-2017

RG flow and BSM

Carosso, Hasenfratz A., Neil,
Rebbi, Witzel: 2018 -

Energy-momentum tensor

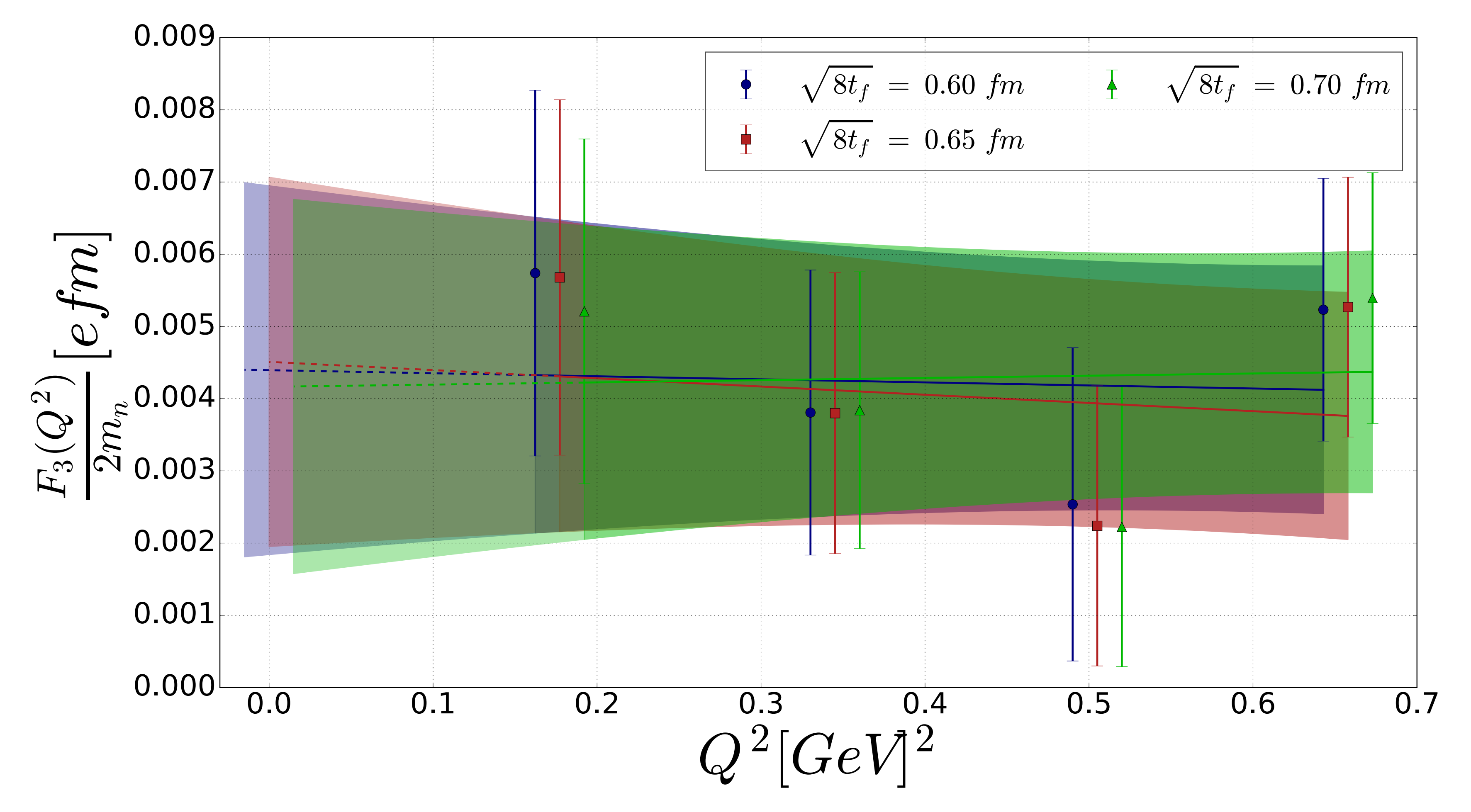
Del Debbio, Patella, Rago:2013
Suzuki et al. : 2013 -

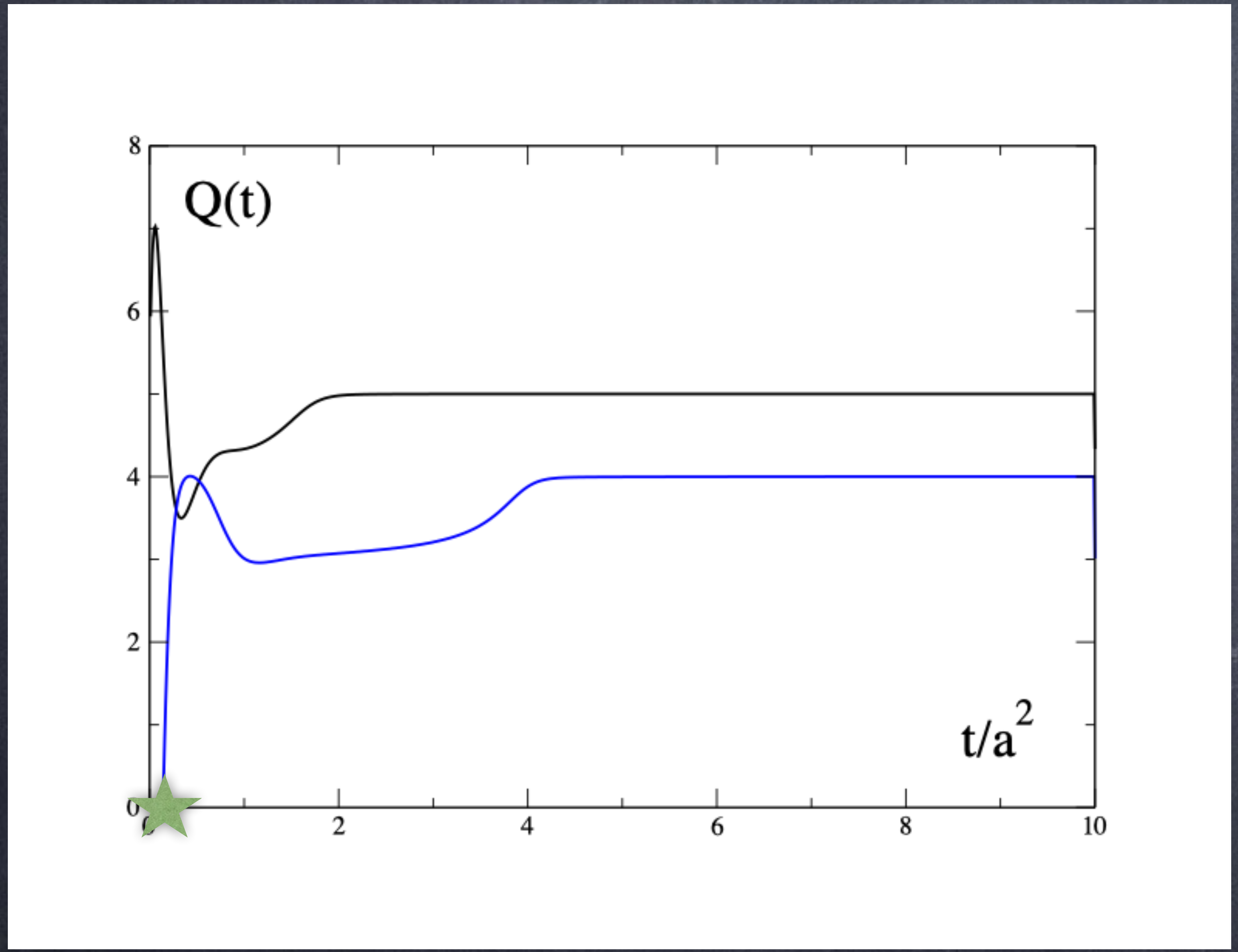
Topological charge

$$q(x,t) = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}\left\{G_{\mu\nu}(x,t)G_{\rho\sigma}(x,t)\right\}$$

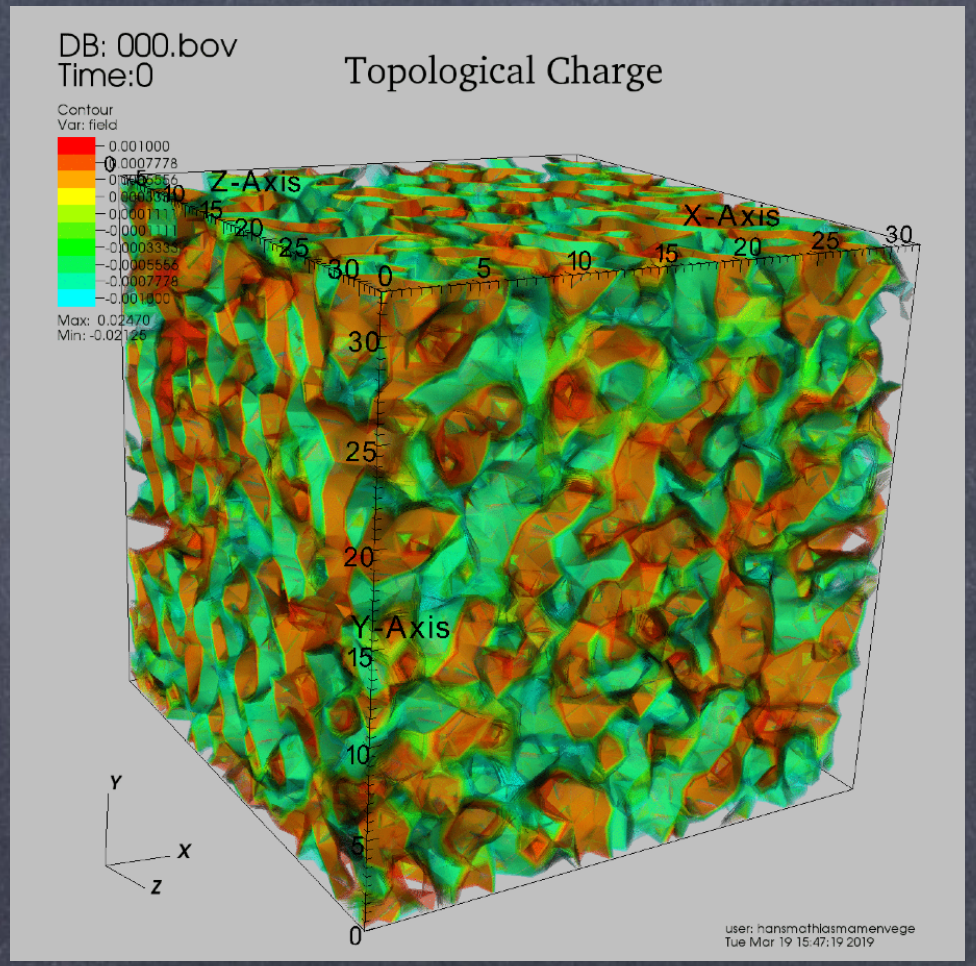
$$Q(t) = \int d^4x\; q(x,t) \qquad S[A] \geq \frac{8\pi^2}{g_0^2}|Q[A]|$$

Dragos, Luu, A.S.,
de Vries, Yousif: 2019





A.S., Luu, de Vries: 2014-2015



Pederiva, Vege: 2018
LatViz

$$\frac{F_3^{P/N}(Q^2)}{2M_N} = d_{P/N} - S_{P/N}Q^2 + H_{P/N}(Q^2)$$

$$\frac{d_P}{d_N} < 0 \qquad \frac{S_P}{S_N} < 0$$

Mereghetti et al.: 2011

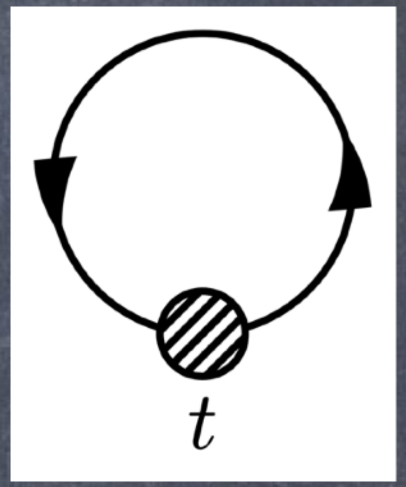
Quark-Chromo EDM

Rizik, Monahan, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer : 2021

$$Z_\chi^{-n/2} \left\langle (\psi)^{n_\psi} \, (\bar\psi)^{n_{\bar\psi}} \, (A_\mu)^{n_A} \, \mathcal{O}_i(t) \right\rangle^{\mathrm{amp}} = c_{ij}(t) \, \left(Z_{jk}^{\mathrm{MS}}\right)^{-1} \left\langle (\psi)^{n_\psi} \, (\bar\psi)^{n_{\bar\psi}} \, (A_\mu)^{n_A} \, \mathcal{O}_k \right\rangle^{\mathrm{amp}}$$

$$c_{ij}(t,\mu) = \delta_{ij} + \frac{\alpha_s(\mu)}{4\pi} c_{ij}^{(1)}(t,\mu) + O(\alpha_s^2)$$

$$\left\langle \mathring{\bar\chi}(x;t) \, \overleftrightarrow{\not{D}} \, \mathring{\chi}(x;t) \right\rangle = -\frac{2N_c N_f}{(4\pi)^2 t^2}$$



Makino, Suzuki: 2014

$$\chi_R(x;t) = (8\pi t)^{\varepsilon/2} \zeta_\chi^{1/2} \mathring{\chi}(x;t)$$

$$\bar\chi_R(x;t) = (8\pi t)^{\varepsilon/2} \zeta_\chi^{1/2} \mathring{\bar\chi}(x;t)$$

$$\zeta_\chi = 1 - \frac{\alpha_s C_F}{4\pi} \left(3 \log\left(8\pi\mu^2 t\right) - \log(432)\right) + O(\alpha_s^2)$$

2-loops

Harlander, Kluth, Lange :2018

Artz, Harlander, Lange,
Neumann, Prausa: 2019

Summary Lattice QCD θ-term EDM

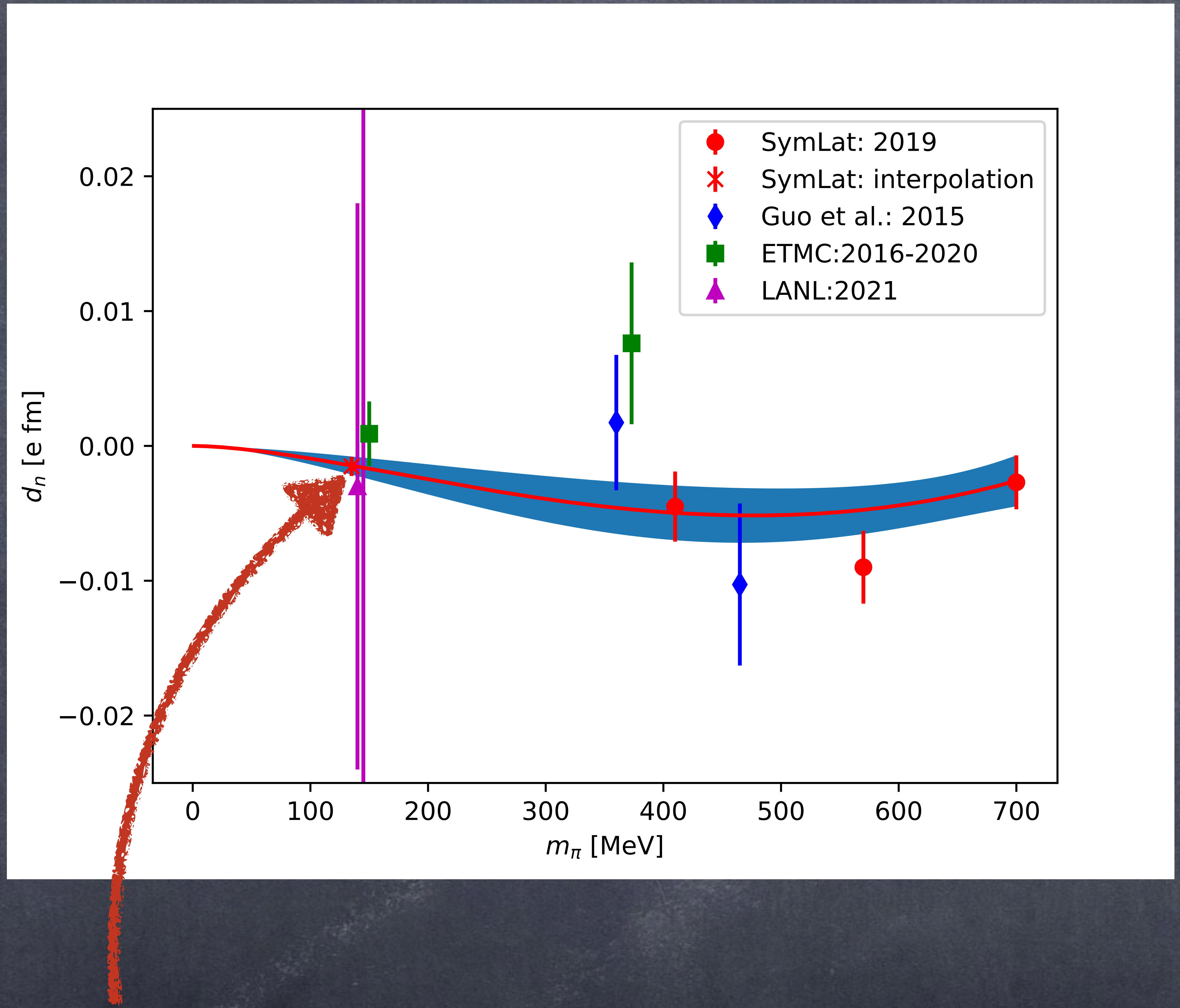
Abramczyk et al. : 2017

Shintani et al.
DWF+Iwasaki+Cooling

Guo et al.
Clover+Iwasaki+Imaginary theta +
Anomalous rotation

Alexandrou et al.
Clover-tm + Iwasaki +
Cooling/Spectral

Bhattacharya et al.
tree(TI)-Clover/HISQ +
Gradient Flow



Dragos, Luu, A.S., de Vries, Yousif
Clover+Wilson+Gradient flow: 2019

$$d_{{}^2\mathrm{H}} = 0.94(1)(d_n + d_p) + \left[0.18(2)\,\bar{g}_1\right] e\,\mathrm{fm}$$

$$d_{{}^3\mathrm{H}} = -0.03(1)d_n + 0.92(1)d_p - [0.11(1)\bar{g}_0 - 0.14(2)\bar{g}_1]\ e\,\mathrm{fm}$$

$$d_{{}^3\mathrm{He}} = 0.90(1)d_n - 0.03(1)d_p + [0.11(1)\bar{g}_0 + 0.14(2)\bar{g}_1]\ e\,\mathrm{fm}$$

de Vries et al.: 2011
Bsaisou et al. :2015

$$d_{{}^2\mathrm{H}}(\bar{\theta}) = \quad 0.2(1.2) \times 10^{-3}\,\bar{\theta}\,e\,\mathrm{fm}$$

$$d_{{}^3\mathrm{H}}(\bar{\theta}) = \quad 3.2(1.0) \times 10^{-3}\,\bar{\theta}\,e\,\mathrm{fm}$$

$$d_{{}^3\mathrm{He}}(\bar{\theta}) = -2.5(0.8) \times 10^{-3}\,\bar{\theta}\,e\,\mathrm{fm}$$

Dragos, Luu, A.S.,
de Vries, Yousif: 2019

Quark-Chromo EDM

$$Z_\chi^{-n/2}\left\langle \left(\psi^{(0)}\right)^{n_\psi} \left(\bar\psi^{(0)}\right)^{n_{\bar\psi}} \left(G_\mu^{(0)}\right)^{n_G} \mathcal{O}_i^t[\chi^{(0)},\bar\chi^{(0)},B^{(0)}]\right\rangle^{\mathrm{amp}} =$$

Rizik, Monahan, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer : 2021

$$= c_{ij}(t)\ \left(Z_{jk}^{\mathrm{MS}}\right)^{-1}\left\langle \left(\psi^{(0)}\right)^{n_\psi} \left(\bar\psi^{(0)}\right)^{n_{\bar\psi}} \left(G_\mu^{(0)}\right)^{n_G} \mathcal{O}_k^{(0)}[\psi^{(0)},\bar\psi^{(0)},G^{(0)}]\right\rangle^{\mathrm{amp}}$$

$$c_{ij}(t,\mu)=\delta_{ij}+\frac{\alpha_s(\mu)}{4\pi}c_{ij}^{(1)}(t,\mu)+O(\alpha_s^2)$$

$$\left\langle \mathring{\bar\chi}(x;t)\overleftrightarrow{\not D}\,\mathring{\chi}(x;t)\right\rangle = -\frac{2N_cN_f}{(4\pi)^2t^2}$$

Makino, Suzuki: 2014

$$\chi(x;t)=(8\pi t)^{\varepsilon/2}\zeta_\chi^{1/2}\mathring{\chi}(x;t)$$

$$\zeta_\chi = 1-\frac{\alpha_s C_F}{4\pi}\left(3\log\left(8\pi\mu^2 t\right)-\log(432)\right)+O(\alpha_s^2)$$

Harlander, Kluth, Lange :2018

$$\bar\chi(x;t)=(8\pi t)^{\varepsilon/2}\zeta_\chi^{1/2}\mathring{\bar\chi}(x;t)$$

Artz, Harlander, Lange,
Neumann, Prausa: 2019

Perturbation theory with flowed fields

Lüscher, Weisz:2010,2011
Lüscher: 2013

Rizik, Monhahan, A.S.:
2018, 2020

$$B_\mu(x;t) = \int \mathrm{d}^d y \Big[K_{\mu\nu}(x-y;t)A_\nu(y) + \int_0^t \mathrm{d}s K_{\mu\nu}(x-y;t-s)R_\nu(y;s)\Big],$$

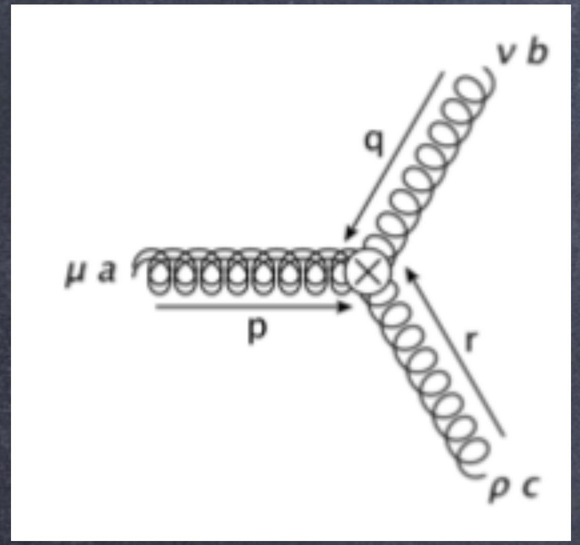
$$\chi(x,t) = \int \mathrm{d}^d y \left[J(x-y;t)\psi(y) + \int_0^t \mathrm{d}s\, J(x-y;t-s)\Delta'\chi(y;s)\right],$$

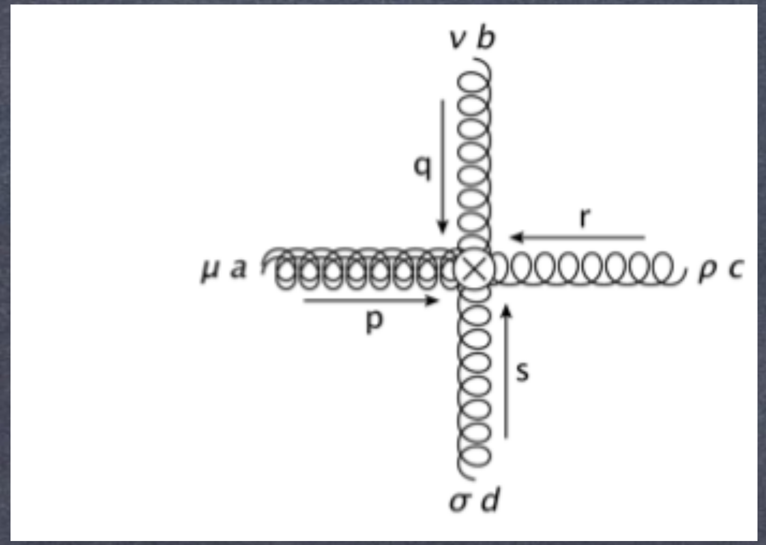
$$\overline{\chi}(x,t) = \int \mathrm{d}^d y \left[\overline{\psi}(y)\bar{J}(x-y;t) + \int_0^t \mathrm{d}s\, \overline{\chi}(y;s)\overleftarrow{\Delta}'\bar{J}(x-y;t-s)\right].$$

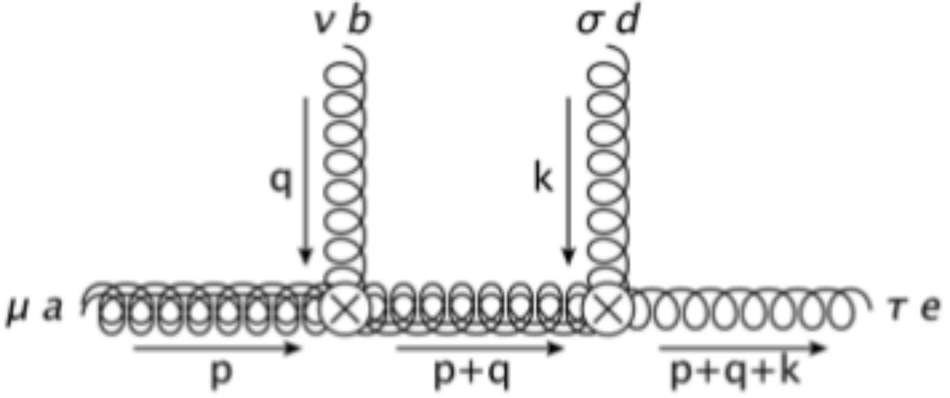
$$\partial_t \chi_t = \Delta \chi_t \qquad \partial_t \overline{\chi_t} = \overline{\chi_t}\overleftarrow{\Delta}$$

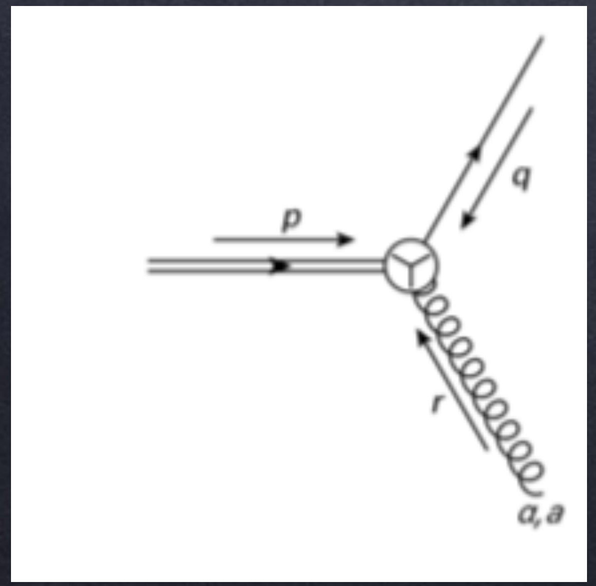
$$\chi_t(x)|_{t=0} = \psi(x)$$

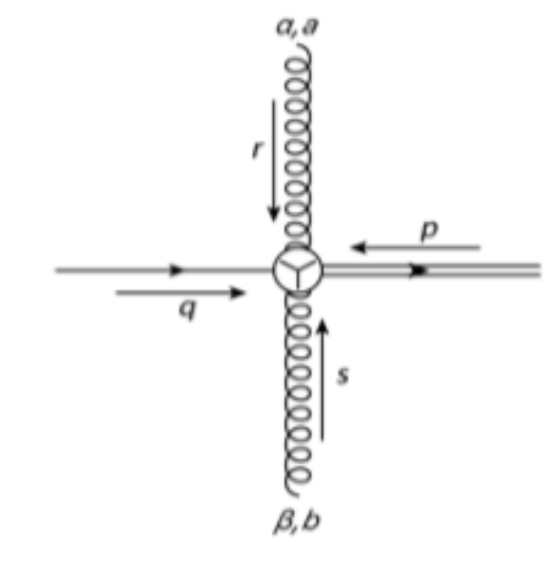
$$\overline{\chi}_t(x)|_{t=0} = \overline{\psi}(x)$$

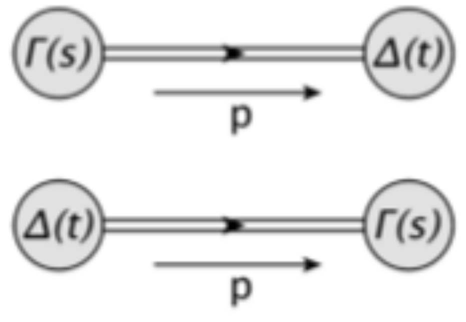












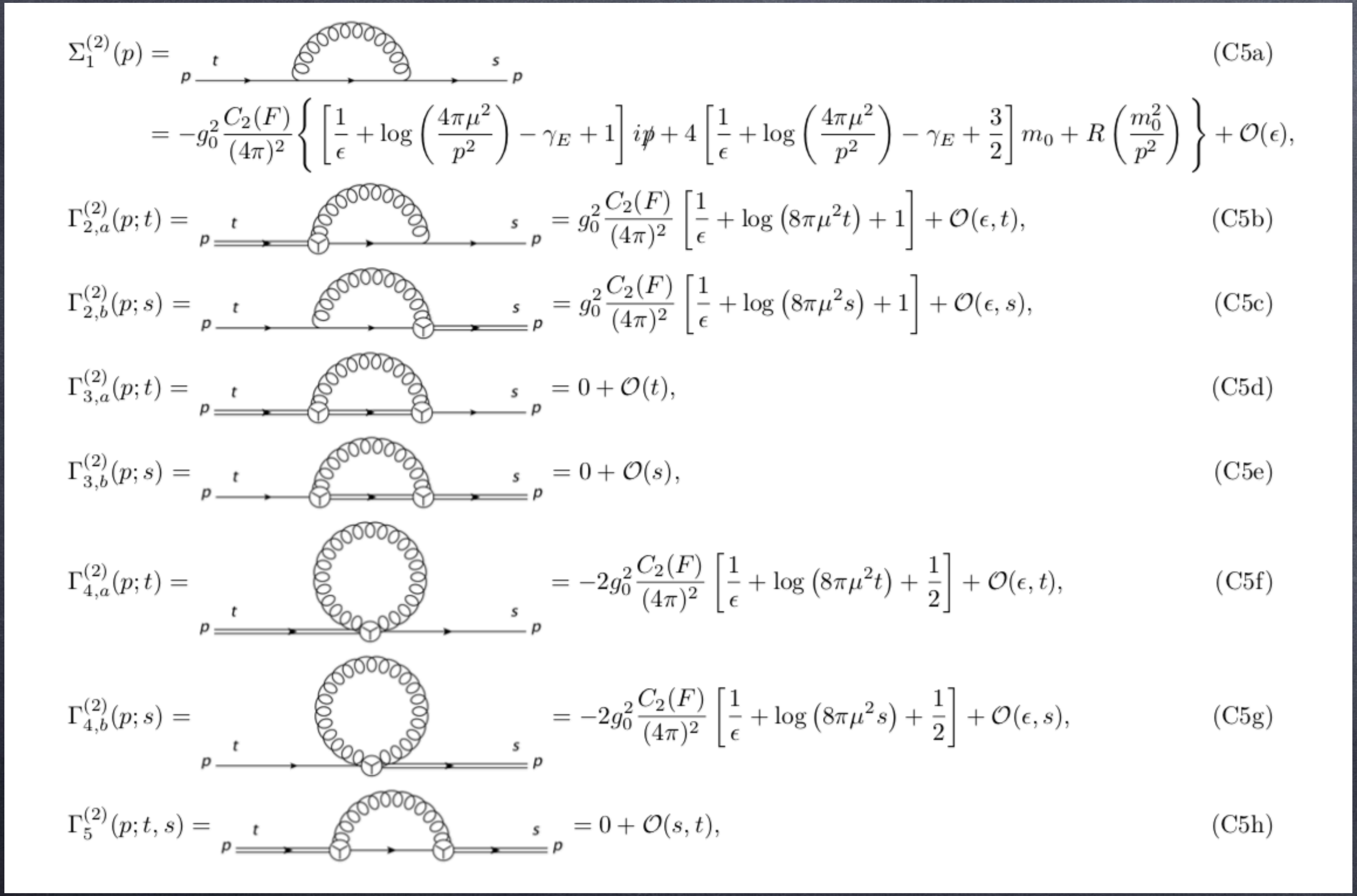
$$= \int_0^\infty ds\ \theta(t-s)\Delta(t)\tilde{J}_{t-s}(p)\Gamma(s)\,,$$

$$= \int_0^\infty ds\ \theta(t-s)\Gamma(s)\tilde{\bar{J}}_{t-s}(p)\Delta(t)\,,$$

Sample calculation: quark propagator

Lüscher: 2013

Rizik, Monhahan, A.S.:
2018, 2020



$$\Sigma_1^{(2)}(p) = \qquad \text{(C5a)}$$

$$= -g_0^2 \frac{C_2(F)}{(4\pi)^2} \left\{ \left[\frac{1}{\epsilon} + \log\left(\frac{4\pi\mu^2}{p^2}\right) - \gamma_E + 1 \right] i\not{p} + 4 \left[\frac{1}{\epsilon} + \log\left(\frac{4\pi\mu^2}{p^2}\right) - \gamma_E + \frac{3}{2} \right] m_0 + R\left(\frac{m_0^2}{p^2}\right) \right\} + \mathcal{O}(\epsilon),$$

$$\Gamma_{2,a}^{(2)}(p;t) = \qquad = g_0^2 \frac{C_2(F)}{(4\pi)^2} \left[\frac{1}{\epsilon} + \log\left(8\pi\mu^2 t\right) + 1 \right] + \mathcal{O}(\epsilon, t), \qquad \text{(C5b)}$$

$$\Gamma_{2,b}^{(2)}(p;s) = \qquad = g_0^2 \frac{C_2(F)}{(4\pi)^2} \left[\frac{1}{\epsilon} + \log\left(8\pi\mu^2 s\right) + 1 \right] + \mathcal{O}(\epsilon, s), \qquad \text{(C5c)}$$

$$\Gamma_{3,a}^{(2)}(p;t) = \qquad = 0 + \mathcal{O}(t), \qquad \text{(C5d)}$$

$$\Gamma_{3,b}^{(2)}(p;s) = \qquad = 0 + \mathcal{O}(s), \qquad \text{(C5e)}$$

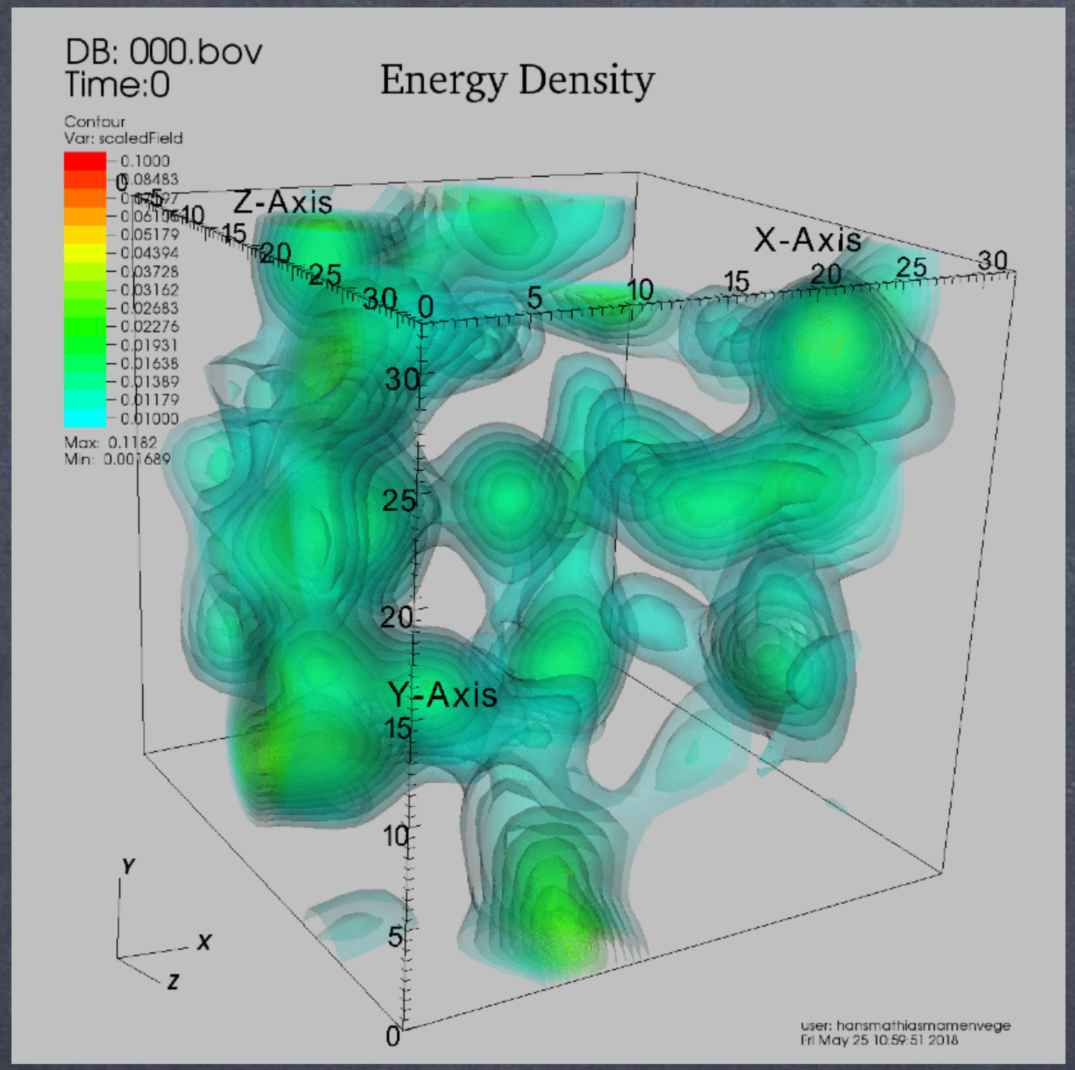
$$\Gamma_{4,a}^{(2)}(p;t) = \qquad = -2g_0^2 \frac{C_2(F)}{(4\pi)^2} \left[\frac{1}{\epsilon} + \log\left(8\pi\mu^2 t\right) + \frac{1}{2} \right] + \mathcal{O}(\epsilon, t), \qquad \text{(C5f)}$$

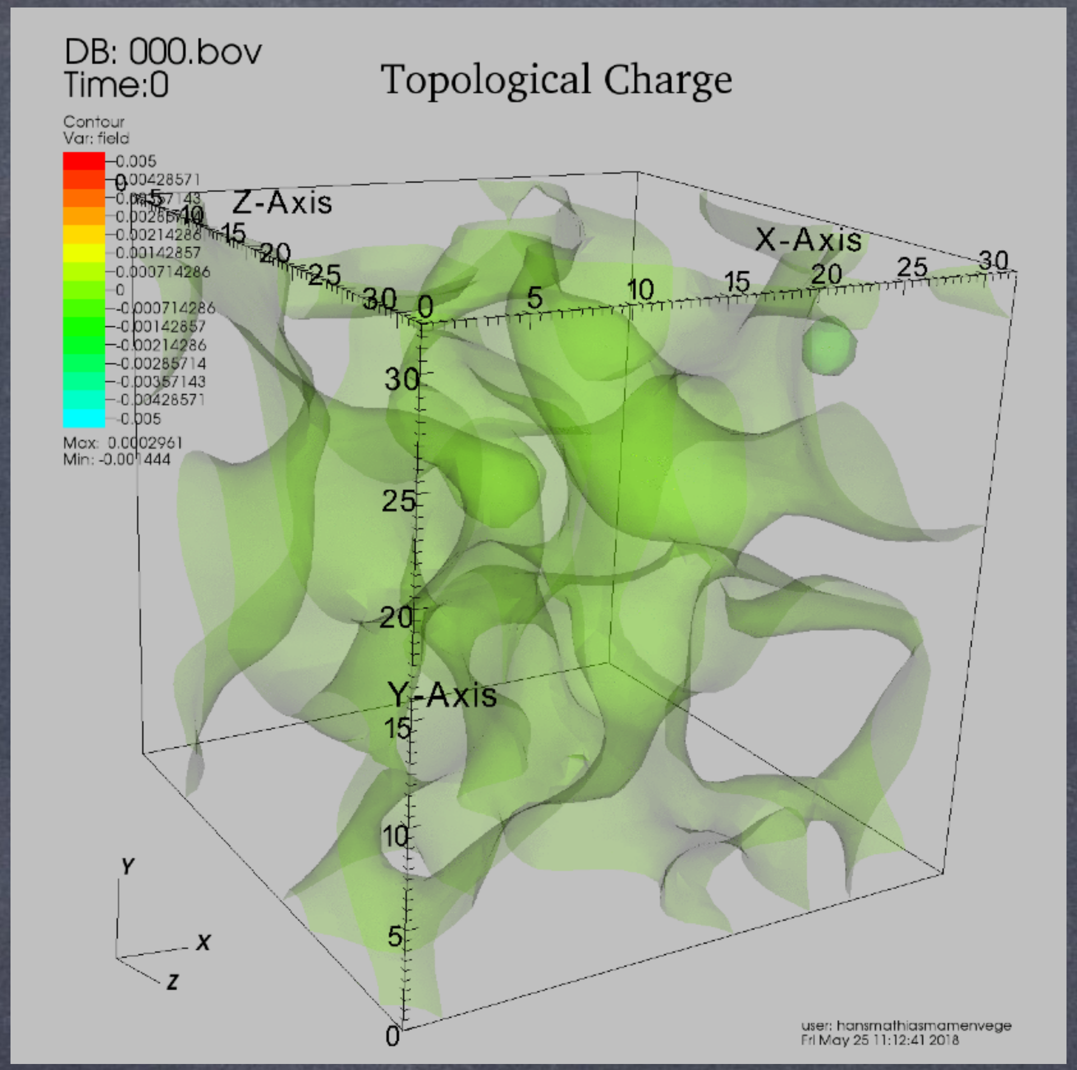
$$\Gamma_{4,b}^{(2)}(p;s) = \qquad = -2g_0^2 \frac{C_2(F)}{(4\pi)^2} \left[\frac{1}{\epsilon} + \log\left(8\pi\mu^2 s\right) + \frac{1}{2} \right] + \mathcal{O}(\epsilon, s), \qquad \text{(C5g)}$$

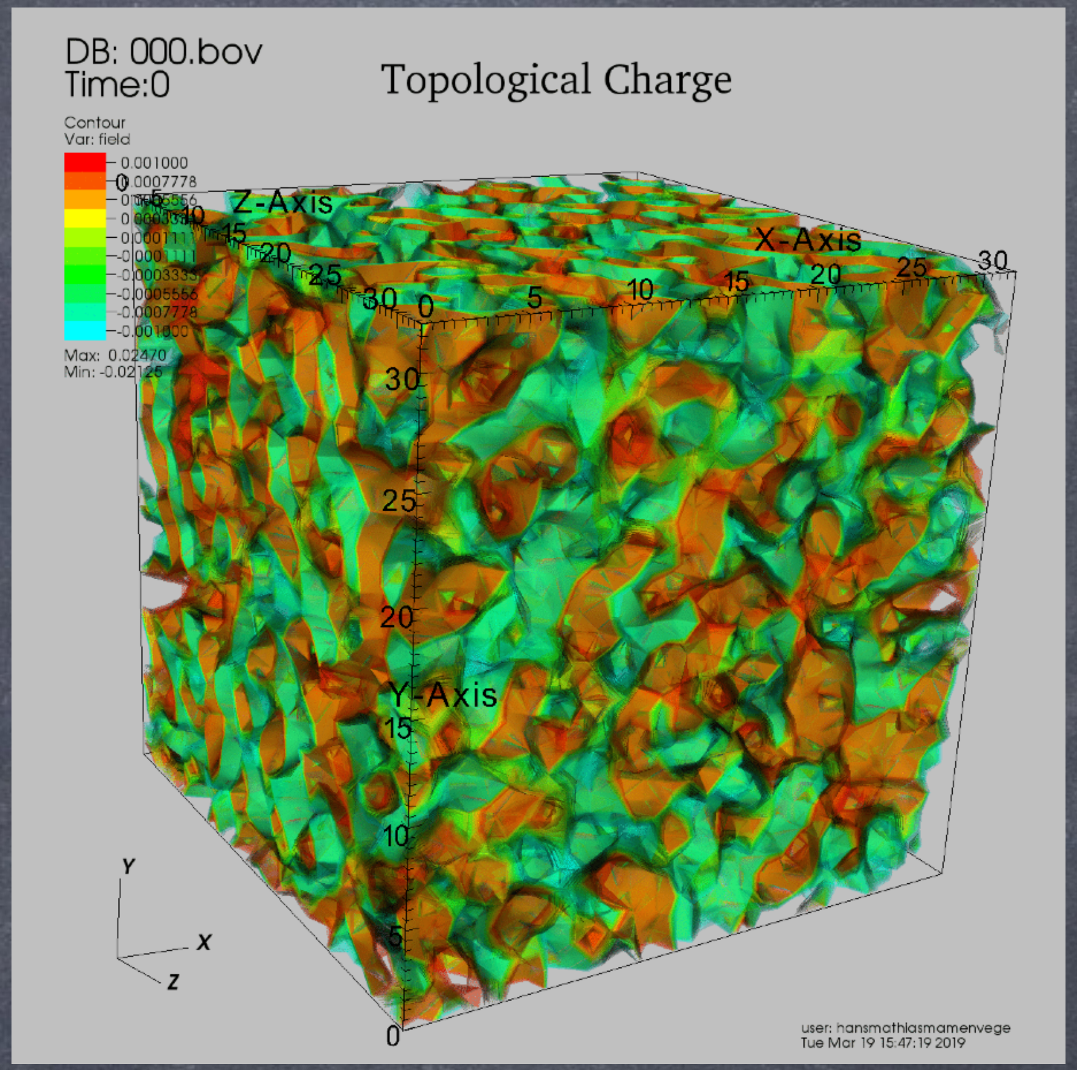
$$\Gamma_5^{(2)}(p;t,s) = \qquad = 0 + \mathcal{O}(s,t), \qquad \text{(C5h)}$$

$$Z_\chi = 1 + g_0^2 \frac{C_2(F)}{(4\pi)^2} \left\{ \frac{3}{\epsilon} + \log(4\pi) - \gamma_E + 1 \right\}$$

LatViz







Pederiva, Vege: 2018

Numerical details

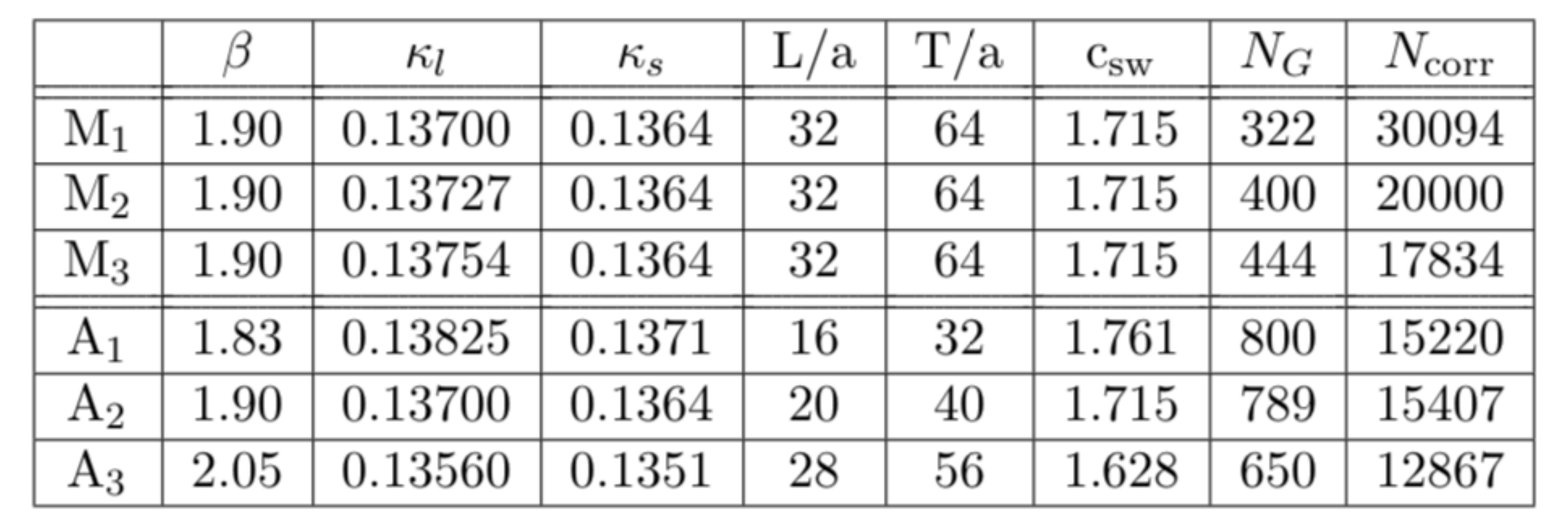
Dragos, Luu, A.S.,
de Vries, Yousif: 2019

NP improved Wilson +
Iwasaki gauge

a=0.1-0.068 fm
mpi=400-700 MeV

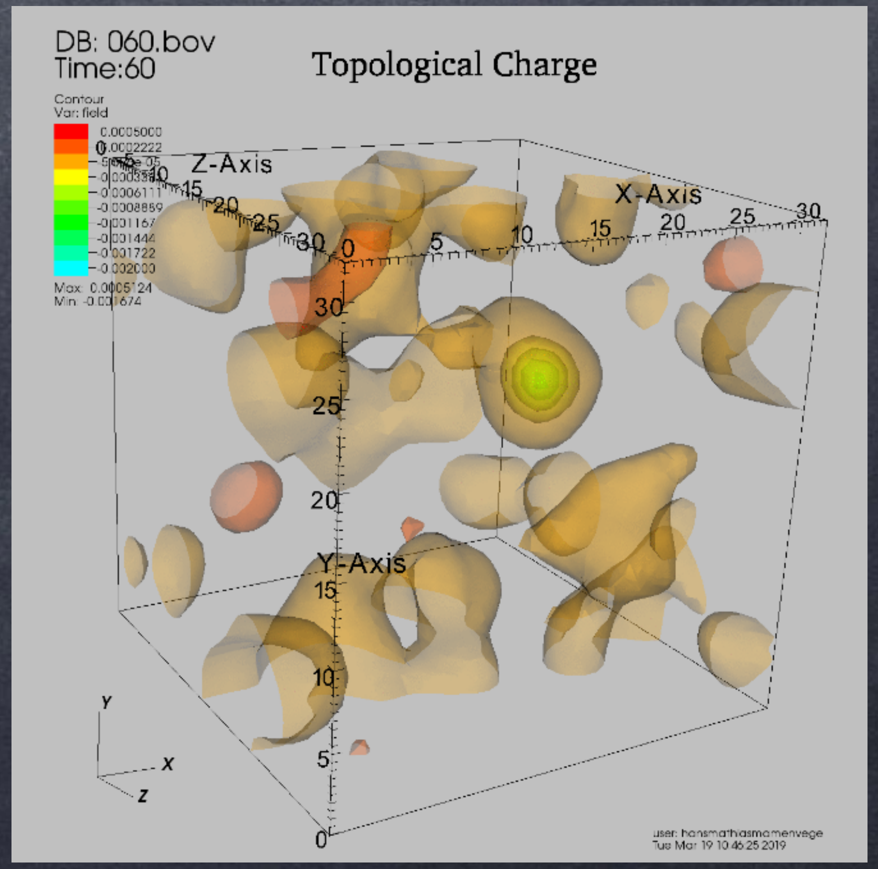
O(L/2a) Stochastic
source locations

3 Gaussian smearings



	β	κ_l	κ_s	L/a	T/a	c_{sw}	N_G	N_{corr}
M_1	1.90	0.13700	0.1364	32	64	1.715	322	30094
M_2	1.90	0.13727	0.1364	32	64	1.715	400	20000
M_3	1.90	0.13754	0.1364	32	64	1.715	444	17834
A_1	1.83	0.13825	0.1371	16	32	1.761	800	15220
A_2	1.90	0.13700	0.1364	20	40	1.715	789	15407
A_3	2.05	0.13560	0.1351	28	56	1.628	650	12867

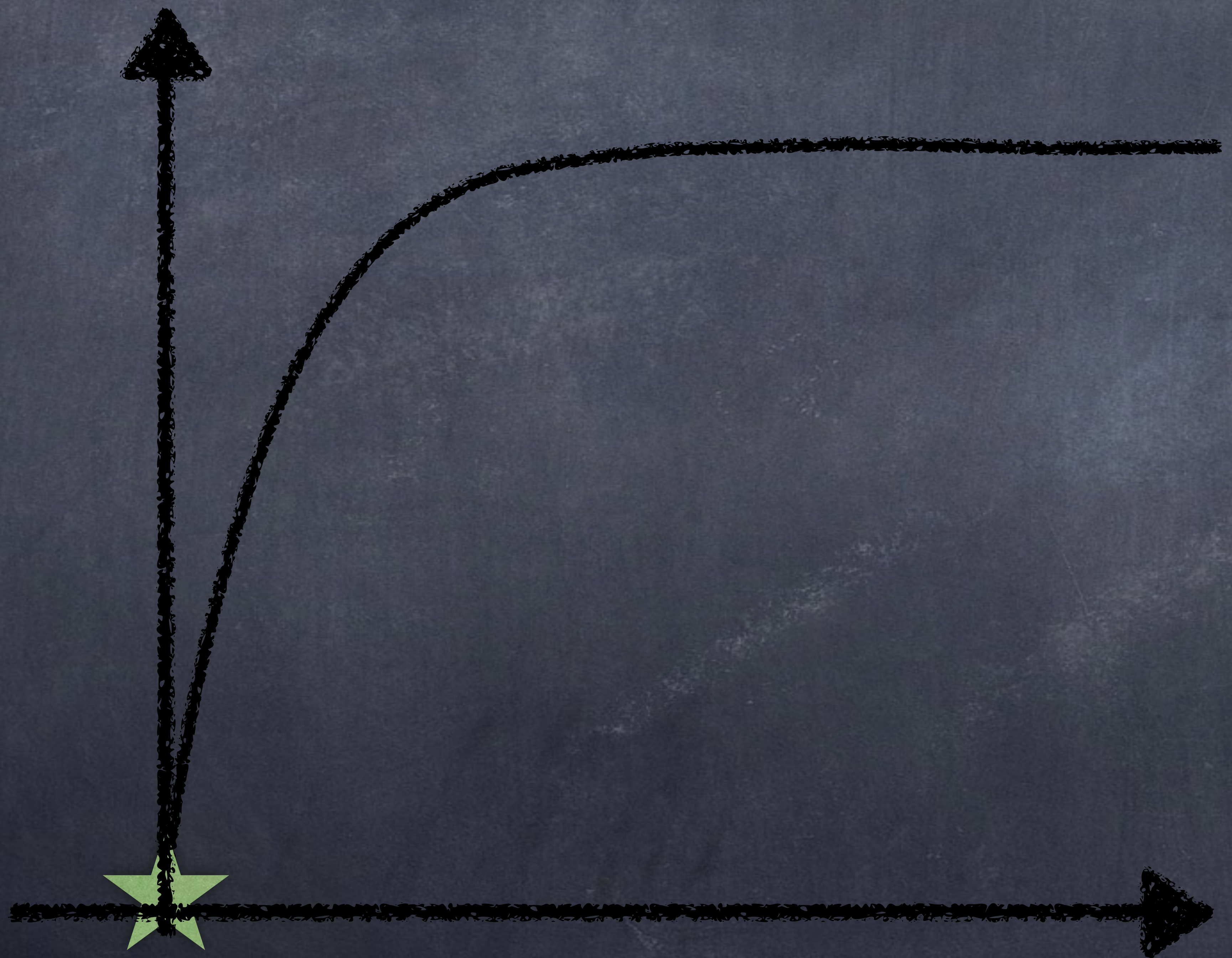
PACS-CS: 2009



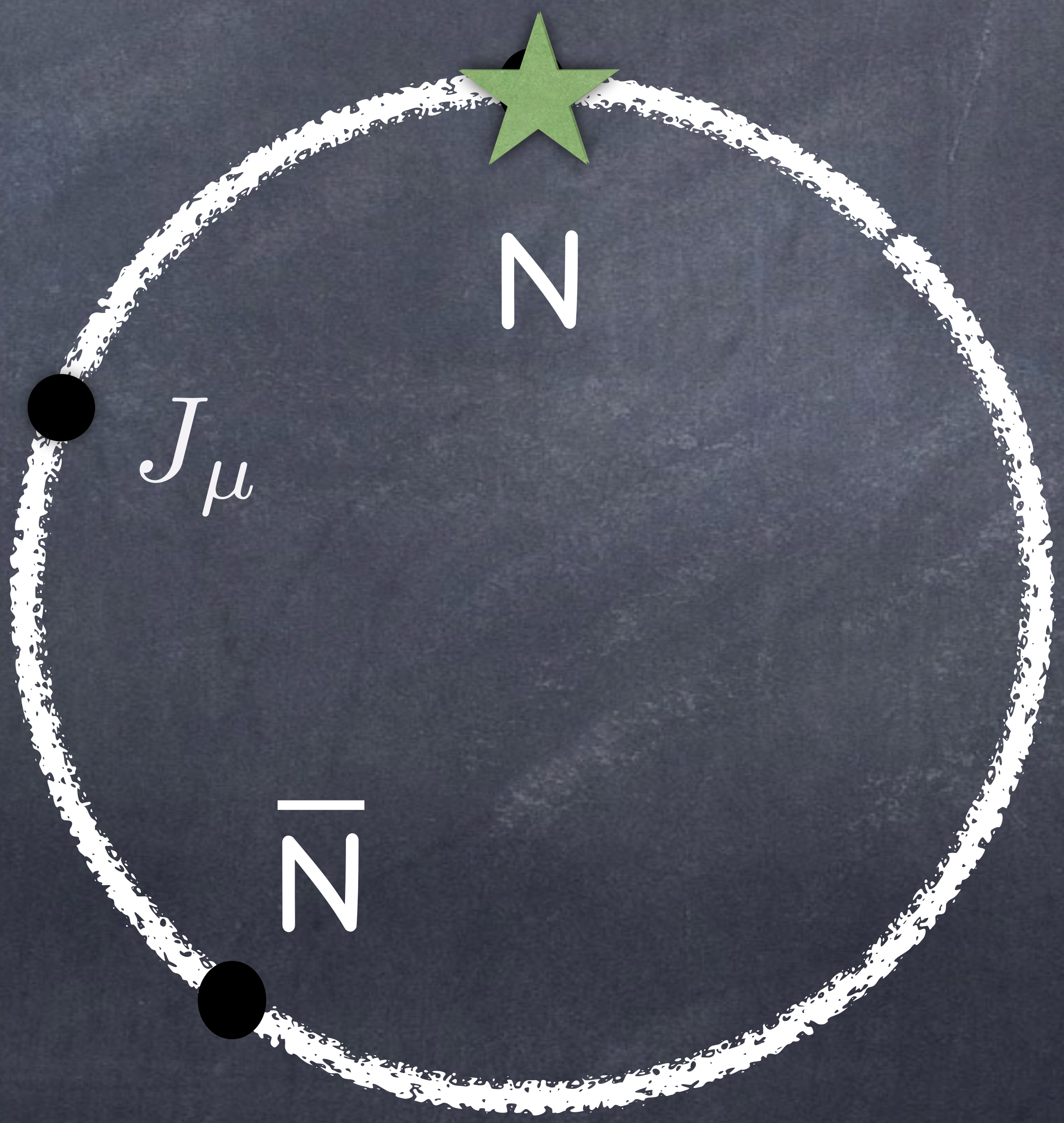
Signal-to-noise improvement

Is there a space-time region dominated by noise that can be neglected in the 4-d integration?

★ = q(t)



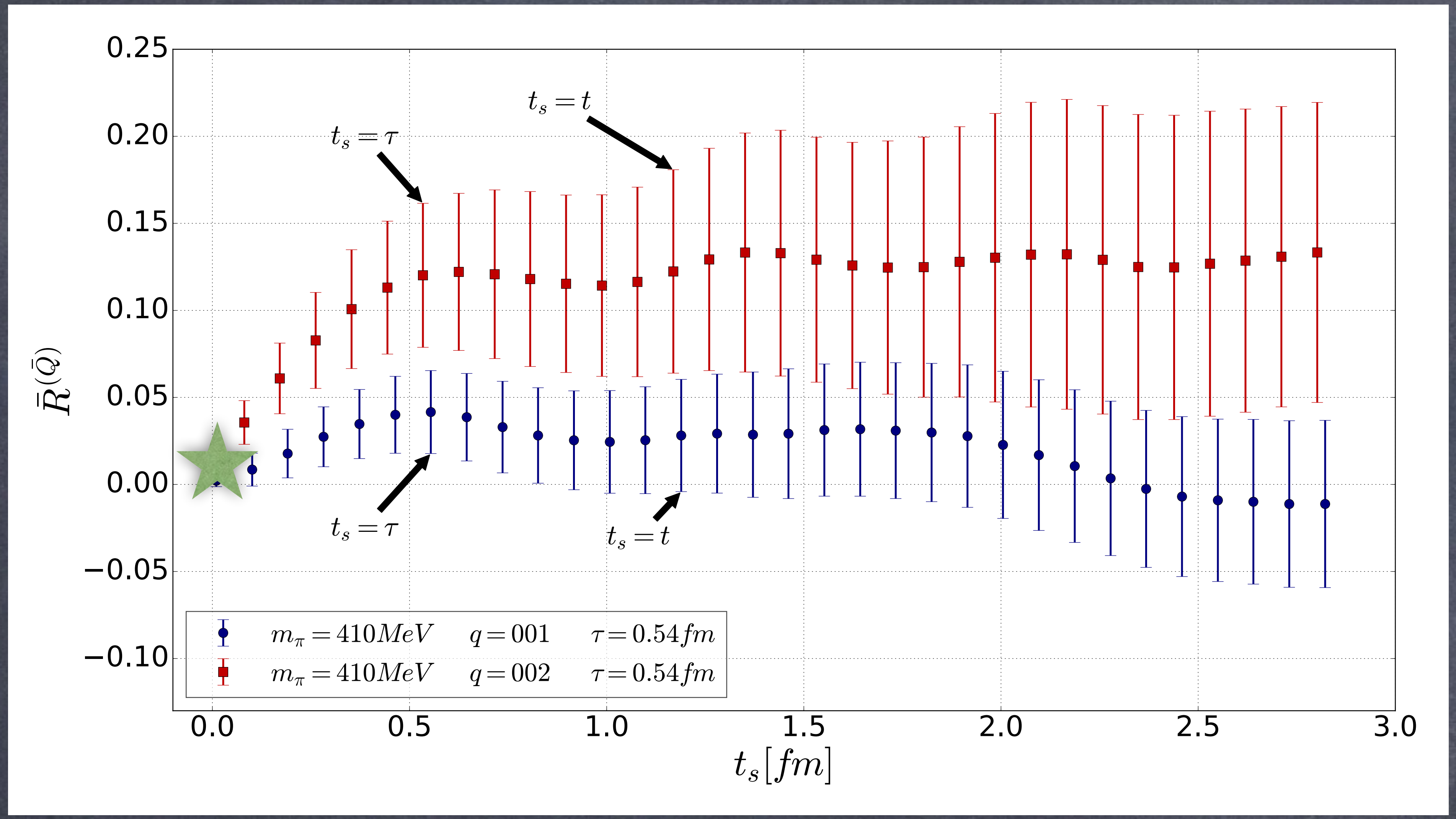
Dragos, Luu, A.S.,
de Vries, Yousif: 2019

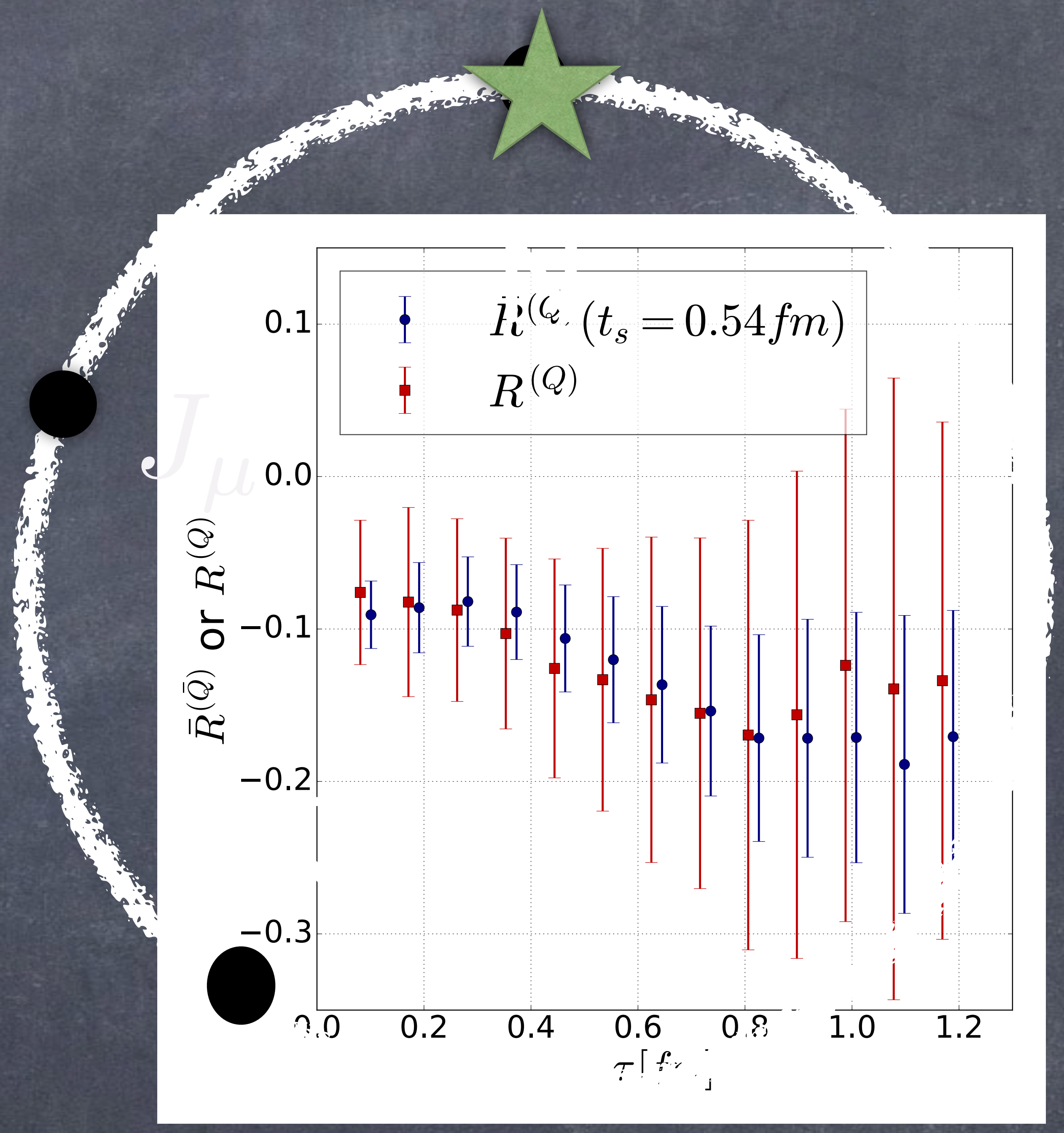


$$\langle N | J_\mu \mathcal{O}_{\cancel{CP}} | N \rangle \rightarrow d \; \underline{E} \cdot \underline{S}$$

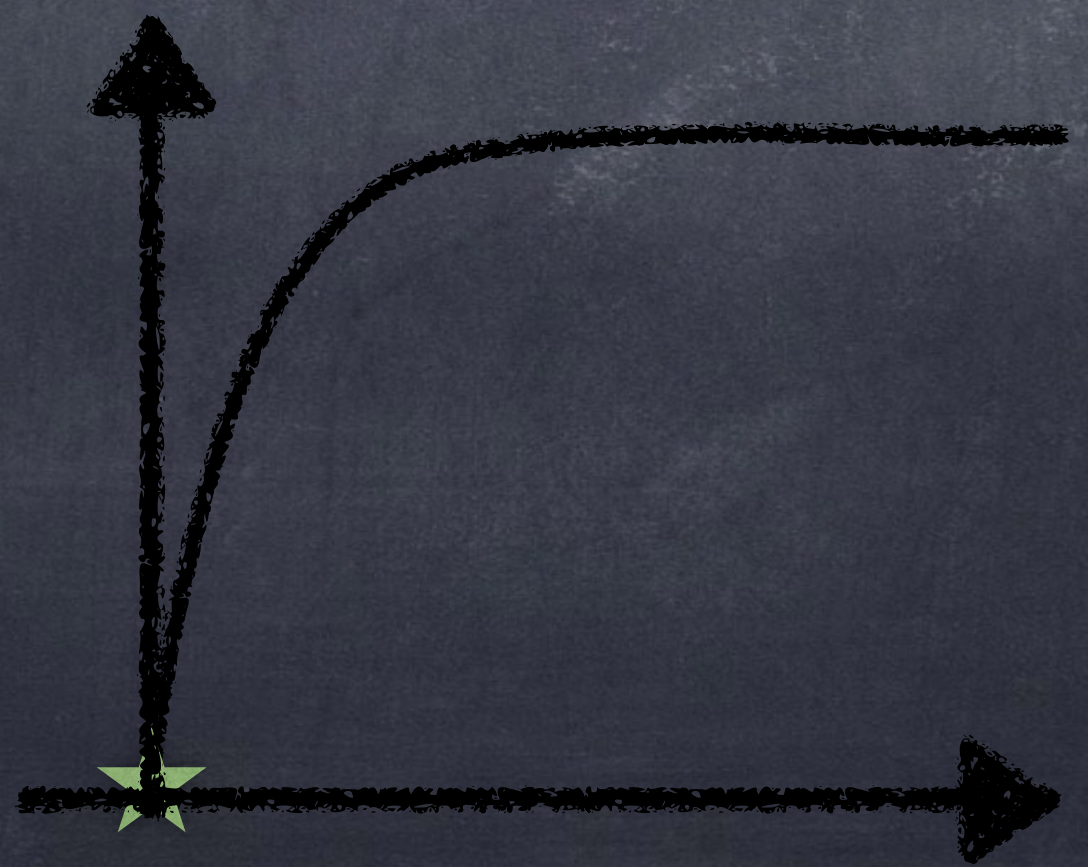
Signal-to-noise improvement

★ = q(t)



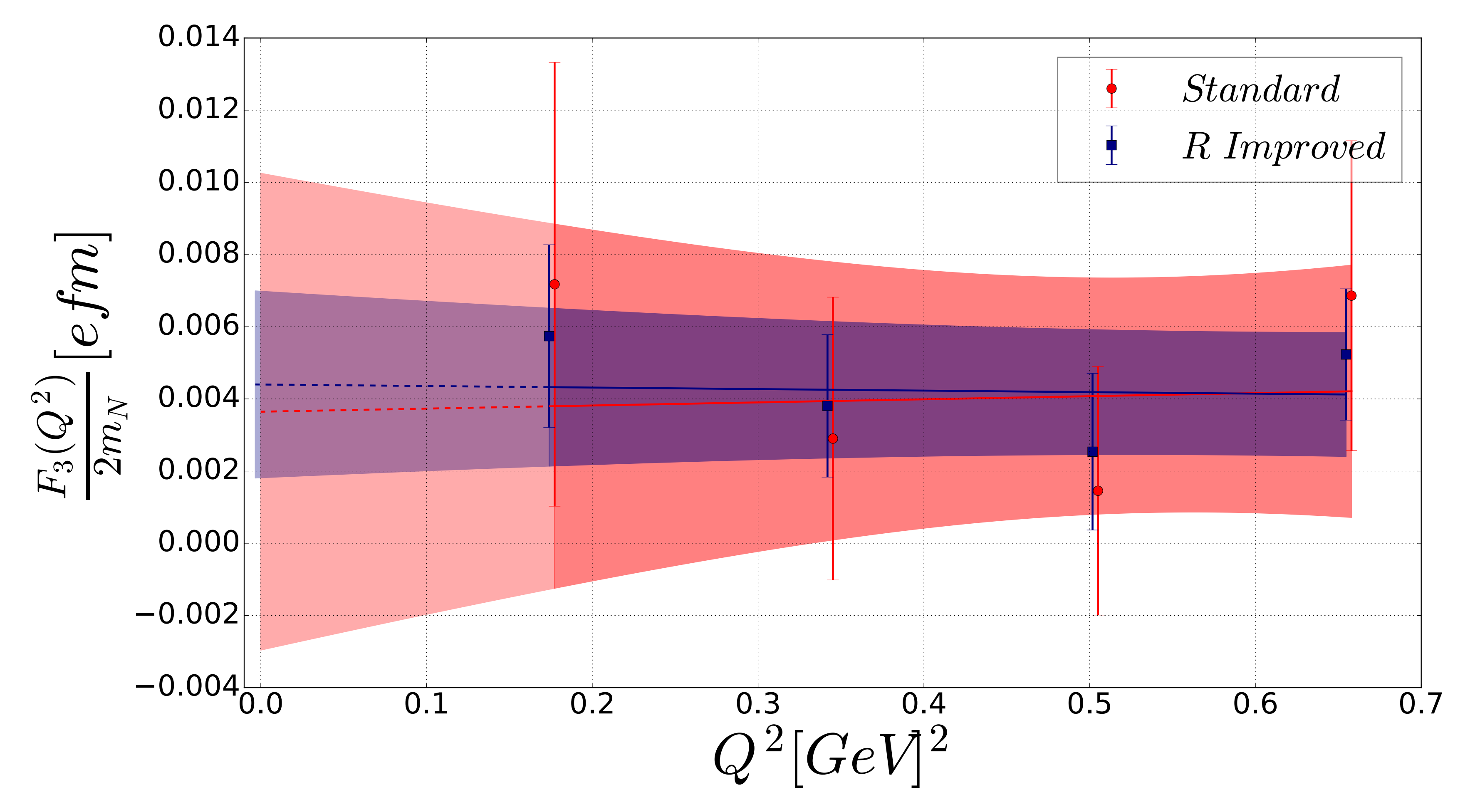


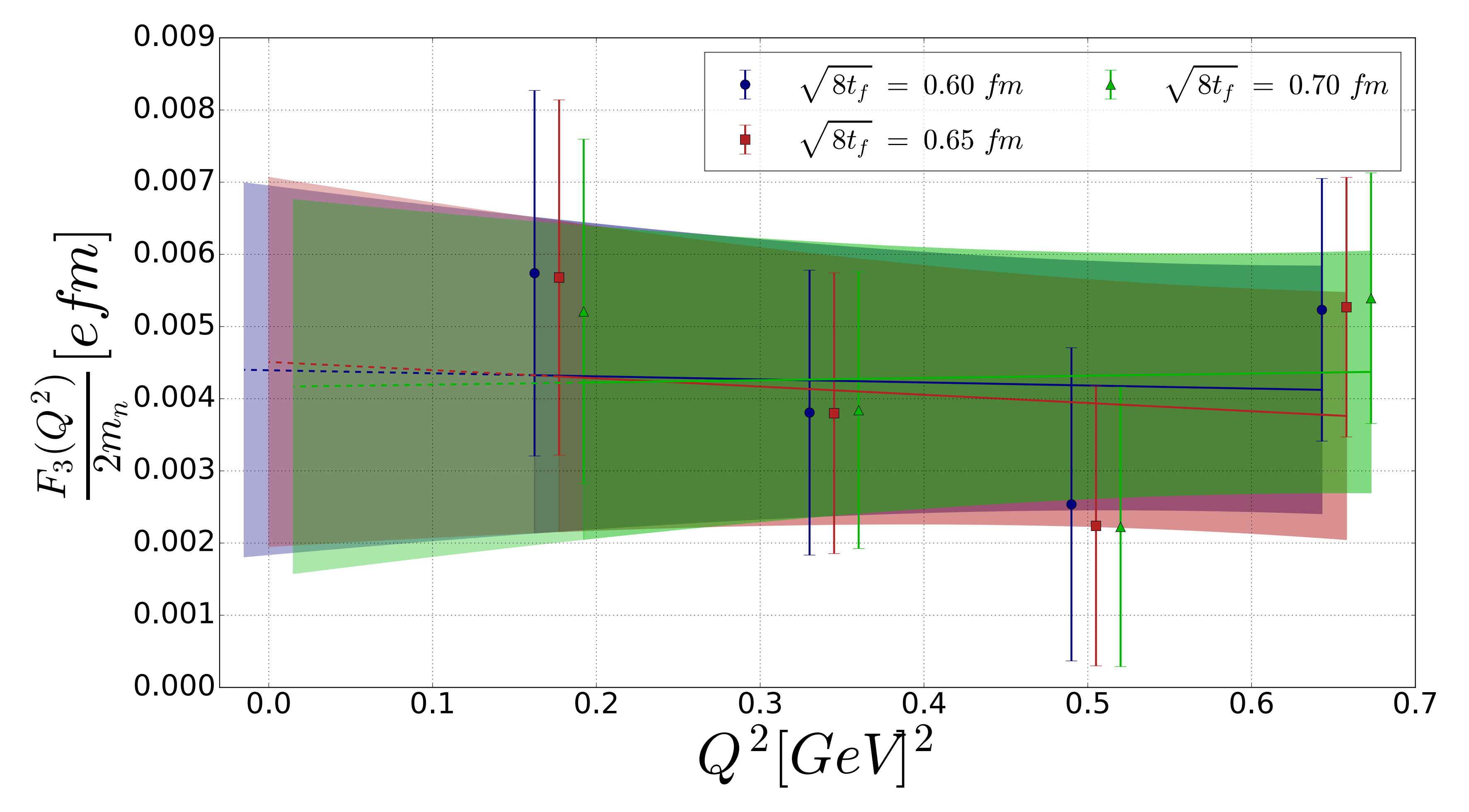
$$R^Q = \frac{G_3^Q(t,\tau,t_f;\underline{p}',\underline{q})}{G_2(\underline{p}',t)} \cdot K(t,\tau;\underline{p}',\underline{q})$$



Dragos, Luu, A.S.,
de Vries, Yousif: 2019

CP-odd form factor



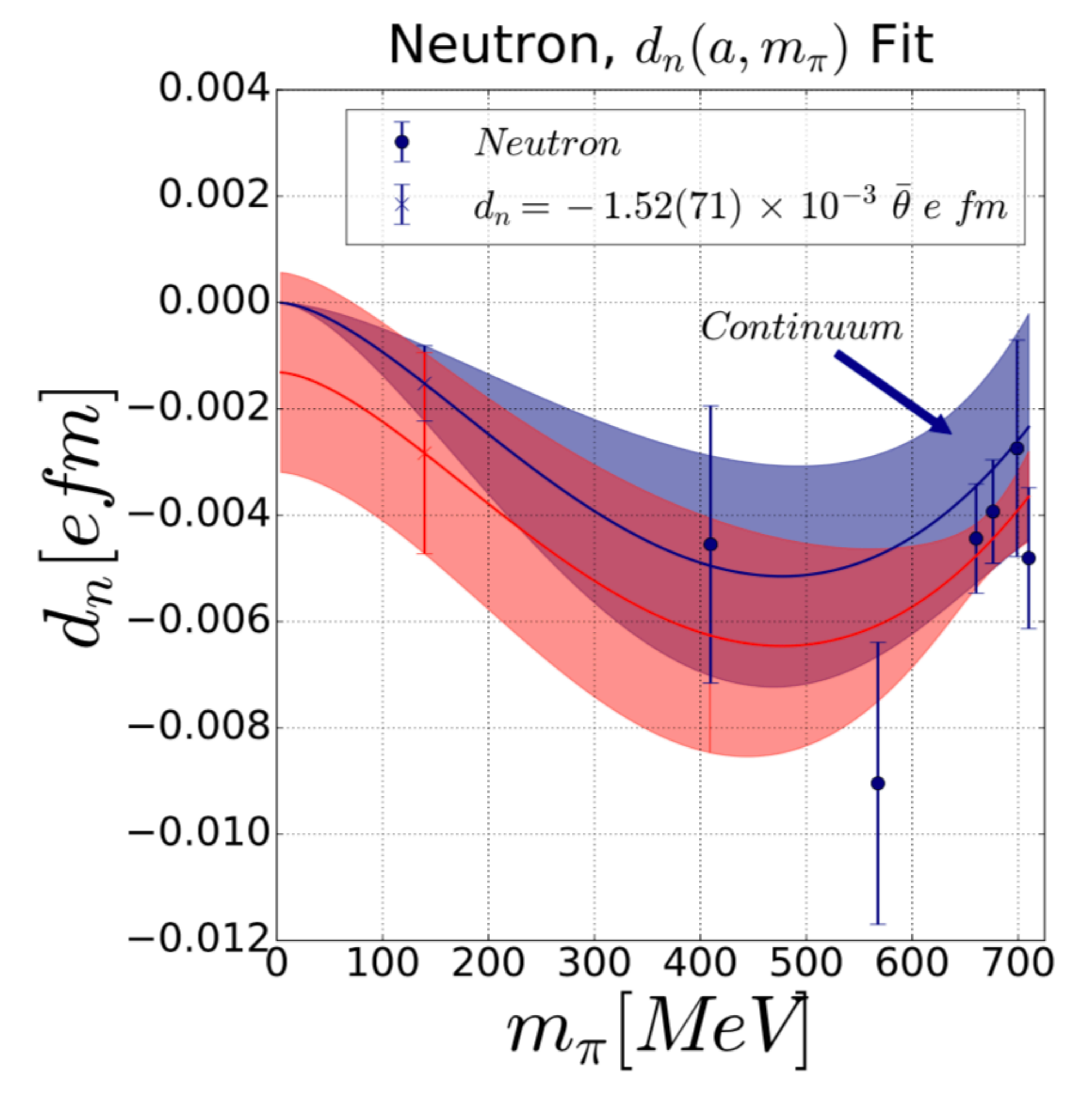


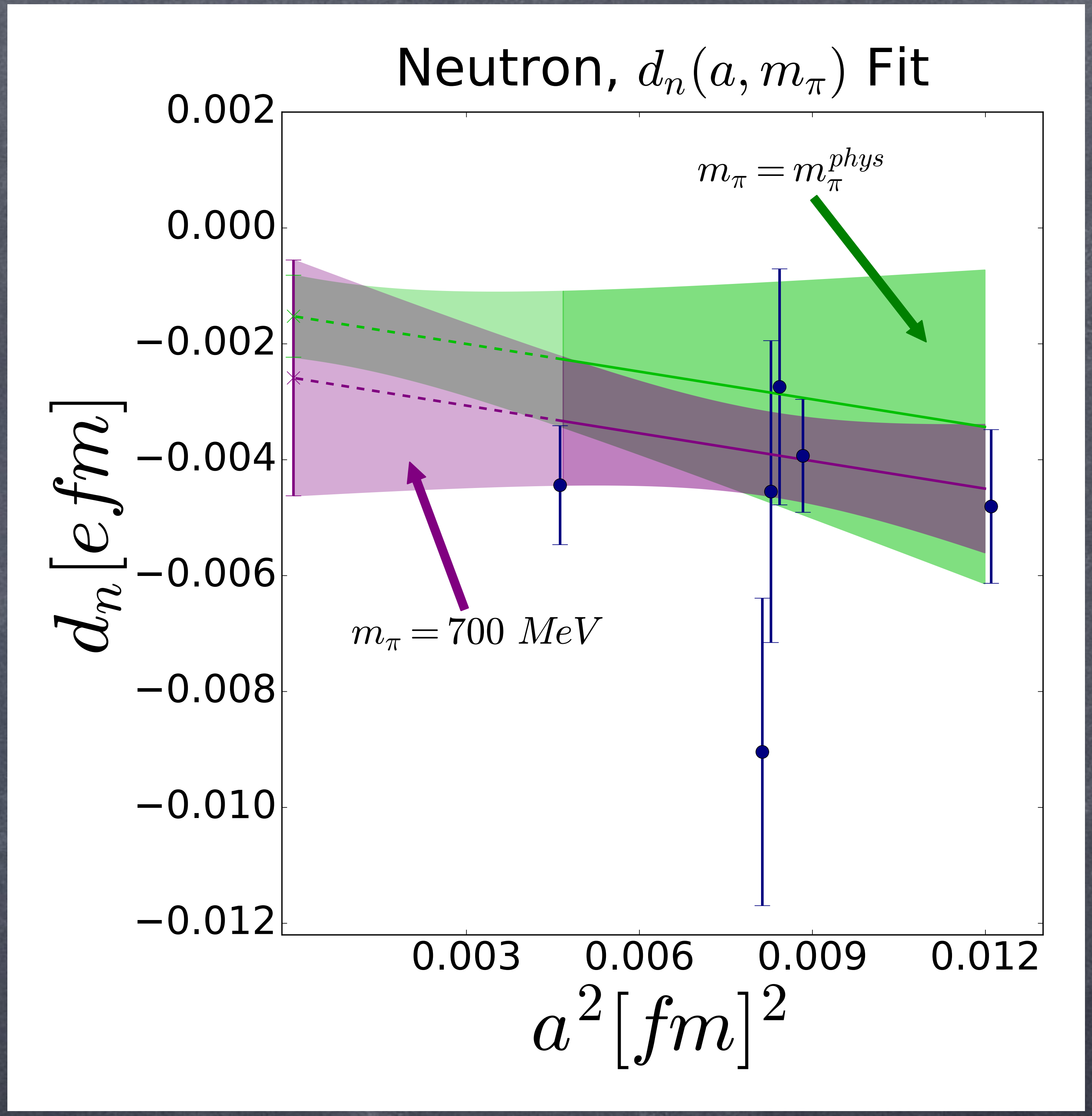
$$\frac{F_3^{P/N}(Q^2)}{2M_N} = d_{P/N} - S_{P/N}Q^2 + H_{P/N}(Q^2)$$

$$\frac{d_P}{d_N} < 0 \qquad \frac{S_P}{S_N} < 0$$

Mereghetti et al.: 2011

Chiral interpolation





$$d_{n/p}(a, m_\pi) = C_1^{n/p} m_\pi^2 + C_2^{n/p} m_\pi^2 \ln \frac{m_\pi^2}{M_N^2} + C_3^{n/p} a^2$$

$$d_n(\bar\theta) = \bar d_n - \frac{e g_A \bar g_0^{\bar\theta}}{8\pi^2 F_\pi} \ln \frac{m_\pi^2}{M_N^2}$$

Dragos, Luu, A.S.,
de Vries, Yousif: 2019

Ottnad et al.: 2010
Mereghetti et al.: 2011

Quark-Chromo EDM

Mereghetti, Monahan, Rizik, A.S., Stoffer : 2021

$$\mathcal{O}^R_{CE}(x;t) = \mathring{\bar{\chi}}(x;t)\tilde{\sigma}_{\mu\nu}t^a\mathring{\chi}(x;t)G^a_{\mu\nu}(x;t)$$

$$\begin{aligned}\mathcal{O}^R_{CE}(x;t) = c_P(t,\mu)\mathcal{O}^{\mathrm{MS}}_P(x;\mu) + c_{m\theta}(t,\mu)\mathcal{O}^{\mathrm{MS}}_{m\theta}(x;\mu) + c_E(t,\mu)\mathcal{O}^{\mathrm{MS}}_E(x;\mu)\\ + c_{CE}(t,\mu)\mathcal{O}^{\mathrm{MS}}_{CE}(x;\mu) + c_{mP^2}(t,\mu)\mathcal{O}^{\mathrm{MS}}_P(x;\mu) + O(t)\end{aligned}$$

$$\begin{aligned}c_{CE}(t,\mu) &= \zeta_\chi^{-1} + \frac{\alpha_s}{4\pi}\left[2(C_F - C_A)\log(8\pi\mu^2 t) - \frac{1}{2}\Big((4+5\delta_{\mathrm{HV}})C_A + (3-4\delta_{\mathrm{HV}})C_F\Big)\right]\\ &= 1 + \frac{\alpha_s}{4\pi}\Bigg[(5C_F - 2C_A)\log(8\pi\mu^2 t)\\ &\qquad - \frac{1}{2}\Big((4+5\delta_{\mathrm{HV}})C_A + (3-4\delta_{\mathrm{HV}})C_F\Big) - \log(432)C_F\Bigg]\end{aligned}$$

$$c_P(t,\mu) = \frac{\alpha_s C_F}{4\pi}\frac{6i}{t} \qquad c_E(t,\mu) = \frac{\alpha_s C_F}{4\pi}\left(4\log(8\pi\mu^2 t) + 3 + 2\delta_{\mathrm{HV}}\right) \qquad c_{m^2P}(t,\mu) = \frac{\alpha_s C_F}{4\pi}i\left(12\log(8\pi\mu^2 t) + \frac{1}{2}(33 - 16\delta_{\mathrm{HV}})\right)$$