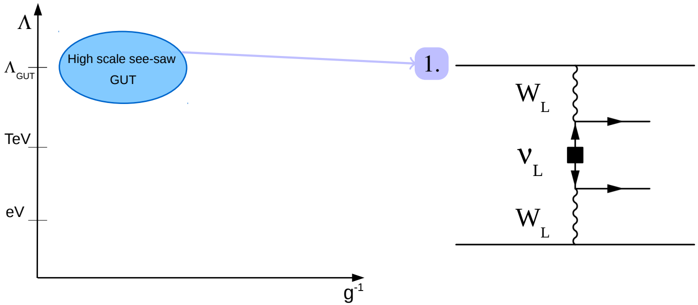
# Bridging particle and nuclear physics for $0\nu\beta\beta$ with EFTs

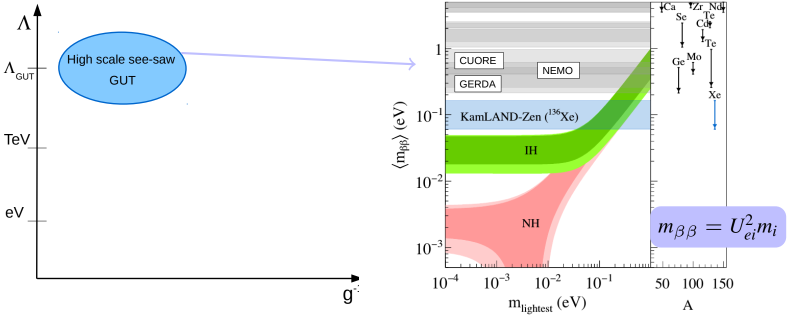
Emanuele Mereghetti

Conference on the Intersections of Particle and Nuclear Physics
September 3, 2022




$0\nu\beta\beta$ is the most sensitive probe of lepton number violation (LNV)

1. LNV originates at very high scales
   direct connection between $\nu$ oscillations and $0\nu\beta\beta$

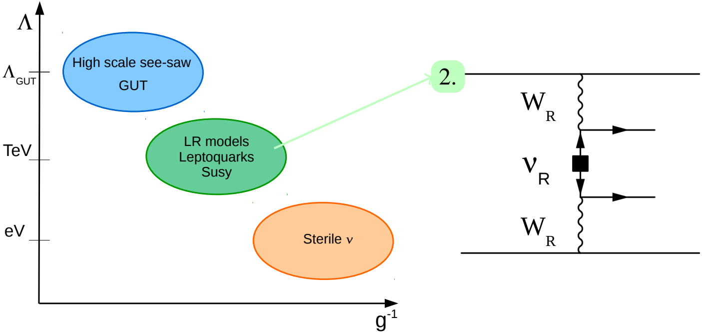



$0\nu\beta\beta$ is the most sensitive probe of lepton number violation (LNV)

1. LNV originates at very high scales

   direct connection between $\nu$ oscillations and $0\nu\beta\beta$

   clear interpretative framework and goals




$0\nu\beta\beta$ is the most sensitive probe of lepton number violation (LNV)

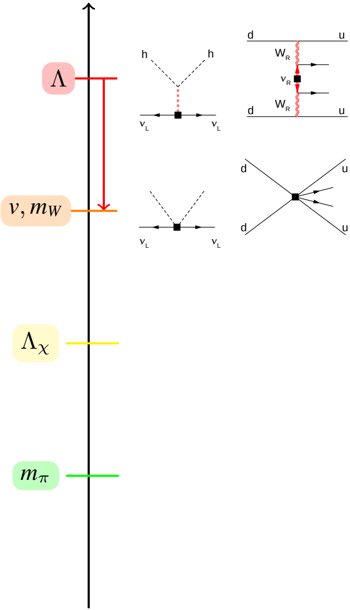
2. LNV at intermediate scales

   $0\nu\beta\beta$ is mediated by new particles, accessible at colliders?

3. very light and weakly coupled new physics

general framework to interpret $0\nu\beta\beta$ exp.?
with controlled uncertainties ?

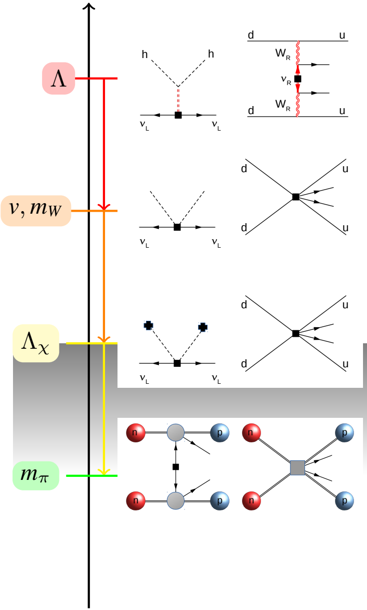
# Effective Field Theories approach to LNV




new physics $\Lambda \gg v$

SMEFT operators

# Effective Field Theories approach to LNV




new physics $\Lambda \gg v$

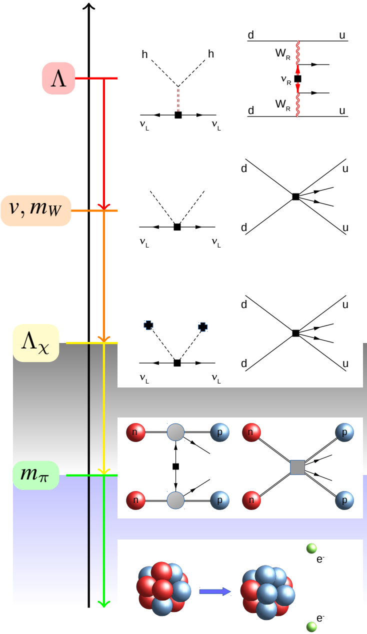
SMEFT operators

$SU(3)_c \times U(1)_{\text{em}}$ operators

non perturbative QCD

Chiral Effective Theory

# Effective Field Theories approach to LNV




new physics $\Lambda \gg v$

SMEFT operators

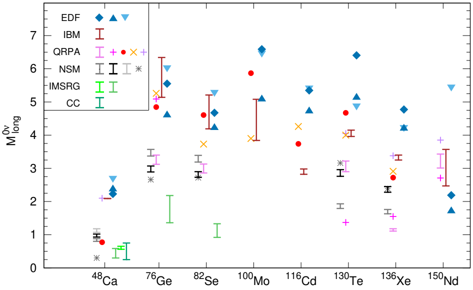
$SU(3)_c \times U(1)_{em}$ operators

non perturbative QCD

Chiral Effective Theory

strong nuclear interactions

Many body




M. Agostini, G. Benato, J. Detwiler, J. Menendez, F. Vissani ‘22

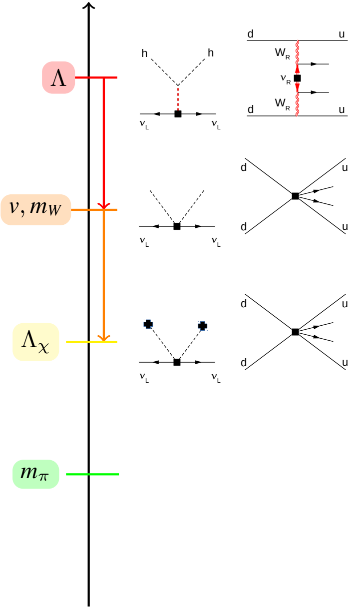
- can *ab initio* methods help quantify/reduce theory uncertainties on $M^{0\nu}$?

open question!

see H. Hergert and S. Gandolfi

Neutrinoless Double-Beta Decay: A Roadmap for Matching Theory to Experiment

# LNV in the Standard Model EFT



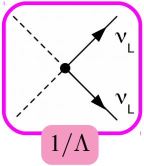

new physics $\Lambda \gg v$

SMEFT operators

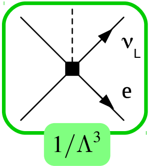
$SU(3)_c \times U(1)_{\rm em}$ operators

# SMEFT/$\nu$SMEFT/LEFT

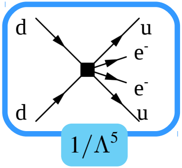
Dim. 5




Dim. 7




Dim. 9




- Weinberg operator

$$\frac{1}{\Lambda}\varepsilon_{ij}\varepsilon_{mn}L_i^T C L_m\, H_j H_n \to \frac{v^2}{\Lambda}\nu_L^T C \nu_L$$

neutrino masses and mixings

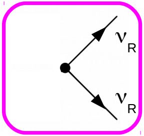
- dimension-7: $\beta$ decays with “wrong” neutrino & generic Lorentz structure

  K. Babu and C. Leung ‘01, A. de Gouvêa and J. Jenkins, ‘08, L. Lehman ‘14

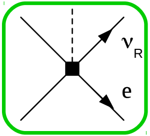
- dimension-9: $\bar{u}d\,\bar{u}d\,ee$ operators

  M. Graesser ‘16, Y. Liao and X. D. Ma, ‘20

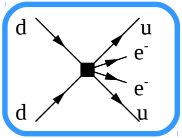
Dim. 3




Dim. 7




Dim. 9




- Weinberg operator

$$\frac{1}{\Lambda}\varepsilon_{ij}\varepsilon_{mn}L_i^T CL_m H_j H_n \to \frac{v^2}{\Lambda}\nu_L^T C\nu_L$$

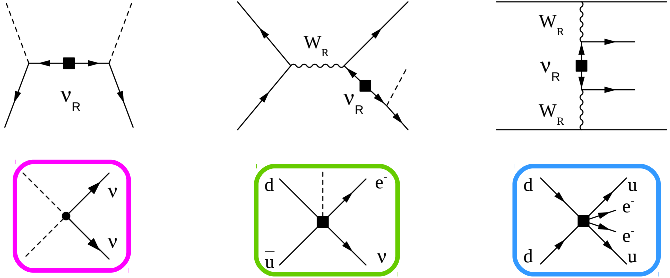
neutrino masses and mixings

- dim-7: $\beta$ decays with "wrong" neutrino & **really** generic Lorentz structure
- dimension-9 $\bar{u}d\,\bar{u}d\,ee$ operators

If $\nu_R$ lighter than EW scale

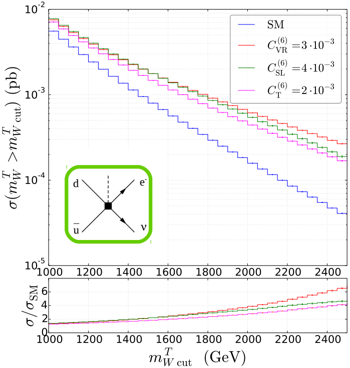
- dim. 3 Majorana mass

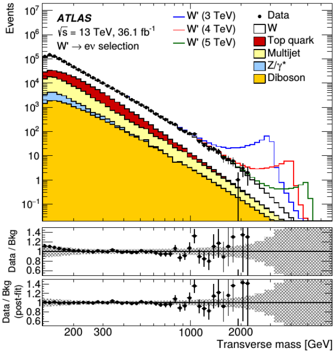
# Connection to models




- specific models will match onto one or several operators
- e.g. Left-Right symmetric model
  dim. 5, 7 & 9 (with different Yukawas)

can match any model to EFT
straightforward to add light particles

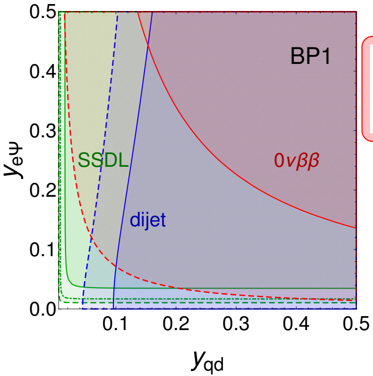


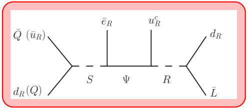




- dimension-5 hard to probe, $m_{\mu\mu} \sim 10$ GeV

  B. Fuks, J. Neundorf, K. Peters, R. Ruiz, M. Saimpert, '21; CMS '22
- $pp \to e\nu$ probes dimension-7 operators with $\Lambda \sim 2.5$ TeV
- no dim-9 analysis

# LNV at LHC



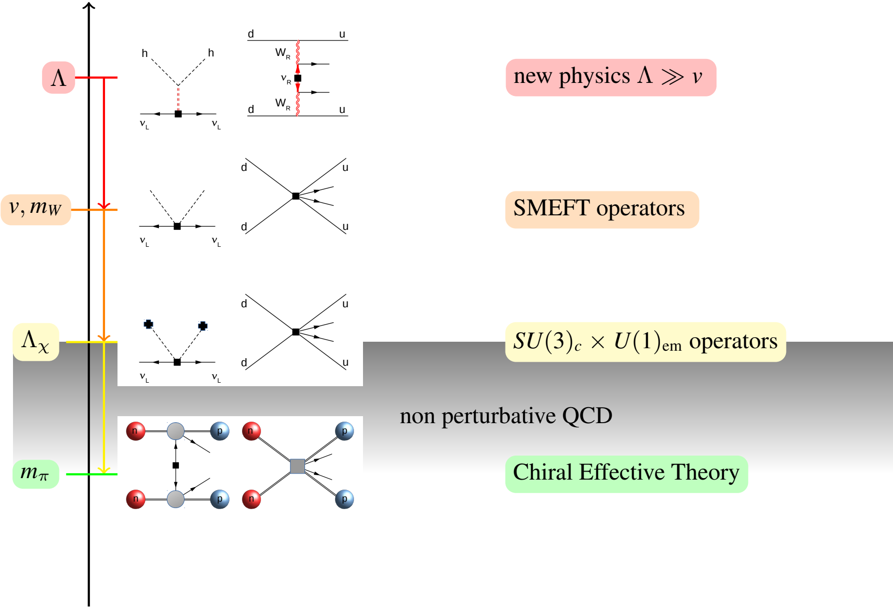




$m_\psi = 1$ TeV,
$m_S = m_R = 2$ TeV

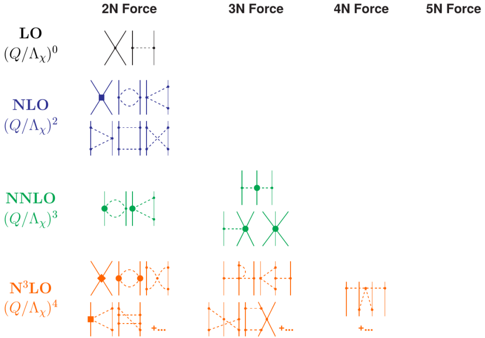
M. Graesser, G. Li, M. Ramsey-Musolf, T. Shen, S. Urrutia-Quiroga, ‘22

- alternatively, directly study (simplified) models
- LHC can gain an edge with resonant production
- SMEFT/models complementary, as LHC push to higher luminosity

# From quark to nucleons and nuclei



# From quark to nucleons: Chiral EFT




from D. R. Entem and R. Machleidt, ‘17

see also:

P. Reinert, H. Krebs, E. Epelbaum, ‘18

M. Piarulli *et al*, ‘16

M. Piarulli *et al*, ‘14

A. Nogga, R. Timmermans, B. van Kolck, ‘05

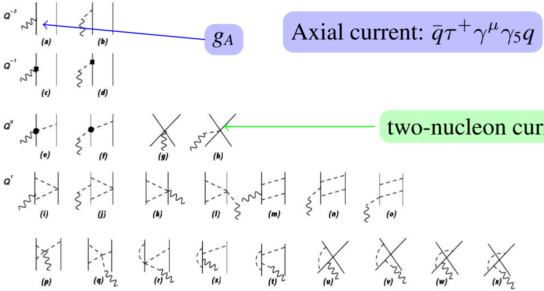
D. Kaplan, M. Savage, M. Wise, ‘96

Exploit QCD symmetries & scale separation in hadronic/nuclear physics

$$Q \sim m_\pi \ll \Lambda_\chi = 4\pi F_\pi \sim 1 \text{ GeV}$$

- expand *NN* potential and external currents in $Q/\Lambda_\chi$
- LECs are fit to data in 2- and 3-nucleon systems
- reproduce well light-nuclear systems

# External currents in chiral EFT




from A. Baroni *et al*, ‘16

- external currents/weak potentials consistent w. nuclear potential

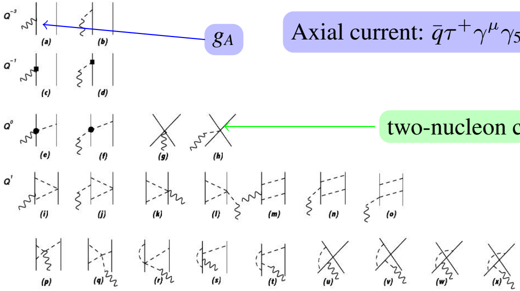
  e.g. vector, axial, scalar, pseudoscalar, tensor

- used to derive $0\nu\beta\beta$ “master formula”

V. Cirigliano, W. Dekens, J. de Vries, M. Graesser, EM, ‘18;
W. Dekens, J. de Vries, K. Fuyuto, EM, G. Zhou, ‘20;

# External currents in chiral EFT

Axial current: $\bar{q}\tau^{+}\gamma^{\mu}\gamma_5 q$



$g_A$

two-nucleon currents

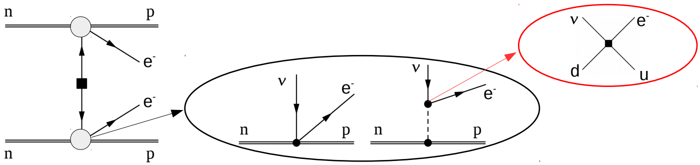
from A. Baroni *et al*, ‘16

1. **predictive power**
   for SM operators/currents, LECs extracted from data
   for BSM operators, need input from Lattice QCD

   same problem in other “hadronization” approaches

2. **power counting**
   unambigous in meson and one-nucleon sector, issues in A $\geq$ 2
3. **convergence**

# Light Majorana-$\nu$ exchange in $\chi$EFT (Weinberg's style)

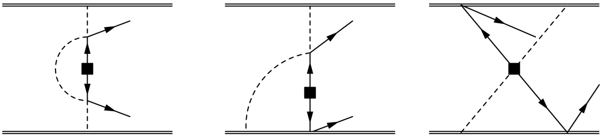



- long range $\nu$-exchange, mediated by V, A 1-nucleon weak current
- Coulomb-like neutrino potential

$$V_\nu = G_F^2 m_{\beta\beta} \tau^{(1)+} \tau^{(2)+} \frac{1}{\mathbf{q}^2} \left\{ \mathbf{1}^{(a)} \times \mathbf{1}^{(b)} - \frac{2}{3} g_A^2 \boldsymbol{\sigma}^{(a)} \cdot \boldsymbol{\sigma}^{(b)} + \dots \right\} .$$

F. Šimkovic *et al*, '99

# Light Majorana-$\nu$ exchange in $\chi$EFT (Weinberg's style)



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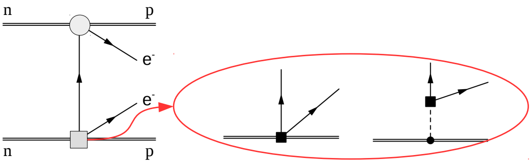
F. Šimkovic *et al*, '99

At N$^2$LO $\mathcal{O}(Q^2/\Lambda_\chi^2)$

1. correction to the one-body currents (magnetic moment, radii, ...)
2. pion-neutrino loops, local counterterms & "closure corrections"
3. two-body corrections to V and A currents

J. Menendez, D. Gazit, A. Schwenk, '11; L. J. Wang, J. Engel, J. M. Yao, '18.

# Dim. 7 operators in $\chi$EFT (Weinberg's style)




$g_A = 1.27$

$g_S = 1.02 \pm 0.10$

$g_M = 4.7$

$g_T = 0.99 \pm 0.03$

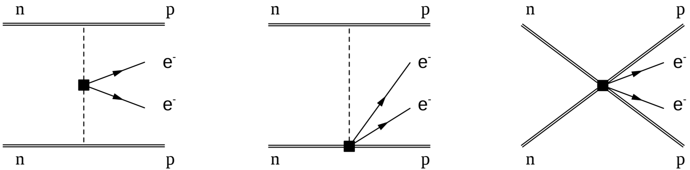
$g'_T = \mathcal{O}(1)$

$B = 2.7$ GeV

$$\begin{aligned}\mathcal{L}^{(6)} = \; & \frac{2G_F}{\sqrt{2}} \Big\{ \bar{u}_L \gamma^\mu d_L \left[ \bar{e}_R \gamma_\mu C^{(6)}_{\rm VLR} \, \nu + \bar{e}_L \gamma_\mu C^{(6)}_{\rm VLL} \, \nu \right] + \bar{u}_R \gamma^\mu d_R \left[ \bar{e}_R \, \gamma_\mu C^{(6)}_{\rm VRR} \, \nu + \bar{e}_L \, \gamma_\mu C^{(6)}_{\rm VRL} \, \nu \right] \\ & + \bar{u}_L d_R \left[ \bar{e}_L \, C^{(6)}_{\rm SRR} \nu + \bar{e}_R \, C^{(6)}_{\rm SRL} \nu \right] + \bar{u}_R d_L \left[ \bar{e}_L \, C^{(6)}_{\rm SLR} \nu + \bar{e}_R \, C^{(6)}_{\rm SLL} \nu \right] \\ & + \bar{u}_L \sigma^{\mu\nu} d_R \, \bar{e}_L \sigma_{\mu\nu} C^{(6)}_{\rm TRR} \, \nu + \bar{u}_R \sigma^{\mu\nu} d_L \, \bar{e}_R \sigma_{\mu\nu} C^{(6)}_{\rm TLL} \, \nu \Big\} + \text{h.c.}\end{aligned}$$

- need axial, vector, scalar, pseudoscalar and tensor one-body currents
- nucleon matrix elements are well determined experimentally or on the lattice (with one exception)

# Dim. 9 operators in $\chi$EFT (Weinberg's style)




- often very large $\pi\pi$ couplings

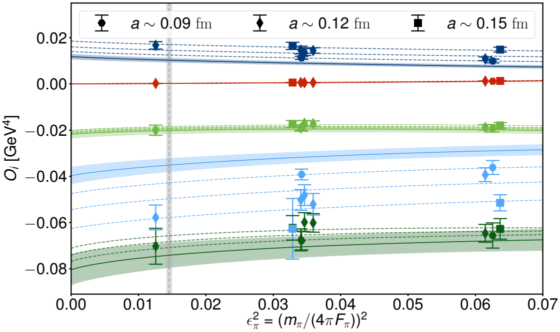
G. Prezeau, M. Ramsey-Musolf, P. Vogel, '03;
A. Faessler, S. Kovalenko, F. Simkovic, J. Schwiger, '97

- $\pi$-N and *NN* couplings @ N2LO
- factorization is a bad approximation!
  e.g $\mathcal{O}_4$

$$\langle pp|\bar{u}_L\gamma^\mu d_L\,\bar{u}_R\,\gamma_\mu d_R|nn\rangle \neq \langle p|\bar{u}_L\gamma^\mu d_L|n\rangle\,\langle p|\bar{u}_R\,\gamma_\mu d_R|n\rangle$$

error from neglecting $\pi\pi$ couplings $\gg$ than from NME

# $\pi\pi$ matrix elements




$g_1^{\pi\pi} = +0.4$

$g_2^{\pi\pi} = -(1.8\,\text{GeV})^2$

$g_3^{\pi\pi} = +(1.0\,\text{GeV})^2$

$g_4^{\pi\pi} = -(1.7\,\text{GeV})^2$

$g_5^{\pi\pi} = -(3.6\,\text{GeV})^2$

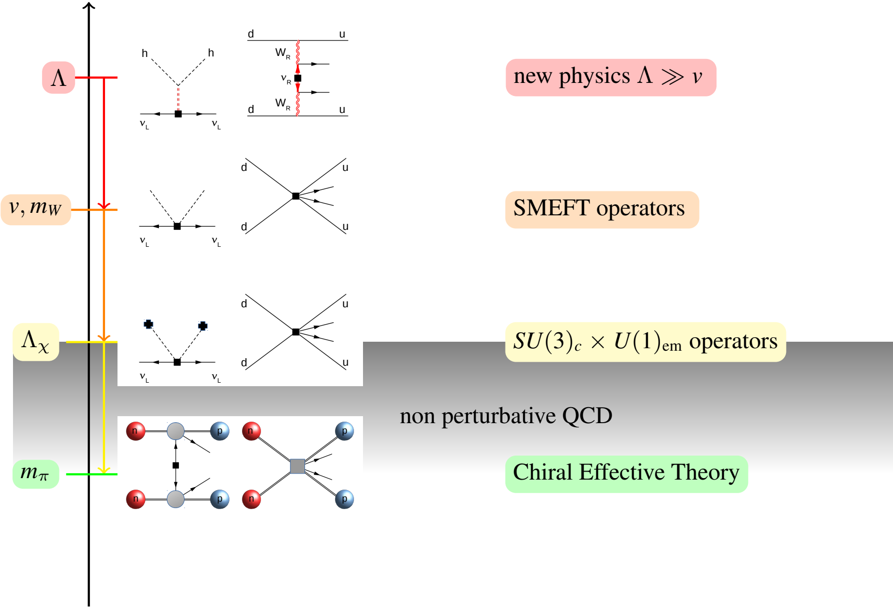
A. Nicholson *et al.*, CalLat collaboration, '18; W. Detmold *et al.*, '22

- $\pi\pi$ matrix elements well determined in LQCD
- NME differ dramatically from factorization

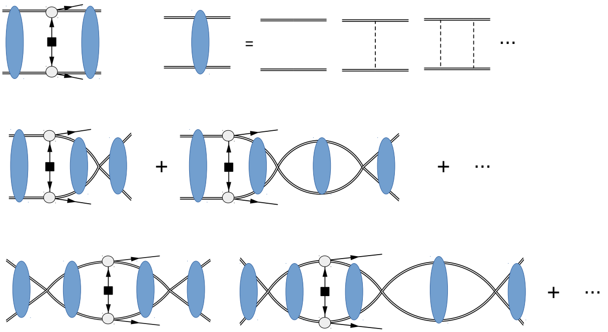
$$M_{\pi\pi} = -\frac{g_4^{\pi\pi} C_4^{(9)}}{2m_N^2}\left(\frac{1}{2}M^{GT}_{AP,sd} + M^{GT}_{PP,sd}\right) \sim -0.60 C_4^{(9)}$$

$$M_{\text{fact}} = -\frac{3g_A^2 - 1}{2g_A^2}\frac{m_\pi^2}{m_N^2} C_4^{(9)} M_{F,sd} \sim -0.04 C_4^{(9)}$$

# From quark to nucleons and nuclei. Beyond Weinberg



# Does Weinberg's counting work for $0\nu\beta\beta$?



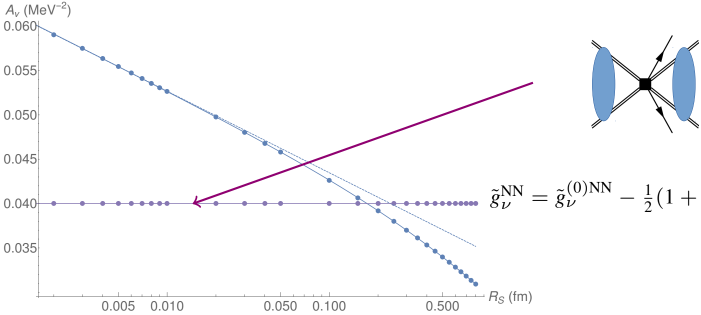

V. Cirigliano, W. Dekens, J. de Vries, M. Graesser, EM, S. Pastore, U. van Kolck, '18

Can address the question in a simple system $nn \to ppe^- e^-$

- solve the Schroedinger equation with LO chiral potential

$$V_{NN}^{^1S_0}(\mathbf{q}) = \tilde{C} - \frac{g_A^2}{4F_\pi^2} \frac{m_\pi^2}{\mathbf{q}^2 + m_\pi^2}$$

# Light Majorana-$\nu$ exchange in $\chi$EFT




$$\tilde{g}_\nu^{\rm NN} = \tilde{g}_\nu^{(0){\rm NN}} - \frac{1}{2}(1 + 2g_A^2)\log R_S$$

V. Cirigliano, W. Dekens, J. de Vries, M. Graesser, EM, S. Pastore, U. van Kolck, '18

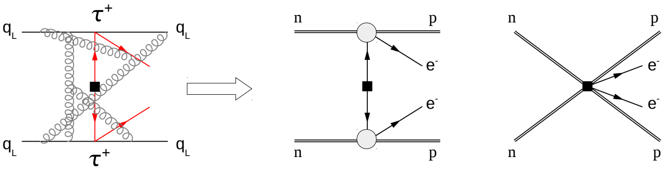
- $nn \to ppee$ amplitude from $V_\nu$ has logarithmic cut-off dependence
- renormalization requires to modify the LO $\nu$ potential

$$V_{\rm LNV} = V_\nu - 2g_\nu^{\rm NN}\tau^{(1)+}\tau^{(2)+}$$

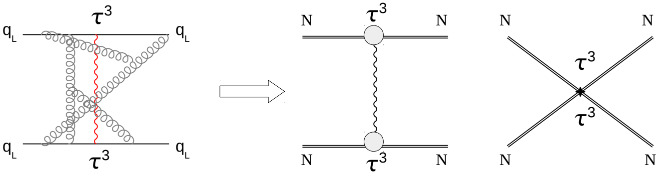
- the coupling $g_\nu^{\rm NN}$ is larger than NDA

$$g_\nu^{\rm NN} \sim \frac{1}{F_\pi^2} \gg \frac{1}{(4\pi F_\pi)^2}$$

# Relation between $0\nu\beta\beta$ and EM isospin breaking

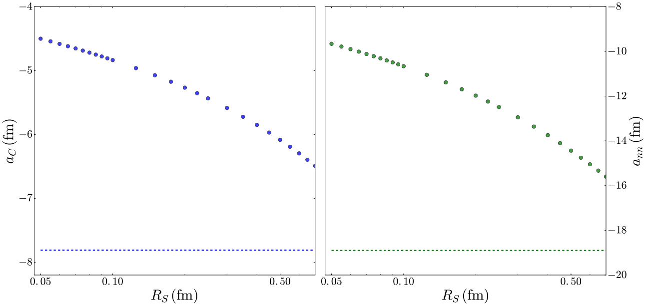



- the dynamics of QCD seems to imply a short-range component for $V_\nu$
- does this happen anywhere else?
  Charge independence breaking in NN scattering$^\dagger$




$^\dagger$equivalence spolied by $R - L$ component of vector current

# Relation between $0\nu\beta\beta$ and charge-independence breaking



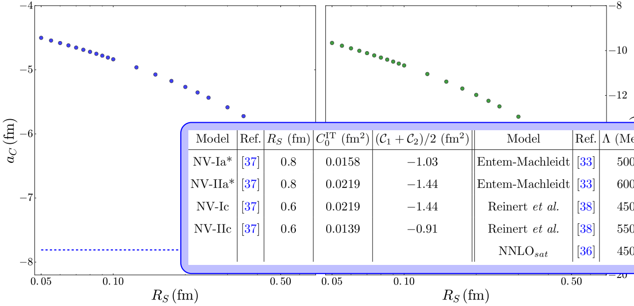

- fit one charge-independent $\tilde{C}$ in $np$, then compute $a_{nn}$ and $a_C$

log divergence! need a ct in each channel

1. $\chi$EFT extraction

$$\frac{C_1 + C_2}{2} = \left(\frac{m_N \tilde{C}}{4\pi}\right)^2 \frac{\tilde{C}_1 + \tilde{C}_2}{2} \sim_{R_S=0.5} \frac{16}{(4\pi F_\pi)^2} = 0.46\,\mathrm{fm}^2$$

# Relation between $0\nu\beta\beta$ and charge-independence breaking




| Model | Ref. | $R_S$ (fm) | $C_0^{\mathrm{IT}}$ (fm$^2$) | $(\mathcal{C}_1 + \mathcal{C}_2)/2$ (fm$^2$) | Model | Ref. | $\Lambda$ (MeV) | $(\mathcal{C}_1 + \mathcal{C}_2)/2$ (fm$^2$) |
|---|---|---|---|---|---|---|---|---|
| NV-Ia* | [37] | 0.8 | 0.0158 | −1.03 | Entem-Machleidt | [33] | 500 | −0.47 |
| NV-IIa* | [37] | 0.8 | 0.0219 | −1.44 | Entem-Machleidt | [33] | 600 | −0.14 |
| NV-Ic | [37] | 0.6 | 0.0219 | −1.44 | Reinert *et al.* | [38] | 450 | −0.67 |
| NV-IIc | [37] | 0.6 | 0.0139 | −0.91 | Reinert *et al.* | [38] | 550 | −1.01 |
| | | | | | NNLO$_{sat}$ | [36] | 450 | −0.39 |

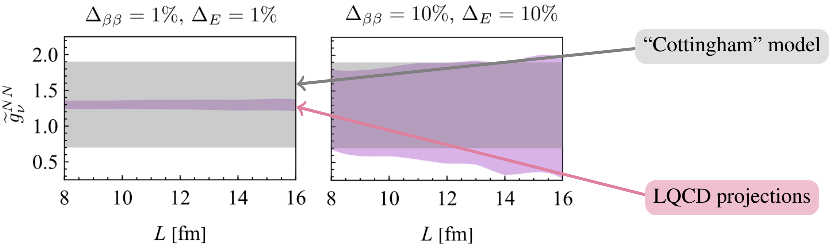
- fit one charge-independent $\tilde{C}$ in $np$, then compute $a_{nn}$ and $a_C$

log divergence! need a ct in each channel

2. all high-quality chiral & pheno $NN$ potentials include short-range CIB

$\Longrightarrow$ renormalization analysis is a **diagnostic** tool
uncertainties might be underestimated

# Evaluation of the LECs




Z. Davoudi and S. Kadam, '20, '21

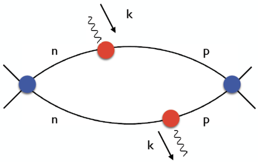
- LQCD offers the most direct avenue
- long distance contributions to $\pi$ $0\nu\beta\beta$ already computed

| | | | |
|---|---|---|---|
| $\lvert g_\nu^{\pi\pi}(\mu)\rvert_{\mu=m_\rho}$ | $=$ | $-10.89 \pm 0.79$ | X.-Y. Tuo, X. Feng and L.-C. Jin, '19 |
| $\lvert g_\nu^{\pi\pi}(\mu)\rvert_{\mu=m_\rho}$ | $=$ | $-10.78 \pm 0.52$ | W. Detmold and D. Murphy, '20 |

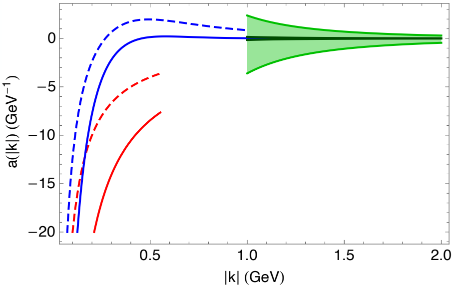
- formalism to match LQCD to pionless EFT developed
- accuracy requirements on LQCD not too steep
- preliminary calculations at heavy quark mass underway

A. Grebe, W. Detmold, Z. Fu, D. Murphy, *in preparation*

# "Cottingham" method




$$\mathcal{A}_\nu = \int_0^\Lambda d|\mathbf{k}|\, a_<(|\mathbf{k}|) + \int_\Lambda^{+\infty} d|\mathbf{k}|\, a_>(|\mathbf{k}|)$$




V. Cirigliano, W. Dekens, J. de Vries, M. Hoferichter, EM, '20, '21

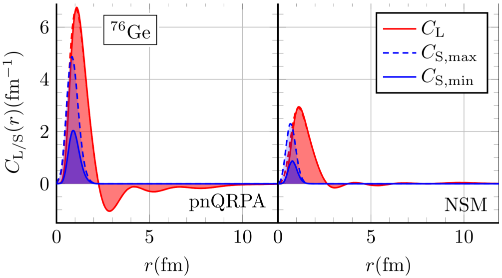
- model the "forward Compton" scattering amplitude $W^+ nn \to W^- pp$
- use NLO $\chi$EFT + half-off-shell "form factors" at small $|\mathbf{k}|$, OPE at large $|\mathbf{k}|$ & connect smoothly

$$\tilde{g}_\nu^{\rm NN}(\mu = m_\pi) = 1.32(50)_{\rm inel}(20)_{\rm r}(5)_{\rm par} = 1.3(6)$$

compares well with "naive" assumption $LL \sim LR$ in CIB

- translate in scheme-independent amplitude and provide "synthetic datum" that can be fit to any potential

# Impact on $0\nu\beta\beta$ nuclear matrix elements




| Nucleus | pnQRPA $M_L^{0\nu}$ | $M_S^{0\nu}$ | $M_S^{0\nu}/M_L^{0\nu}(\%)$ |
|---|---|---|---|
| $^{48}$Ca | | | |
| $^{76}$Ge | 4.72 – 5.22 | 1.49 – 3.80 | 32 – 73 |
| $^{82}$Se | 4.20 – 4.61 | 1.27 – 3.24 | 30 – 70 |
| $^{96}$Zr | 4.22 – 4.63 | 1.24 – 3.19 | 29 – 69 |
| $^{100}$Mo | 3.40 – 3.95 | 1.66 – 4.26 | 49 – 108 |
| $^{116}$Cd | 4.24 – 4.57 | 1.10 – 2.80 | 26 – 61 |
| $^{124}$Sn | 4.72 – 5.29 | 1.69 – 4.28 | 36 – 81 |
| $^{128}$Te | 3.92 – 4.50 | 1.37 – 3.45 | 35 – 77 |
| $^{130}$Te | 3.46 – 3.89 | 1.18 – 3.05 | 34 – 77 |
| $^{136}$Xe | 2.53 – 2.80 | 0.76 – 1.95 | 30 – 70 |

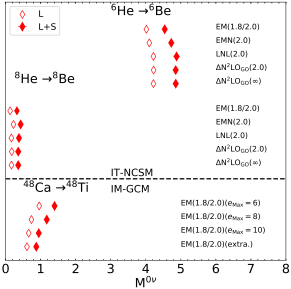
| Nucleus | NSM $M_L^{0\nu}$ | $M_S^{0\nu}$ | $M_S^{0\nu}/M_L^{0\nu}(\%)$ |
|---|---|---|---|
| $^{48}$Ca | 0.96 – 1.05 | 0.22 – 0.65 | 23 – 62 |
| $^{76}$Ge | 3.34 – 3.54 | 0.52 – 1.49 | 15 – 42 |
| $^{82}$Se | 3.20 – 3.38 | 0.48 – 1.38 | 15 – 41 |
| $^{96}$Zr | | | |
| $^{100}$Mo | | | |
| $^{116}$Cd | | | |
| $^{124}$Sn | 3.20 – 3.41 | 0.54 – 1.58 | 17 – 46 |
| $^{128}$Te | 3.56 – 3.80 | 0.61 – 1.76 | 17 – 46 |
| $^{130}$Te | 3.26 – 3.48 | 0.57 – 1.64 | 17 – 47 |
| $^{136}$Xe | 2.62 – 2.79 | 0.45 – 1.31 | 17 – 47 |

L. Jokiniemi, J. Menendez and P. Soriano, ‘21

- similar behavior observed in heavy nuclei with pheno approaches ...

  somewhat smaller impact in shell model
- treatment of strong interaction/weak operator not fully consistent

# Impact on $0\nu\beta\beta$ nuclear matrix elements

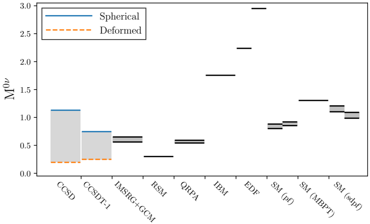



R. Wirth, J. M. Yao, H. Hergert, '21

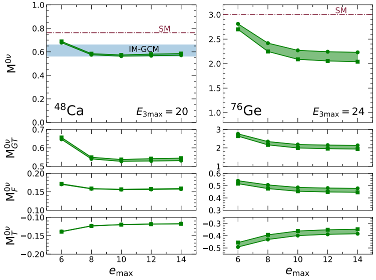
- . . . and *ab initio* methods
- fit the "synthetic" amplitude to 3 different chiral potentials
- SRG-evolve strong and weak potential & calculate $^{48}$Ca NME

43% shift in $^{48}$Ca

# *Ab initio* calculations of $0\nu\beta\beta$ ME




S. J. Novario, P. Gysbers, J. Engel, G. Hagen, '20

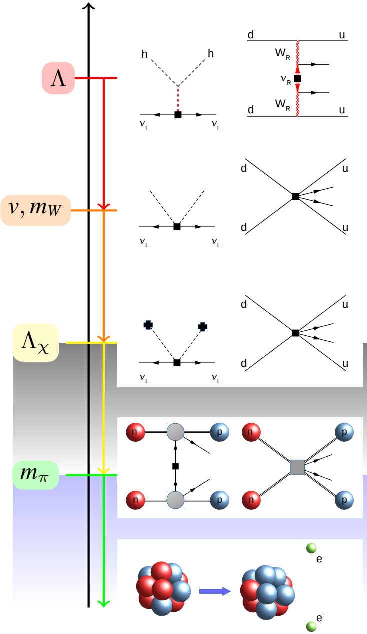



A. Belley, C. G. Payne, S. R. Stroberg, T. Miyagi, J. D. Holt, '20

- first *ab initio* calculations in $0\nu\beta\beta$ candidates
- coupled-cluster calculation of $^{48}$Ca $\rightarrow$ $^{48}$Ti
- in medium similarity ren. group (IMSRG) for $^{48}$Ca, $^{76}$Ge and $^{82}$Se

promising step towards controlled calculations!
need more work to control all systematics
see H. Hergert's talk

# $0\nu\beta\beta$ and particle physics




new physics $\Lambda \gg v$

SMEFT operators

$SU(3)_c \times U(1)_{em}$ operators

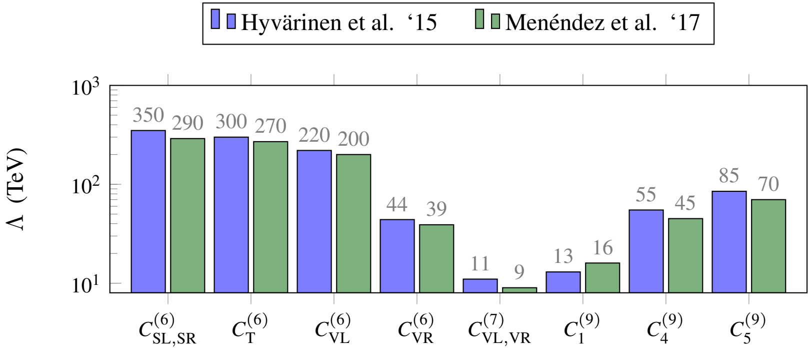
non perturbative QCD

Chiral Effective Theory

strong nuclear interactions

Many body

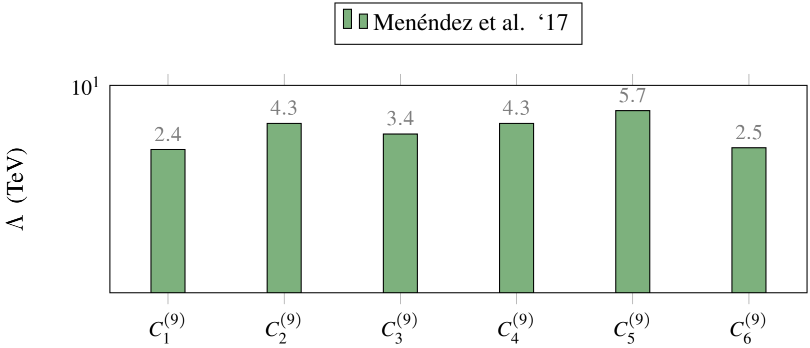
# Bounds on effective operators




- $0\nu\beta\beta$ puts strong limits on dim. 7 operators
- dim. 9 in the TeV range

pattern can be understood from effective dimension
& chiral properties of $0\nu\beta\beta$ operator

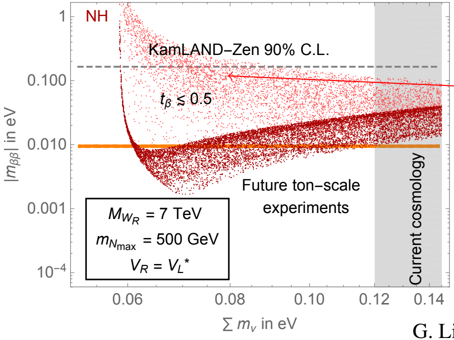
# Bounds on effective operators




- $0\nu\beta\beta$ puts strong limits on dim. 7 operators
- dim. 9 in the TeV range

pattern can be understood from effective dimension
& chiral properties of $0\nu\beta\beta$ operator

# Left-right symmetric models




enhancement from $\pi\pi$ operators

G. Li, M. Ramsey-Musolf, J. C. Vasquez, '20

Correct matching to low-energy important for pheno
E.g. LR symmetric model

- $W_L$-$W_R$ mixing contribution enhanced by $g_4^{\pi\pi}$, $g_5^{\pi\pi}$
- possible signal in tonne-scale experiments even with NH

# Conclusion

- $0\nu\beta\beta$ extremely powerful probe of LNV
- ... but connection with fundamental LNV parameters requires accurate NMEs

Chiral EFT

- systematic organization of $\nu$ potentials in powers of $Q/\Lambda_\chi$
- power counting (and matrix elements) can be tested in simpler systems
- consistent $0\nu\beta\beta$ operators in 2-body sector, need 3-body
- unknown short-range couplings at LO!

LQCD:

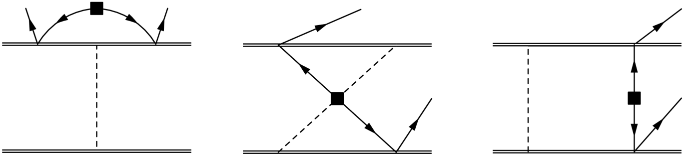
- needed to provide the required nonperturbative QCD input
- great progress in $\pi\pi$ sector, for both standard and non-standard mechanisms
- $N\pi$ and $NN$ sectors coming next

Many body

- first *ab initio* calculations!

# Backup

# Loop corrections to the standard mechanism



axial-axial component

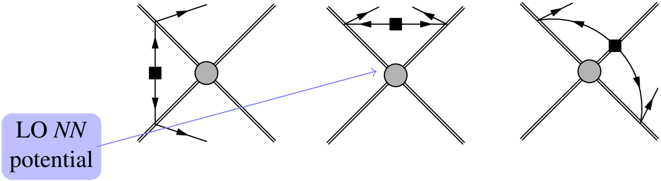
$$\mathcal{V}_{AA}^{a,b} = \frac{g_A^4}{(4\pi F_\pi)^2} \frac{2}{\mathbf{q}^2 + m_\pi^2} \left\{ \boldsymbol{\sigma}^{(a)} \cdot \mathbf{q}\, \boldsymbol{\sigma}^{(b)} \cdot \mathbf{q} \log \frac{m_\pi^2}{m_\nu^2} + \mathbf{1}^{(a)} \times \mathbf{1}^{(b)}\, \mathbf{q}^2 \log \frac{m_\pi^2}{m_\nu^2} \right) + \ldots$$

- the infrared dependence does not drop out!

what's going on?

- ultrasoft neutrinos are still propagating!

# Loop corrections to the standard mechanism




- to define $V_\nu$, need to subtract usoft neutrinos

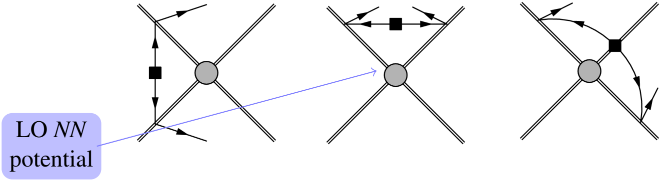
$$V^{(a,b)}_{\nu,2} = \tau^{(a)+}\tau^{(b)+}\left(\mathcal{V}^{(a,b)}_{VV} + \mathcal{V}^{(a,b)}_{AA} + \tilde{\mathcal{V}}^{(a,b)}_{AA}\,\log\frac{m_\pi^2}{\mu_{\rm us}^2} + \mathcal{V}^{(a,b)}_{CT}\right).$$

- the $\mu_{\rm usoft}$ dependent piece

$$\tilde{\mathcal{V}}^{(a,b)}_{AA} = 2\,\frac{g_A^4}{(4\pi F_\pi)^2}\,\frac{\boldsymbol{\sigma}^{(a)}\cdot\mathbf{q}\,\boldsymbol{\sigma}^{(b)}\cdot\mathbf{q} \,+\, \mathbf{q}^2\,\mathbf{1}^{(a)}\times\mathbf{1}^{(b)}}{\mathbf{q}^2+m_\pi^2} + \frac{g_A^2}{(4\pi)^2}\,16 C_T\,\boldsymbol{\sigma}^{(a)}\cdot\boldsymbol{\sigma}^{(b)}$$

- $\mathcal{V}^{(a,b)}_{AA}$ is some ugly function of $|\mathbf{q}|/m_\pi$

# Loop corrections to the standard mechanism




- to define $V_\nu$, need to subtract usoft neutrinos

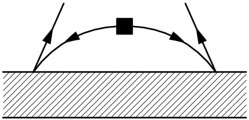
$$V_{\nu,2}^{(a,b)} = \tau^{(a)+}\tau^{(b)+}\left(\mathcal{V}_{VV}^{(a,b)} + \mathcal{V}_{AA}^{(a,b)} + \tilde{\mathcal{V}}_{AA}^{(a,b)} \log\frac{m_\pi^2}{\mu_{\rm us}^2} + \mathcal{V}_{CT}^{(a,b)}\right).$$

- the $\mu_{\rm usoft}$ dependent piece

$$\tilde{\mathcal{V}}_{AA}^{(a,b)} = 2\,\frac{g_A^4}{(4\pi F_\pi)^2}\,\frac{\boldsymbol{\sigma}^{(a)}\cdot\mathbf{q}\,\boldsymbol{\sigma}^{(b)}\cdot\mathbf{q} \,+\, \mathbf{q}^2\,\mathbf{1}^{(a)}\times\mathbf{1}^{(b)}}{\mathbf{q}^2+m_\pi^2} + \frac{g_A^2}{(4\pi)^2}\,16 C_T\,\boldsymbol{\sigma}^{(a)}\cdot\boldsymbol{\sigma}^{(b)}$$

- $\mathcal{V}_{AA}^{(a,b)}$ is some ugly function of $|\mathbf{q}|/m_\pi$

## Usoft contribution to the amplitude



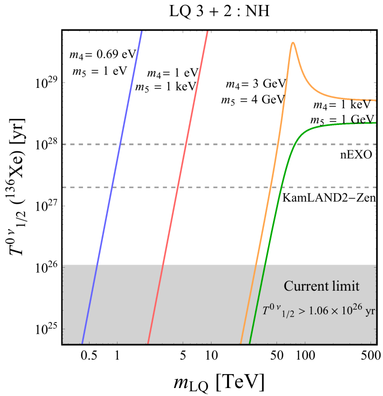
overlap $\langle n | J_\mu | i \rangle$
same as in $2\nu\beta\beta$!

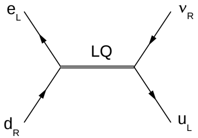
- usoft neutrinos couple to the nuclear bound states

$$T_{\rm usoft}(\mu_{\rm us}) = \frac{T_{\rm lept}}{8\pi^2} \sum_n \langle f | J_\mu | n \rangle \langle n | J^\mu | i \rangle \left\{ (E_2 + E_n - E_i) \left( \log \frac{\mu_{\rm us}}{2(E_2 + E_n - E_i)} + 1 \right) + 1 \leftrightarrow 2 \right\}$$

- $|i\rangle$, $|f\rangle$, $|n\rangle$ eigenvector of the LO strong Hamiltonian
- $\mu_{\rm usoft}$ dependence cancel with $V_{\nu,2}^{(a,b)}$
- contrib. suppressed by $E/(4\pi k_F)$

# Sterile neutrinos with non-minimal interactions




- 3+2 scenario with light Majorana $\nu_R$ interacting with LQ
- probe $m_{\rm LQ} \sim$ 50-100 TeV if $m_{4,5}$ in MeV-GeV region

# The two-body neutrino potential at N$^2$LO

- one LEC at leading order

$$V_\nu^{(0)} = \tau^{(1)+}\tau^{(2)+}\left\{\frac{1}{\mathbf{q}^2}\left(1-\frac{2}{3}g_A^2\boldsymbol{\sigma}^{(a)}\cdot\boldsymbol{\sigma}^{(b)}+\dots\right)-2g_\nu^{\rm NN}\right\}$$

- three more LECs at N$^2$LO

$$\begin{aligned} V_\nu^{(2)} &= \tau^{(1)+}\tau^{(2)+}\left\{-g_{2\nu}^{\rm NN}\left(\mathbf{p}^2+\mathbf{p}'^2\right)+\frac{5}{6}\frac{g_A^2 g_\nu^{\pi\pi}}{(4\pi F_\pi)^2}\boldsymbol{\sigma}^{(a)}\cdot\mathbf{q}\,\boldsymbol{\sigma}^{(b)}\cdot\mathbf{q}\frac{\mathbf{q}^2}{(\mathbf{q}^2+m_\pi^2)^2}\right. \\ &\left. -\frac{g_A^2 g_\nu^{N\pi}}{(4\pi F_\pi)^2}\boldsymbol{\sigma}^{(a)}\cdot\mathbf{q}\,\boldsymbol{\sigma}^{(b)}\cdot\mathbf{q}\frac{1}{\mathbf{q}^2+m_\pi^2}+f(m_\pi,\mathbf{q}^2)\right\}\end{aligned}$$

- "closure corrections" can also be incorporated in the formalism

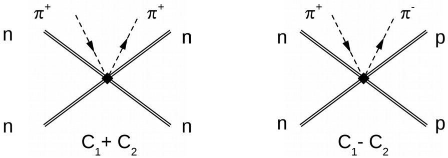
V. Cirigliano, W. Dekens, EM, A. Walker-Loud, '17

- no full N$^2$LO analysis of the $nn \to pp$ amplitude yet,
short-range interactions in higher waves?

# Summary: LNV hadronic couplings

| $n \to pe\nu$, $\pi \to e\nu$ | | $\pi\pi \to ee$ : $\mathcal{O}^{(9)}$ | |
|---|---|---|---|
| $g_A$ | $1.271 \pm 0.002$ | $g_1^{\pi\pi}$ | $0.36 \pm 0.02$ |
| $g_S$ | $1.02 \pm 0.10$ | $g_2^{\pi\pi}$ | $2.0 \pm 0.2$ GeV$^2$ |
| $g_M$ | $4.7$ | $g_3^{\pi\pi}$ | $-0.62 \pm 0.06$ GeV$^2$ |
| $g_T$ | $0.99 \pm 0.03$ | $g_4^{\pi\pi}$ | $-1.9 \pm 0.2$ GeV$^2$ |
| $\lvert g'_T \rvert$ | $\mathcal{O}(1)$ | $g_5^{\pi\pi}$ | $-8.0 \pm 0.6$ GeV$^2$ |
| $B$ | 2.7 GeV | | |
| $n \to p\pi ee$ : $\mathcal{O}^{(9)}$, $\mathcal{O}^{(6,7)}$ | | $\pi\pi \to ee$ : $\mathcal{O}^{(6,7)} \otimes \mathcal{O}^{(6,7)}$ | |
| $\lvert g_i^{\pi N} \rvert$ | $\mathcal{O}(1)$ | $\lvert g^{\pi\pi}_{\mathrm{T,VLL}} \rvert$, $\lvert g^{\pi\pi}_{\mathrm{S,VLL}} \rvert$, $\lvert g^{\pi\pi}_{\mathrm{T,VRL}} \rvert$, $\lvert g^{\pi\pi}_{\mathrm{S,VRL}} \rvert$ | $\mathcal{O}(1)$ |
| | | $\lvert g^{\pi\pi}_{\mathrm{LR}} \rvert$, $\lvert g^{\pi\pi}_{\mathrm{S1,S2}} \rvert$ | $\mathcal{O}(F_\pi^2)$ |
| | | $\lvert g^{\pi\pi}_{\mathrm{TT}} \rvert$, $\lvert g^{\pi\pi}_{\mathrm{TL}} \rvert$, $\lvert g^{\pi\pi}_{\mathrm{TL,TR}} \rvert$ | $\mathcal{O}(F_\pi^2)$ |
| $nn \to pp\, ee$ : $\mathcal{O}^{(9)}$ | | $nn \to pp\, ee$ : $\mathcal{O}^{(6,7)} \otimes \mathcal{O}^{(6,7)}$ | |
| $\lvert g^{NN}_{1,6,7} \rvert$ | $\mathcal{O}(1)$ | $\lvert g^{NN}_{\nu} \rvert$, $\lvert g^{NN}_{\mathrm{LR}} \rvert$, $\lvert g^{NN}_{\mathrm{S1}} \rvert$ | $\mathcal{O}(1/F_\pi^2)$ |
| $\lvert g^{NN}_{2,3,4,5} \rvert$ | $\mathcal{O}((4\pi)^2)$ | $\lvert g^{NN}_{\mathrm{S2}} \rvert$, $\lvert g^{NN}_{\mathrm{TT}} \rvert$, $\lvert g^{NN}_{\mathrm{SLL,VLL}} \rvert$ | $\mathcal{O}(1/F_\pi^2)$ |
| | | $\lvert g^{NN}_{\mathrm{TLL,VLL}} \rvert$, $\lvert g^{NN}_{\mathrm{TL}} \rvert$, $\lvert g^{NN}_{\mathrm{TL,TR}} \rvert$ | $\mathcal{O}(1/F_\pi^2)$ |
| | | $\lvert g^{NN}_{\mathrm{TL,T}} \rvert$, $\lvert g^{NN}_{\mathrm{TR,T}} \rvert$ | $\mathcal{O}(1/\Lambda_\chi^2)$ |
| | | $\lvert g^{NN}_{\mathrm{S,VLL}} \rvert$, $\lvert g^{NN}_{\mathrm{T,VLL}} \rvert$ $\lvert g^{NN}_{\mathrm{VLL,VLR}} \rvert$ | $\mathcal{O}(1)$ |
| | | $\lvert g^{NN}_{\mathrm{S,VRL}} \rvert$, $\lvert g^{NN}_{\mathrm{T,VRL}} \rvert$ | $\mathcal{O}(1)$ |
| | | $\lvert g^{NN}_{\mathrm{T,SRL}} \rvert$, $\lvert g^{NN}_{\mathrm{T,SLL}} \rvert$, $\lvert g^{NN}_{\mathrm{TL,V}} \rvert$, $\lvert g^{NN}_{\mathrm{TR,V}} \rvert$ | $\mathcal{O}((4\pi)^2)$ |

# Isospin breaking in few-body systems

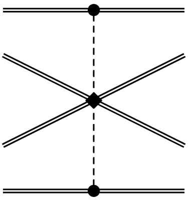



- LL and LR contact differ at the pion level
- only $C_1 - C_2$ contributes to pion DCX reactions

$$\pi^- \, {}^3\mathrm{He} \to \pi^+ 3n, \; \pi^+ \, {}^{12}\mathrm{C} \to \pi^- \, {}^{12}\mathrm{O}$$

not much data at low-energy, not very precise

## Isospin breaking in few-body systems



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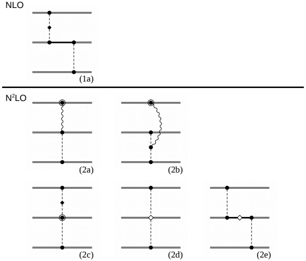
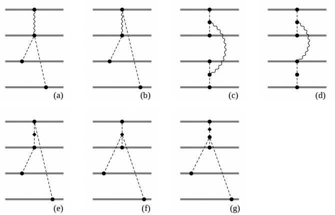
not much data at low-energy, not very precise

- $C_1$ and $C_2$ generate different 4-body forces
- could contribute CIB in light nuclei

$$a_2(A=6, T=1) \propto B(^6\mathrm{He}, 0^+) + B(^6\mathrm{Be}, 0^+) - 2B(^6\mathrm{Li}, 0^+) \qquad \frac{(a_2)_{\chi EFT}}{(a_2)_{\mathrm{exp}}} \sim 1.2$$

# Isospin breaking in few-body systems

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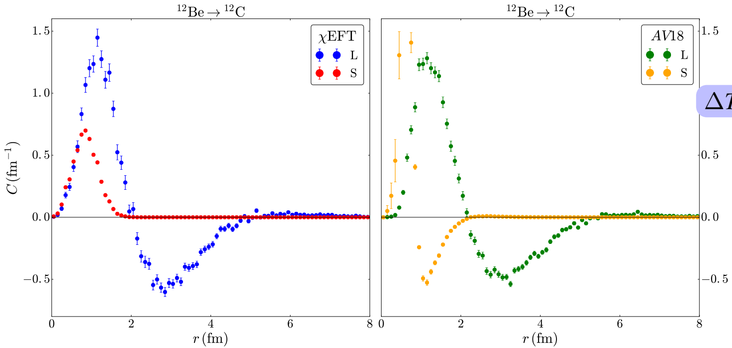
- $C_1$ and $C_2$ generate different 4-body forces
- could contribute CIB in light nuclei

$$a_2(A=6, T=1) \propto B({}^6\text{He}, 0^+) + B({}^6\text{Be}, 0^+) - 2B({}^6\text{Li}, 0^+) \qquad \frac{(a_2)_{\chi EFT}}{(a_2)_{\text{exp}}} \sim 1.2$$

several contributions at the same order... in progress with **J. Lieffers** @UoA

# Impact on $0\nu\beta\beta$ nuclear matrix elements

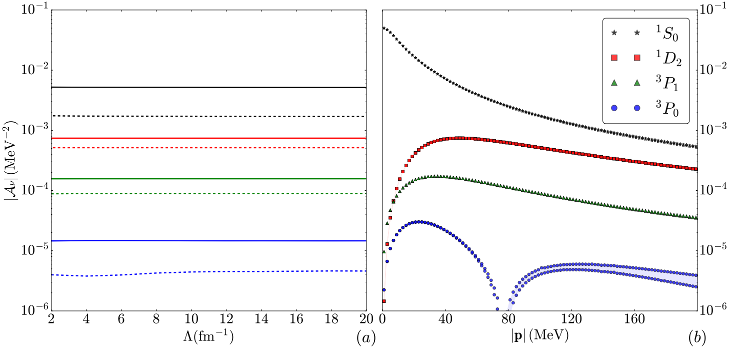



$\Delta T = 2$

thanks to S. Pastore, M. Piarulli and B. Wiringa

- *ab initio* calculations of $^{12}$Be $\rightarrow$ $^{12}$C
- size $g_\nu^{\rm NN}$ inspired by EM isospin-breaking potential
- large 50% - 70% corrections to $\Delta T = 2$ transitions

# Higher partial waves

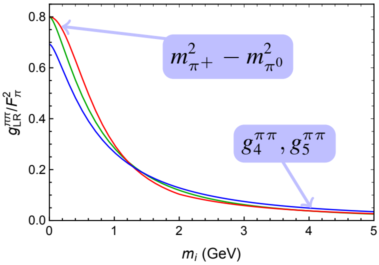


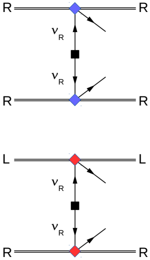

- Weinberg's counting leads to problems in $^3P_{0,2}$ waves

  $\Longrightarrow$ need LO counterterms in the strong interaction
- neutrino potential in $P$ waves does not require further renormalization
- NLO ($\mathcal{O}(Q/\Lambda_\chi)$) also free from problems

V. Cirigliano, W. Dekens, J. de Vries, M. Graesser, EM, S. Pastore, M. Piarulli, U. van Kolck, R. Wiringa, '19

# $\chi$EFT with sterile neutrinos



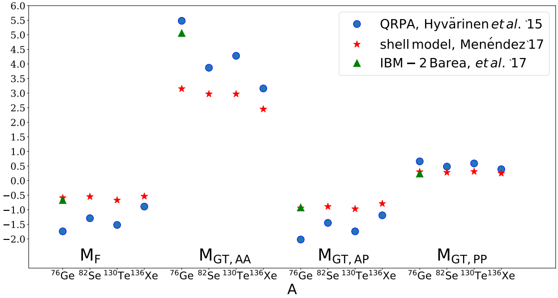

- $0\nu\beta\beta$ rate can be generalized to $\nu_R$, with generic interactions in $\nu$SMEFT

W. Dekens, J. de Vries, K. Fuyuto, EM, G. Zhou, '20;

- solid tools for light $m_{\nu_R} \lesssim 0.4$ GeV and heavy $m_{\nu_R} \gtrsim 2$ GeV neutrinos
- intermediate region trickier

need to track $m_\nu$ dependence of hadronic couplings
one more task for LQCD...

# Nuclear matrix elements




- all LO NMEs already exist
- 8 long-range NME

contribute to light $\nu$ exchange

- 6 short-range NME

contribute to heavy Majorana $\nu$ exchange

- same level of discrepancy as standard mechanism