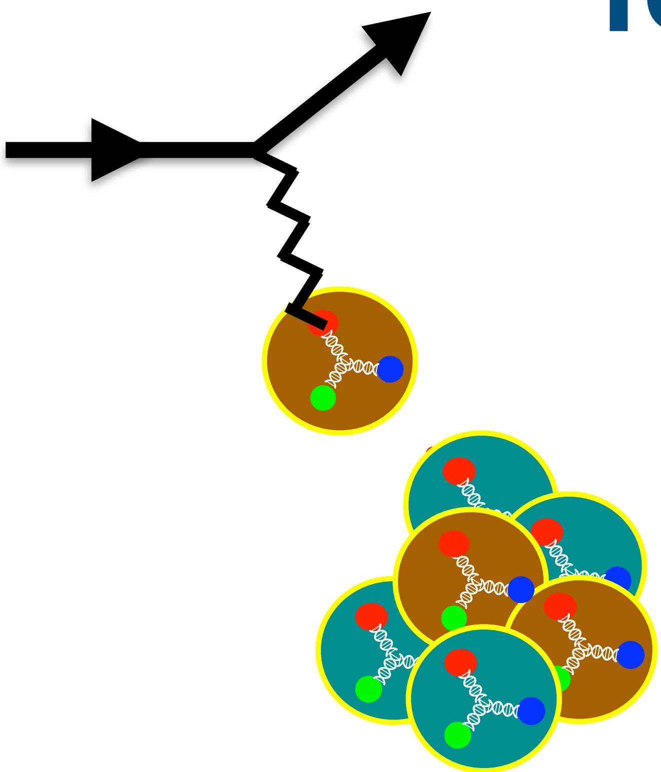
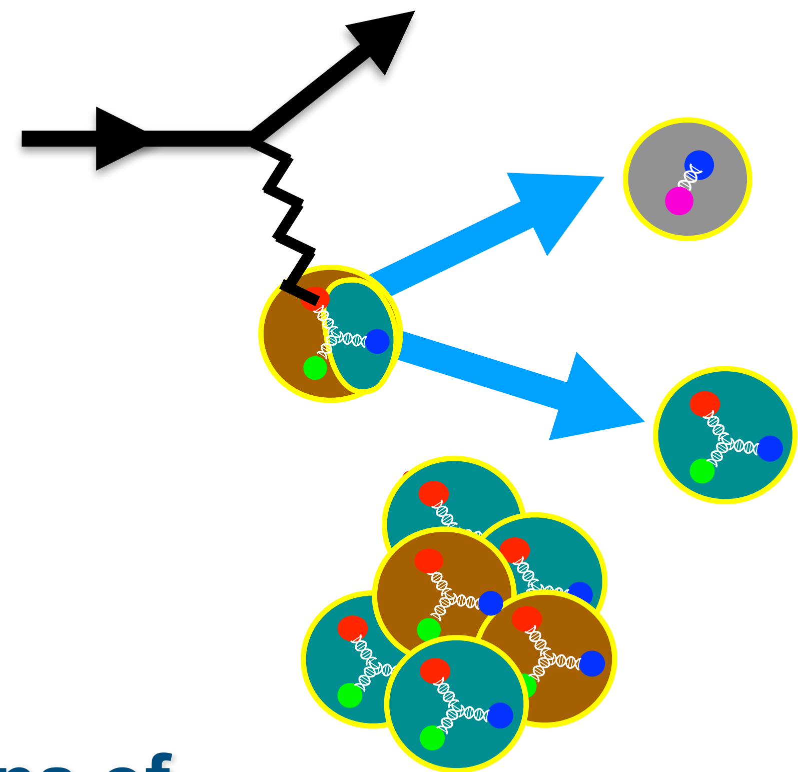
Precise lattice calculations for neutrino scattering





Michael Wagman

14th Conference on the Intersections of Particle and Nuclear Physics (CIPANP 2022)

Fermilab

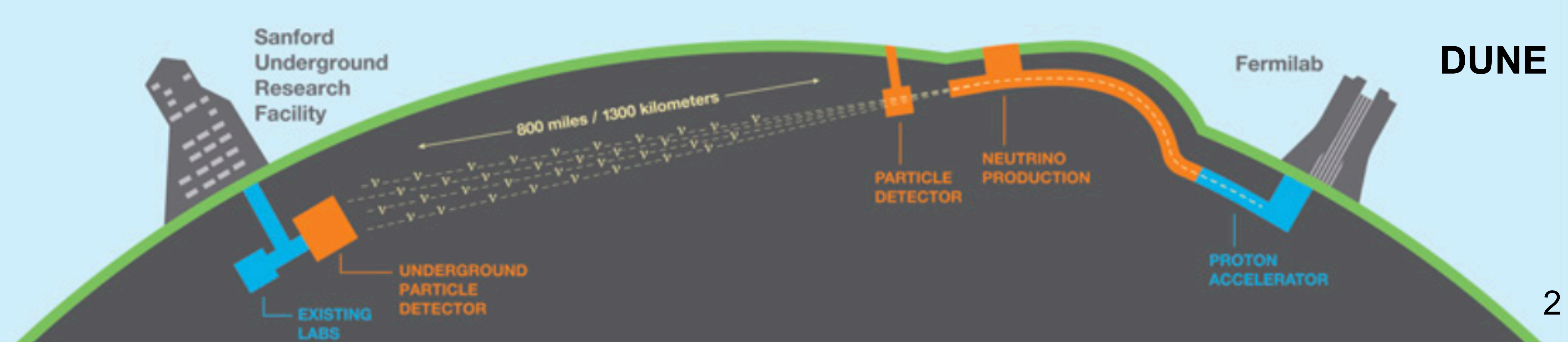
September 1, 2022

Neutrinos and new physics

Neutrino oscillations demonstrate that neutrinos have mass, but the fundamental interactions giving rise to neutrino masses are not well understood

- *CP* violation?
- Dirac or Majorana?
- Sterile neutrinos?
- Mass ordering?

Next-generation neutrino experiments will measure oscillation parameters with unprecedented accuracy to shed light on neutrino masses



DUNE

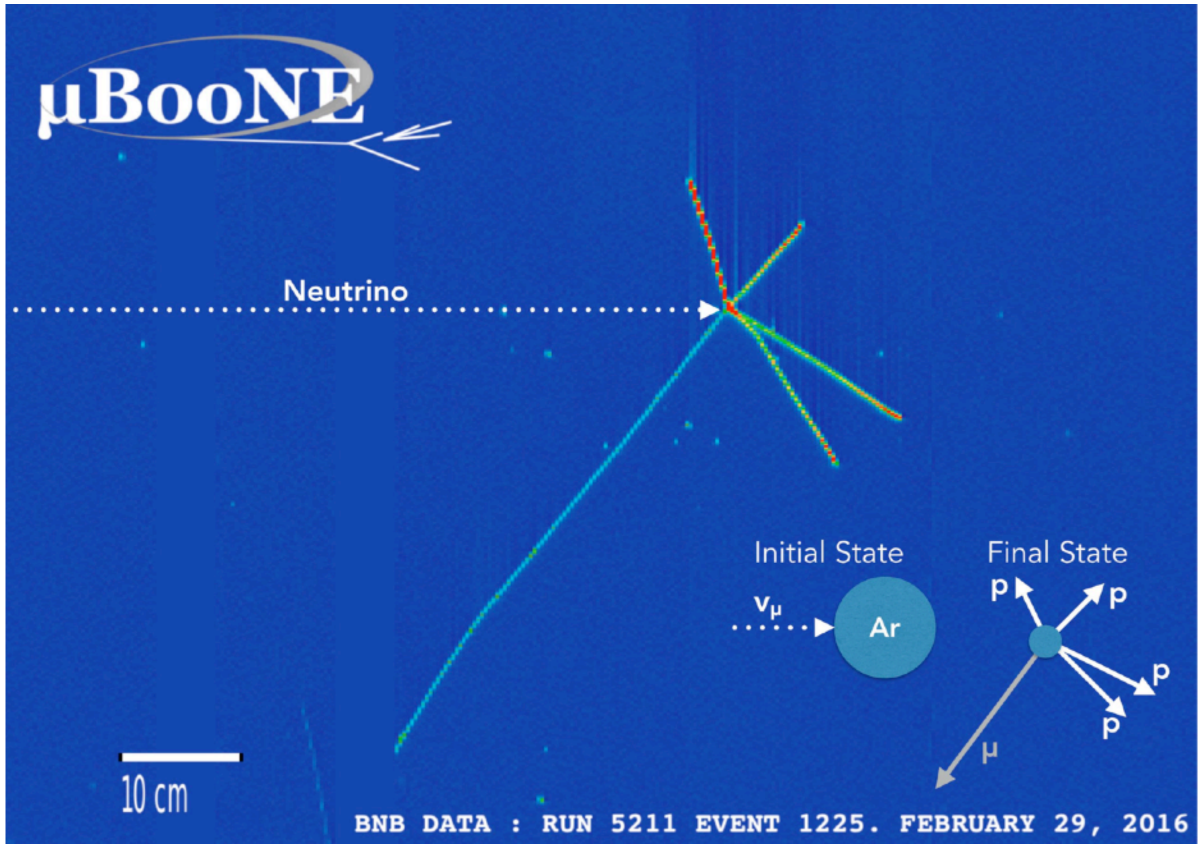
Quarks, neutrinos, and new physics

DUNE and other accelerator neutrino experiments use nuclear targets

Inferring incident neutrino energies from measured final state event rates requires theoretical interaction model

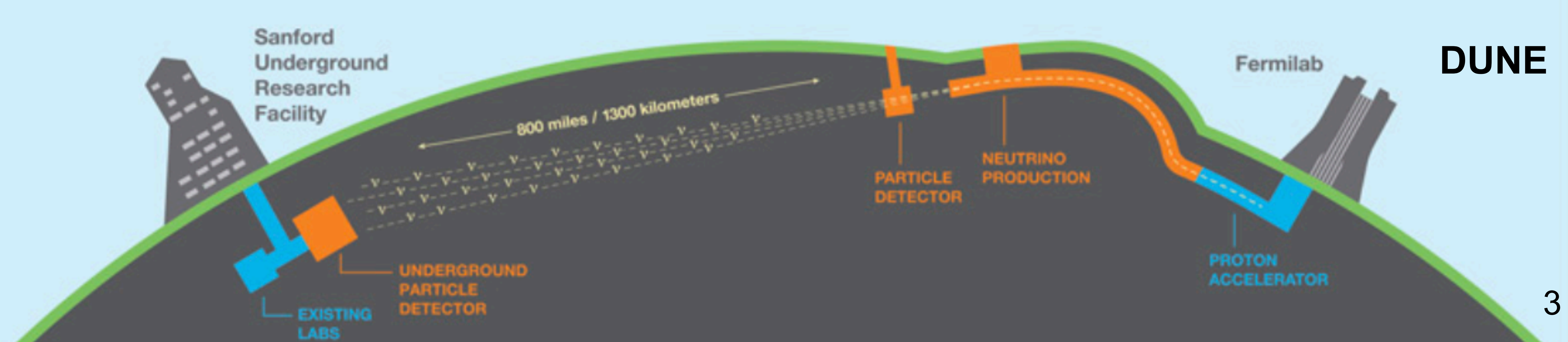
- Near detector tuning is essential but theory still needed to extrapolate to far detector kinematics, find BSM physics, …

See talk by Coyle



Maximizing the discovery potential of next-generation neutrino experiments will require a coordinated theory effort:

High-energy theory, nuclear many-body theory, lattice QCD, event generators, …



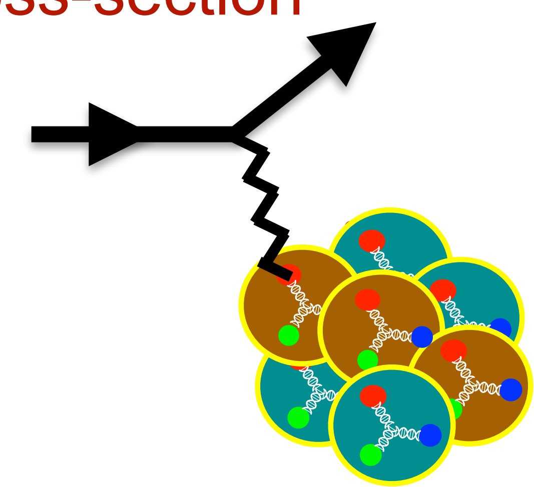
Precision neutrino physics

Relating measured final-state event rates to neutrino fluxes entering oscillation analyses requires knowledge of νA cross-section

Near-detector neutrino flux and acceptance

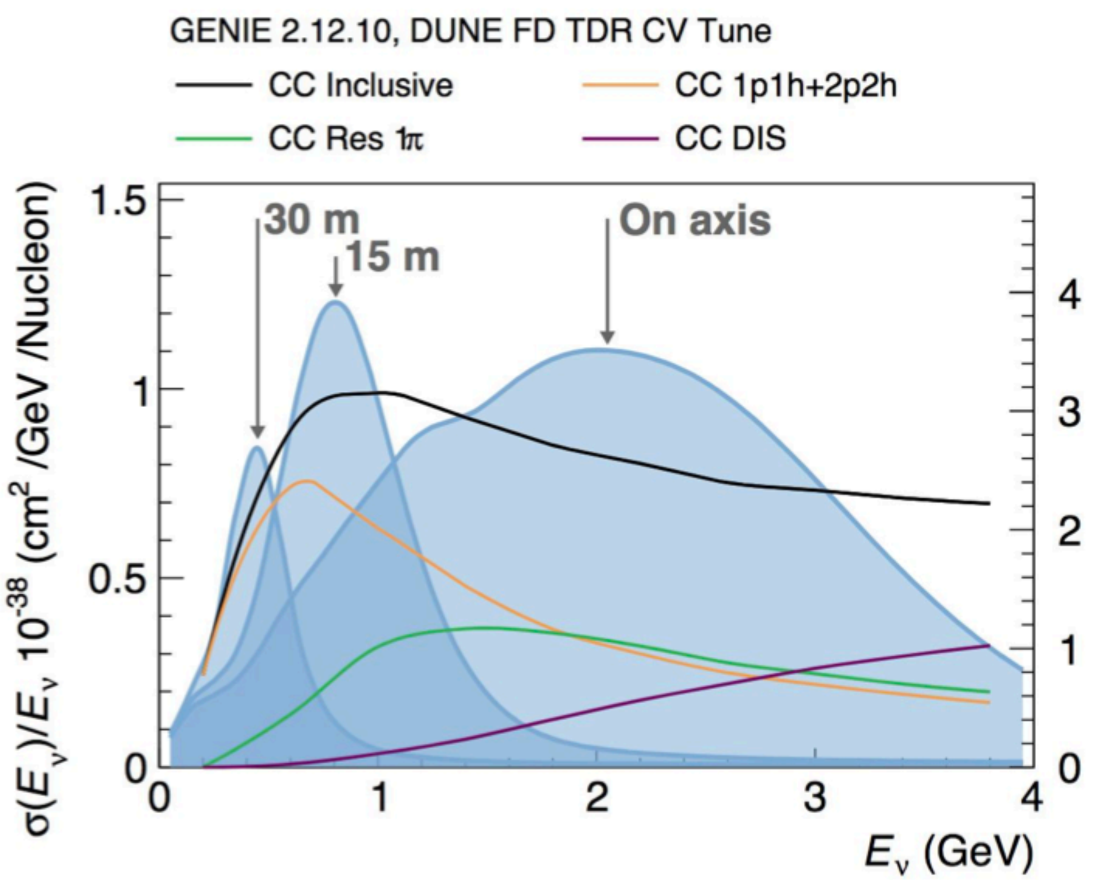
$$\frac{N_{\rm near}}{N_{\rm far}} = \frac{\int dE_\nu \Phi_{\rm near}(E_\nu)\sigma(E_\nu)}{\int dE_\nu \Phi_{\rm far}(E_\nu)\sigma(E_\nu)}$$

Cross-section



Experimentally measured event rates

Far-detector flux (depends on oscillation parameters)



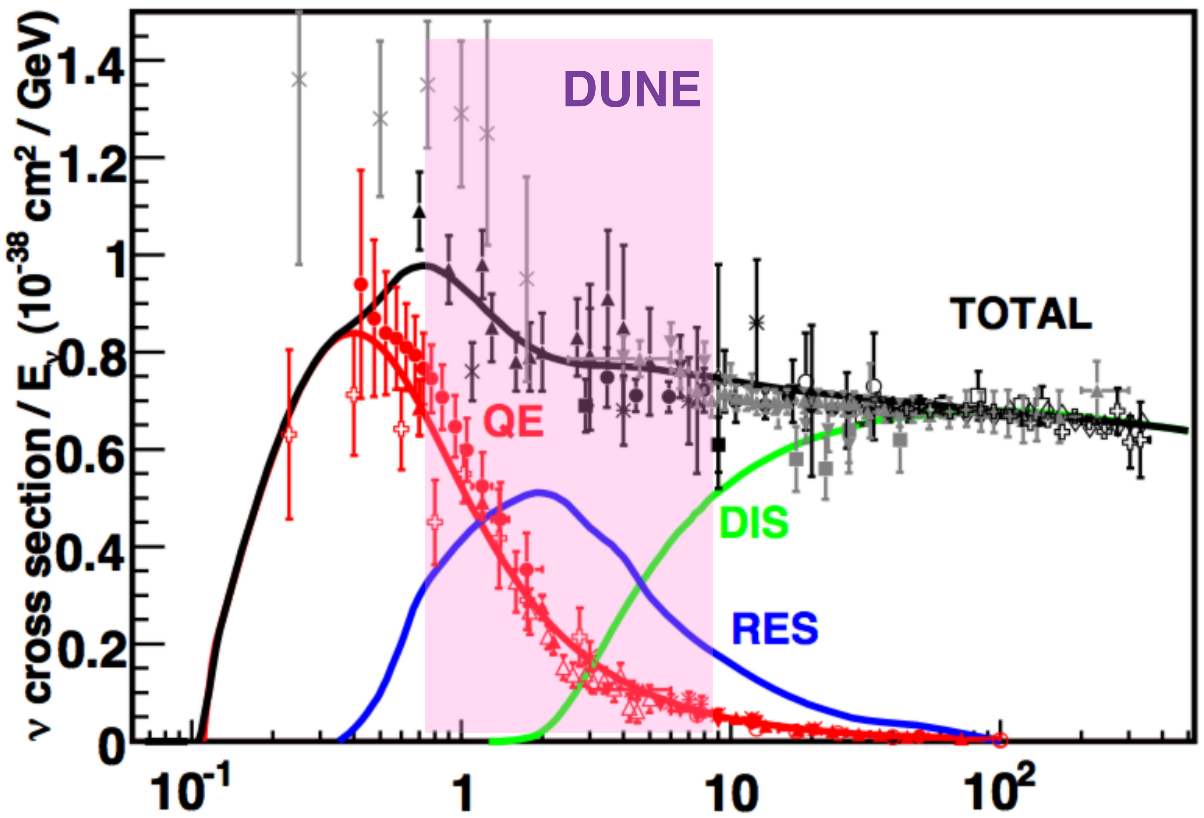
DUNE near detector CDR, arXiv:2013.13910

The DUNE near detector and especially PRISM design will provide a wealth of data for informing cross-section models

Theory input and models will also be essential:

- **BSM searches** require Standard Model predictions or else new physics will be absorbed into near detector x-sec tunes
- Neutrino energies are not directly measured, **energy reconstruction** requires a theoretical cross-section model

Neutrino-nucleus scattering



Formaggio, Zeller, Rev. Mod. Phys. 84 (2012)

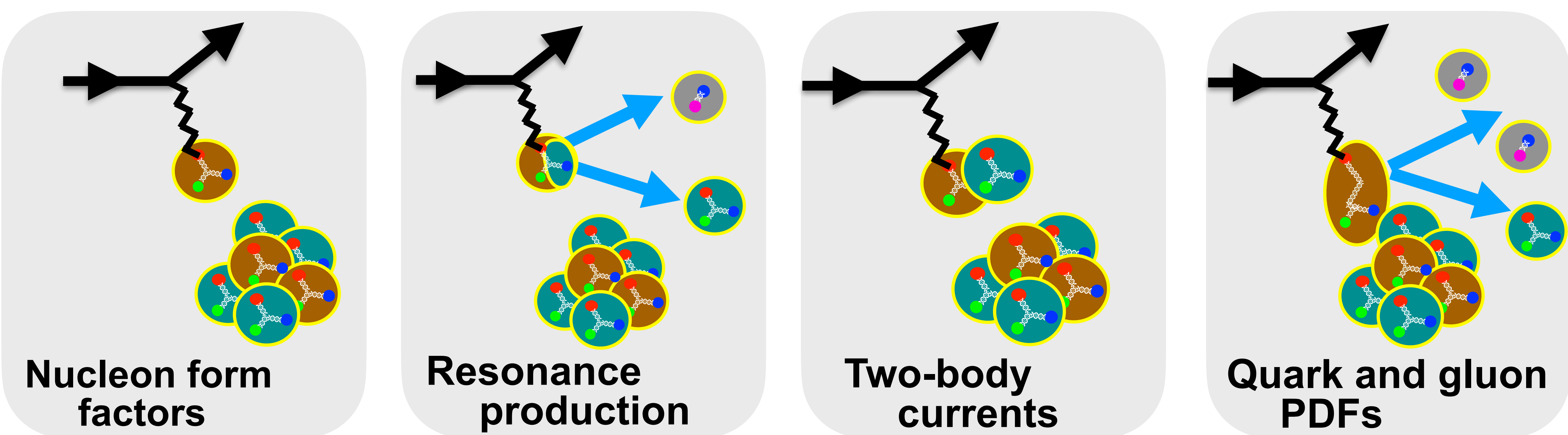
Accelerator neutrino fluxes cover a wide range of energies where different processes dominate cross-section:

- Quasi-elastic nucleon scattering
- Resonance production
- Deep inelastic scattering

Theory input required to decompose cross section into such processes and therefore predict its energy dependence

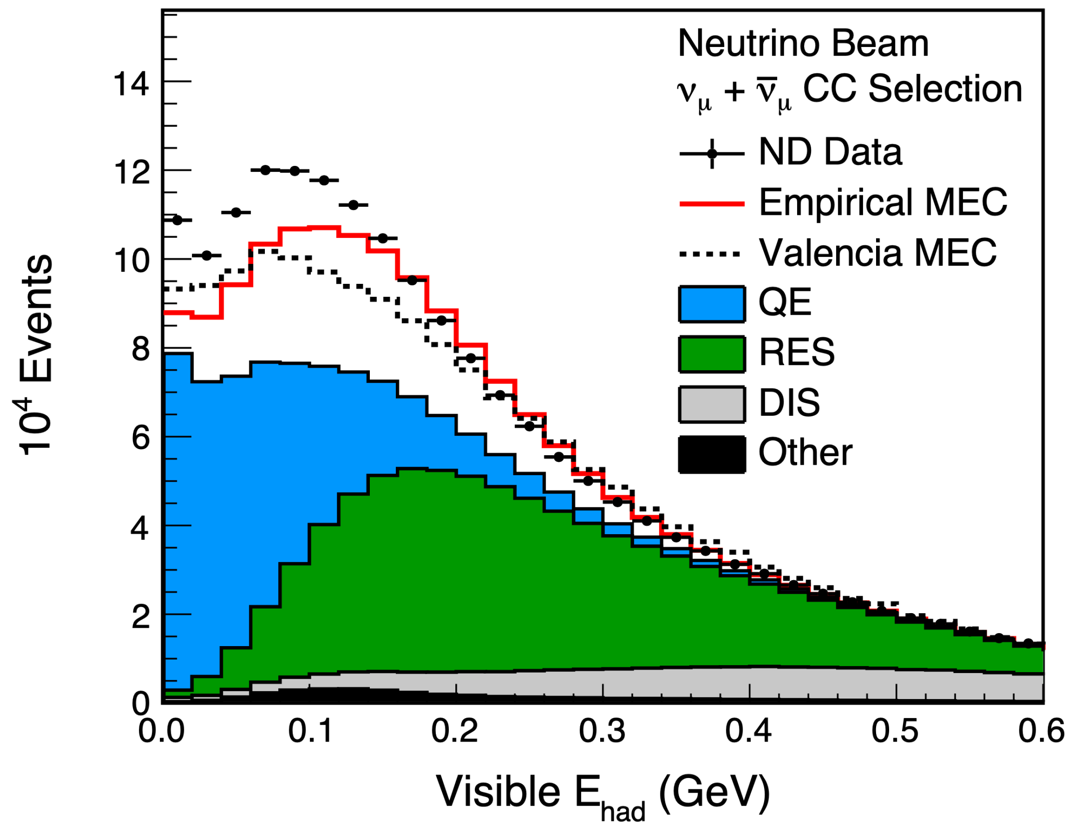
See talks by Lin, Martinez-Casales, Pandey, Tomalak, Vohra, …

Effective theories for different energies require different inputs



Predicting νA cross-sections

Acero et al [NOvA] Eur. Phys. J. C 80 (2020)



Predictions for experimentally relevant nuclei made using **event generators** combining models for different reaction mechanisms

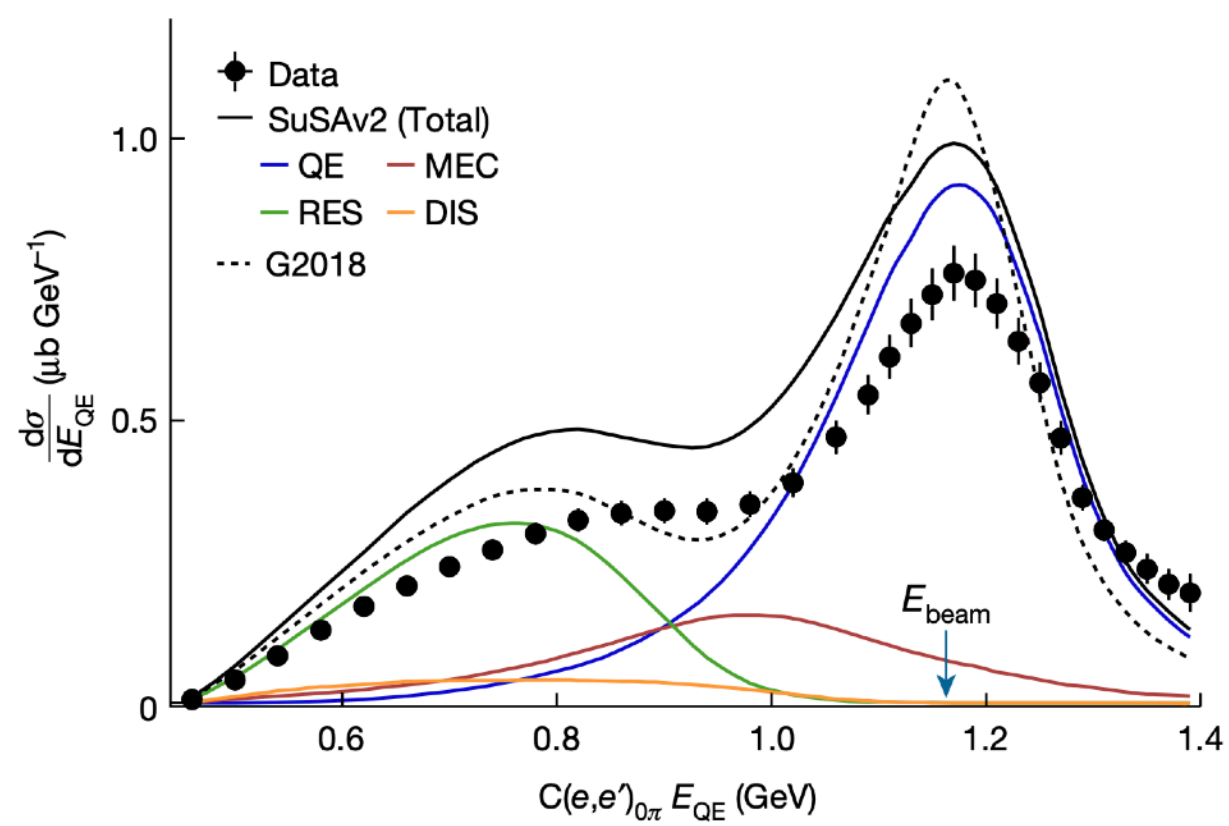
Snowmass WP: Campell et al, arXiv:2203.11110

Discrepancies between generators and data often corrected by tuning an empirical model of the least well known mechanism: MEC ("meson exchange"/two-body currents)

Neutrino event generators can be validated by comparing predictions electron scattering with precise data

See talk by Barrow

Khachatryan et al [Clas and e4v] Nature 599 (2021)

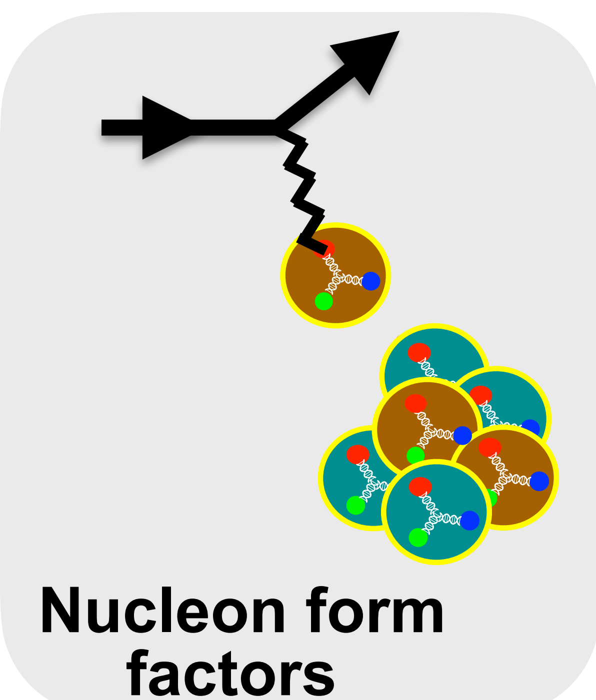


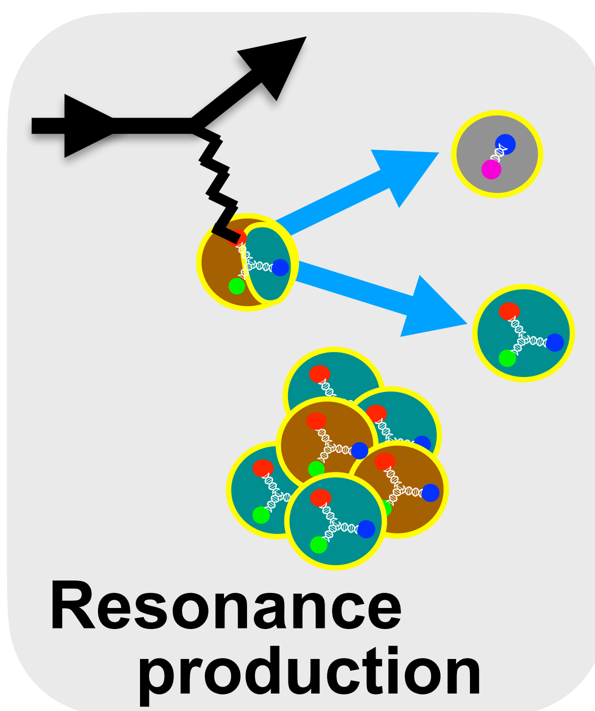
- Significant discrepancies presently visible
- **Tuning to reproduce one process does not mean other processes/energies will be accurately predicted**
- Contributions from different reaction mechanisms must be isolated

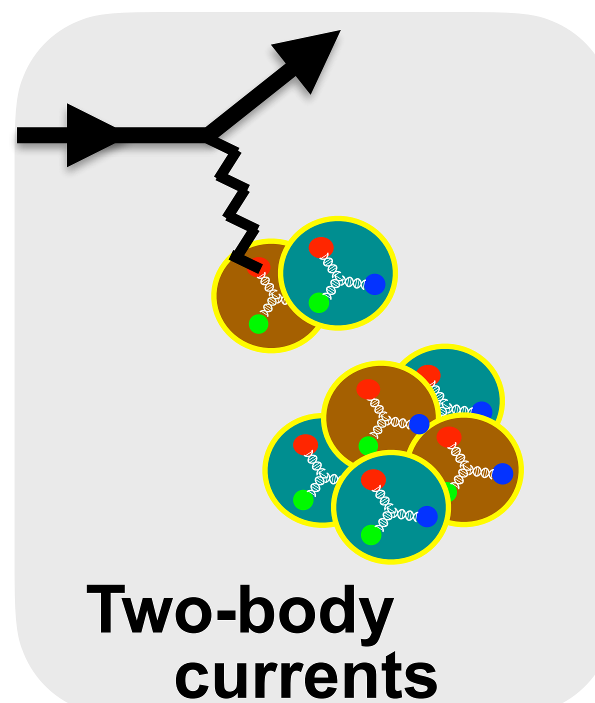
Reaction mechanisms

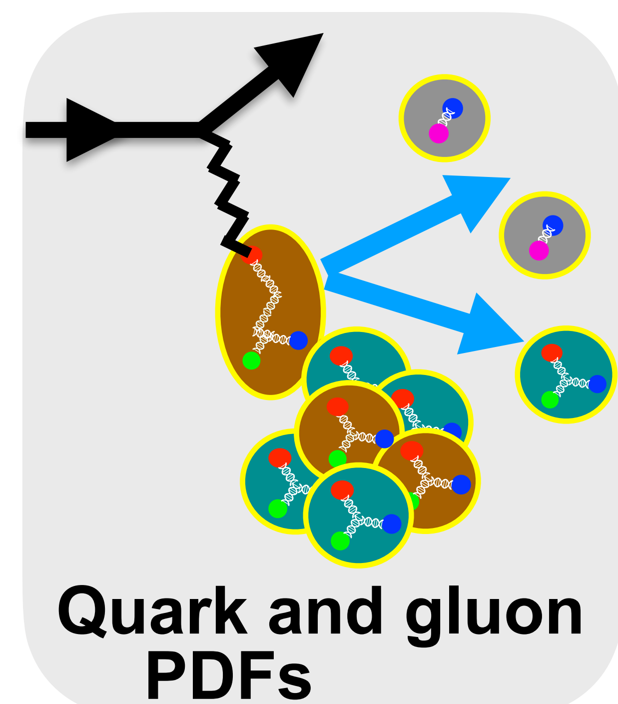
Process	Neutrino Energy Range	Example Final State
Coherent Elastic Scattering	$\lesssim 50$ MeV	$\nu + A$
Inelastic Scattering	$\lesssim 100$ MeV	$e + {}^{\mathrm{A}}(Z+1)^{*}(\rightarrow {}^{\mathrm{A}}(Z+1) + n\gamma)$
Quasi-Elastic Scattering	100 MeV–1 GeV	$l + p + X$
Two-Nucleon Emission	1 GeV	$l + 2N + X$
Resonance Production	1–3 GeV	$l + \Delta(\rightarrow N + \pi) + X$
Shallow Inelastic Scattering	3–5 GeV	$l + n\pi + X$
Deep Inelastic Scattering	$\gtrsim 5$ GeV	$l + n\pi + X$

Snowmass WP: “Theoretical Tools for Neutrino Scattering” arXiv:2203.09030









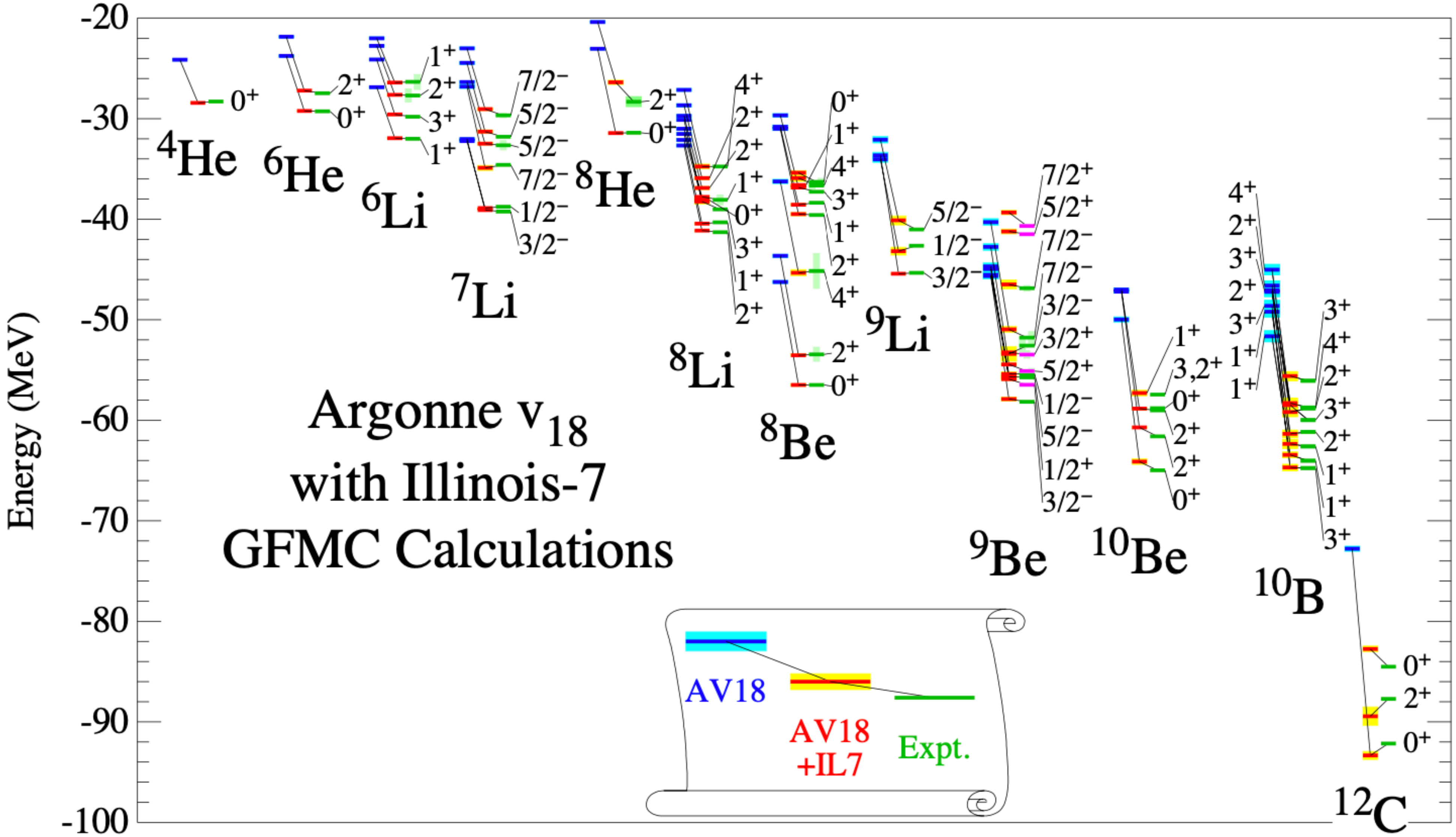
Nuclear effective theories

Nuclear effective theories can accurately describe light nuclei up to $^{12}\mathrm{C}$ with Quantum Monte Carlo many-body methods and microscopic interactions:

- Phenomenological potentials such as AV18 + Illinois-7
- Chiral EFT with Weinberg power counting scheme

Phenomenologically successful despite renormalization challenges — see talks by Epelbaum, Gandolfi, Hergert, Piarulli, …

Review: van Kolck, Front. in Phys. 8 (2020)

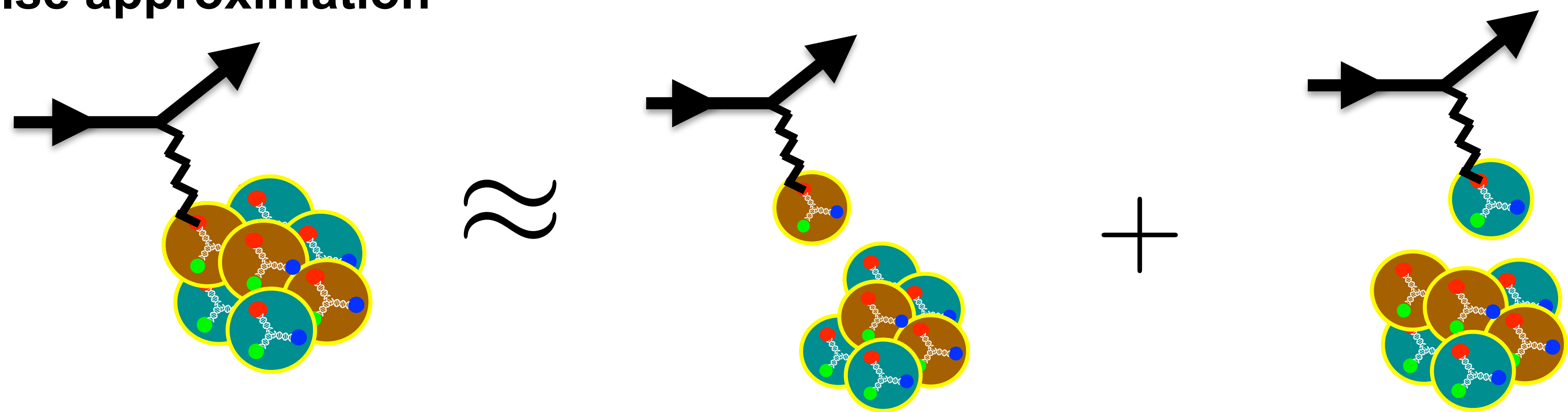


Carlson et al, Rev. Mod. Phys. 87 (2015)

Electroweak currents

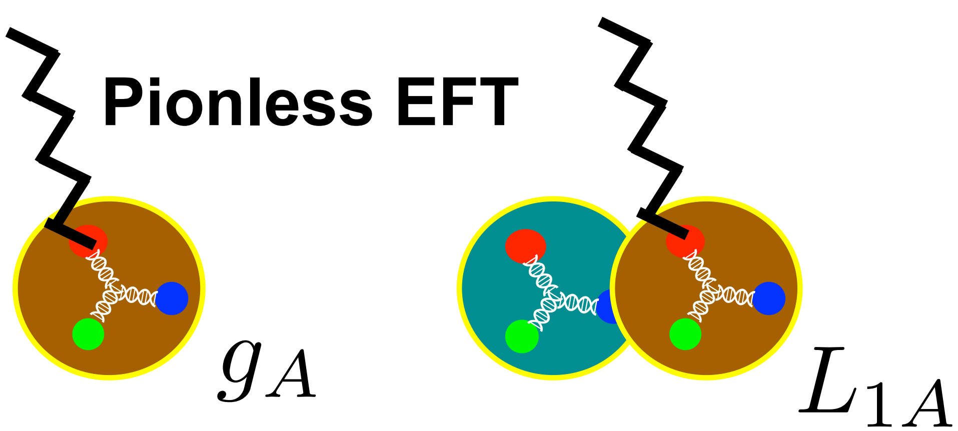
Single-nucleon currents dominate at low energies and can be used to predict nuclear response functions in EFTs of light nuclei, shell models of larger nuclei, …

Impulse approximation

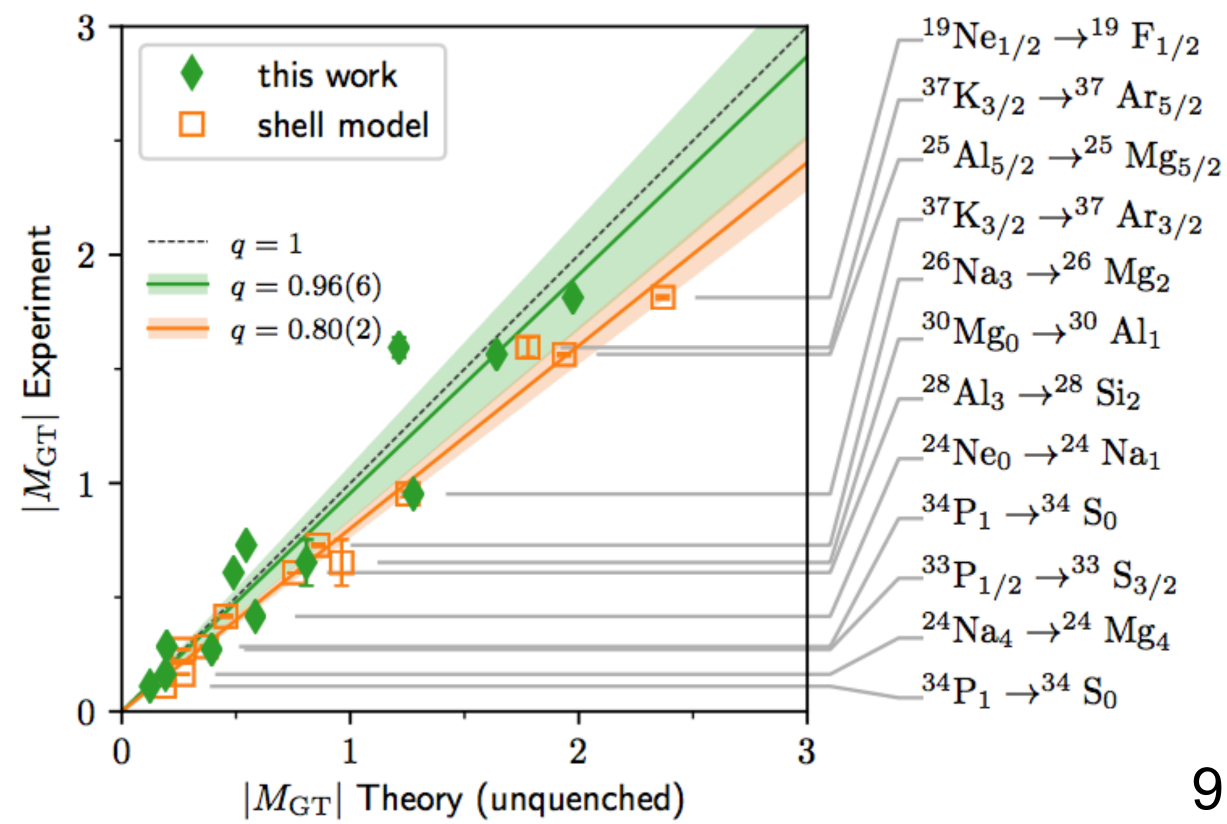


Nucleon vector and axial form factors are key inputs to nuclear EFTs / models

Two-body currents and correlations lead to significant corrections in medium-mass nuclei, 30% decrease in beta-decay rates



Gysbers et al, Nature Phys. 15 (2019)



Lattice QCD, EFT, and νA

LQCD can provide results for few-nucleon observables that can be matched to nuclear EFTs and models that can make predictions for larger nuclei

LQCD → Nuclear EFT and many-body → Event generators

Results provided by LQCD and experiment are complementary

Easy for LQCD:

- Axial vs vector currents
- Isovector vs isoscalar
- Pions

Hard for LQCD:

- Large baryon number
- Real-time dynamics
- Multi-hadron states
- (Light quark masses)

Lattice QCD and νA

νA scattering amplitudes factorize into leptonic and hadronic parts

$$\mathcal{M}_{\nu A \to \ell f} \propto (\overline{u}_\ell \gamma_\mu \gamma_5 u_\nu) \langle f | \, \overline{q} \gamma_\mu \gamma_5 q \, | A \rangle + \ldots$$

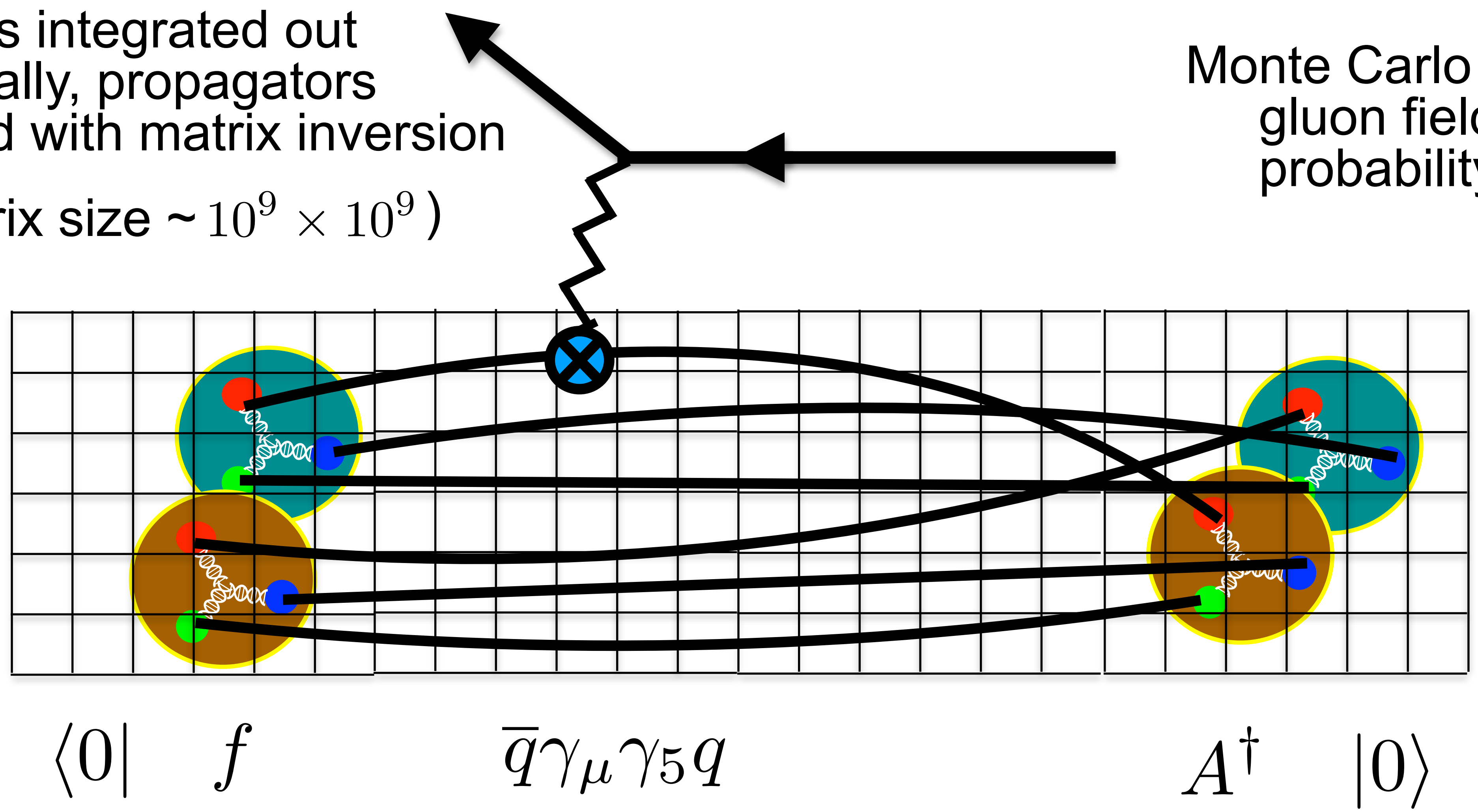
Generic Euclidean hadronic matrix elements calculable (in principle) using lattice QCD

$$\langle \mathcal{O} \rangle = \int \mathcal{D}U \mathcal{D}\overline{q} \mathcal{D}q \; e^{-S_{QCD}(U,q,\overline{q})} \; \mathcal{O}(U,q,\overline{q}) \approx \frac{1}{N_{\text{cfg}}} \sum_{i=1}^{N_{\text{cfg}}} \mathcal{O}(U_i)$$

Quark fields integrated out analytically, propagators obtained with matrix inversion

(Dirac matrix size ~ $10^9 \times 10^9$)

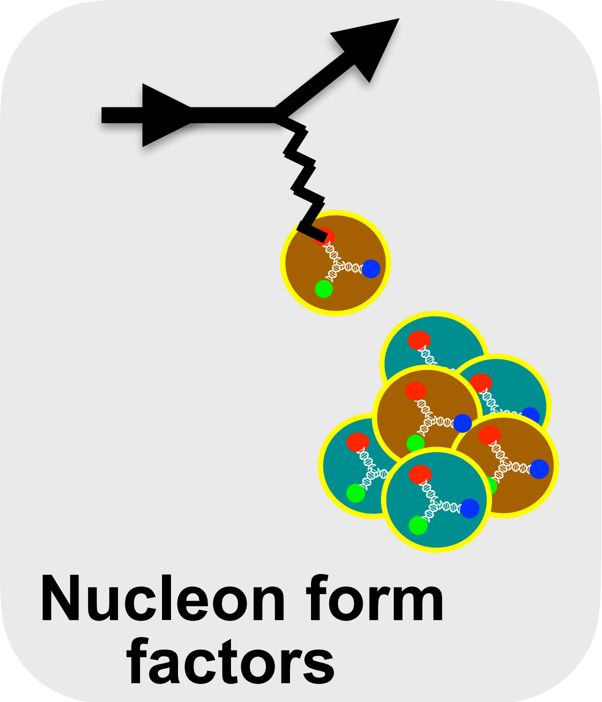
Monte Carlo sample gluon fields with probability $\propto e^{-S}$

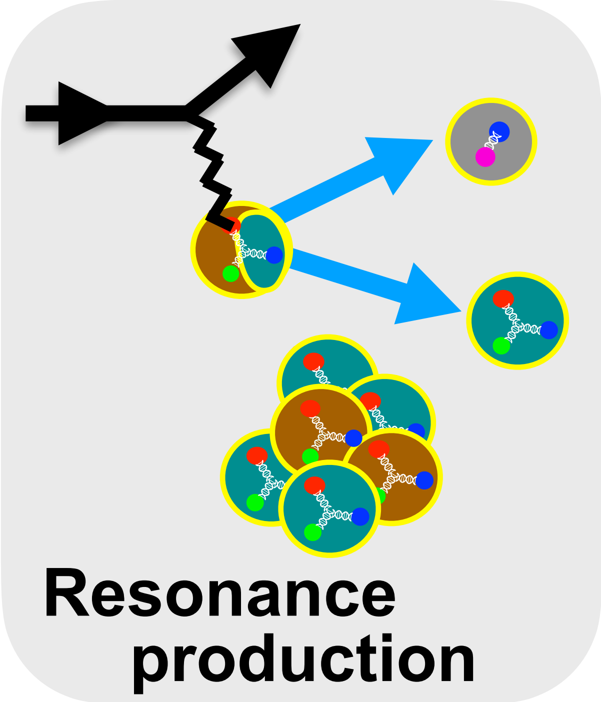


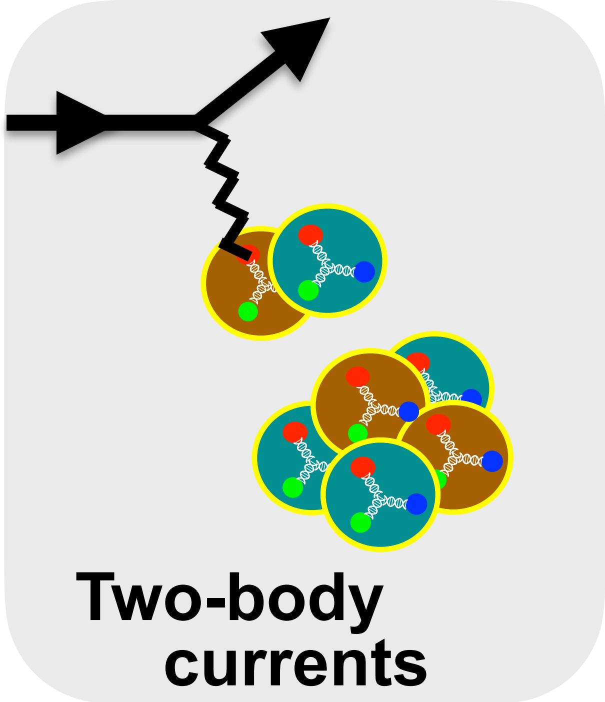
Reaction mechanisms

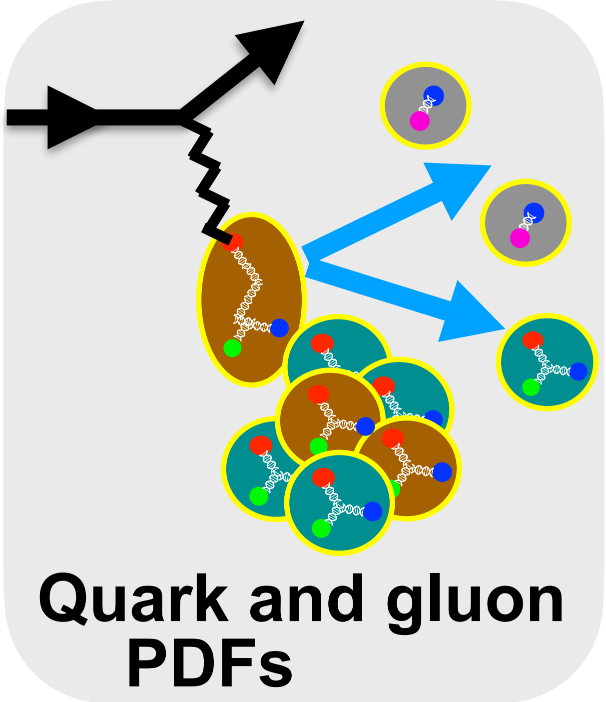
Process	Neutrino Energy Range	Example Final State
Coherent Elastic Scattering	$\lesssim 50$ MeV	$\nu + A$
Inelastic Scattering	$\lesssim 100$ MeV	$e + {}^{A}(Z+1)^{*}(\rightarrow {}^{A}(Z+1) + n\gamma)$
Quasi-Elastic Scattering	100 MeV–1 GeV	$l + p + X$
Two-Nucleon Emission	1 GeV	$l + 2N + X$
Resonance Production	1–3 GeV	$l + \Delta(\rightarrow N + \pi) + X$
Shallow Inelastic Scattering	3–5 GeV	$l + n\pi + X$
Deep Inelastic Scattering	$\gtrsim 5$ GeV	$l + n\pi + X$

Snowmass WP: "Theoretical Tools for Neutrino Scattering" arXiv:2203.09030



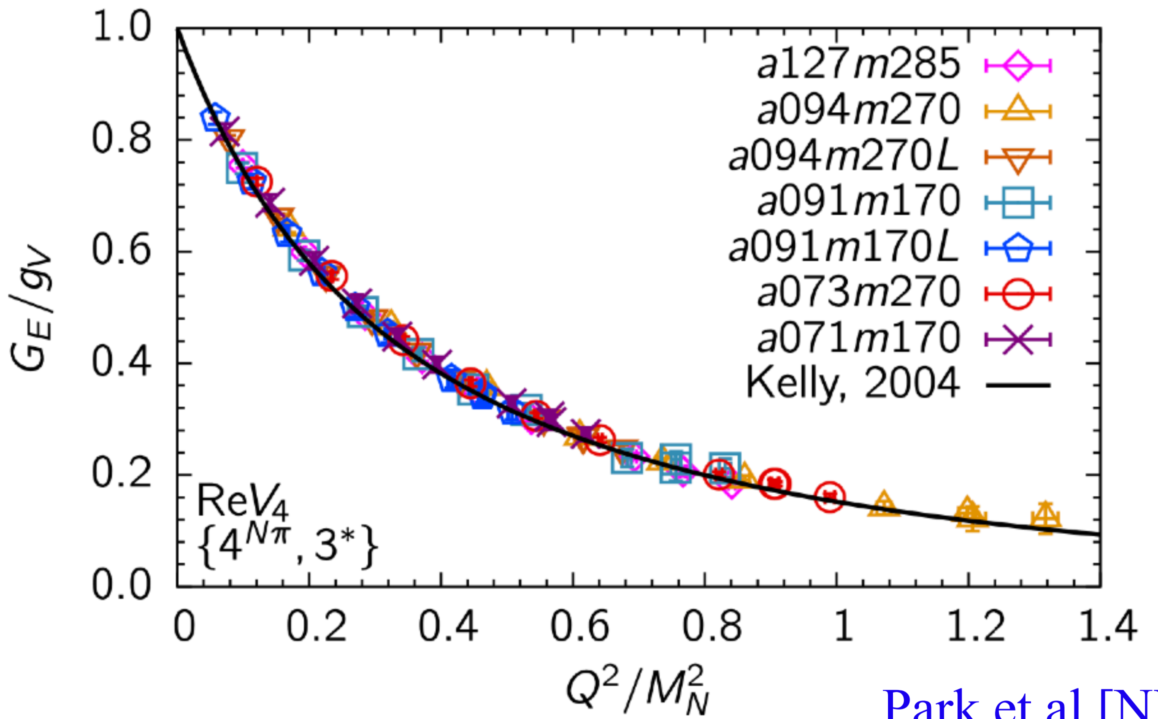


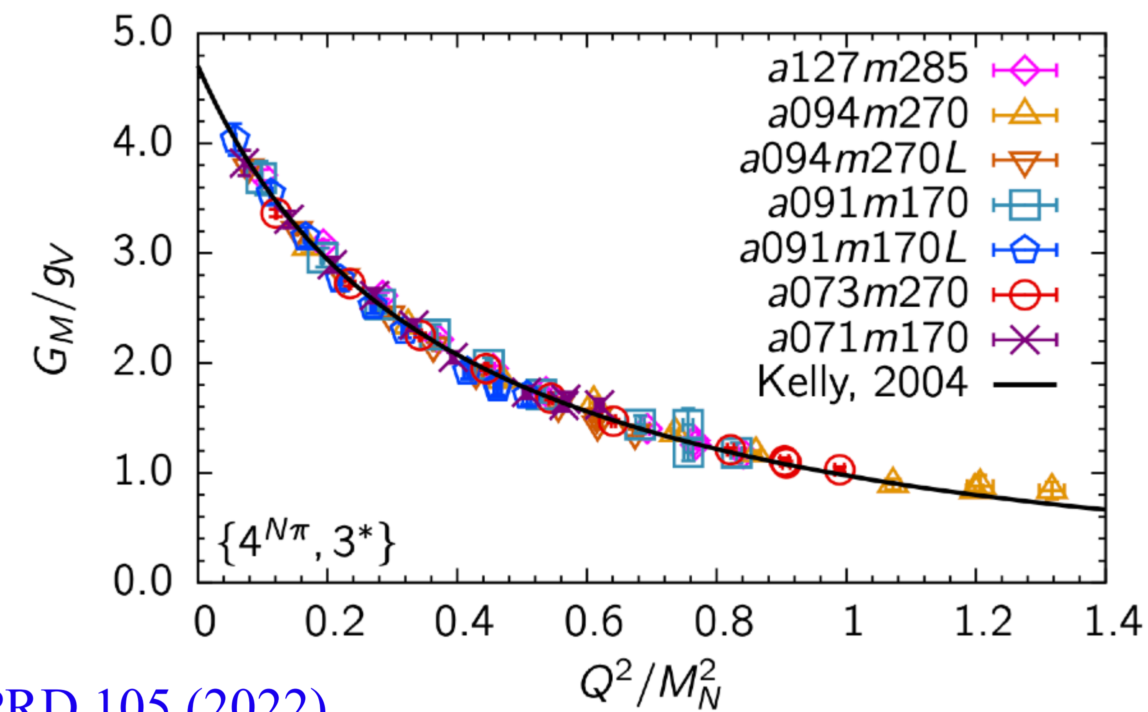




Form factors and LQCD

Vector and axial form factors recently calculated using nearly physical quark masses:

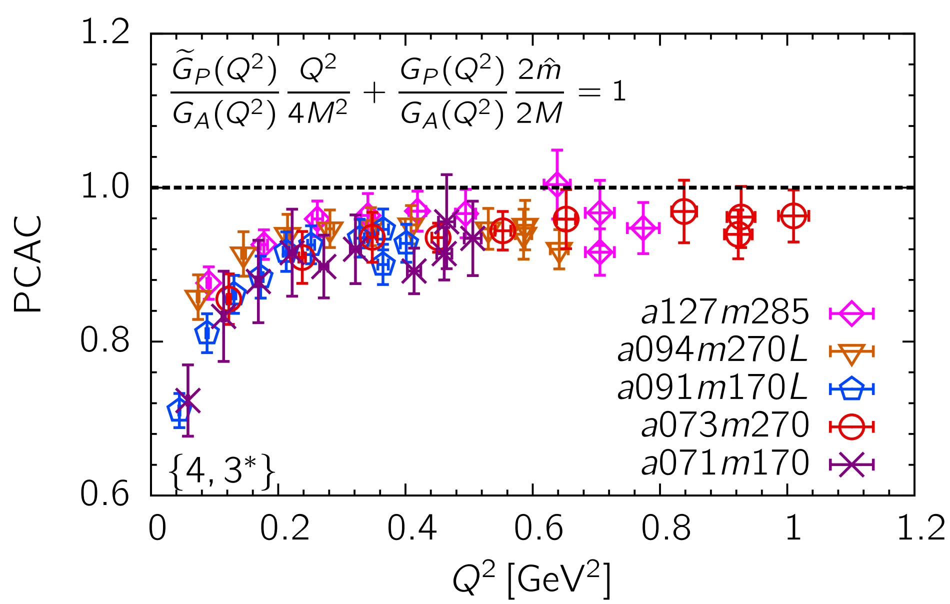


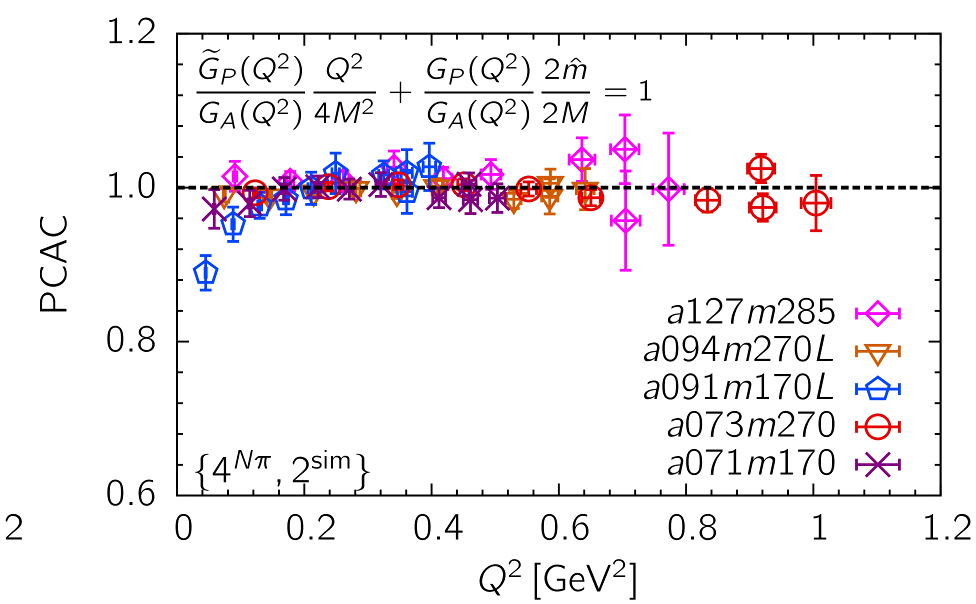


Park et al [NME], PRD 105 (2022)

LQCD nucleon electric and magnetic form factor results agree with phenomenological parameterizations after accounting for excited-state and discretization effects

Careful treatment of $N\pi$ excited states required to reproduce consequences of axial ward identities that assume ground-state dominance

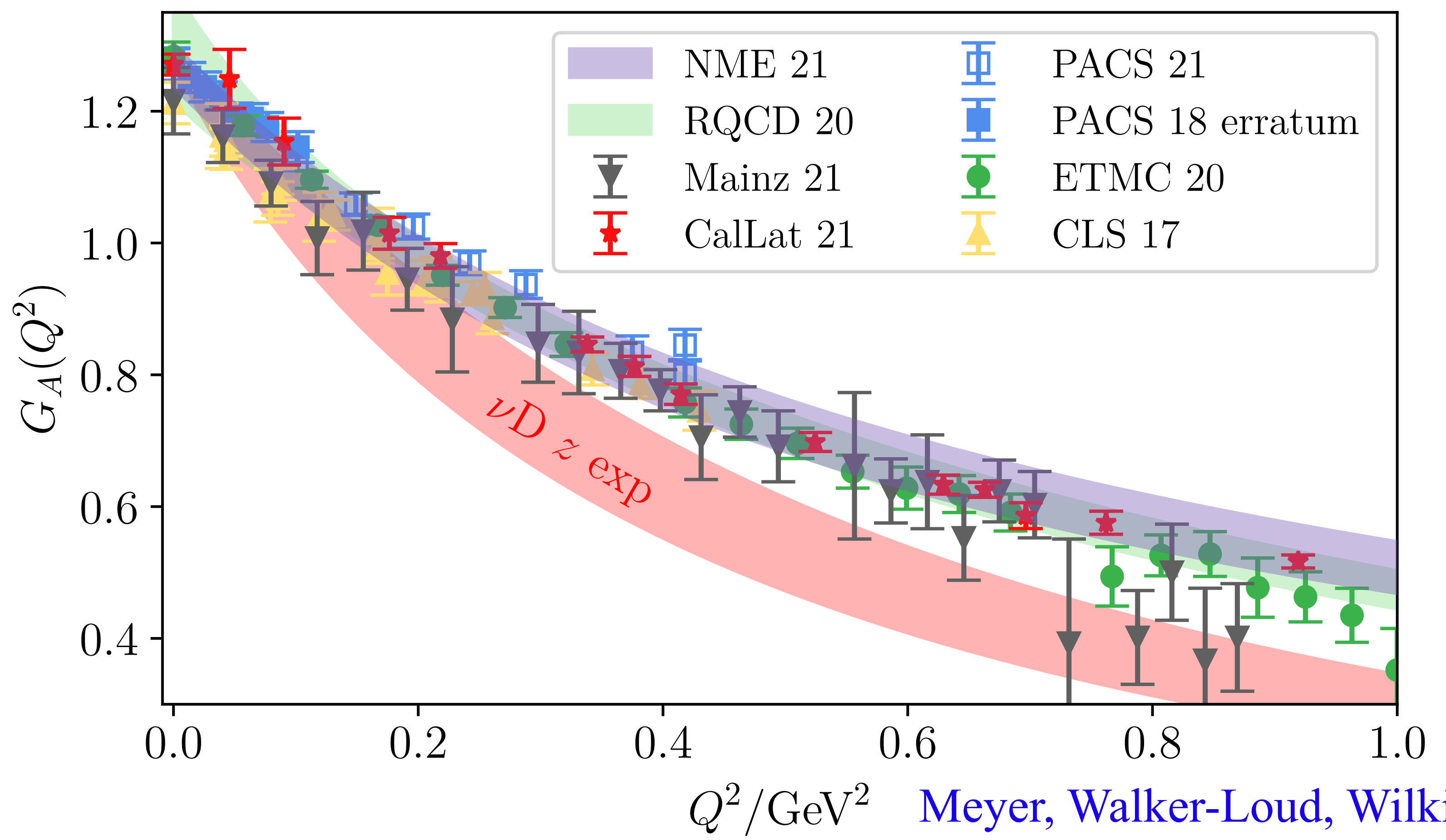




Park et al [NME], PRD 105 (2022)

Axial form factors

Recent axial form factor calculations include physical quark masses, continuum / infinite-volume extrapolations, and excited-state fits that account explicitly for $N\pi$ states



Meyer, Walker-Loud, Wilkinson, arXiv:2201.01839

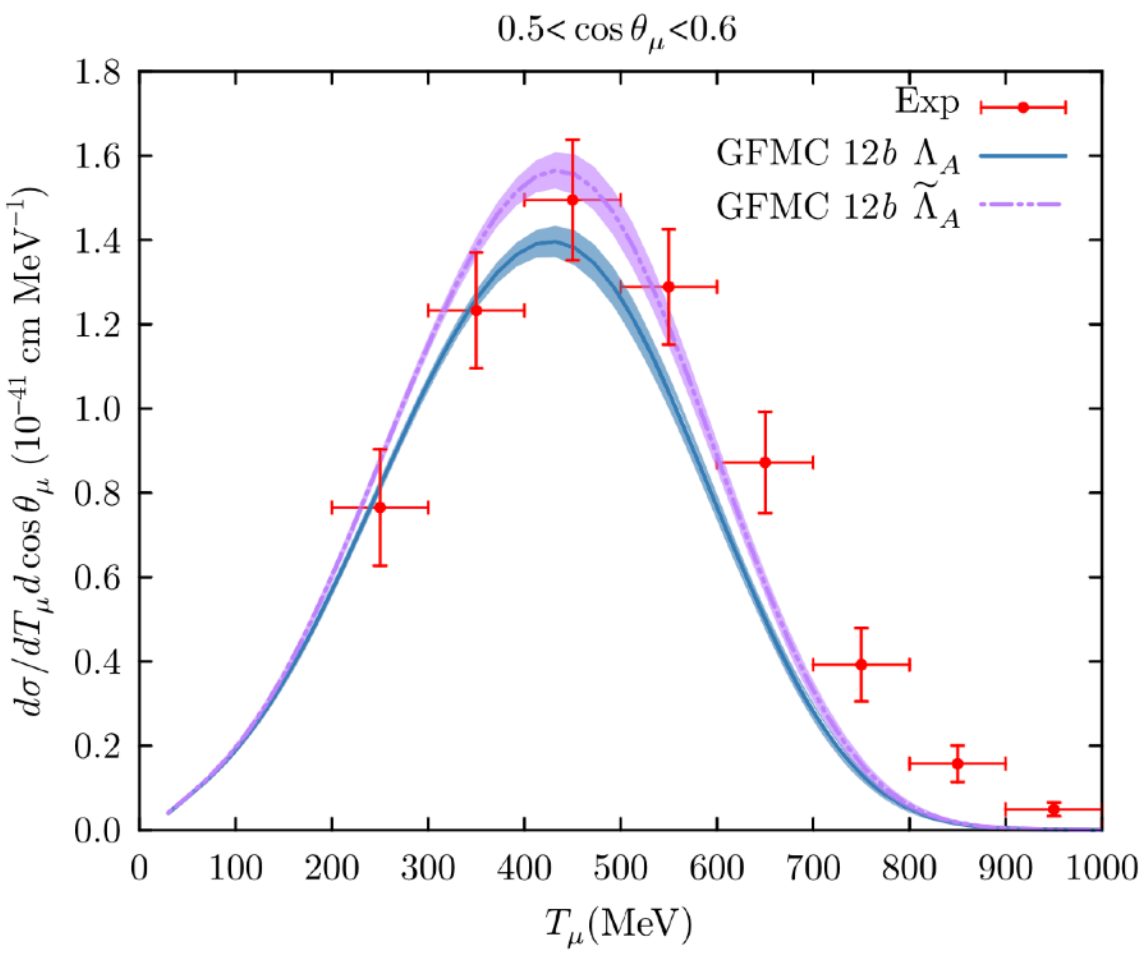
Differences between LQCD and experimental axial form factor determinations could arise from challenging LQCD systematic uncertainties (excited states, lattice spacing, …)

Differences could also arise from underestimated uncertainties in phenomenological form factor determinations using deuterium bubble chamber data

Meyer, Betancourt, Gran, and Hill, PRD 93 (2016)

GFMC form factor uncertainties

Quasi-elastic cross-section is sensitive to nucleon axial form factors used as input



Lovato, Carlson, Gandolfi, Rocco, and Schiavilla, PRX 10 (2020)

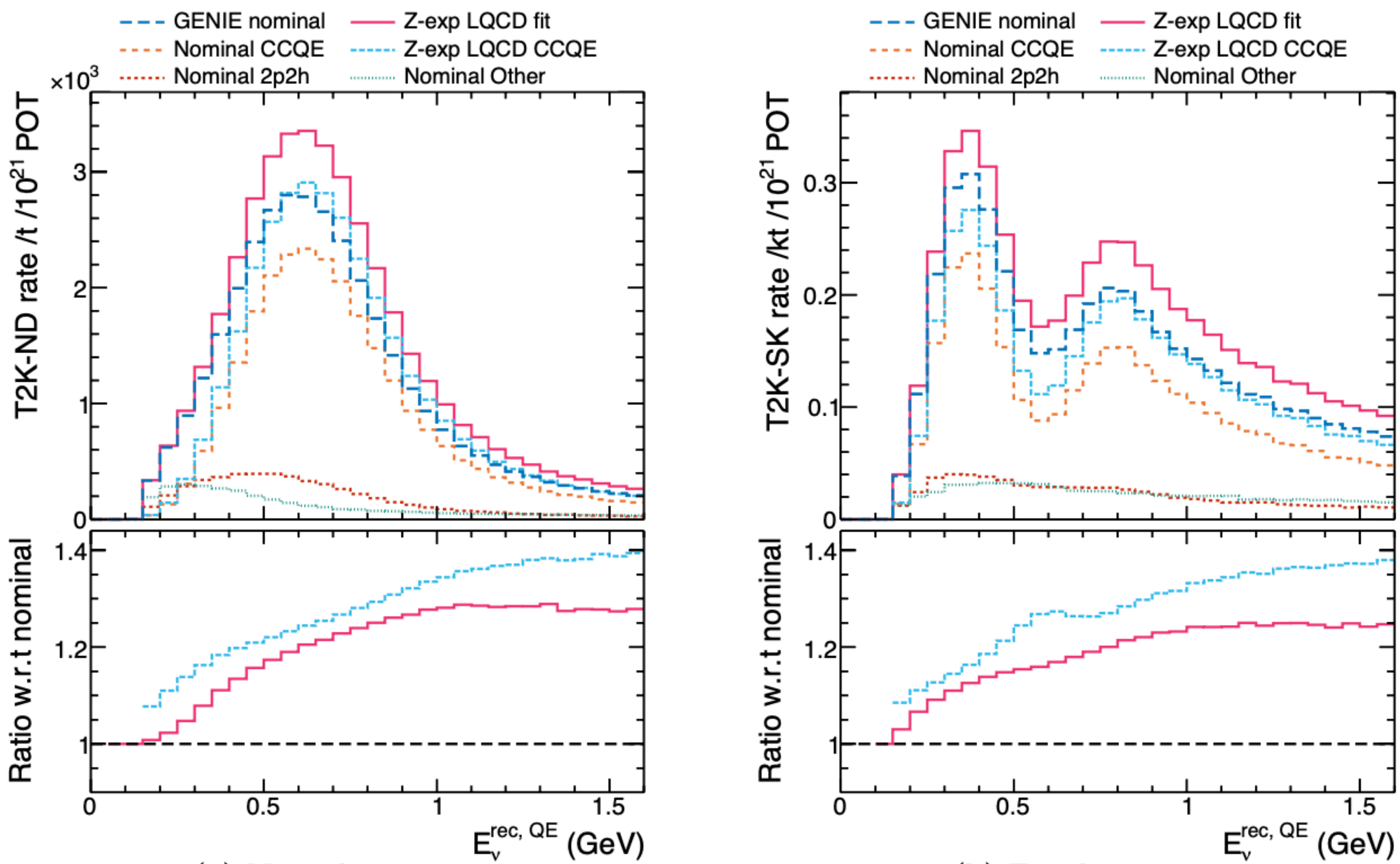
Dipole parameterization example: 15% change in axial dipole mass leads to ~15% change in quasi-elastic peak of MiniBooNE flux-folded cross section

Achieving few-percent cross-section uncertainties will require precise knowledge of nucleon axial form factors

GENIE form factor uncertainties

GENIE predictions for T2K event rate using deuterium bubble chamber vs recent LQCD axial form factors differ by ~20%

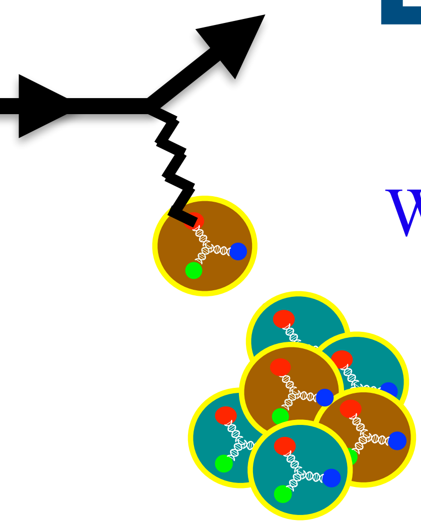
- Understanding discrepancy essential for achieving few-percent cross-section uncertainties



(a) Near detector (b) Far detector

Meyer, Walker-Loud, Wilkinson, arXiv:2201.01839

What precision do we need from LQCD for neutrino physics?



Work in progress: Lovato, Meurice, Rocco, **Simons**, **Steinberg**, MW

First step — axial form factor precision needs

- **Quantitative precision targets** essential for high-performance computing campaigns
- **Cannot assume an overly restrictive form factor shape**, e.g. dipole; assumptions lead to underestimated precision needs
- **Theoretically consistent nuclear models** needed to disentangle axial form factor from multi-nucleon effects
- **Comparison between multiple nuclear models** needed to study nuclear uncertainties in the role of axial form factors in νA

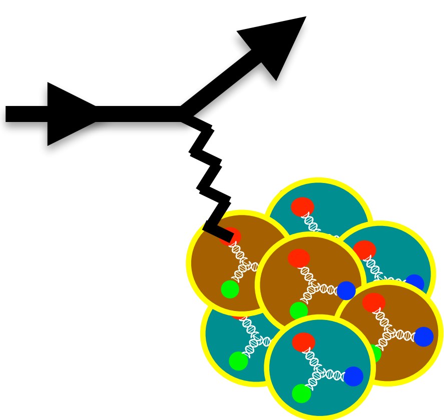
Predicting νA cross sections

Quantum Monte Carlo methods can accurately solve nuclear many-body problem given a Hamiltonian and electroweak current operators

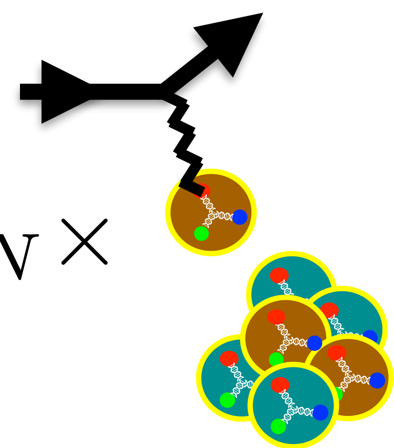
- Cost grows rapidly with nucleon number, computationally limited to $A \lesssim 12$

More computationally tractable: extended factorization scheme using approximate spectral functions — distributions of nucleons (+ *NN* pairs + …) in nucleus

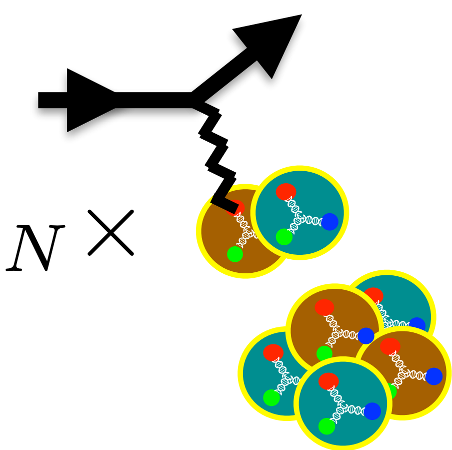
Benhar et al, PRD (2005) Rocco, Lovato, Benhar PRL 116 (2016) Rocco et al, PRC 99 (2019) …



$\approx \int P_N \times$

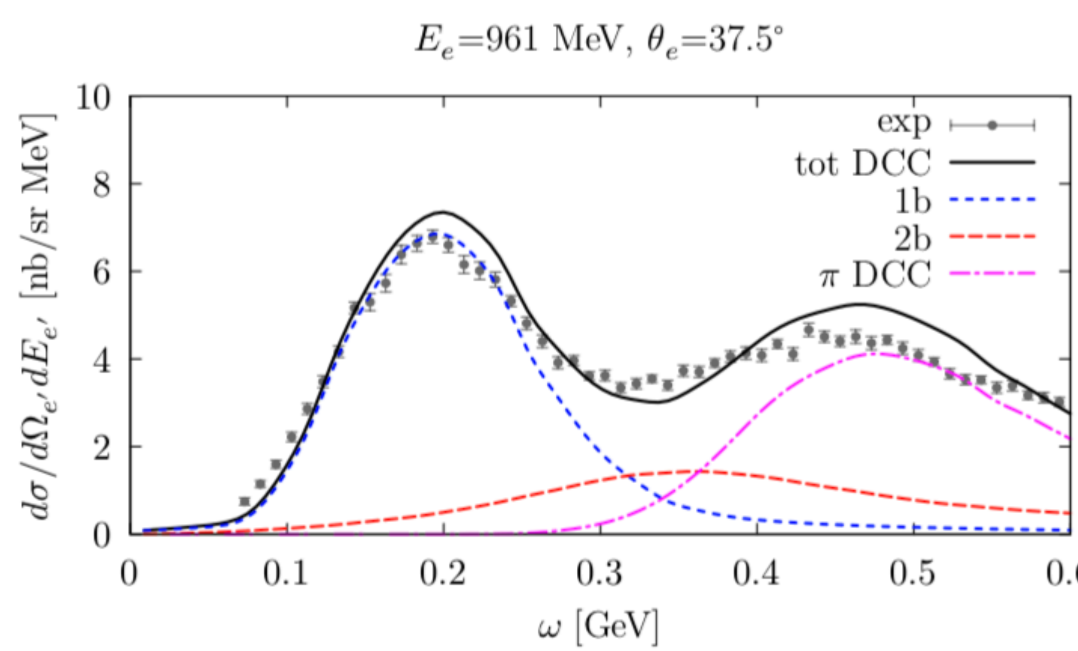


$+ \int P_{NN} \times$



$+ \dots$

Allows inclusion of 2-body currents and resonance production, computationally feasible for medium-mass nuclei

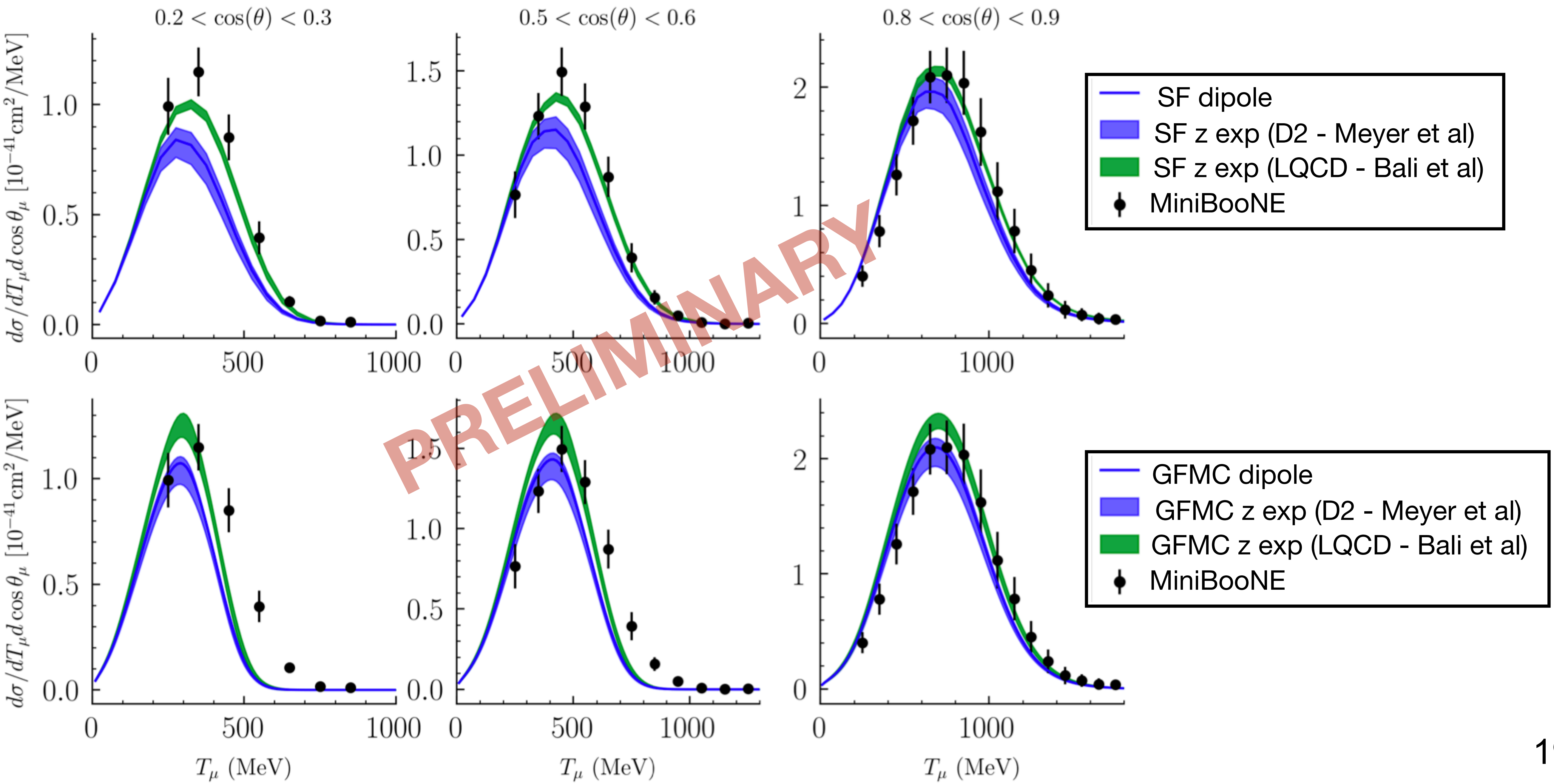


Rocco, Nakamura, Lee, Lovato, PRC 100 (2019)

MiniBooNE results

Comparison of GFMC and spectral function (SF) results for $^{12}\mathrm{C}$ with experimental data and one another provides validation of nuclear many-body methods

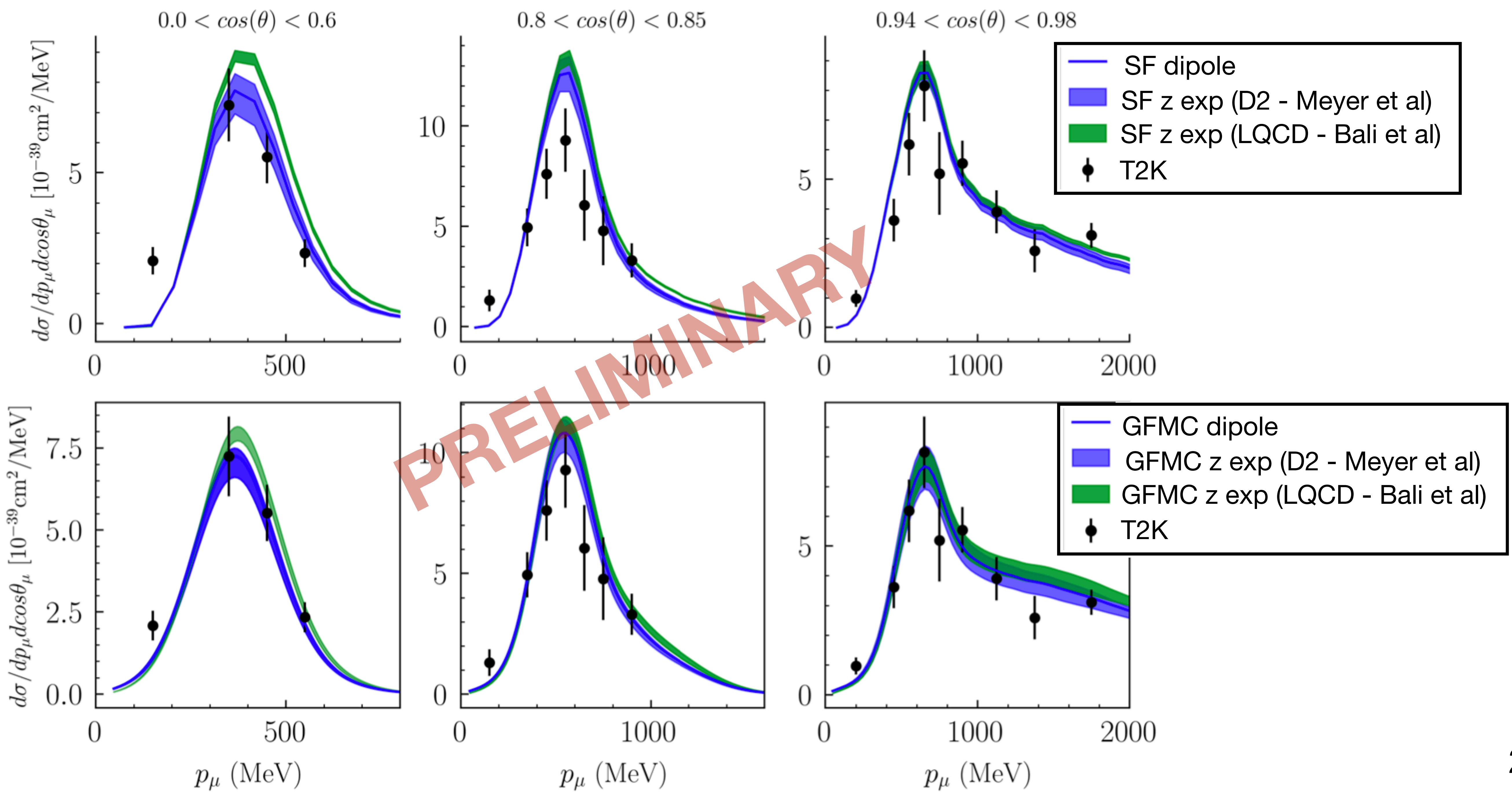
Differences between LQCD and deuterium bubble chamber axial form factor determinations visible but comparable to differences between many-body methods with different two-body current treatment (interference effects not yet included in SF)



T2K results

Similar results seen for T2K flux-folded cross sections

- Consistent treatment of axial FF and nuclear many-body effects essential when fitting to neutrino scattering data



Quantifying axial FF uncertainties

z expansion provides a model independent parameterization of axial (and other) form factors that only assumes basic field theory / QCD properties

Hill, eConf C060409, 027 (2006)

Hill and Paz, PRD 82 (2010)

Bhattacharya, Hill, and Paz, PRD 84 (2011)

…

$$F_A(Q^2) = \sum_{k=0}^{\infty} a_k \, z(Q^2)^k$$

Known analytic function

z expansion can also be used to provide a model independent relation between nucleon axial FF uncertainties and nuclear cross section uncertainties

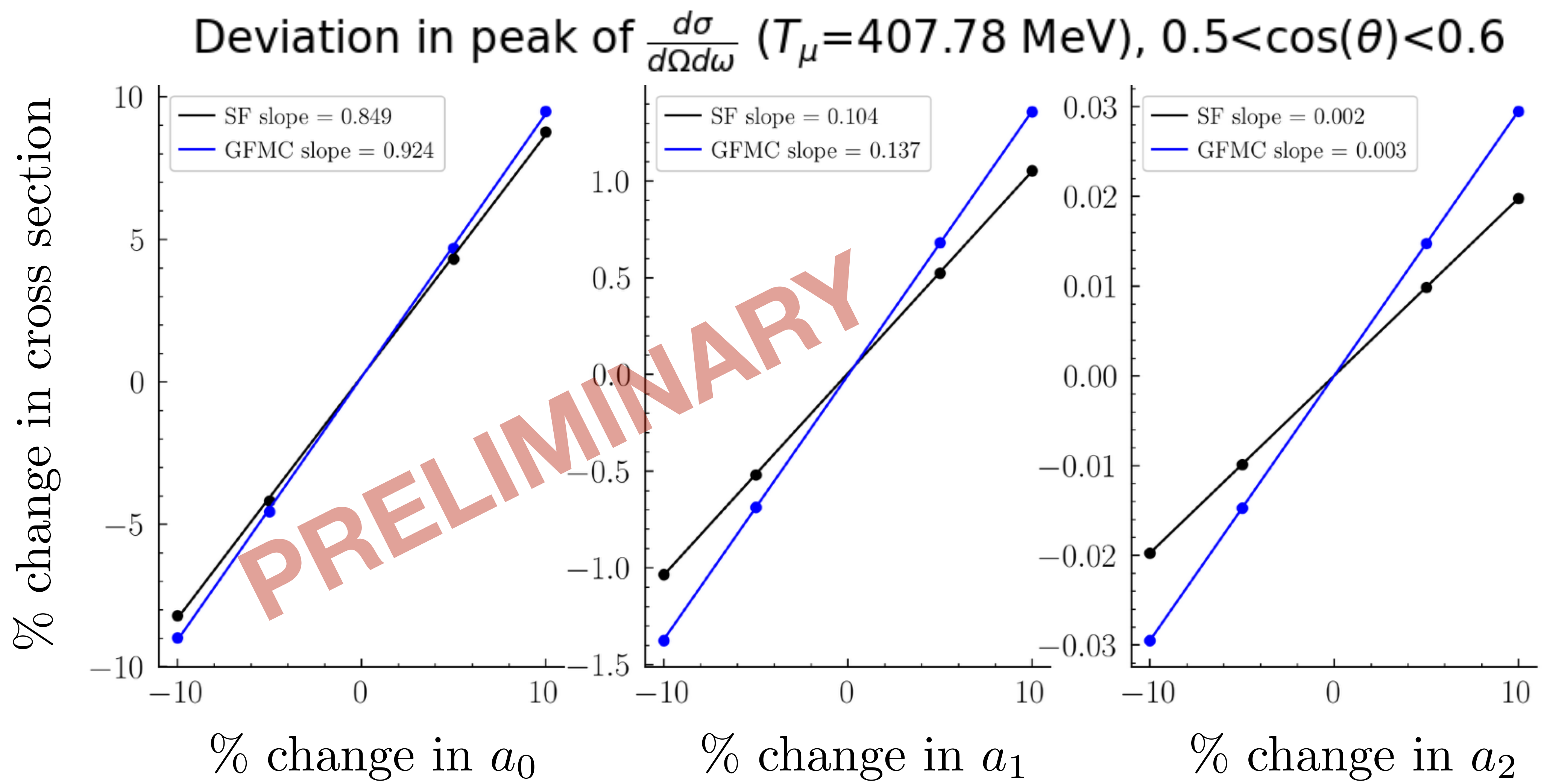
$$\frac{\delta\sigma}{\sigma} \geq \left(\left| \frac{\partial\sigma}{\partial a_k} \right| \frac{|a_k|}{\sigma} \right) \frac{\delta a_k}{|a_k|}$$

Derivative required to relate uncertainties can be estimated straightforwardly using additional cross-section calculations with shifted parameters and finite difference

$$\frac{\partial\sigma}{\partial a_k} \approx \frac{\sigma(a_k + \delta a_k) - \sigma(a_k)}{\delta a_k}$$

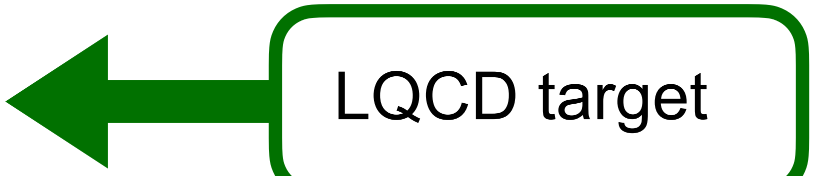
Axial FF uncertainty needs

GFMC and SF methods provide similar estimates of uncertainty relations



Achieving 2% cross-section precision for MiniBooNE kinematics requires:

- ~ 2% precision in $a_0 = g_A$ ✔
- ~ 10% precision in a_1 (related to axial radius)
- Relatively little knowledge of $a_2, \ldots$

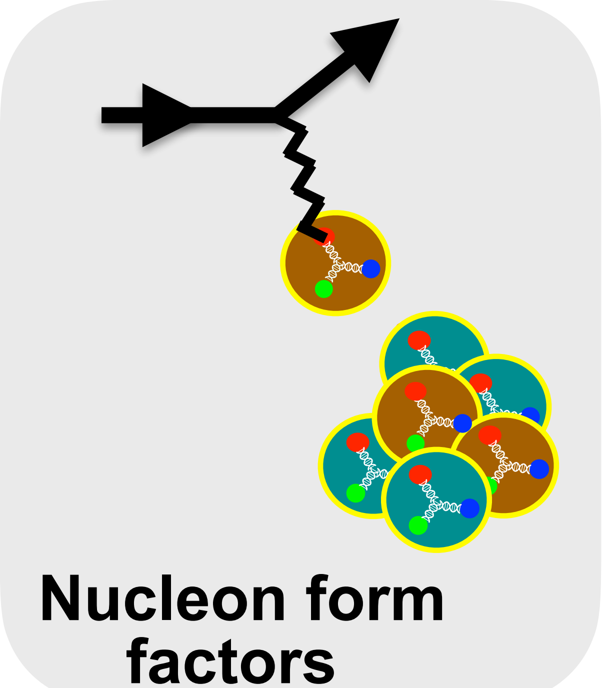


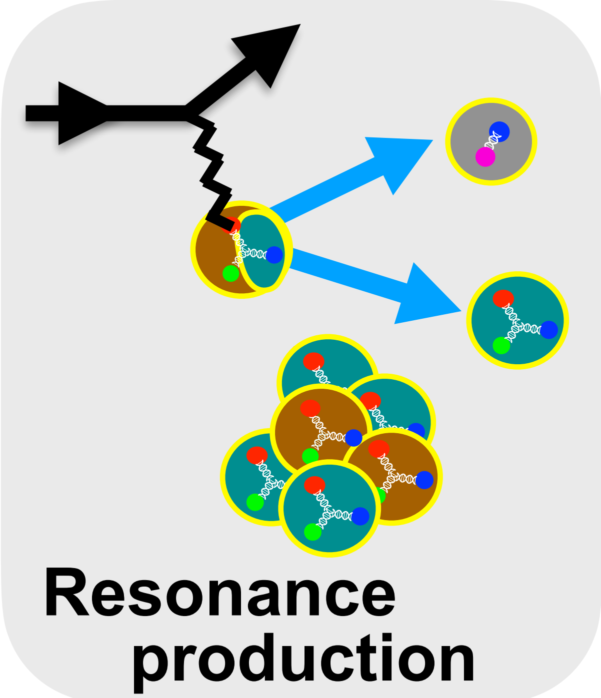
DUNE will be more sensitive to $a_1, a_2, \ldots$, further dedicated studies needed

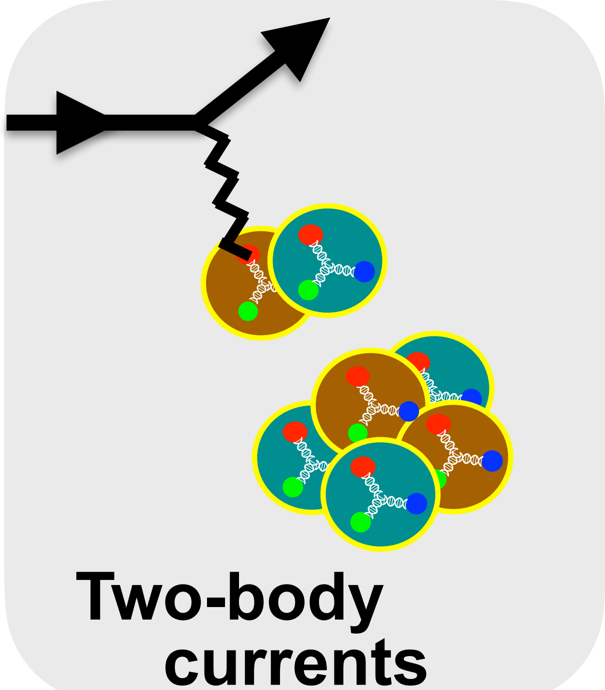
Beyond the axial form factor

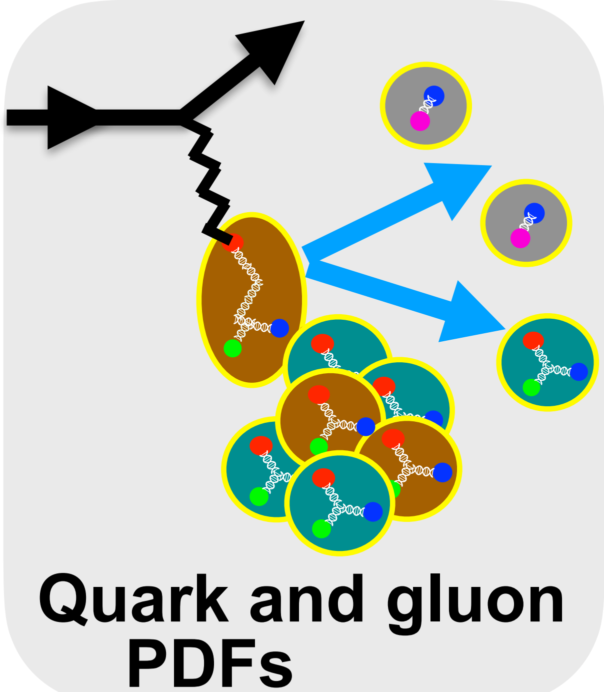
Process	Neutrino Energy Range	Example Final State
Coherent Elastic Scattering	$\lesssim 50$ MeV	$\nu + A$
Inelastic Scattering	$\lesssim 100$ MeV	$e + {}^{A}(Z+1)^{*}(\to {}^{A}(Z+1) + n\gamma)$
Quasi-Elastic Scattering	100 MeV–1 GeV	$l + p + X$
Two-Nucleon Emission	1 GeV	$l + 2N + X$
Resonance Production	1–3 GeV	$l + \Delta(\to N + \pi) + X$
Shallow Inelastic Scattering	3–5 GeV	$l + n\pi + X$
Deep Inelastic Scattering	$\gtrsim 5$ GeV	$l + n\pi + X$

Snowmass WP: "Theoretical Tools for Neutrino Scattering" arXiv:2203.09030,







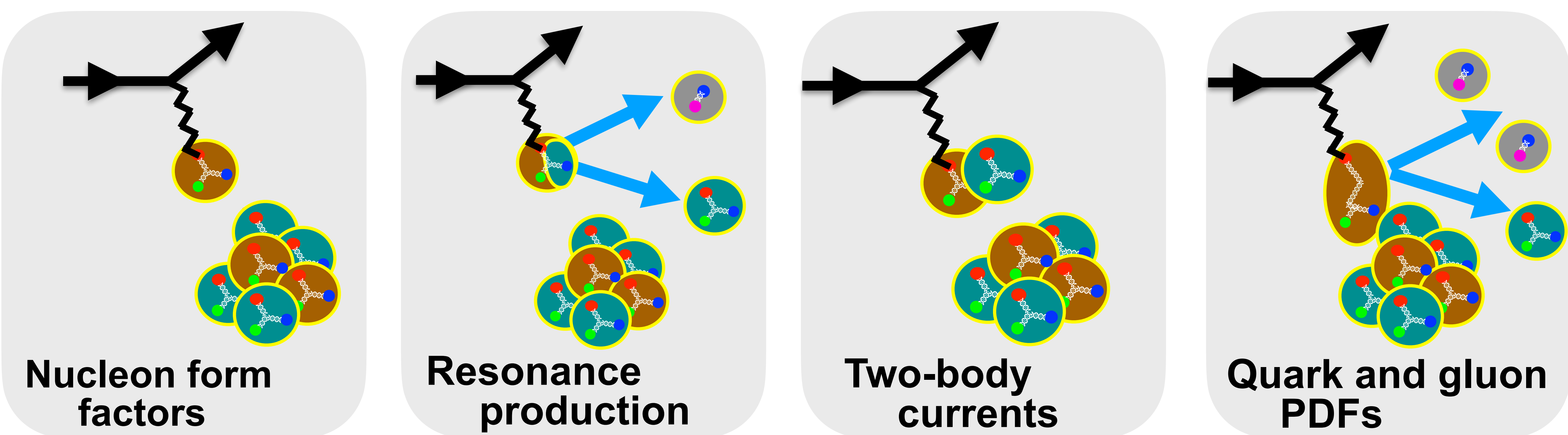


Exploratory

Precise lattice calculations for neutrino scattering

Two-Nucleon Emission	1 GeV	$l + 2N + X$
Resonance Production	1–3 GeV	$l + \Delta(\rightarrow N + \pi) + X$
Shallow Inelastic Scattering	3–5 GeV	$l + n\pi + X$
Deep Inelastic Scattering	$\gtrsim 5$ GeV	$l + n\pi + X$

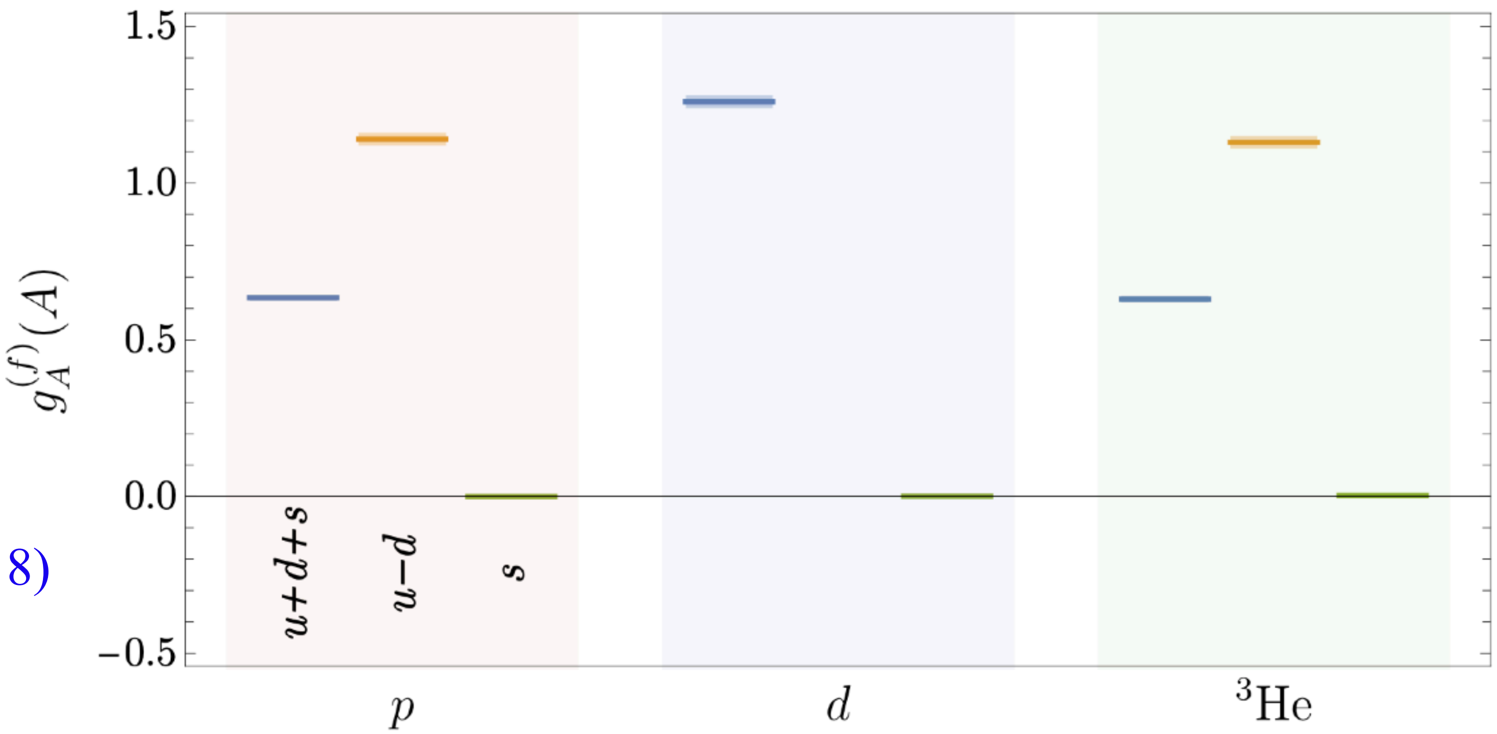
Snowmass WP: “Theoretical Tools for Neutrino Scattering” arXiv:2203.09030,

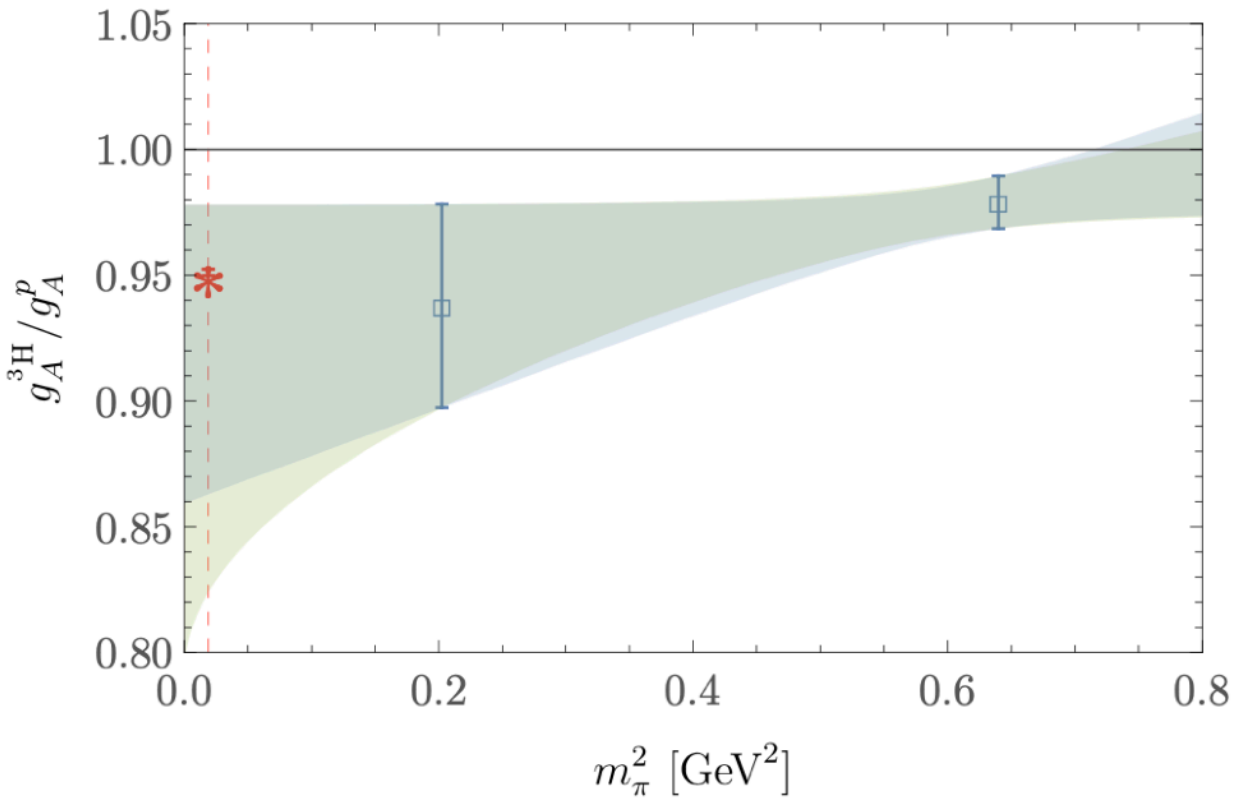


Two-body currents in LQCD

Flavor decomposition of axial matrix elements of two and three nucleon systems computed with
$m_\pi = 806$ MeV

Chang, MW et al [NPLQCD], PRL 120 (2018)





Parreño, MW et al [NPLQCD] PRD 103 (2021)

Axial current matrix element calculations with $m_\pi = 450$ MeV permit preliminary extrapolation of triton axial charge to physical point

Several systematic uncertainties remain, but encouraging agreement with experiment seen

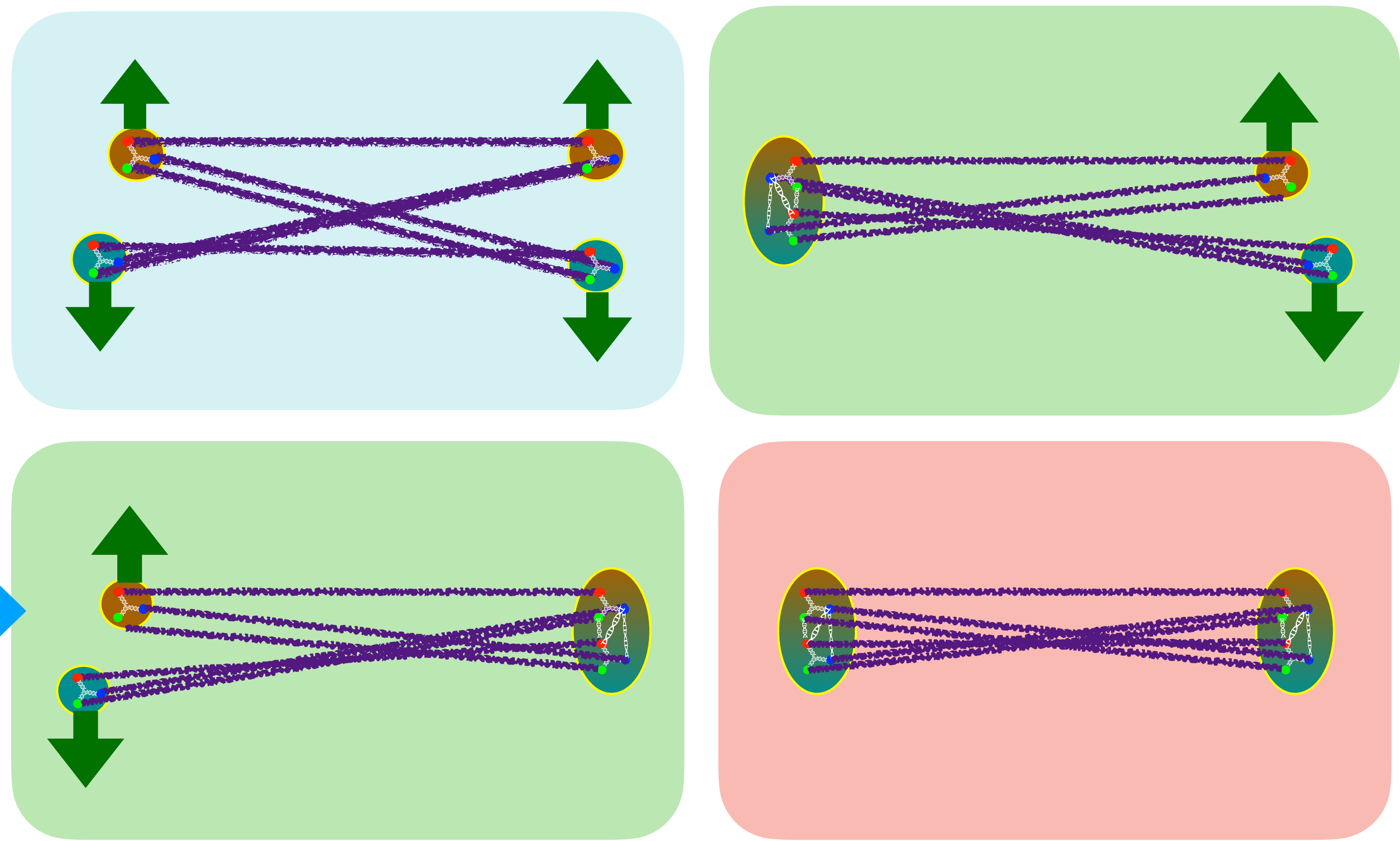
Matching to finite-volume pionless EFT used to constrain L_{1A}

Detmold and Shanahan, PRD 103 (2021)

Variational methods

Excited-state effects from unbound multi-nucleon scattering states are not effectively suppressed in computationally feasible LQCD calculations

LQCD nuclear matrix element calculations so far



Variational methods involving diagonalization of symmetric correlation-function matrices demonstrate that excited-state effects are significant for *NN* systems; provide a path towards robust future LQCD studies of multi-nucleon systems

Francis et al, PRD 99 (2019)

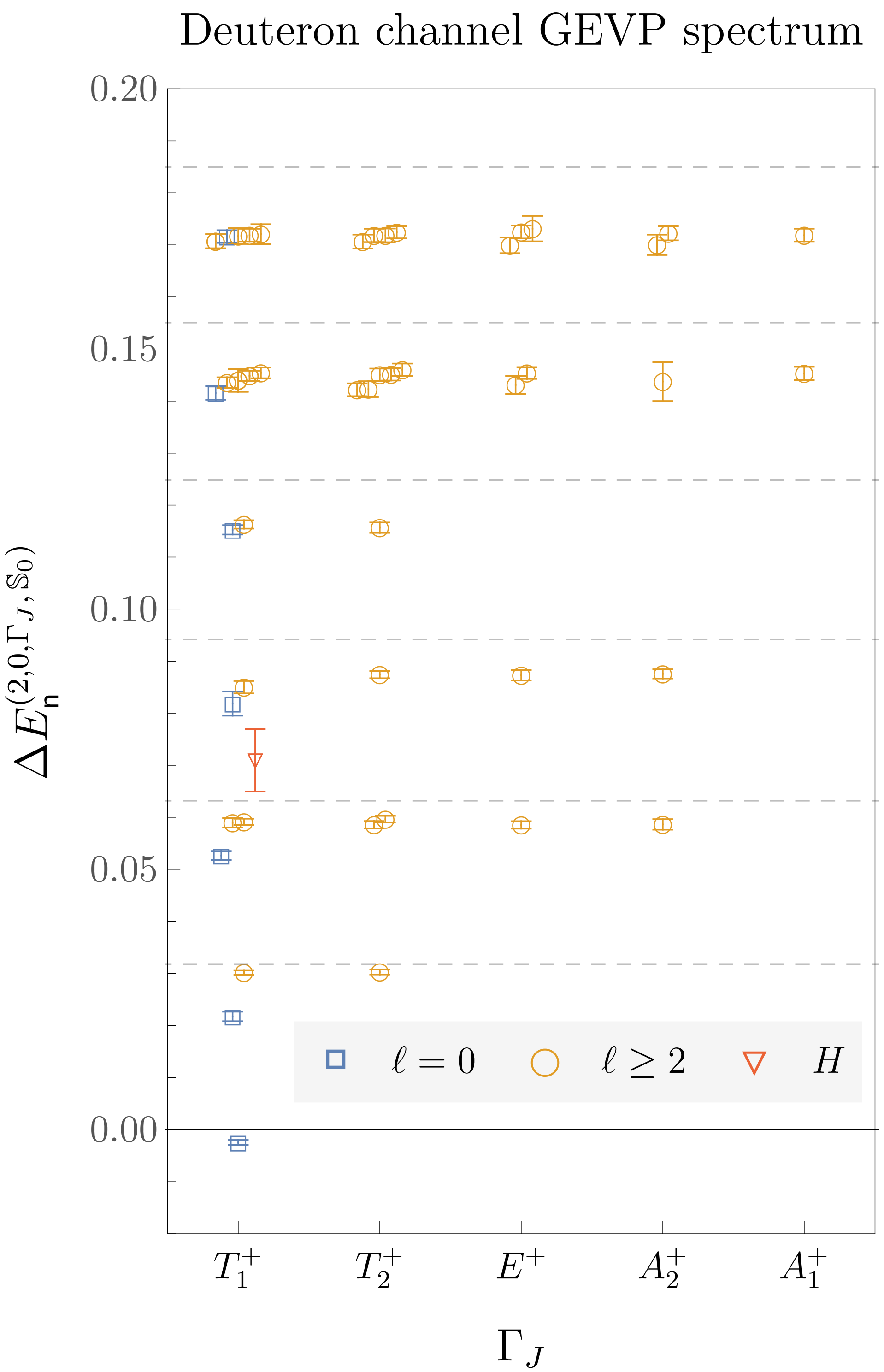
Green et al, PRL 127 (2021)

Hörz et al, PRC 103 (2021)

Amarasinghe, MW et al, arXiv:2108.10835

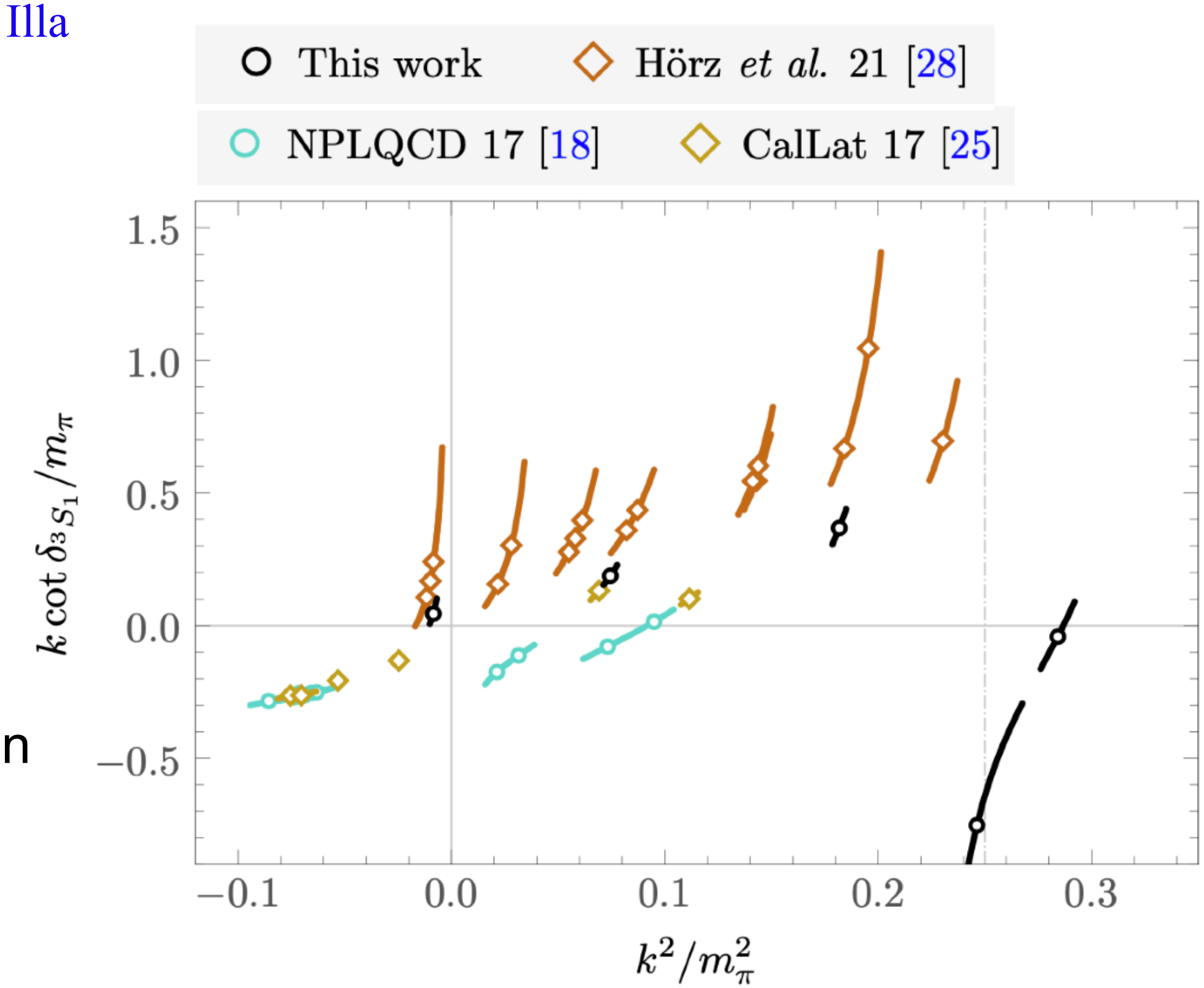
Towards *NN* scattering from LQCD

Variational calculations including a wide range of two-nucleon operators lead to precise determinations of *NN* energy spectra, constraints on *NN* phase shifts



See talk by Illa

Lüscher quantization condition



- Consistency among studies with similar interpolating operators
- Significant discrepancies between calculations with different operators

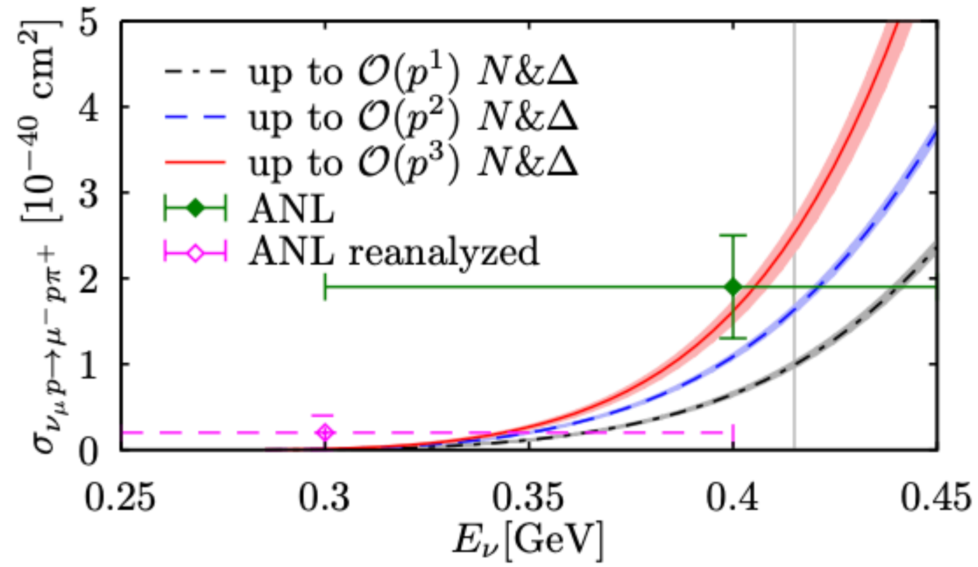
Amarasinghe, MW et al, arXiv:2108.10835

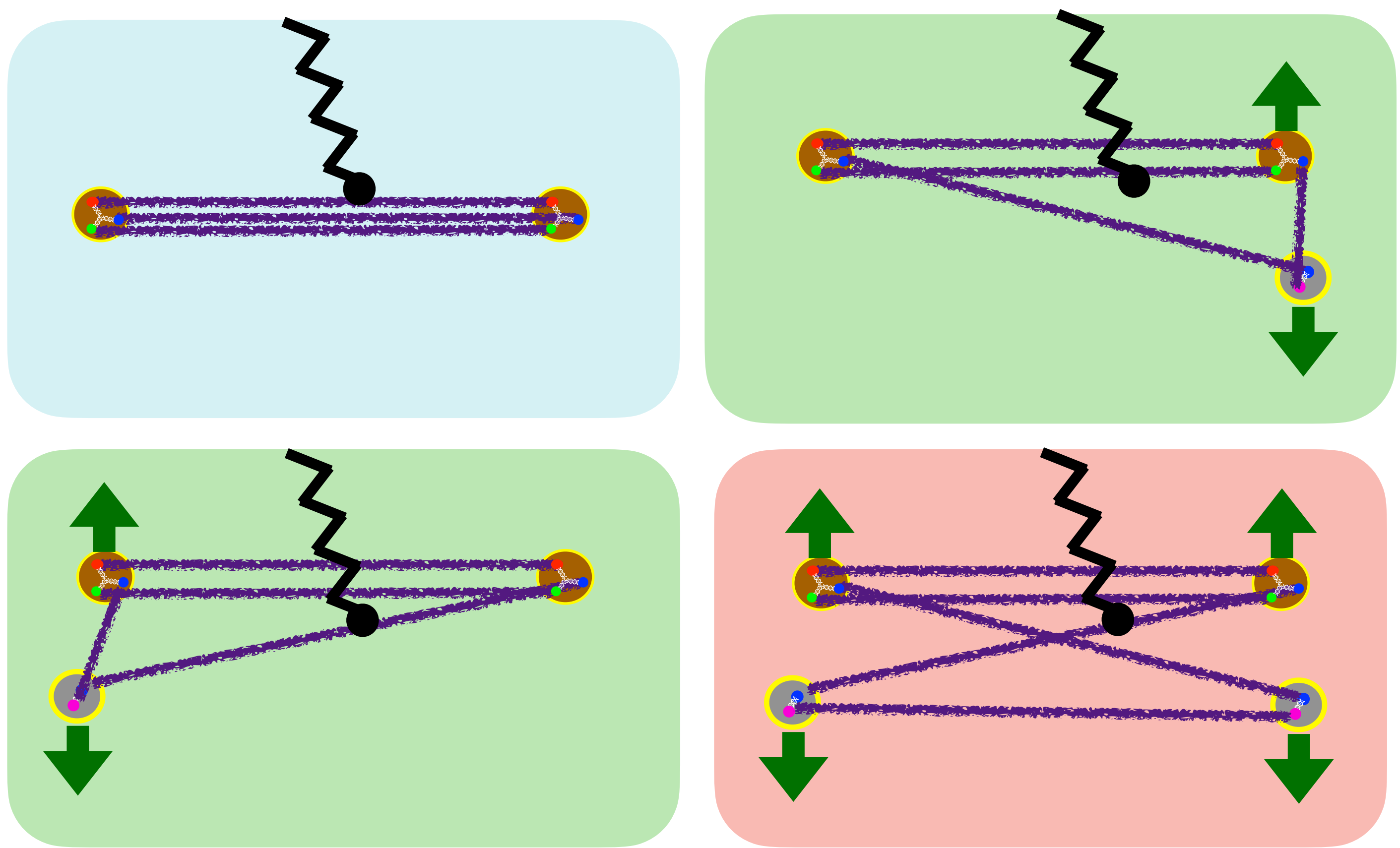
Exciting progress, further studies needed

Pion production

Experimental data on neutrino-induced pion production are scarce

Theoretical assumptions required to extract axial-current $N \to \Delta$ transition form factors





Variational methods can be used to compute nucleon pion production amplitudes with LQCD

Pion production amplitudes can be accessed while simultaneously allowing $N\pi$ excited states to be removed from elastic nucleon form factor calculations

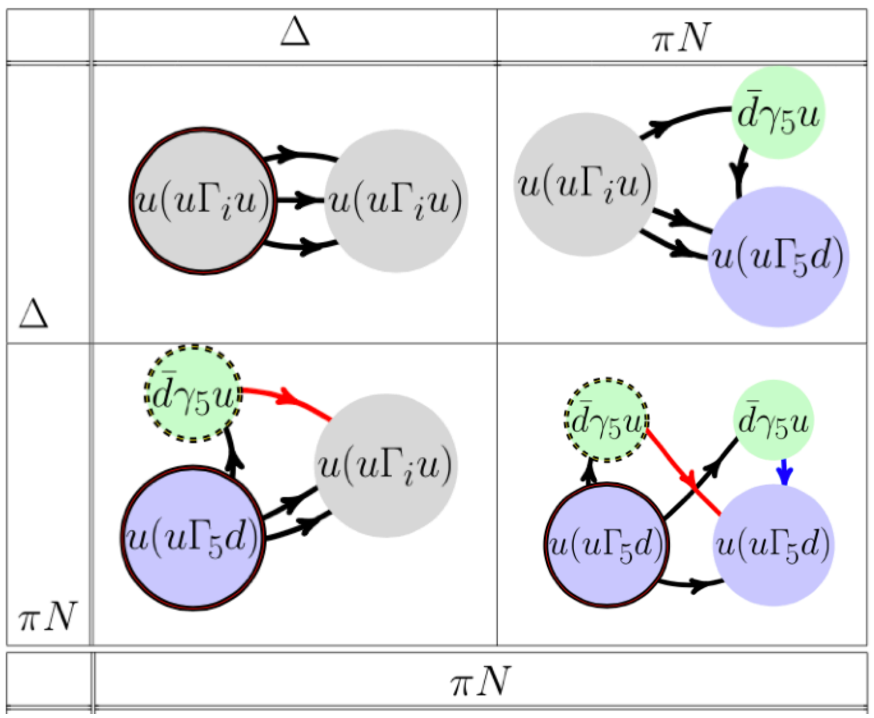
$N \to \Delta$ transition form factors can be calculated using similar methods

$N\pi$ systems in LQCD

Variational methods have been used to study $N\pi$ and Δ systems in LQCD

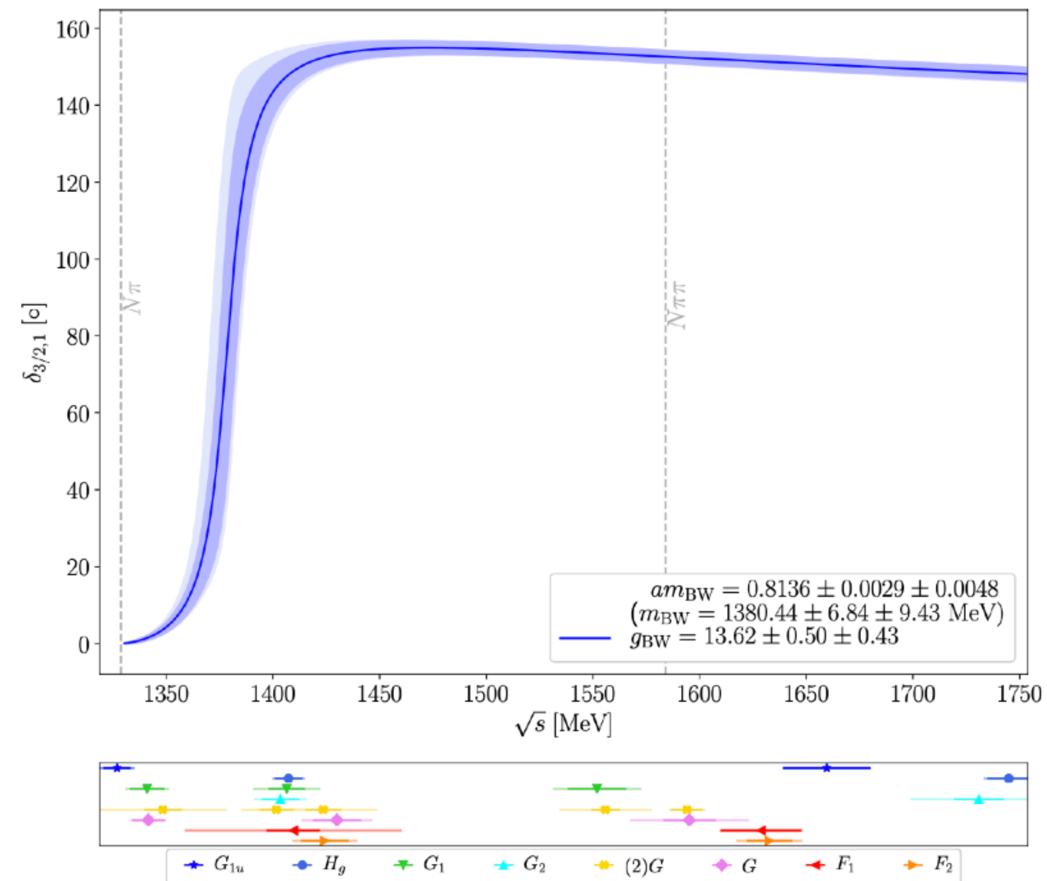
Andersen, Bulava, Hörz, Morningstar, PRD 97 (2018)

Silvi, Paul et al, PRD 23 (2021)



Lüscher quantization condition

$N\pi$ p-wave phase shift



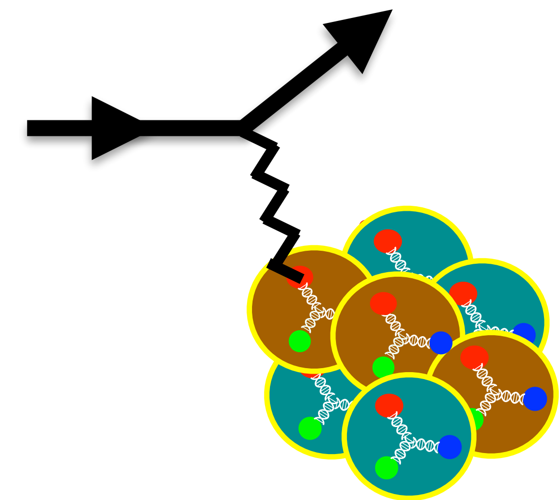
First preliminary calculations of $N \to \Delta$ transition amplitudes have been performed

Barca, Bali, and Collins, *PoS* LATTICE2021 (2022)

Inclusion of $N\pi$ operators significantly reduces excited-state contamination in nucleon axial form factor calculations

Barca, LATTICE2022

Higher-energy resonances



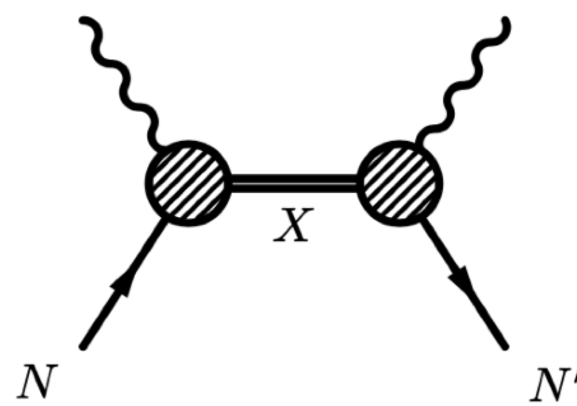
LQCD methods for calculating exclusive cross-sections break down above multi-particle thresholds (≤ 3 hadrons is state-of-the-art)

Reviews: Briceño, Dudek, and Young, Rev. Mod. Phys. 90 (2018)

Hansen and Sharpe, Ann. Rev. Nucl. Part. Sci. 69 (2019)

How can we use LQCD to constrain higher-energy cross sections, e.g. shallow inelastic scattering region?

The hadron tensor:



Liu and Dong, PRL 72 (1994)

Aglietti et al, Phys. Lett. B 432 (1998)

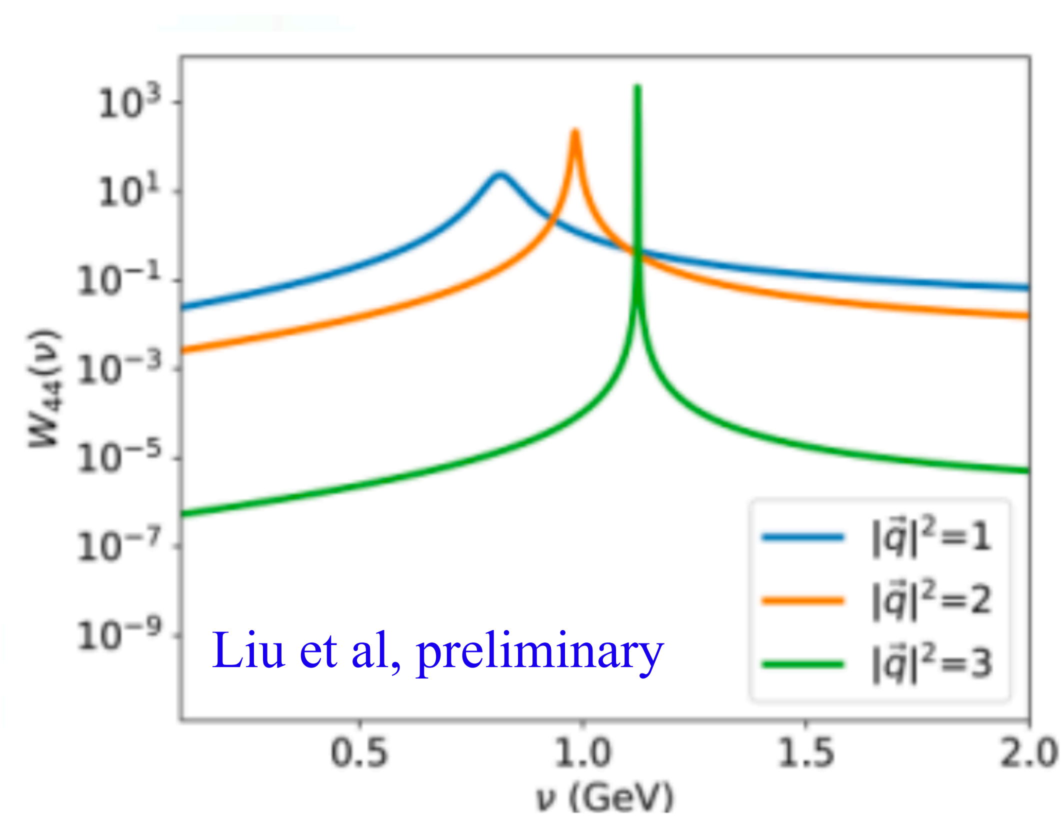
Liu, PRD 62 (2000)

Inverse Laplace transform challenging, calculations exploring different methods underway

Liang, Draper, Liu, Rothkopf, and Yang [χQCD] PRD 101 (2020)

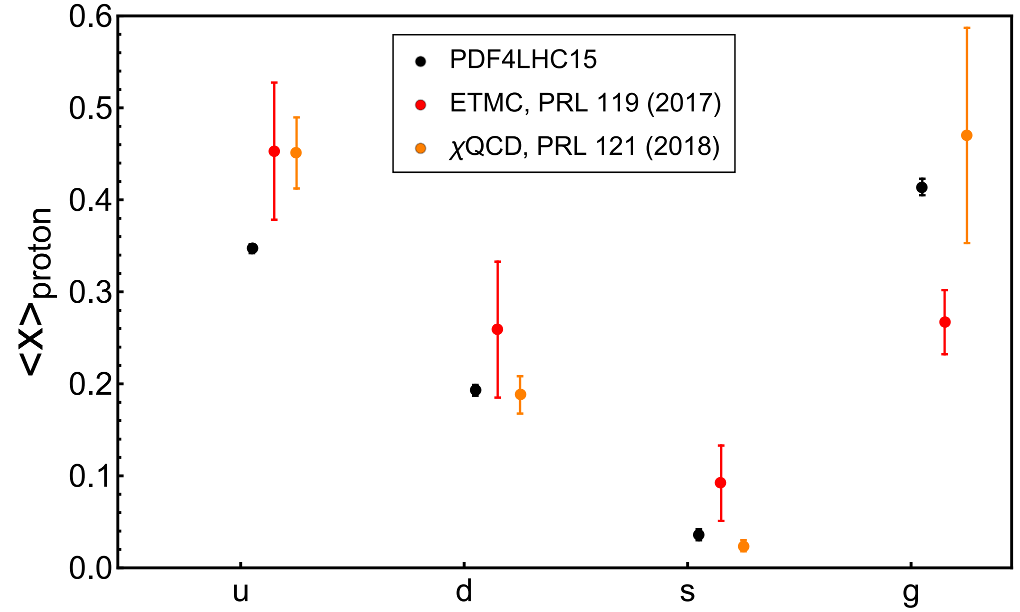
Fukaya, Hashimoto, Kaneko, and Ohki, PRD 102 (2020)

After subtracting elastic contributions, resonant behavior consistent with Roper contributions observed in vector-current hadron tensor



DIS and LQCD

LQCD can also compute quantities relevant to neutrino DIS



$\langle x \rangle^q_{\rm proton}$ and $\langle x \rangle^g_{\rm proton}$ calculated by several groups

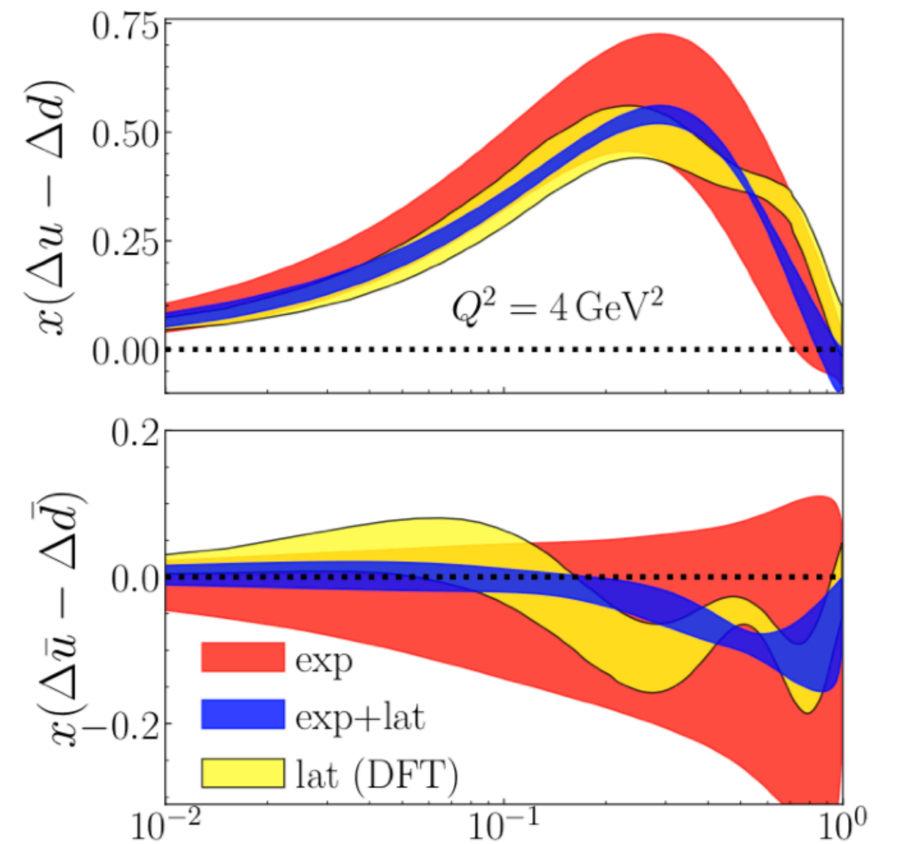
Review: Lin et al, Prog. Part. Nucl. Phys. 100 (2018)

Large momentum effective theory connects Euclidean matrix elements to light-cone PDFs

Review: Ji et al, Rev. Mod. Phys. 93 (2021)

Current LQCD results can improve global analyses of isovector polarized PDFs that are relevant for weak interactions in neutrino DIS

Chen, Cohen, Ji, Lin, Zhang, Nucl. Phys. B 911 (2016)



Bringewatt et al [JAM], PRD 103 (2021)

Alexandrou, et al, PRL 121 (2018)   Alexandrou et al, PRL 126 (2021)

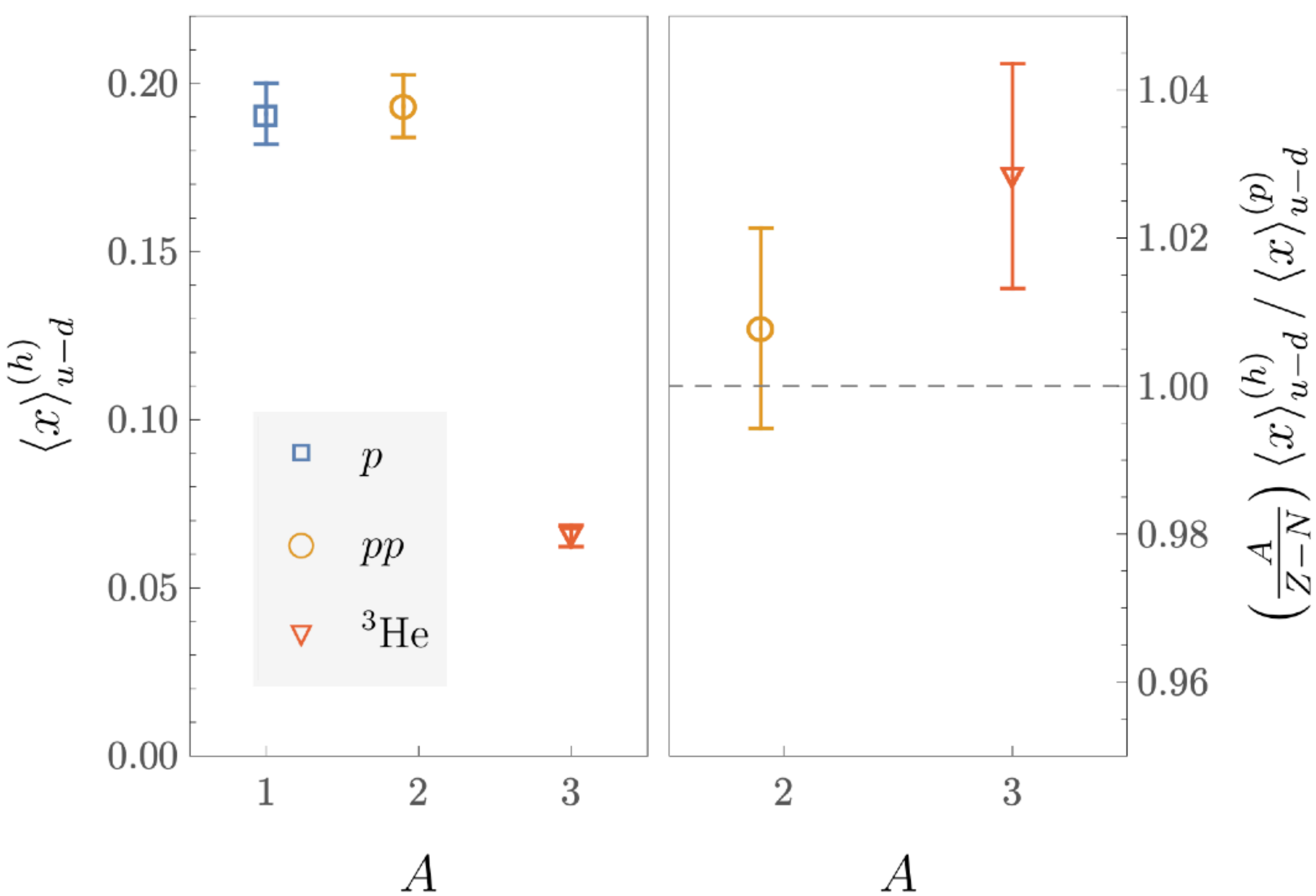
Nuclear momentum fractions

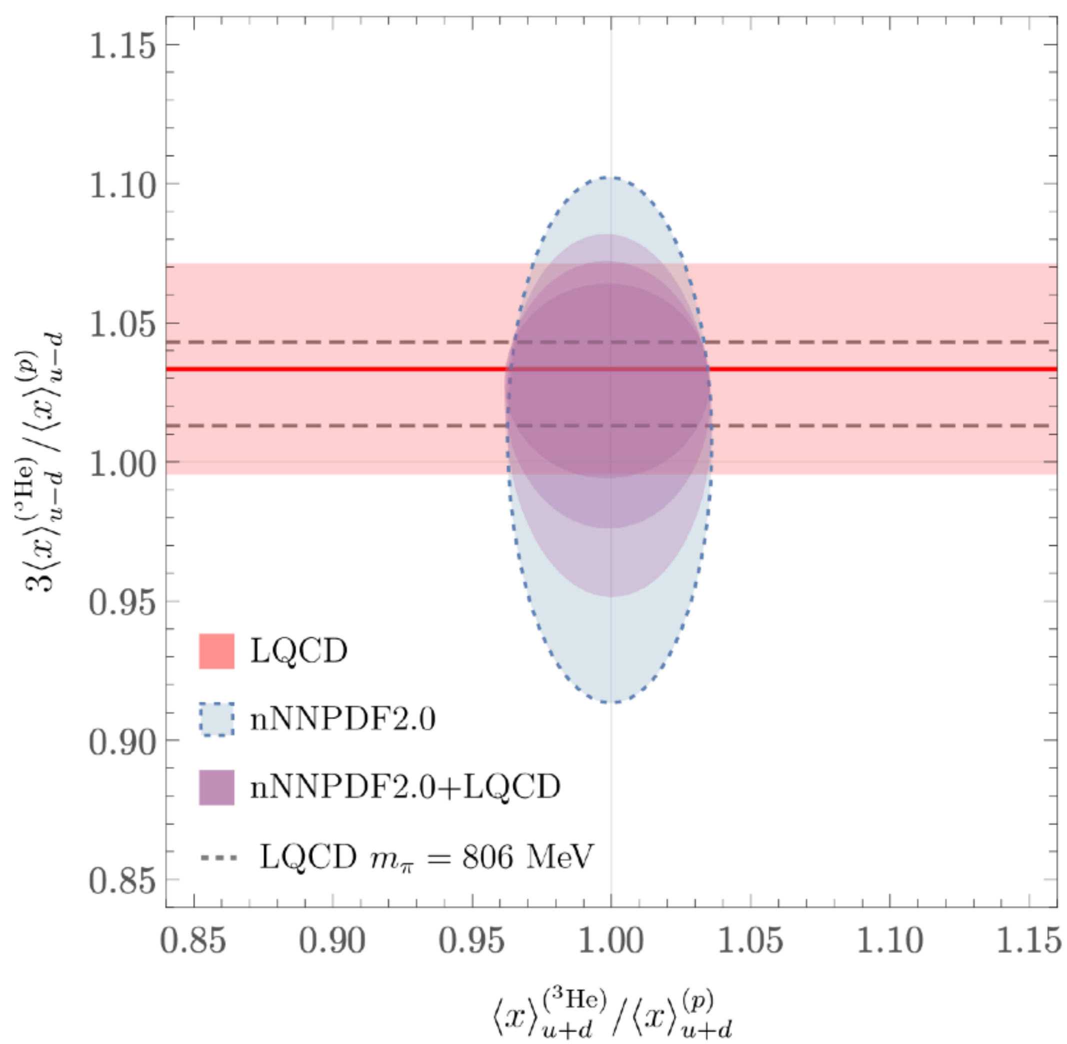
First calculations of gluon and isovector quark momentum fractions of light nuclei

Winter, MW et al [NPLQCD], PRD 96 (2017) Detmold, MW et al [NPLQCD] PRL 126 (2021)

Results matched to poinless EFT to determine two-body current operator relevant to isovector EMC effects

Although systematic uncertainties are not fully controlled (one lattice spacing, volume, quark mass, …) demonstrates potential for LQCD to usefully constrain nuclear PDFs





Conclusions

Disentangling the νA reaction mechanisms relevant for DUNE:

- Is essential for connecting near- and far-detector measurements where fluxes differ

- Requires a robust theory pipeline from the Standard Model to event generators with cooperation between many groups: HEP / NP, theory / experiment / computing, …