# Semileptonic $B \to D^*$ Decay with Lattice QCD

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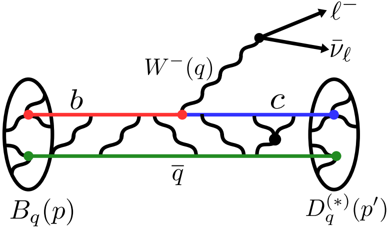
HPQCD

# Motivation

Many interesting $B$ semileptonic decays currently under active investigation

- Here, focus on $B_{(s)} \to D^*_{(s)}\ell\nu$
  - Complementary determinations of $V_{cb}$,
  - Comparison of observables sensitive to lepton flavor universality violation (LFUV) to experiment

# Theory




Kinematic variables:

$$q^2 = (p - p')^2$$

$$w = \frac{p' \cdot p}{M_{B_q} M_{D_q^{(*)}}}$$

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$q = u/d \implies B \to D^*$$

$$q = s \implies B_s \to D_s^*$$

$$q = c \implies B_c \to J/\psi$$

# Differential Decay Rate

Pseudoscalar to vector decay has the following structure in the SM:

$$\frac{d\Gamma}{dq^2} = \chi(q^2) \times \mathcal{F}^2(q^2)|V_{cb}|^2$$

$$\mathcal{F}^2(q^2) = \left[\left(1 + \frac{m_\ell^2}{2q^2}\right)\left(H_+^2(q^2) + H_-^2(q^2) + H_0^2(q^2)\right) + \frac{3m_\ell^2}{2q^2} H_t^2(q^2)\right]$$

Helicity amplitudes expressed in terms of form factors

$$\left\{H_+(q^2), H_-(q^2), H_0(q^2), H_t(q^2)\right\} \leftrightarrow \left\{h_{A_1}(q^2), h_{A_2}(q^2), h_{A_3}(q^2), h_V(q^2)\right\}$$

Similar expressions for angular differential rates.

# Form Factors

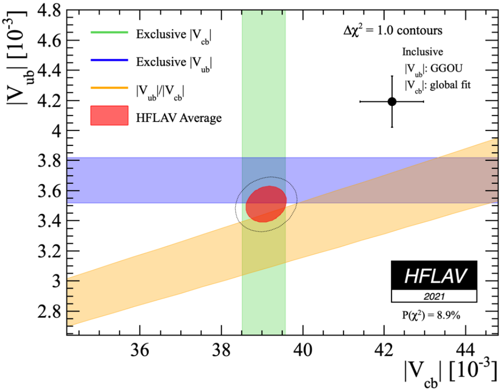
The form factors parameterise the matrix elements

$$\langle D^* | \bar{c}\gamma^\mu b | \overline{B} \rangle = i\sqrt{M_B M_{D^*}} \varepsilon^{\mu\nu\alpha\beta} \epsilon^*_\nu v'_\alpha v_\beta h_V$$

$$\langle D^* | \bar{c}\gamma^\mu \gamma^5 b | \overline{B} \rangle = \sqrt{M_B M_{D^*}} \left[ h_{A_1}(w+1)\epsilon^{*\mu} - h_{A_2}(\epsilon^* \cdot v) v^\mu - h_{A_3}(\epsilon^* \cdot v) v'^\mu \right]$$

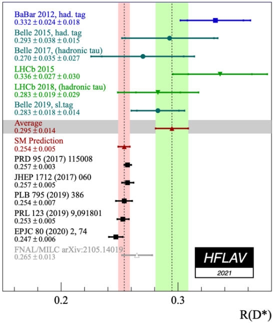
# $V_{cb}$

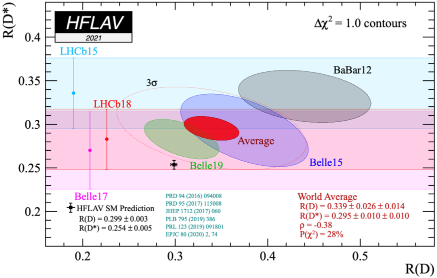
- Exclusive $V_{cb}$
  - Compare experimental value of $\eta_{\mathrm{EW}}\mathcal{F}(q^2_{\max})|V_{cb}|$, extracted from fits to data using BGL/CLN, to lattice calculations of $\mathcal{F}(q^2_{\max})$
  - $B \to D^*$ preferred over $B \to D$ due to favorable kinematics near zero-recoil.
- Inclusive $V_{cb}$
  - Uses total rate to any charmed final state $B \to X_c \ell \bar{\nu}$, together with OPE.



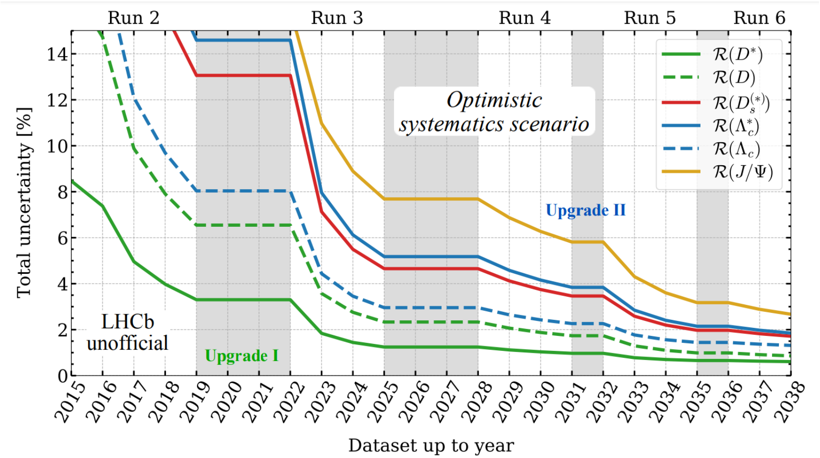
# $R(D^*)$

- $R(D^*) = \Gamma(B \to D^* \tau \bar{\nu}_\tau)/\Gamma(B \to D^* \mu \bar{\nu}_\mu)$
  - Sensitive to LFUV
  - Theory for $R(D^*)$ relies on experimental fits + HQET for $A_0$



# Experimental Outlook



# Tensor Form Factors

Most general effective Hamiltonian for $b \to c\ell\bar{\nu}$, assuming only left handed neutrinos, is:

$$\begin{aligned}\mathcal{H}_{\text{eff}} = \sqrt{2} G_F V_{cb} \Big[ & g_V \bar{c}\gamma_\mu b \bar{\ell}_L \gamma^\mu \nu_L + g_A \bar{c}\gamma_\mu\gamma_5 b \bar{\ell}_L \gamma^\mu \nu_L \\ & + g_S \bar{c} b \bar{\ell}_R \nu_L \\ & + g_P \bar{c}\gamma_5 b \bar{\ell}_R \nu_L \\ & + g_T \bar{c}\sigma_{\mu\nu} b \bar{\ell}_R \sigma^{\mu\nu} \nu_L \\ & + g_{T5} \bar{c}\sigma_{\mu\nu}\gamma^5 b \bar{\ell}_R \sigma^{\mu\nu} \nu_L + \text{h.c.} \Big]\end{aligned}$$

- Need tensor form factors

$$\begin{aligned}\langle D^* | \bar{c}\sigma^{\mu\nu} b | \overline{B} \rangle = - \sqrt{M_B M_{D^*}} \varepsilon^{\mu\nu\alpha\beta} \big[ & h_{T_1} \epsilon^*_\alpha (v + v')_\beta \\ & + h_{T_2} \epsilon^*_\alpha (v - v')_\beta + h_{T_3} (\epsilon^* \cdot v) v_\alpha v'_\beta \big]\end{aligned}$$

# We will need...

- Precise SM form factors across full kinematic range
  - Resolve discrepancy between inclusive and exclusive determinations of $V_{cb}$.
  - Make first principles predictions for $R(D^*_{(s)})$ independent of experimental measurements.
- Tensor form factors
  - Disentangle possible new physics effects.
  - Indicate which angular observables are most sensitive to new physics effects.

# Challenges with $B \to D^*$ on the Lattice

Lattice calculation of FFs for $B_c \to J/\psi \ < \ B_s \to D_s^* \ < \ B \to D^*$

- Computational cost of propagators for $c < s << u/d$
- $J/\psi$ and $D_s^*$ are 'gold-plated'
  - Lattice calculations of form factors for both decays available across full $q^2$ range from HPQCD[1]
- $B \to D^*$ requires careful treatment of chiral effects
  - On the lattice, typically use unphysically heavy pions and treat $D^* \to D\pi$ resonance using $\chi\mathrm{PT}$

[1] 2105.11433, 2007.06957

# Overview of $B_{(s)} \to D_{(s)}^{(*)}$ Lattice Results

- SM FFs for $B \to D\ell\nu$ available away from zero recoil[2]
- SM FFs for $B_s \to D_s\ell\nu$ now available across the full kinematic range, tensor FF available close to zero-recoil, with work also ongoing[3]
- SM FFs for $B \to D^*\ell\nu$ recently became available from Fermilab-MILC away from zero-recoil[4], with lattice calculations also underway by JLQCD as well as HPQCD.
- (Preliminary) FFs for $B \to D^*\ell\nu$ across full kinematic range from HPQCD

[2] e.g. 1503.07237,1505.03925
[3] 1906.00701,1310.5238,2110.10061
[4] 2105.14019

# Current Results

| | Lattice only | Lattice+Exp[5] | Experiment | Tension |
|---|---|---|---|---|
| $R(D)$ | 0.293(4)[6] | 0.299(3) | 0.340(30) | $1.4\sigma$ |
| $R(D^*)$ | 0.265(13) | 0.2483(13) | 0.295(14) | $3.3\sigma$ |
| $R(D_s)$ | 0.299(5) | – | – | – |
| $R(D_s^*)$ | 0.249(7) | – | – | – |

HFLAV average, Fermilab-MILC, HPQCD.

| | $V_{cb}$ | |
|---|---|---|
| $B \to D$ | $39.58(94)_{\mathrm{exp}}(37)_{\mathrm{th}} \times 10^{-3}$ | HFLAV |
| $B \to D^*$ | $38.76(42)_{\mathrm{exp}}(55)_{\mathrm{th}} \times 10^{-3}$ | |
| $B_s \to D_s^{(*)}$ | $42.3(1.2)_{\mathrm{exp}}(1.2)_{\mathrm{th}} \times 10^{-3}$ | LHCb (2001.03225) |
| $B \to X_c \ell \nu$ | $42.16(51) \times 10^{-3}$ | Bordone et al.(2107.00604) |

[5] Assumes new physics only possible in semitauonic mode

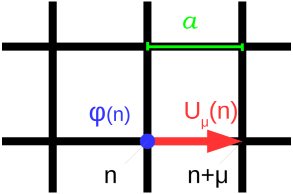
[6] FLAG review

# A Brief Summary of Lattice QCD

We want to extract matrix elements, amplitudes and energies from Euclidean correlation functions computed in the path integral formalism,

$$\int \mathcal{D}[\psi, \overline{\psi}, A]\mathcal{O}^1(t)\mathcal{O}^2(0)e^{-S^E[\psi,\overline{\psi},A]} = \sum_n \langle 0|\hat{\mathcal{O}}^1|n\rangle\langle n|\hat{\mathcal{O}}^2|0\rangle e^{-E_n t},$$

- discretise QCD onto a lattice, $A_\mu \to U_\mu$
- Fermion integrals exact $\to$ need to invert dirac operator
- Monte-carlo integral over gauge fields $U$




To simulate bottom quarks precisely need $am_q < 1$, but typical modern lattices have lattice spacing $a > 1/m_b$, on these lattices cannot simulate physical $b$ quarks directly.

# Form Factors Across the Full $q^2$ Range with Lattice QCD[8]

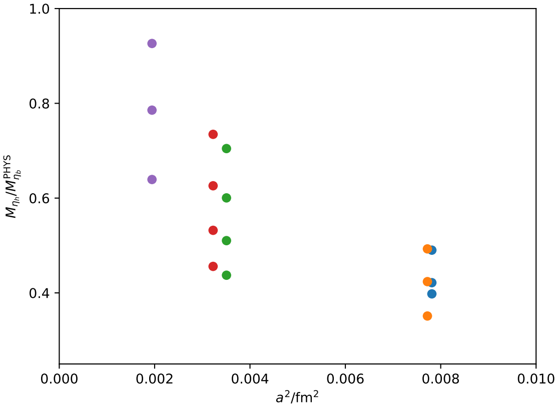
Use "Heavy-HISQ" approach:

- Compute form factors using multiple heavy masses ranging up to close to the physical b-quark mass
- Use **H**ighly **I**mproved **S**taggered **Q**uark action[7] for all quarks - fully relativistic, small discretisation effects
- Nonperturbatively renormalised currents, using PCVC and PCAC relations for vector and axial-vector, RI-SMOM for tensor
- Fit the form factor data including $am_h$ discretisation effects, physical heavy mass dependence, and lattice spacing dependence

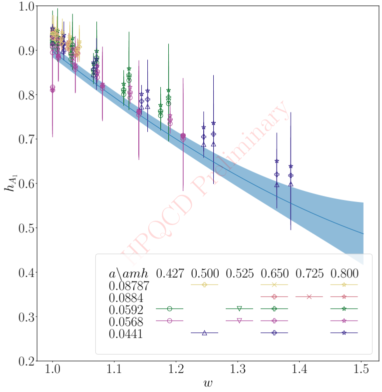
$$F = \sum_{nijk} a^n_{ijk}(w-1)^n \left(\frac{am_c}{\pi}\right)^i \left(\frac{am_h}{\pi}\right)^j \left(\frac{\Lambda_{\mathrm{QCD}}}{M_B}\right)^k \mathcal{N}_n + X_{\log}(M_\pi/\Lambda_\chi) + A\left(\frac{M_\pi}{\Lambda_\chi}\right)^2$$

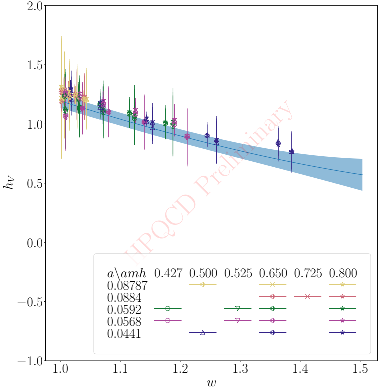
[7] hep-lat/0610092
[8] $B_s \to D_s^*$:2105.11433, $B_c \to J/\psi$ :2007.06957

- We use the second generation MILC HISQ gauge configurations with $u/d$, $s$ and $c$ quarks in the sea.
- The subset of configurations we use include physical $u/d$ quark masses, and have small lattice spacings allowing us to come very close to the physical $b$ mass.



# Preliminary results for $B \to D^*$




We include data from $B_s \to D_s^*$ in our chiral extrapolation.

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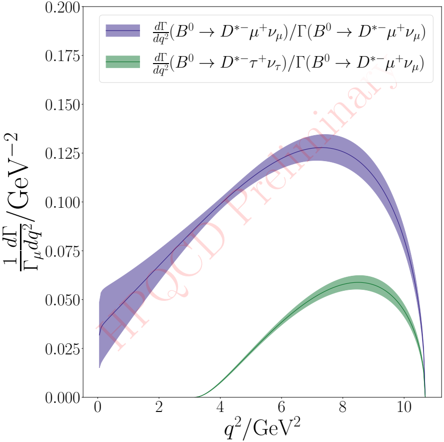
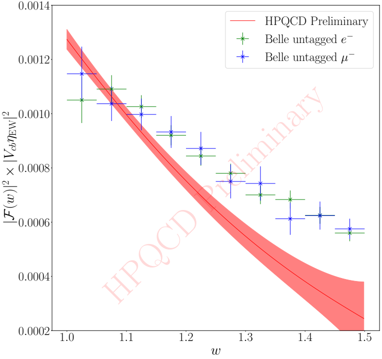



$$R(D^*) = 0.280(13)$$

# Preliminary results for $B \to D^*$

We can compare rate to experimental data from Belle[9]

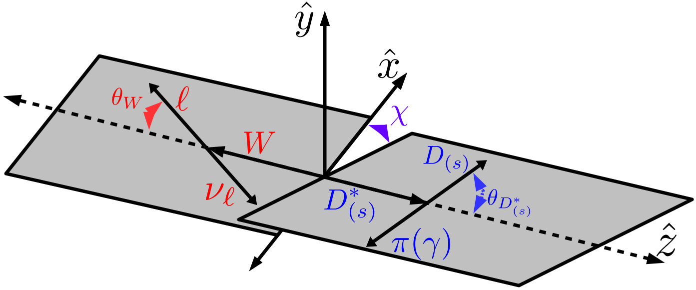



$$\chi^2/\mathrm{dof} = 1.3$$

$$V_{cb} = 39.2(0.6)_{\mathrm{latt}}(0.5)_{\mathrm{exp}} \times 10^{-3} \qquad \text{(PRELIMINARY)}$$

[9] 1809.03290

# Angular Shape of the Decay



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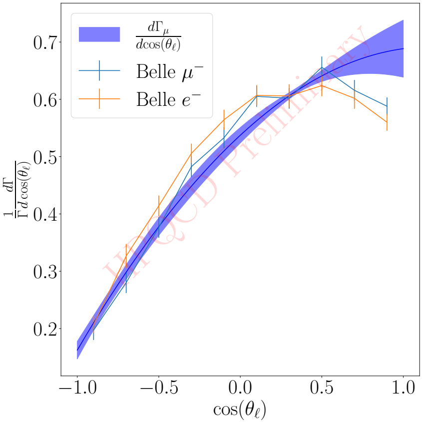

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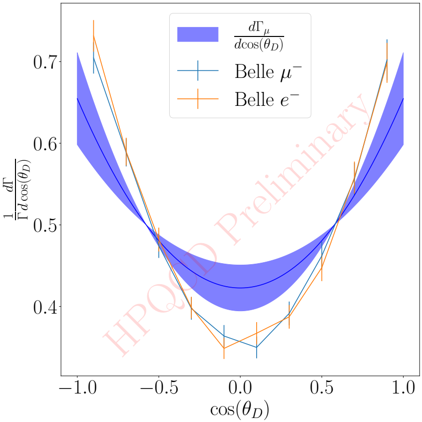

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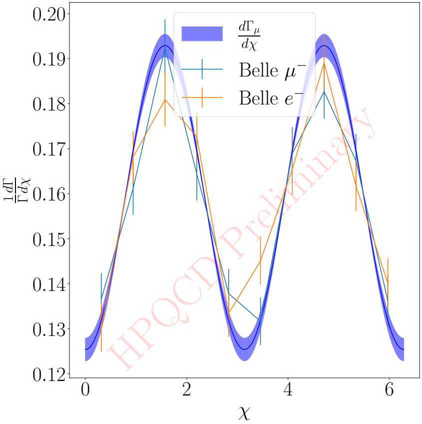



Joint fit to HPQCD lattice and Belle untagged data - $\chi^2/\mathrm{dof} = 1.3$, $Q = 0.09$, $R(D^*) = 0.2464(19)$

# Current Results

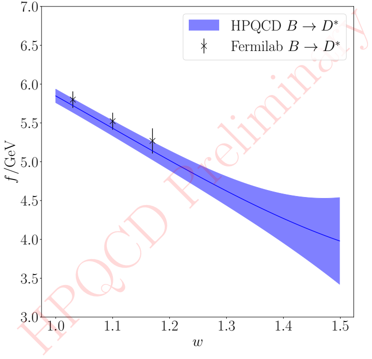
| | Lattice only | Lattice+Exp[10] | Experiment | Tension |
|---|---|---|---|---|
| $R(D^*)$ | 0.265(13) | 0.2483(13) | 0.295(14) | $3.3\sigma$ |
| This work | 0.280(13) | 0.2464(19) | – | – |

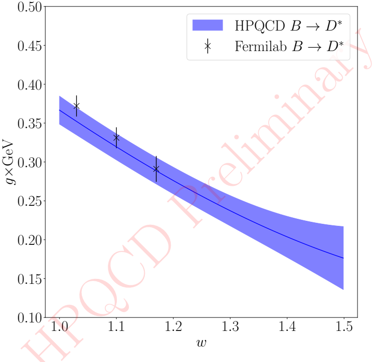
HFLAV average, Fermilab-MILC, HPQCD.

[10] Assumes new physics only possible in semitauonic mode

# Comparison to Fermilab-MILC (2105.14019)

Compare form factors in helicity basis



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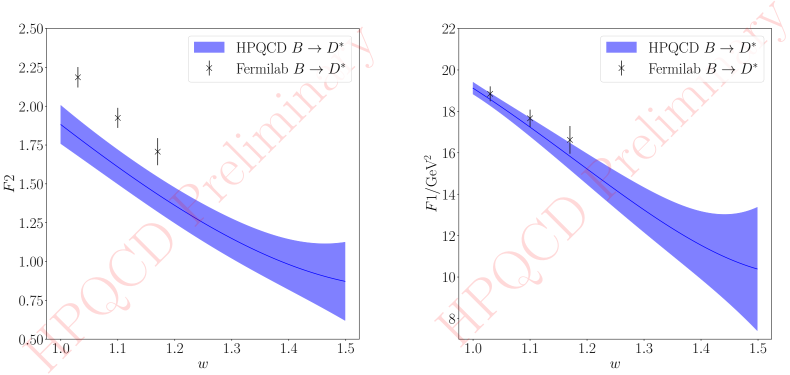
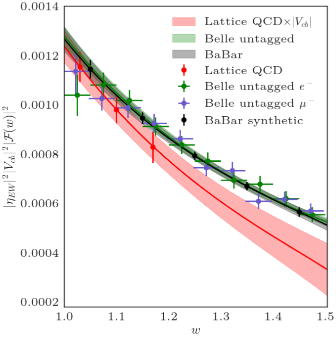


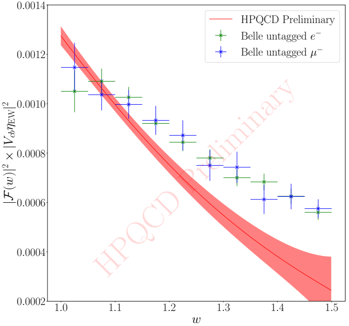

(Preliminary) indications of disagreement in pseudoscalar form factor $F_2$, good agreement otherwise

$$\chi^2/\mathrm{dof} = 1.3, \quad Q = 0.22$$

# Comparison to Fermilab-MILC (2105.14019)

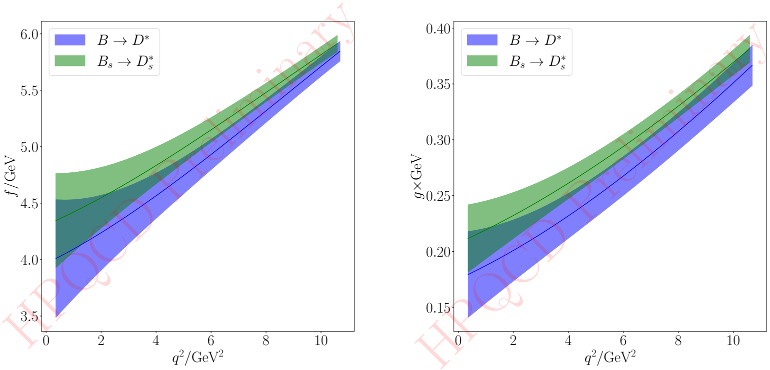
Agreement on shape $\mathcal{F}$, which only depends on $g$, $f$ and $F_1$ for light leptons, both in disagreement with untagged Belle results



# $SU(3)_F$ Breaking Effects

Since we include $B_s \to D_s^*$ data in fit, can study breaking effects.



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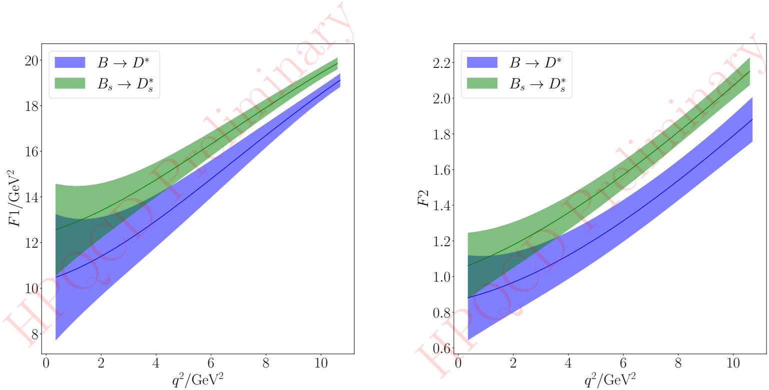
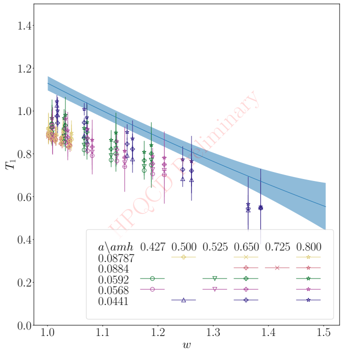
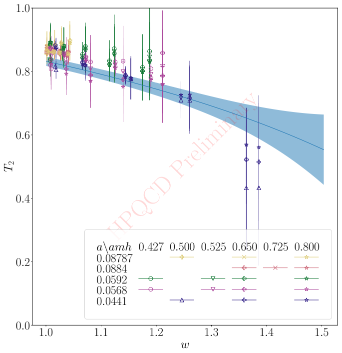


# $B \to D^*$ Tensor Form Factors

Finished collecting data for tensor form factors. More conventional to express these in terms of $T_1$, $T_2$ and $T_{23}$.



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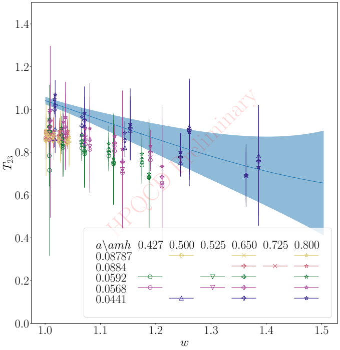

# Summary

- Computed preliminary $B \to D^*$ form factors
  - Agree with Fermilab for shape and relevant form factors, some disagreement for pseudoscalar $F_2$
  - Fully model independent determination of $R(D^*)$ much closer to experiment, but discrepancy in shape
  - $SU(3)_F$ breaking effects at the level of $\leq 10\%$
  - Tensor Form Factors also computed for the first time.

Thanks for listening!