Lattice Calculation of the Proton Charge Radius

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[on behalf of Nucleon Matrix Elements (NME) Collaboration]

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PNDME Collaboration: Thirteen 2+1+1-flavor HISQ ensembles = clover-on-HISQ formulation

NME Collaboration: Thirteen 2+1-flavor clover ensembles = clover-on-clover formulation

PNDME and NME members

- Tanmoy Bhattacharya (LANL)
- Vincenzo Cirigliano (LANL->INT, UW)
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- Sungwoo Park (Jlab)
- Santanu Mondal (MSU)
- Huey-Wen Lin (MSU)
- Balint Joo (ORNL)
- Frank Winter (Jlab)

References

PNDME

- Charges: Gupta et al, PRD.98 (2018) 034503
- AFF: Gupta et al, PRD 96 (2017) 114503
- AFF: Jang et al, PRL 124 (2020) 072002
- VFF: Jang et al, PRD 100 (2020) 014507
- $\sigma_{\pi N}$ Gupta et al, PRL 127 (2021) 242002
- d_n from Θ-term Bhattacharya etal, PRD 103 (2021) 114507
- d_n from qEDM Gupta et al, PRD 98 (2018) 091501
- Moments of PDFs Mondal et al, PRD 102 (2020) 054512
- Proton spin: Lin et al, PRD 98 (2018) 094512

NME

- Charges, FF: Park et al, PRD 105 (2022) 054505
- Moments of PDFs Mondal et al, JHEP 04 (2021) 044

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Electromagnetic Form Factors, G_E and G_M

- Dirac/Pauli Form factors, $\boldsymbol{F_{1,2}}$ parametrize the matrix element of the electromagnetic current $V_\mu^{\text{em}} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d$ within nucleon ground states

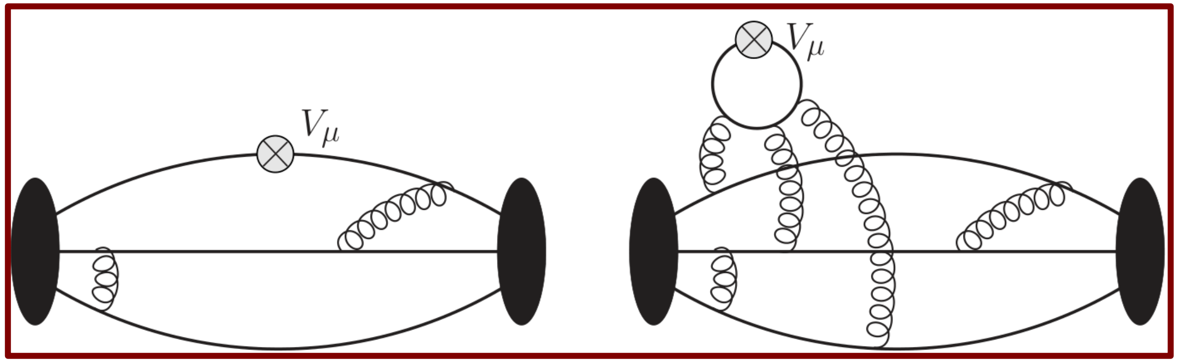
$$\langle N(\boldsymbol{p}',s')|V_\mu^{\text{em}}|N(\boldsymbol{p},s)\rangle = \bar{u}(\boldsymbol{p}',s')\left(\boldsymbol{F_1(q^2)}\gamma_\mu + \frac{i\sigma_{\mu\nu}}{2M_N}\boldsymbol{F_2(q^2)}\right)u(\boldsymbol{p},s)$$

- Sachs form factors, $\boldsymbol{G_E} = F_1 - \frac{Q^2}{4M_N^2}F_2, \quad \boldsymbol{G_M} = F_1 + F_2$

- Isovector form factors $\boldsymbol{G_{E,M}^{u-d} = G_{E,M}^{p} - G_{E,M}^{n}}$ assuming flavor SU(2) isospin symmetry,

$$\langle p|\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d|p\rangle = \langle p|V_\mu^{\text{em}}|p\rangle - \langle n|V_\mu^{\text{em}}|n\rangle$$

Isovector Charge Radii



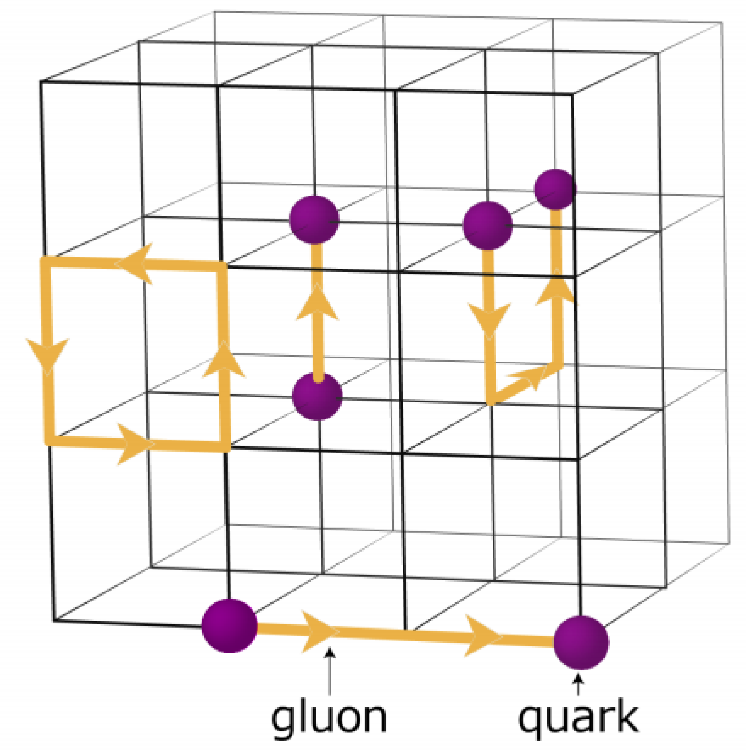
- Proton electromagnetic form factor $\boldsymbol{G_{E,M}^{p}}$ has contribution from both connected and disconnected diagrams
- We focus on the isovector form factor $\boldsymbol{G_{E,M}^{u-d} = G_{E,M}^{p-n}}$ where the disconnected contribution cancels in the isospin symmetric limit.
- We calculate $\boldsymbol{G_{E,M}^{u-d}(Q^2)}$ as a function of $Q^2 = -q^2$ **using lattice QCD!**

- Charge radii squared

$$\langle r_E^2 \rangle^{u-d} = -6 \frac{d}{dQ^2} \left(\frac{G_{E,M}^{u-d}(Q^2)}{G_{E,M}^{u-d}(0)} \right) \Big|_{Q^2=0}$$

Lattice QCD

[Formulated by K. Wilson (1974). Numerical computation field opened by M. Creutz (1979)]



Lattice QCD is QCD defined on a 4-dimensional Euclidean space-time lattice

- Finite lattice spacing: (a)
- Quark fields $(q, \bar{q})$, Gauge fields (gluons): (U_μ)
- Perturbative & **Numerical (nonperturbative) calculations**

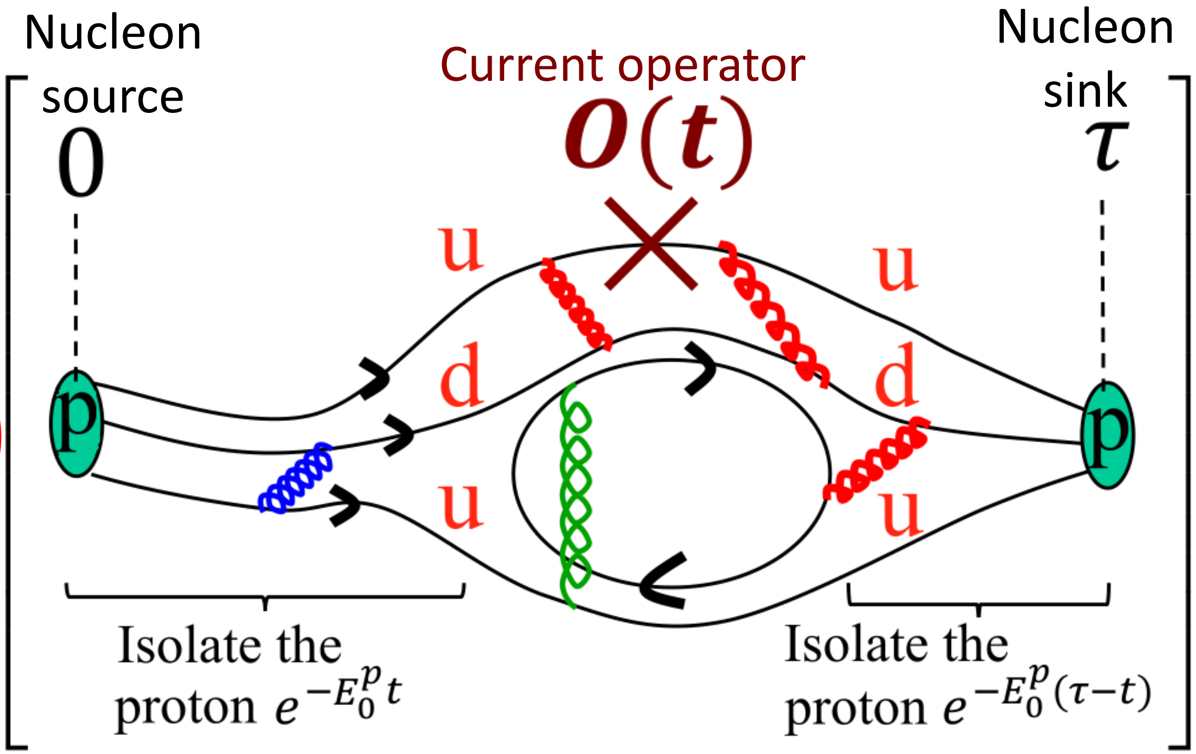
The simulation allows **ab initio** calculations of nonperturbative QCD interactions of quarks and gluons using the **Feynman path integral** formulation of QFT.

Calculation of Nucleon Matrix Element

- Properties of nucleons ($\langle p|O|p\rangle$, Form Factor) are extracted from the **<u>3-point correlation function</u>** $\boldsymbol{C(t,\tau) \equiv \langle N^p(\tau) O(t) \bar{N}^p(0)\rangle}$:

Average over the "Gauge ensemble" generated using Markov Chain Monte-Carlo

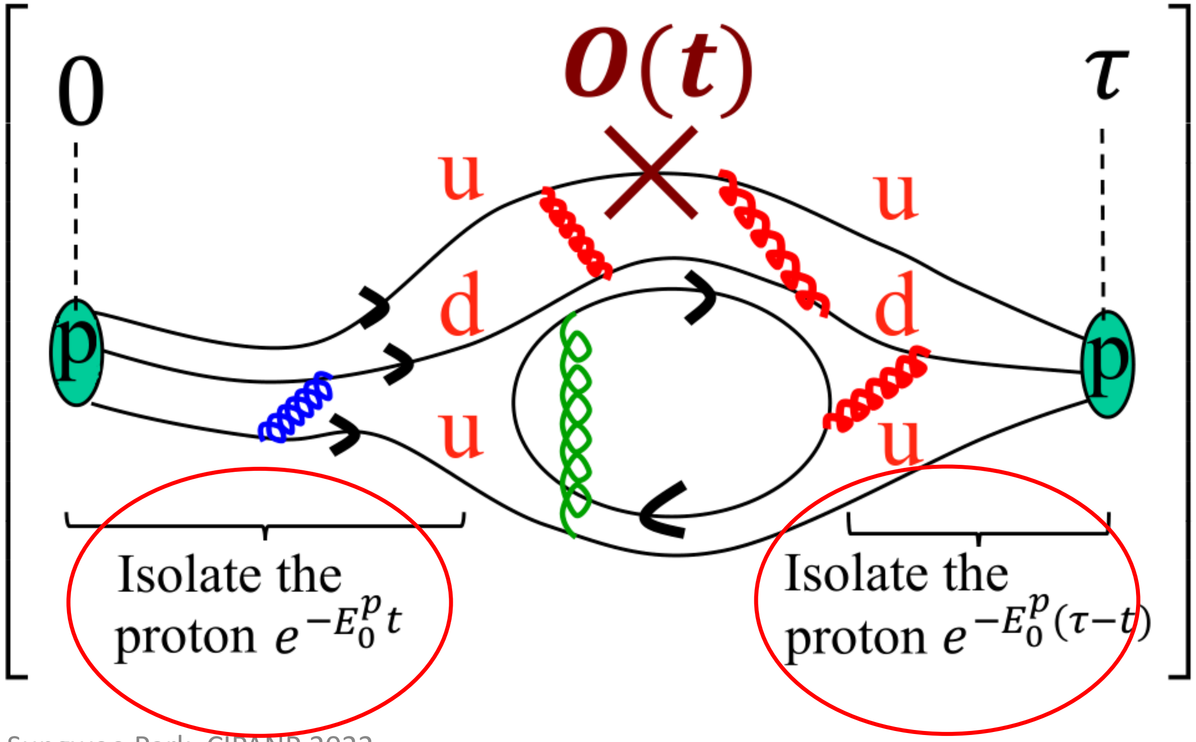
$$C(t,\tau) = \frac{1}{N_{\mathrm{samp}}} \sum_{\{U\}}$$



Excited State (ES) effect

- Nucleon signal/noise decays $\propto e^{-(E-1.5M_\pi)\tau}$ with Euclidean time τ.
- Nucleon operator creates ground state nucleons (N) plus all excited states with the same quantum number, including $N\pi, N\pi\pi, N\rho$, $N^*(1440), N^*(1710)$, $\cdots$.

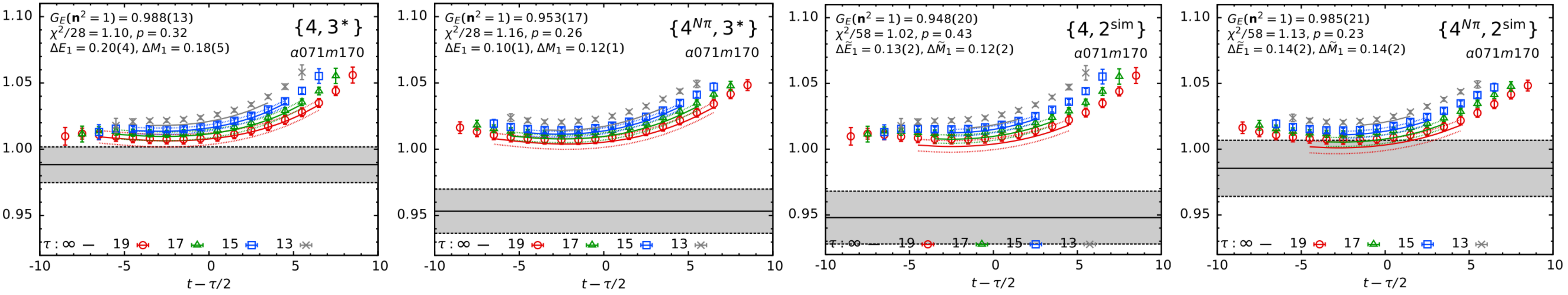
$$C(t,\tau) = \frac{1}{N_{\text{samp}}} \sum_{\{U\}}$$



[NME (2021), PRD 105 054505]
$a \approx 0.071$ fm, $M_\pi \approx 170$ MeV
At $\vec{q} = \frac{2\pi}{L}(1,0,0)$

G_E^{u-d}: Excited State (ES) effect

- Over 4 different strategies to control the ES effect, $G_E^{u-d}(\vec{q})$ has $\approx 4\%$ variation
- At larger momentum transfer $\vec{q}$, the data and fit become less sensitive to ES



- Data displayed: 3-point/2-point ratio of correlation functions showing dependence on t, τ due to ES
- Gray band: $G_E^{u-d}(\vec{q})$ determined from the ES fit.

Gauge Ensemble: 2+1-flavor Clover Ensembles

Ensemble ID	a [fm]	M_π [MeV]	$M_\pi L$	N_{conf}
a127m285	0.127	285	5.87	2002
a094m270	0.094	269	4.09	2469
a094m270L	0.094	269	6.15	4510
a093m220	0.093	216	4.95	2000
a093m220X	0.093	214	4.81	2005
a091m170	0.091	169	3.35	4012
a091m170L	0.091	170	5.01	3000
a073m270	0.073	272	4.81	4720
a072m220	0.072	223	5.10	2000
a071m170	0.071	166	4.28	2500
a070m130	0.070	127	4.37	980
a056m280	0.056	281	5.10	2700
a056m220	0.056	214	4.38	2049

- **13 gauge ensembles** generated by the Jlab/W&M/LANL/MIT collaborations
- Simulations are being done over a range of the three free parameters $(a, M_\pi, M_\pi L)$
- Results obtained by extrapolation to the physical values $(\boldsymbol{a = 0, M_\pi^{\text{Phys}}, M_\pi L = \infty})$

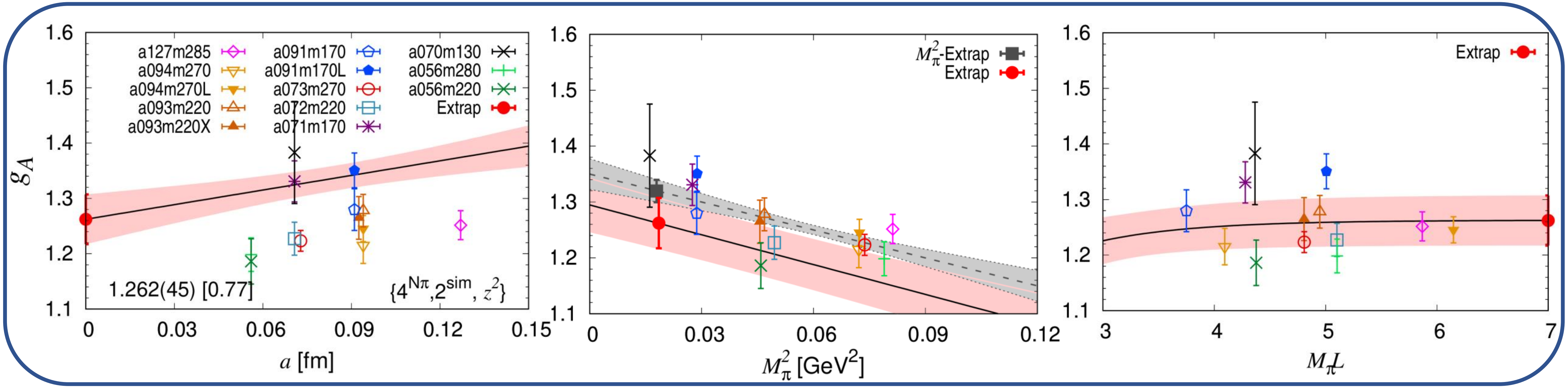
Summit (GPU) at OLCF

Major Uncertainties in Lattice QCD calculations

- Finite lattice spacing a (UV cut-off effect)
- Chiral fit to get value at physical pion mass
- Finite Volume

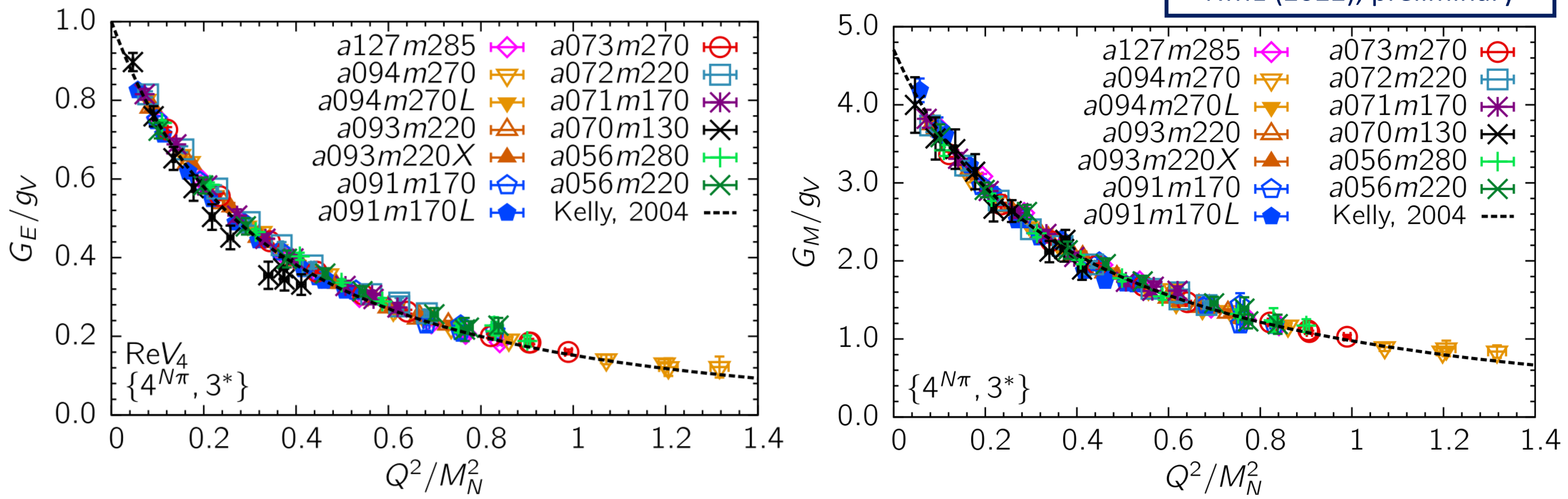
- Statistical errors
- Excited state contaminations
- Renormalization

Chiral-Continuum Finite volume extrapolation
of nucleon axial charge g_A [NME (2022), preliminary]



G_E^{u-d} , G_M^{u-d} and Kelly parametrization

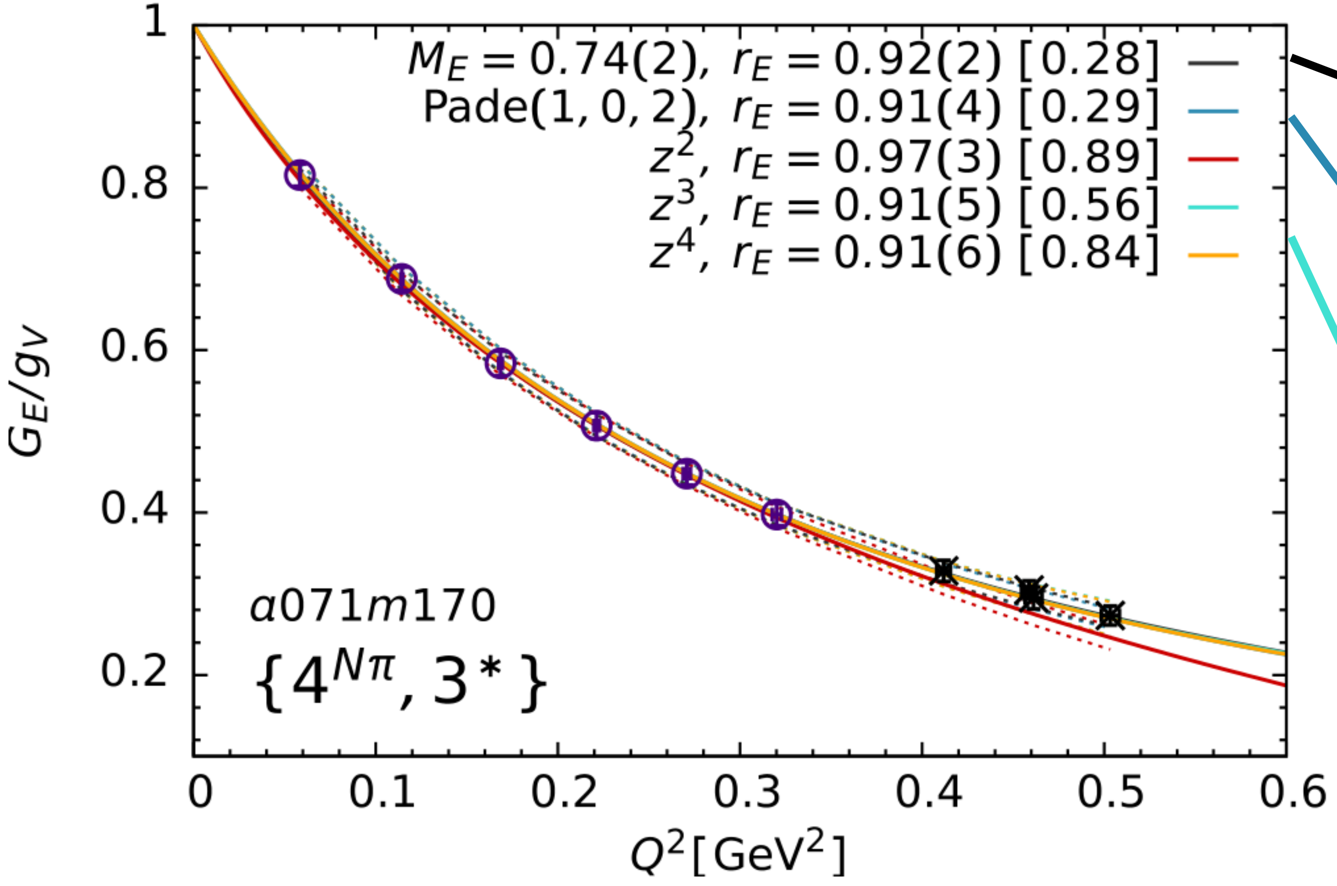
NME (2022), preliminary



- Lattice data agree with the Kelly parametrization of the experimental data
- Errors in lattice data for G_E^{u-d} , G_M^{u-d} much (>10X) larger than experimental data

[J.J.Kelly, PRC 70, 068202 (2004)]

G_E^{u-d} : Examined Dipole, Pade and z-expansion fits

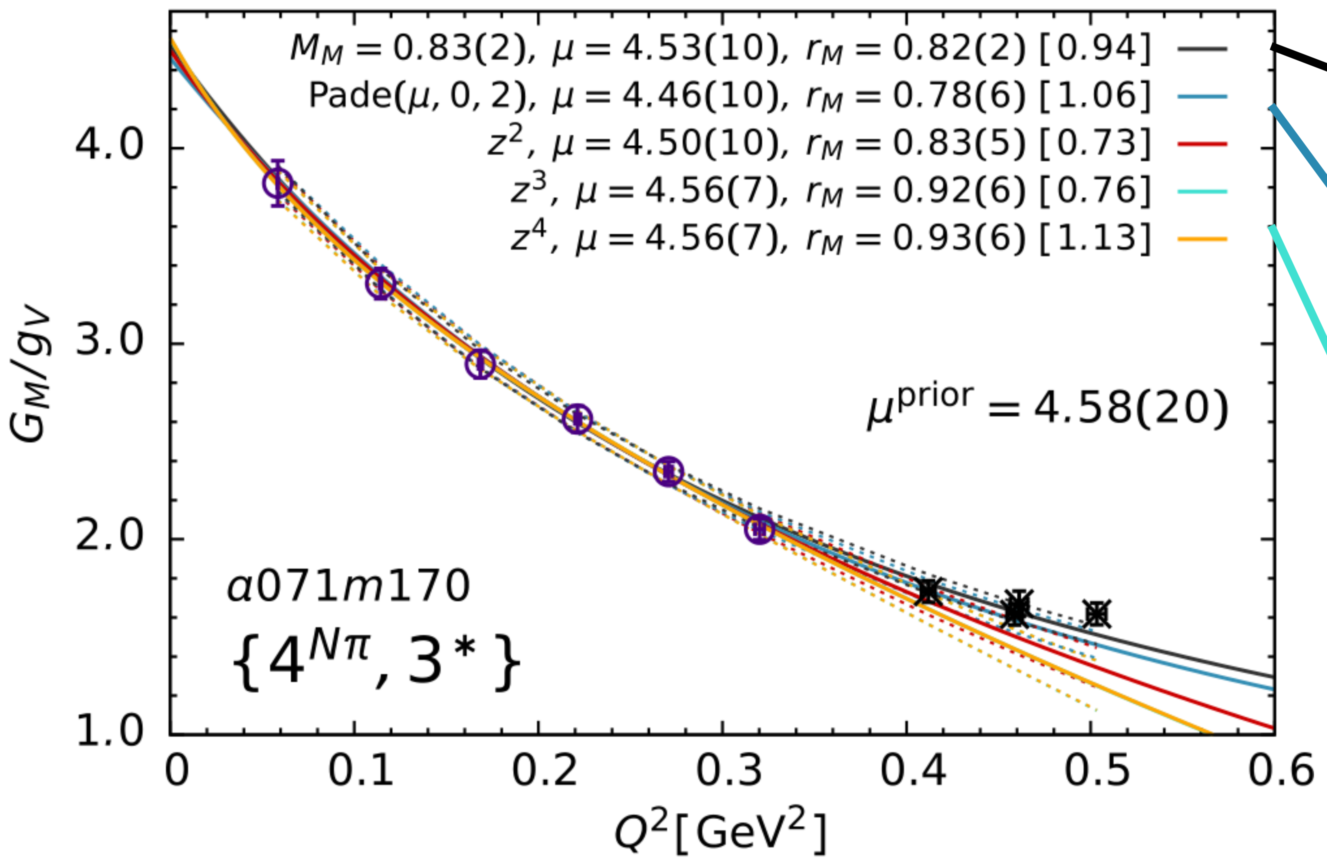


- Dipole: $\frac{G_E(0)/g_V}{\left(1+\frac{Q^2}{M_E^2}\right)^2}$
- Pade: $\frac{g}{1+b_1Q^2+b_2Q^4}$
- z^n-expansion: $\sum_{k=0}^{n} a_k z(Q^2)^k$

  Need z^3 to fit the data

[NME (2021), PRD 105 054505]

G_M^{u-d} : Examined Dipole, Pade and z-expansion fits



- Dipole: $\frac{G_M(0)/g_V}{\left(1+\frac{Q^2}{M_M^2}\right)^2}$
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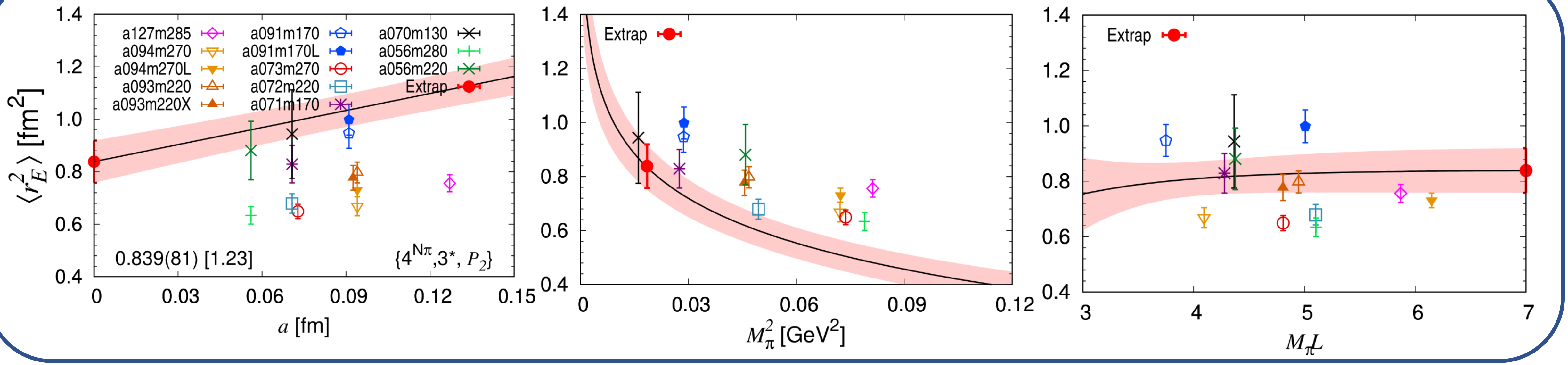
  Need z^3 to fit the data

[NME (2021), PRD 105 054505]

$\langle r_E^2 \rangle^{u-d}$: Chiral-Continuum-Finite Volume extrapolation

NME (2022), preliminary

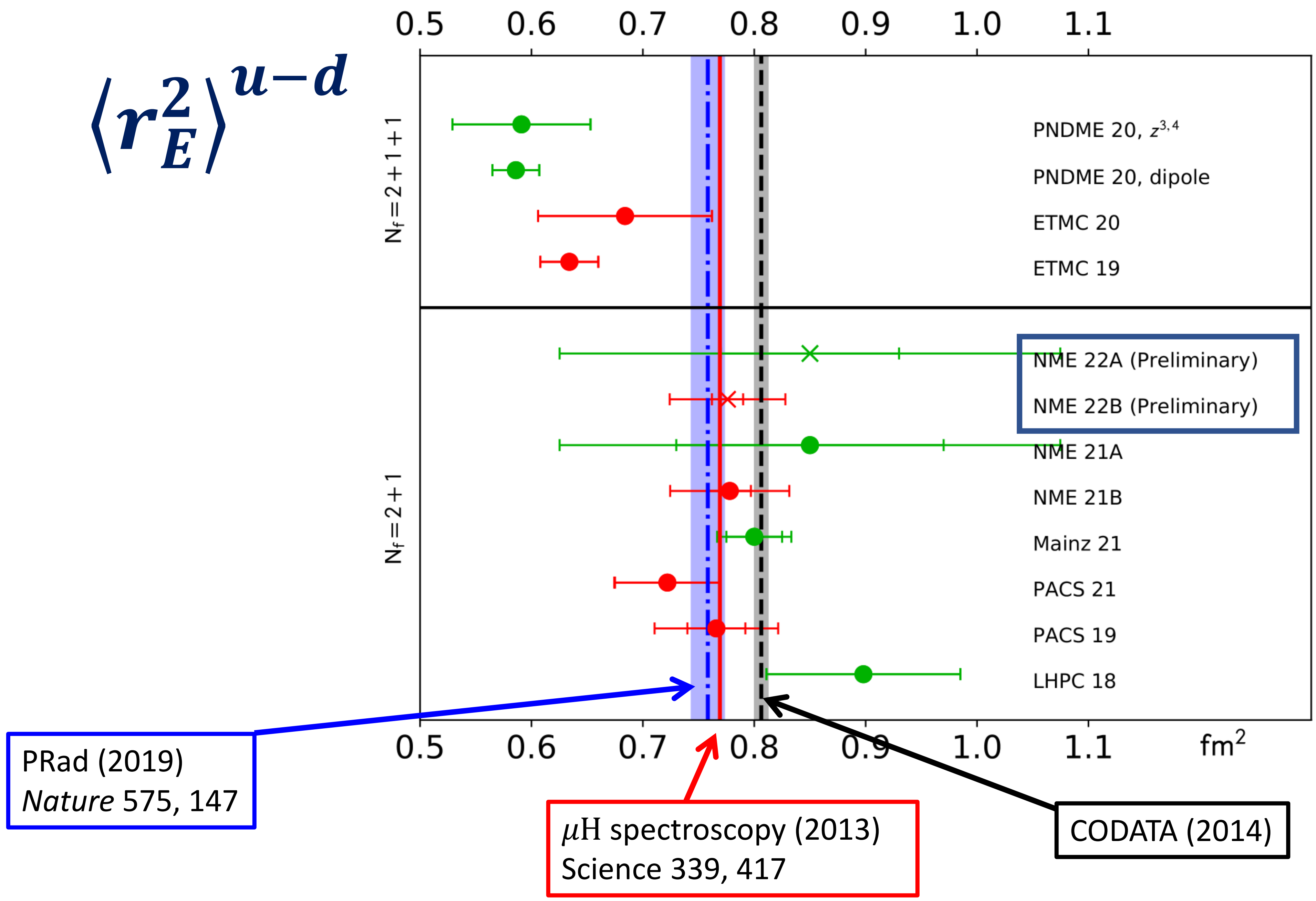


Leading corrections included in the fit ansatz

- $c_1 + c_2 a + c_3 \log\frac{M_\pi^2}{\lambda^2} + c_4 \log\frac{M_\pi^2}{\lambda^2} e^{-M_\pi L}$

Nucl.Phys.A635, 121 (1998)
Nucl.Phys.A679, 698 (2001)
Phys.Rev.D71, 034508 (2005)

$$\langle r_E^2 \rangle^{u-d}$$



Large systematic uncertainty from the excited state effect

Summary

- We are calculating isovector electromagnetic form factors $\boldsymbol{G_{E,M}^{u-d}(Q^2)}$ as part of a comprehensive analysis of nucleon structure
- Form factors presented as a function of Q^2 over $0.04 < Q^2 < 1\ \mathrm{GeV}^2$.
- The chiral-continuum-finite volume extrapolation of the isovector electric charge radius $\boldsymbol{\langle r_E^2 \rangle^{u-d}}$ is presented. This preliminary result is compared with other lattice and experimental results.
- Dominant systematic uncertainty in our preliminary results is from accounting for residual excited state (ES) effect
  - Need higher statistics to resolve the ES at M_π^{Phys} and on finer lattices (smaller a)
  - Higher order ES fits are under investigation

Acknowledgements

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