with Will Parrott and Christine Davies

$B \to K$ form factors and associated phenomenology

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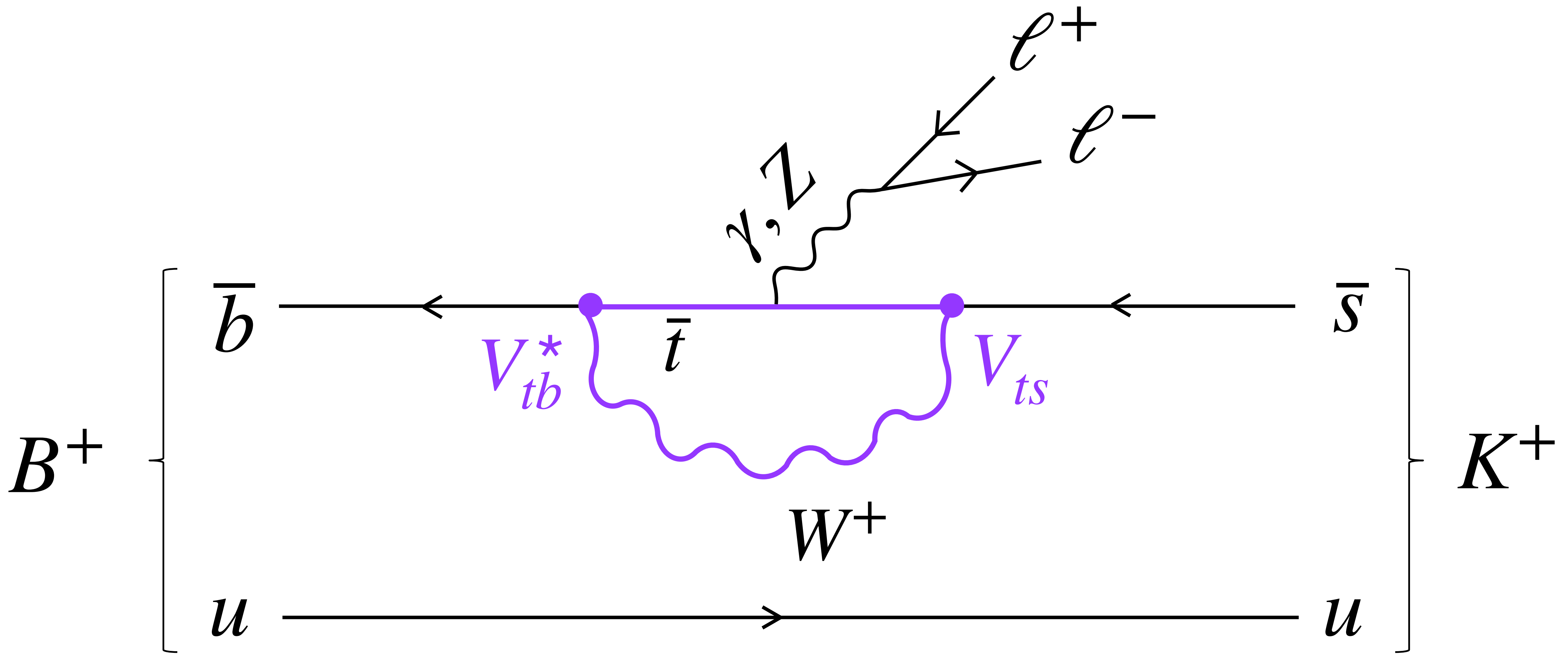
CIPANP 2022

Orlando, FL, 28 Aug - 5 Sep

I. Motivation
II. Form factor calculation via lattice QCD Parrot, Bouchard, and Davies, 2207.12468
III. Phenomenology Parrot, Bouchard, and Davies, 2207.1337
IV. Conclusion and outlook

Motivation: SM contribution small

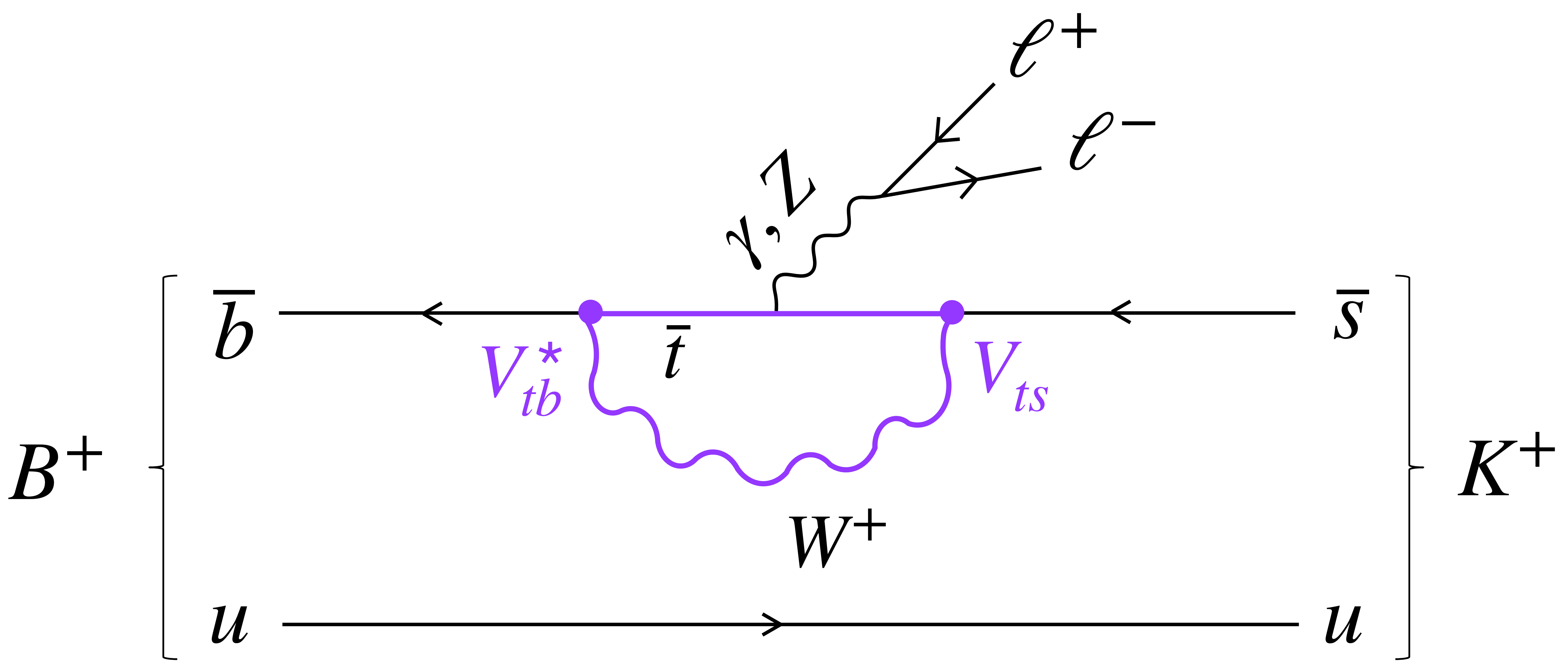


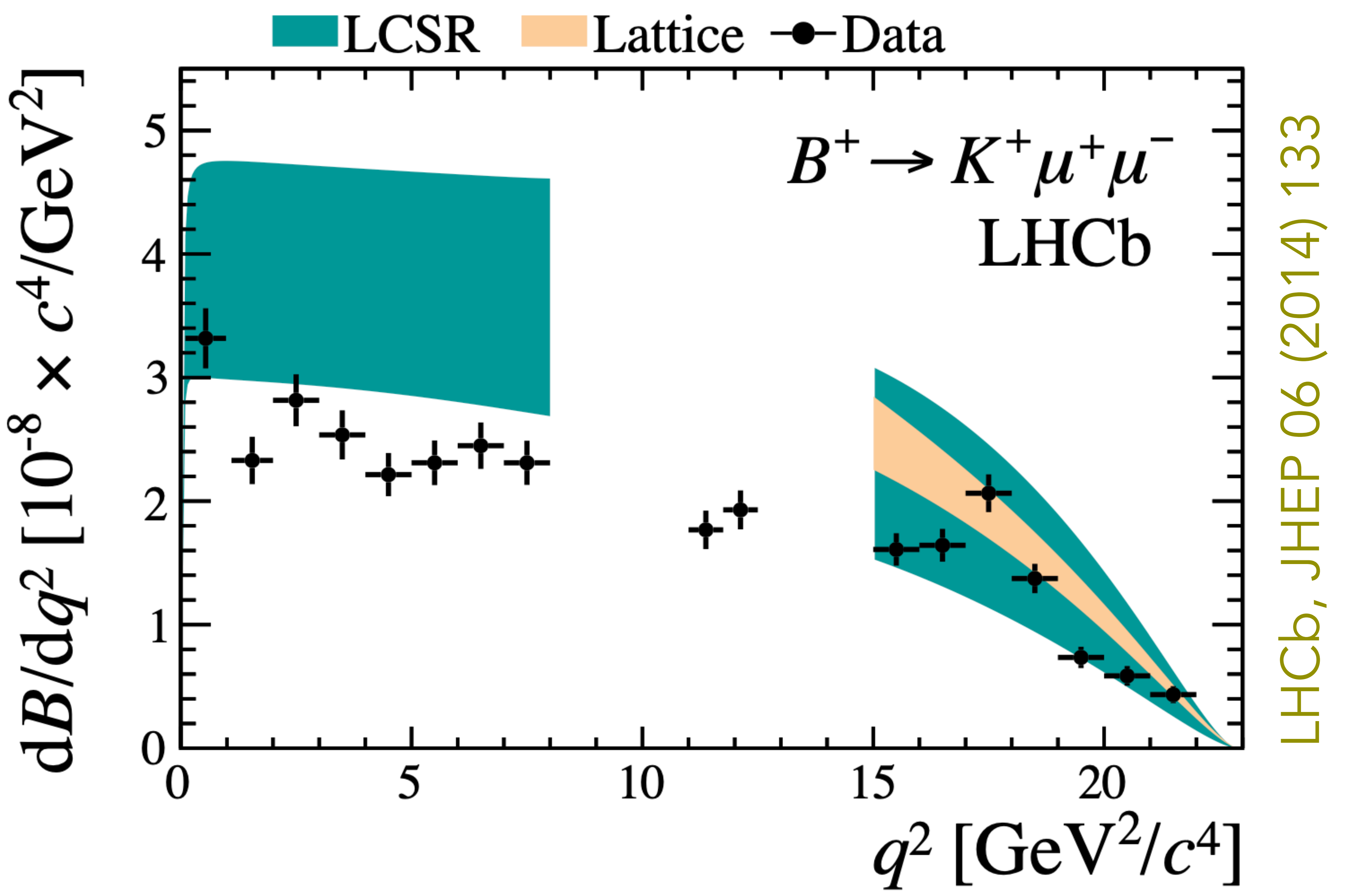
rare decay

- loop suppressed, amplitude $\propto G_F \sim 10^{-5}\,\mathrm{GeV}^{-2}$
- CKM suppressed, amplitude $\propto |V_{tb}V_{ts}| \sim 0.04$

SM suppression makes new physics effects potentially visible.

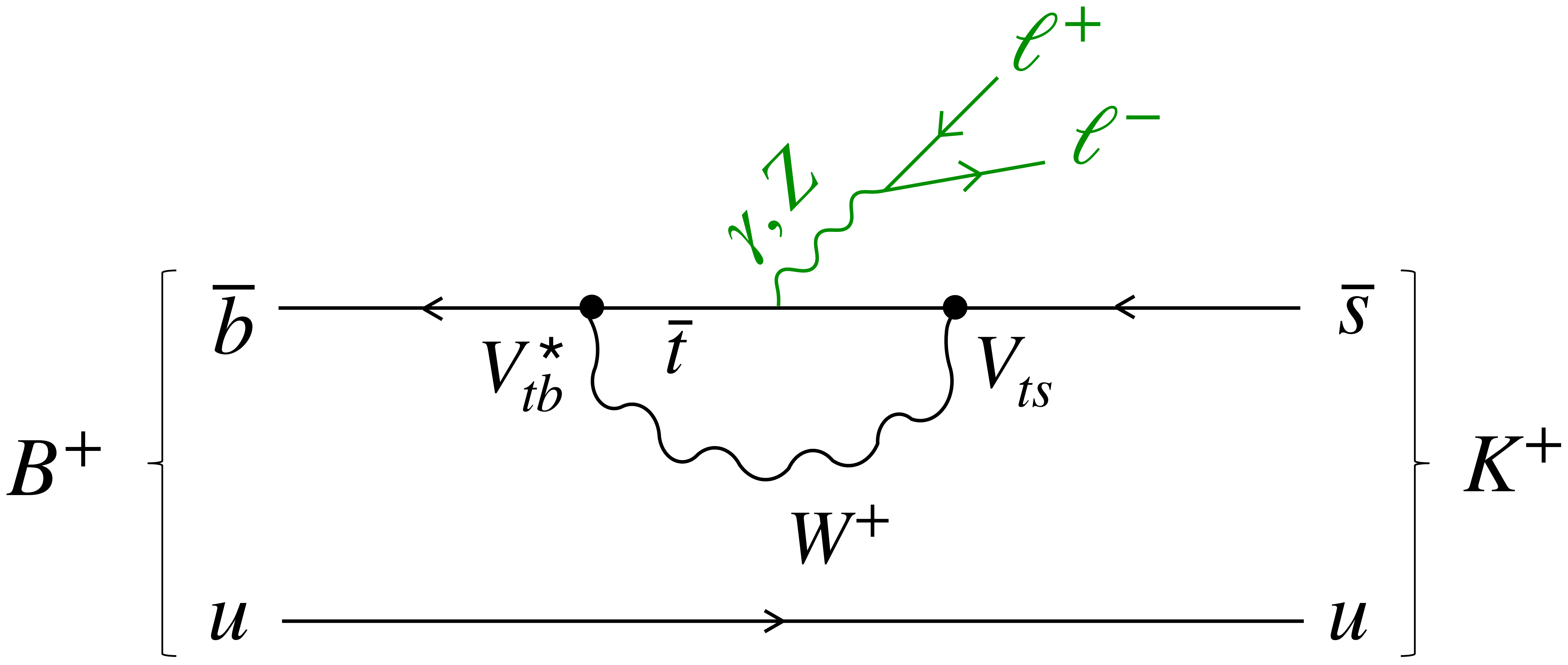
Motivation: SM contribution small





- measured by LHCb and will be measured by Belle-II
- persistent tension at low q^2, need improved form factors

Form Factor calculation: preliminaries

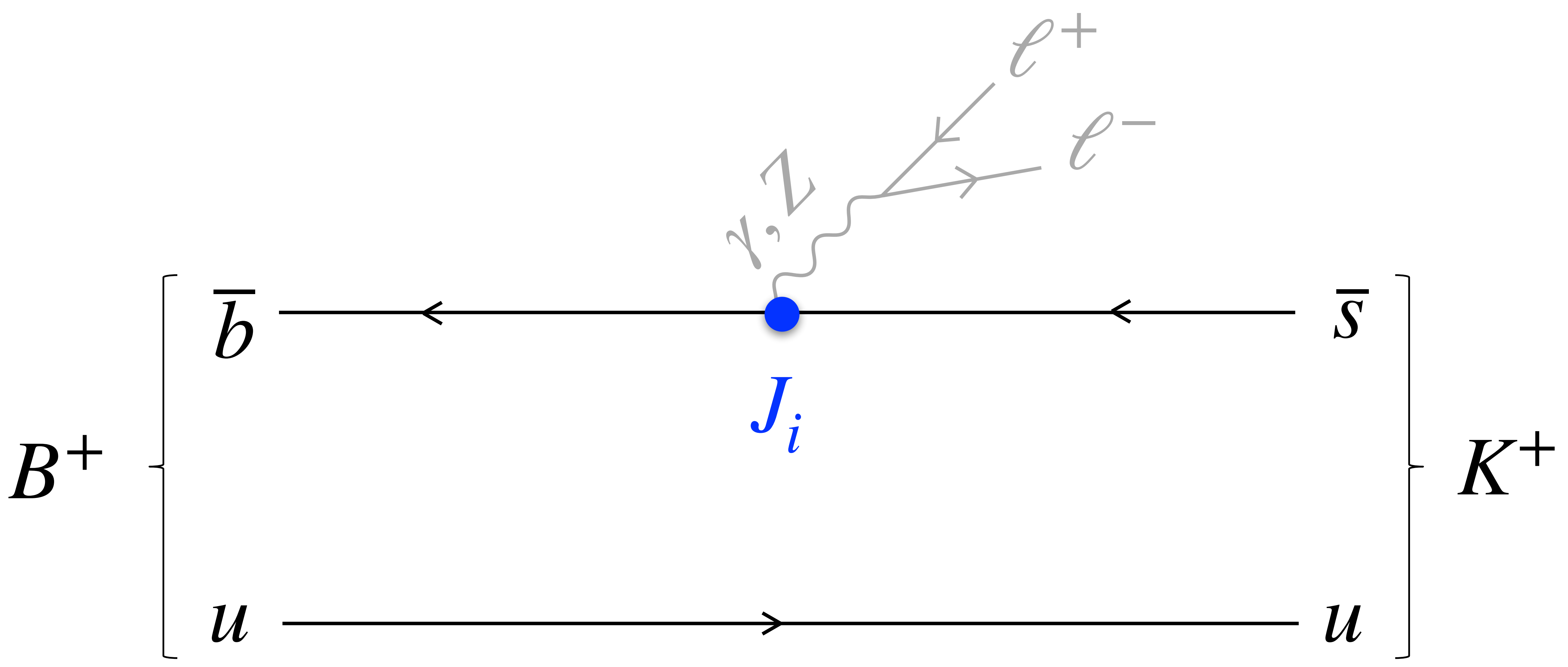


$\langle K | J_i | B \rangle$

hadronic matrix elements have:

- momentum transfer dependence, $0 \leq q^2 \leq q^2_{\max} = (M_B - M_K)^2$

Form Factor calculation: preliminaries



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- short distance weak interactions: M_t, $M_W \sim \mathcal{O}(100\,\text{GeV})$

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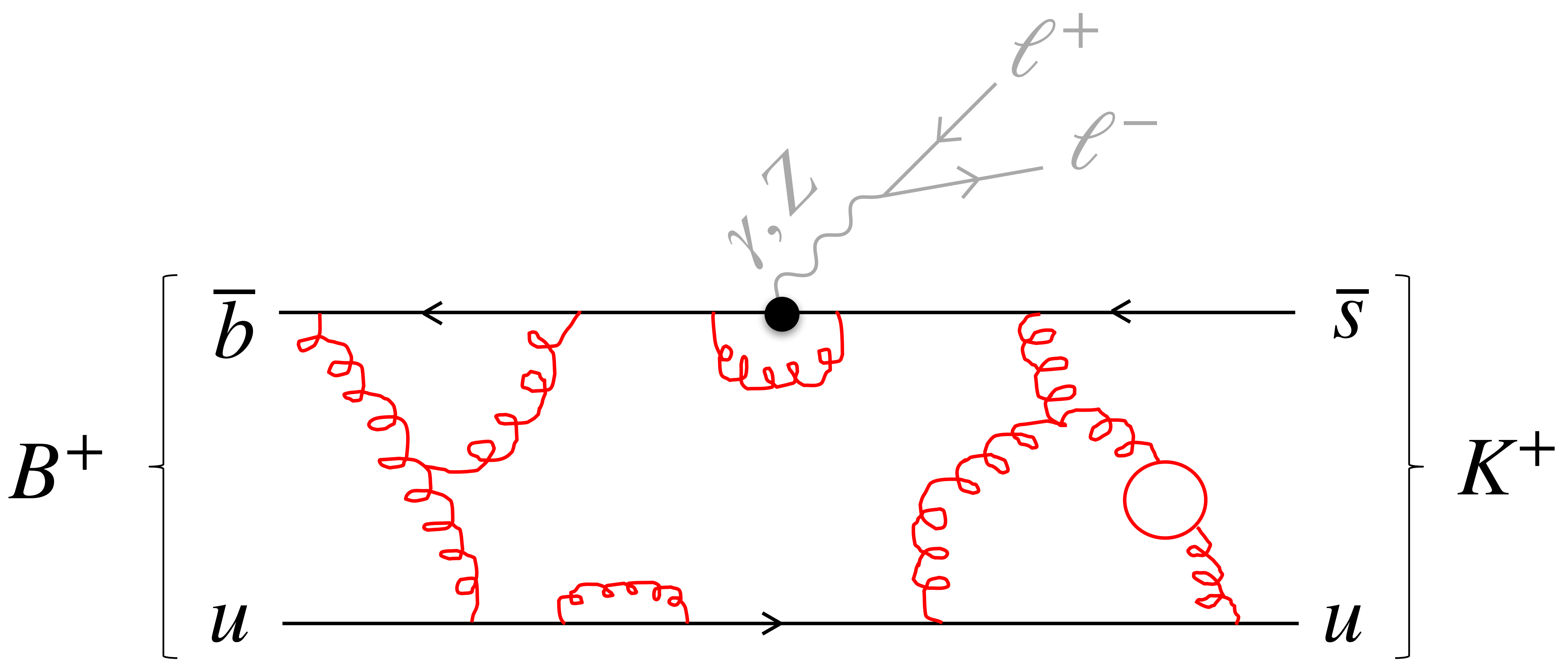
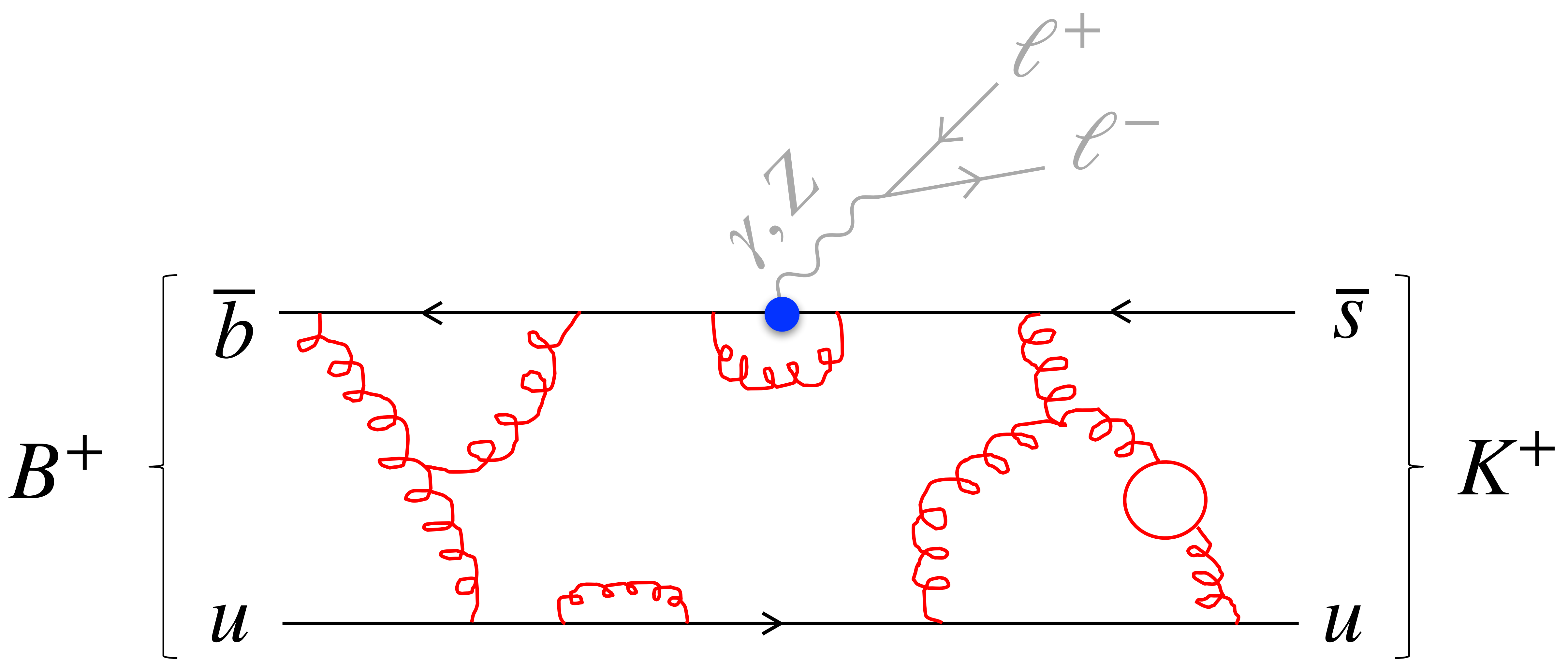


$\langle K | J_i | B \rangle$

hadronic matrix elements have:

- momentum transfer dependence: $0 \leq q^2 \leq q^2_{\text{max}} = (M_B - M_K)^2$
- short distance weak interactions: $M_t\,,\ M_W \sim \mathcal{O}(100\,\text{GeV})$
- long distance QCD interactions: $\Lambda_{\text{QCD}} \sim 0.5\,\text{GeV}$

Form Factor calculation: preliminaries

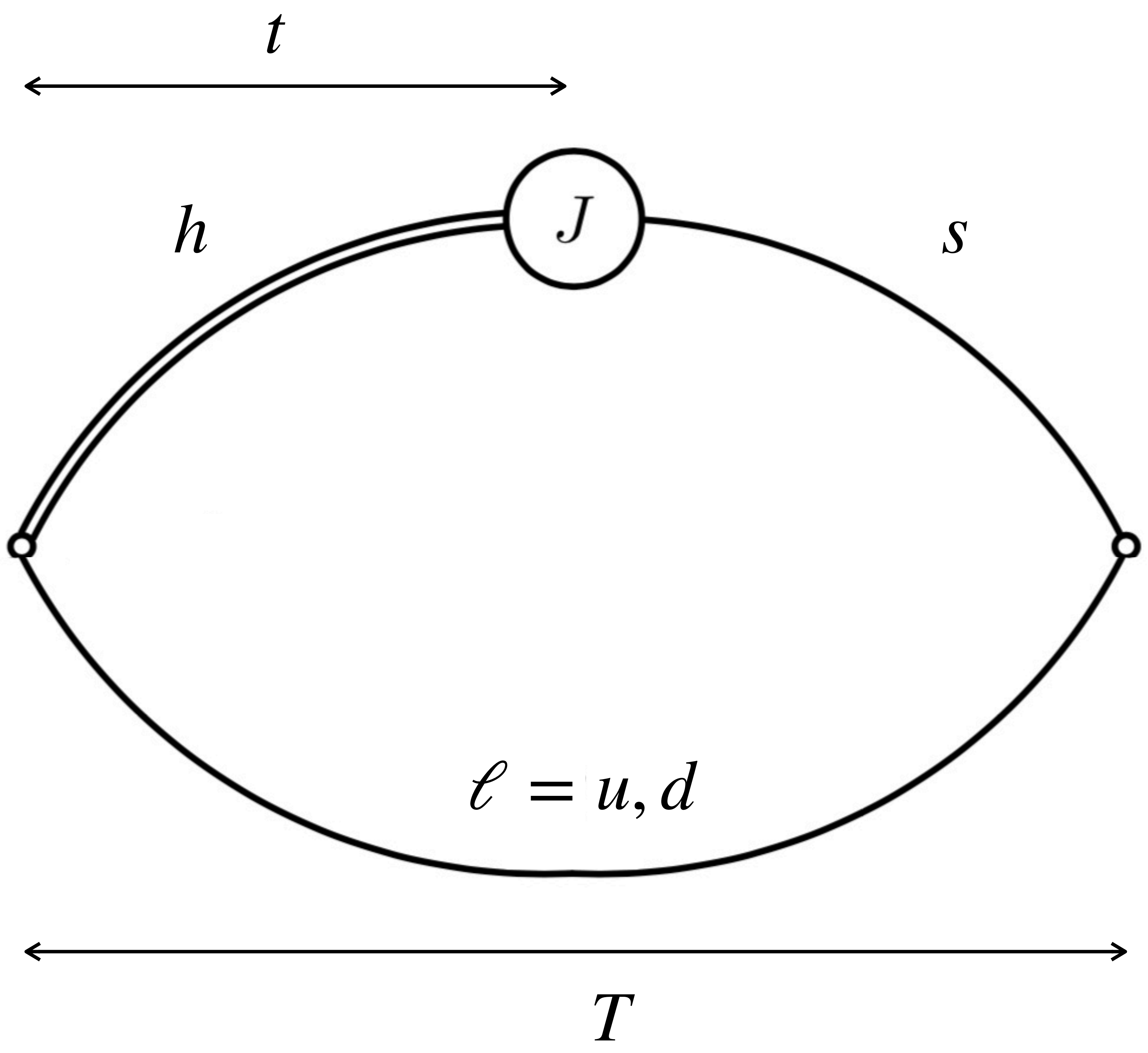


Physics at disparate scales factorizes (up to small corrections)

$$\frac{d\mathscr{B}}{dq^2} = \left| \sum_i C_i \langle K | J_i | B \rangle \right|^2 + \ldots$$

- Wilson coefficients: short distance, *perturbative*
- hadronic matrix elements: long distance, *nonperturbative*

Form Factor calculation: matrix element via LQCD



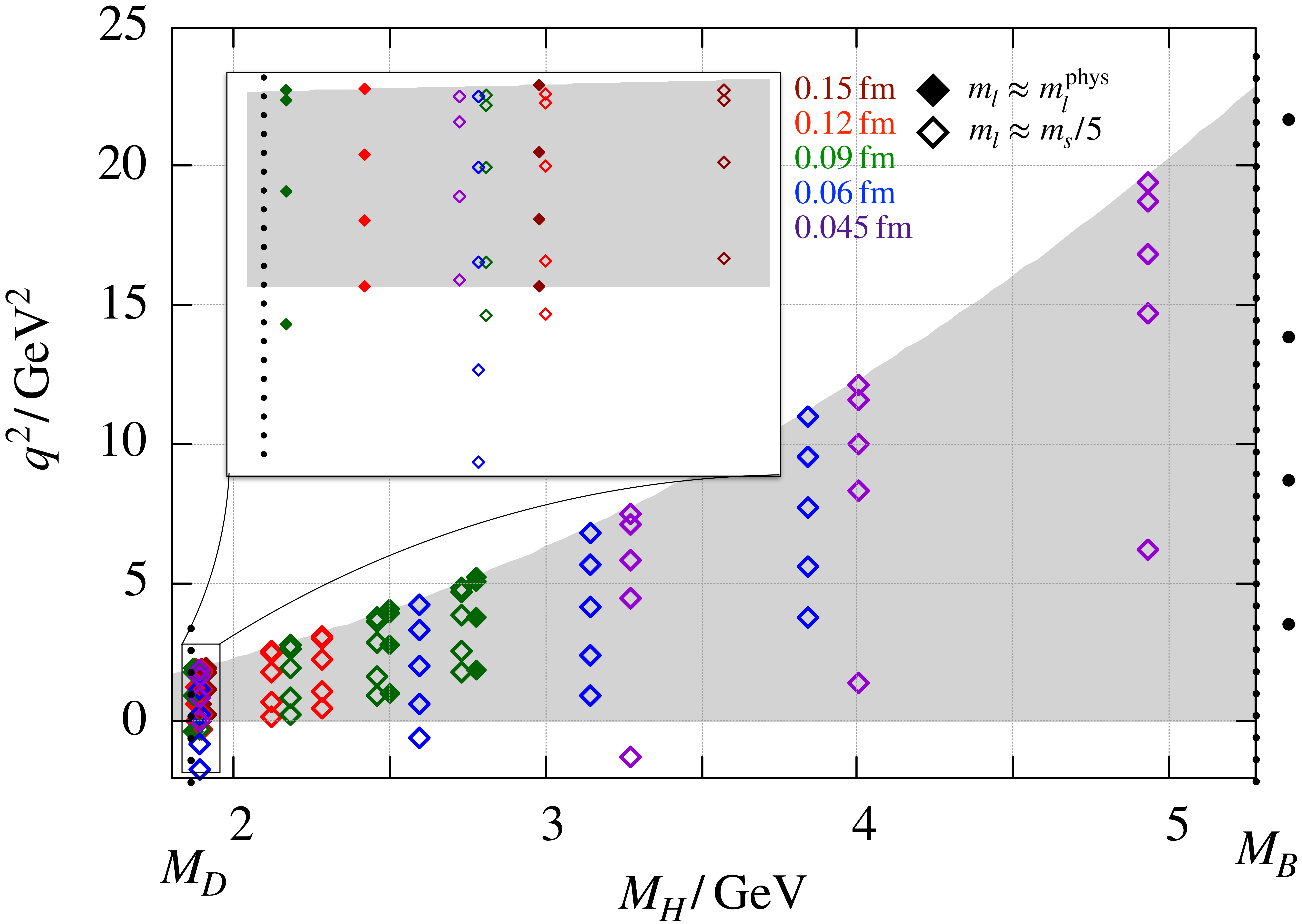
- numerically evaluate path integral representation of 3pt correlator

$$\langle K(T)\, J(t)\, H(0)^\dagger \rangle$$

  - H a proxy for heavy meson, $M_D \leq M_H \leq M_B$
  - ranges of t and T (also momenta, quark masses, lattice spacings, and volumes)
  - produce data for 3pt correlator at each combination of t and T

- J specifies matrix element (scalar, vector, or tensor)

Form Factor calculation: matrix element via LQCD



- For large range of M_H, cover all $0 \leq q^2 \leq q^2_{\mathrm{max}} = (M_H - M_K)^2$
- Near M_B on finest lattice
- Analysis gives results for B and D
- MILC's HISQ $n_f = 2+1+1$ lattices

Bazavov et al., PRD 82, 074501 (2010);
Bazavov et al., PRD 87, 054505 (2012)

Form Factor calculation: matrix element via LQCD

- analyze t and T dependence of data to extract hadronic matrix element

$$\langle K(T)\, J(t)\, H(0)^\dagger \rangle = \sum_{l,m=0}^{\infty} \langle K | E_l^{(K)} \rangle \langle E_l^{(K)} | J | E_m^{(H)} \rangle \langle E_m^{(H)} | H^\dagger \rangle \frac{1}{\sqrt{2E_l^{(K)}}} \frac{1}{\sqrt{2E_m^{(H)}}}\, e^{-E_l^{(K)}(T-t)}\, e^{-E_m^{(H)}t}$$

for $l, m = 0$, gives $\langle K | J | H \rangle$

- form factors parameterize matrix elements

$$\langle K | S | H \rangle = \frac{M_H^2 - M_K^2}{m_h - m_s} f_0(q^2) \qquad Z_T(\overline{\mathrm{MS}}, M_H)\, \langle K | T^{jo} | H \rangle = \frac{2iM_H p_K^j}{M_H + M_K} f_T(\overline{\mathrm{MS}}, M_H; q^2)$$

$$Z_V \langle K | V^\mu | H \rangle = f_+(q^2)\left(p_H^\mu + p_K^\mu - \frac{M_H^2 - M_K^2}{q^2} q^\mu\right) + f_0(q^2)\, \frac{M_H^2 - M_K^2}{q^2} q^\mu$$

Form Factor calculation: matrix element via LQCD

- analyze t and T dependence of data to extract hadronic matrix element

$$\langle K(T)\, J(t)\, H(0)^\dagger \rangle = \sum_{l,m=0}^{\infty} \langle K | E_l^{(K)} \rangle \langle E_l^{(K)} | J | E_m^{(H)} \rangle \langle E_m^{(H)} | H^\dagger \rangle \frac{1}{\sqrt{2E_l^{(K)}}} \frac{1}{\sqrt{2E_m^{(H)}}}\, e^{-E_l^{(K)}(T-t)}\, e^{-E_m^{(H)} t}$$

- form factors parameterize matrix elements

$$\langle K | S | H \rangle = \frac{M_H^2 - M_K^2}{m_h - m_s} f_0(q^2)$$

$Z_T(\overline{\mathrm{MS}}, M_H)$ Calculated via RI-SMOM at 2 GeV (accounting for nonperturbative contributions)

Hatton, Davies, Lepage, Lytle, PRD 102, 094509 (2020)

Z_V Calculated via PCVC relation, $Z_V = \left. \frac{m_h - m_s \langle K | S | H \rangle}{(M_H - M_K) \langle K | V^0 | H \rangle} \right|_{\vec{p}_K = 0}$

Na, Davies, Follana, Lepage, PRD 82, 114506 (2010)

Form Factor calculation: extrapolate to real world

- trade q^2 for: $z(q^2) = \left(\sqrt{t_+ - q^2} - \sqrt{t_+}\right) / \left(\sqrt{t_+ - q^2} + \sqrt{t_+}\right)$, where $t_+ = (M_H + M_K)^2$
  - $|z| \ll 1$, allows series expansion of form factor (once pole removed)

$$f(q^2)\left(1 - \frac{q^2}{M_{\rm pole}^2}\right) = \sum_n a_n z^n$$

- modified z-expansion fit
  - extrapolate to $a \to 0$, volume $\to \infty$, and quark masses $\to$ physical
  - interpolate over full range of q^2

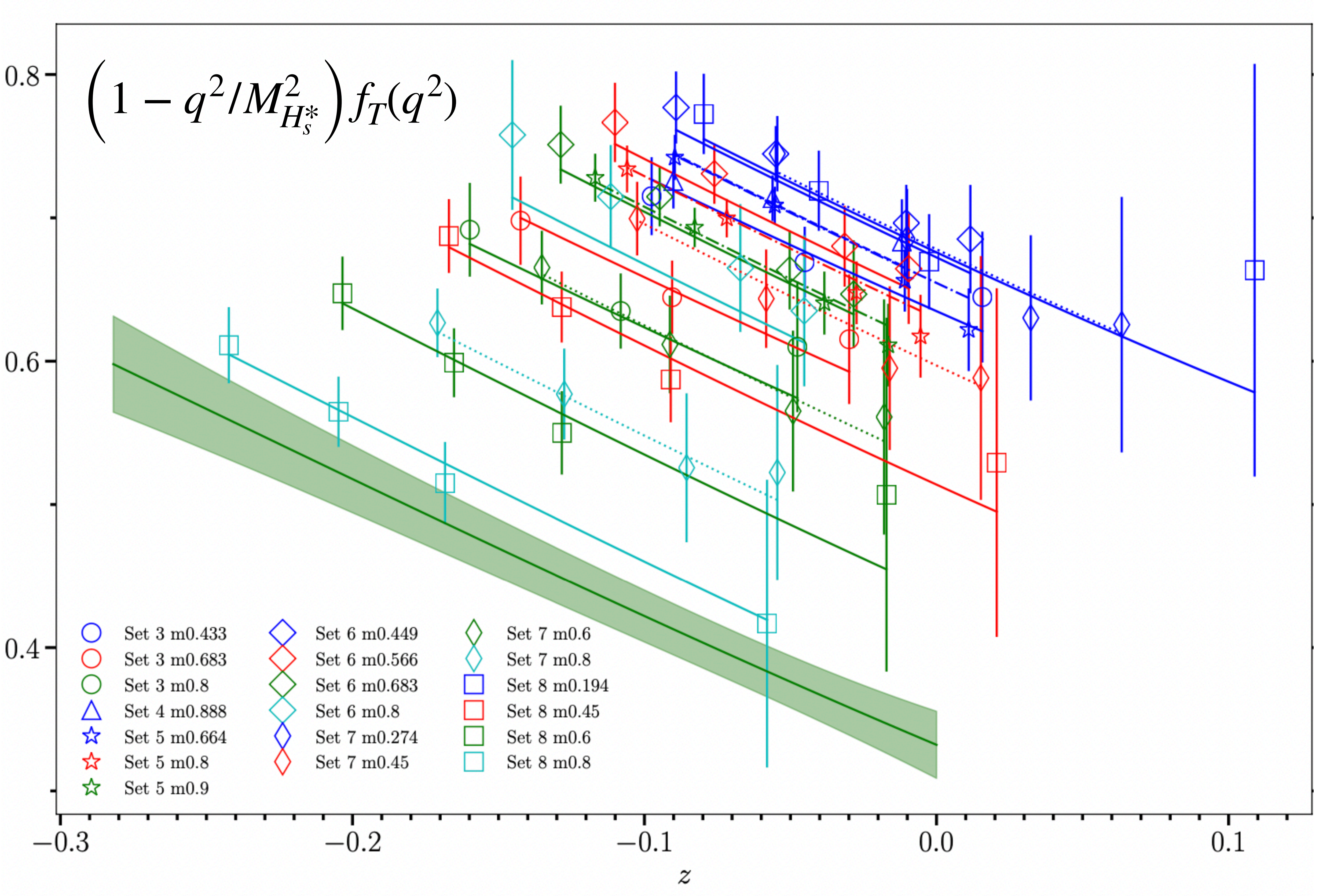
a_n contains chiral, mistuning, heavy quark expansion, and discretization terms

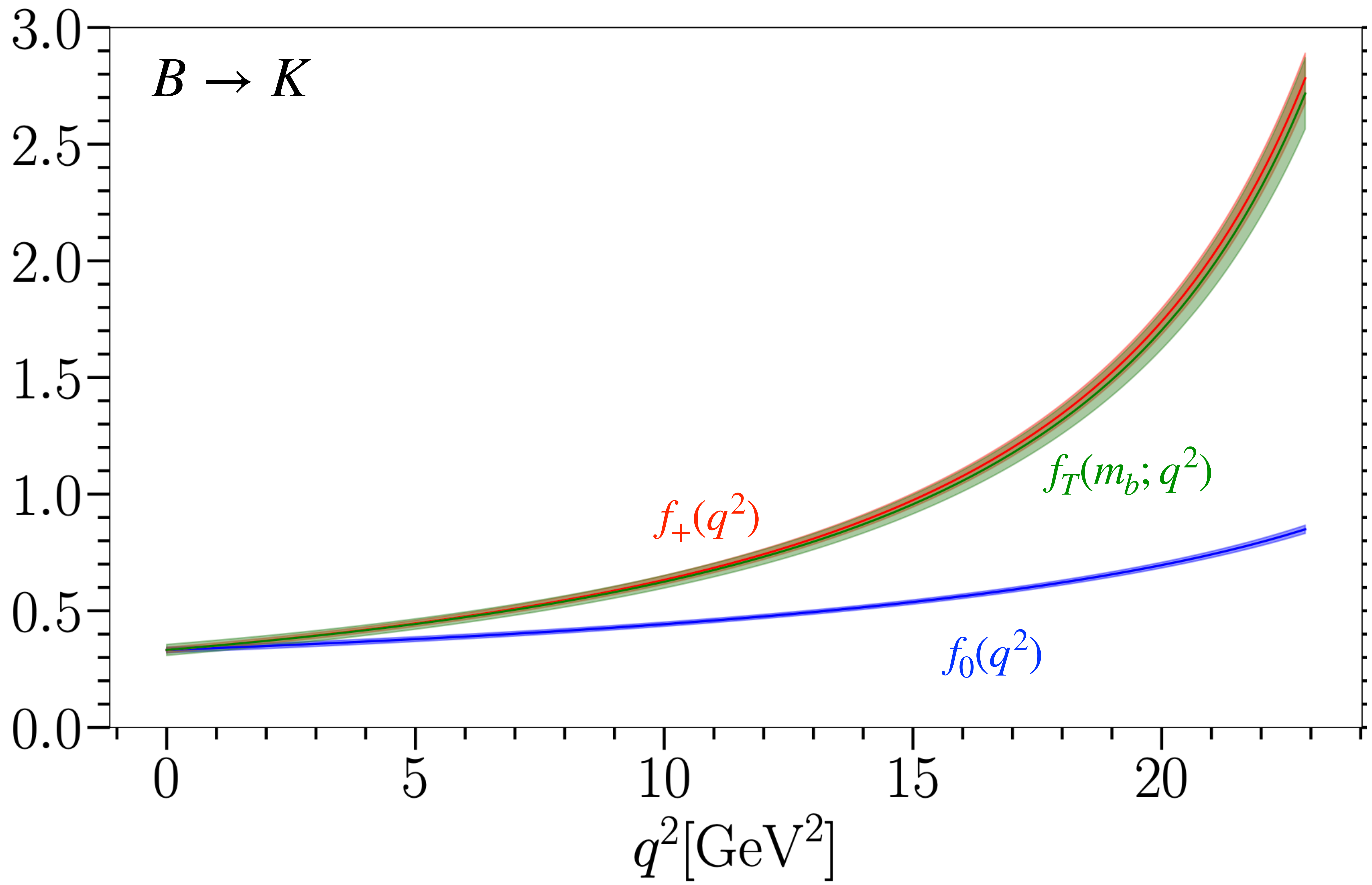
$$a_n = (1 + L(m_l, V))\Big(1 + \epsilon_n\Big)\Big(1 + \rho_n \log\big(\frac{M_H}{M_D}\big)\Big) \sum_{i,j,k,l=0}^{N_{ijkl}-1} d_{ijkln} \left(\frac{\Lambda}{M_H}\right)^i \left(\frac{am_h}{\pi}\right)^{2j} \left(\frac{a\Lambda}{\pi}\right)^{2k} \left(\frac{m_\pi^2 - (m_\pi^{\rm phys})^2}{(4\pi f_\pi)^2}\right)^l$$

Form Factor calculation: extrapolate to real world

- bands show form factors in continuum, infinite volume, with physical quark masses, and for $m_h = m_b$

Form Factor calculation: extrapolate to real world





- improved precision, especially at low q^2, where it is needed
- errors statistics dominated, so improvement straightforward

Phenomenology: $B \to K\ell^+\ell^-$

- differential decay rate (or branching fraction $\mathscr{B} = \tau_B \Gamma$) is measured

$$\frac{d\Gamma(B \to K\ell^+\ell^-)}{dq^2} = 2a_\ell + \frac{2}{3}c_\ell$$

$$a_\ell = \mathscr{C}\left[q^2 |F_P|^2 + \frac{\lambda(q, M_B, M_K)}{4}(|F_A|^2 + |F_V|^2) + 4m_\ell^2 M_B^2 |F_A|^2 + 2m_\ell(M_B^2 - M_K^2 + q^2)\mathrm{Re}(F_P F_A^*)\right]$$

$$c_\ell = -\frac{\mathscr{C}\lambda(q, M_B, M_K)\beta_\ell^2}{4}(|F_A|^2 + |F_V|^2)$$

- prediction depends on $F_{P,A,V}$ - functions of form factors and Wilson coefficients

Phenomenology: $B \to K\ell^+\ell^-$

$$F_P = -m_\ell C_{10}\left[f_+ - \frac{M_B^2 - M_K^2}{q^2}(f_0 - f_+)\right]$$

$$F_A = C_{10} f_+$$

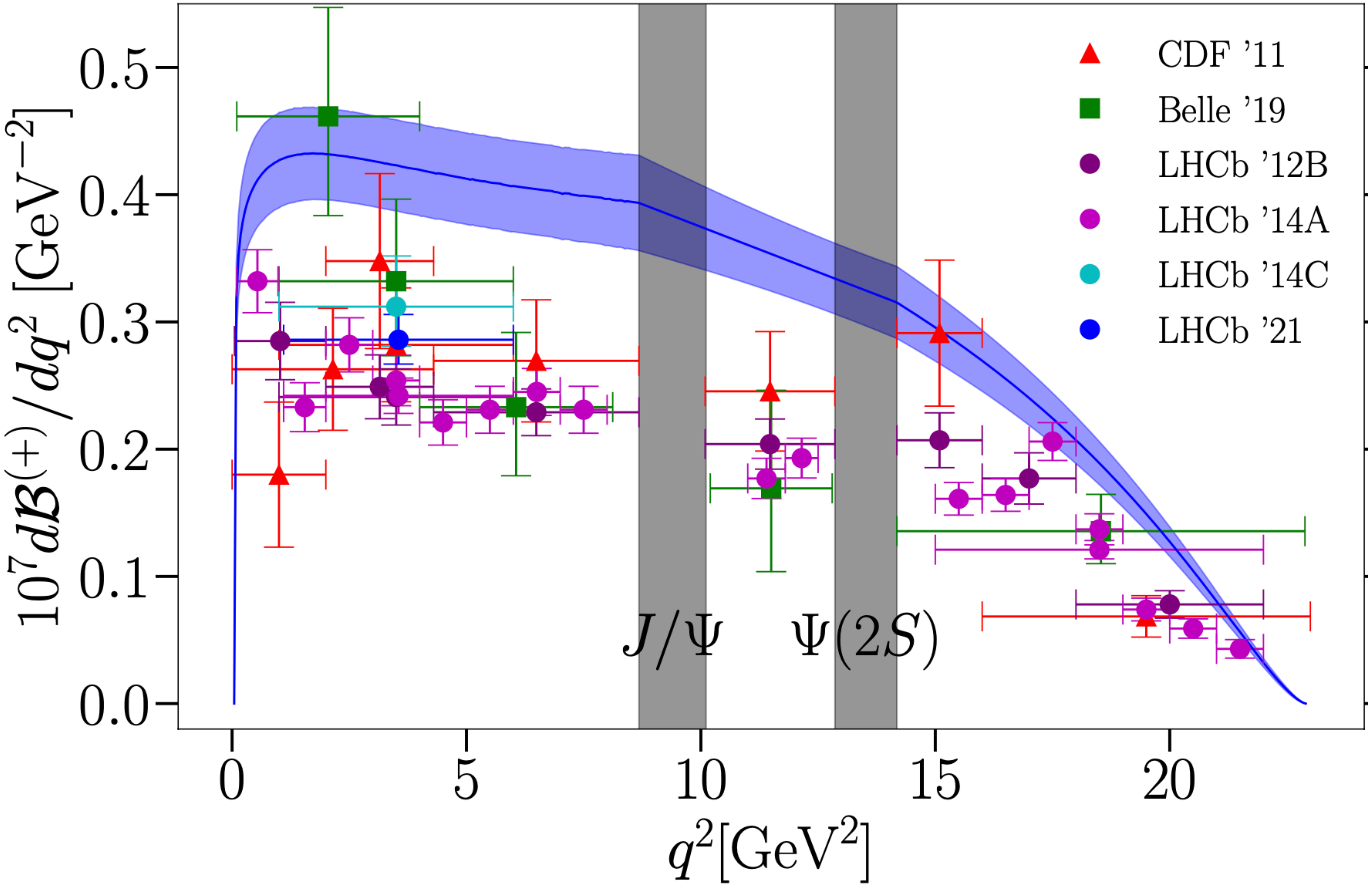
$$F_V = C_9^{\mathrm{eff},1} f_+ + \frac{2m_b^{\overline{\mathrm{MS}}}(\mu_b)}{M_B + M_K} C_7^{\mathrm{eff},1} f_T(\mu_b)$$

- $C_9^{\mathrm{eff},1}$ includes nonfactoriazable and $\mathcal{O}(\alpha_s)$ perturbative QCD corrections
- $C_7^{\mathrm{eff},1}$ includes $\mathcal{O}(\alpha_s)$ corrections

FNAL/MILC, PRD 93, 034005 (2016)

- corrections amount to $< 1\sigma$ shift, slightly reducing tension with experiment

Phenomenology: $B \to K\ell^+\ell^-$

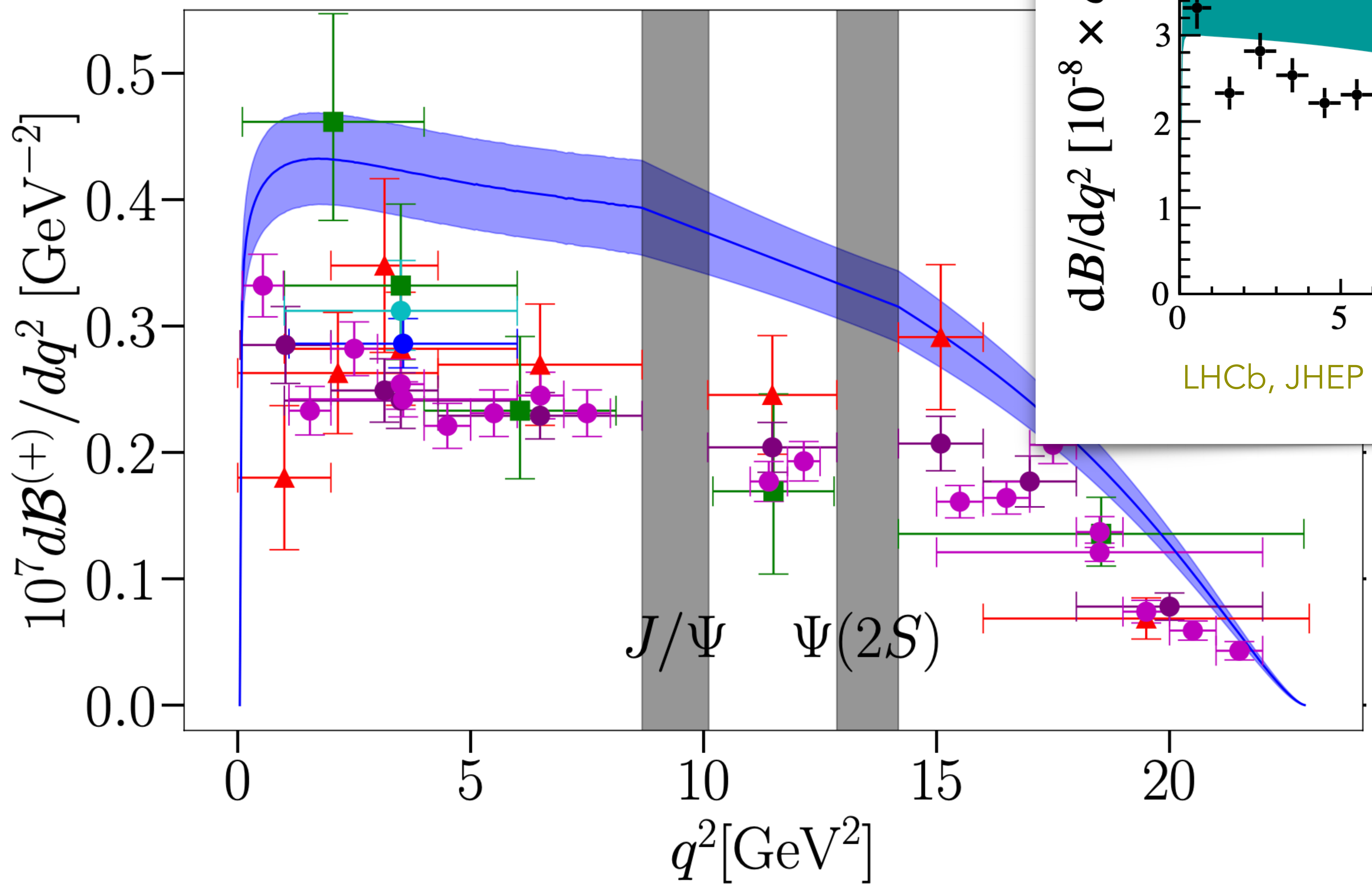


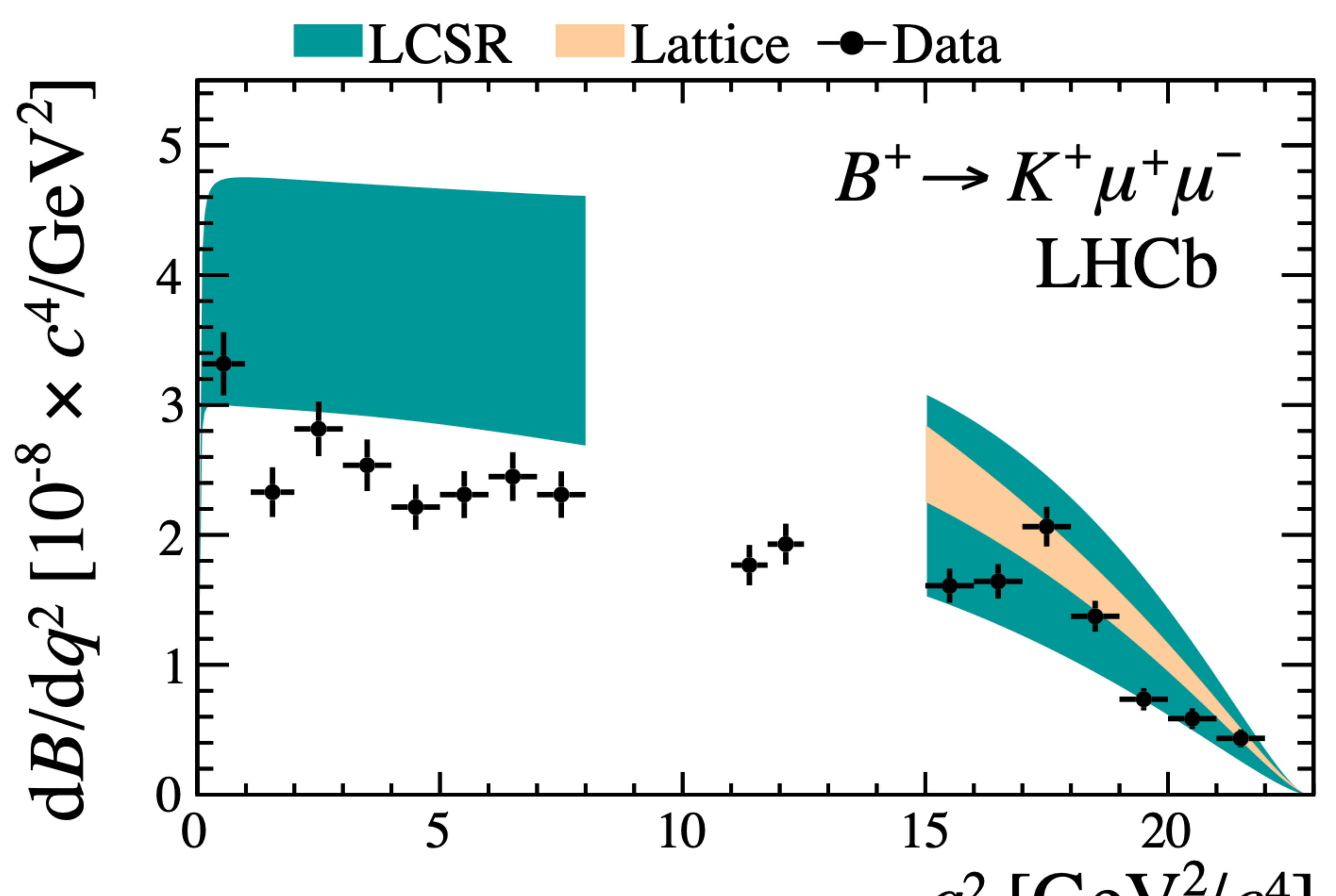
Focus on two well-behaved regions:

- $1.1 \leq q^2/\mathrm{GeV}^2 \leq 6$: below $c\bar{c}$ resonances; improved precision and increased tension
- $15 \leq q^2/\mathrm{GeV}^2 \leq 22$: above (dominant) $c\bar{c}$ resonances, include 2% uncertainty for others

LHCb, Eur. Phys. J. C 77, 161 (2017)

Phenomenology: $B \to K\ell$





LHCb, JHEP 06 (2014) 133

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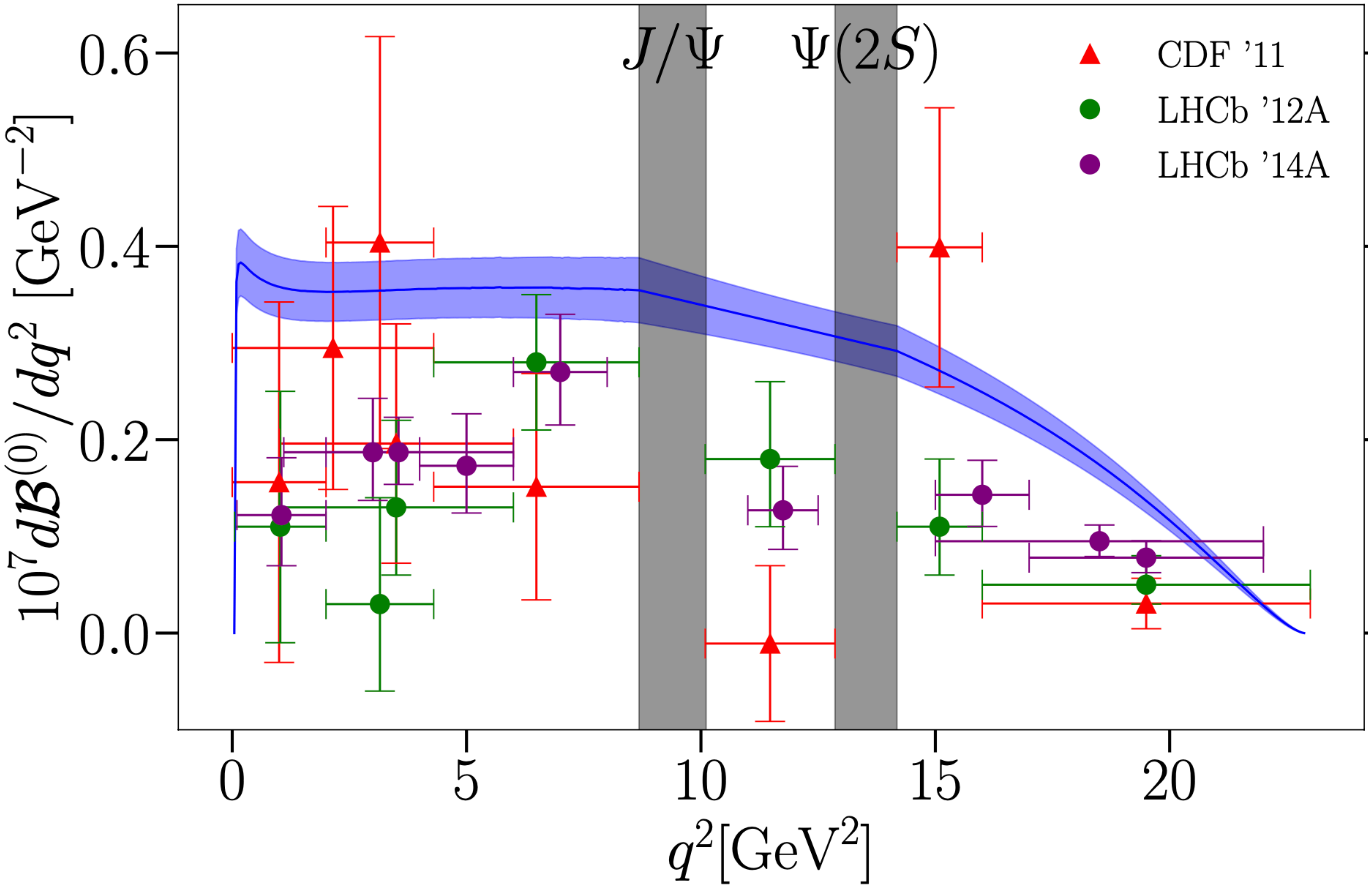
Phenomenology: $B \to K\ell^+\ell^-$



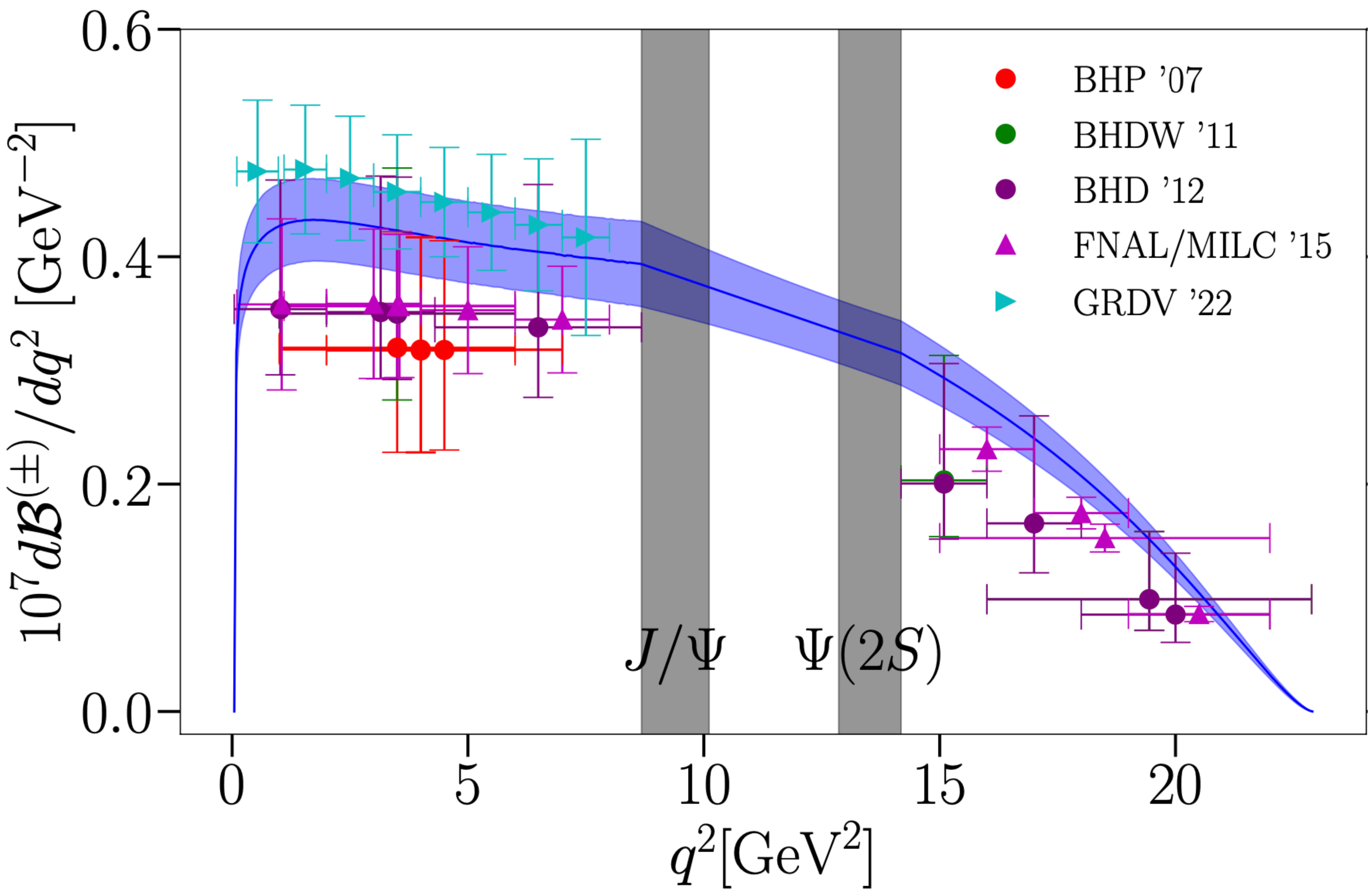
- LQCD calculation omits QED and in isospin limit, with $m_l = (m_u + m_d)/2$
- Differentiate between charged and neutral cases
    - 0.5% for form factor m_l
    - Missing final state radiation in experiment and no QED in form factors: 5% (2%) for e (μ) decay rates; 1% for R_K

Phenomenology: $B \to K\ell^+\ell^-$

Channel	Result	q^2/GeV^2 range	$\mathcal{B} \times 10^7$	Tension with HPQCD '22
$B^+ \to K^+e^+e^-$	LHCb '21	$(1.1, 6)$	$1.401^{+0.074}_{-0.069} \pm 0.064$	-3.3σ (-3.0σ)
$B^+ \to K^+e^+e^-$	HPQCD '22	$(1.1, 6)$	$2.07 \pm 0.17(\pm 0.10)_{\mathrm{QED}}$	-
$B^+ \to K^+e^+e^-$	Belle '19	$(1, 6)$	$1.66^{+0.32}_{-0.29} \pm 0.04$	-1.2σ (-1.2σ)
$B^+ \to K^+e^+e^-$	HPQCD '22	$(1, 6)$	$2.11 \pm 0.18(\pm 0.11)_{\mathrm{QED}}$	-
$B^0 \to K^0\mu^+\mu^-$	LHCb '14A	$(1.1, 6)$	$0.92^{+0.17}_{-0.15} \pm 0.044$	-3.6σ (-3.5σ)
$B^0 \to K^0\mu^+\mu^-$	HPQCD '22	$(1.1, 6)$	$1.74 \pm 0.15(\pm 0.04)_{\mathrm{QED}}$	-
$B^0 \to K^0\mu^+\mu^-$	LHCb '14A	$(15, 22)$	$0.67^{+0.11}_{-0.11} \pm 0.035$	-3.2σ (-3.1σ)
$B^0 \to K^0\mu^+\mu^-$	HPQCD '22	$(15, 22)$	$1.16 \pm 0.10(\pm 0.02)_{\mathrm{QED}}$	-
$B^+ \to K^+\mu^+\mu^-$	Belle '19	$(1, 6)$	$2.30^{+0.41}_{-0.38} \pm 0.05$	$+0.4\sigma$ $(+0.4\sigma)$
$B^+ \to K^+\mu^+\mu^-$	HPQCD '22	$(1, 6)$	$2.11 \pm 0.18(\pm 0.04)_{\mathrm{QED}}$	-
$B^+ \to K^+\mu^+\mu^-$	LHCb '14A	$(1.1, 6)$	$1.186 \pm 0.034 \pm 0.059$	-4.7σ (-4.6σ)
$B^+ \to K^+\mu^+\mu^-$	HPQCD '22	$(1.1, 6)$	$2.07 \pm 0.17(\pm 0.04)_{\mathrm{QED}}$	-
$B^+ \to K^+\mu^+\mu^-$	LHCb '14A	$(15, 22)$	$0.847 \pm 0.028 \pm 0.042$	-3.4σ (-3.3σ)
$B^+ \to K^+\mu^+\mu^-$	HPQCD '22	$(15, 22)$	$1.26 \pm 0.11(\pm 0.03)_{\mathrm{QED}}$	-

- consistent tension with LHCb
- single experiment (LHCb '14A, $B^+ \to K^+\mu^+\mu^-$, $1.1 \le q^2/\mathrm{GeV}^2 \le 6$) approaching 5σ

Phenomenology: $B \to K\ell^+\ell^-$ vs other theory



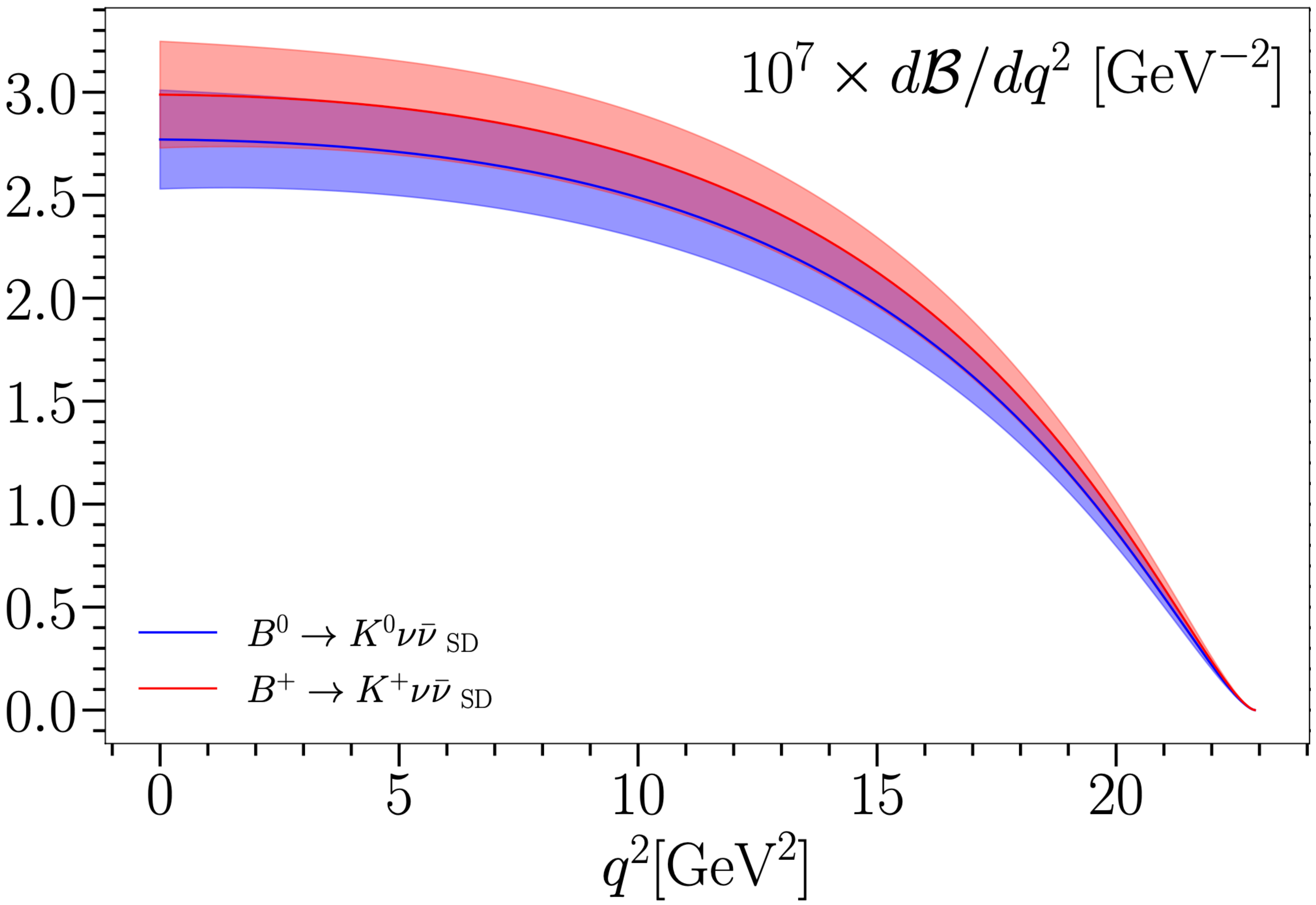
C. Bobeth, G. Hiller, and G. Piranishvili

C. Bobeth, G. Hiller, D. van Dyk, and C. Wacker

C. Bobeth, G. Hiller, and D. van Dyk

N. Gubernari, M. Reboud, D. van Dyk, and J. Virto

Phenomenology: $B \to K\nu\bar{\nu}$

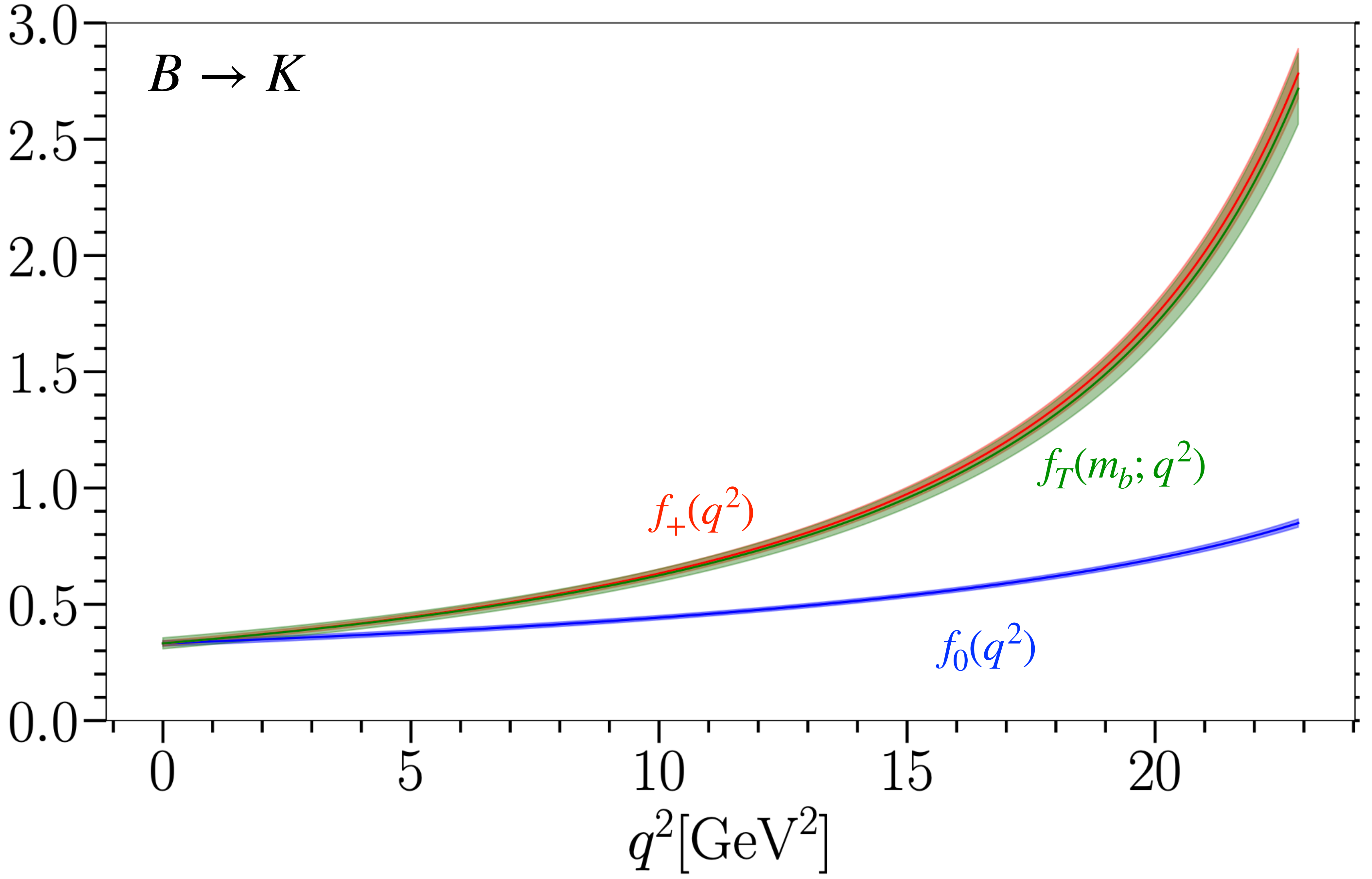


Decay	$\mathcal{B} \times 10^6$	Reference	
$B^0 \to K^0_S\nu\bar{\nu}$	< 13 (90% CL) Exp.	[32]	Belle '17
$B^0 \to K^0\nu\bar{\nu}$	< 49 (90% CL) Exp.	[34]	BaBar '13
	4.01(49)	[9]	FNAL '16
	$4.1^{+1.3}_{-1.0}$	[37]	Wang, Xiao '12
	4.67(35)	HPQCD '22	
$B^+ \to K^+\nu\bar{\nu}$	< 16 (90% CL) Exp.	[34]	
	< 19 (90% CL) Exp.	[32]	
	< 41 (90% CL) Exp.	[33]	Belle II '21
	5.10(80)	[75, 78]	Altmanshoffer et al '09; Kamenik, Smith '09
	$4.4^{+1.4}_{-1.1}$	[37]	
	3.98(47)	[76]	Buras et al '14
	4.94(52)	[9]	
	4.53(64)	[83]	Buras, Venturini '21
	4.65(62)	[84]	Buras, Venturini '22
	5.67(38)	HPQCD '22	

- modest improvement in precision
- matches expected Belle-II precision at 50 ab^{-1}

Conclusions and Outlook

- 'Heavy HISQ' form factors most precise to date at low q^2 Parrot, Bouchard, and Davies, 2207.12468
  - statistics limited
  - other groups (e.g. FNAL/MILC) have calculations underway



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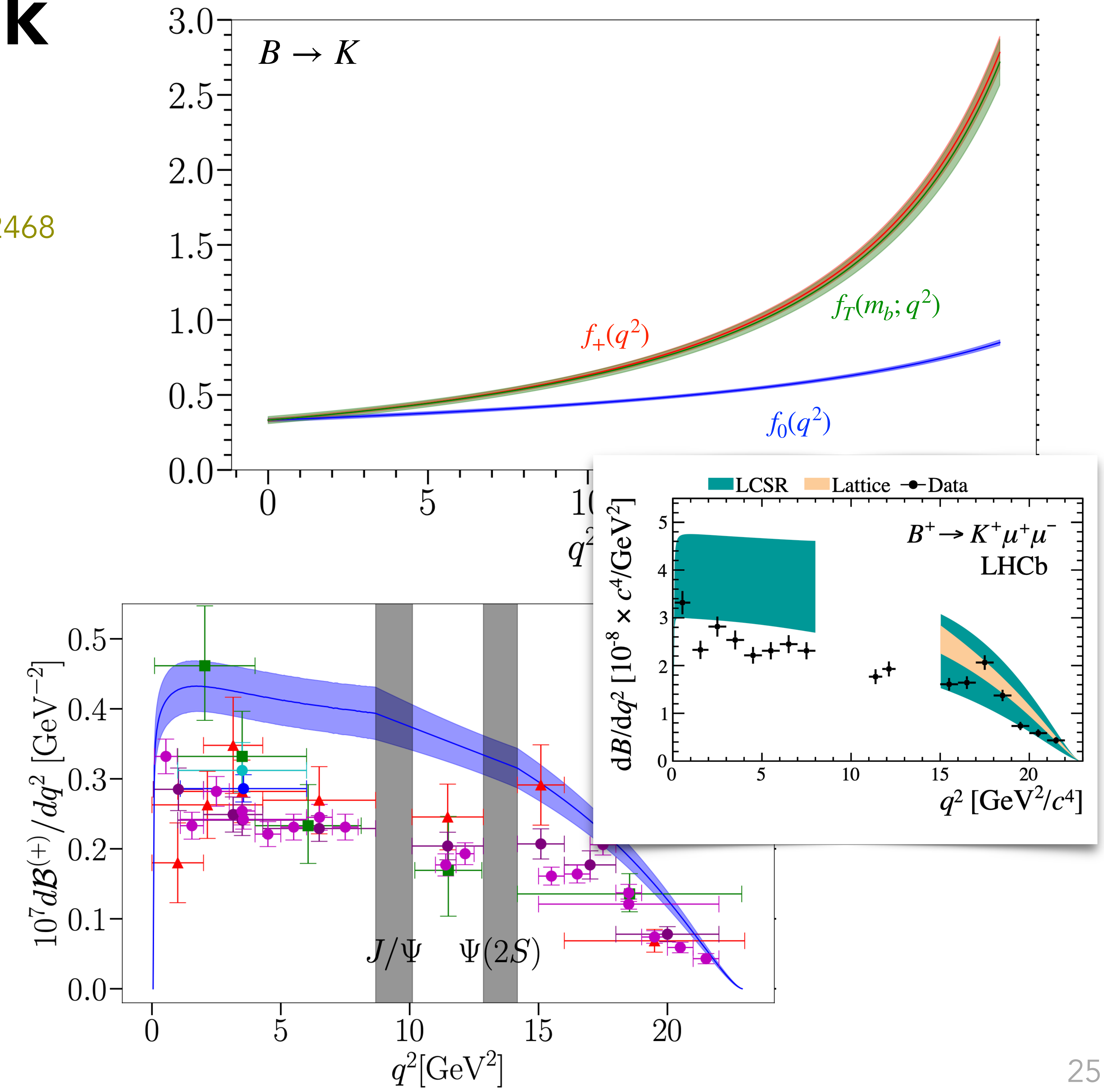
  - approaching 5σ for single experiment



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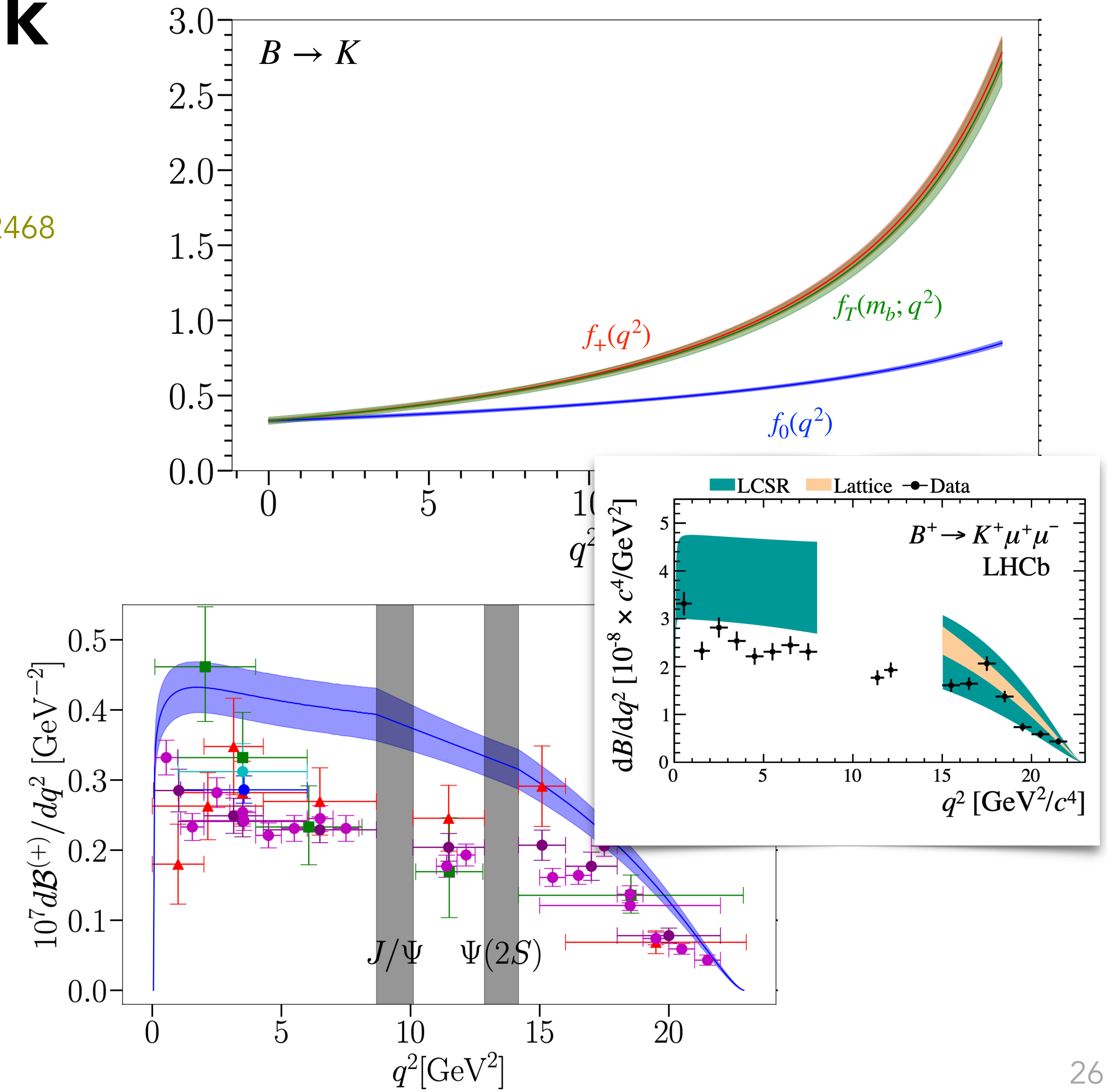
- increased precision for phenomenology Parrot, Bouchard, and Davies, 2207.1337
  - approaching 5σ for single experiment
- fully relativistic b quark removes EFT matching and improves q^2 coverage
  - form factor precision to match Belle-II expectations



Backup Slides

$H \to K\ell^+\ell^-$

- MILC HISQ $n_f = 2 + 1 + 1$ gauge field configurations; all HISQ valence quarks

- am_b generates large discretization effects unless $a \lesssim 0.04$ fm
- Instead, simulate over range of m_h, then extrapolate to m_b using HQET
- "Heavy HISQ" method

Set	a (fm)	$N_x^3 \times N_t$	$n_{cfg} \times n_{src}$	$am_l^{sea/val}$	am_h^{val}
1	0.15	$32^3 \times 48$	998×16	0.00235	0.8605
2	0.12	$48^3 \times 64$	985×16	0.00184	0.643
3	0.09	$64^3 \times 96$	620×8	0.00120	0.433, 0.683, 0.8
4	0.15	$16^3 \times 48$	1020×16	0.013	0.888
5	0.12	$24^3 \times 64$	1053×16	0.0102	0.664, 0.8, 0.9
6	0.09	$32^3 \times 96$	499×16	0.0074	0.449, 0.566, 0.683, 0.8
7	0.06	$48^3 \times 144$	415×8	0.0048	0.274, 0.45, 0.6, 0.8
8	0.044	$64^3 \times 192$	375×4	0.00316	0.194, 0.45, 0.6, 0.8

Bazavov et al., PRD 82, 074501 (2010); Bazavov et al., PRD 87, 054505 (2012)

Form Factor calculation: correlator fit stability

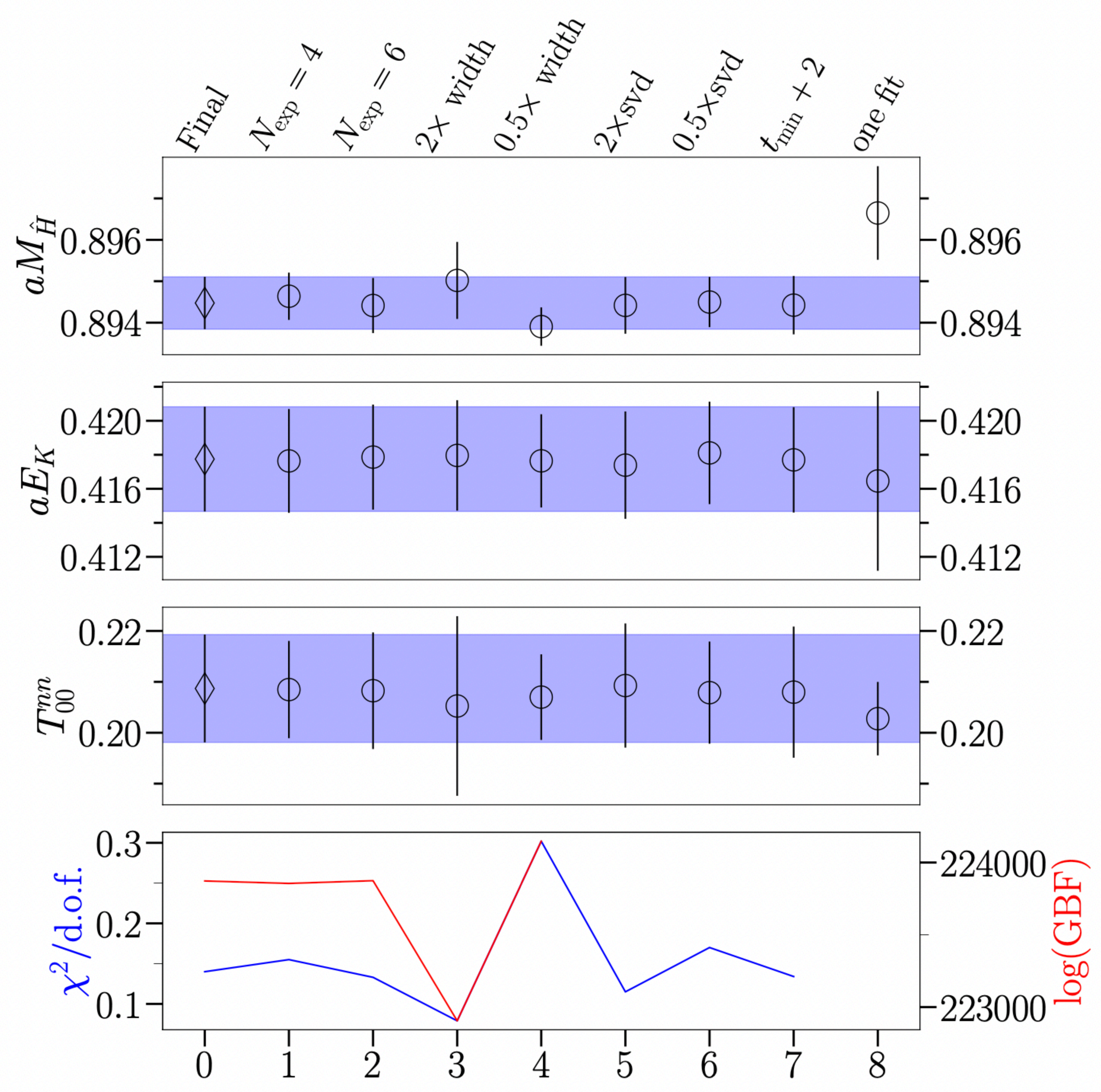


FIG. 2. Stability plot for different correlator fit choices on set 8, showing the mass of the ground-state non-goldstone $\hat{H}$ meson for $am_h = 0.6$, the ground-state energy of the K with twist $\theta = 4.705$ and T_{00}^{nn} for $am_h = 0.45$, $\theta = 2.235$. Test 0 is the final result, corresponding to $N_{\mathrm{exp}} = 5$ exponentials.

Form Factor calculation: variation with m_h

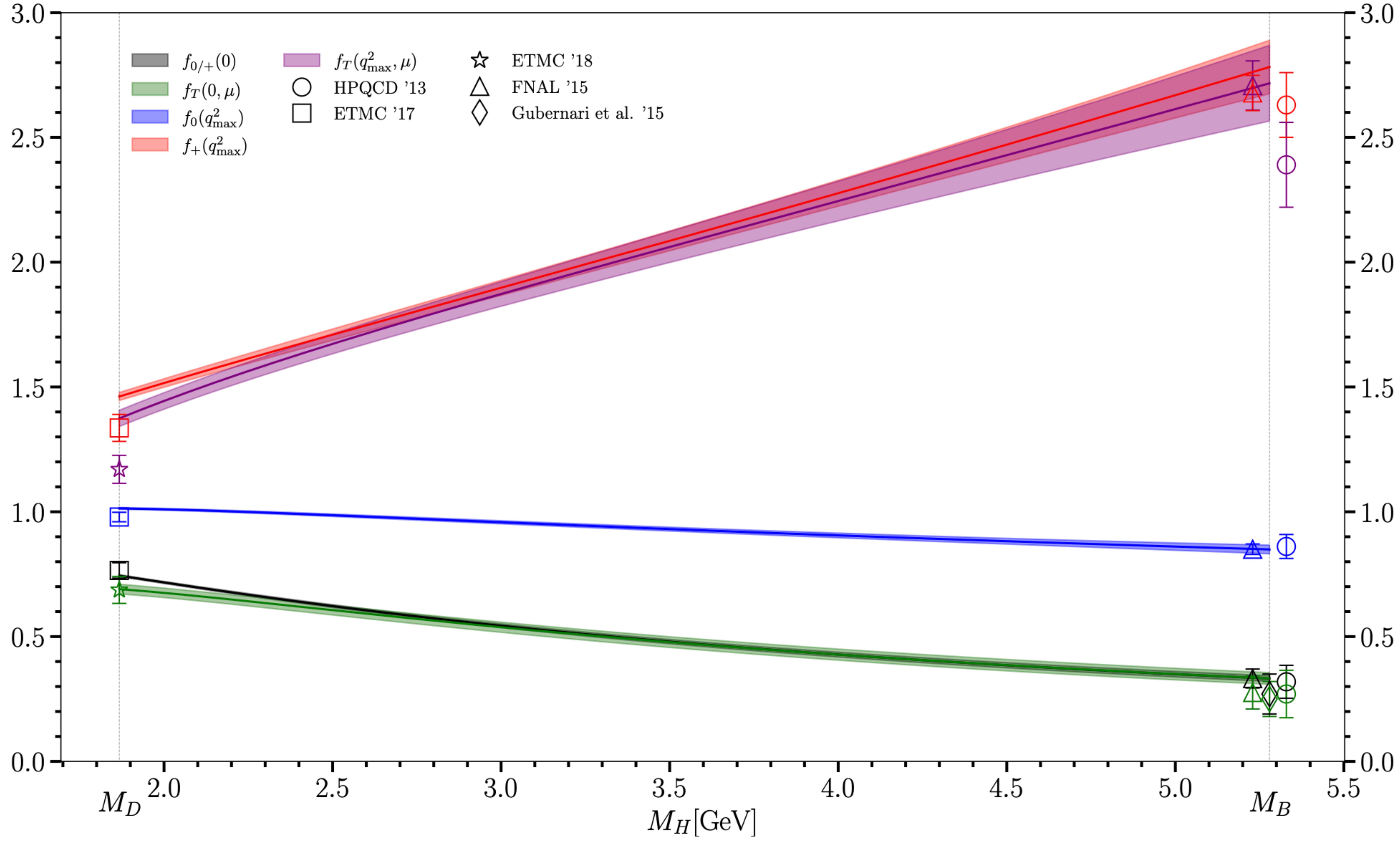
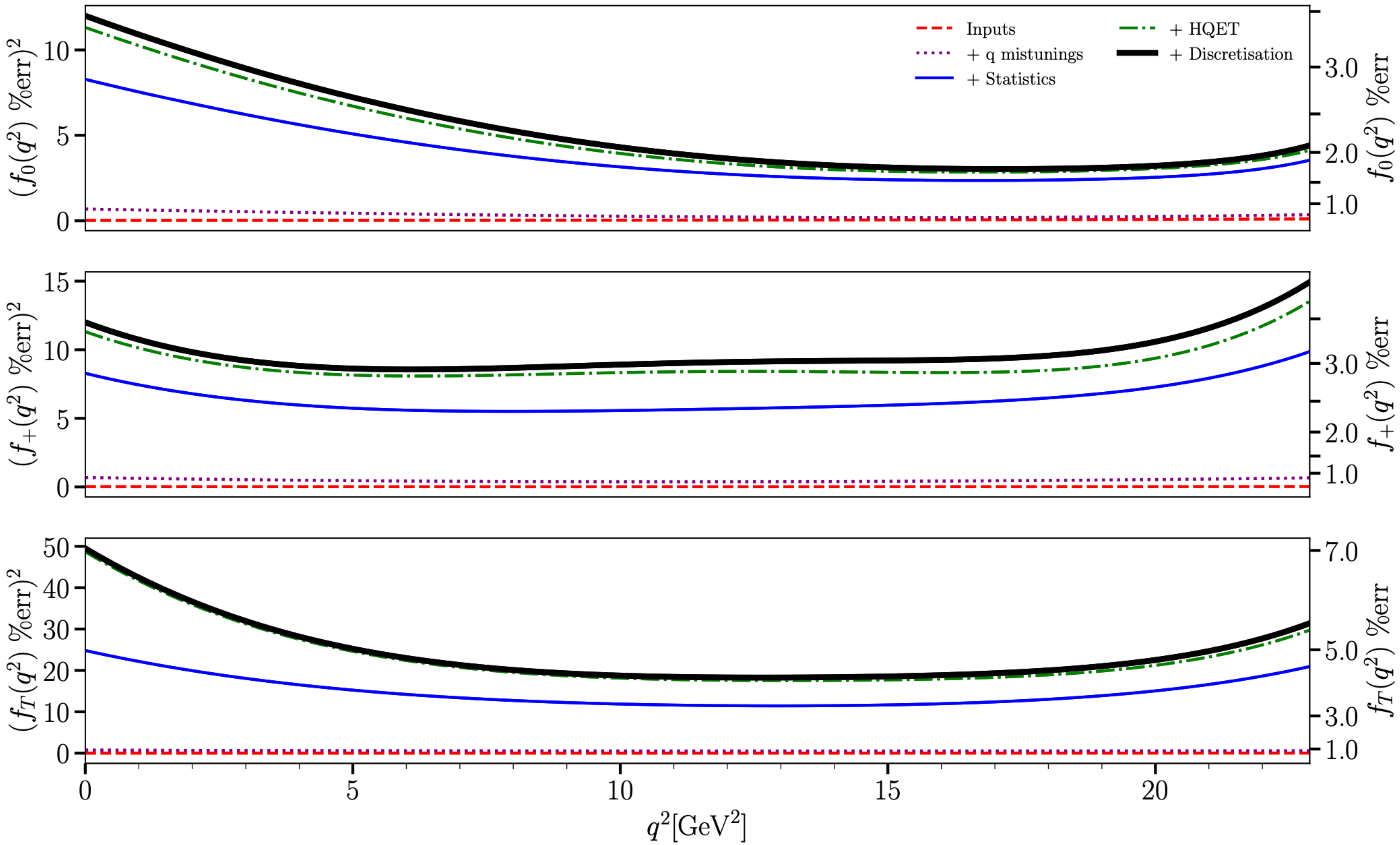
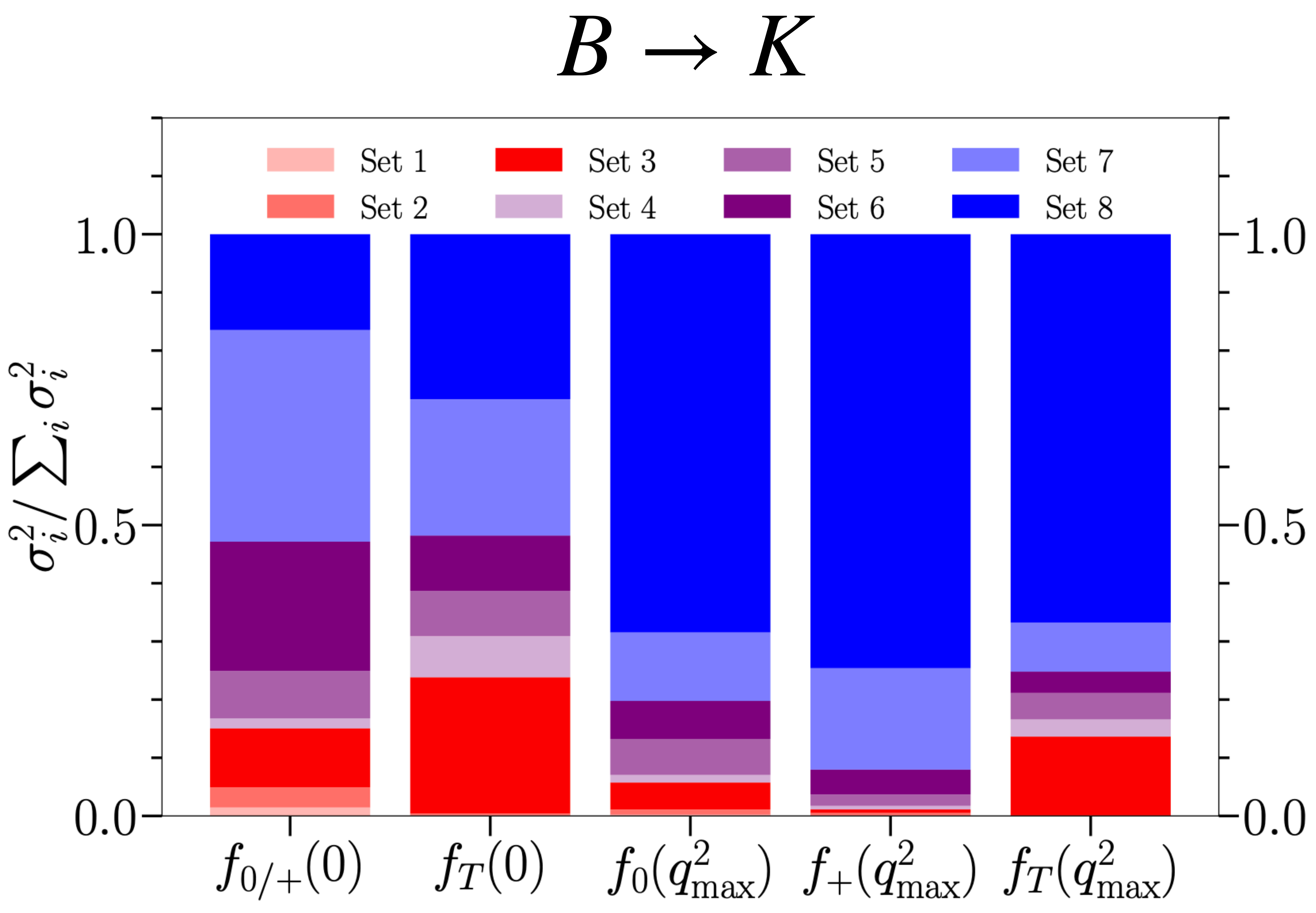


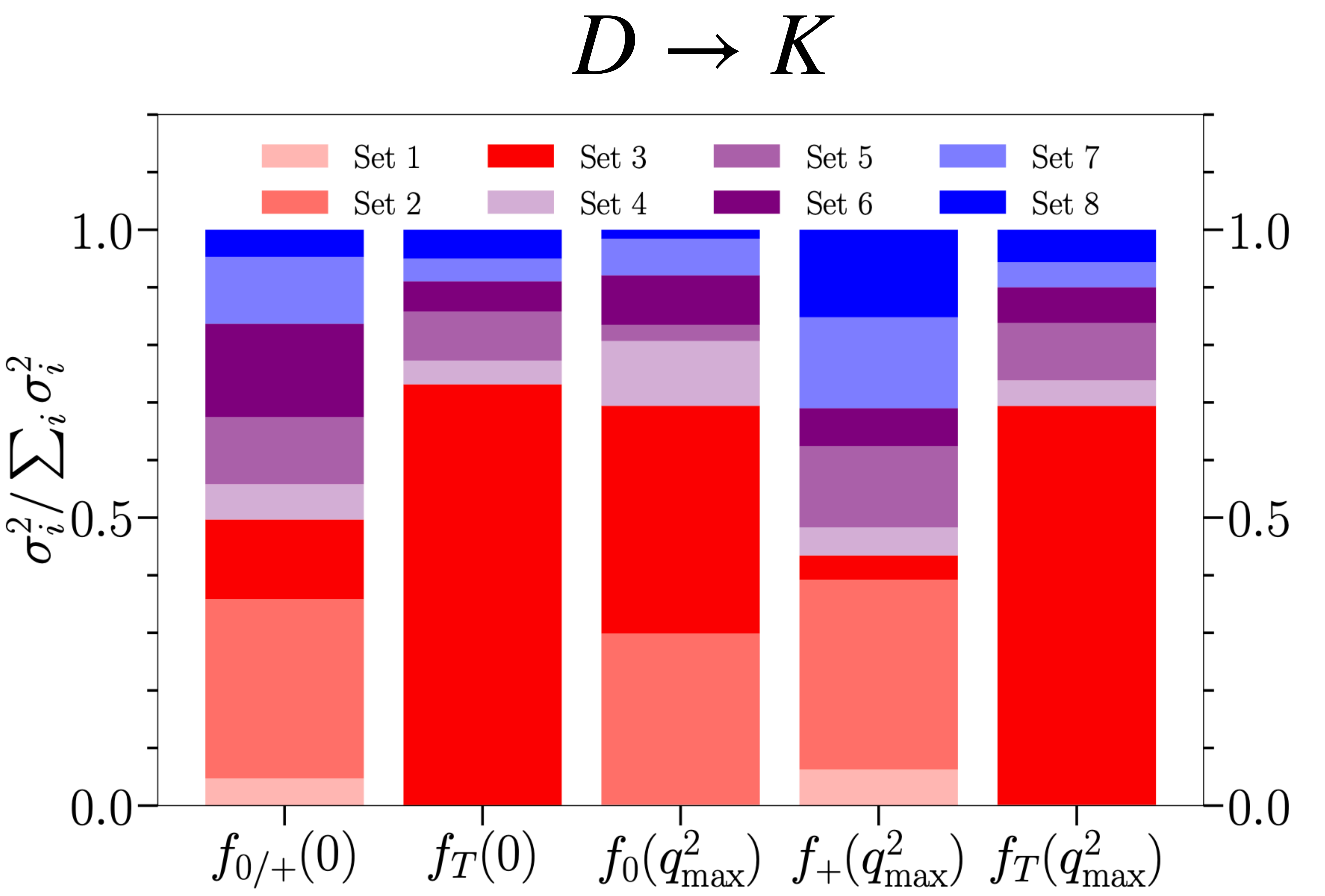
FIG. 10. The form factors at q^2_{max} and $q^2 = 0$ evaluated across the range of physical heavy masses from the D to the B. Other lattice studies [25, 28, 68, 69] of both $D \to K$ and $B \to K$ are shown for comparison. We also include some $B \to K$ results at $q^2 = 0$ from Gubernari et al. [70], a calculation using light cone sum rules. We do not include HPQCD's $D \to K$ results that share data with our calculation here [36]; see text for a discussion of that comparison. At the B end, data points are offset from M_B for clarity. Note that we have run Z_T to scale μ in this plot, where μ is defined linearly between 2 GeV and $m_b = 4.8$ GeV, according to Equation (26). The full running to 2 GeV from m_b results in a factor of 1.0773(17), applied to $f_T^{D\to K}$.

Form Factor calculation: error budget vs. q^2



Form Factor calculation: error budget by ensemble



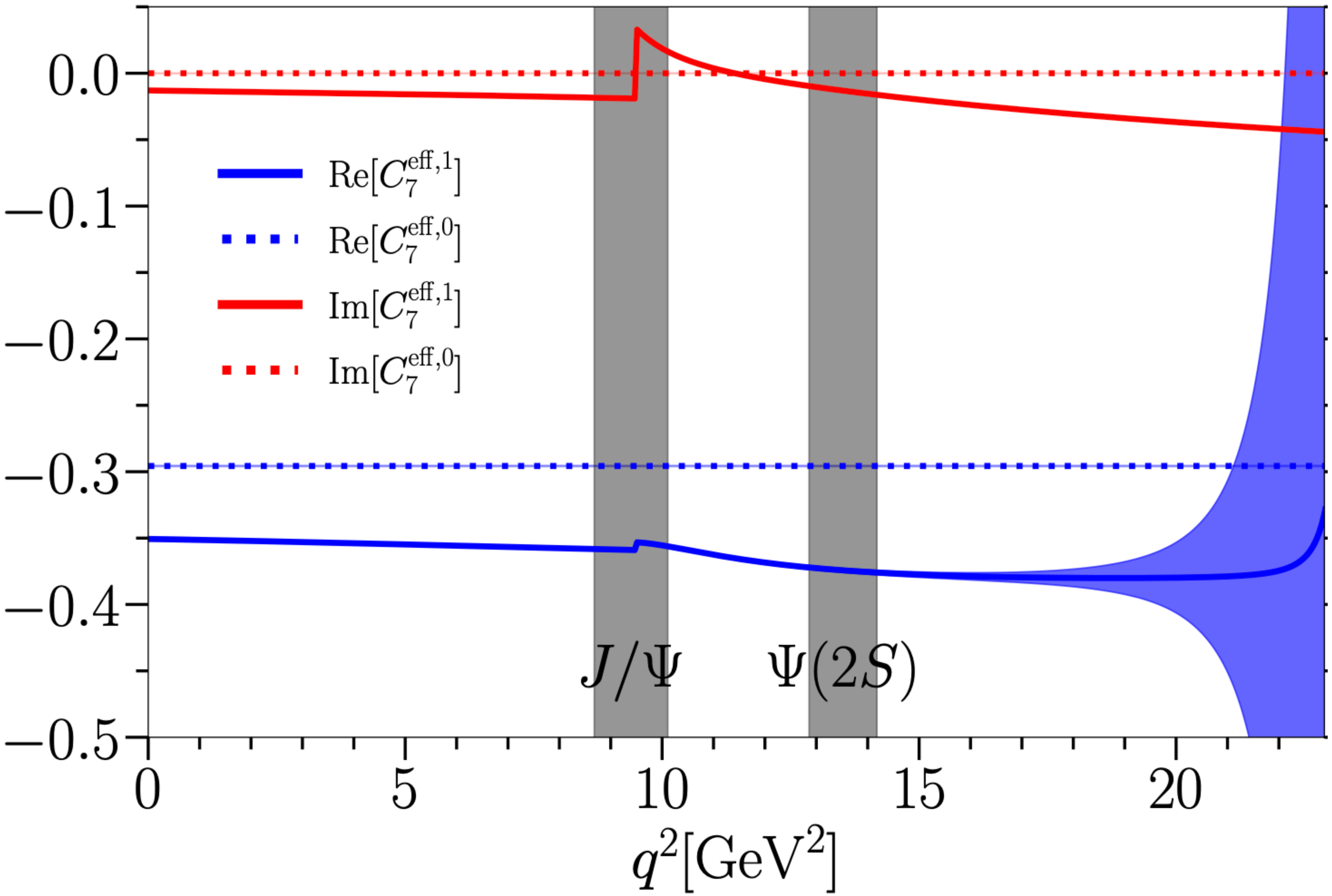


- Blue are lattices with finest lattice spacing, needed to reach m_b
- Red are lattices with physical light quark mass

Phenomenology: inputs

Parameter	Value	Reference
$\eta_{\mathrm{EW}} G_F$	$1.1745(23) \times 10^{-5}\,\mathrm{GeV}^{-2}$	[45], Eq. (7)
$m_c^{\overline{\mathrm{MS}}}(m_c^{\overline{\mathrm{MS}}})$	1.2719(78) GeV	See caption
$m_b^{\overline{\mathrm{MS}}}(\mu_b)$	4.209(21) GeV	[46]
m_c	1.68(20) GeV	-
m_b	4.87(20) GeV	-
f_{K^+}	0.1557(3) GeV	[47–50]
f_{B^+}	0.1894(14) GeV	[51]
τ_{B^0}	1.519(4) ps	[52]
$\tau_{B^\pm}$	1.638(4) ps	[52]
$1/\alpha_{\mathrm{EW}}(M_Z)$	127.952(9)	[45]
$\sin^2\theta_W$	0.23124(4)	[45]
$\lvert V_{tb}V_{ts}^*\rvert$	0.04185(93)	[53]
$C_1(\mu_b)$	-0.294(9)	[54]
$C_2(\mu_b)$	1.017(1)	[54]
$C_3(\mu_b)$	-0.0059(2)	[54]
$C_4(\mu_b)$	-0.087(1)	[54]
$C_5(\mu_b)$	0.0004	[54]
$C_6(\mu_b)$	0.0011(1)	[54]
$C_7^{\mathrm{eff},0}(\mu_b)$	-0.2957(5)	[54]
$C_8^{\mathrm{eff}}(\mu_b)$	-0.1630(6)	[54]
$C_9(\mu_b)$	4.114(14)	[54]
$C_9^{\mathrm{eff},0}(\mu_b)$	$C_9(\mu_b) + Y(q^2)$	-
$C_{10}(\mu_b)$	-4.193(33)	[54]

Phenomenology: $B \to K\ell^+\ell^-$ corrections

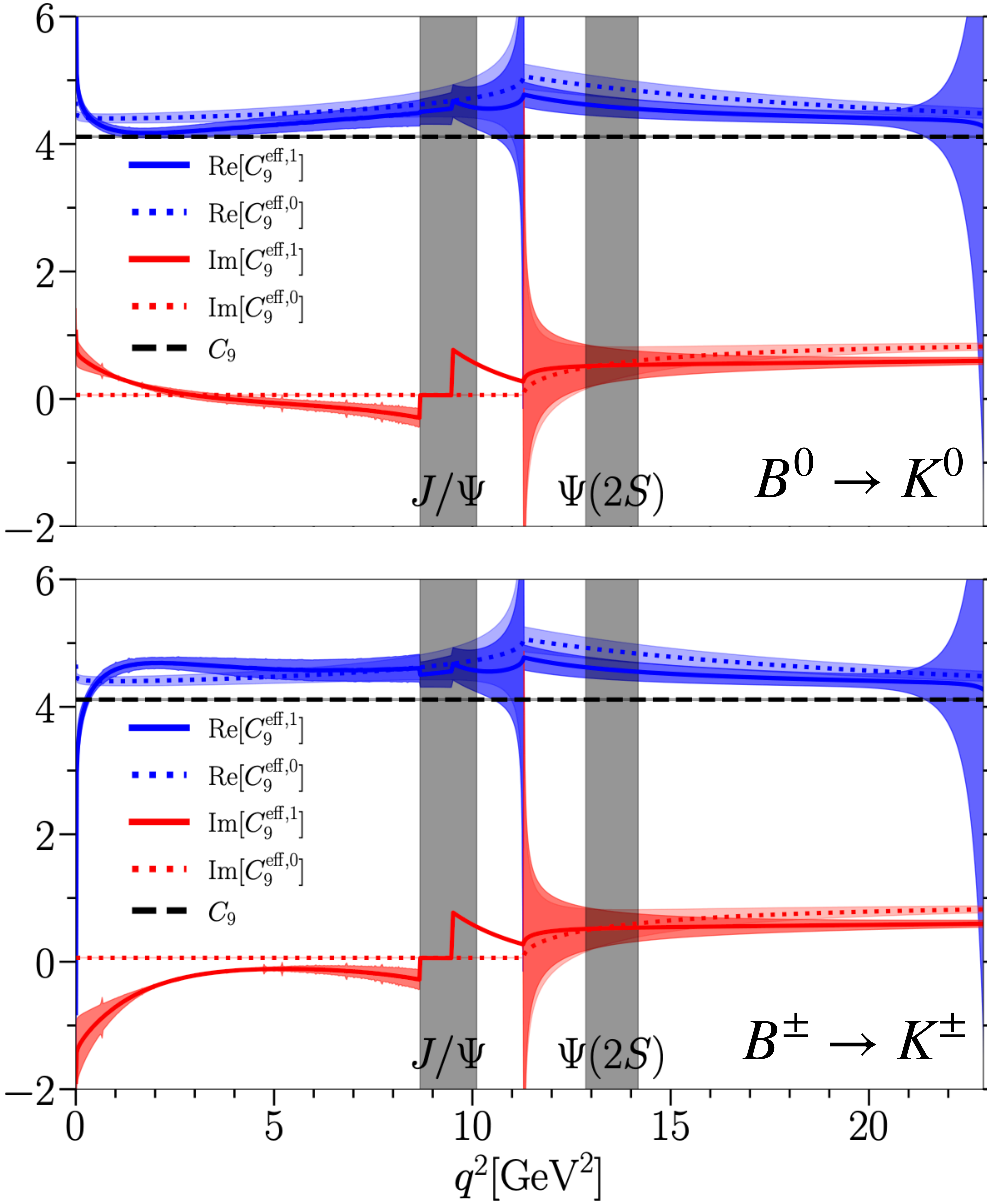


uncorrected C_7^{eff}

corrected C_7^{eff}

Corrections to C_7^{eff} are $\mathcal{O}(\alpha_s)$ perturbative QCD effects for all q^2

Phenomenology: $B \to K\ell^+\ell^-$ corrections



uncorrected C_9^{eff}

corrected C_9^{eff}

corrections to C_9^{eff} include:

- $\mathcal{O}(\alpha_s)$ perturbative QCD effects for all q^2
- non-factorizable corrections at low q^2

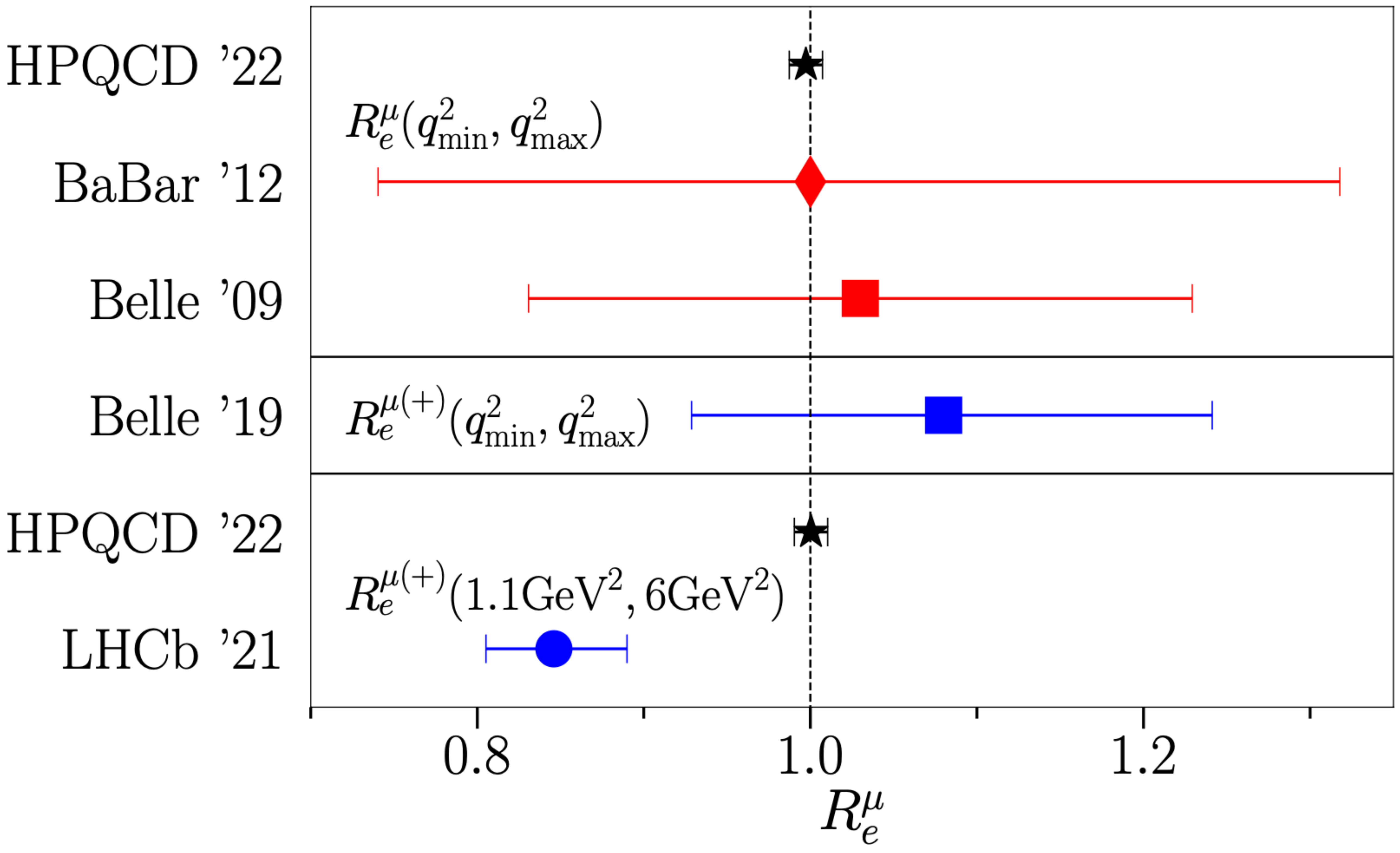
Beneke, Feldmann, Seidel, NPB 612, 25-58 (2001)

Phenomenology: R_K

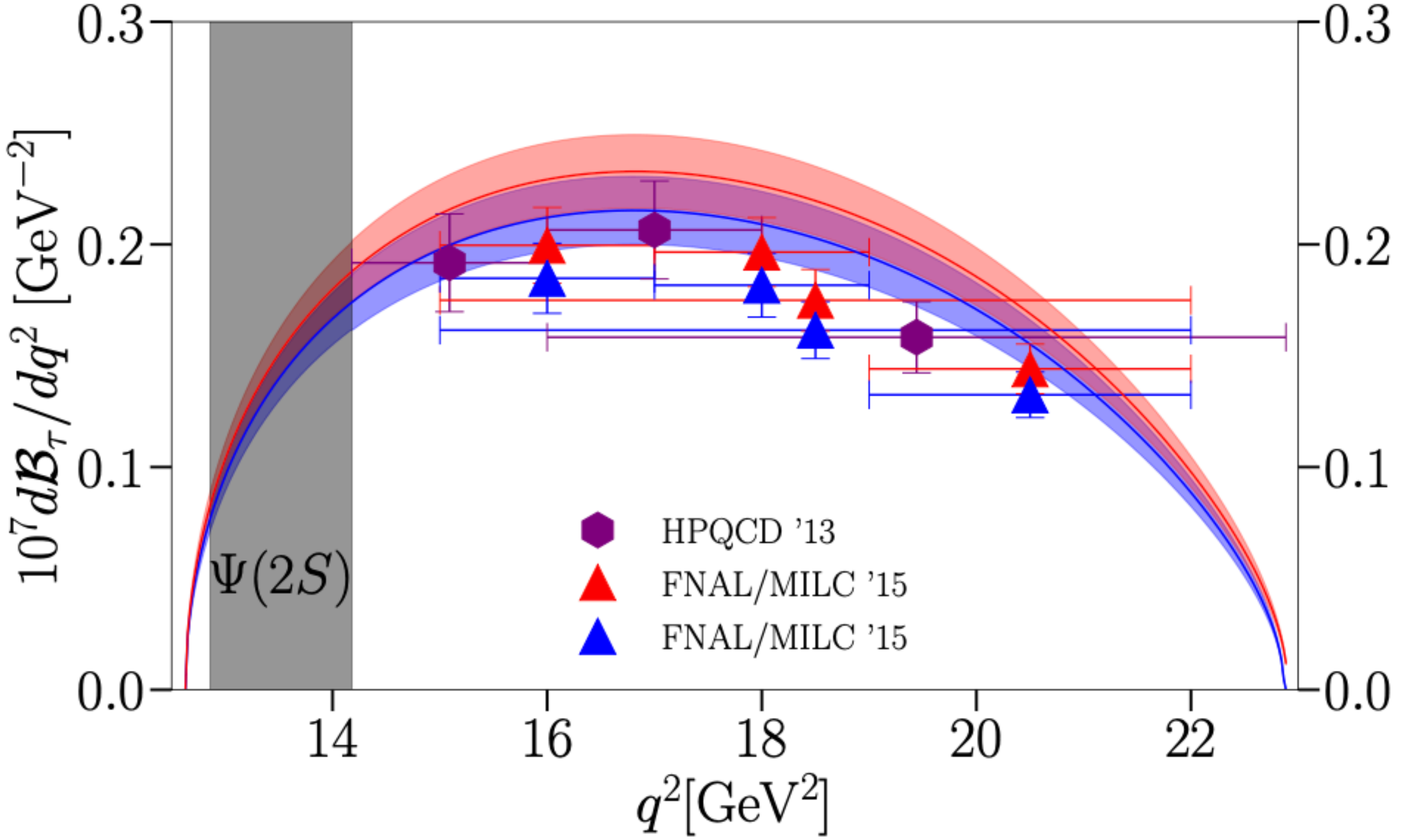
- Ratio of differential branching fractions

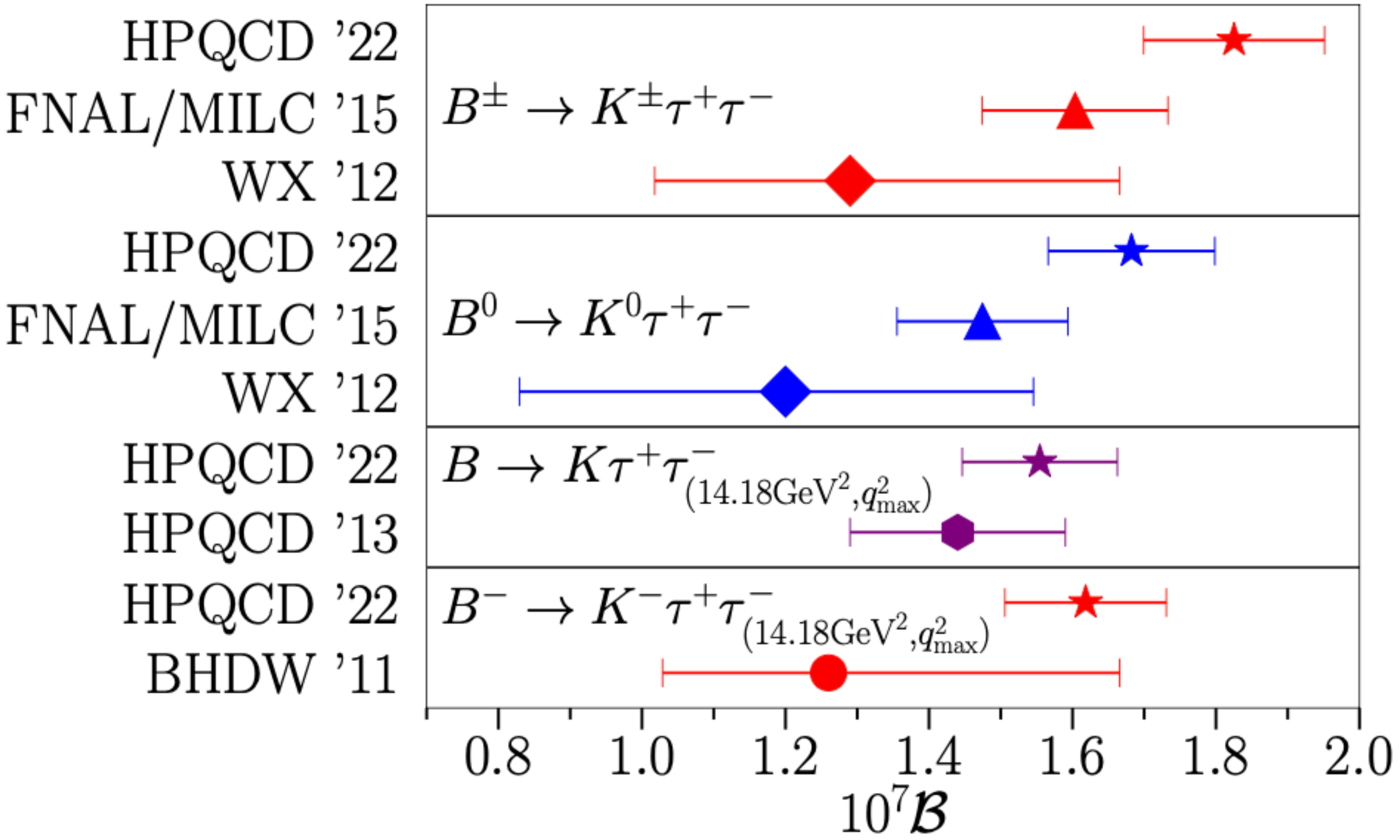
$$R^{\ell_1}_{\ell_2}(q^2_{\text{low}}, q^2_{\text{upp}}) = \frac{\int_{q^2_{\text{low}}}^{q^2_{\text{upp}}} \frac{d\mathscr{B}_{\ell_1}}{dq^2} dq^2}{\int_{q^2_{\text{low}}}^{q^2_{\text{upp}}} \frac{d\mathscr{B}_{\ell_2}}{dq^2} dq^2}$$

- Hadronic uncertainties largely cancel
- LHCb '21 is 3.1σ from SM



Phenomenology: $B \to K\tau^+\tau^-$





- improved precision over previous predictions