# New Techniques for the Reduction of One-Loop Scattering Amplitudes

**Giovanni Ossola**

**New York City College of Technology**
City University of New York (CUNY)

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# Outline

1. LHC needs NLO
2. A walk through the OPP method
3. Applications and Results

# LHC needs NLO

- The experimental programs of LHC require high precision predictions for multi-particle processes
- The current need of precision goes beyond tree order. At LHC, most analyses require at least next-to-leading order calculations (NLO)
- The search and the interpretation of new physics requires a precise understanding of the Standard Model backgrounds. We need accurate predictions and reliable error estimates

**In summary:**

One-loop corrections for multi-particle processes!

# Did NLO need LHC?

**Some recent calculations → Cross Sections available**

- $pp \to Z\,Z\,Z$ and $pp \to t\bar{t}Z$ [Lazopoulos, Melnikov, Petriello]
- $p\bar{p} \to b\bar{b}Z$ [Febres Cordero, Reina, Wackeroth]
- $pp \to H + 2$ jets, $pp \to WW+$ jet [Campbell, Ellis, Giele, Zanderighi]
- $pp \to VV + 2$ jets via VBF [Bozzi, Jäger, Oleari, Zeppenfeld]
- $pp \to H\,H\,H$ [Binoth, Karg, Kauer, Ruckl]
- $pp \to t\bar{t}$+jet [Ciccolini, Denner and Dittmaier]
- $pp \to VVV$ [Binoth, G.O., Papadopoulos, Pittau]
- $pp \to VVV$ with leptonic decays [Campanario, Hankele, Oleari *et al*]
- $pp \to W+$ 3 jets [Berger *et al*, Ellis *et al*]
- $pp \to t\bar{t}b\bar{b}$ [Bredenstein, Denner, Dittmaier, and Pozzorini]

**A lot of progress on $2 \to 4$**

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- $pp \to H$ … Zanderighi]
- …
- $pp$ …
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Also many **New Techniques**

**A lot of progress on $2 \to 4$**

# OPP Method

In 2007, we proposed a new method for the numerical evaluation of scattering amplitudes, based on a decomposition at the **integrand level**.

**G. O., C. G. Papadopoulos and R. Pittau**
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Some of the advantages:

- Universal - applicable to any process
- Simple - based on basic algebraic properties
- Automatizable - easy to implement in a computer code

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**Final Task**

Produce a MULTI-PROCESS fully automatized NLO generator

1. **Passarino-Veltman Reduction to Scalar Integrals**

$$
\begin{aligned}
\mathcal{M} &= \sum_i d_i \,\mathrm{Box}_i + \sum_i c_i \,\mathrm{Triangle}_i \\
&+ \sum_i b_i \,\mathrm{Bubble}_i + \sum_i a_i \,\mathrm{Tadpole}_i + \mathrm{R}\,,
\end{aligned}
$$

## 1 Passarino-Veltman Reduction to Scalar Integrals

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- Set the basis for our NLO calculations
- Exploits the Lorentz structure

$$B^{\mu} = \int d^4q \frac{q^{\mu}}{[q^2 - m_0^2][(q+\mathbf{p_1})^2 - m_1^2]} = \mathbf{p_1}^{\mu}\; B_1(p_1, m_0, m_1)$$

# “Standing on the shoulders of giants”

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**2 Pittau/del Aguila Recursive Tensorial Reduction**

- Express $q^\mu = \sum_i G_i \, \ell_i{}^\mu \; , \; \ell_i{}^2 = 0$
- The generated terms might reconstruct denominators $D_i$ or vanish upon integration

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   Aim at the direct extraction of the coefficients that multiply the scalar integral

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3. **“Cut-based” Techniques**

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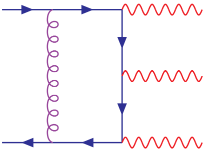
*Pigmaei gigantum humeris impositi plusquam ipsi gigantes vident*

# One-loop – Definitions



Any $m$-point one-loop amplitude can be written, before integration, as

$$A(\bar{q}) = \frac{N(\bar{q})}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

where

$$\bar{D}_i = (\bar{q} + p_i)^2 - m_i^2 \quad , \quad \bar{q}^2 = q^2 + \tilde{q}^2 \quad , \quad \bar{D}_i = D_i + \tilde{q}^2$$

Our task is to calculate, for each phase space point:

$$\mathcal{M} = \int d^n\bar{q}\ A(\bar{q}) = \int d^n\bar{q} \frac{N(\bar{q})}{\bar{D}_0 \bar{D}_1 \ldots \bar{D}_{m-1}}$$

# The traditional "master" formula

$$
\begin{aligned}
\int A \;=\; & \sum_{i_0<i_1<i_2<i_3}^{m-1} d(i_0 i_1 i_2 i_3) D_0(i_0 i_1 i_2 i_3) \\
+ & \sum_{i_0<i_1<i_2}^{m-1} c(i_0 i_1 i_2) C_0(i_0 i_1 i_2) \\
+ & \sum_{i_0<i_1}^{m-1} b(i_0 i_1) B_0(i_0 i_1) \\
+ & \sum_{i_0}^{m-1} a(i_0) A_0(i_0) \\
+ & \text{ rational terms}
\end{aligned}
$$

# OPP "MASTER" FORMULA - I

General expression for the **4-dim** $N(q)$ at the integrand level in terms of $D_i$

$$
\begin{aligned}
N(q) = & \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)\right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
+ & \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
+ & \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] \prod_{i \neq i_0, i_1}^{m-1} D_i \\
+ & \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0)\right] \prod_{i \neq i_0}^{m-1} D_i
\end{aligned}
$$

**This is 4-dimensional Identity**

# OPP "master" formula - II

$$
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N(q) = & \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[ d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1} \left[ c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
+ & \sum_{i_0<i_1}^{m-1} \left[ b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} \left[ a(i_0) + \tilde{a}(q; i_0) \right] \prod_{i \neq i_0}^{m-1} D_i
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The quantities $d$, $c$, $b$, $a$ are the coefficients of all possible scalar functions

The quantities $\tilde{d}$, $\tilde{c}$, $\tilde{b}$, $\tilde{a}$ are the "spurious" terms → vanish upon integration

**It is now an algebraic problem:**

Any N(q) just depends on a set of coefficients, to be determined!

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The quantities $d$, $c$, $b$, $a$ are the coefficients of all possible scalar functions
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## IT IS NOW AN ALGEBRAIC PROBLEM:

Any N(q) just depends on a set of coefficients, to be determined!

## CHOOSE $\{q_i\}$ WISELY

by evaluating N(q) for a set of values of the integration momentum $\{q_i\}$ such that some denominators $D_i$ vanish ("cuts")

# Example: 4-particles process

$$N(q) = d + \tilde{d}(q) + \sum_{i=0}^{3} [c(i) + \tilde{c}(q;i)] D_i + \sum_{i_0<i_1}^{3} \left[ b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] D_{i_0} D_{i_1}$$
$$+ \sum_{i_0=0}^{3} [a(i_0) + \tilde{a}(q; i_0)] D_{i \neq i_0} D_{j \neq i_0} D_{k \neq i_0}$$

We look for a $q$ such that

$$D_0 = D_1 = D_2 = D_3 = 0$$

$\rightarrow$ there are two solutions $q_0^{\pm}$

# Example: 4-particles process

$$N(q) \quad = \quad d + \tilde{d}(q)$$

Our "master formula" for $q = q_0^{\pm}$ is:

$$N(q_0^{\pm}) = [d + \tilde{d}\ T(q_0^{\pm})]$$

$\longrightarrow$ solve to extract the coefficients $d$ and $\tilde{d}$

# Example: 4-particles process

$$N(q) - d - \tilde{d}(q) = \sum_{i=0}^{3} [c(i) + \tilde{c}(q;i)] D_i + \sum_{i_0 < i_1}^{3} \left[ b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] D_{i_0} D_{i_1}$$

$$+ \sum_{i_0=0}^{3} [a(i_0) + \tilde{a}(q; i_0)] D_{i \neq i_0} D_{j \neq i_0} D_{k \neq i_0}$$

Then we can move to the extraction of $c$ coefficients using

$$N'(q) = N(q) - d - \tilde{d}\, T(q)$$

and setting to zero three denominators (ex: $D_1 = 0$, $D_2 = 0$, $D_3 = 0$)

# Example: 4-particles process

$$N(q) - d - \tilde{d}(q) \quad = \quad [c(0) + \tilde{c}(q;0)]\, D_0$$

We have infinite values of $q$ for which

$$D_1 = D_2 = D_3 = 0 \quad \text{and} \quad D_0 \neq 0$$

$\longrightarrow$ Here we need 7 of them to determine $c(0)$ and $\tilde{c}(q;0)$

# A first summary

1. The functional form of the OPP-master formula is **universal** (process independent)

2. To extract **all coefficients** $d$, $c$, $b$, and $a$ we ONLY need the numerator N(q) **numerically** at **fixed given values of** $q$.

3. Strong **test** on the 4-dimensional reduction $\rightarrow$ $N = N$ (no previous or external information required)

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We need to reconstruct **n-dimensional objects**, not 4-dim!

This generates the **rational terms**

# From 4 to N (part I - Denominators)

- We find the decomposition for $N(q)$

$$N(q) = \ldots + c_2 D_2 + \ldots$$

# From 4 to N (part I - Denominators)

- We find the decomposition for $N(q)$, divide by the denominators

$$\frac{N(q)}{\bar{D}_0\bar{D}_1\bar{D}_2\bar{D}_3} = \ldots + \frac{c_2 D_2}{\bar{D}_0\bar{D}_1\bar{D}_2\bar{D}_3} + \ldots$$

# From 4 to N (part I - Denominators)

- We find the decomposition for $N(q)$, divide by the denominators and finally integrate over $q$

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- Using the expression for $\bar{Z}_2$

$$\int \frac{N(q)}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \bar{D}_3} = \ldots + \int \frac{c_2}{\bar{D}_0 \bar{D}_1 \bar{D}_3} + \int \frac{c_2\, \tilde{q}^2}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \bar{D}_3} + \ldots$$

# "Extra Integrals" for $R_1$

The "Extra Integrals" are of the form

$$I^{(n;2\ell)}_{s;\mu_1\cdots\mu_r} \equiv \int d^n q \, \tilde{q}^{2\ell} \frac{q_{\mu_1}\cdots q_{\mu_r}}{\bar{D}(k_0)\cdots\bar{D}(k_s)} \, ,$$

where

$$\bar{D}(k_i) \equiv (\bar{q}+k_i)^2 - m_i^2 \, , k_i = p_i - p_0$$

**These integrals:**

- **have dimensionality** $\mathcal{D} = 2(1+\ell-s)+r$
- **contribute only when** $\mathcal{D} \geq 0$**, otherwise are of** $\mathcal{O}(\epsilon)$

Pittau – arXiv:hep-ph/0406105

G.O., Papadopoulos, Pittau – arXiv:0802.1876 [hep-ph]

# From 4 to N (more about Denominators)

- What about expressing directly the 4-dim $N(q)$ in terms of n-dim $\bar{D}_i$?

$$\bar{D}_i = (\bar{q} + p_i)^2 - m_i^2 \quad , \quad \bar{q}^2 = q^2 + \tilde{q}^2 \quad , \quad \bar{D}_i = D_i + \tilde{q}^2$$

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- For scalar products nothing changes

$$(q.p_1) = \frac{1}{2}(D_1 - D_0 - p_1^2 + m_1^2 - m_0^2) = \frac{1}{2}(\bar{D}_1 - \bar{D}_0 - p_1^2 + m_1^2 - m_0^2)$$

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- The dimension is important when we reconstruct $q^2$ (4-dim!)

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- The dimension is important when we reconstruct $q^2$ (4-dim!)

$$q^2 = D_0 + m_0 = \bar{D}_0 + m_0 - \tilde{q}^2$$

- This can be mimicked by the mass-shift

$$m_i^2 \rightarrow m_i^2 - \tilde{q}^2$$

# A DIFFERENT METHOD FOR $R_1$

Look at the implicit $\tilde{q}^2$-dependence in the coefficients once $\tilde{q}^2$ is reintroduced through the mass shift

$$m_i^2 \to m_i^2 - \tilde{q}^2$$

The coefficients of the OPP expansion depend on $\tilde{q}^2$

$$\begin{aligned}
d(ijkl;\tilde{q}^2) &= d(ijkl) + \tilde{q}^2 d^{(2)}(ijkl) + \tilde{q}^4 d^{(4)}(ijkl) \\
c(ijk;\tilde{q}^2) &= c(ijk) + \tilde{q}^2 c^{(2)}(ijk) \\
b(ij;\tilde{q}^2) &= b(ij) + \tilde{q}^2 b^{(2)}(ij)
\end{aligned}$$

We need the following extra integrals

$$\begin{aligned}
\int d^n\bar{q}\frac{\tilde{q}^2}{\bar{D}_i\bar{D}_j} &= -\frac{i\pi^2}{2}\left[m_i^2 + m_j^2 - \frac{(p_i-p_j)^2}{3}\right] + \mathcal{O}(\epsilon) \\
\int d^n\bar{q}\frac{\tilde{q}^2}{\bar{D}_i\bar{D}_j\bar{D}_k} &= -\frac{i\pi^2}{2} + \mathcal{O}(\epsilon) \\
\int d^n\bar{q}\frac{\tilde{q}^4}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_l} &= -\frac{i\pi^2}{6} + \mathcal{O}(\epsilon)
\end{aligned}$$

# From 4 to N (part II - Numerators)

What if $N(q)$ develops an $\epsilon$-dimensional part?

- Algebra of Dirac matrices
- $(\bar{q}.p)$ is 4-dim but $(\bar{q}.\bar{q}) = q^2 + \tilde{q}^2$

$\bar{N}(\bar{q})$ can be split into a 4-dim plus a $\epsilon$-dimensional part

$$\bar{N}(\bar{q}) = N(q) + \tilde{N}(\tilde{q}^2, q, \epsilon)$$

$\tilde{N}(\tilde{q}^2, q, \epsilon)$ is responsible for the rational term $R_2$

A practical solution: **tree-level like Feynman Rules**

General idea and QED: G. O., Papadopoulos, Pittau - arXiv:0802.1876
Rules for QCD: Draggiotis, Garzelli, Papadopoulos, Pittau - arXiv:0903.0356
Full Standard Model: in progress

# Overview Rational Terms

$$R = R_1 + R_2$$

$R_1$ – The OPP expansion is written in terms of 4-dim $D_i$, while $n$-dim $\bar{D}_i$ appear in scalar integrals.

$$A(\bar{q}) = \frac{N(q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

$R_1$ can be calculated in **two different ways**, both **fully automatized**.

$R_2$ – The numerator $\bar{N}(\bar{q})$ can be also split into a 4-dim plus a $\epsilon$-dim part

$$\bar{N}(\bar{q}) = N(q) + \tilde{N}(\tilde{q}^2, q, \epsilon)\,.$$

Compute $R_2$ using **tree-level like Feynman Rules**.

# One-Loop as a 3 step process

1) Compute the numerator $N(q)$ numerically at given $q$
2) Extract the coefficients with **OPP reduction**
3) Combine with scalar integrals

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# One-Loop as a 3 step process

1) Compute the numerator $N(q)$ numerically at given $q$
2) Extract the coefficients with **OPP reduction** **[IMPLEMENTED]**
3) Combine with scalar integrals **[AVAILABLE]**
   Numerical Codes for the Scalar Integrals are available
   (van Hameren or Ellis/Zanderighi)

$$\begin{aligned}\mathcal{M} &= \sum_i d_i \,\mathrm{Box}_i + \sum_i c_i \,\mathrm{Triangle}_i \\ &+ \sum_i b_i \,\mathrm{Bubble}_i + \sum_i a_i \,\mathrm{Tadpole}_i + \mathrm{R}\,,\end{aligned}$$

# One-Loop as a 3 step process

1) Compute the numerator $N(q)$ numerically at given $q$ **[???]**
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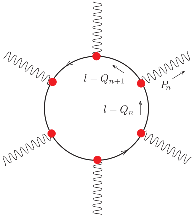
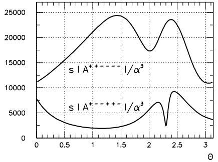
3) Combine with scalar integrals **[AVAILABLE]**
   Numerical Codes for the Scalar Integrals are available
   (van Hameren or Ellis/Zanderighi)

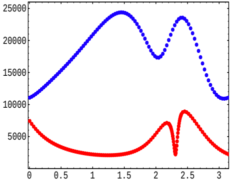
$$\mathcal{M} = \sum_i d_i \,\mathrm{Box}_i + \sum_i c_i \,\mathrm{Triangle}_i + \sum_i b_i \,\mathrm{Bubble}_i + \sum_i a_i \,\mathrm{Tadpole}_i + \mathrm{R}\,,$$

What about **the numerator N(q)** ? ...just wait for a few slides

# A first check: 6 photon amplitudes




$$N(q) \approx T[\not{q}\,\not{\epsilon}\;\not{q}\,\not{\epsilon}\;\not{q}\not{\epsilon}\;\not{q}\,\not{\epsilon}\;\not{q}\,\not{\epsilon}\;\not{q}\,\not{\epsilon}]$$

A "simple" numerator (but try to "reduce" it with standard methods...)

This application showed that

- The OPP method is working
- Internal/External Masses are not a problem

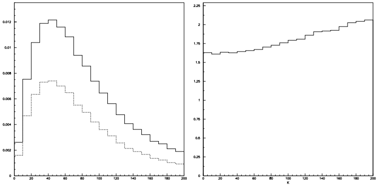
**Mahlon – hep-ph/9412350 Nagy and Soper – hep-ph/0610028**
**Binoth, Heinrich, Gehrmann, and Mastrolia, hep-ph/0703311**
**G.O., Papadopoulos, Pittau – arXiv:0704.1271**

# A second check: VVV production

We obtained cross sections for $pp \to VVV$

Example: $pp \to W^+W^-W^+$

| scale | $\sigma_0$ | $\sigma_{NLO}$ | K |
|---|---|---|---|
| $\mu = 3/2\, M_w$ | 82.7(5) | 153.2(6) | 1.85 |
| $\mu = 3\, M_w$ | 81.4(5) | 144.5(6) | 1.77 |
| $\mu = 6\, M_w$ | 81.8(5) | 139.1(6) | 1.70 |



**Binoth, G.O., Papadopoulos, Pittau – arXiv:0804.0350**

## This calculation showed that

Positive: We can build OPP-powered cross sections

Negative: $N(q)$ "by hand" is heavy $\to$ **need for automatization** !

# A real proof of concept

**van Hameren, Papadopoulos, Pittau – arXiv:0903.4665 [hep-ph]**

1) numerator $N(q)$ numerically with **HELAC**
2) coefficients via **OPP reduction** with **CutTools**
3) scalar integrals with **OneLOop/QCDloop**

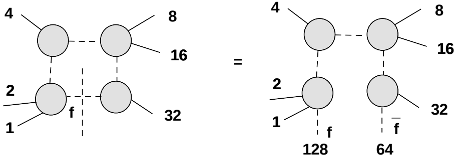
Fully Automated numerical evaluation of **ANY** one-loop amplitude

All **6-particle** processes in the Les Houches 2007 "Wish List"

$$
\begin{array}{rcl}
u\bar{u} \to t\bar{t}b\bar{b} & \quad & gg \to t\bar{t}b\bar{b} \\
u\bar{u} \to W^+W^-b\bar{b} & \quad & gg \to W^+W^-b\bar{b} \\
u\bar{u} \to b\bar{b}b\bar{b} & \quad & gg \to b\bar{b}b\bar{b} \\
u\bar{d} \to W^+ggg & \quad & u\bar{u} \to Zggg \\
u\bar{u} \to t\bar{t}gg & \quad & gg \to t\bar{t}gg
\end{array}
$$

# $N(q)$ WITH HELAC-1LOOP

- HELAC is capable to compute any tree-order amplitude for the full Standard Model, using Dyson-Schwinger recursive equations
- After fixing the integration momentum $q$, any n-point one-loop amplitude is an $(n+2)$-point tree level amplitude




- HELAC reconstructs the amplitude as in the tree-order calculation.
- The complete treatment of the color degrees of freedom is included

Kanaki, Papadopoulos – hep-ph/0002082 and hep-ph/0012004
Cafarella, Papadopoulos, Worek – arXiv:0710.2427 [hep-ph]
van Hameren, Papadopoulos, Pittau – arXiv:0903.4665 [hep-ph]

# $N(q)$ WITH HELAC-1LOOP

- HELAC is capable to compute any tree-order amplitude for the full Standard Model, using Dyson-Schwinger recursive equations
- After fixing the integration momentum $q$, any n-point one-loop amplitude is an … litude

**NEW**

**Czakon, Papadopoulos, Worek – arXiv:0905.0883**

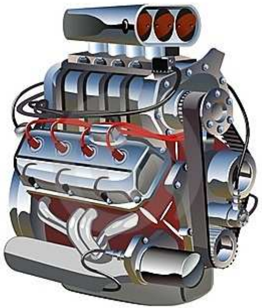
**Automated Dipoles within HELAC**

- HELAC reconstructs the amplitude as in the tree-order calculation.
- The complete treatment of the color degrees of freedom is included

Kanaki, Papadopoulos – hep-ph/0002082 and hep-ph/0012004
Cafarella, Papadopoulos, Worek – arXiv:0710.2427 [hep-ph]
van Hameren, Papadopoulos, Pittau – arXiv:0903.4665 [hep-ph]

# THE OPP METHOD IS A REDUCTION ENGINE

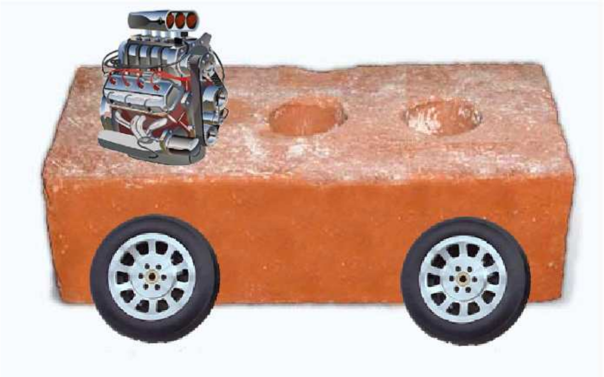
However, it requires a car around it
Numerators, Scalar Functions, Dipoles, . . .



It is the core for the reduction of virtual parts!

# Early Applications

We did not invest on "design", we wanted to test the reduction engine!



**It works !**

Even a "rough" implementation is competitive, when OPP-powered

# Temptation...

To go as fast as possible...


...but let's not forget **versatility**, **precision**, and **stability**

**It goes everywhere!**

# What is next?

**Work in progress**

**Phenomenology**
**New Codes**
**Optimization**

# What is next?

## Work in progress

**Phenomenology**
**New Codes**
**Optimization**

Example: Improve the system-solving algorithm in the OPP-equations for triangles and bubbles by exploiting:

- polynomial structure of the integrand
- freedom in choosing the solutions for the cuts

Do we gain by using DFT? (work in collaboration with P. Mastrolia)

# What is next?

**Work in progress → New results are coming, stay tuned!**

**Phenomenology**
**New Codes**
**Optimization**

**Put an OPP-engine in YOUR Calculations !!**