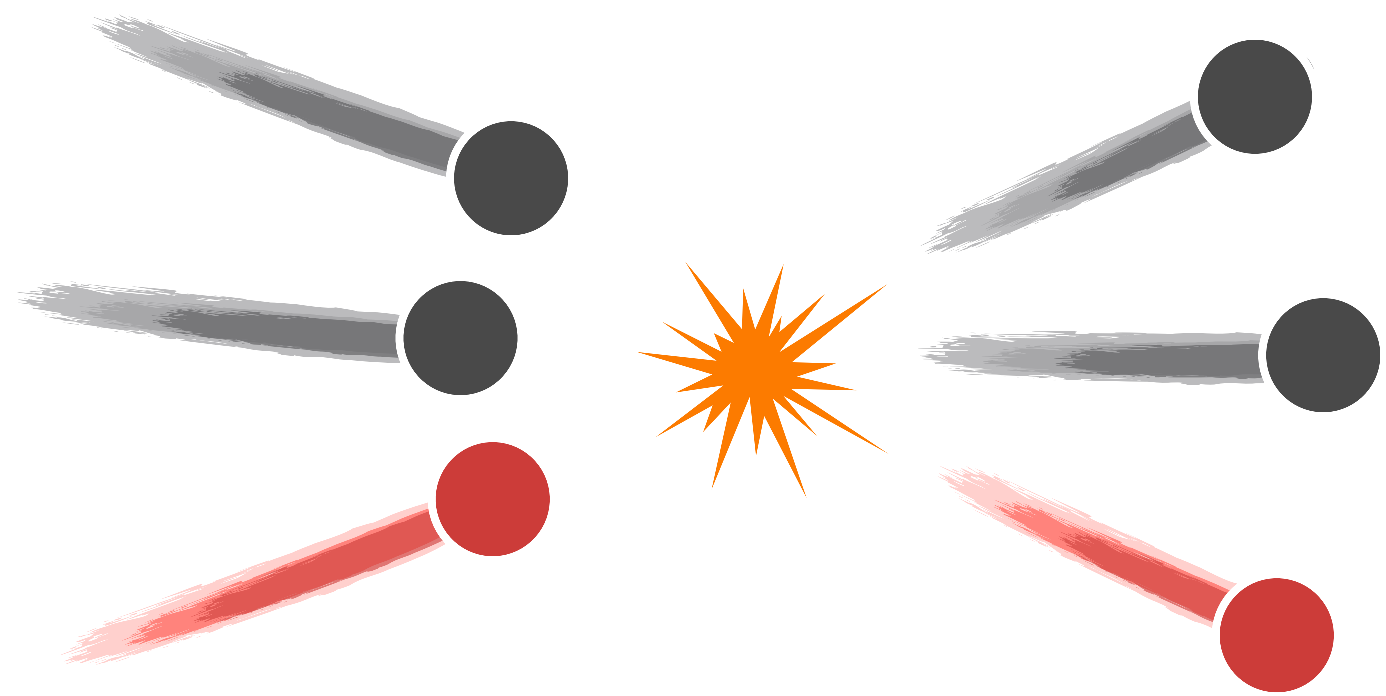
# Few-Body Dynamics from QCD

Andrew W. Jackura

14th Conference on the Intersections of Particle and Nuclear Physics (CIPANP 2022)

Saturday, September 3rd, 2022

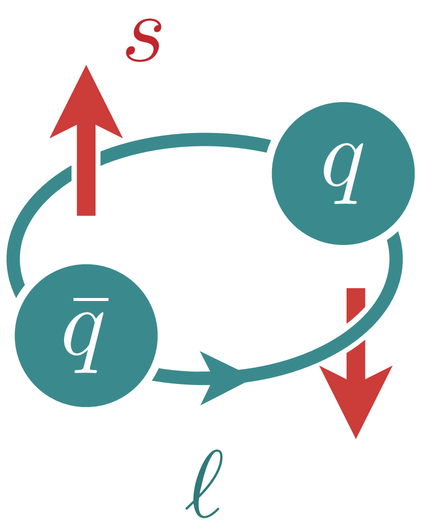


Old Dominion University
Jefferson Lab
Thomas Jefferson National Accelerator Facility

# The Hadron Spectrum

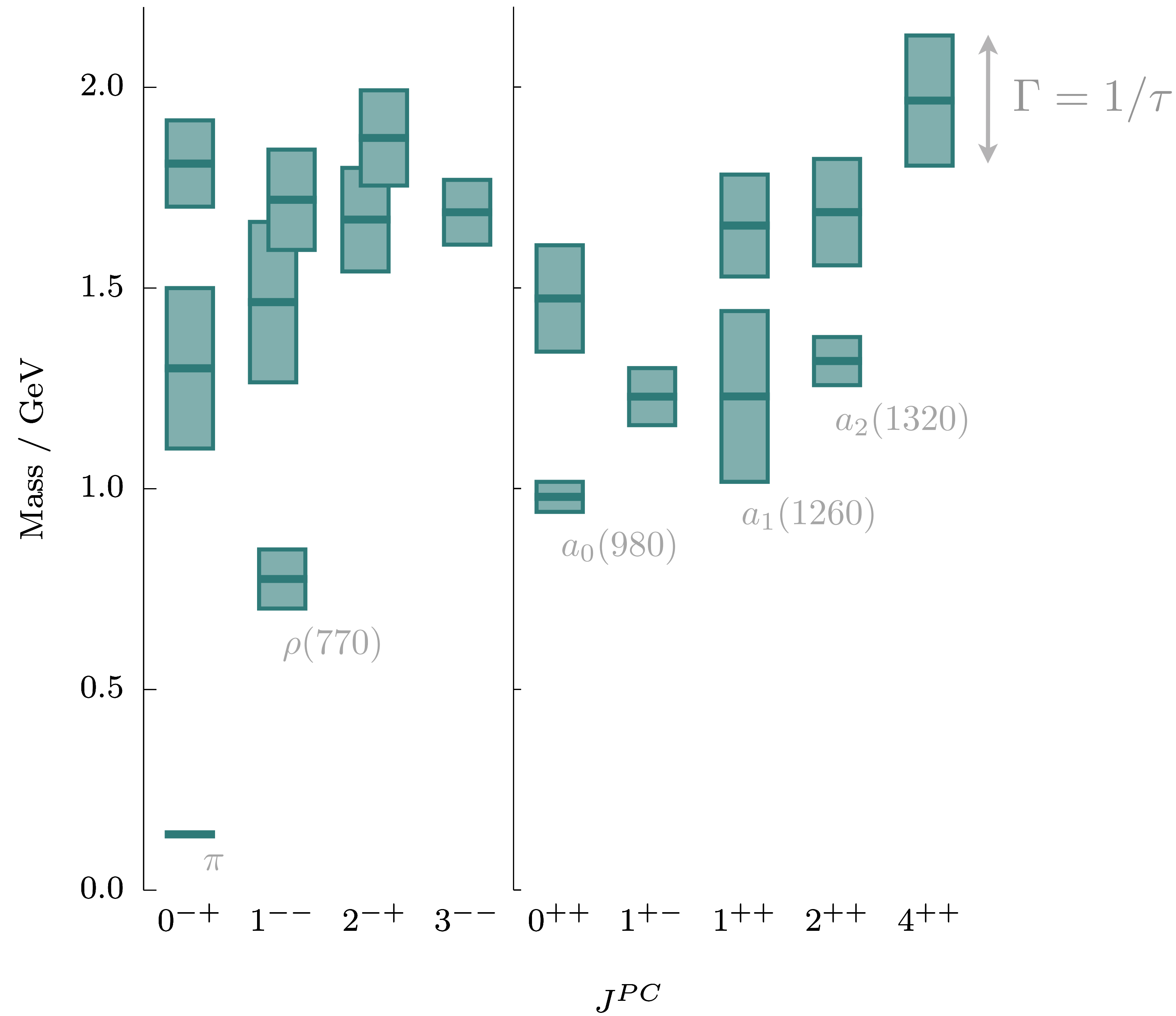
Quark models give gross structure of the hadron spectrum

*e.g. light isovector mesons*




$$q\bar{q}\ \left(n^{2s+1}\ell_J\right)$$

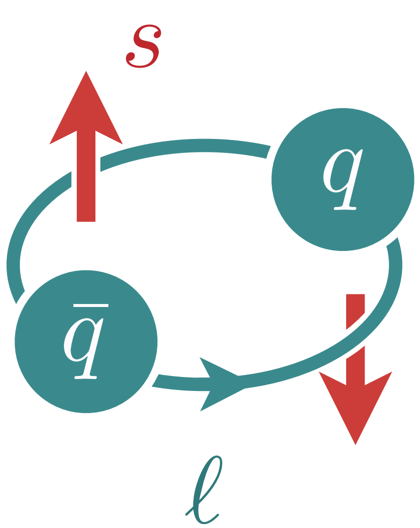
| | $J^{PC}$ | |
|---|---|---|
| | singlet | triplet |
| $\ell = 0$ | $0^{-+}$ | $1^{--}$ |
| $\ell = 1$ | $1^{+-}$ | $(0,1,2)^{++}$ |
| $\ell = 2$ | $2^{-+}$ | $(1,2,3)^{--}$ |
| $\vdots$ | $\vdots$ | $\vdots$ |



# The Hadron Spectrum

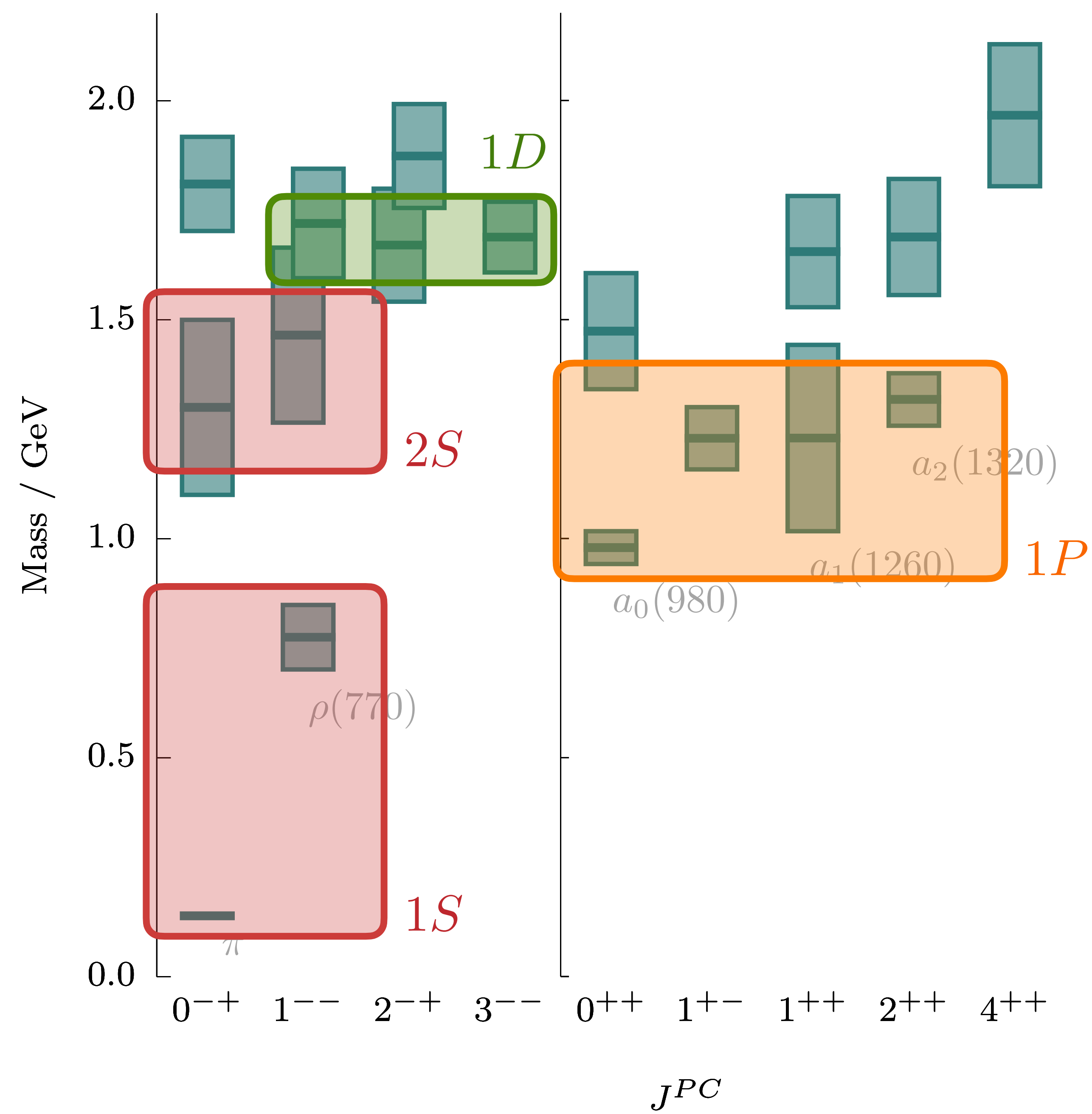
Quark models give gross structure of the hadron spectrum

*e.g. light isovector mesons*




$$q\bar{q}\ \left(n^{2s+1}\ell_J\right)$$

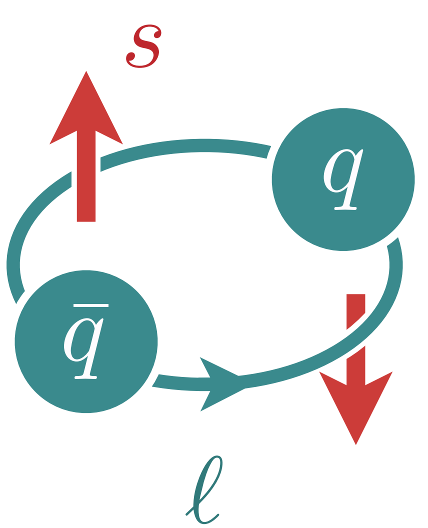
| | $J^{PC}$ singlet | $J^{PC}$ triplet |
|---|---|---|
| $\ell = 0$ | $0^{-+}$ | $1^{--}$ |
| $\ell = 1$ | $1^{+-}$ | $(0,1,2)^{++}$ |
| $\ell = 2$ | $2^{-+}$ | $(1,2,3)^{--}$ |
| $\vdots$ | $\vdots$ | $\vdots$ |



# The Hadron Spectrum

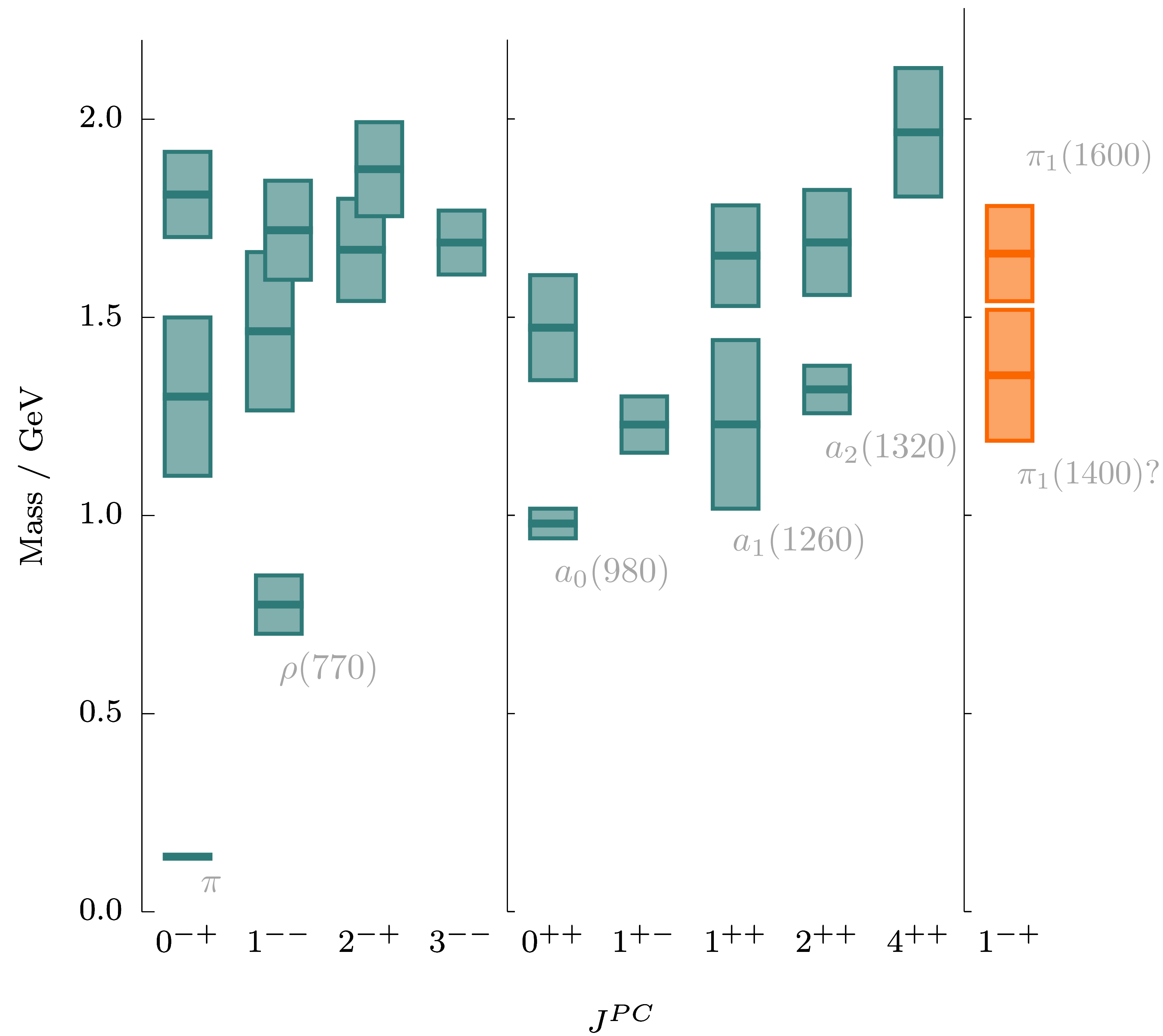
Quark models give gross structure of the hadron spectrum

*e.g. light isovector mesons*




$$q\bar{q}\ \left(n^{2s+1}\ell_J\right)$$

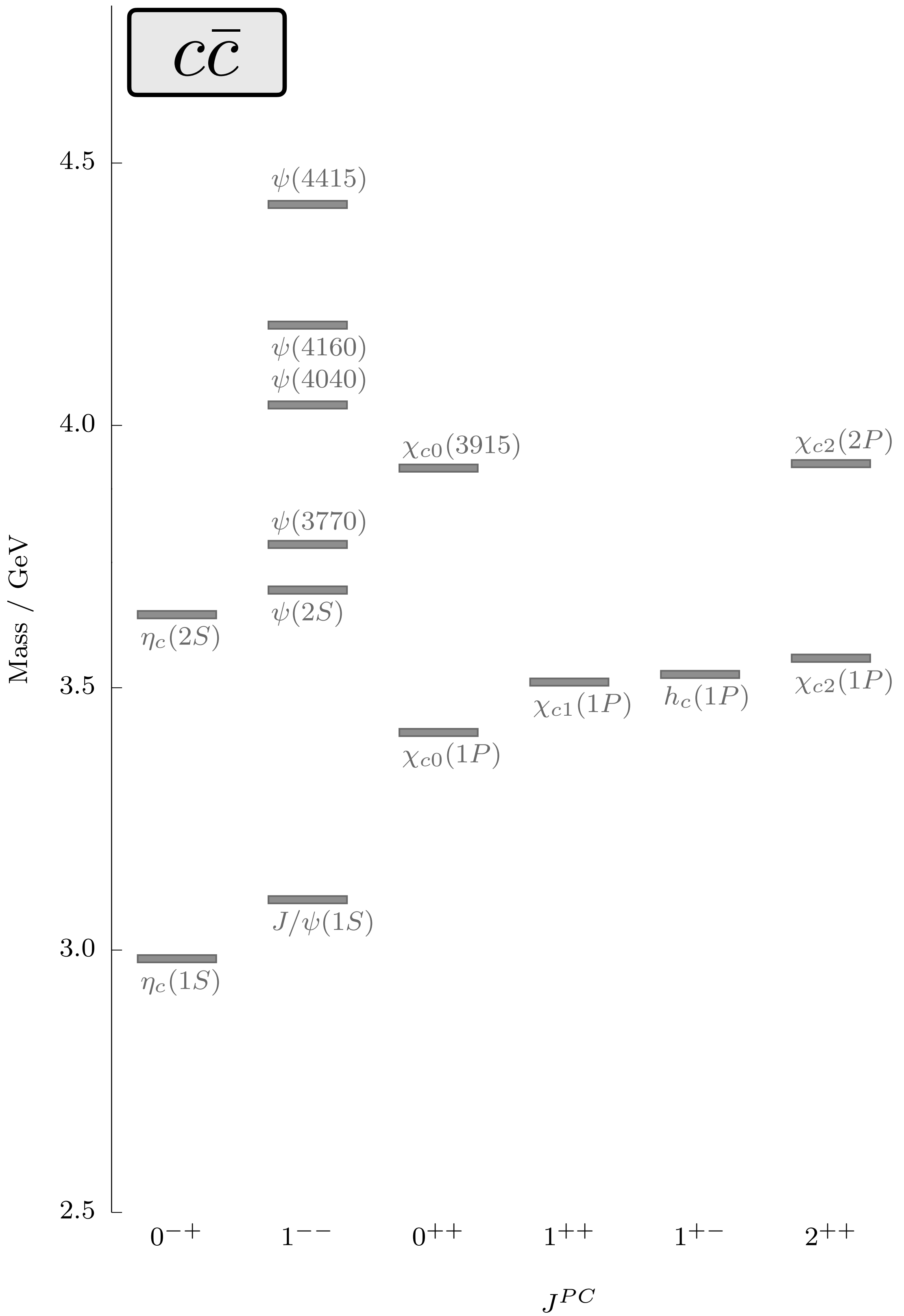
| | $J^{PC}$ singlet | $J^{PC}$ triplet |
|---|---|---|
| $\ell = 0$ | $0^{-+}$ | $1^{--}$ |
| $\ell = 1$ | $1^{+-}$ | $(0,1,2)^{++}$ |
| $\ell = 2$ | $2^{-+}$ | $(1,2,3)^{--}$ |
| $\vdots$ | $\vdots$ | $\vdots$ |




***Forbidden quantum numbers :*** $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, \ldots$

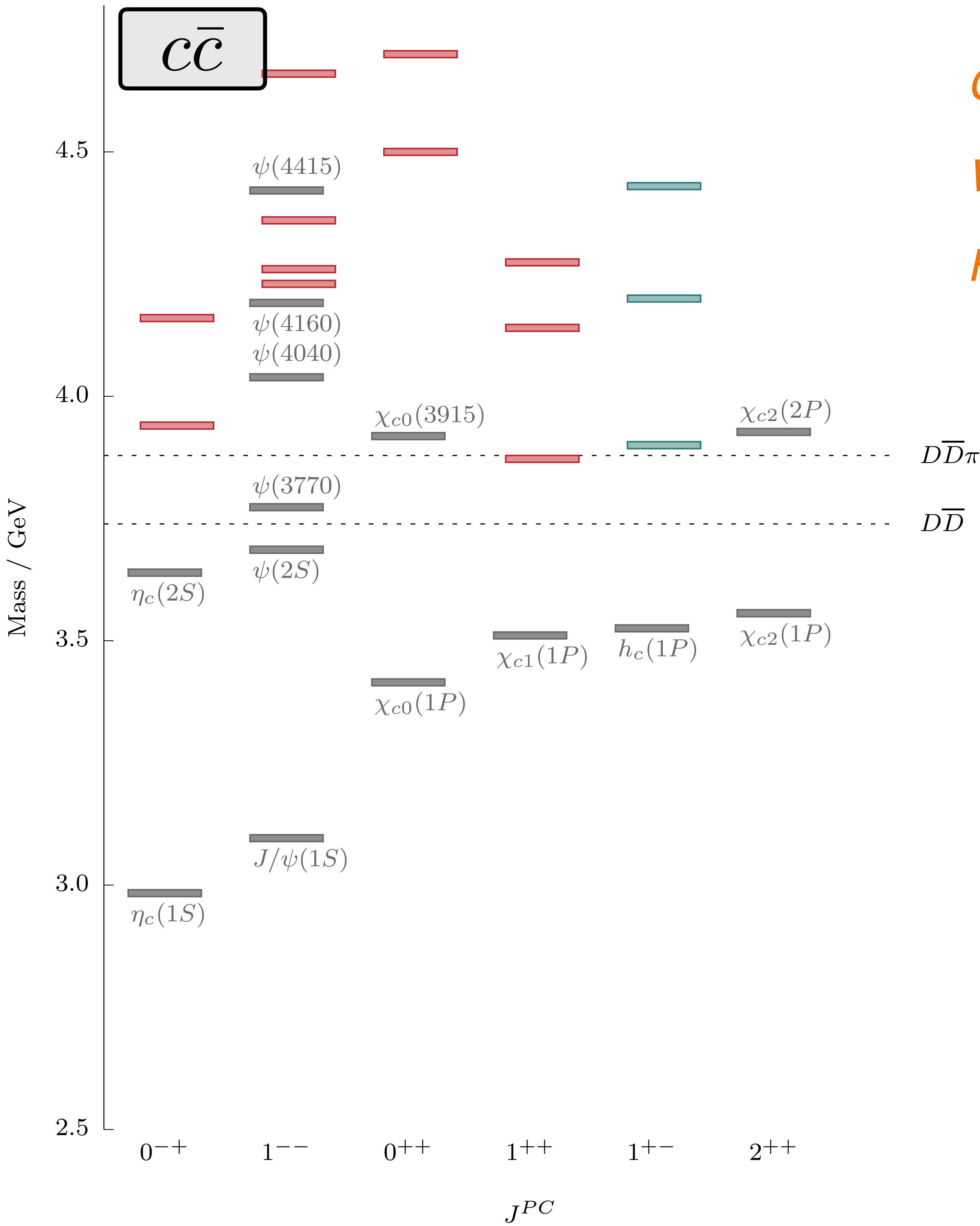
# The Hadron Spectrum

Modern experiments have been finding new states which don't fit the conventional quark models



# The Hadron Spectrum

Modern experiments have been finding new states which don't fit the conventional quark models



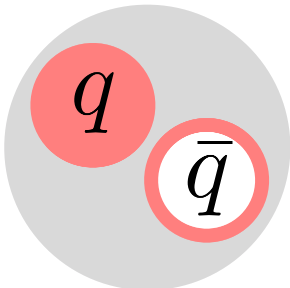

*QCD allows for non-quark model hadrons*

*What is the nature of these states?*

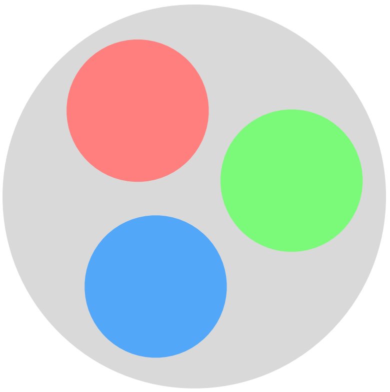
*How do we being to find a global explanation?*

# The Hadron Spectrum

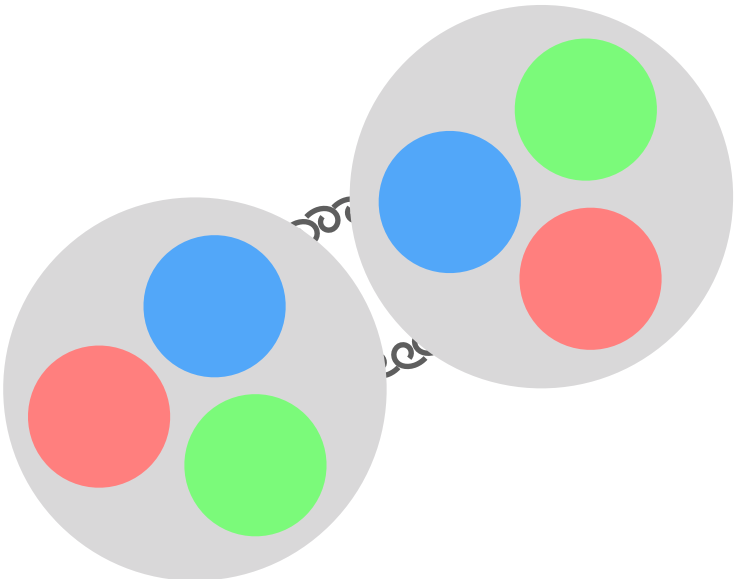
How to connect QCD to the hadron spectrum?




Meson



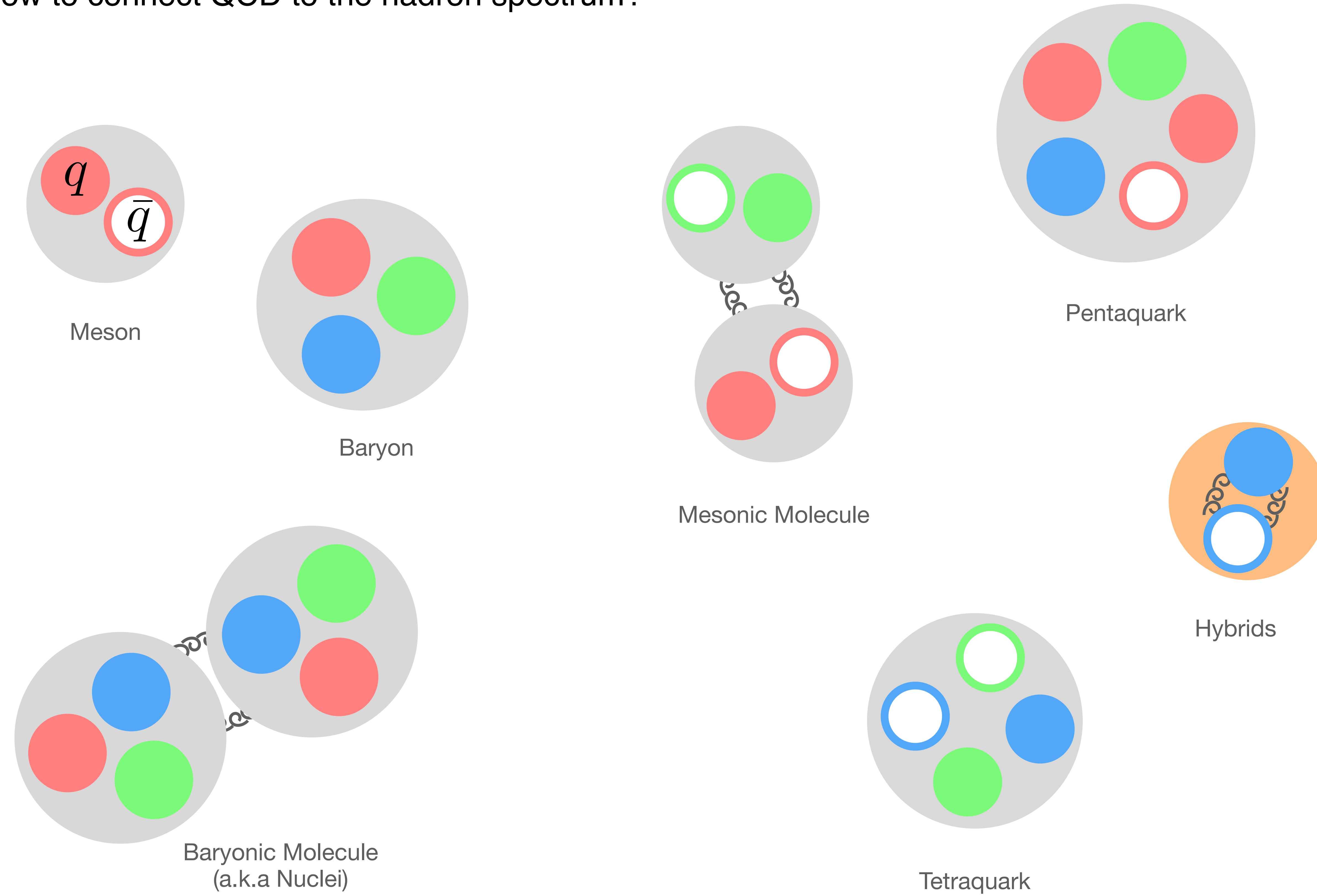
Baryon



Baryonic Molecule
(a.k.a Nuclei)

# The Hadron Spectrum

How to connect QCD to the hadron spectrum?

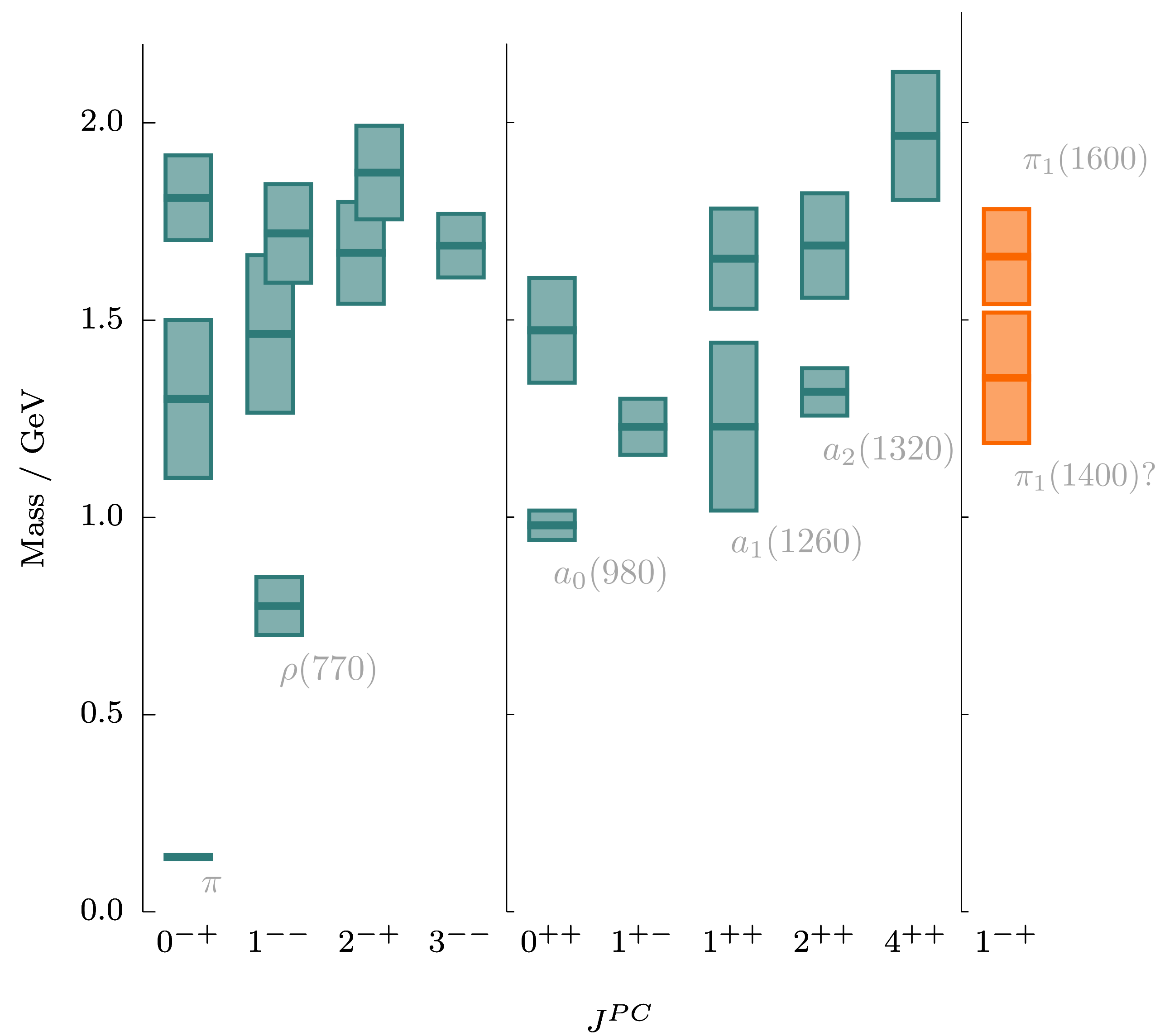


# The Hadron Spectrum

How to connect QCD to the hadron spectrum?

- Need to understand how to quantify what the hadrons are in nature
- Need to find non-perturbative approach to access these hadrons ***rigorously***

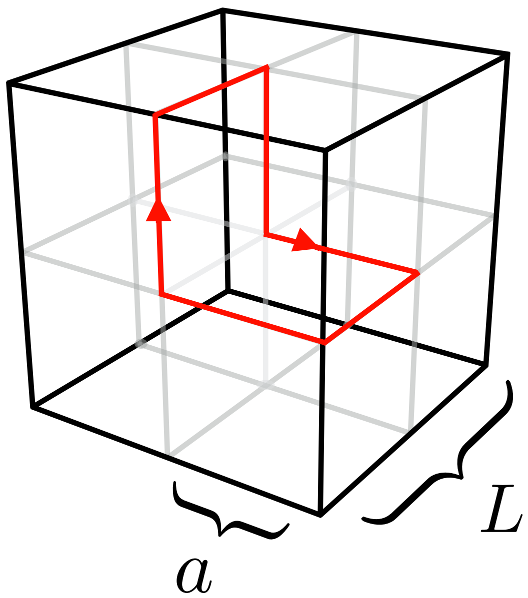
$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_f \left(i\not{D} - m_f\right) \psi_f - \frac{1}{2}\text{tr}\left(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}\right)$$



# Few-Body Physics from QCD

Lattice QCD offers a systematic approach to compute hadrons from QCD

- Numerically evaluate QCD path integral via Monte Carlo sampling



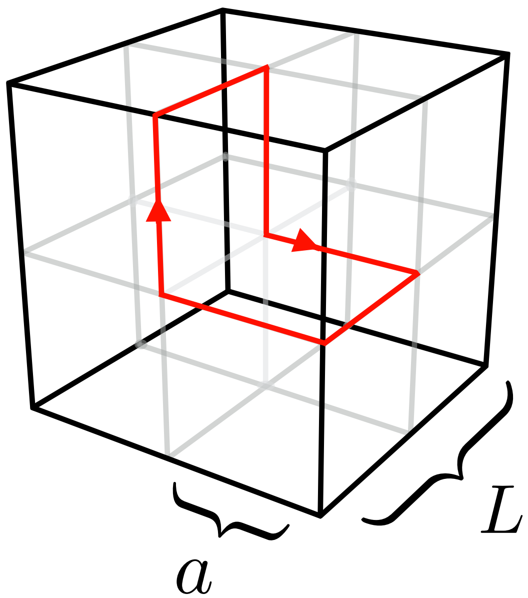

$$Z_{\text{QCD}} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu \, e^{iS_{\text{QCD}}(\psi,\bar{\psi},A_\mu)}$$

- *Euclidean spacetime,* $t \to -i\tau$
- *Finite volume,* $L$
- *Discrete spacetime,* $a$
- *Heavier than physical quark mass,* $m > m_{\text{phys.}}$

# Few-Body Physics from QCD

Lattice QCD offers a systematic approach to compute hadrons from QCD

- Numerically evaluate QCD path integral via Monte Carlo sampling

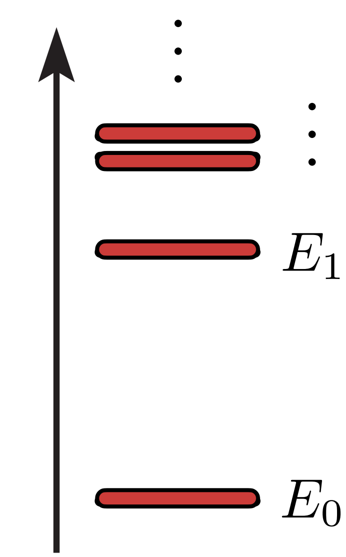



$$Z_{\text{QCD}} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu \, e^{iS_{\text{QCD}}(\psi,\bar{\psi},A_\mu)}$$

- *Euclidean spacetime,* $t \to -i\tau$
- *Finite volume,* $L$
- *Discrete spacetime,* $a$
- *Heavier than physical quark mass,* $m$

*Correlation functions yield discrete spectrum*

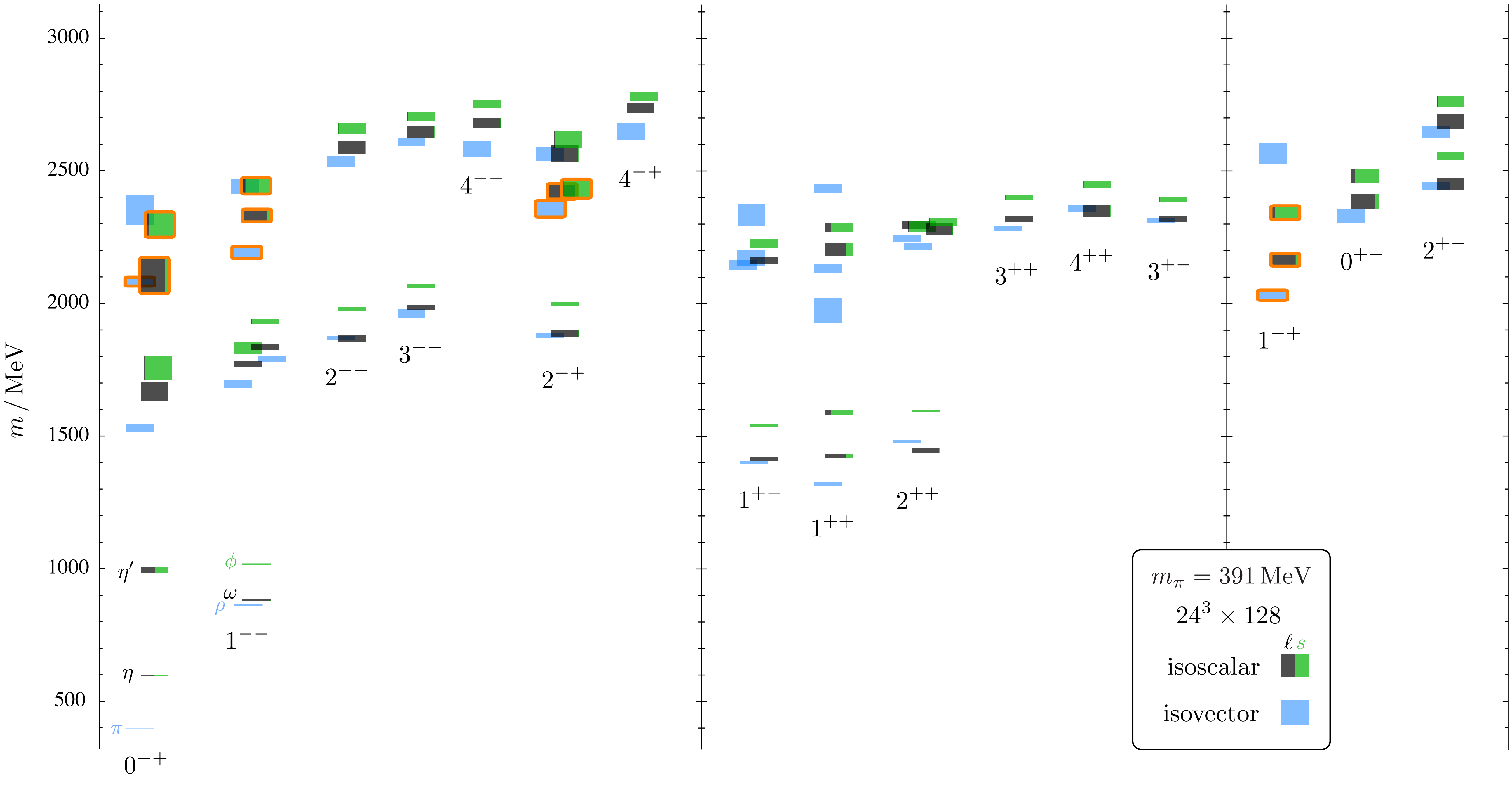
$$\langle \mathcal{O}(\tau)\mathcal{O}^\dagger(0)\rangle = \sum_{\mathfrak{n}} \left|\langle 0| \mathcal{O} |\mathfrak{n}\rangle\right|^2 \, e^{-E_{\mathfrak{n}}\tau}$$



# Few-Body Physics from QCD

Lattice QCD offers a systematic approach to compute hadrons from QCD

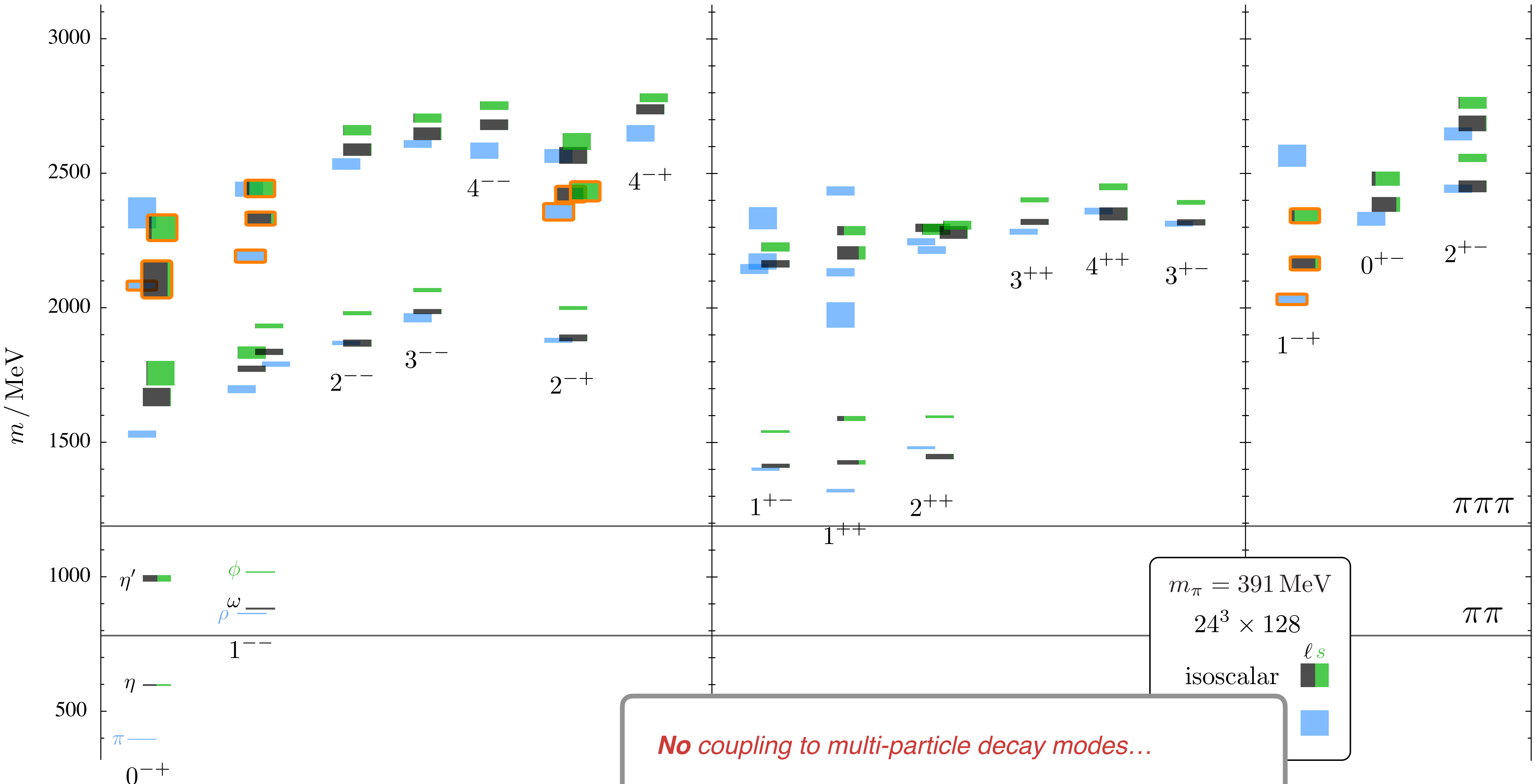
- Numerically evaluate QCD path integral via Monte Carlo sampling




J. Dudek, R. Edwards, P. Guo, and C. Thomas
Phys.Rev.D **88**, 094505 (2013)

Lattice QCD offers a systematic approach to compute hadrons from QCD

- Numerically evaluate QCD path integral via Monte Carlo sampling



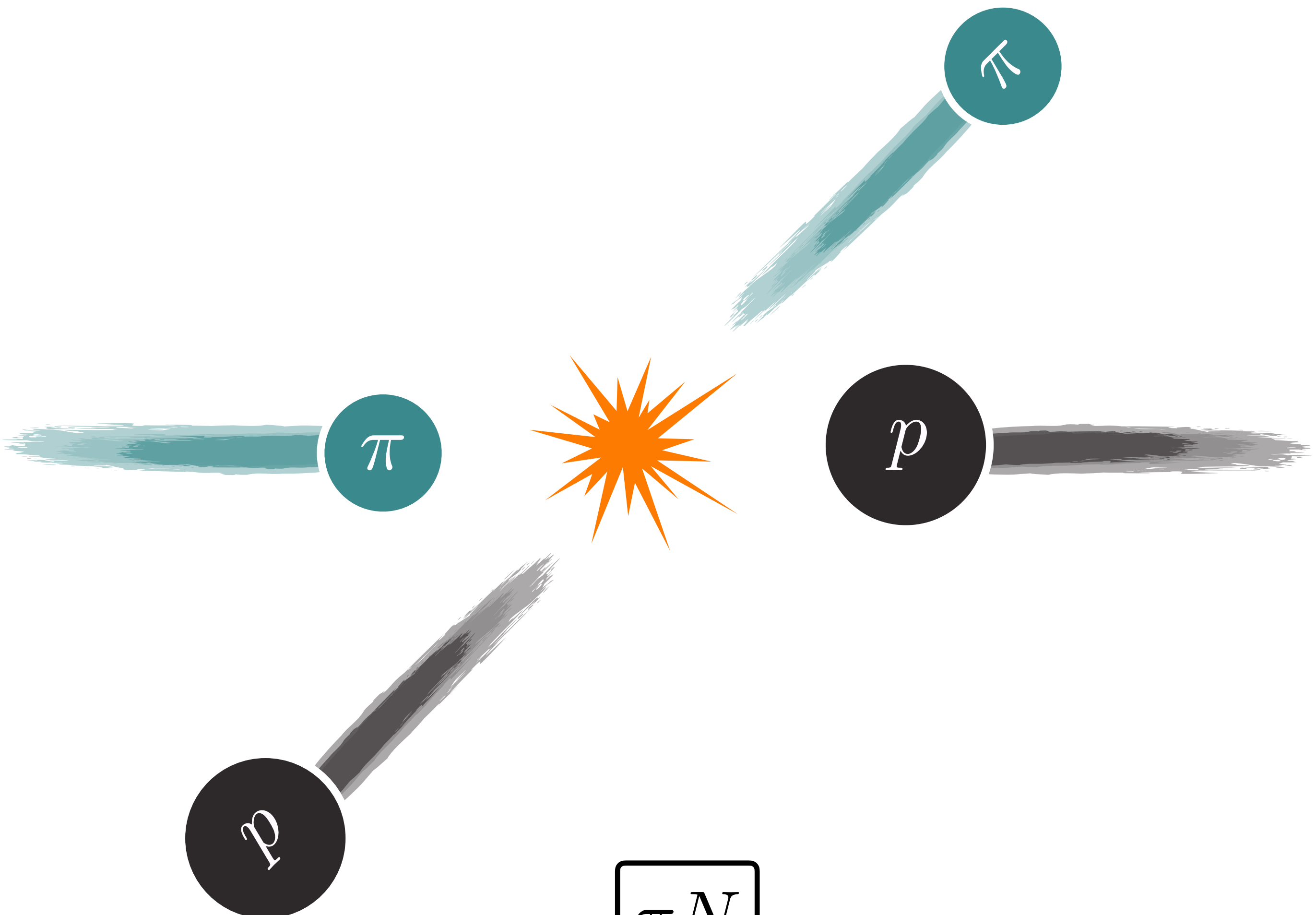

***No** coupling to multi-particle decay modes…*

*…An incomplete picture of QCD spectrum*

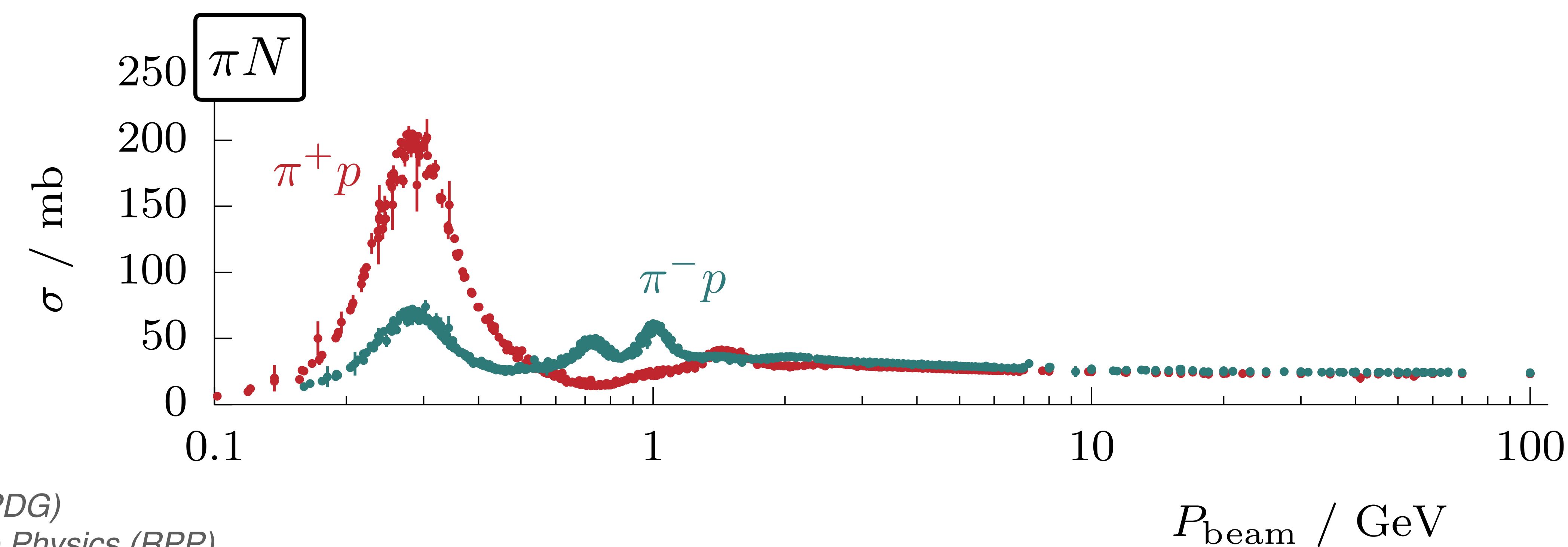
J. Dudek, R. Edwards, P. Guo, and C. Thomas
Phys.Rev.D **88**, 094505 (2013)

# Few-Body Physics from QCD

All our knowledge of the hadrons comes from ***scattering experiments***




How to quantify such a process with Lattice QCD?

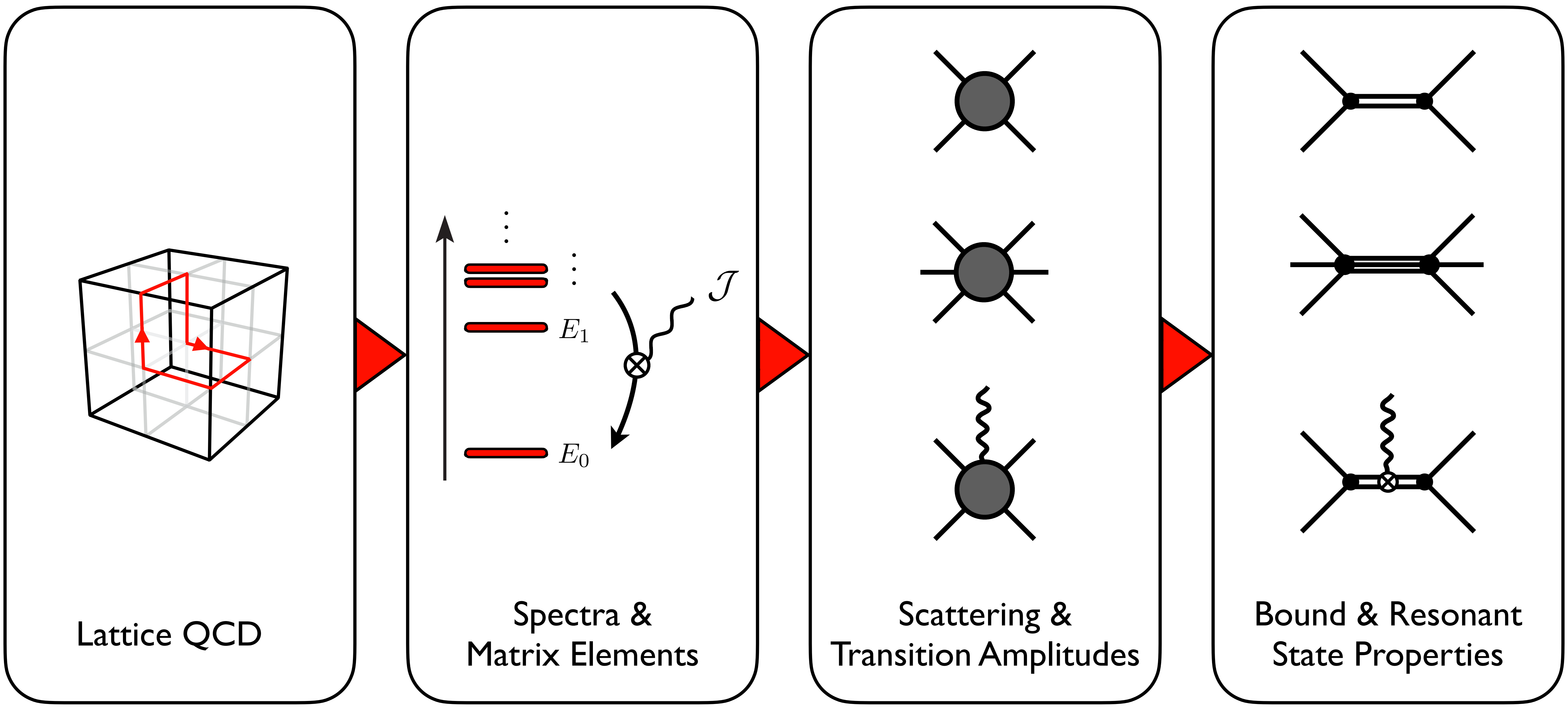



*Particle Data Group (PDG)*
*The Review of Particle Physics (RPP)*

# Few-Body Physics from QCD

Path to few-body physics from QCD

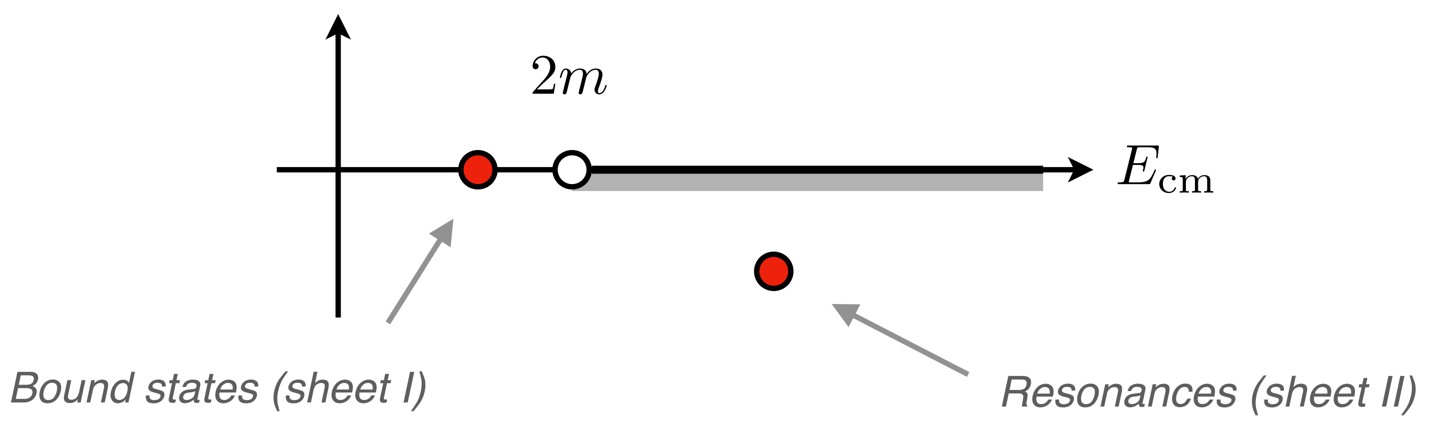
- Link finite-volume spectra and matrix elements to scattering amplitudes
- Tools: *Lattice QCD*, *Scattering Theory*, & *Effective Field Theory*



# Scattering Theory & QCD Spectrum

Resonances & Bound states are pole singularities of scattering amplitudes

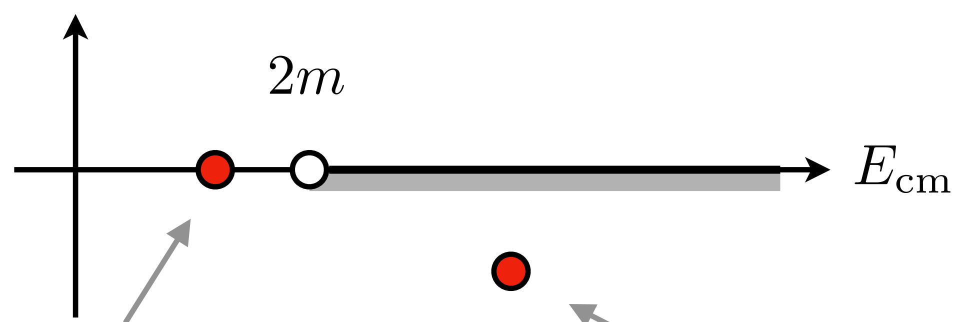
$$\mathcal{M}_2 = \mathcal{K}_2 \, \frac{1}{1 - i\rho\,\mathcal{K}_2}$$



# Scattering Theory & QCD Spectrum

Resonances & Bound states are pole singularities of scattering amplitudes

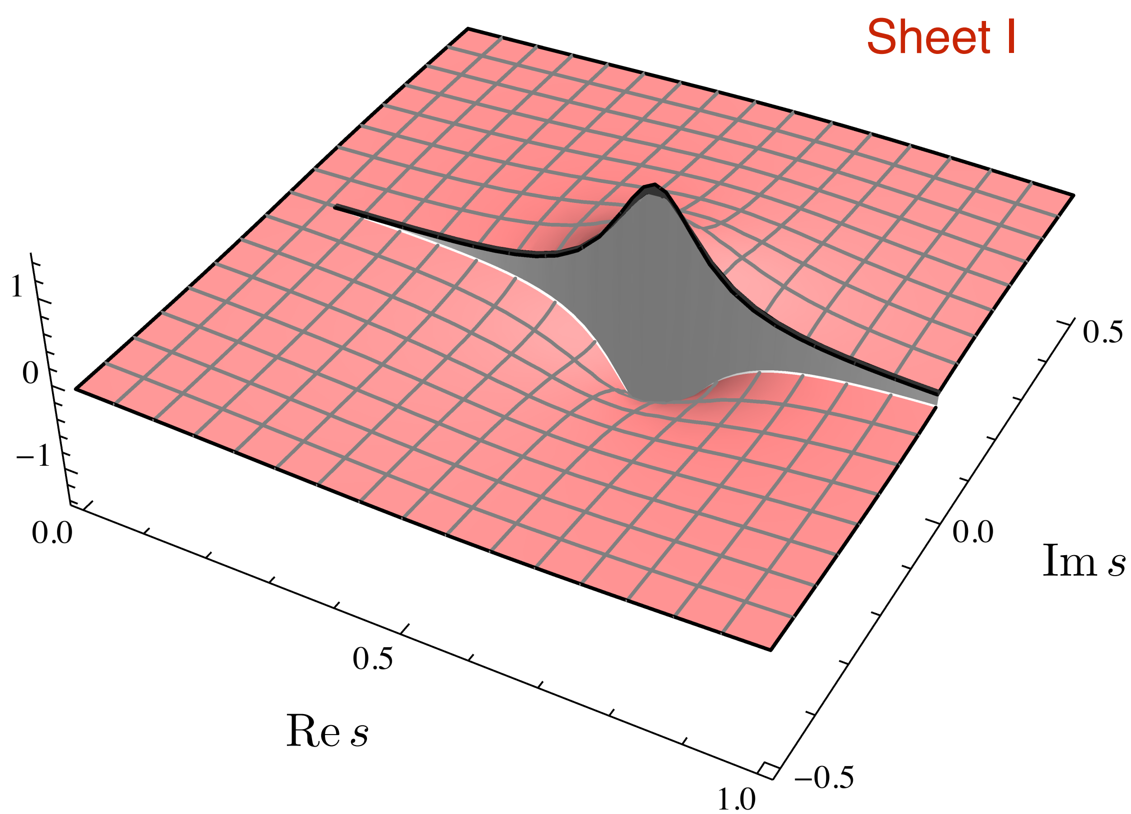
$$\mathcal{M}_2 = \mathcal{K}_2 \frac{1}{1 - i\rho\,\mathcal{K}_2}$$

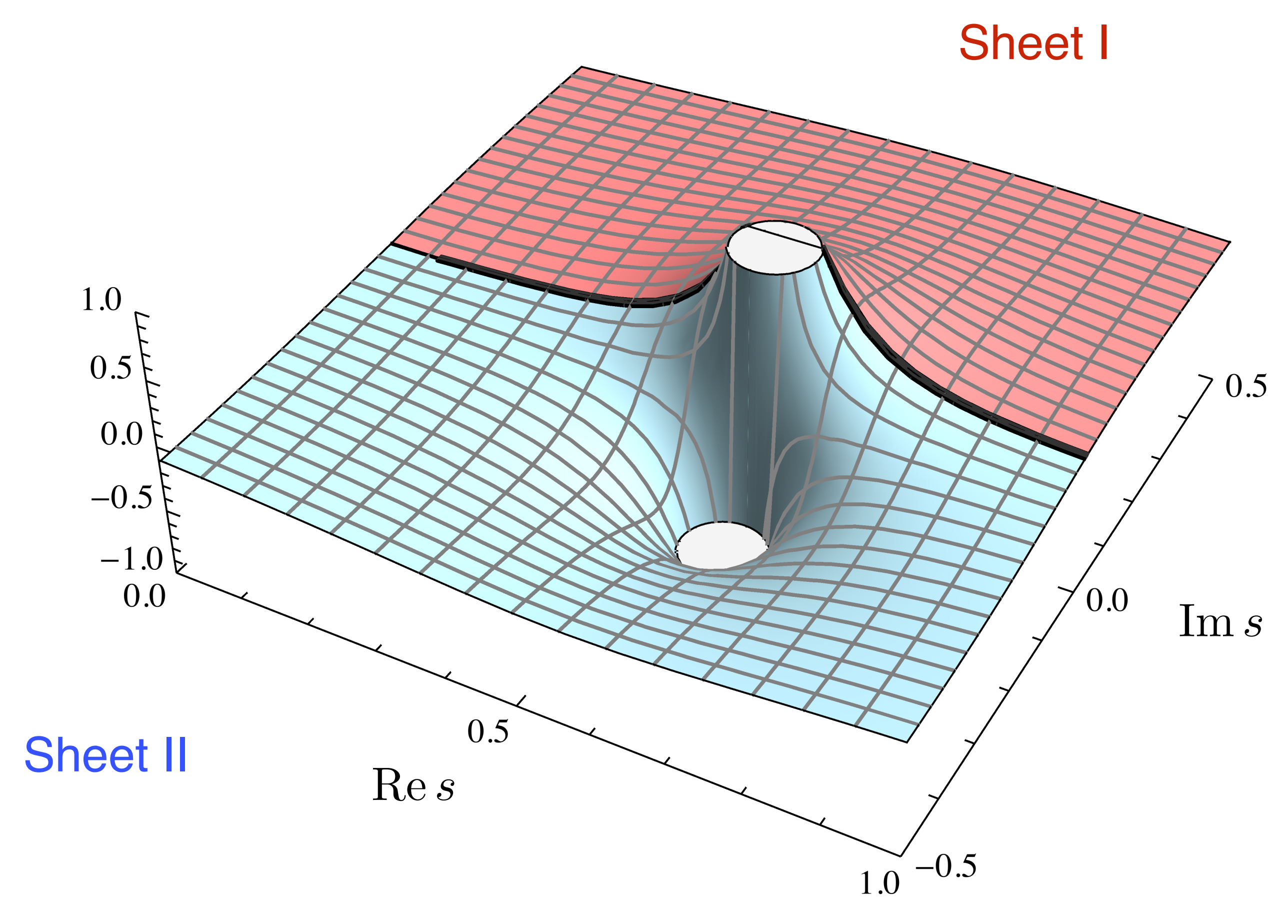



*Bound states (sheet I)*

*Resonances (sheet II)*

*e.g., Narrow resonance*



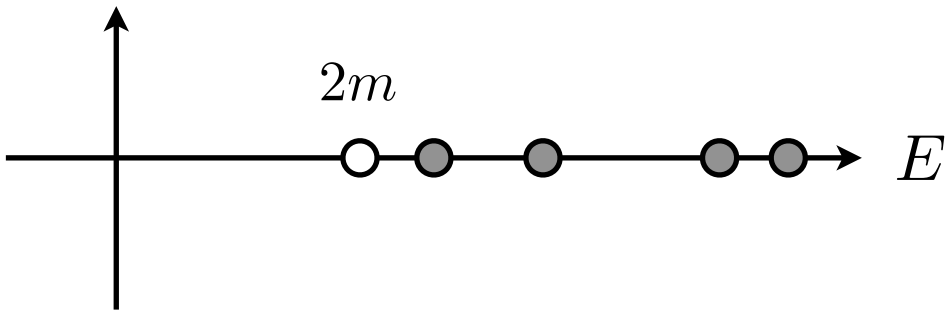




$s = E_{\rm cm}^2$

# Connecting Scattering Physics to QCD

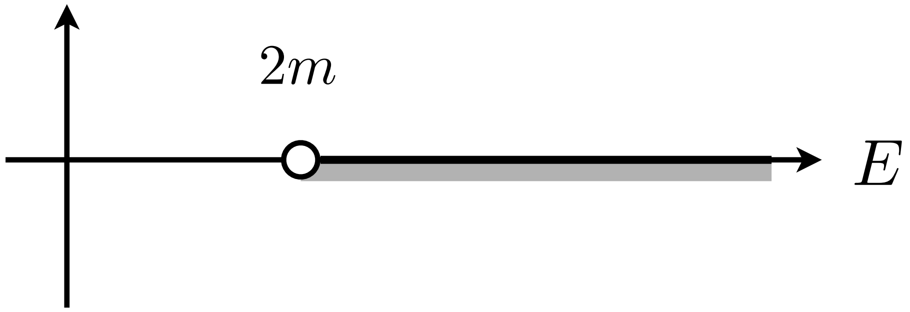
**Q:** How do we connect a finite-volume spectrum computed from QCD…

$$\int_L \mathrm{d}^4 x \, e^{iP\cdot x} \, \langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \sum_{\mathfrak{n}} \frac{i \, |\langle 0| \, \mathcal{O} \, |\mathfrak{n}\rangle|^2}{E - E_{\mathfrak{n}}}$$




…to infinite-volume scattering amplitudes?

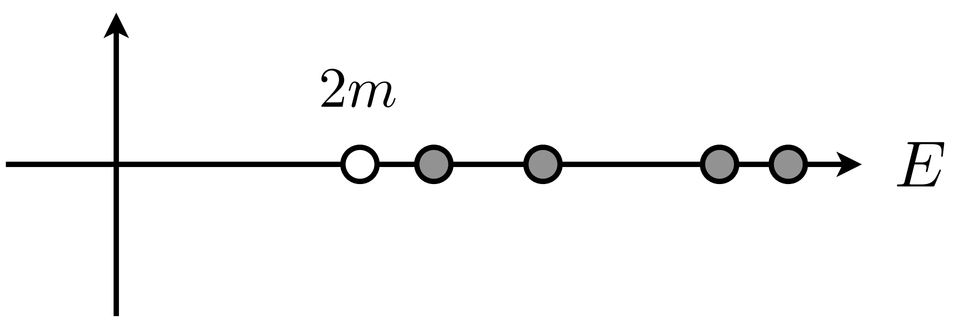
$$\mathcal{M}_2 = \mathcal{K}_2 \, \frac{1}{1 - i\rho \, \mathcal{K}_2}$$



# Connecting Scattering Physics to QCD

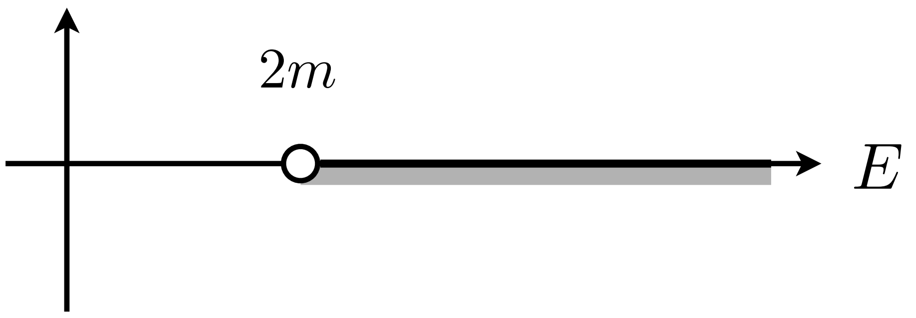
**Q:** How do we connect a finite-volume spectrum computed from QCD…

$$\int_L \mathrm{d}^4x\, e^{iP\cdot x}\, \langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \sum_{\mathfrak{n}} \frac{i\,|\langle 0|\,\mathcal{O}\,|\mathfrak{n}\rangle|^2}{E - E_{\mathfrak{n}}}$$




…to infinite-volume scattering amplitudes?

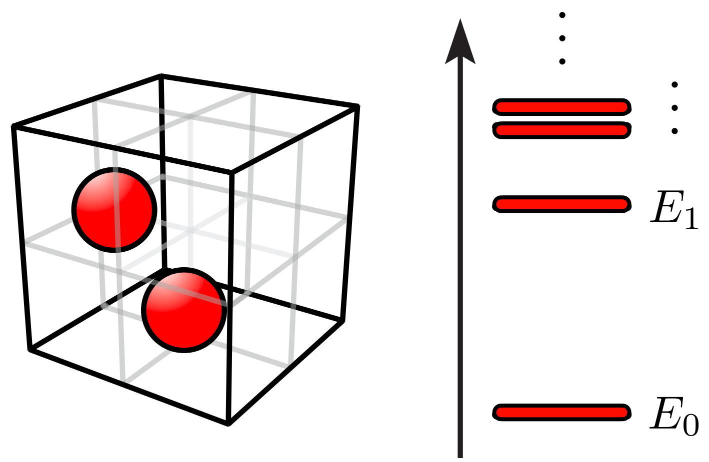
$$\mathcal{M}_2 = \mathcal{K}_2 \frac{1}{1 - i\rho\,\mathcal{K}_2}$$




**A:** Correct analytic structure of finite-volume correlators

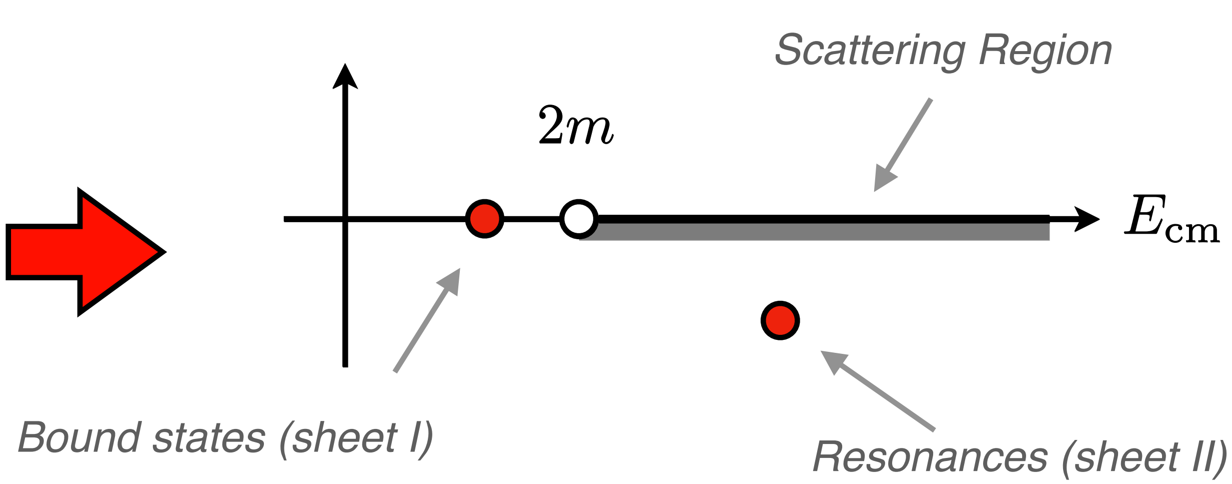
# Connecting Scattering Physics to QCD

Employing scattering theory and EFTs to all-orders connects lattice QCD spectra to scattering observables




$$\det\left(1 + \mathcal{K}_2 F_L\right)_{E=E_n} = 0$$

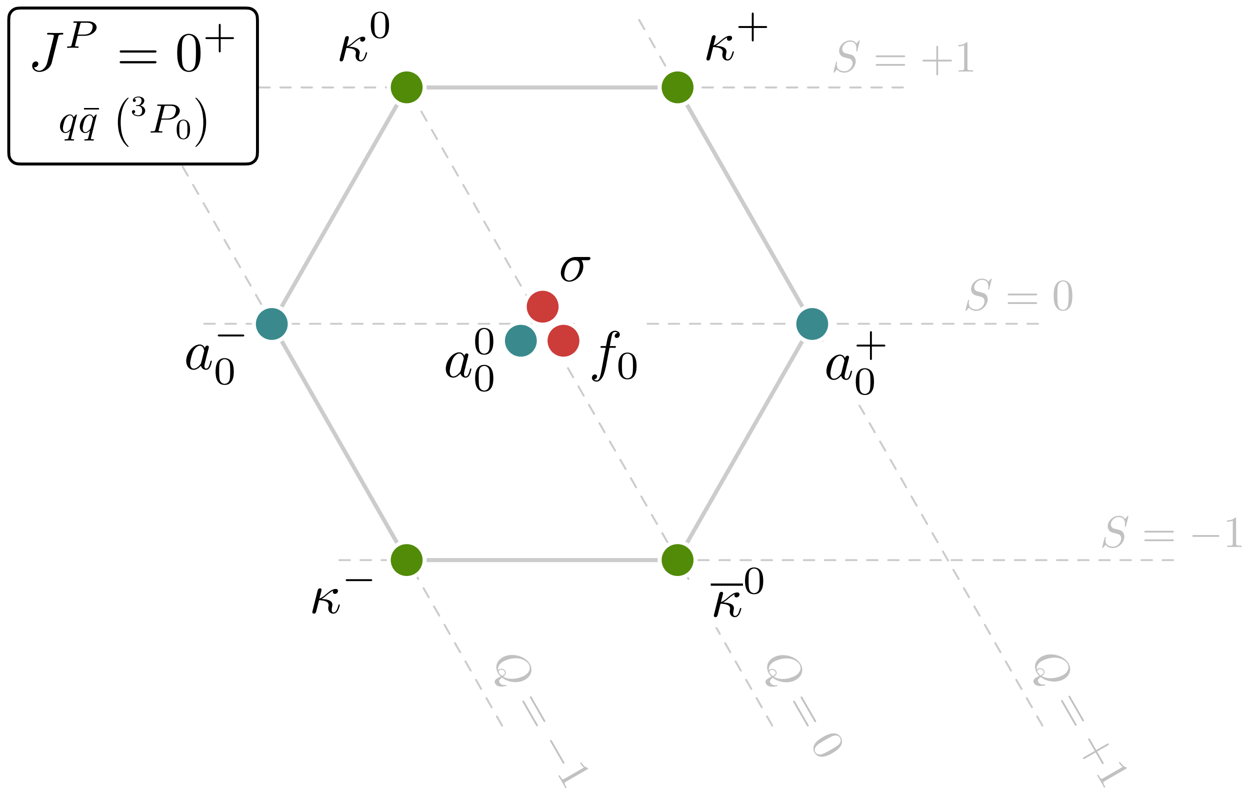
$$\mathcal{M}_2 = \mathcal{K}_2 \frac{1}{1 - i\rho\,\mathcal{K}_2}$$



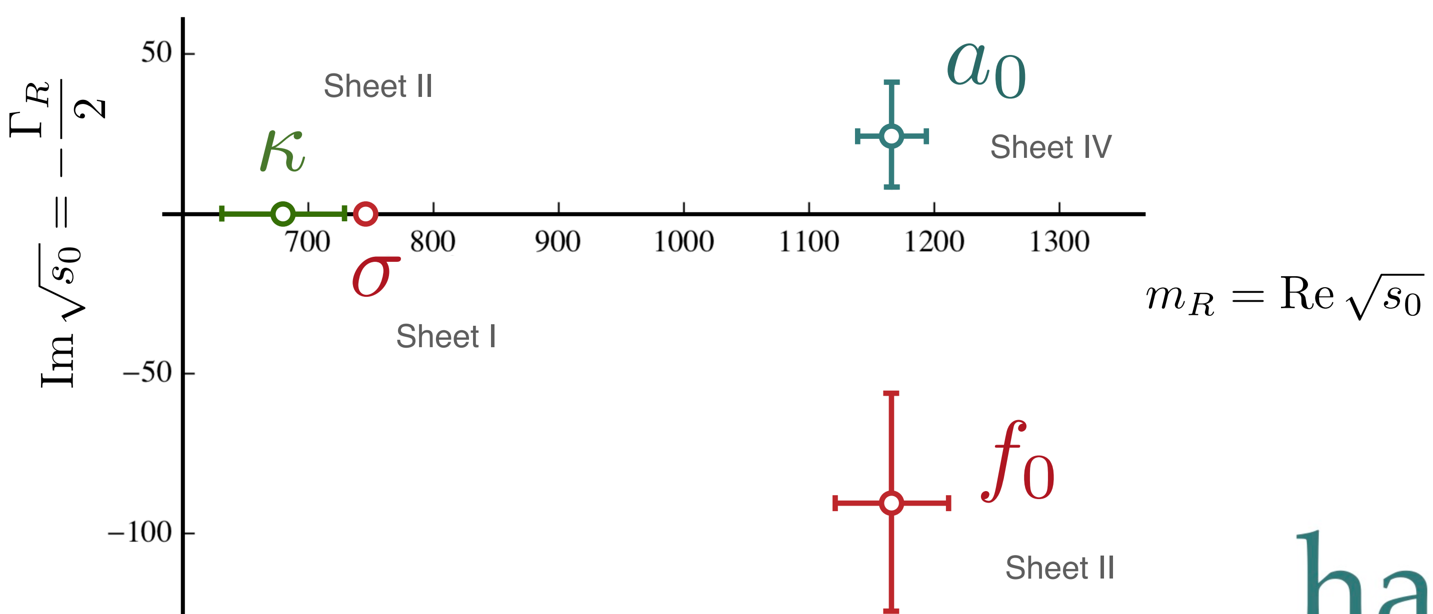

M. Lüscher
Commun.Math.Phys. **105**, 153 (1986)
Nucl.Phys. **B354**, 531 (1991)

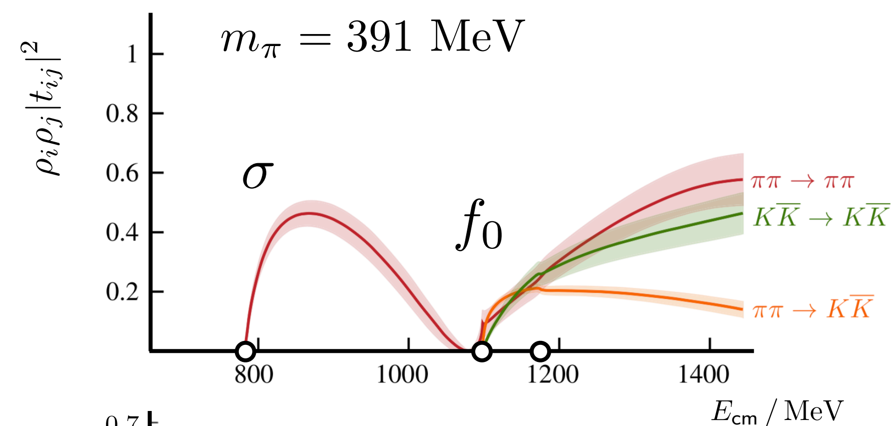
Many others…

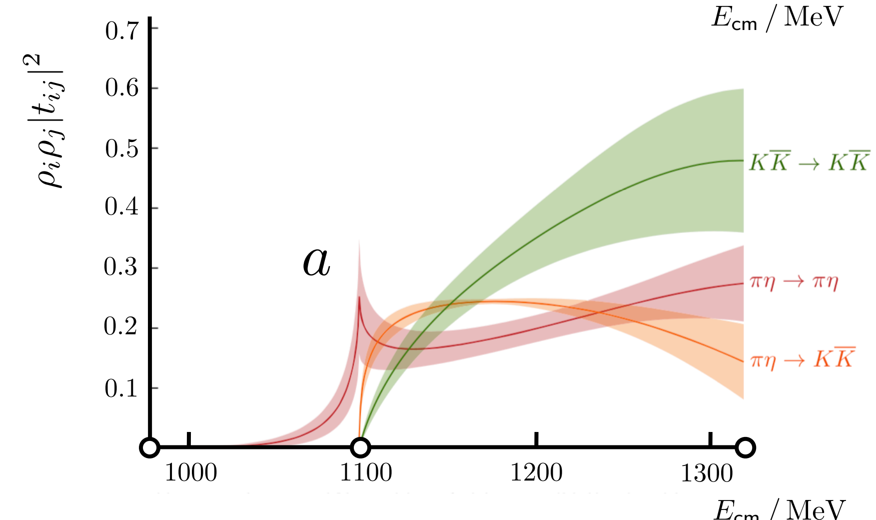
# Connecting Scattering Physics to QCD

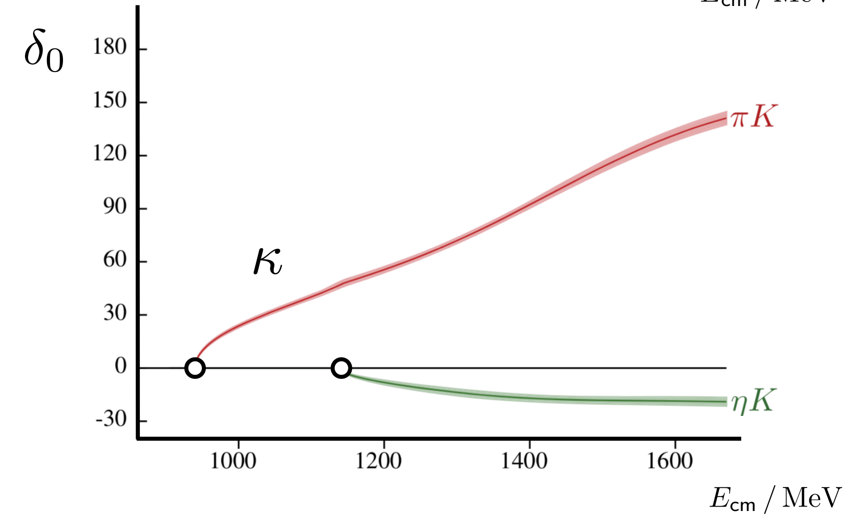



$m_\pi = 391$ MeV



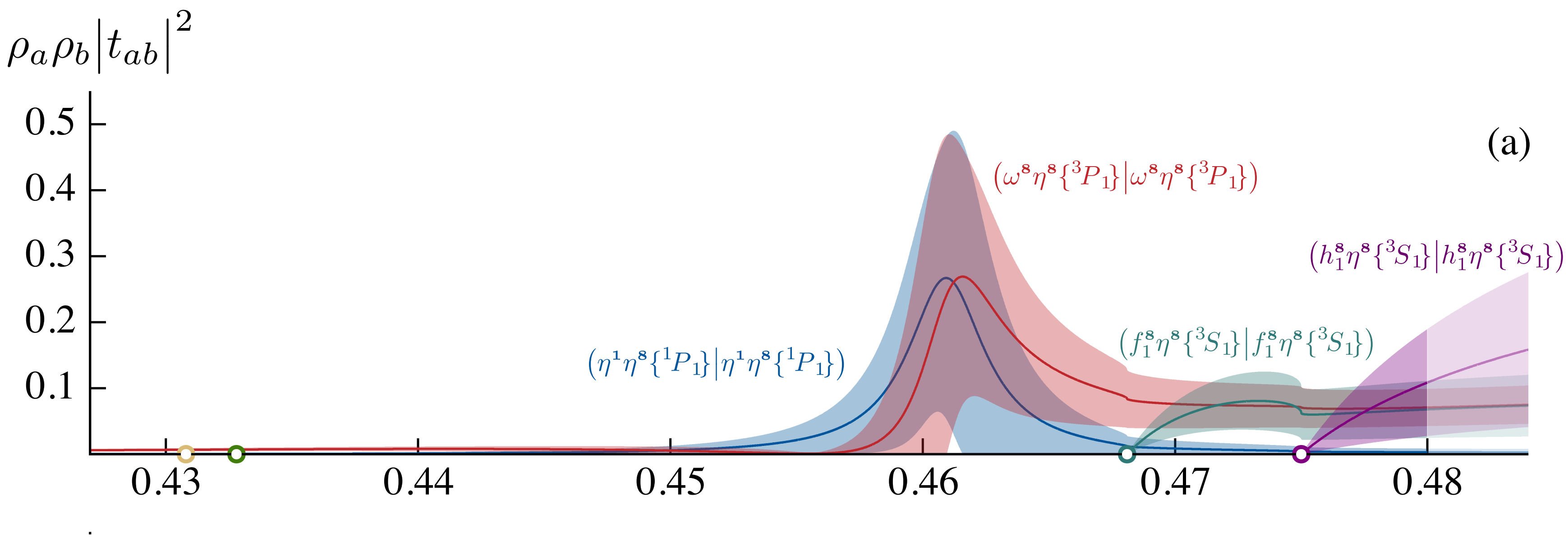

R.A. Briceño et al. [HadSpec]
Phys. Rev. **D97**, 054513 (2018)

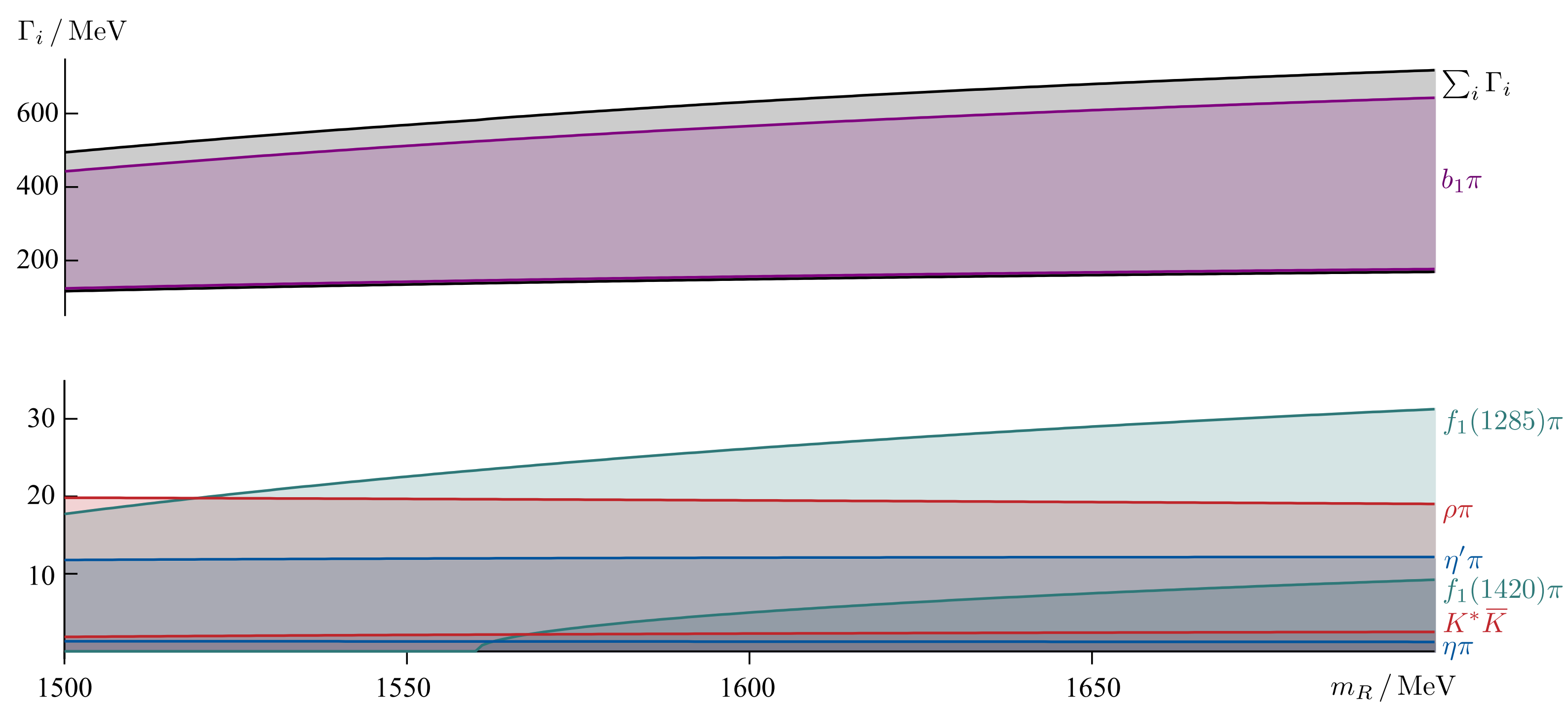
J.J. Dudek et al. [HadSpec]
Phys. Rev. **D93**, 094506 (2016)

J.J. Dudek et al. [HadSpec]
Phys. Rev. Lett. **113**, 182001 (2014)

# Connecting Scattering Physics to QCD

First determination of hybrid candidate, $J^{PC} = 1^{-+}$, $m_\pi \sim 700\,\mathrm{MeV}$

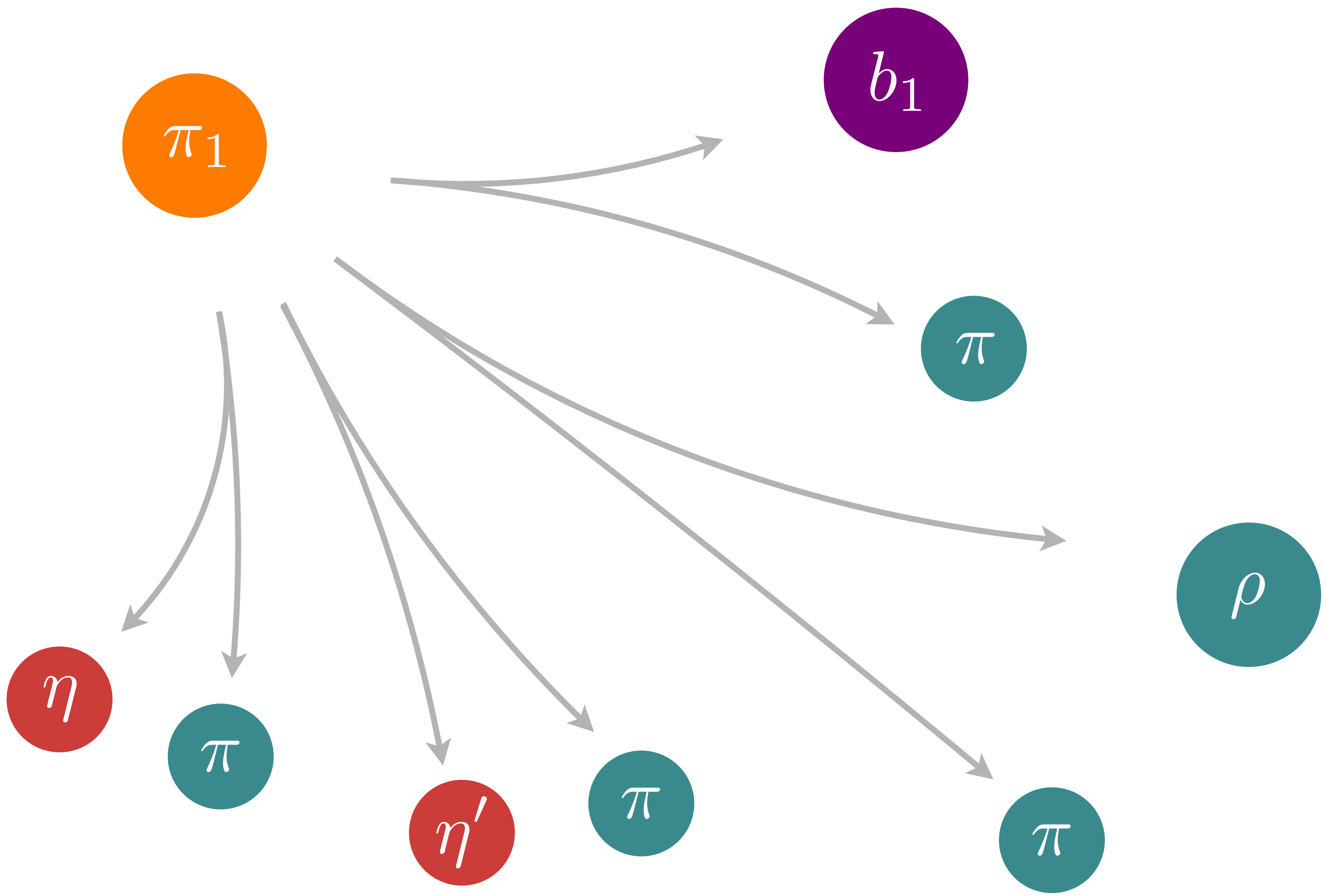



A.J. Woss et al. [HadSpec]
Phys. Rev. **D103**, 054502 (2021)

# Connecting Scattering Physics to QCD

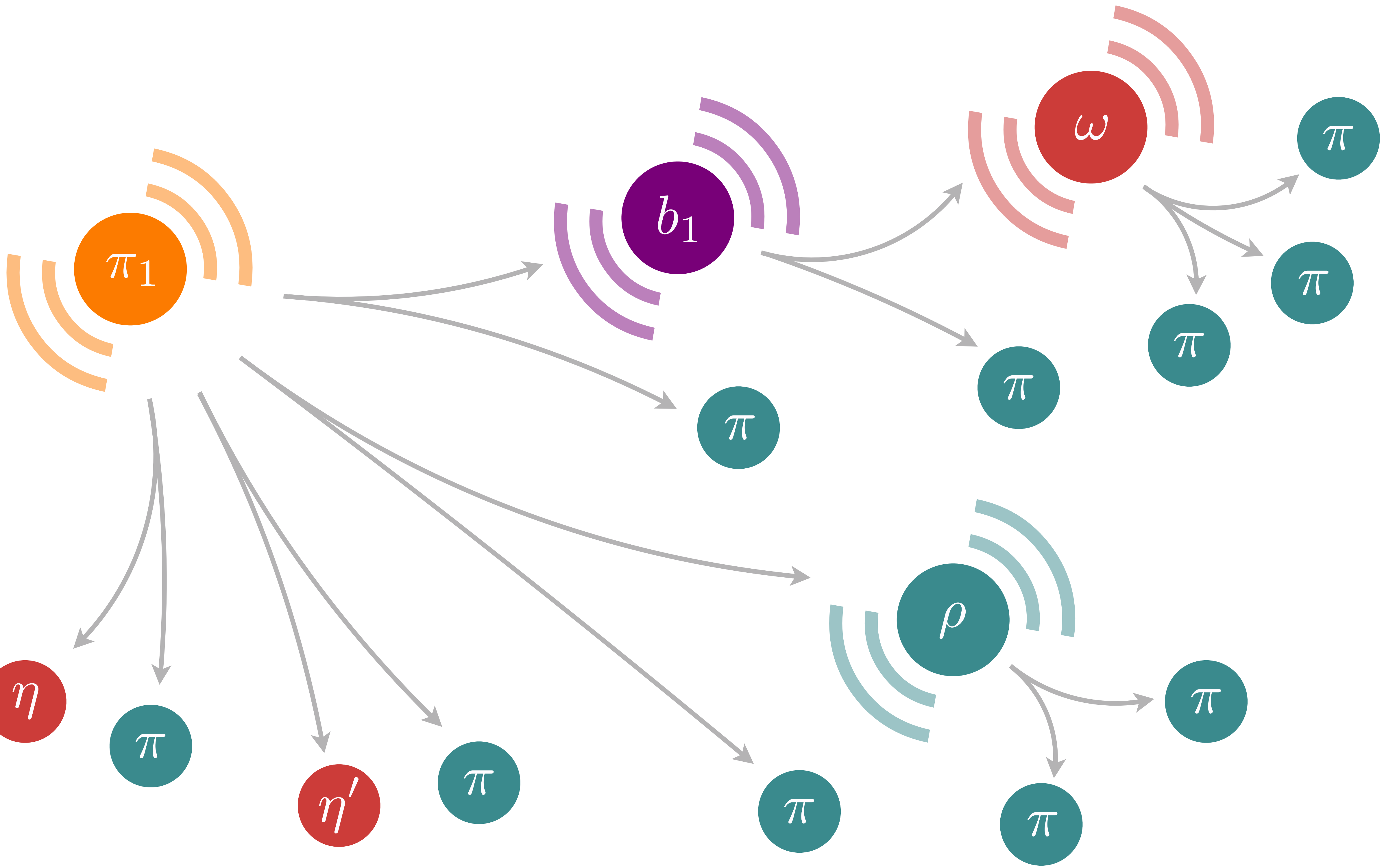
First determination of hybrid candidate, $J^{PC} = 1^{-+}$, $m_\pi \sim 700\,\mathrm{MeV}$



# Connecting Scattering Physics to QCD

First determination of hybrid candidate, $J^{PC} = 1^{-+}$, $m_\pi \sim 700\,\mathrm{MeV}$

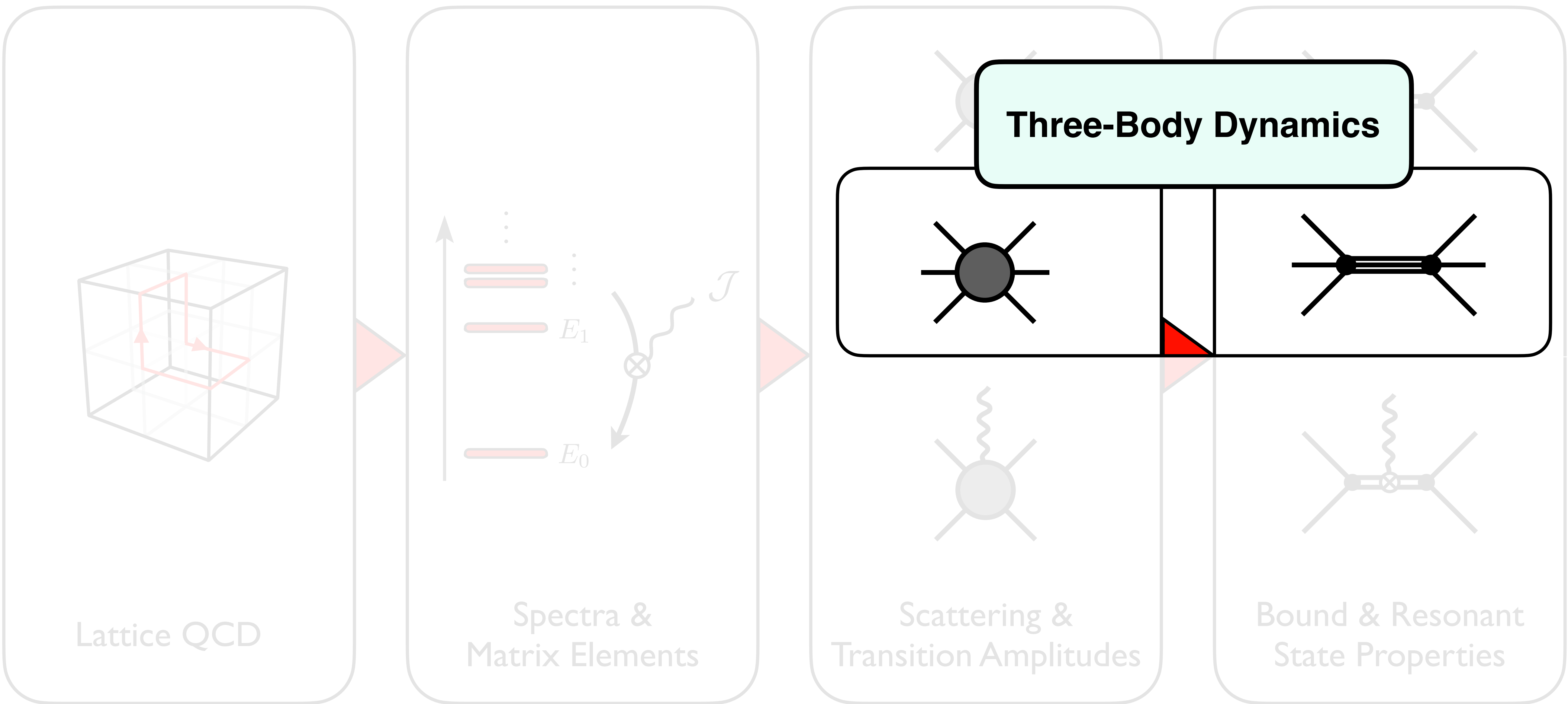
At physical point, 3, 4,.. body decays



# Few-Body Physics from QCD

Path to few-body physics from QCD

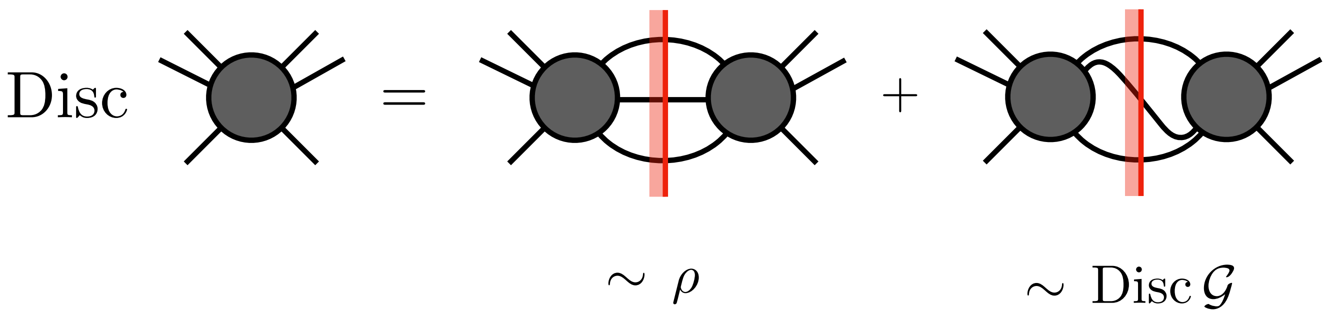
- Link finite-volume spectra and matrix elements to scattering amplitudes
- Tools: *Lattice QCD*, *Scattering Theory*, & *Effective Field Theory*



# Three-Body Dynamics

## On-shell scattering relations

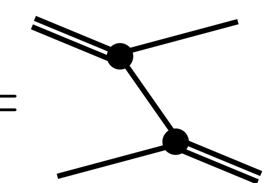
*Unitarity condition*




*On-shell scattering equation*

$$\mathcal{M}_3 = \mathcal{K}_3 + \int \mathcal{K}_2 \cdot i\rho \cdot \mathcal{M}_3 + \iint \mathcal{K}_3 \cdot \mathcal{G} \cdot \mathcal{M}_3$$

$\mathcal{K}_3$ *Unknown!*
*Obtained from Lattice QCD*

$\mathcal{G} =$ 

M. Mai, B. Hu, M. Döring, A. Pilloni, and A. Szczepaniak
Eur. Phys. J. A **53**, 177 (2017)

AJ et al. [JPAC]
Eur. Phys. J. C **79**, no. 1, 56 (2019)

AJ et al. [JPAC]
Phys. Rev. D **100**, 034508 (2019)

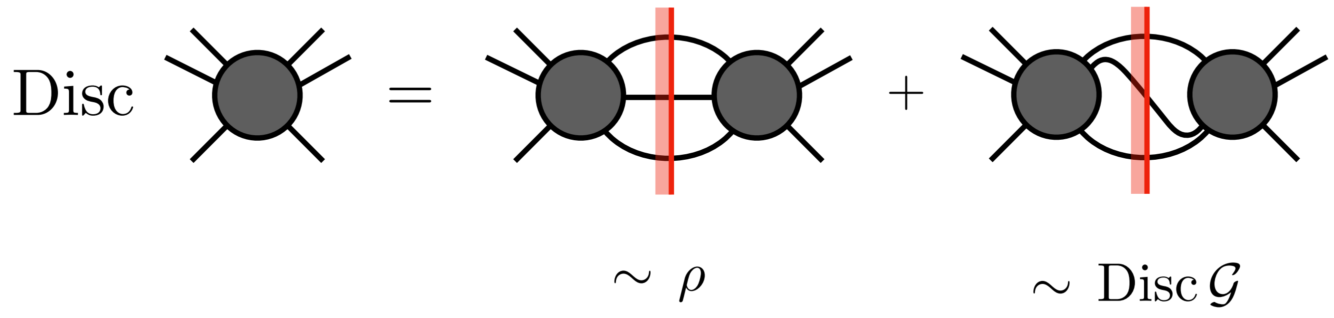
AJ, arxiv:2208.10587

*cf. two-body case:*

$$\mathcal{M}_2 = \mathcal{K}_2 + \mathcal{K}_2 \cdot i\rho \cdot \mathcal{M}_2$$

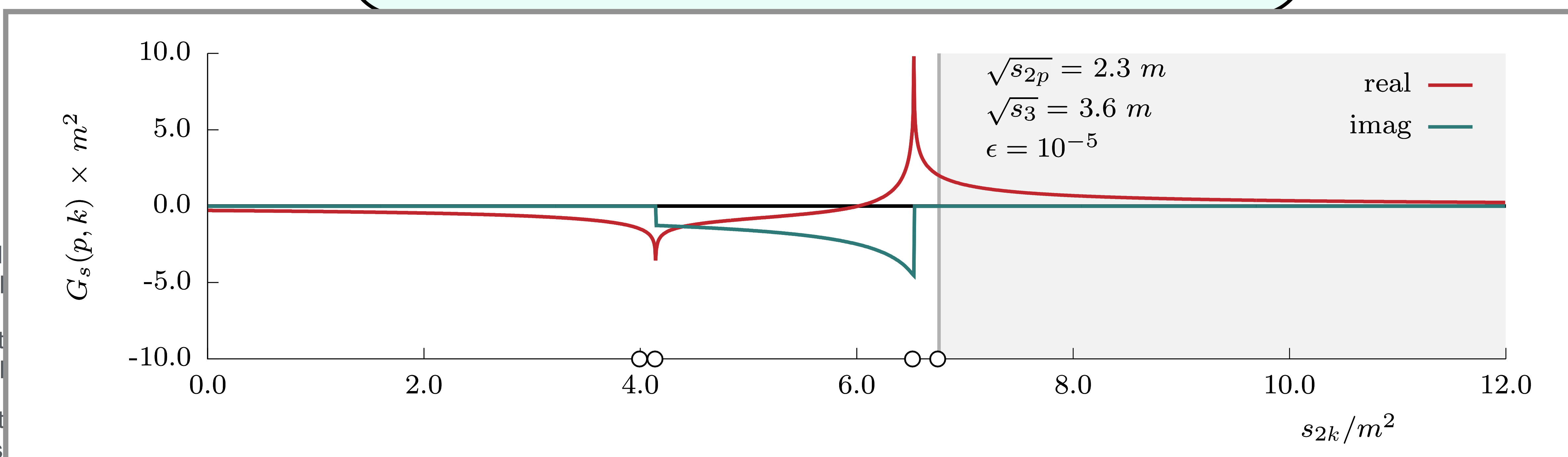
## On-shell scattering relations

*Unitarity condition*




*On-shell scattering equation*

$$\mathcal{M}_3 = \mathcal{K}_3 + \int \mathcal{K}_2 \cdot i\rho \cdot \mathcal{M}_3 + \iint \mathcal{K}_3 \cdot \mathcal{G} \cdot \mathcal{M}_3$$




M. M… Eur. …

AJ et… Eur. …

AJ et… Phys…

…CD

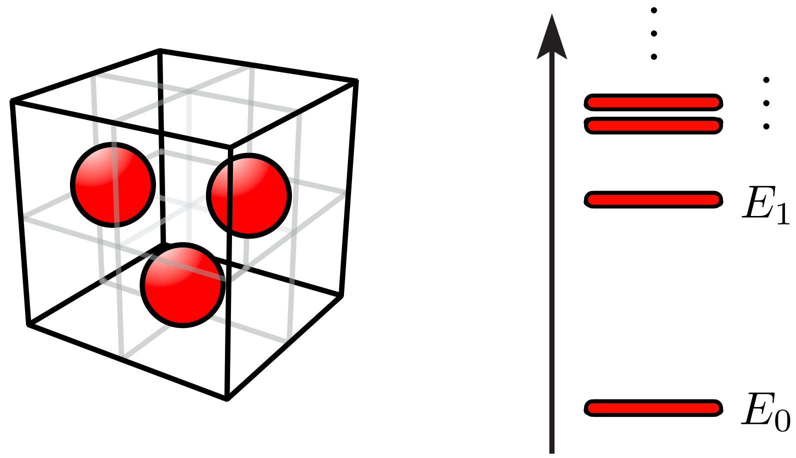
AJ, arxiv:2208.10587

$$\mathcal{M}_2 = \mathcal{K}_2 + \mathcal{K}_2 \cdot i\rho \cdot \mathcal{M}_2$$

# Three-Body Dynamics

## Connecting to finite-volume spectra

*Finite-volume quantization condition*




$$\det\Big(1+\mathcal{K}_3\,\left(\mathcal{F}_L+\mathcal{G}_L\right)\Big)_{E=E_{\mathfrak{n}}}=0$$

$$\mathcal{M}_3=\mathcal{K}_3+\int\mathcal{K}_2\cdot i\rho\cdot\mathcal{M}_3+\iint\mathcal{K}_3\cdot\mathcal{G}\cdot\mathcal{M}_3$$

M. Hansen and S. Sharpe
Phys. Rev. D **90**, 116003 (2014), Phys. Rev. D **95**, 034501 (2017)

M. Mai and M. Döring
Eur. Phys. J. A **53**, 240 (2017), Phys. Rev. Lett. **122**, 062503 (2019)

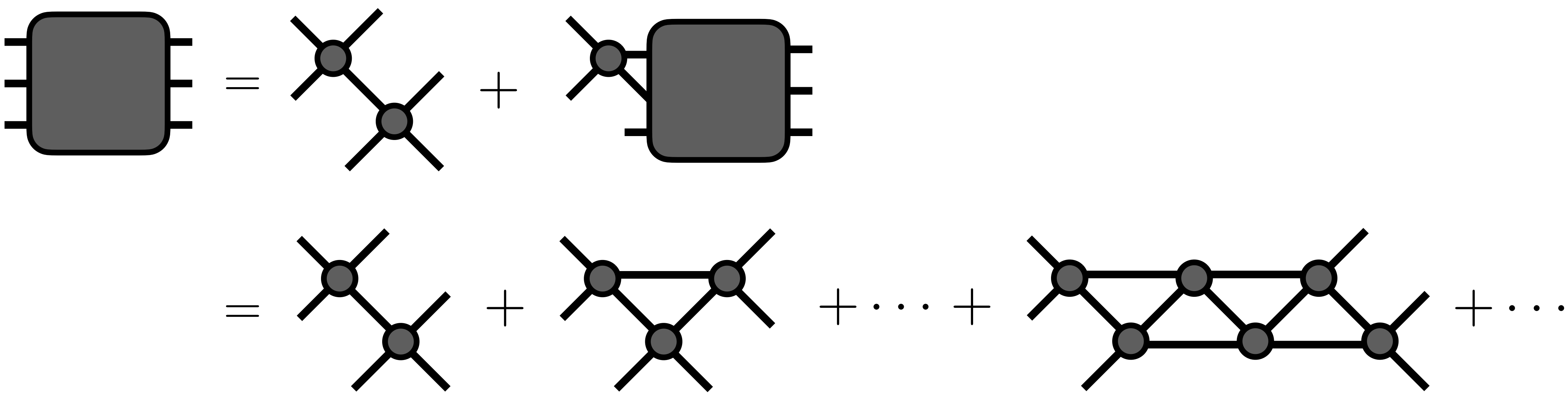
AJ, arxiv:2208.10587

# Three-Body Dynamics

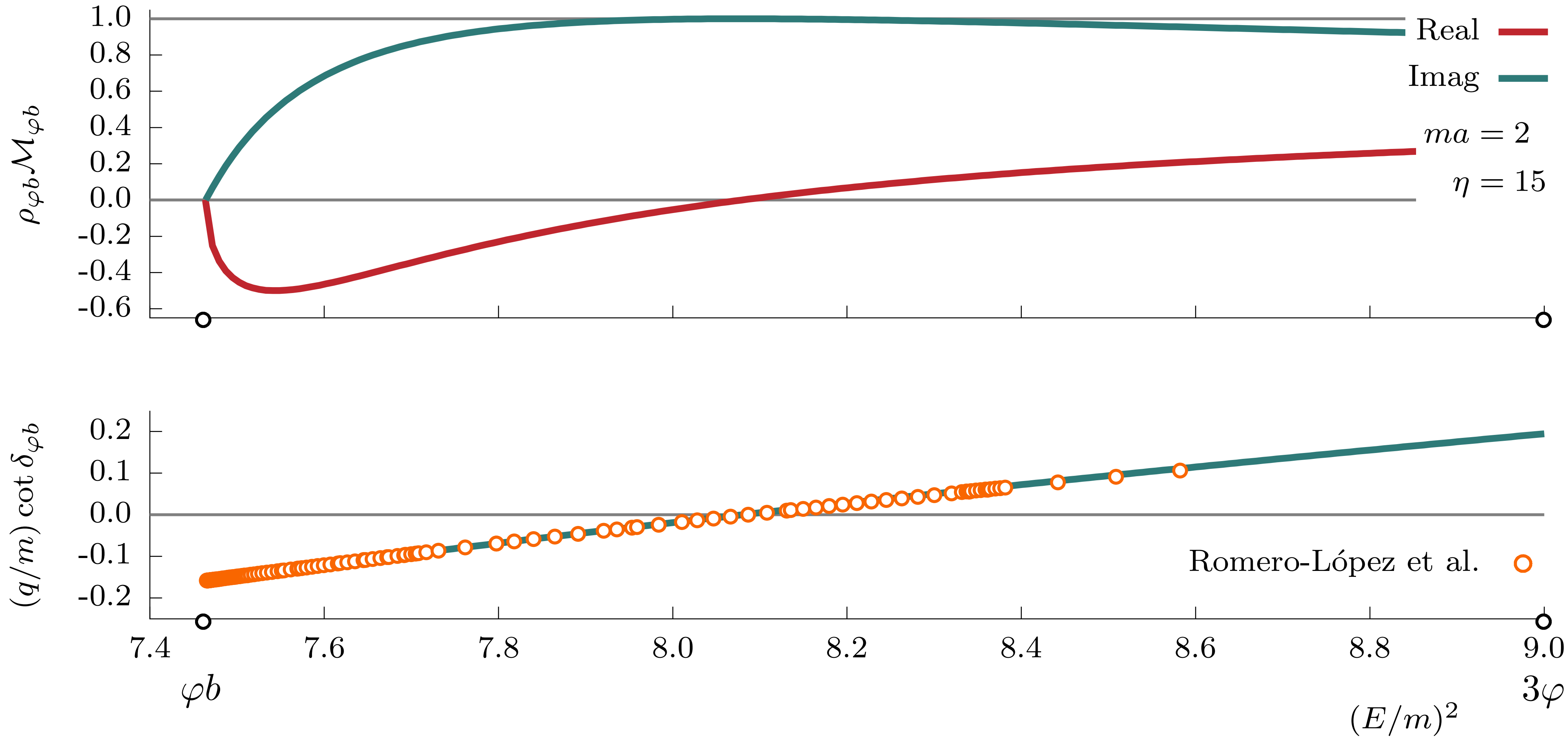
Examine toy-model — $3\varphi \to 3\varphi$

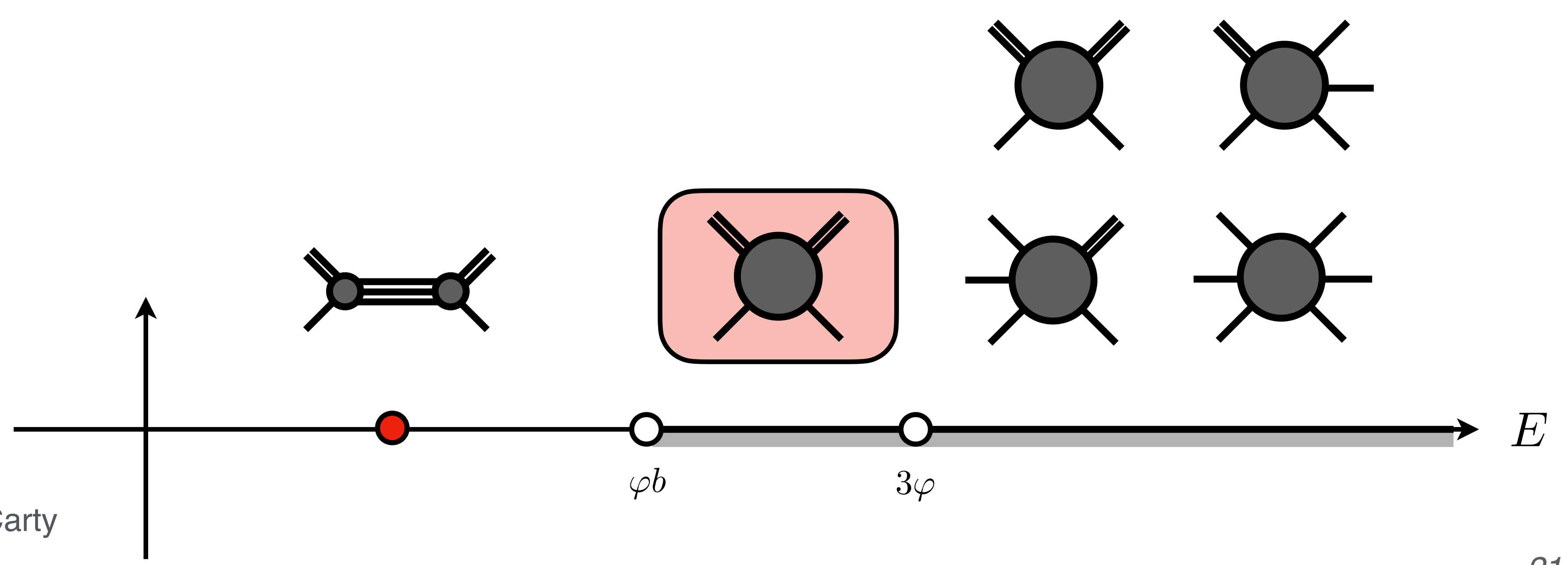
- Assume exchange dominance — ***No short-range three-body forces***
- Scalar system — $J = 0$
- Two-hadron pair forms bound state — $2\varphi \to b$

*Toy model version of* $3N \to 3N$ *with* $2N \to d$



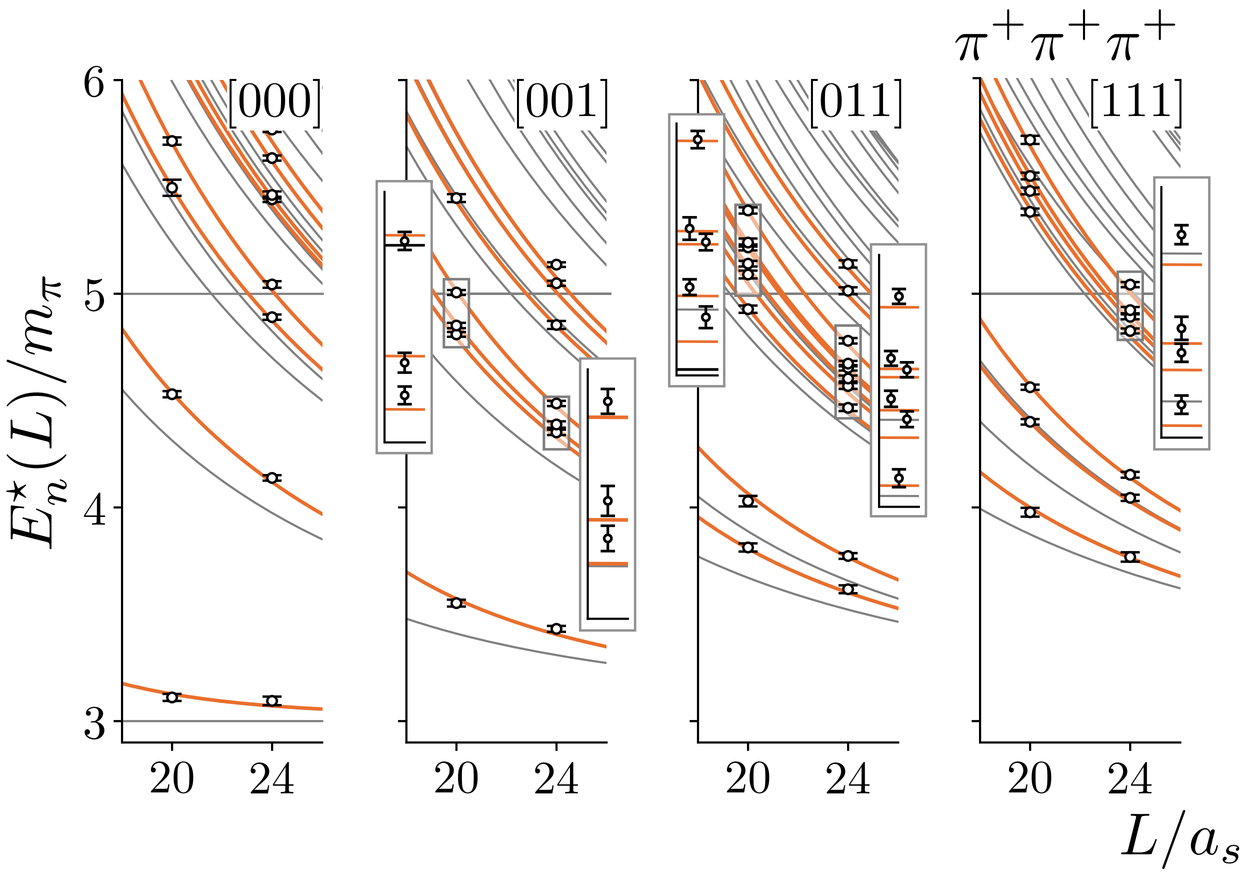
AJ, R. Briceño, S. Dawid, M.H. Islam, and C. McCarty
Phys. Rev. D **104** 014507 (2021)

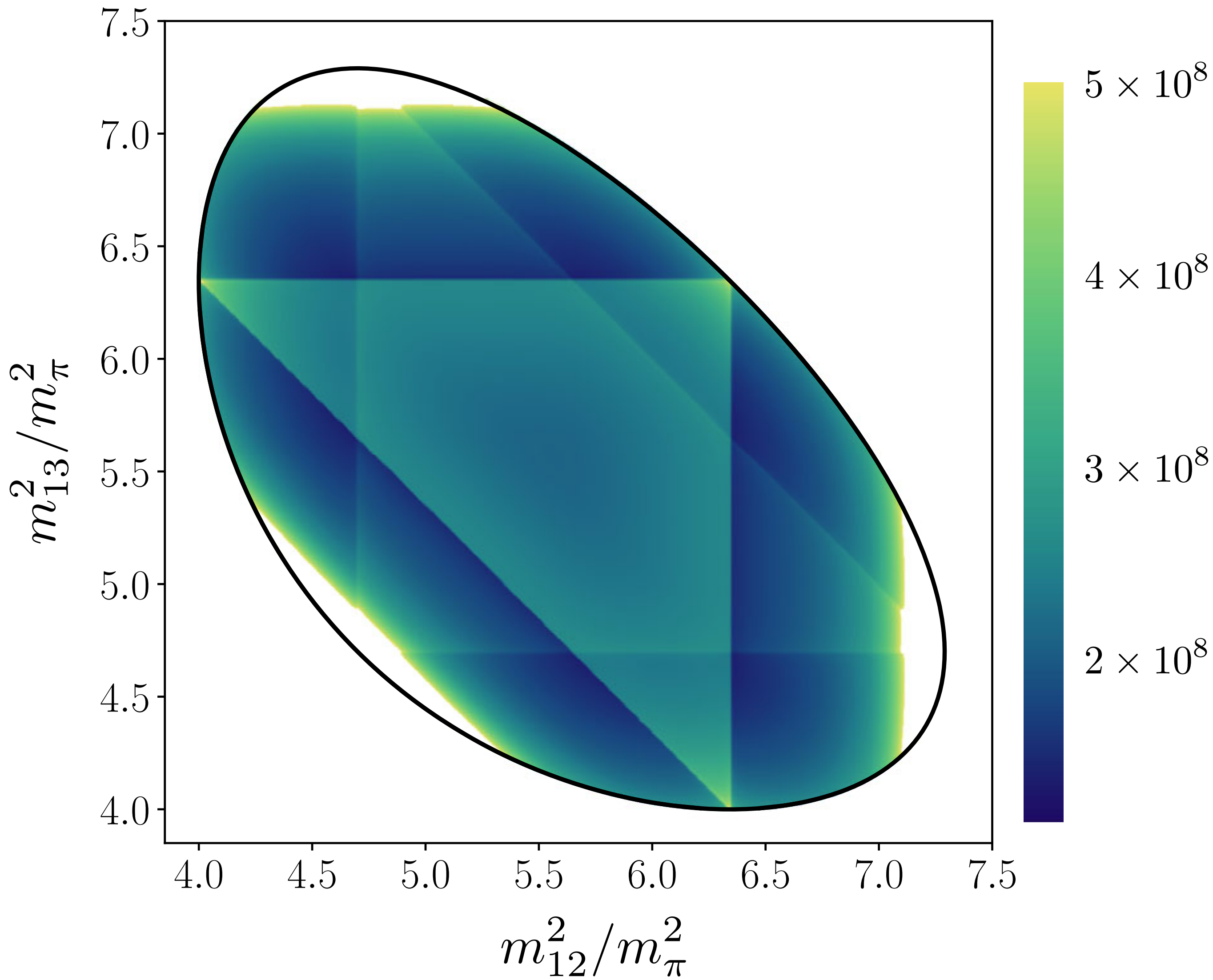



AJ, R. Briceño, S. Dawid, M.H. Islam, and C. McCarty
Phys. Rev. D **104** 014507 (2021)

# Applications to $3\pi^+$



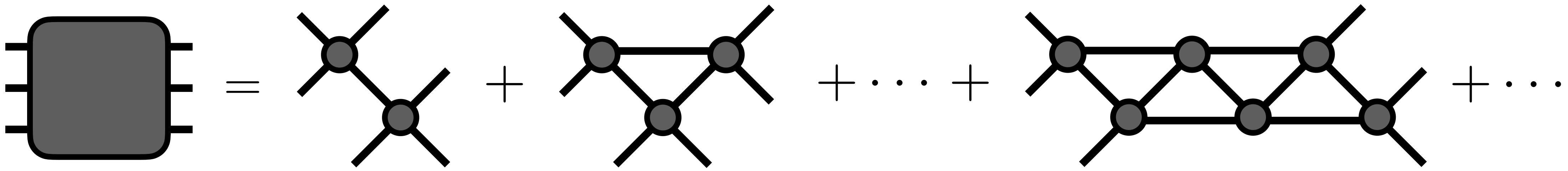




M. Hansen et al. [HadSpec]
Phys. Rev. Lett. **126**, (2021) 012001

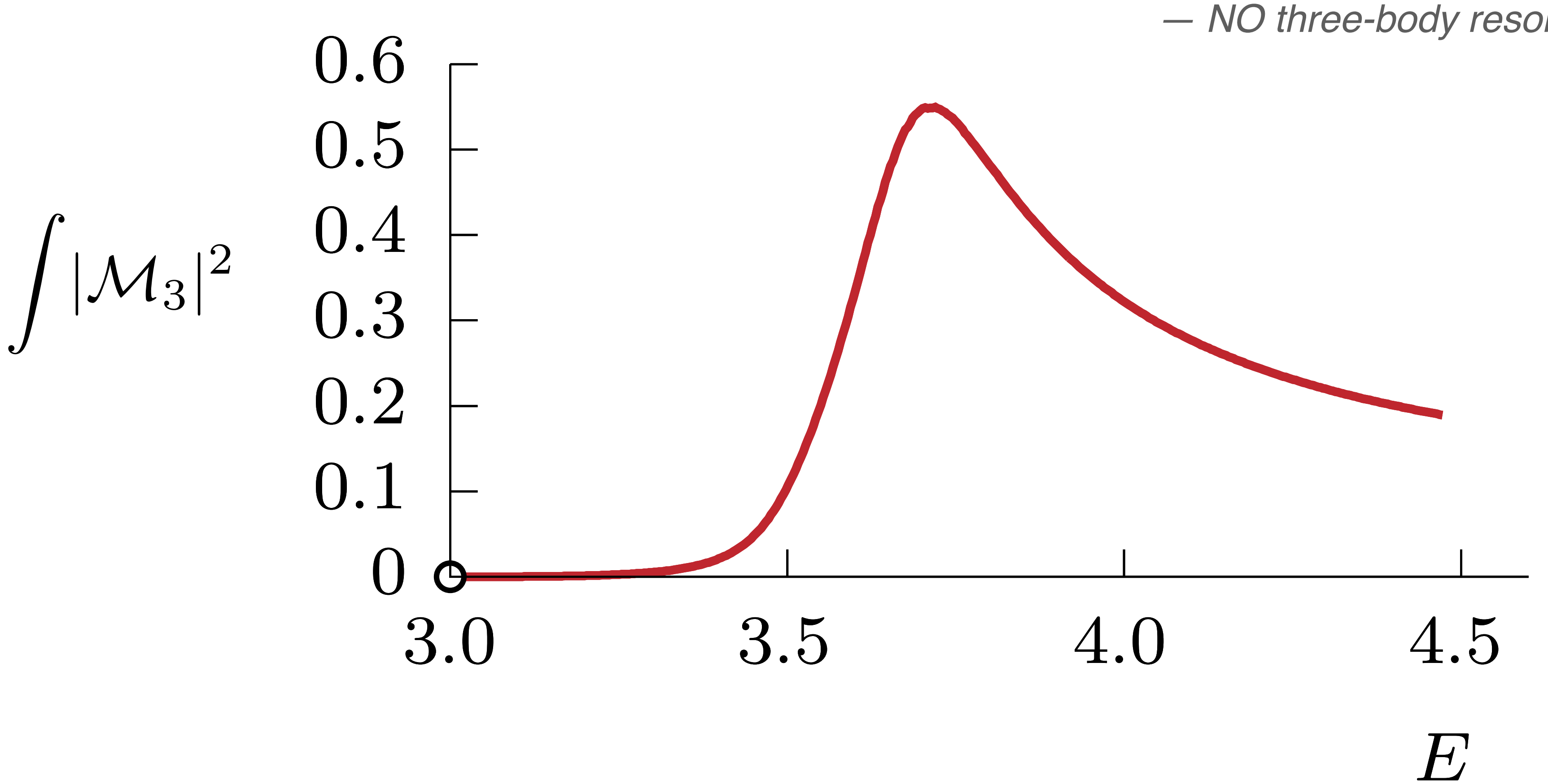
# Three-Body Dynamics

Three-body physics contains more degrees-of-freedom

- Find new features not encountered in two-body systems



*Bump in the spectrum due to exchange — NO three-body resonance*

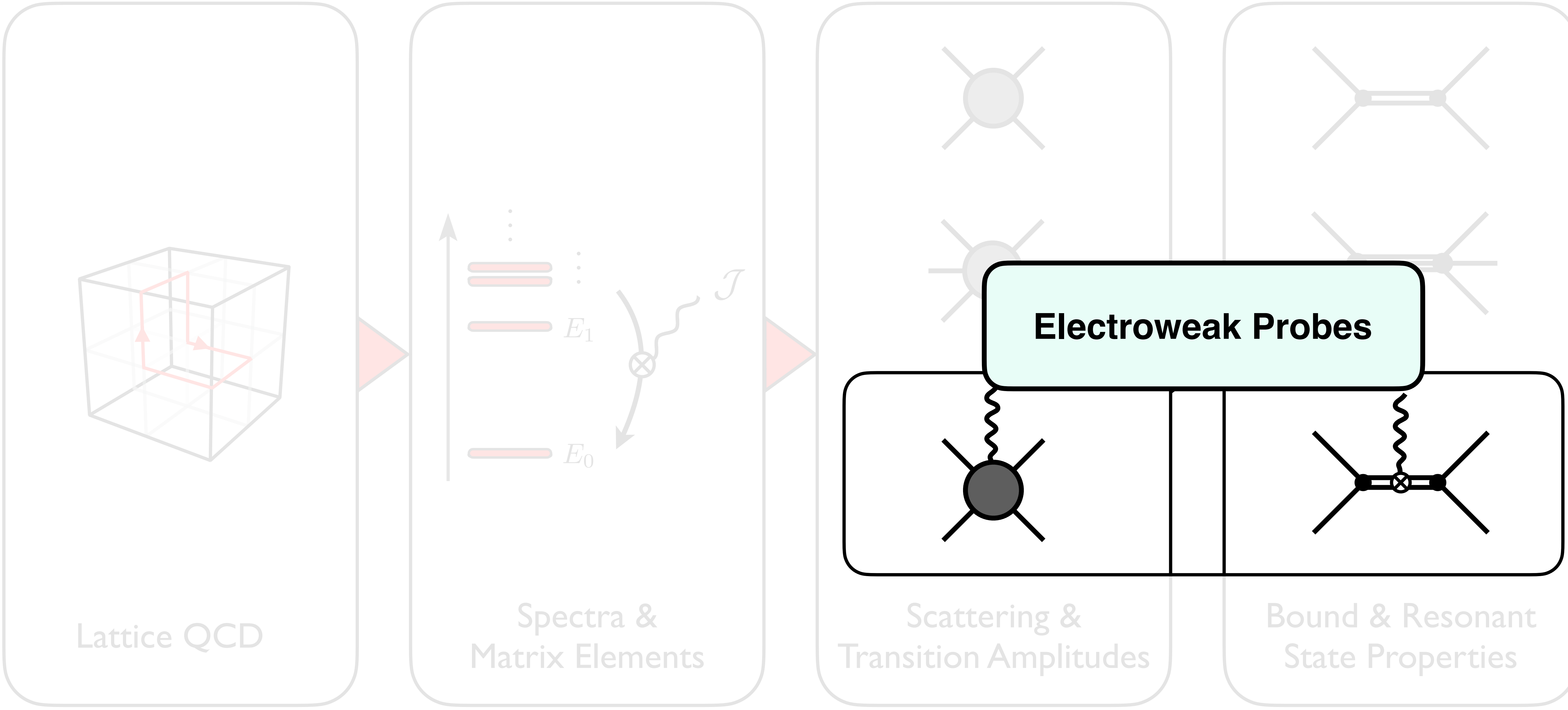



*Taylor Powell (W&M)*

# Few-Body Physics from QCD

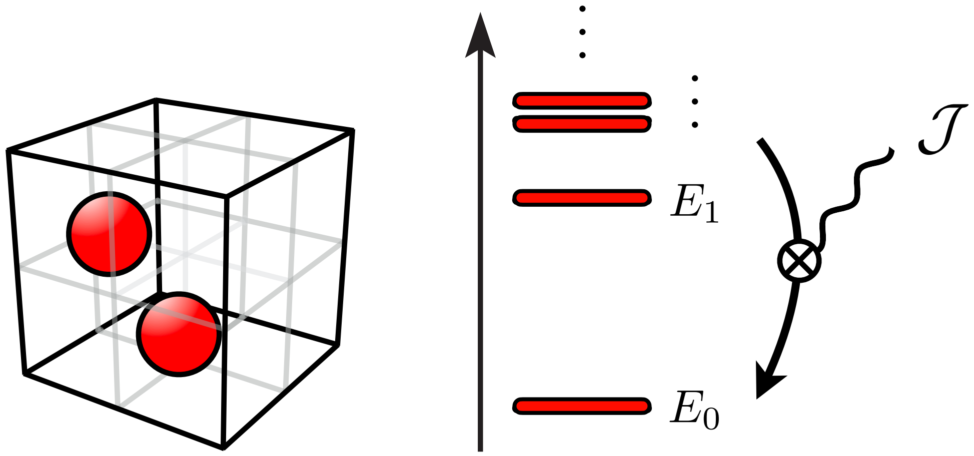
Path to few-body physics from QCD

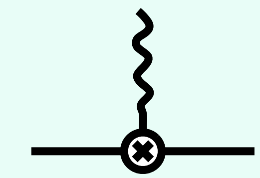
- Link finite-volume spectra and matrix elements to scattering amplitudes
- Tools: *Lattice QCD*, *Scattering Theory*, & *Effective Field Theory*

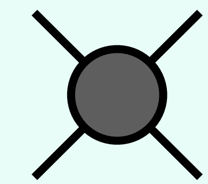


# Hadronic Structure & Electroweak Probes

Mapping between matrix elements and $\mathbf{2} + \mathcal{J} \to \mathbf{2}$ amplitudes



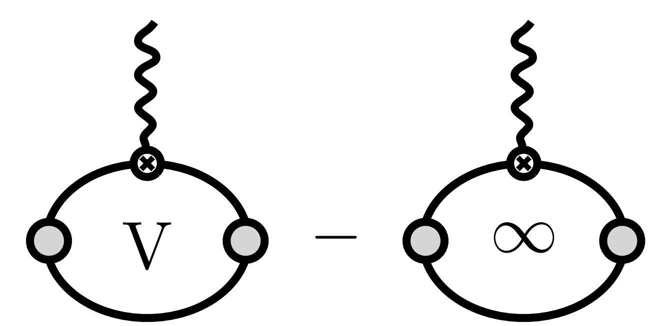

*Single hadron form-factors* $f =$ 

*Hadronic scattering amplitude* $\mathcal{M} =$ 

$$\langle \mathfrak{m} | \, \mathcal{J} \, | \mathfrak{n} \rangle_L = \frac{1}{L^3} \mathcal{W}_{L,\mathrm{df}} \cdot \sqrt{\mathcal{R}_{L,\mathfrak{m}} \cdot \mathcal{R}_{L,\mathfrak{n}}}$$

*FV conversion factors*

$$\mathcal{W}_{L,\mathrm{df}} = \mathcal{W}_{\mathrm{df}} + \mathcal{M} \cdot f \cdot G_L \cdot \mathcal{M}$$

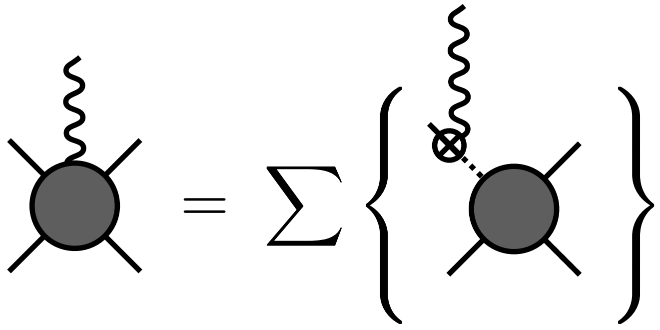
$G_L =$ 


*FV geometric function*

R. Briceño, M. Hansen,
Phys. Rev. D **94** 13008 (2016)

A. Baroni, R. Briceño, M. Hansen, F. Ortega-Gama,
Phys. Rev. D **100** 034511 (2019)

On-shell representation of $\mathbf{2} + \mathcal{J} \to \mathbf{2}$ amplitudes



$$= \sum \left\{ \quad \right\} + i\mathcal{W}_{\rm df}$$

*After considerable manipulations…*

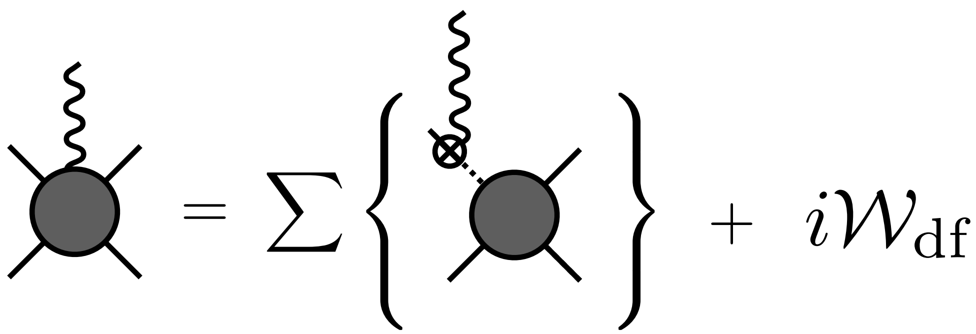
$$\mathcal{W}_{\rm df} = \mathcal{M} \cdot ( \mathcal{A} + f \cdot G ) \cdot \mathcal{M}$$

AJ, R. Briceño, F. Ortega-Gama, K. Sherman,
Phys. Rev. D **103** 114512 (2021)

$$\langle \mathfrak{m} | \, \mathcal{J} \, | \mathfrak{n} \rangle_L \sim \mathcal{W}_{\rm df} + \mathcal{M} \cdot f \cdot G_L \cdot \mathcal{M}$$

# Hadronic Structure & Electroweak Probes

On-shell representation of $\mathbf{2} + \mathcal{J} \to \mathbf{2}$ amplitudes




*After considerable manipulations…*

$$\mathcal{W}_{\rm df} = \mathcal{M} \cdot ( \mathcal{A} + f \cdot G ) \cdot \mathcal{M}$$

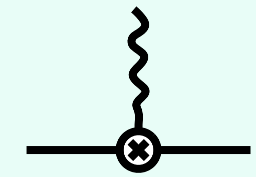
*Unknown short-distance function*

*— Constrain using Lattice QCD*

*— Constrained by Ward-Takahashi identity*

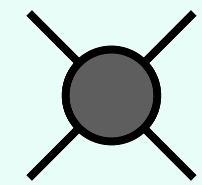
*Single hadron form-factors*

$f =$



*Hadronic scattering amplitude*

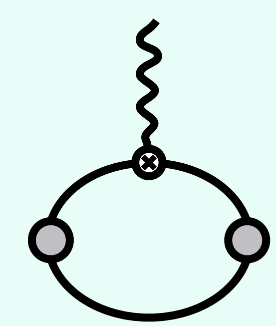
$\mathcal{M} =$



*Triangle diagram*

*Contains normal and anomalous singularities from intermediate on-shell particles*

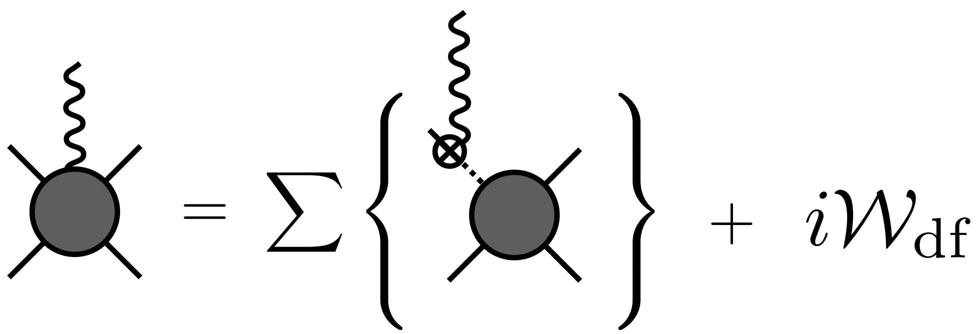
$G =$



AJ, R. Briceño, F. Ortega-Gama, K. Sherman, Phys. Rev. D **103** 114512 (2021)

$$\langle \mathfrak{m} | \mathcal{J} | \mathfrak{n} \rangle_L \sim \mathcal{W}_{\rm df} + \mathcal{M} \cdot f \cdot G_L \cdot \mathcal{M}$$

On-shell representation of $\mathbf{2} + \mathcal{J} \to \mathbf{2}$ amplitudes

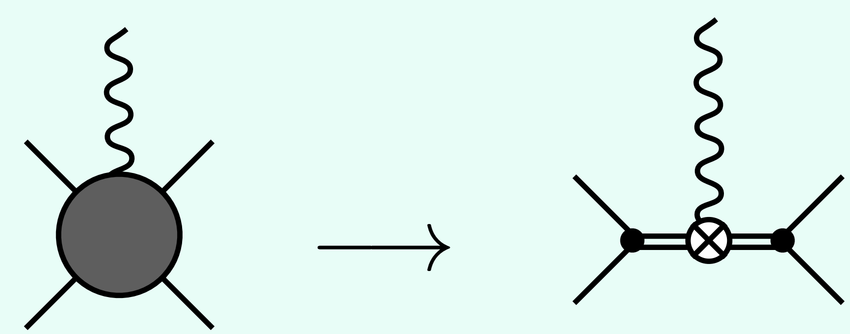



*After considerable manipulations…*

$$\mathcal{W}_{\rm df} = \mathcal{M} \cdot ( \mathcal{A} + f \cdot G ) \cdot$$

*Rigorous definition for resonance form factors*

$$\mathcal{W}_{\rm df} \sim \frac{g}{s_f - s_p} \cdot f_p \cdot \frac{g}{s_i - s_p}$$



$$f_p = g^2 ( \mathcal{A} + f \cdot G ) \Big|_{s_f = s_i = s_p}$$

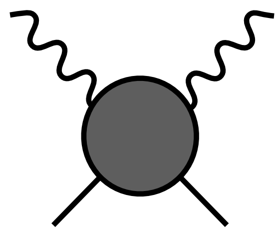
AJ, R. Briceño, F. Ortega-Gama, K. Sherman, Phys. Rev. D **103** 114512 (2021)

# Two-current systems

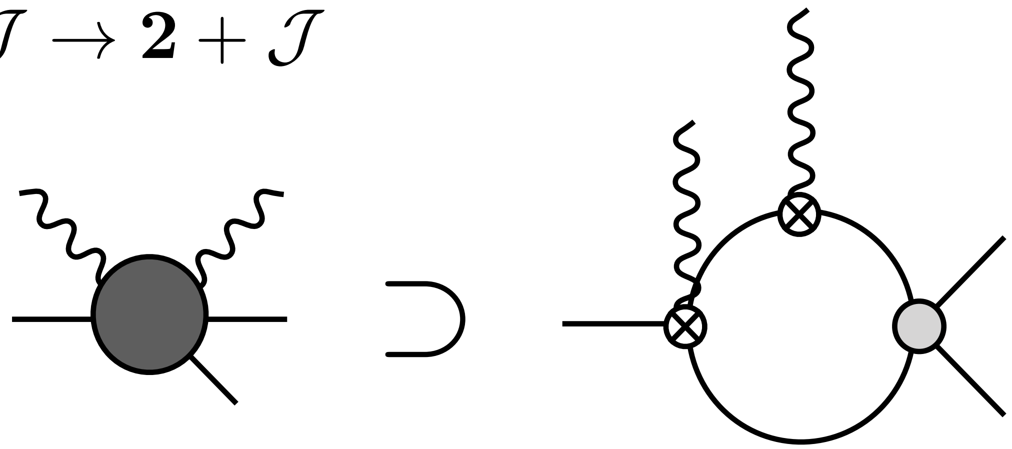
Coupling two currents to hadronic systems — Compton-like processes

☑ $\mathbf{1} + \mathcal{J} \to \mathbf{1} + \mathcal{J}$

R. Briceño, Z. Davoudi, M. Hansen, M. Schindler, A. Baroni
Phys. Rev. D **101** 014509 (2020)

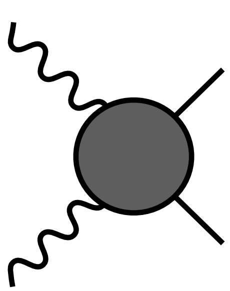


☑ $\mathbf{1} + \mathcal{J} \to \mathbf{2} + \mathcal{J}$



F. Ortega-Gama, K. Sherman, AJ, R. Briceño,
Phys. Rev. D **105** (2022)

☐ $\mathcal{J} + \mathcal{J} \to \mathbf{2}$



AJ, R. Briceño, A. Rodas, J. Guerrero
*In preparation*

# Summary

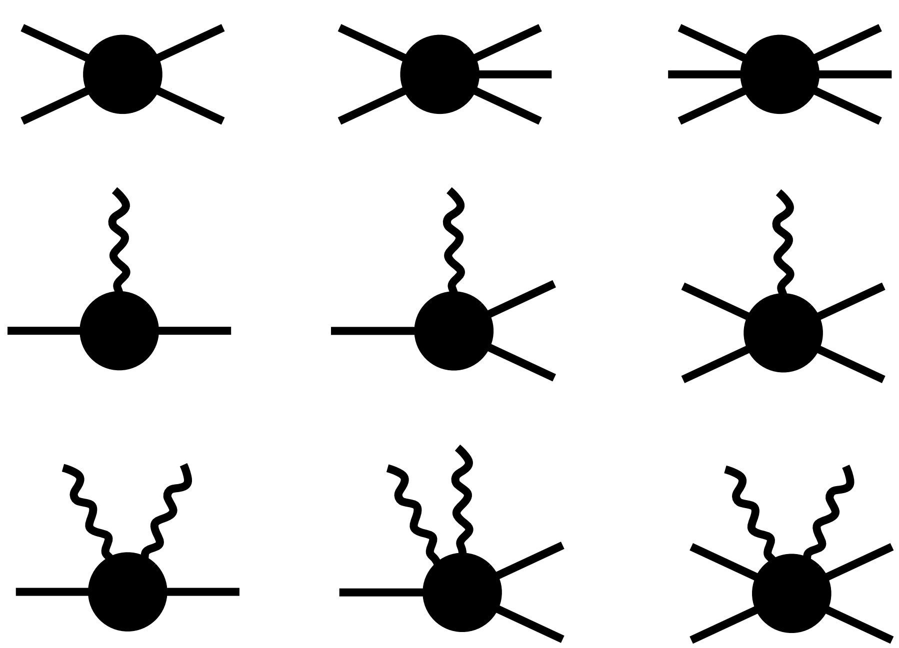
Few-body interactions play a key role in many outstanding problems in nuclear & hadron physics

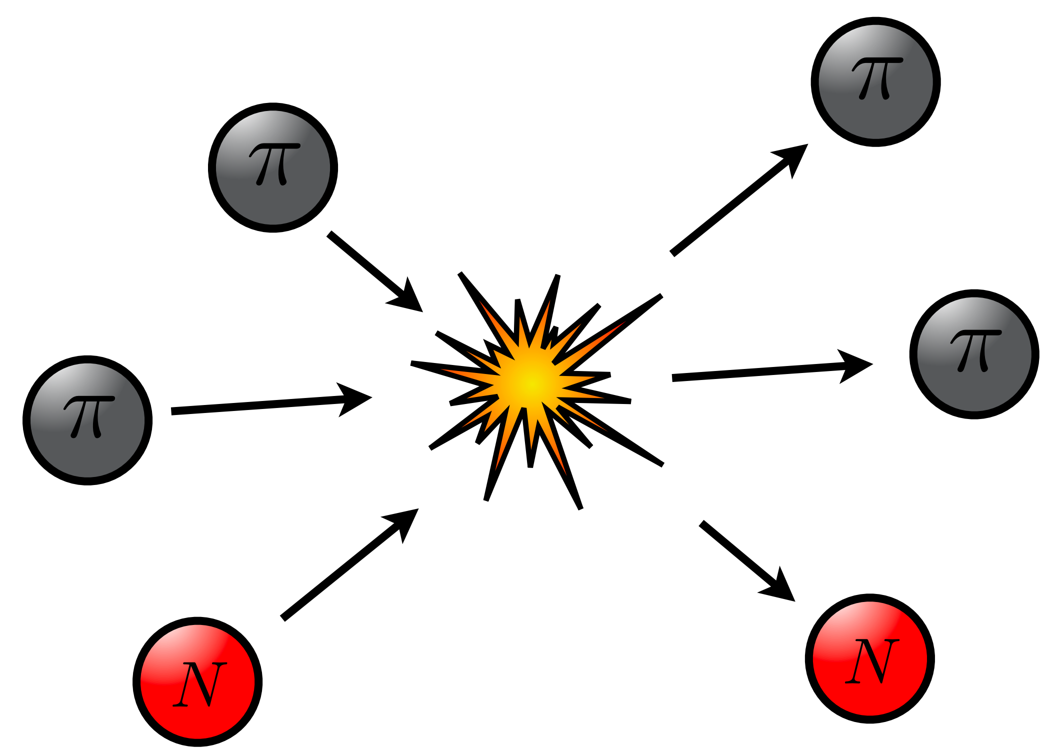
Lattice QCD, EFTs, & Scattering theory combined provide useful tools to extract physics from QCD

- Rapid development in formalisms relating lattice QCD observables to amplitudes
- Scattering phenomenology is advancing in tandem

Latest developments in three-body scattering & two-body matrix elements

- First applications appearing in literature
- Can address increasingly complicated processes






**Much more to come!**