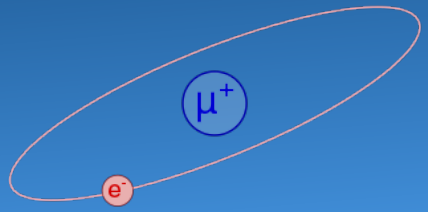

Muonium-antimuonium oscillations

Photo: LANGAN

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Alexey A. Petrov
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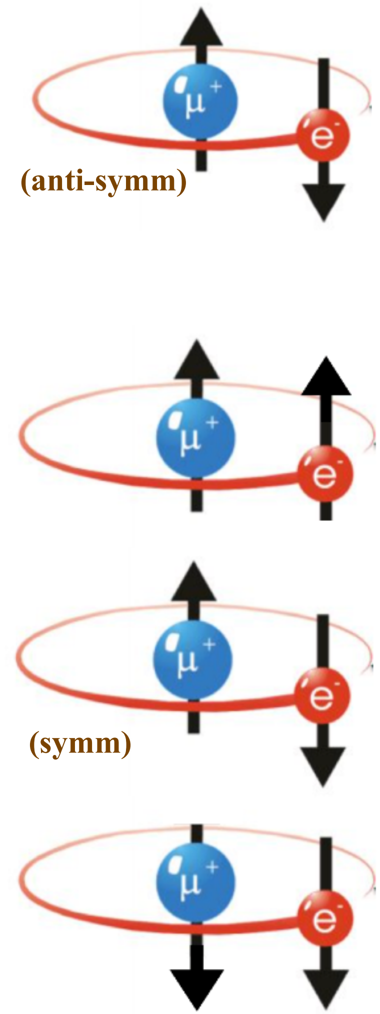
The simplest bound states: muonium

- Muonium: a bound state of μ^+ and e^-
  - $(\mu^+\mu^-)$ bound state is *true muonium*

- Muonium lifetime $\tau_{M_\mu} = 2.2\ \mu s$
  - main decay mode: $M_\mu \to e^+ e^- \bar{\nu}_\mu \nu_e$
  - annihilation: $M_\mu \to \bar{\nu}_\mu \nu_e$

- Muonium's been around since 1960's
  - used in chemistry
  - QED bound state physics, etc.
  - New Physics searches (oscillations)



Spin-0 (singlet) paramuonium

Spin-1 (triplet) orthomuonium

Hughes (1960)

The masses of singlet and triplet are almost the same!

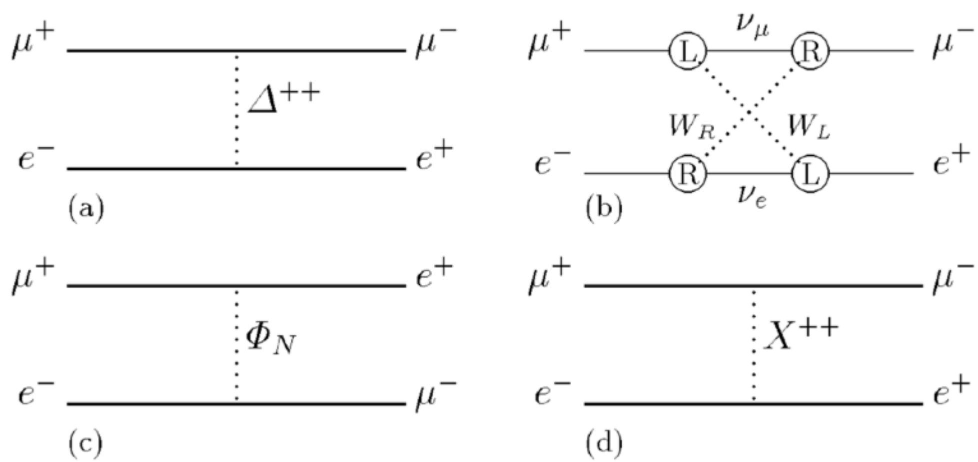
Muonium oscillations: just like $B^0\bar{B}^0$ mixing, but simpler!

★ Lepton-flavor violating interactions can change $M_\mu \to \overline{M}_\mu$

Pontecorvo (1957)
Feinberg, Weinberg (1961)

- Such transition amplitudes are tiny in the Standard Model
  - … but there are plenty of New Physics models where it can happen

Clark, Love; Cvetic et al,
Li, Schmidt; Endo, Iguro, Kitahara;
Fukuyama, Mimura, Uesaka; …



$$\sim (\bar{\mu}\Gamma e)(\bar{\mu}\Gamma e)$$

effective operator

  - theory: compute transition amplitudes for ALL New Physics models!
  - experiment: produce M_μ but look for the decay products of $\overline{M}_\mu$

Combined evolution = flavor oscillations

- If there is an interaction that couples M_μ and $\overline{M}_\mu$ (both SM or NP)
  - combined time evolution: non-diagonal Hamiltonian!

$$i\frac{d}{dt}\begin{pmatrix}|M(t)\rangle \\ |\overline{M}(t)\rangle\end{pmatrix} = \left(m - i\frac{\Gamma}{2}\right)\begin{pmatrix}|M(t)\rangle \\ |\overline{M}(t)\rangle\end{pmatrix}$$

  - diagonalization: new mass eigenstates:

$$|M_{\mu_{1,2}}\rangle = \frac{1}{\sqrt{2}}\left[|M_\mu\rangle \mp |\overline{M}_\mu\rangle\right]$$

  - new mass eigenstates: mass and lifetime differences

$$\left.\begin{array}{l}\Delta m \equiv M_1 - M_2, \\ \Delta\Gamma \equiv \Gamma_2 - \Gamma_1.\end{array}\right\} \quad x = \frac{\Delta m}{\Gamma}, \qquad y = \frac{\Delta\Gamma}{2\Gamma} \quad \text{(small)}$$

These mass and width difference are observable quantities

Combined evolution = flavor oscillations

- Study oscillations via decays: amplitudes for $M_\mu \to f$ and $\overline{M}_\mu \to \bar{f}$
  - possibility of flavor oscillations ($M_\mu \to \overline{M}_\mu \to \bar{f}$)

$$\begin{aligned} |M(t)\rangle &= g_+(t)\,|M_\mu\rangle + g_-(t)\,\left|\overline{M}_\mu\right\rangle, \\ \left|\overline{M}(t)\right\rangle &= g_-(t)\,|M_\mu\rangle + g_+(t)\,\left|\overline{M}_\mu\right\rangle, \end{aligned}$$

with

$$\begin{aligned} g_+(t) &= e^{-\Gamma_1 t/2} e^{-i m_1 t} \left[1 + \frac{1}{8}\,(y - ix)^2\,(\Gamma t)^2\right], \\ g_-(t) &= \frac{1}{2} e^{-\Gamma_1 t/2} e^{-i m_1 t}\,(y - ix)\,(\Gamma t)\,. \end{aligned}$$

  - time-dependent width: $\Gamma(M_\mu \to \overline{f})(t) = \frac{1}{2} N_f\,|A_f|^2\,e^{-\Gamma t}\,(\Gamma t)^2\,R_M(x,y)$
  - oscillation probability: $\boxed{P(M_\mu \to \overline{M}_\mu) = \frac{\Gamma(M_\mu \to \overline{f})}{\Gamma(M_\mu \to f)} = R_M(x,y) = \frac{1}{2}\left(x^2 + y^2\right)}$

R. Conlin and AAP

Oscillation parameters: introduction

- Mixing parameters are related to off-diagonal matrix elements
  - heavy and light intermediate degrees of freedom

$$\left(m - \frac{i}{2}\Gamma\right)_{12} = \frac{1}{2M_M}\left\langle \overline{M}_\mu \left|\mathcal{H}_{\rm eff}\right| M_\mu \right\rangle + \frac{1}{2M_M}\sum_n \frac{\left\langle \overline{M}_\mu \left|\mathcal{H}_{\rm eff}\right| n \right\rangle \left\langle n \left|\mathcal{H}_{\rm eff}\right| M_\mu \right\rangle}{M_M - E_n + i\epsilon}$$

Local at scale $\mu = M_\mu$: only Δm
lepton number change $\Delta L_\mu = 2$

Bi-local at scale $\mu = M_\mu$: both Δm and $\Delta\Gamma$
lepton number changes: $(\Delta L_\mu = 1)^2$
or $(\Delta L_\mu = 0)(\Delta L_\mu = 2)$

  - each term has contributions from different effective Lagrangians

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm eff}^{\Delta L_\mu=0} + \mathcal{L}_{\rm eff}^{\Delta L_\mu=1} + \mathcal{L}_{\rm eff}^{\Delta L_\mu=2}$$

  - ... all of which have a form $\mathcal{L}_{\rm eff} = -\frac{1}{\Lambda^2}\sum_i c_i(\mu) Q_i,$ with $\Lambda \sim \mathcal{O}(TeV)$

Mass difference = real (dispersive) part; width difference: imaginary (absorptive) part

Mass difference

- Mass difference comes from the dispersive part

$$x = \frac{1}{2M_M\Gamma}\mathrm{Re}\left[2\langle\overline{M}_\mu\left|\mathcal{H}_{\text{eff}}\right|M_\mu\rangle + \cancel{\langle\overline{M}_\mu\left|i\int d^4x\ \mathrm{T}\left[\mathcal{H}_{\text{eff}}(x)\mathcal{H}_{\text{eff}}(0)\right]\right|M_\mu\rangle}\right]$$

  - consider only $\Delta L_\mu = 2$ Lagrangian contributions (largest?)

$$\mathcal{L}_{\text{eff}}^{\Delta L_\mu=2} = -\frac{1}{\Lambda^2}\sum_i C_i^{\Delta L=2}(\mu)Q_i(\mu)$$

  - leading order: all heavy New Physics models are encoded in (the Wilson coefficients of) the five dimension-6 operators

$$\begin{aligned}
Q_1 &= \left(\overline{\mu}_L\gamma_\alpha e_L\right)\left(\overline{\mu}_L\gamma^\alpha e_L\right), \quad Q_2 = \left(\overline{\mu}_R\gamma_\alpha e_R\right)\left(\overline{\mu}_R\gamma^\alpha e_R\right), \\
Q_3 &= \left(\overline{\mu}_L\gamma_\alpha e_L\right)\left(\overline{\mu}_R\gamma^\alpha e_R\right), \quad Q_4 = \left(\overline{\mu}_L e_R\right)\left(\overline{\mu}_L e_R\right), \\
Q_5 &= \left(\overline{\mu}_R e_L\right)\left(\overline{\mu}_R e_L\right).
\end{aligned}$$

  - need to compute matrix elements for both singlet and triplet states

Mass difference: matrix elements

- QED bound state: know leading order wave function!
  - spacial part is the same as in Hydrogen atom

$$\varphi(r) = \frac{1}{\sqrt{\pi a_{M_\mu}^3}} e^{-\frac{r}{a_{M_\mu}}}$$

  - can unambiguously compute decay constants and mixing MEs (QED)

$$\langle 0| \overline{\mu}\gamma^\alpha\gamma^5 e \left| M_\mu^P \right\rangle = i f_P p^\alpha, \quad \langle 0| \overline{\mu}\gamma^\alpha e \left| M_\mu^V \right\rangle = f_V M_M \epsilon^\alpha(p),$$
$$\langle 0| \overline{\mu}\sigma^{\alpha\beta} e \left| M_\mu^V \right\rangle = i f_T \left(\epsilon^\alpha p^\beta - \epsilon^\beta p^\alpha\right),$$

  - in the non-relativistic limit all decay constants $f_P = f_V = f_T = f_M$

$$f_M^2 = 4\frac{|\varphi(0)|^2}{M_M}$$ (QED version of Van Royen-Weisskopf)

  - NR matrix elements: “vacuum insertion” = direct computation

Mass difference: results

- Spin-singlet muonium state:
  - matrix elements:

$$\langle \bar{M}_\mu^P | Q_1 | M_\mu^P \rangle = f_M^2 M_M^2, \quad \langle \bar{M}_\mu^P | Q_2 | M_\mu^P \rangle = f_M^2 M_M^2,$$
$$\langle \bar{M}_\mu^P | Q_3 | M_\mu^P \rangle = -\frac{3}{2} f_M^2 M_M^2, \quad \langle \bar{M}_\mu^P | Q_4 | M_\mu^P \rangle = -\frac{1}{4} f_M^2 M_M^2,$$
$$\langle \bar{M}_\mu^P | Q_5 | M_\mu^P \rangle = -\frac{1}{4} f_M^2 M_M^2.$$

$$\boxed{x_P = \frac{4(m_{red}\alpha)^3}{\pi \Lambda^2 \Gamma} \left[C_1^{\Delta L=2} + C_2^{\Delta L=2} - \frac{3}{2} C_3^{\Delta L=2} - \frac{1}{4} \left(C_4^{\Delta L=2} + C_5^{\Delta L=2} \right) \right]}$$

- Spin-triplet muonium state:
  - matrix elements

$$\langle \bar{M}_\mu^V | Q_1 | M_\mu^V \rangle = -3 f_M^2 M_M^2, \quad \langle \bar{M}_\mu^V | Q_2 | M_\mu^V \rangle = -3 f_M^2 M_M^2,$$
$$\langle \bar{M}_\mu^V | Q_3 | M_\mu^V \rangle = -\frac{3}{2} f_M^2 M_M^2, \quad \langle \bar{M}_\mu^V | Q_4 | M_\mu^V \rangle = -\frac{3}{4} f_M^2 M_M^2,$$
$$\langle \bar{M}_\mu^V | Q_5 | M_\mu^V \rangle = -\frac{3}{4} f_M^2 M_M^2.$$

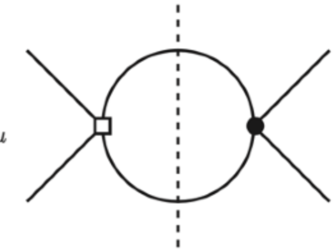
$$\boxed{x_V = -\frac{12(m_{red}\alpha)^3}{\pi \Lambda^2 \Gamma} \left[C_1^{\Delta L=2} + C_2^{\Delta L=2} + \frac{1}{2} C_3^{\Delta L=2} + \frac{1}{4} \left(C_4^{\Delta L=2} + C_5^{\Delta L=2} \right) \right]}$$

Experimental constraints on x result in experimental constraints on Wilson coefficients $C_k^{\Delta L=2}$ that encode all information about possible New Physics contributions

R. Conlin and AAP, Phys.Rev.D 102 (2020) 9, 095001

Width difference and muonium decays

- Width difference comes from the absorptive part
  - light SM intermediate states ($e^+e^-, \gamma\gamma, \bar{\nu}\nu, etc.$)
  - $\bar{\nu}\nu$ state gives parametrically largest contribution



$$\Gamma(M_\mu^V \to \bar{\nu}_\mu \nu_e) = \frac{G_F^2 f_M^2 M_M^3}{12\pi} \qquad Br(M_\mu^V \to \bar{\nu}_\mu \nu_e) = 8.8 \times 10^{-12}$$

AAP, R. Conlin, C. Grant

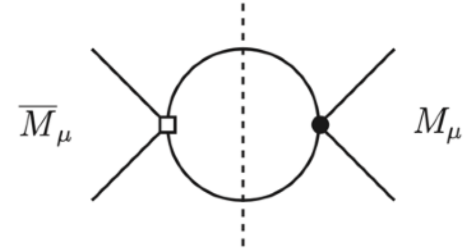
- Muonium two- and there-body decays
  - two-body decays ($M_\mu^{V,P} \to e^+e^-, \gamma\gamma, etc$) are dominated by New Physics
  - probe different combinations of SM EFT Wilson coefficients
    - e.g. $\mu \to 3e$ vs. $M_\mu \to e^+e^-$ (also phase space enhancement)

R. Conlin, J. Osborne, AAP

  - can $M_\mu \to invisible$ (SM: $M_\mu \to \nu_e \bar{\nu}_\mu$) be measured?

Gninenko, Krasnikov, Matveev. Phys.Rev. D87 (2013) 015016

- Width difference comes from the absorptive part
  - light SM intermediate states ($e^+e^-, \gamma\gamma, \bar{\nu}\nu, etc.$)
  - $\bar{\nu}\nu$ state gives parametrically largest contribution



$$y = \frac{1}{2M_M\Gamma}\mathrm{Im}\left[\langle\overline{M}_\mu\left|i\int d^4x\ \mathrm{T}\left[\mathcal{H}_{\rm eff}(x)\mathcal{H}_{\rm eff}(0)\right]\right|M_\mu\rangle\right]$$
$$= \frac{1}{M_M\Gamma}\mathrm{Im}\left[\langle\overline{M}_\mu\left|i\int d^4x\ \mathrm{T}\left[\mathcal{H}_{\rm eff}^{\Delta L_\mu=2}(x)\mathcal{H}_{\rm eff}^{\Delta L_\mu=0}(0)\right]\right|M_\mu\rangle\right]$$

New Physics $\Delta L_\mu = 2$ contribution

Standard Model $\Delta L_\mu = 0$ contribution

$$\mathcal{L}_{\rm eff}^{\Delta L_\mu=2} = -\frac{1}{\Lambda^2}\sum_i C_i^{\Delta L=2}(\mu)Q_i(\mu)$$

$$\mathcal{L}_{\rm eff}^{\Delta L_\mu=0} = -\frac{4G_F}{\sqrt{2}}\left(\overline{\mu}_L\gamma_\alpha e_L\right)\left(\overline{\nu_e}_L\gamma^\alpha \nu_{\mu L}\right)$$

$$Q_6 = \left(\overline{\mu}_L\gamma_\alpha e_L\right)\left(\overline{\nu_\mu}_L\gamma^\alpha\nu_{eL}\right),$$
$$Q_7 = \left(\overline{\mu}_R\gamma_\alpha e_R\right)\left(\overline{\nu_\mu}_L\gamma^\alpha\nu_{eL}\right)$$

$$\Gamma(M_\mu^V \to \overline{\nu}_e\nu_\mu) = \frac{f_M^2 M_M^3}{9\pi\Lambda^4}\left|C_6^{\Delta L_\mu=2} + C_7^{\Delta L_\mu=2}\right|^2 \qquad \Big\| \qquad \Gamma(M_\mu^V \to \overline{\nu}_\mu\nu_e) = \frac{G_F^2 f_M^2 M_M^3}{12\pi}$$

Width difference: results

- Spin-singlet muonium state:

$$y_P = \frac{G_F}{\sqrt{2}\Lambda^2}\frac{M_M^2}{\pi^2\Gamma}(m_{red}\alpha)^3\left(C_6^{\Delta L=2} - C_7^{\Delta L=2}\right)$$

- Spin-triplet muonium state:

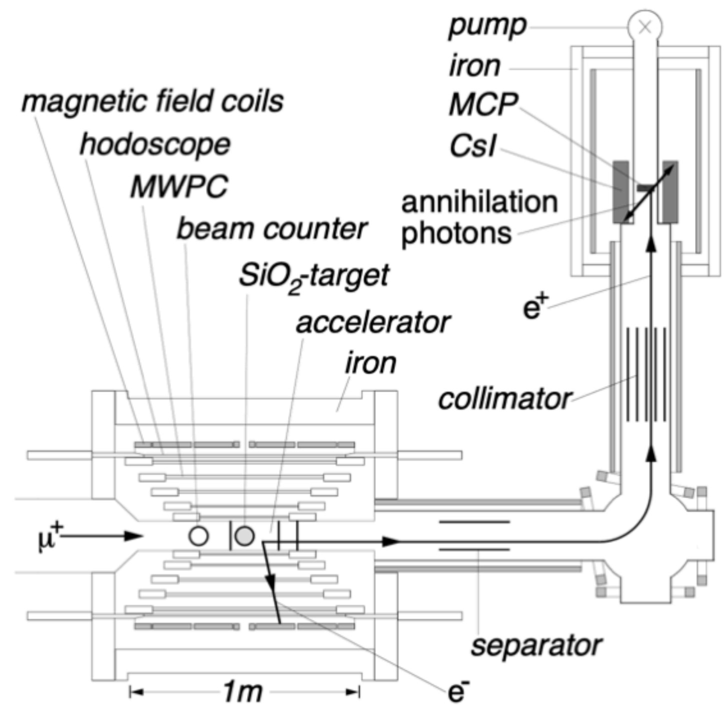
$$y_V = -\frac{G_F}{\sqrt{2}\Lambda^2}\frac{M_M^2}{\pi^2\Gamma}(m_{red}\alpha)^3\left(5C_6^{\Delta L=2} + C_7^{\Delta L=2}\right)$$

- Note: y has the same $1/\Lambda^2$ suppression as the mass difference!

R. Conlin and AAP, Phys.Rev.D 102 (2020) 9, 095001

Experimental setup and constraints

- Similar experimental set ups for different experiments
  - example: MACS at PSI
  - idea: form M_μ by scattering muon (μ^+) beam on SiO_2 powder target
- A couple of "little inconveniences":
  - ➡ how to tell f apart from $\bar{f}$?
    - $M_\mu \to f$ decay: $M_\mu \to e^+ e^- \bar{\nu}_\mu \nu_e$
    - $\overline{M}_\mu \to \bar{f}$ decay: $\overline{M}_\mu \to e^+ e^- \bar{\nu}_e \nu_\mu$
    - $\bar{f}$: fast e^- (~53 MeV), slow e^+ (13.5 eV)
  - ➡ oscillations happen in magnetic field
    - … which selects M_μ vs. $\overline{M}_\mu$



Muonium-Antimuonium Conversion Spectrometer (MACS)

L. Willmann, et al. PRL 82 (1999) 49

The most recent experimental data comes from 1999! Time is ripe for an update!

- MACS: observed 5.7×10^{10} muonium atoms after 4 months of running
  - magnetic field is taken into account (suppression factor)

Interaction type	2.8 μT	0.1 T	100 T
SS	0.75	0.50	0.50
PP	1.0	0.9	0.50
$(V \pm A) \times (V \pm A)$ or $(S \pm P) \times (S \pm P)$	0.75	0.35	0.0
$(V \pm A) \times (V \mp A)$ or $(S \pm P) \times (S \mp P)$	0.95	0.78	0.67

L. Willmann, et al. PRL 82 (1999) 49

  - no oscillations have been observed (yet!)

Experimental constraints

- We can now put constraints on the Wilson coefficients of effective operators from experimental data (assume single operator dominance)
  - presence of the magnetic field

$$P(M_\mu \to \overline{M}_\mu) \leq 8.3 \times 10^{-11}/S_B(B_0)$$

  - no separation of spin states: average

$$P(M_\mu \to \overline{M}_\mu)_{\rm exp} = \sum_{i=P,V} \frac{1}{2S_i+1} P(M_\mu{}^i \to \overline{M}_\mu{}^i)$$

  - set Wilson coefficients to one, set constraints on the scale probed

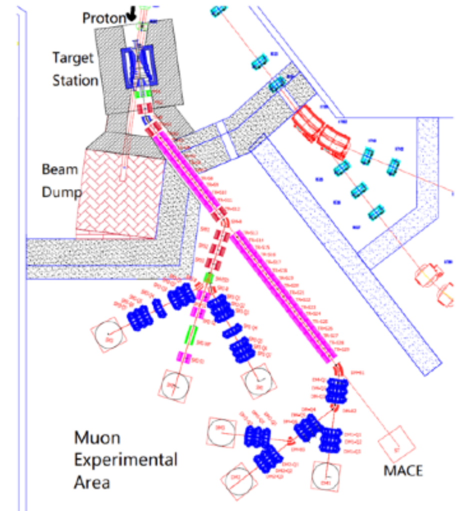
Operator	Interaction type	$S_B(B_0)$ (from [9])	Constraints on the scale Λ, TeV
Q_1	$(V-A)\times(V-A)$	0.75	5.4
Q_2	$(V+A)\times(V+A)$	0.75	5.4
Q_3	$(V-A)\times(V+A)$	0.95	5.4
Q_4	$(S+P)\times(S+P)$	0.75	2.7
Q_5	$(S-P)\times(S-P)$	0.75	2.7
Q_6	$(V-A)\times(V-A)$	0.75	0.58×10^{-3}
Q_7	$(V+A)\times(V-A)$	0.95	0.38×10^{-3}

R. Conlin and AAP, Phys.Rev.D 102 (2020) 9, 095001

New muon sources: CSNS

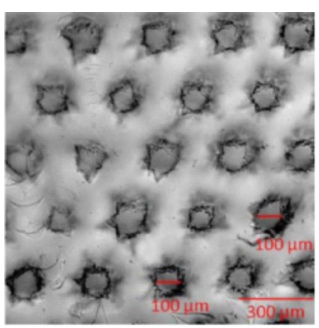
- Experimental Muon Source (EMuS) at Chinese Spallation Neutron Source
  - CSNS proton driver can be used to produce muons

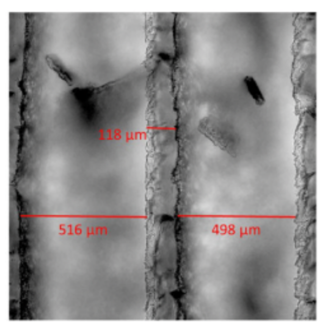
	Proton driver [MW]	Intensity [$\times 10^6$/s]	Polarization[%]	Spread [%]
PSI	1.30	420	90	10
ISIS	0.16	1.5	95	≤ 15
RIKEN/RAL	0.16	0.8	95	≤ 15
JPARC	1.00	100	95	15
TRIUMF	0.075	1.4	90	7
EMuS	0.025	83	50	10



  - EMuS will produce up to $10^9\ \mu^+/s$, which will be transported to MACE

- Muonium states will be formed in laser-ablated silica aerogel target





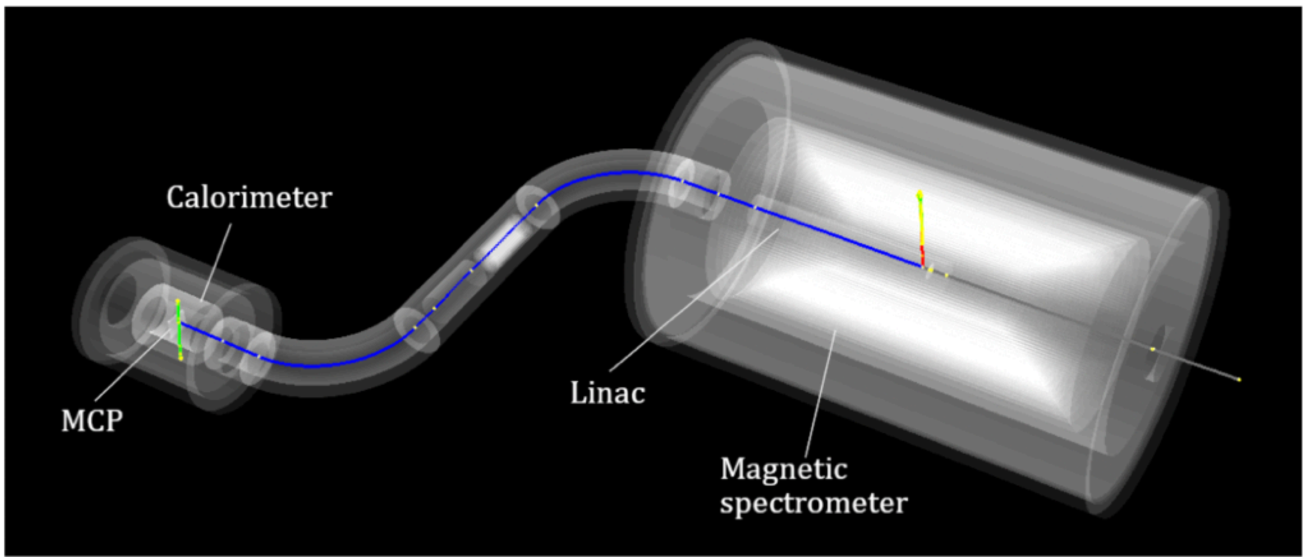
  - the muonium emission rate of aerogel target with holes is up to 36 times higher than that of silica powder target used in MACS

J. Beare et al, Prog. Theor. Exp. Phys. 2020, 123C01

- Muonium-to-Antimuonium Conversion Experiment (MACE)

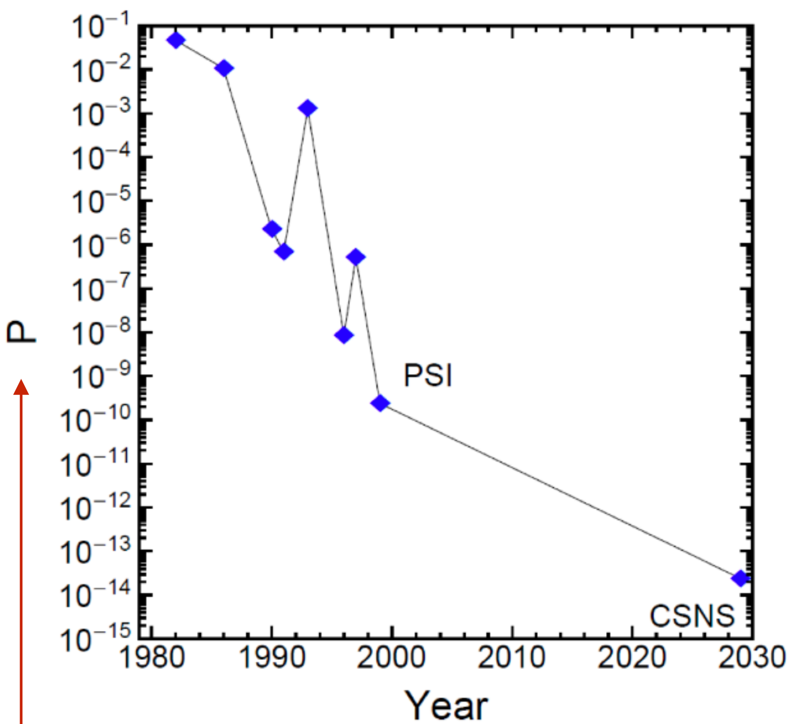
A.-Y. Bai, et al arXiv:2203.11406 [hep-ph]

  - MACE uses the same kinematical tag as MACS
    - $M_\mu \to f$ decay: $M_\mu \to e^+ e^- \bar{\nu}_\mu \nu_e$
    - $\overline{M}_\mu \to \bar{f}$ decay: $\overline{M}_\mu \to e^+ e^- \bar{\nu}_e \nu_\mu$
    - $\bar{f}$: fast (Michel) e^- of 52.8 MeV and slow (shell) e^+ of 13.5 eV



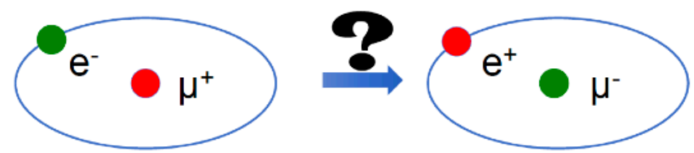
  - Triple coincidence: The Michel electron is detected by the drift chamber. The atomic-shell positron is accelerated and transported to the MCP and annihilates into two photons. The photons are detected by the electromagnetic calorimeter.

Fundamental science with EMuS (China)



$$P(M_\mu \to \overline{M}_\mu) = \frac{\Gamma(M_\mu \to \overline{f})}{\Gamma(M_\mu \to f)} = R_M(x,y)$$

$$R_M(x,y) = \frac{1}{2}\left(x^2 + y^2\right)$$



- The latest bound was done at PSI more than 20 years ago with a muon intensity $8 \times 10^6 \mu^+/s$ and high-precision magnetic spectrometer.
- Timing resolution in detector: ~ ns
- Position resolution in detector: ~ mm
- EMuS plan to offer $10^9 \mu^+/s$
- Current timing resolution in detector: ~ ps
- Current position resolution in detector:~μs
- Expect to be improved by > O(10^2)?

MACE experiment at EMuS (Chinese SNS)
Jian Tang, talk at RPPM meeting (Snowmass 2021)

Snowmass2021 Whitepaper: Muonium to antimuonium conversion
A.-Y. Bai, et al arXiv:2203.11406 [hep-ph]

Conclusions and things to take home

- There is no indication from high energy studies where the NP show up
  - this makes indirect searches the most valuable source of information
- Muonium is the simplest atom: atomic physics
  - level splitting (Lamb shift): probe NP w/out QCD complications

MuSEUM experiment (J-PARC)

- Muons are ideal tools to probe fundamental physics
  - flavor-conserving quantities (g-2, EDM)
  - flavor-changing neutral current decays
  - flavor oscillations (muonium-antimuonium conversion)
  - muon transitions already probe the LHC energy domain and can do better!

Prospects for precise predictions of a_μ in the Standard Model
G. Colangelo, et. al., arXiv:2203.15810 [hep-ph]

- New experimental facilities: MACE at CSNS
  - similar domestic experiment at AMF (FNAL) or SNS (Oak Ridge)?
  - possible muonium oscillation experiment at J-PARC (Japan)?

Snowmass2021 Whitepaper: Muonium to antimuonium conversion
A.-Y. Bai, …, AAP, …, arXiv:2203.11406 [hep-ph]


The Evening Herald
EXTRA!

Effective Lagrangians and particular models

- Effective Lagrangian approach encompasses all models
  - lets look at an example of a model with a doubly charged Higgs Δ^{--}
  - this is common for the left-right models, etc.

$$\mathcal{L}_R = g_{\ell\ell} \overline{\ell}_R \ell^c \Delta + H.c.,$$

  - integrate out Δ^{--} to get

$$\mathcal{H}_\Delta = \frac{g_{ee} g_{\mu\mu}}{2M_\Delta^2} \left(\overline{\mu}_R \gamma_\alpha e_R\right) \left(\overline{\mu}_R \gamma^\alpha e_R\right) + H.c.,$$

  - match to $\mathscr{L}_{\text{eff}}^{\Delta L=2}$ to see that $M_\Delta = \Lambda$ and

$$C_2^{\Delta L=2} = g_{ee} g_{\mu\mu}/2.$$

Chang, Keung (89);
Schwartz (89);
Han, Tang, Zhang (21)

Is it better than/worse than/complimentary to $\mu \to 3e$?

- A possibility of using muon beams at CMP facilities

Jian Tang, talk at RPPM meeting (Snowmass 2021)

	Proton driver [MW]	Surface muons			Decay muons		
		Intensity [1E6/s]	Polarization [%]	Spread [%]	energy [MeV/c]	Intensity [1E6/s]	Spread [%]
PSI	1.3	420	90	10	85-125	240	3
ISIS	0.16	1.5	95	<15	20-120	0.4	10
RIKEN/RAL	0.16	0.8	95	<15	65-120	1	10
JPARC	1	100	95	15	33-250	10	15
TRIUMF	0.075	1.4	90	7	20-100	0.0014	10
EMuS	0.005	83	50	10	50-450	16	10
Baby EMuS	0.005	1.2	95	10			

×5 CSNS-II upgrade

Facility	Source Type	Intensity (μ+/sec)*
ISIS	pulsed	1.5×10^6
J-PARC	continuous	1.8×10^6
PSI	continuous	7.0×10^4
TRIUMF	pulsed	5.0×10^6
SEEMS	pulsed	1.9×10^8

- Muonium Antimuonium Conversion Experiment (MACE) EMuS at CSNS

- Muon decay $\mu \to 3e$:

$$
\begin{aligned}
\Gamma & \left(\mu \to 3e\right) = \\
= & \frac{\alpha m_\mu^5}{3\Lambda^4 (4\pi)^2} \left(|C_{DL}|^2 + |C_{DR}|^2\right) \left(8 \log\left[\frac{m_\mu}{m_e}\right] - 11\right) \\
+ & \frac{4 m_\mu^5}{3\Lambda^4 (16\pi)^3} \left(m_e^4 G_F^2 \left(|C_{SR}^e|^2 + |C_{SL}^e|^2\right)\right. \\
+ & 2\left(2\left(|C_{VR}^e|^2 + |C_{VL}^e|^2 + |C_{AR}^e|^2 + |C_{AL}^e|^2\right) + |C_{AR}^e + C_{VR}^e|^2 + |C_{AL}^e - C_{VL}^e|^2\right)\Big) \\
- & \frac{\sqrt{4\pi\alpha} m_\mu^5}{3\Lambda^4 (4\pi)^3} \left(\Re\left[C_{DL}\left(3C_{VR}^e + C_{AR}^e\right)^*\right] + \Re\left[C_R^D\left(3C_{VL}^e - C_{AL}^e\right)^*\right]\right)
\end{aligned}
$$

- Muonium decay $M_\mu^V \to e^+ e^-$:

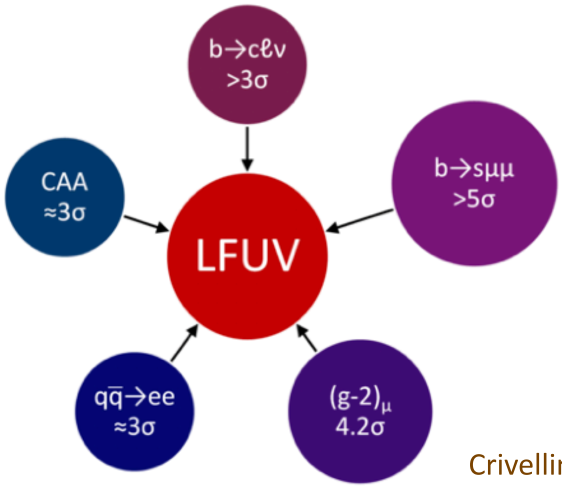
$$
\begin{aligned}
\Gamma\left(M_\mu^V \to e^+ e^-\right) = & \frac{f_M^2 M_M^3}{48\pi\Lambda^4} \left\{\frac{3}{2}|C_{VR}^e + C_{AR}^e|^2 - \frac{3}{2}|C_{VL}^e + C_{AL}^e|^2 \right. \\
& \left. + |2C_{VL}^e + C_{VR}^e|^2 + |2C_{AL}^e + C_{AR}^e|^2\right\}
\end{aligned}
$$

- Note: different combination of Wilson coefficients!

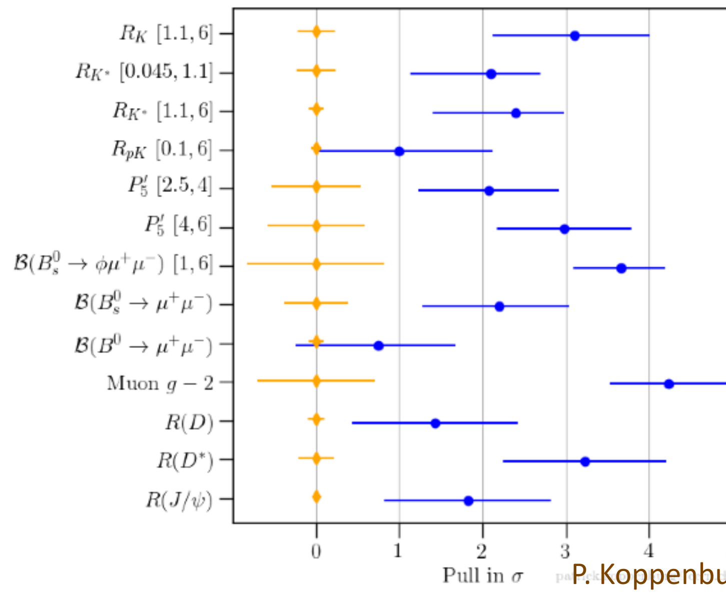
R. Conlin, J. Osborne, AAP

Muons and recent experimental anomalies

★ Many experimental anomalies involve interactions with muons and taus

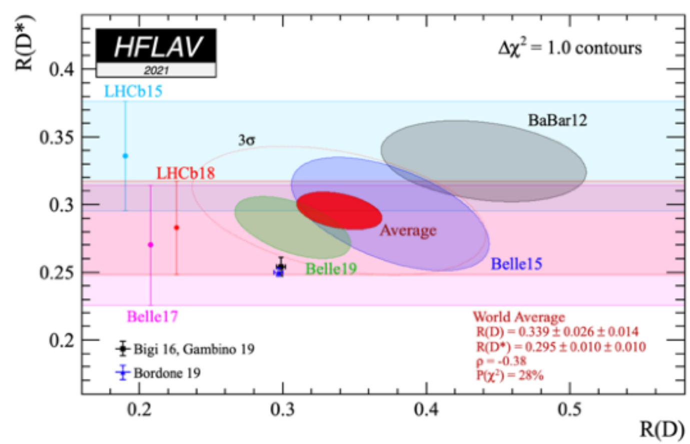


Crivellin, Hoferichter



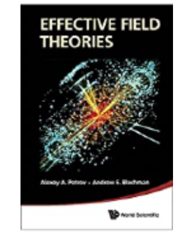
P. Koppenburg

- other lepton-flavor conserving processes
  - magnetic properties: muon g-2
    - currently a discrepancy theory/exp
  - electric properties: muon EDM
    - probes CP-violation in leptons
  - muonic hydrogen
    - proton size/QED/New Physics

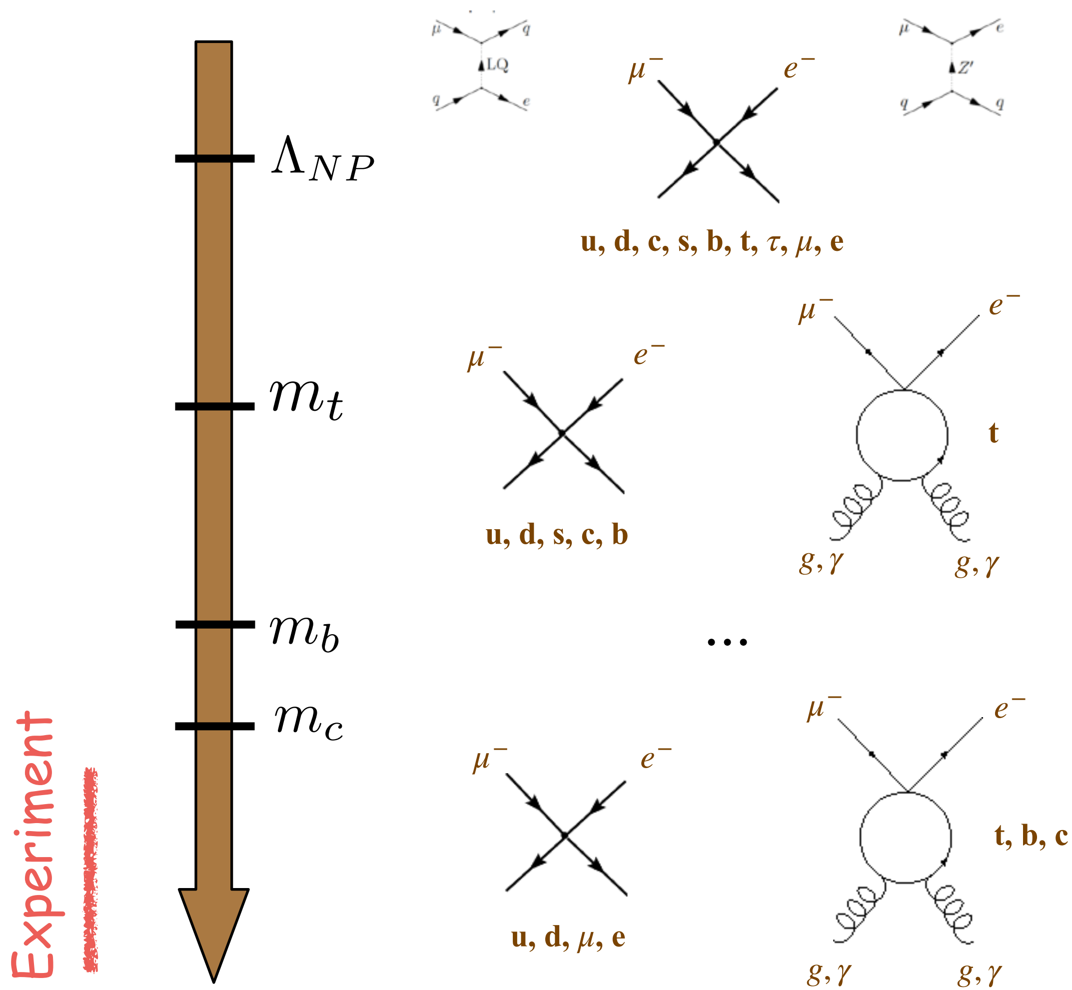


Flavor violation and effective Lagrangians

★ Modern approach to flavor physics calculations: effective field theories



★ It is important to understand ALL relevant energy scales for the problem at hand



New Physics generates lepton FCNC

Scales associated with heavy SM particles (quarks, leptons)

...

heavy quarks decouple

...

Scales associated with experiment

Flavor violation and effective Lagrangians

★ Systematic approach: Standard Model Effective Field Theory (SMEFT)

- effective Lagrangian

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} Q^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} Q_i^{(6)} + \ldots$$

with the Weinberg operator $Q^{(5)}$

$$Q^{(5)} = \epsilon_{jk}\epsilon_{mn} H^j H^m \left(L_p^k\right)^T \mathcal{C} L_r^n$$

and lots (59+5) of $Q_i^{(6)}$ operators

- the strategy of identifying an NP model involves fitting C_i from experimental data and/or matching of $\mathscr{L}$ to UV-completed NP models

TABLE 2.3 Operators with H^n, sets X^3, H^6, H^4D^2, and $\psi^2 H^3$.

X^3		H^6 and H^4D^2		ψ^2H^3+ h.c.	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_H	$\left(H^\dagger H\right)^3$	Q_{eH}	$\left(H^\dagger H\right)\left(\overline{L}_p e_r H\right)$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{H\Box}$	$\left(H^\dagger H\right)\Box\left(H^\dagger H\right)$	Q_{uH}	$\left(H^\dagger H\right)\left(\overline{Q}_p u_r \widetilde{H}\right)$
Q_W	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	Q_{HD}	$\left(H^\dagger D^\mu H\right)^*\left(H^\dagger D_\mu H\right)$	Q_{dH}	$\left(H^\dagger H\right)\left(\overline{Q}_p d_r H\right)$
$Q_{\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				

TABLE 2.4 Operators with H^n, sets X^2H^2, ψ^2XH, and ψ^2H^2D.

X^2H^2		ψ^2XH+ h.c.		ψ^2H^2D	
Q_{HG}	$H^\dagger H G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$\left(\overline{L}_p\sigma^{\mu\nu}e_r\right)\tau^I H W_{\mu\nu}^I$	$Q_{Hl}^{(1)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{L}_p\gamma^\mu L_r\right)$
$Q_{H\widetilde{G}}$	$H^\dagger H\widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$\left(\overline{L}_p\sigma^{\mu\nu}e_r\right)HB_{\mu\nu}$	$Q_{Hl}^{(3)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu^I H\right)\left(\overline{L}_p\tau^I\gamma^\mu L_r\right)$
Q_{HW}	$H^\dagger H W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$\left(\overline{Q}_p\sigma^{\mu\nu}T^A u_r\right)\widetilde{H}G_{\mu\nu}^A$	Q_{He}	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{e}_p\gamma^\mu e_r\right)$
$Q_{H\widetilde{W}}$	$H^\dagger H\widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$\left(\overline{Q}_p\sigma^{\mu\nu}u_r\right)\tau^I\widetilde{H}W_{\mu\nu}^I$	$Q_{Hq}^{(1)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{Q}_p\gamma^\mu Q_r\right)$
Q_{HB}	$H^\dagger H B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$\left(\overline{Q}_p\sigma^{\mu\nu}u_r\right)\widetilde{H}B_{\mu\nu}$	$Q_{Hq}^{(3)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu^I H\right)\left(\overline{Q}_p\tau^I\gamma^\mu Q_r\right)$
$Q_{H\widetilde{B}}$	$H^\dagger H\widetilde{B}_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$\left(\overline{Q}_p\sigma^{\mu\nu}T^A d_r\right)HG_{\mu\nu}^A$	Q_{Hu}	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{u}_p\gamma^\mu u_r\right)$
Q_{HWB}	$H^\dagger\tau^I H W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$\left(\overline{Q}_p\sigma^{\mu\nu}d_r\right)\tau^I H W_{\mu\nu}^I$	Q_{Hd}	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{d}_p\gamma^\mu d_r\right)$
$Q_{H\widetilde{W}B}$	$H^\dagger\tau^I H\widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$\left(\overline{Q}_p\sigma^{\mu\nu}d_r\right)HB_{\mu\nu}$	Q_{Hud}	$i\left(\widetilde{H}^\dagger D_\mu H\right)\left(\overline{u}_p\gamma^\mu d_r\right)$

TABLE 2.5 Four-fermion operators, classes $(\overline{L}L)(\overline{L}L)$, $(\overline{R}R)(\overline{R}R)$, and $(\overline{L}L)(\overline{R}R)$.

$(\overline{L}L)(\overline{L}L)$		$(\overline{R}R)(\overline{R}R)$		$(\overline{L}L)(\overline{R}R)$	
Q_{ll}	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{L}_s\gamma^\mu L_t\right)$	Q_{ee}	$(\overline{e}_p\gamma^\mu e_r)(\overline{e}_s\gamma^\mu e_t)$	Q_{le}	$\left(\overline{L}_p\gamma^\mu L_r\right)(\overline{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$\left(\overline{Q}_p\gamma^\mu Q_r\right)\left(\overline{Q}_s\gamma^\mu Q_t\right)$	Q_{uu}	$(\overline{u}_p\gamma^\mu u_r)(\overline{u}_s\gamma^\mu u_t)$	Q_{lu}	$\left(\overline{L}_p\gamma^\mu L_r\right)(\overline{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$\left(\overline{Q}_p\gamma^\mu\tau^I Q_r\right)\left(\overline{Q}_s\gamma^\mu\tau^I Q_t\right)$	Q_{dd}	$\left(\overline{d}_p\gamma^\mu d_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$	Q_{ld}	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$
$Q_{lq}^{(1)}$	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{Q}_s\gamma^\mu Q_t\right)$	Q_{eu}	$(\overline{e}_p\gamma^\mu e_r)(\overline{u}_s\gamma^\mu u_t)$	Q_{qe}	$\left(\overline{Q}_p\gamma^\mu Q_r\right)(\overline{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$\left(\overline{L}_p\gamma^\mu\tau^I L_r\right)\left(\overline{Q}_s\gamma^\mu\tau^I Q_t\right)$	Q_{ed}	$(\overline{e}_p\gamma^\mu e_r)\left(\overline{d}_s\gamma^\mu d_t\right)$	$Q_{qu}^{(1)}$	$\left(\overline{Q}_p\gamma^\mu Q_r\right)(\overline{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\overline{u}_p\gamma^\mu u_r)\left(\overline{d}_s\gamma^\mu d_t\right)$	$Q_{qu}^{(8)}$	$\left(\overline{q}_p\gamma^\mu T^A q_r\right)\left(\overline{u}_s\gamma^\mu T^A u_t\right)$
		$Q_{ud}^{(8)}$	$\left(\overline{u}_p\gamma^\mu T^A u_r\right)\left(\overline{d}_s\gamma^\mu T^A d_t\right)$	$Q_{qd}^{(1)}$	$\left(\overline{q}_p\gamma^\mu q_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$
				$Q_{qd}^{(8)}$	$\left(\overline{Q}_p\gamma^\mu T^A Q_r\right)\left(\overline{d}_s\gamma^\mu T^A d_t\right)$

TABLE 2.6 Four-fermion operators, classes $(LR)(RL)$, and B (baryon-number) violating.

$(\overline{L}R)(\overline{R}L)$		B-violating	
Q_{ledq}	$\left((\overline{L}_p^j e_r\right)\left(\overline{d}_s Q_t^j\right)$	Q_{duq}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[\left(d_p^\alpha\right)^T C u_r^\beta\right]\left[\left(Q_s^{\gamma j}\right)^T C L_t^k\right]$
$Q_{quqd}^{(1)}$	$\left((\overline{Q}_p^j u_r\right)\epsilon_{jk}\left(\overline{Q}_s^k d_t\right)$	Q_{qqu}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[\left(Q_p^{\alpha j}\right)^T C Q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$\left((\overline{Q}_p^j T^A u_r\right)\epsilon_{jk}\left(\overline{Q}_s^k T^A d_t\right)$	$Q_{qqq}^{(1)}$	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\epsilon_{mn}\left[\left(Q_p^{\alpha j}\right)^T C Q_r^{\beta k}\right]\left[(Q_s^{\gamma m})^T C L_t^n\right]$
$Q_{lequ}^{(1)}$	$\left((\overline{L}_p^j e_r\right)\epsilon_{jk}\left(\overline{Q}_s^k u_t\right)$	$Q_{qqq}^{(3)}$	$\epsilon^{\alpha\beta\gamma}\left(\tau^I\epsilon\right)_{jk}\left(\tau^I\epsilon\right)_{mn}\left[\left(Q_p^{\alpha j}\right)^T C Q_r^{\beta k}\right]\left[(Q_s^{\gamma m})^T C L_t^n\right]$
$Q_{lequ}^{(3)}$	$\left((\overline{L}_p^j\sigma_{\mu\nu}e_r\right)\epsilon_{jk}\left(\overline{Q}_s^k\sigma^{\mu\nu}u_t\right)$	Q_{duu}	$\epsilon^{\alpha\beta\gamma}\left[\left(d_p^\alpha\right)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

Effective Lagrangians at low energy

- Effective Lagrangians for $\Delta L_\mu = 0$, $\Delta L_\mu = 1$, and $\Delta L_\mu = 2$
  - SM: $$\mathcal{L}_{\rm eff}^{\Delta L_\mu=0} = -\frac{4G_F}{\sqrt{2}} \left(\overline{\mu}_L \gamma_\alpha e_L\right) \left(\overline{\nu_e}_L \gamma^\alpha \nu_{\mu L}\right)$$
  - four-fermion operators (assume no FCNC in quark currents for now)

$$\begin{aligned}
\mathcal{L}_{\rm eff}^{\Delta L_\mu=1} = -\frac{1}{\Lambda^2}\sum_f \Big[& \left(C_{VR}^f \,\overline{\mu}_R \gamma^\alpha e_R + C_{VL}^f \,\overline{\mu}_L \gamma^\alpha e_L\right) \overline{f}\gamma_\alpha f \\
+ & \left(C_{AR}^f \,\overline{\mu}_R \gamma^\alpha e_R + C_{AL}^q \,\overline{\mu}_L \gamma^\alpha e_L\right) \overline{f}\gamma_\alpha \gamma_5 f \\
+ & \; m_e m_f G_F \left(C_{SR}^f \,\overline{\mu}_R e_L + C_{SL}^f \,\overline{\mu}_L e_R\right) \overline{f} f \\
+ & \; m_e m_f G_F \left(C_{PR}^f \,\overline{\mu}_R e_L + C_{PL}^f \,\overline{\mu}_L e_R\right) \overline{f}\gamma_5 f \\
+ & \; m_e m_f G_F \left(C_{TR}^f \,\overline{\mu}_R \sigma^{\alpha\beta} e_L + C_{TL}^f \,\overline{\mu}_L \sigma^{\alpha\beta} e_R\right) \overline{f}\sigma_{\alpha\beta} f + h.c. \Big],
\end{aligned}$$

  - dipole operators $$\mathcal{L}_D = -\frac{m_2}{\Lambda^2}\left[\left(C_{DR}\overline{\ell}_1 \sigma^{\mu\nu} P_L \ell_2 + C_{DR}\overline{\ell}_1 \sigma^{\mu\nu} P_R \ell_2\right) F_{\mu\nu} + h.c.\right]$$
  - gluonic (Rayleigh) operators $$\begin{aligned}\mathcal{L}_G = -\frac{m_2 G_F}{\Lambda^2}\frac{\beta_L}{4\alpha_s}\Big[& \left(C_{GR}\overline{\ell}_1 P_R \ell_2 + C_{GL}\overline{\ell}_1 P_L \ell_2\right) G_{\mu\nu}^a G^{a\mu\nu} \\ + & \left(C_{\tilde{G}R}\overline{\ell}_1 P_R \ell_2 + C_{\tilde{G}L}\overline{\ell}_1 P_L \ell_2\right) G_{\mu\nu}^a \widetilde{G}^{a\mu\nu} + h.c.\Big]\end{aligned}$$

AAP and D. Zhuridov