Z-boson pair + 1-jet production at NLO QCD

Stefan Karg

Institut für Theoretische Physik E
RWTH Aachen University

in collaboration with T. Binoth, N. Kauer, G. Sanguinetti

LoopFest VIII

Why do we need next-to-leading order corrections?

- LO predictions usually have large theoretical uncertainties
- Large impact of higher order corrections due to new channels and experimental cuts possible
- no sensible way to estimate NLO corrections without doing the work!

Why do we need next-to-leading order corrections?

- LO predictions usually have large theoretical uncertainties
- Large impact of higher order corrections due to new channels and experimental cuts possible
- no sensible way to estimate NLO corrections without doing the work!

$\Longrightarrow$ NLO necessary for precision comparisons of data to theory

Status for $2 \rightarrow n$ hadronic collisions at NLO

- $2 \rightarrow 2$: everything you want
- $2 \rightarrow 3$: feasible with current techniques
  $pp \rightarrow 3j$, $V + 2j$, $\gamma\gamma j$, $t\bar{t}H$, $b\bar{b}H$, $H + 2j$, HHH, $t\bar{t}j$, $Wb\bar{b}$, VVV, VVjet
- non-hadronic $2 \rightarrow 4$: $e^+e^- \rightarrow 4f$, $HH\nu\bar{\nu}$, $\gamma\gamma \rightarrow t\bar{t}b\bar{b}$
- hadronic $2 \rightarrow 4$: $pp \rightarrow t\bar{t}b\bar{b}$, $pp \rightarrow W + 3$jets

Difficulties in NLO multi-leg calculations

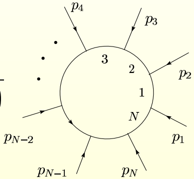
- huge amount of algebra, long expressions
  $\rightarrow$ Computeralgebra (Maple, Mathematica, FORM,...), automation
- complicated structure of singularities: real and virtual corrections
  $\rightarrow$ e.g. Catani-Seymour Dipole method
- numerically stable evaluation of one-loop tensor-integrals when integrating over the multi-dimensional phasespace (bottleneck!)

Difficulties in NLO multi-leg calculations

- huge amount of algebra, long expressions
  $\rightarrow$ Computeralgebra (Maple, Mathematica, FORM,...), automation
- complicated structure of singularities: real and virtual corrections
  $\rightarrow$ e.g. Catani-Seymour Dipole method
- numerically stable evaluation of one-loop tensor-integrals when integrating over the multi-dimensional phasespace (bottleneck!)
- no conceptual issues, no general library available yet
  various solutions exist

Reduction method:

$$I_N^{n,\mu_1\ldots\mu_r} = \int \frac{d^n k}{i\,\pi^{n/2}} \frac{q_1^{\mu_1}\ldots q_r^{\mu_r}}{(q_1^2 - m_1^2 + i\delta)\ldots(q_N^2 - m_N^2 + i\delta)}$$



- main problem: numerically fast and stable evaluation needed
- many proposals exist, only a few are tested in non-trivial problems
- classical approach: Passarino-Veltman method: $I_2^{\mu\nu} = A g^{\mu\nu} + B p^\mu p^\nu$
  $A, B \propto 1/\mathrm{det}G * (\sum \text{scalar integrals } I_N^n)$
- our method (GOLEM-coll.): reduce tensor int. to scalar int. in shifted dimensions (Davydychev 91)
  avoids inverse Gram determinants, algebraic separation of IR poles
  (T. Binoth, et al. hep-ph/0504267)

$$I_N^{n,\mu_1\ldots\mu_r} = \sum \tau^{\mu_1\ldots\mu_r}(r_{j_1},\ldots,r_{j_r},g^{\times m})\, I_N^{n+2m}(j_1,\ldots,j_R)$$

$$I_N^D(j_1,\ldots,j_R) = (-1)^N\,\Gamma(N-D/2)\int_0^\infty d^N z\,\delta(1-\sum_{l=1}^N z_l)\frac{z_{j_1}\ldots z_{j_r}}{(z\cdot S\cdot z/2)^{N-D/2}}$$

$$S_{ij} = (r_i - r_j)^2 - m_i^2 - m_j^2,\; r_j = p_1 + \ldots + p_j$$

- two alternatives for evaluation:
  - further algebraic reduction: occuring basis integrals: I_4^{n+2}, I_3^n, I_2^n
    But $1/\det G$ unavoidable ($G_{ij} = 2\, r_i \cdot r_j$)
  - direct numerical evaluation in critical regions of phasespace feasible
    for VVjet: fully algebraic reduction
- singularities:
  - UV singularities: absorbed in overall $\Gamma(N - D/2)$ factor, at most single pole in $1/\varepsilon \to$ cancelled by renormalization
  - IR singularities: end point zeros of $z \cdot S \cdot z$, at most double pole $1/\varepsilon^2$ contained only in simple three point functions (I_4^6 UV and IR finite)
- formalism valid for arbitrary # of external legs, massless/massive particles
- $N = 5$ rank 2 example: $I_5^{\mu_1 \mu_2}$

$$I_5^{\mu_1 \mu_2} = T_{00}^{5,2} g^{\mu_1 \mu_2} + \sum T_{i,j}^{5,2} r_i^{\mu_1} r_j^{\mu_2}$$

$$T_{00}^{5,2} = -\frac{1}{2} \sum_l b_l I_{4,l}^{n+2} + \mathcal{O}(\varepsilon)$$

$$T_{ij}^{5,2} \to \sum_l I_{4,j}^{n+2}, \sum_{l,m} I_{3,l,m}^n$$

The PP $\rightarrow$ VVjet amplitude

An experimenter's wishlist

- Hadron collider cross-sections one would like to know at NLO

Run II Monte Carlo Workshop, April 2001

Single boson	Diboson	Triboson	Heavy flavour
$W + \leq 5j$	$WW + \leq 5j$	$WWW + \leq 3j$	$t\bar{t} + \leq 3j$
$W + b\bar{b} + \leq 3j$	$WW + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} + \leq 3j$	$WW + c\bar{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 5j$	$ZZ + \leq 5j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 3j$	$ZZ + b\bar{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \leq 2j$
$Z + c\bar{c} + \leq 3j$	$ZZ + c\bar{c} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{b} + \leq 2j$
$\gamma + \leq 5j$	$\gamma\gamma + \leq 5j$		$b\bar{b} + \leq 3j$
$\gamma + b\bar{b} + \leq 3j$	$\gamma\gamma + b\bar{b} + \leq 3j$		
$\gamma + c\bar{c} + \leq 3j$	$\gamma\gamma + c\bar{c} + \leq 3j$		
	$WZ + \leq 5j$		
	$WZ + b\bar{b} + \leq 3j$		
	$WZ + c\bar{c} + \leq 3j$		
	$W\gamma + \leq 3j$		
	$Z\gamma + \leq 3j$		

Les Houches 07 realistic wishlist

process ($V \in \{Z, W, \gamma\}$)	# groups working on
1. $pp \to V\,V$ jet	~ 4
2. $pp \to t\bar{t}\,b\bar{b}$	~ 1
3. $pp \to t\bar{t} + 2$ jets	
4. $pp \to W\,W\,W$	~ 2
5. $pp \to V\,V\,b\bar{b}$	
6. $pp \to V\,V + 2$ jets	
7. $pp \to V + 3$ jets	~ 2
8. $pp \to b\bar{b}b\bar{b}$	~ 1
9. $pp \to 4$ jets	~ 1 (theor. interest)
10. $gg \to W^*W^*$ (NLO, 2 loops)	~ 1
11. NNLO for $t\bar{t}$	~ 1
12. NNLO for Z/γ+jet	(gluon pdfs)

- Goal: $PP \to VVjet$ at NLO (T. Binoth, SK, N. Kauer, G. Sanguinetti)
- physical motivation: background process to $H \to VV + \text{jet}$, anomalous gauge boson couplings, part of $PP \to VV$ at NNLO
- $\mathcal{O}(100)$ Feynman diagrams:

- 6 scales: $s_{12}, s_{23}, s_{34}, s_{45}, s_{15}, M_Z^2$
- regularisation scheme: 'tHooft/Veltman (anti-commuting γ_5)
- trivial colour structure: $\mathcal{M} \propto (N_c, 1/N_c, N_F)\, T^a_{ij}$
- 36 helicity amplitudes, related by bose symmetry, charge conjugation and parity transformation
  $\mathcal{M}^{\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5} = \epsilon^{\lambda_3}_{3,\mu_3}\epsilon^{\lambda_4}_{4,\mu_4}\epsilon^{\lambda_5}_{5,\mu_5}\, \langle 2^{\lambda_2}|\Gamma^{\mu_3\mu_4\mu_5}|1^{\lambda_1}\rangle$

Helicity projection for $q\bar{q}VVg \to 0$

- replace momenta of the massive vectorbosons ($p_{3,4}$) with light-like momenta ($k_{3,4}$) to apply spinor formalism

$$k_{3,4} = \frac{1}{2\beta}\left[(1+\beta)p_{3,4} - (1-\beta)p_{4,3}\right] \quad \text{with } k_{3,4}^2 = 0$$

$$\epsilon^{+}_{3,\mu} = \frac{1}{\sqrt{2}}\frac{\langle 4^-|\mu|3^-\rangle}{\langle 43\rangle}, \quad \epsilon^{0}_{3,\mu} = \frac{1}{\sqrt{2}}\frac{(1+\beta)k_{3,\mu} - (1-\beta)k_{4,\mu}}{2M_V}$$

Use to define projectors on helicity amplitudes, schematically:

$$\begin{aligned}\mathcal{M}^{\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5} &= \mathcal{P}^{\lambda_3\lambda_4\lambda_5}_{\mu_3\mu_4\mu_5}\langle 2^{\lambda_2}|\Gamma^{\mu_3\mu_4\mu_5}|1^{\lambda_1}\rangle \\ &= (\text{global spinorial factor}) \times (\text{contracted tensor integrals})\end{aligned}$$

- Lorentz indices saturated, at most rank 1 pentagons (+ rank 3 boxes)
- spinor products can be treated as global factors
- further simplifications in analytical expressions possible and performed

Tuned comparison of results

[Dittmaier, Kallweit, Uwer 07], [Campbell, Ellis, Zanderighi 07],
[Binoth, Guillet, SK, Kauer, Sanguinetti] → [NLM Les Houches report 08]

- Integrated LO results

$PP \to W^+W^-$+jet	$\sigma_{LO}[fb]$
DKU	10371.7(12)
CEZ	10372.26(97)
BGKKS	10371.7(11)

Tuned comparison of results

[Dittmaier, Kallweit, Uwer 07], [Campbell, Ellis, Zanderighi 07],
[Binoth, Guillet, SK, Kauer, Sanguinetti] → [NLM Les Houches report 08]

- Integrated LO results

$PP \to W^+W^-$+jet	$\sigma_{LO}\,[fb]$
DKU	10371.7(12)
CEZ	10372.26(97)
BGKKS	10371.7(11)

- Results for virtual corrections checked at one phase space point:

$$2\,\mathrm{Re}\{\mathcal{M}_V^* \cdot \mathcal{M}_{LO}\} = e^4 g_s^2\, \Gamma(1+\epsilon) \left(\frac{4\pi\mu^2}{M_W^2}\right)^2 \left(\frac{1}{\epsilon^2} c_{-2} + \frac{1}{\epsilon} c_{-1} + c_0\right)$$

$u\bar{u} \to W^+W^-g$	c_{-2}	c_{-1}	c_0
DKU	$-1.0806993055087 \cdot 10^{-4}$	$7.8428619052630 \cdot 10^{-4}$	$-3.3829109154253 \cdot 10^{-3}$
CEZ	$-1.0806993055058 \cdot 10^{-4}$	$7.8428619052767 \cdot 10^{-4}$	$-3.3829109154640 \cdot 10^{-3}$
BGKKS	$-1.0806993055088 \cdot 10^{-4}$	$7.8428619052632 \cdot 10^{-4}$	$-3.3829109156162 \cdot 10^{-3}$

Tuned comparison of results

[Dittmaier, Kallweit, Uwer 07], [Campbell, Ellis, Zanderighi 07],
[Binoth, Guillet, SK, Kauer, Sanguinetti] → [NLM Les Houches report 08]

- Integrated LO results

$PP \to W^+W^-$+jet	σ_{LO} [fb]
DKU	10371.7(12)
CEZ	10372.26(97)
BGKKS	10371.7(11)

- Results for virtual corrections checked at one phase space point:

$$2\,\mathrm{Re}\{\mathcal{M}_V^* \cdot \mathcal{M}_{LO}\} = e^4 g_s^2\, \Gamma(1+\epsilon) \left(\frac{4\pi\mu^2}{M_W^2}\right)^2 \left(\frac{1}{\epsilon^2} c_{-2} + \frac{1}{\epsilon} c_{-1} + c_0\right)$$

$u\bar{u} \to W^+W^- g$	c_{-2}	c_{-1}	c_0
DKU	$-1.0806993055087 \cdot 10^{-4}$	$7.8428619052630 \cdot 10^{-4}$	$-3.3829109154253 \cdot 10^{-3}$
CEZ	$-1.0806993055058 \cdot 10^{-4}$	$7.8428619052767 \cdot 10^{-4}$	$-3.3829109154640 \cdot 10^{-3}$
BGKKS	$-1.0806993055088 \cdot 10^{-4}$	$7.8428619052632 \cdot 10^{-4}$	$-3.3829109156162 \cdot 10^{-3}$

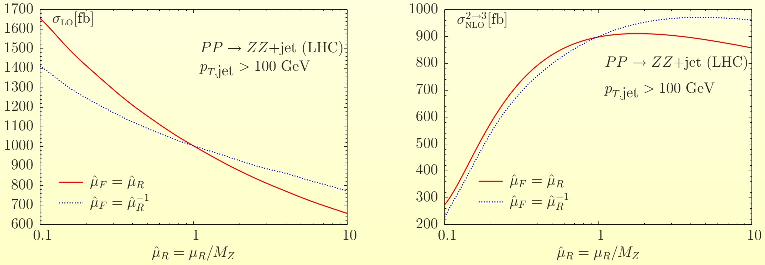
- three different methods, impressive agreement!

Intermediate results for $ZZ + \text{jet}$

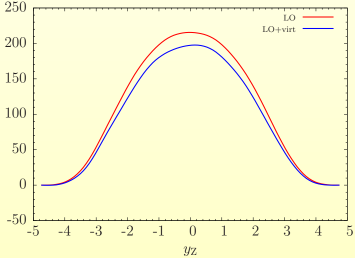
Setup:
$N_F = 5$, $m_q = 0$, $M_Z = 91.188$ GeV, $\sin^2\theta_W = 0.222247$
PDFs: Cteq6L1 for LO, Cteq6m for NLO
Cuts: $p_{T\text{jet}} > 100$ GeV
Scales: $\mu = \mu_F = \mu_R = \xi \cdot M_Z$

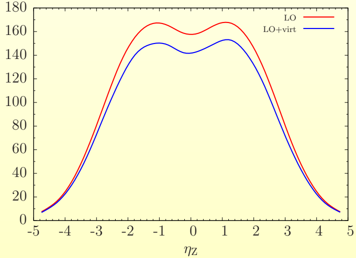
	$\sigma(\mu = 0.5M_Z)[\text{fb}]$	$\mu = M_Z$	$\mu = 2\,M_Z$
LO	1156	1006	874
LO+virt	836	899	909
NLO: real	117	88	67
NLO: col	?	?	?

Scale variations:

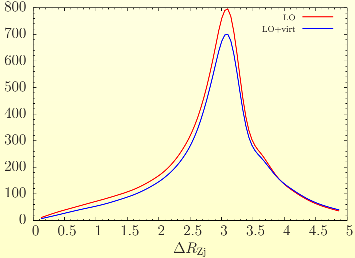


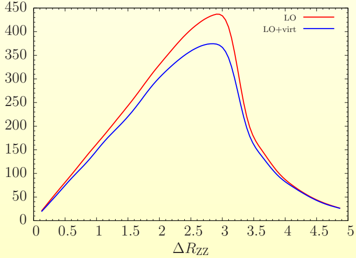
Distributions:



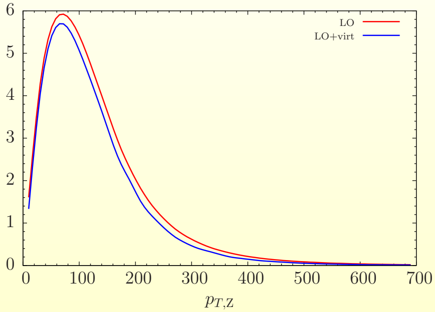


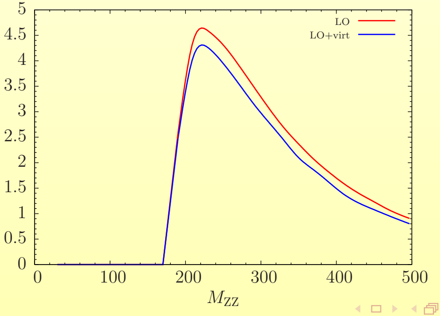
$y = \frac{1}{2} \log \frac{E+p_z}{E-p_z}$, $\eta = \frac{1}{2} \log \frac{1+\cos\theta}{1-\cos\theta}$

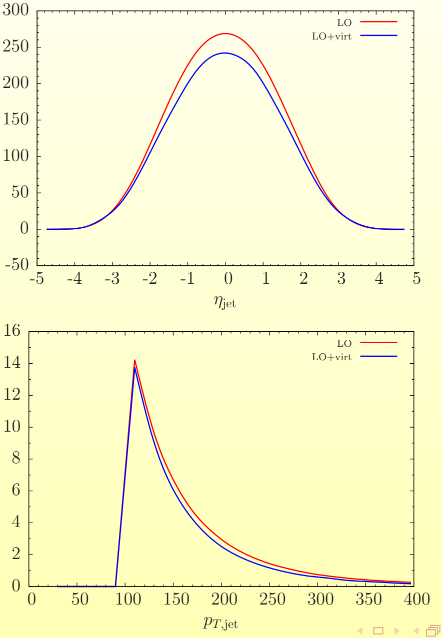




$$\Delta R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}$$


LO
LO+virt
0
1
2
3
4
5
6
0
100
200
300
400
500
600
700
$p_{T,\mathrm{Z}}$


LO
LO+virt
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
100
200
300
400
500
M_{ZZ}


300
250
200
150
100
50
0
-50
-5 -4 -3 -2 -1 0 1 2 3 4 5
η_{jet}
LO
LO+virt
16
14
12
10
8
6
4
2
0
0 50 100 150 200 250 300 350 400
$p_{T,\text{jet}}$
LO
LO+virt

Summary of results and Conclusion

- Calculation of $PP \to WW/ZZ+$jet at NLO as an important background process for Higgs searches almost complete
- analytic expressions for virtual corrections available and tested
- real corrections available
- collinear term under construction

Summary of results and Conclusion

- Calculation of $PP \rightarrow WW/ZZ+$jet at NLO as an important background process for Higgs searches almost complete
- analytic expressions for virtual corrections available and tested
- real corrections available
- collinear term under construction
- LHC phenomenology needs at least NLO precision
- NLO multi particle processes are still challenging
- our methods are completely general
  $\rightarrow$ apply to other interesting processes

Summary of results and Conclusion

- Calculation of $PP \rightarrow WW/ZZ$+jet at NLO as an important background process for Higgs searches almost complete
- analytic expressions for virtual corrections available and tested
- real corrections available
- collinear term under construction
- LHC phenomenology needs at least NLO precision
- NLO multi particle processes are still challenging
- our methods are completely general
  $\rightarrow$ apply to other interesting processes
- LHC = Long and Hard Calculations
  but results are coming in and we contribute