Precision QCD with Jet observables

Aude Gehrmann-De Ridder

(with R. Boughezal, A. Daleo, T. Gehrmann, N. Glover,

G. Heinrich, G. Luisoni, D. Maître, M. Ritzmann)

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Precision physics with jets

Jet observables

- testing ground for QCD: perturbation theory, logarithmic resummation, power corrections
- enable a precise determination of the strong coupling constant α_s
  - in e^+e^- from $e^+e^- \to 3j$ and event shapes
  - in ep from $ep \to (2+1)j$
  - in $p\bar{p}$ from $p\bar{p} \to 1j + X$
- Determination of α_s so far dominated by theoretical uncertainty
  - example: in e^+e^- from jets:

$$\alpha_s(M_Z) \quad = \quad 0.1202 \pm 0.0003(\text{stat}) \pm 0.0009(\text{sys}) \pm 0.0009(\text{had}) \pm 0.0047(\text{scale})$$

- enable a better knowledge of the gluon distribution in the proton from $ep \to (2+1)j$ or $pp, \; p\bar{p} \to 1j + X$
- multijet-signatures often part of signals or backgrounds to new physics searches at present and future colliders

$\longrightarrow$ Jet observables needed as precisely as possible: at NNLO

NNLO Subtraction

Structure of NNLO m-jet cross section:

$$\begin{aligned}
d\sigma_{NNLO} &= \int_{d\Phi_{m+2}} \left(d\sigma^{R}_{NNLO} - d\sigma^{S}_{NNLO} \right) \\
&+ \int_{d\Phi_{m+1}} \left(d\sigma^{V,1}_{NNLO} - d\sigma^{VS,1}_{NNLO} \right) \\
&+ \int_{d\Phi_{m}} d\sigma^{V,2}_{NNLO} + \int_{d\Phi_{m+2}} d\sigma^{S}_{NNLO} + \int_{d\Phi_{m+1}} d\sigma^{VS,1}_{NNLO} ,
\end{aligned}$$

- $d\sigma^{S}_{NNLO}$: real radiation subtraction term for $d\sigma^{R}_{NNLO}$
- $d\sigma^{VS,1}_{NNLO}$: one-loop virtual subtraction term for $d\sigma^{V,1}_{NNLO}$
- $d\sigma^{V,2}_{NNLO}$: two-loop virtual corrections

- Subtraction terms constructed using the antenna subtraction method at NNLO
  T. Gehrmann, N. Glover, AG.
- Each line above is finite numerically and free of infrared ϵ-poles

$e^+e^- \to 3$ jets and event shapes

Application of NNLO antenna subtraction

- implemented as parton-level event generator: EERAD3
  T. Gehrmann, E.W.N. Glover, G. Heinrich, AG
- allows to compute jet cross sections and event shapes through to α_s^3 in e^+e^- collisions

$e^+e^- \to 3$ jets and event shapes

Event shape variables

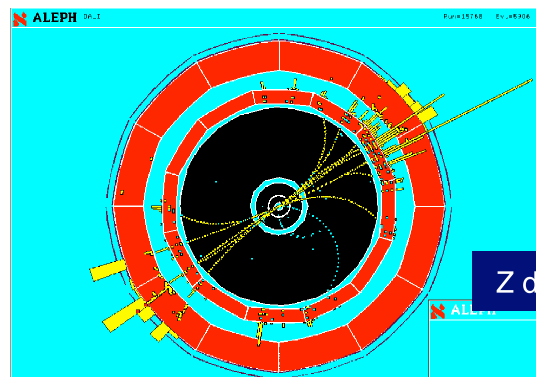
- characterize the geometrical properties of final state events, are based on the particle momenta and are infrared safe
- easily accesible experimentally

e.g. Thrust in e^+e^-

$$T = \max_{\vec{n}} \frac{\sum_{i=1}^{n} |\vec{p_i} \cdot \vec{n}|}{\sum_{i=1}^{n} |\vec{p_i}|}$$

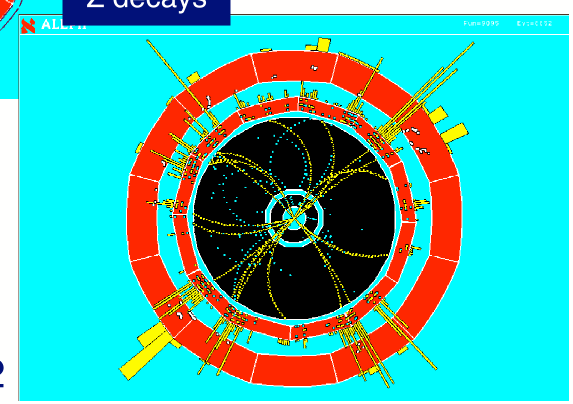
limiting values:

- back-to-back (two-jet) limit: $T = 1$
- spherical limit: $T = 1/2$



Thrust → 1

Z decays



Thrust → 1/2

Event shapes variables

Standard Set of LEP

- Thrust (E. Farhi)

$$T = \max_{\vec{n}} \left(\sum_{i=1}^{n} |\vec{p_i} \cdot \vec{n}| \right) / \left(\sum_{i=1}^{n} |\vec{p_i}| \right)$$

- Heavy jet mass (L. Clavelli, D. Wyler)

$$\rho = M_i^2/s = \frac{1}{E_{\mathrm{vis}}^2} \left(\sum_{k \in H_i} |\vec{p_k}| \right)^2$$

- C-parameter: eigenvalues of the tensor (G. Parisi)

$$\Theta^{\alpha\beta} = \frac{1}{\sum_k |\vec{p_k}|} \frac{\sum_k p_k^\alpha p_k^\beta}{\sum_k |\vec{p_k}|}$$

- Jet broadenings (S. Catani, G. Turnock, B. Webber)

$$B_i = \left(\sum_{k \in H_i} |\vec{p_k} \times \vec{n}_T| \right) / \left(2 \sum_k |\vec{p_k}| \right)$$

$$B_W = \max(B_1, B_2) \qquad B_T = B_1 + B_2$$

- $3j \to 2j$ transition parameter in Durham algorithm y_{23}^D
  S.Catani, Y.L.Dokshitzer, M.Olsson, G.Turnock, B.Webber

Event shapes at NNLO

Event shape observables : $\frac{1}{\sigma_{had}} y \frac{\mathrm{d}\sigma}{\mathrm{d}y}$ (y: event shape variable)

- NNLO corrections sizeable, non uniform
- theoretical uncertainty reduced
- Perturbative results of all event shape observables reliable between
  - 2 jet region: ($y \to 0$), the observables diverge like $1/y \ln^a y$, ($a = 2$ (NLO) and $a = 3$ (NNLO))
  - multi-jet region: (large y), the observables vanish, the event shape variables y are bound kinematically (example: (LO) Thrust $(1 - T) < 1/3$)
- Applications of event shapes calculated at NNLO
  - matching onto NLLA possible to improve results towards the 2-jet region
    G. Luisoni, H. Stenzel, T. Gehrmann
  - new extraction of α_s, based on NNLO or NNLO+NLLA
    G. Dissertori, T. Gehrmann, E.W.N. Glover, G. Heinrich, G. Luisoni, H. Stenzel, AG, work in progress
  - Moments of event shapes
  - but: . . .

$e^+e^- \to 3$ jets and event shapes

Comparison with other groups

- comparison with SCET-based calculation of logarithmically enhanced terms: discrepancy in two colour factors in two-jet region (kinematic limit)
  T. Becher, M. Schwartz
- independent implementation of antenna subtraction uncovered oversubtraction of large-angle soft gluon emission (S. Weinzierl)
- corrected by introducing soft antenna function in N^2 and N^0 colour factors

$$\begin{aligned}
\mathcal{S}_{ac;ik} &= \int \mathrm{d}\Phi_{X_{ijk}} S_{ajc} \\
&= (s_{IK})^{-\epsilon} \frac{\Gamma^2(1-\epsilon)e^{\epsilon\gamma}}{\Gamma(1-3\epsilon)} \left(-\frac{2}{\epsilon}\right) \left[-\frac{1}{\epsilon} + \ln\left(x_{ac,IK}\right) + \epsilon \operatorname{Li}_2\left(-\frac{1-x_{ac,IK}}{x_{ac,IK}}\right)\right] \\
x_{ac,IK} &= \frac{s_{ac}s_{IK}}{(s_{aI}+s_{aK})(s_{cI}+s_{cK})}
\end{aligned}$$

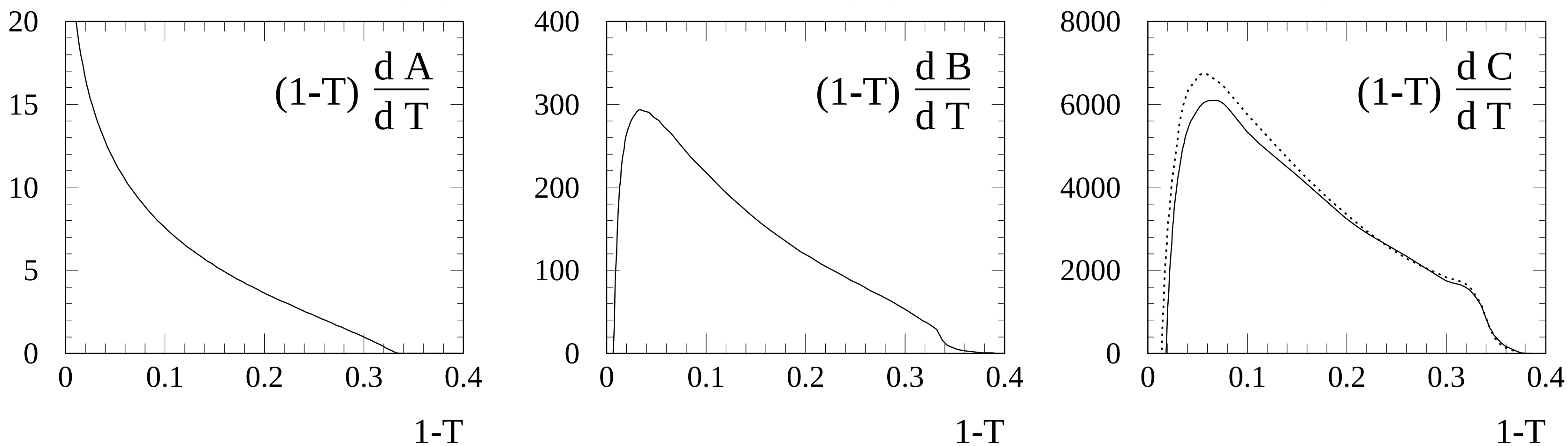
- now: numerical agreement with S. Weinzierl , discrepancy with SCET resolved

Event shapes at NNLO

NNLO expression for Thrust

$$(1-T)\,\frac{1}{\sigma_{\rm had}}\,\frac{{\rm d}\sigma}{{\rm d}T} = \left(\frac{\alpha_s}{2\pi}\right) A(T) + \left(\frac{\alpha_s}{2\pi}\right)^2 (B(T) - 2A(T)) + \left(\frac{\alpha_s}{2\pi}\right)^3 (C(T) - 2\,B(T) - 1.64\,A(T))$$

with LO contribution $A(T)$, NLO contribution $B(T)$, NNLO contribution $C(T)$



- for all event shapes
  - In the three-jet region (relevant for phenomenology) changes have only minor numerical impact
  - In the 2-jet region, changes lift the discrepancies

NNLO moments of event shapes

The n^{th} moment of an event-shape variable y

$$\langle y^n \rangle = \frac{1}{\sigma_{\rm had}} \int_0^{y_{\rm max}} y^n \frac{{\rm d}\sigma}{{\rm d}y} {\rm d}y$$

with

$$\langle y^n \rangle = \langle y^n \rangle_{\rm pt} + \langle y^n \rangle_{\rm np}$$

- Moments with $1 \leq n \leq 5$ have beeen measured by JADE and OPAL for $Q = 10 - 206$ GeV
  Aim: extract non-perturbative part $\langle y^n \rangle_{\rm np}$ by comparing the data with the calculated perturbative part $\langle y^n \rangle_{\rm pt}$
  - large range of energies considered to disentangle the nature of the corrections needed: power-like $(1/Q)$, perturbative $(1/\ln(Q))$
- the perturbative part $\langle y^n \rangle_{\rm pt}$ is computed to NNLO (with EERAD3) using

$$\langle y^n \rangle (s, \mu^2 = s) = \left(\frac{\alpha_s}{2\pi}\right) \bar{\mathcal{A}}_{y,n} + \left(\frac{\alpha_s}{2\pi}\right)^2 \bar{\mathcal{B}}_{y,n} + \left(\frac{\alpha_s}{2\pi}\right)^3 \bar{\mathcal{C}}_{y,n} + \mathcal{O}(\alpha_s)^4$$

- coefficients: dimensionless numbers for each moment and event shape variable

NNLO moments of event shapes

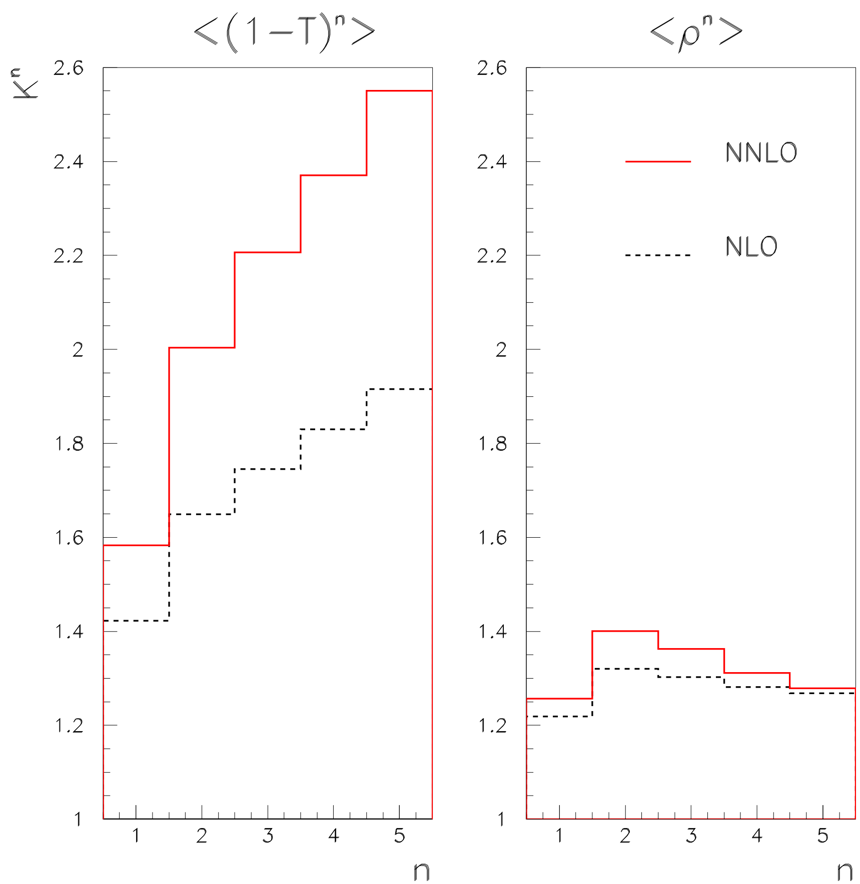
Moments require integration over full phase space

$$\langle y^n \rangle = \frac{1}{\sigma_{\rm had}} \int_0^{y_{\rm max}} y^n \frac{{\rm d}\sigma}{{\rm d}y} {\rm d}y$$

- all event shapes diverge like $1/y \ln^a y$ for $y \to 0$, ($a = 2$ (NLO) and $a = 3$ (NNLO))
- moments are finite for $n \geq 1$
- the evaluation of the first moment is particularly challenging
  - receives sizable contribution from the $y \to 0$ region
  - contains integrable logarithmic singularity
  - in practise: introduce technical cut-offs on the event shape variable and the phase space invariants
- the higher n is, the more the moments are sensitive on the multi-jet region (large y)

NNLO moments of event shapes

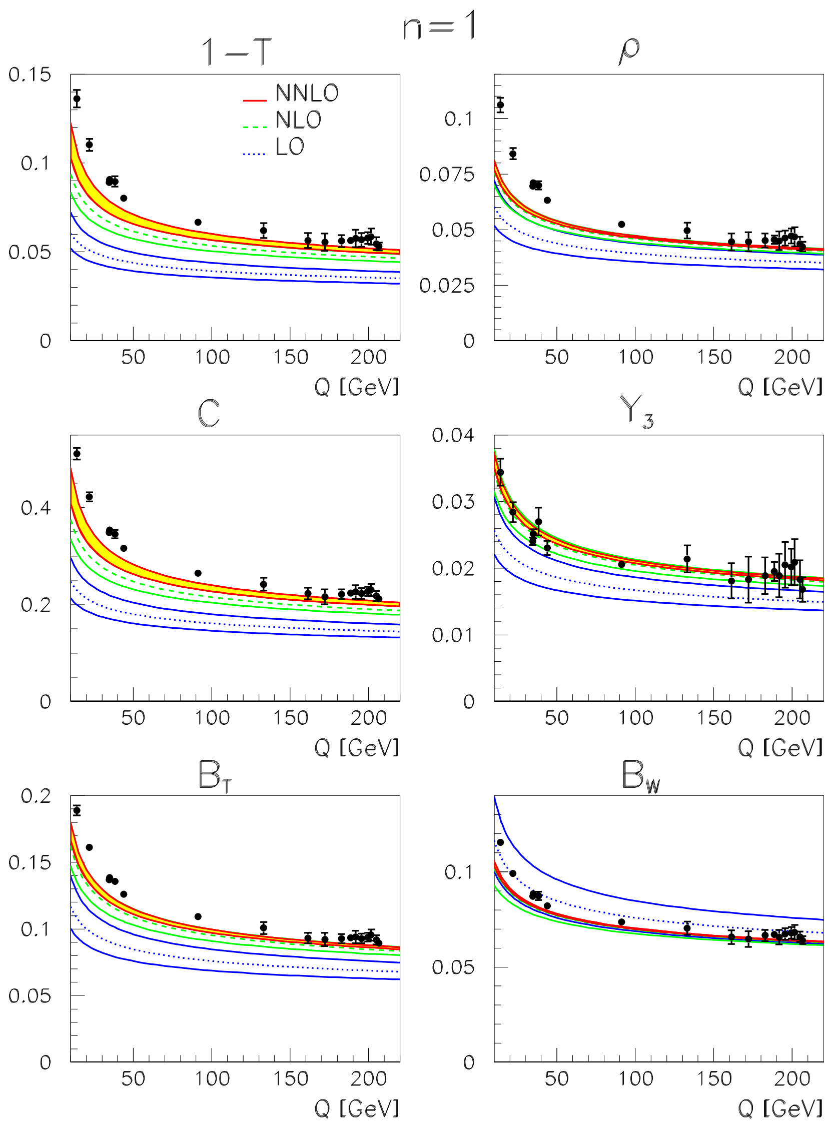
Behaviour of NNLO corrections



K-factors for $\mu = Q$
normalised to LO, for $\alpha_s = 0.124$

- size of corrections increases with n for $(1 - T)$, C, B_T
- size of corrections constant ($\mathcal{K}_{\rm NNLO} \approx 1.3$) with n for ρ, Y_3, B_W

NNLO moments of event shapes



Energy dependence of the first moment

[Data: JADE and OPAL collaborations]

Results:

- reduced theoretical uncertainty in all variables
- describe Y_3 and B_W largely without power corrections
- potentially large power corrections, especially in ρ and $(1-T)$ for low Q
- same features observed for higher moments $\longrightarrow$ full kinematical range concerned

Next step:

Precision jet observables at hadron colliders
HERA (ep), Tevatron ($p\bar{p}$) and LHC (pp)

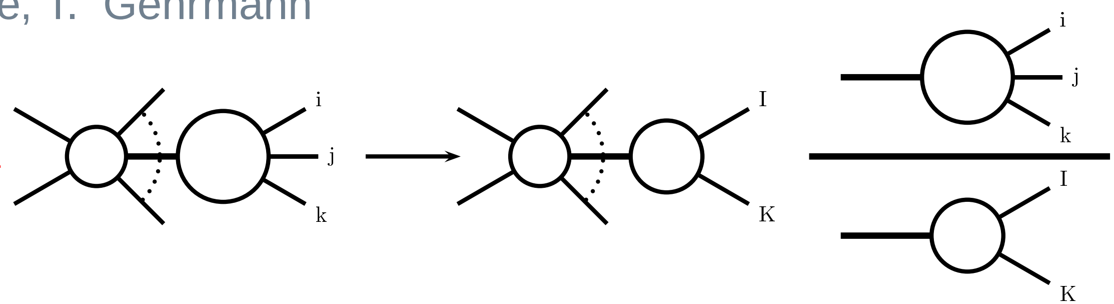
Towards NNLO antenna subtraction with hadronic initial states

Incoming hadrons

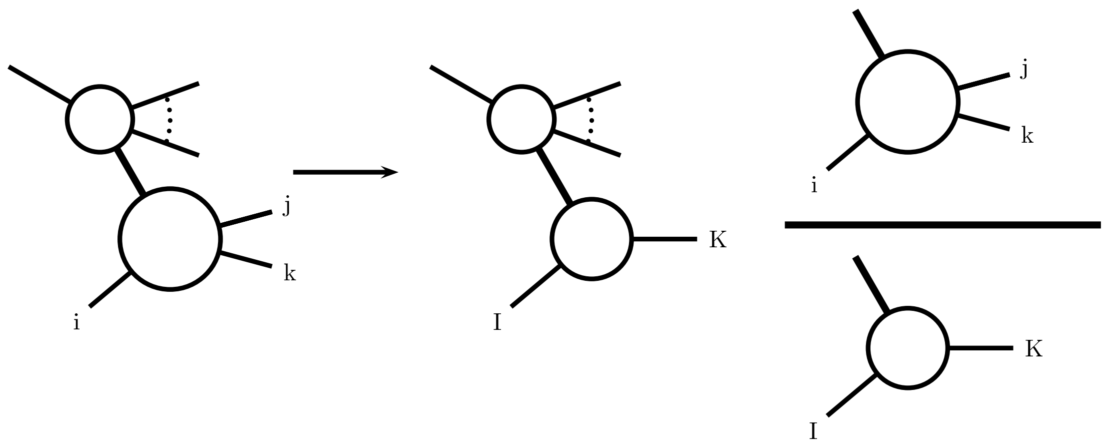
Three antenna types

NLO: A. Daleo, D. Maître, T. Gehrmann

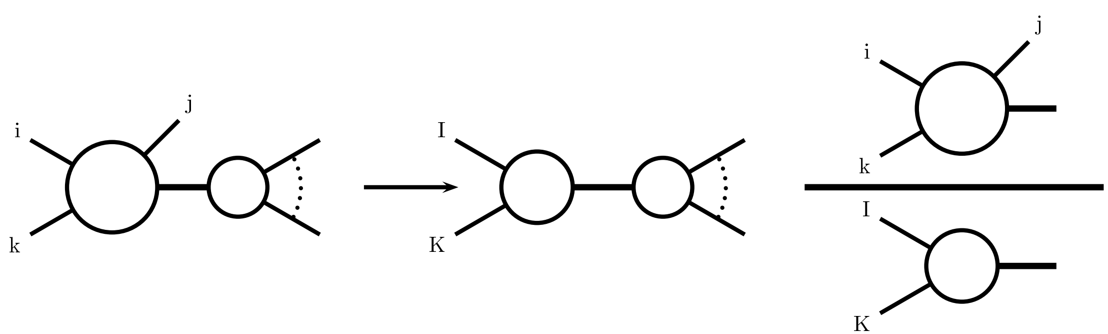
- final-final antenna



- initial-final antenna



- initial-initial antenna



Colour-ordered antenna functions

Antenna Functions

- colour-ordered pair of hard partons (radiators) with radiation in between
  - hard quark-antiquark pair
  - hard quark-gluon pair
  - hard gluon-gluon pair
- three-parton antenna $\longrightarrow$ one unresolved parton
- four-parton antenna $\longrightarrow$ two unresolved partons
- can be at tree level or at one loop
- all three-parton and four-parton antenna functions can be derived from physical matrix elements, normalised to two-parton matrix elements
  - $q\bar{q}$ from $\gamma^* \to q\bar{q} + X$
  - qg from $\tilde{\chi} \to \tilde{g}g + X$
  - gg from $H \to gg + X$

Initial–final antenna functions

Real Radiation: $2 \to 3$

- obtain antenna functions by crossing $1 \to 4$ NNLO antennae
- kinematics: $q + p \to k_1 + k_2 + k_3$, with $q^2 < 0$.
- phase space factorization:

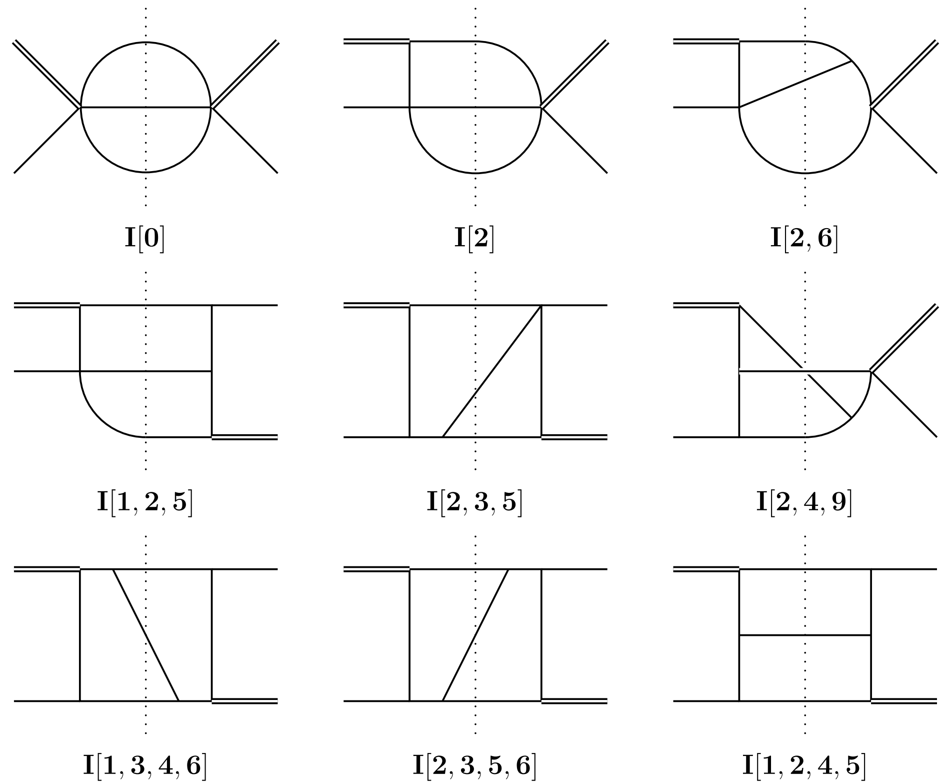
$$
\begin{aligned}
&\mathrm{d}\Phi_{m+2}(k_1, \ldots, k_j, k_k, k_l, \ldots, k_{m+2}; p, r) = \\
&\qquad \mathrm{d}\Phi_m(k_1, \ldots, K_L, \ldots, k_{m+2}; xp, r) \frac{Q^2}{2\pi} \mathrm{d}\Phi_3(k_j, k_k, k_l; p, q) \frac{\mathrm{d}x}{x}
\end{aligned}
$$

A. Daleo, D. Maître, T. Gehrmann

- integrated antenna functions: inclusive three-particle phase space integrals with q^2 and $z = -q^2/(2q \cdot p)$ fixed
- similar to NNLO deep-inelastic coefficient functions
  W.L. van Neerven, E.B. Zijlstra

Initial–final antenna functions

Real Radiation: $2 \to 3$



- reduce phase space integrals to master integrals
  C. Anastasiou, K. Melnikov
- compute using differential equations
- boundary conditions: very similar to inclusive $1 \to 4$ phase space master intergals
  T. Gehrmann, G. Heinrich, AG

Initial–final antenna functions

Real Radiation at One Loop: $2 \to 2$

- obtain antenna functions by crossing one-loop $1 \to 3$ NNLO antennae
- kinematics: $q + p \to k_1 + k_2$, with $q^2 < 0$.
- phase space factorization:

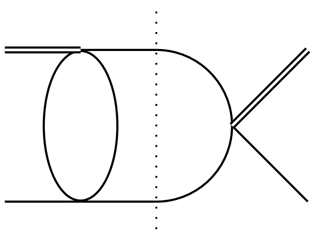
$$
\begin{aligned}
&\mathrm{d}\Phi_{m+1}(k_1, \ldots, k_j, k_k, \ldots, k_{m+1}; p, r) = \\
&\qquad \mathrm{d}\Phi_m(k_1, \ldots, K_K, \ldots, k_{m+1}; xp, r) \frac{Q^2}{2\pi} \mathrm{d}\Phi_2(k_j, k_k; p, q) \frac{\mathrm{d}x}{x}
\end{aligned}
$$

- integrated antenna functions: inclusive two-particle phase space integrals of one-loop matrix elements with q^2 and $z = -q^2/(2q \cdot p)$ fixed

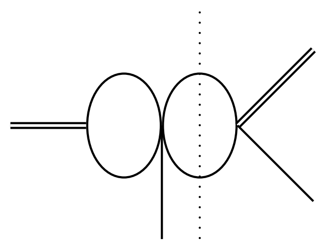
Initial–final antenna functions

Real Radiation at One Loop: $2 \to 2$

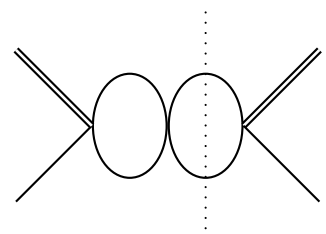
- reduce to master integrals
- most yield trivial Γ-functions
- non-trivial ones computed using differential equations



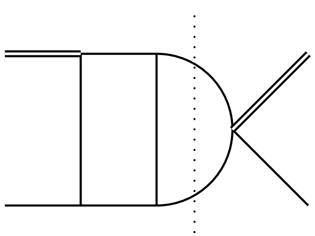
$\mathbf{V}[\mathbf{1},\mathbf{3}]$



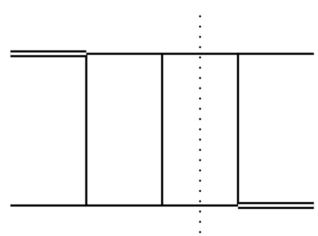
$\mathbf{V}[\mathbf{1},\mathbf{4}]$



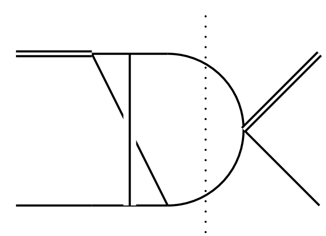
$\mathbf{V}[\mathbf{2},\mathbf{4}]$



$\mathbf{V}[\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4}]$



$\mathbf{V}[\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4},\mathbf{5}]$



$\mathbf{C}[\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4}]$

Initial–final antenna functions

NNLO results

A. Daleo, T. Gehrmann, G. Luisoni, AG

- obtain full set of integrated $2 \to 3$ tree-level and $2 \to 2$ one-loop antenna functions
- check in progress: combination of those yields NNLO deep inelastic coefficient functions for quarks and gluons
  E.B. Zijlstra, W.L. van Neerven; S. Moch, J. Vermaseren
- pole terms combine into two-loop deep inelastic splitting functions
- all ingredients for NNLO corrections to (2+1)-jet production in deep inelastic scattering

Initial–initial antenna functions

Real Radiation: $2 \longrightarrow 3$

- obtain antenna functions by crossing $1 \to 4$ NNLO antennae
- kinematics: $p_a + p_b \to k_1 + k_2 + q$, with $q^2 > 0$.
- phase space factorization: (A. Daleo,T. Gehrmann, D. Maître)

$$\begin{aligned} & \mathrm{d}\Phi_{m+2}(k_1,\ldots,k_i,k_j,k_k,k_l,\ldots,k_{m+2};p_1,p_2) \\ & = \quad \mathrm{d}\Phi_m(\tilde{k}_1,\ldots,\tilde{k}_i,\tilde{k}_l,\ldots,\tilde{k}_{m+2};x_1p_1,x_2p_2) \\ & \qquad \times\delta(x_1-\hat{x}_1)\,\delta(x_2-\hat{x}_2)\,[\mathrm{d}k_j]\,[\mathrm{d}k_k]\,\mathrm{d}x_1\,\mathrm{d}x_2 \end{aligned}$$

$$\hat{x}_1 = \left(\frac{s_{12}-s_{j2}-s_{k2}}{s_{12}}\ \frac{s_{12}-s_{1j}-s_{1k}-s_{j2}-s_{k2}+s_{jk}}{s_{12}-s_{1j}-s_{1k}}\right)^{\frac{1}{2}}$$

$$\hat{x}_2 = \left(\frac{s_{12}-s_{1j}-s_{1k}}{s_{12}}\ \frac{s_{12}-s_{1j}-s_{1k}-s_{j2}-s_{k2}+s_{jk}}{s_{12}-s_{j2}-s_{k2}}\right)^{\frac{1}{2}}$$

- integration: inclusive three-particle phase space integrals with q^2 and x_1, x_2 fixed
- similar to NNLO coefficient functions for differential Drell-Yan production
  C. Anastasiou, L.J. Dixon, K. Melnikov, F. Petriello
- work in progress: 32 Master integrals (R. Boughezal, M. Ritzmann, AG)

Summary and outlook

Antenna subtraction method

- based on collecting all radiation between a colour-ordered pair of two hard emitters
- allows subtraction of infrared divergences at NLO and NNLO
- three configurations for emitters: final-final, initial-final, initial-initial
- final-final fully solved and applied to $e^+e^- \to 3$ jets, phenomenology still ongoing
- Newest results: Moments of event shapes

Initial-final antenna functions

- established phase space factorization
- computed by reduction to master integrals and differential equations
- will allow to compute $ep \to e + (2+1)j$ in DIS

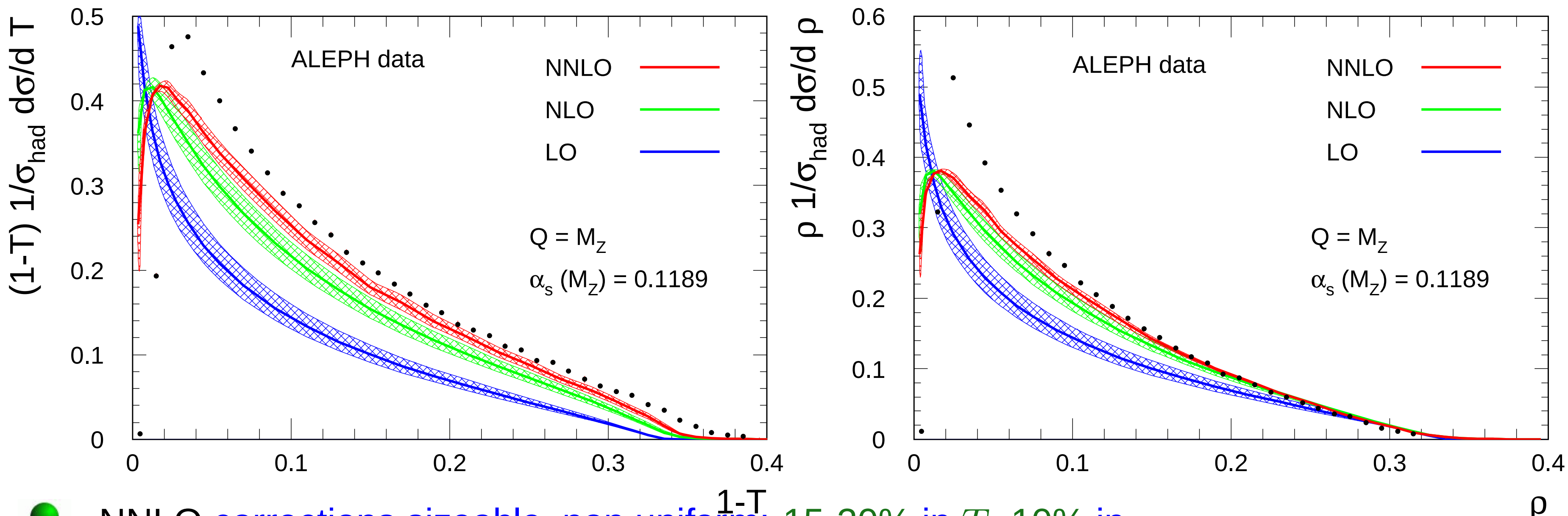
Initial-initial antenna functions

- established phase space factorization
- work in progress

Back-up slides

Event shapes at NNLO

NNLO thrust and heavy mass distributions



- NNLO corrections sizeable, non uniform: 15-20% in T, 10% in ρ
- theory uncertainty reduced by about 40 %
- large $1 - T, \rho > 0.33$: kinematically forbidden at LO
- small $1 - T, \rho$: two-jet region, need matching onto NLL resummation

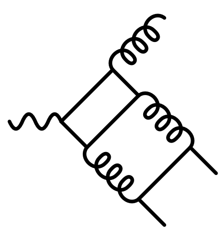
T. Gehrmann, G. Luisoni, H. Stenzel

Colour structure of NNLO 3-jet

Decomposition into leading and subleading colour terms

$$\sigma_{NNLO} = (N^2-1)\left[N^2\, A_{NNLO} + B_{NNLO} + \frac{1}{N^2}\, C_{NNLO} + N N_F\, D_{NNLO} \right.$$
$$\left. + \frac{N_F}{N}\, E_{NNLO} + N_F^2\, F_{NNLO} + N_{F,\gamma}\left(\frac{4}{N} - N\right)\, G_{NNLO}\right]$$

- last term: closed quark loop coupling to vector boson, numerically tiny



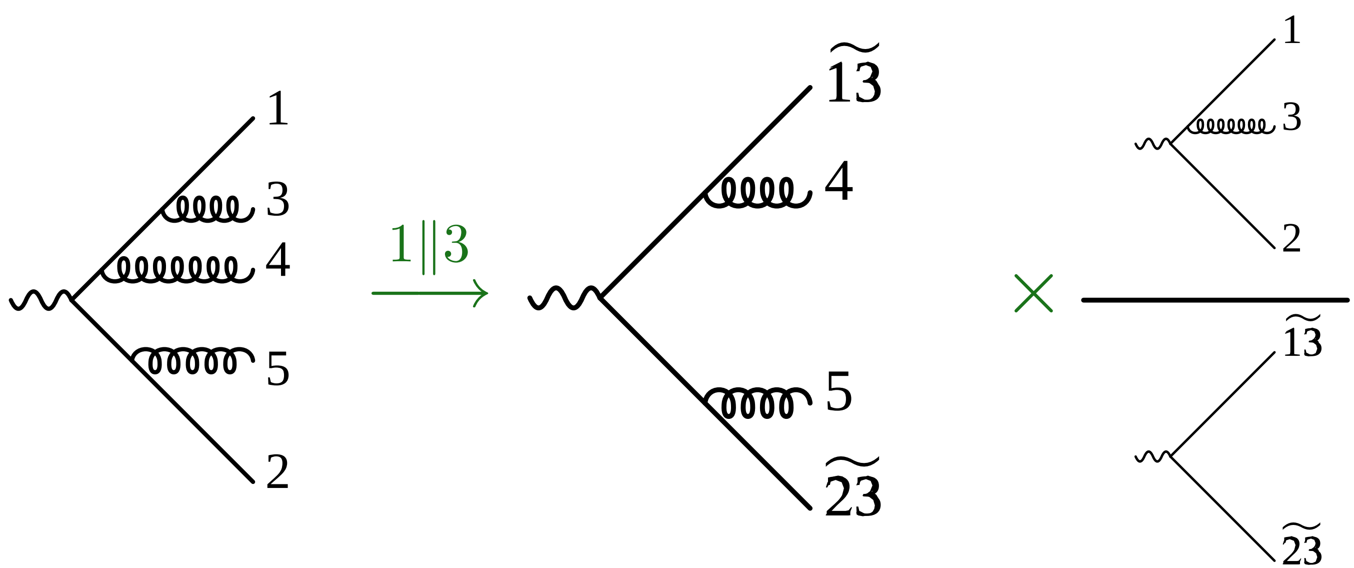
$$N_{F,\gamma} = \frac{\left(\sum_q e_q\right)^2}{\sum_q e_q^2}$$

- most subleading colour: C_{NNLO}, E_{NNLO}, F_{NNLO}, (G_{NNLO})
  QED-type contributions: gluons $\rightarrow$ photons
- simplest term: F_{NNLO}, only 3 parton and 4 parton contributions

Antenna functions

Quark-antiquark

consider subleading colour (gluons photon-like)



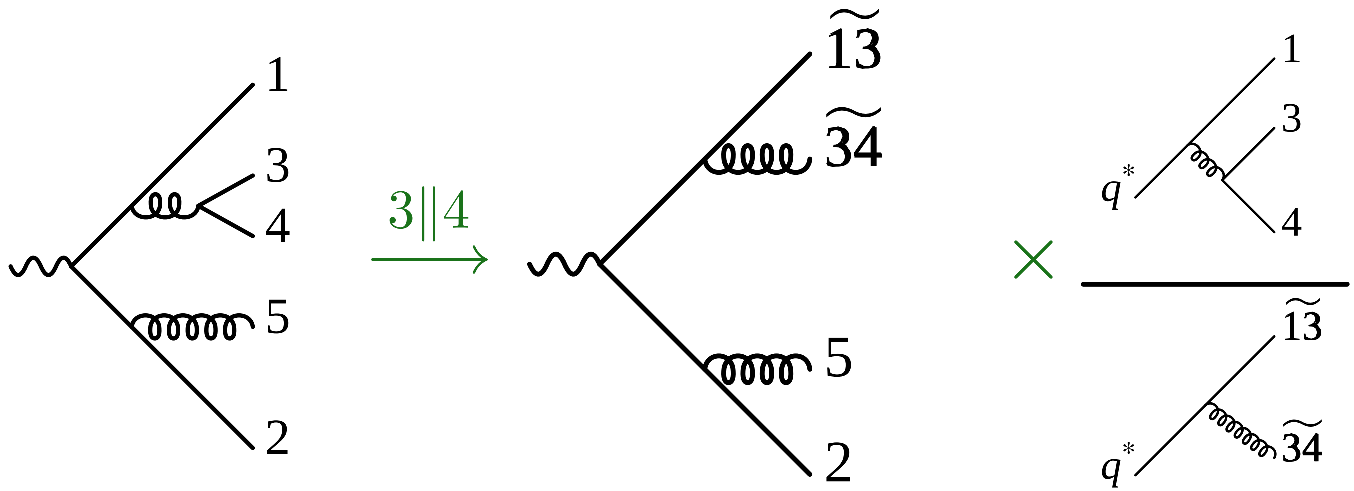
$$|M_{q\bar{q}ggg}|^2(1,3,4,5,2) \xrightarrow{1\|3} |M_{q\bar{q}gg}|^2(\widetilde{13},4,5,\widetilde{23}) \times X_{132}$$

with

$$X_{132} = \frac{|M_{q\bar{q}g}|^2}{|M_{q\bar{q}}|^2} \equiv A_3^0(1_q, 3_g, 2_{\bar{q}})$$

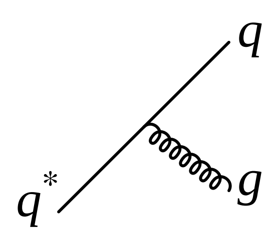
Antenna functions

Quark-gluon



$$|M_{q\bar{q}q\bar{q}g}|^2(1,3,4,5,2) \xrightarrow{3\|4} |M_{q\bar{q}gg}|^2(\widetilde{13},\widetilde{34},5,2) \times X_{134}$$

with hard radiators:
quark $(\widetilde{13})$ and gluon $(\widetilde{34})$



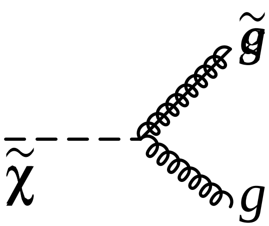
q^*	:	spin $1/2$, colour triplet
$q(\widetilde{13})$	:	spin $1/2$, colour triplet
$g(\widetilde{34})$	:	spin 1, colour octet

Off-shell matrix element: violates $SU(3)$ gauge invariance

Antenna functions

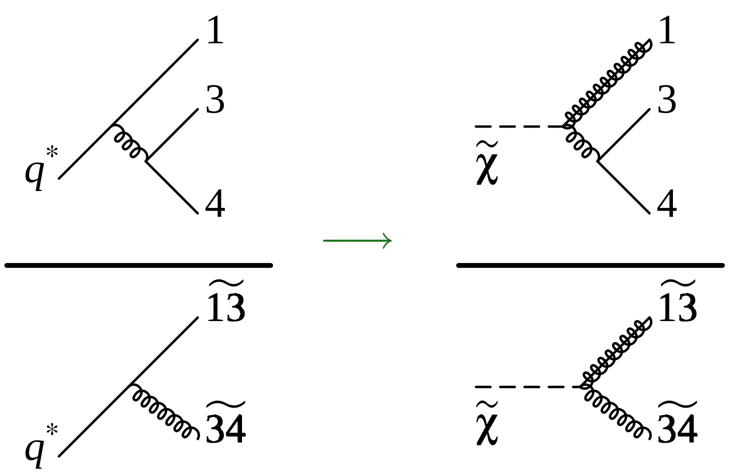
Quark-gluon

Construct colour-ordered qg antenna function from $SU(3)$ gauge-invariant decay:
neutralino $\longrightarrow$ gluino + gluon (T. Gehrmann, E.W.N. Glover, AG)



$$\begin{array}{lcl} \tilde{\chi} & : & \text{spin } 1/2, \text{colour singlet} \\ \tilde{g} & : & \text{spin } 1/2, \text{colour octet} \\ g & : & \text{spin } 1, \text{colour octet} \end{array}$$

Gluino $\tilde{g}$ mimics quark and antiquark (same Dirac structure), but is octet in colour space



$\tilde{\chi} \longrightarrow \tilde{g}g$ described by effective Lagrangian
H. Haber, D. Wyler

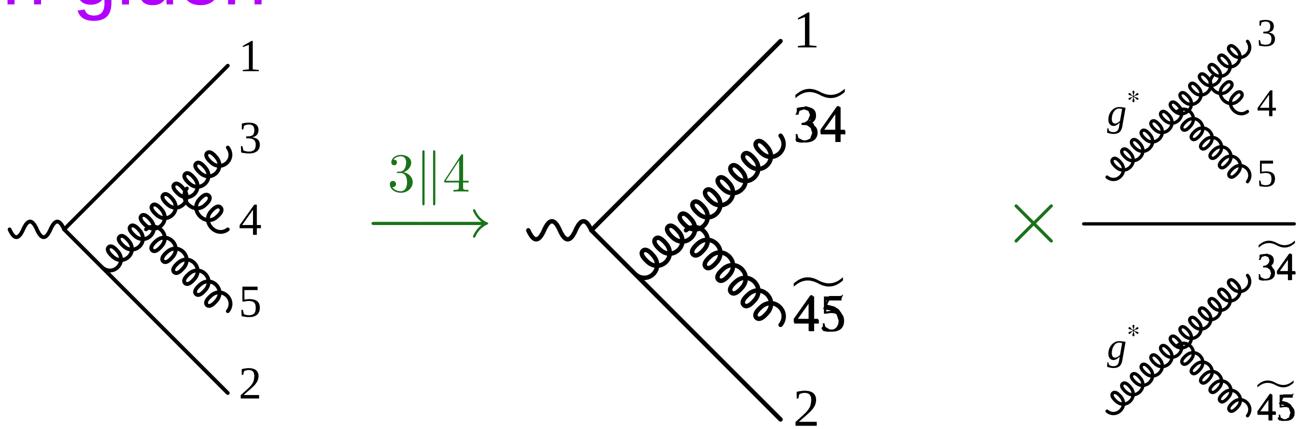
$$\mathcal{L}_{\text{int}} = i\eta\overline{\psi}^a_{\tilde{g}}\sigma^{\mu\nu}\psi_{\tilde{\chi}}F^a_{\mu\nu} + (\text{h.c.})$$

Antenna function

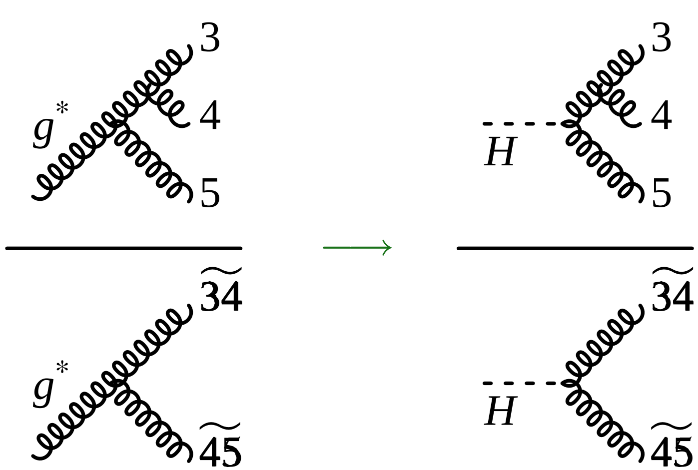
$$X_{134} = \frac{|M_{\tilde{g}q'\bar{q}'}|^2}{|M_{\tilde{g}g}|^2} \equiv E_3^0(1_q, 3_{q'}, 4_{\bar{q}'})$$

Antenna functions

Gluon-gluon



$$|M_{q\bar{q}ggg}|^2(1,3,4,5,2) \xrightarrow{3\|4} |M_{q\bar{q}gg}|^2(1,\widetilde{34},\widetilde{45},2) \times X_{345}$$



$H \rightarrow gg$ described by effective Lagrangian

F. Wilczek; M. Shifman, A. Vainshtein, V. Zakharov

$$\mathcal{L}_{\rm int} = \frac{\lambda}{4} H F^a_{\mu\nu} F^{\mu\nu}_a$$

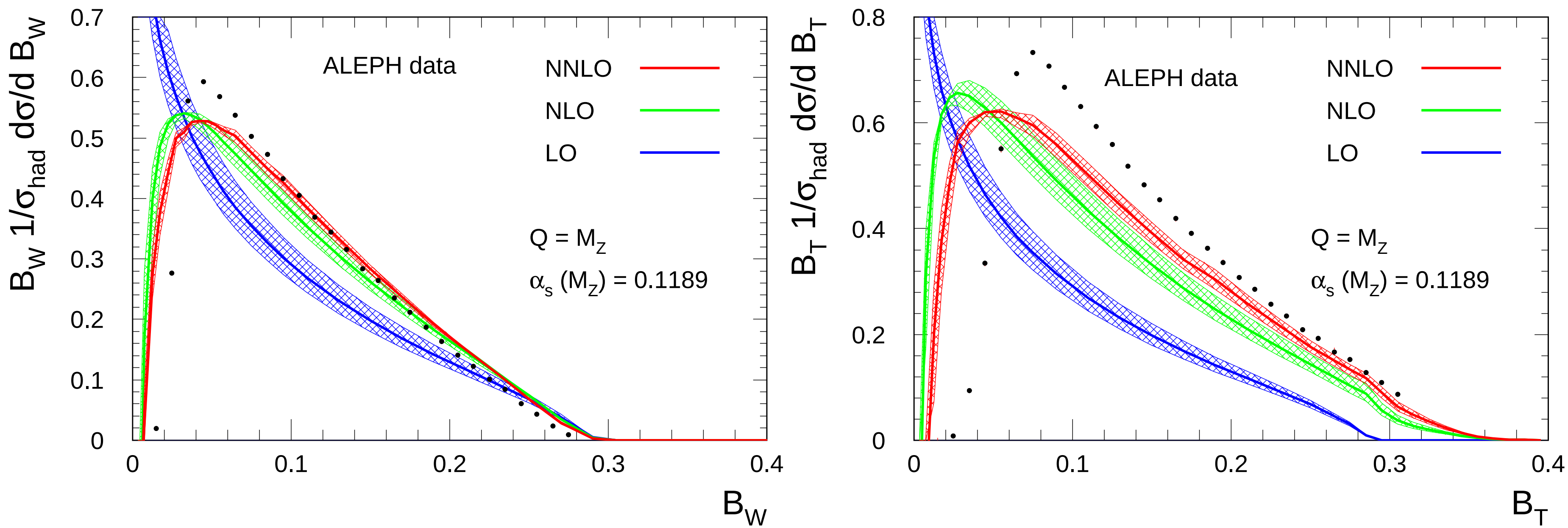
Antenna function

$$X_{345} = \frac{|M_{ggg}|^2}{|M_{gg}|^2} \equiv F_3^0(3_g, 4_g, 5_g)$$

Event shapes at NNLO

NNLO corrections: broadenings

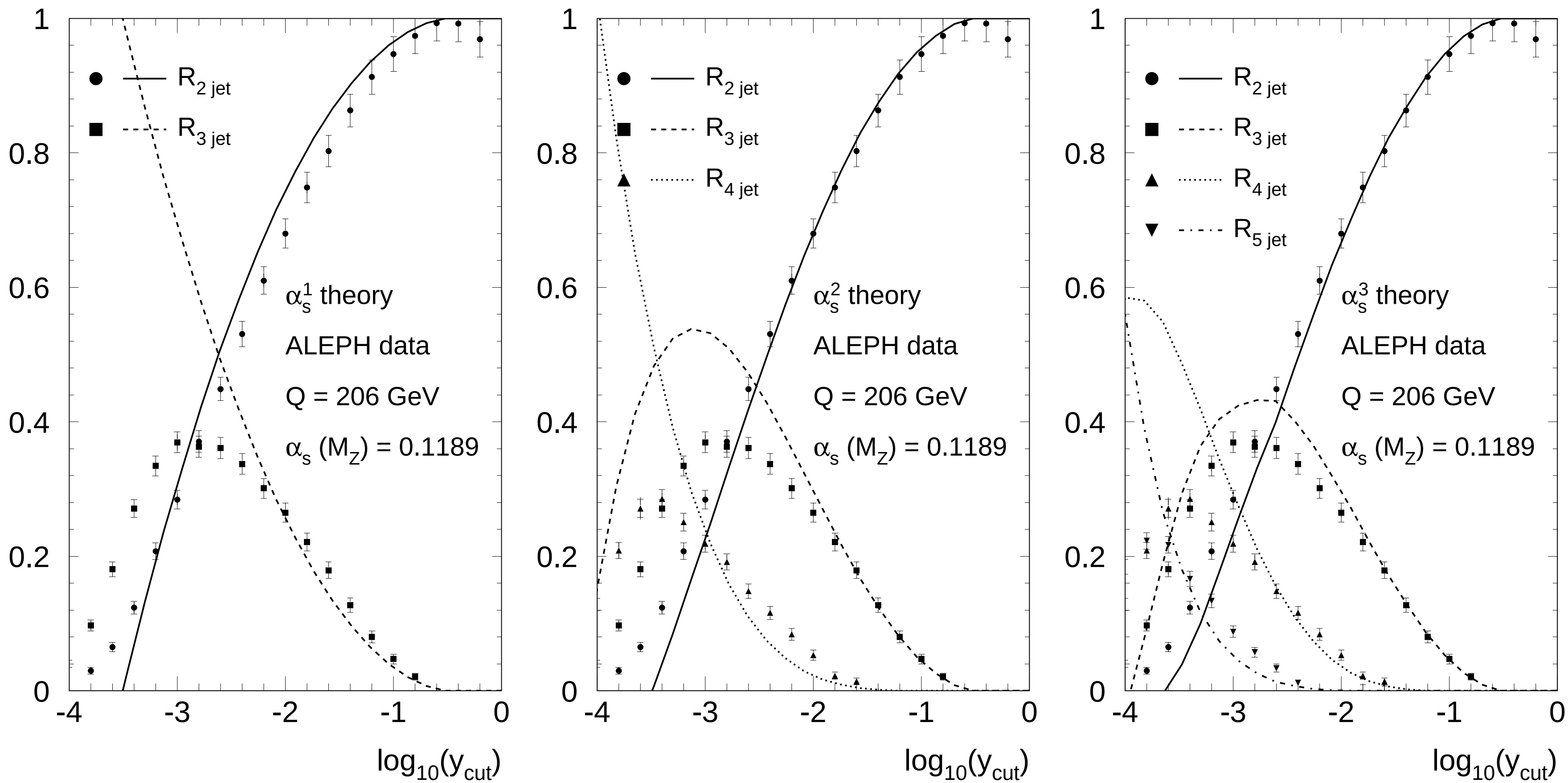
wide jet boadening B_W total jet boadening B_T



- NNLO corrections for B_W smaller than for B_T
- again require matching onto NLL resummation and hadronization corrections
- observe: small corrections for Y_3; large corrections for C
- reduction of dependence on renormalisation scale by 30–60%

Three-jet cross section at NNLO

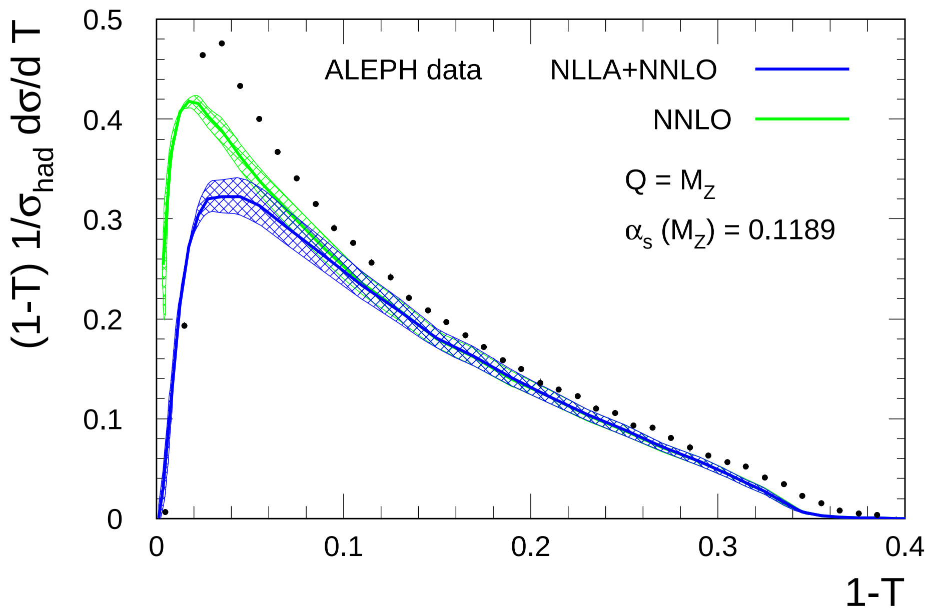
NNLO corrections: jet rates

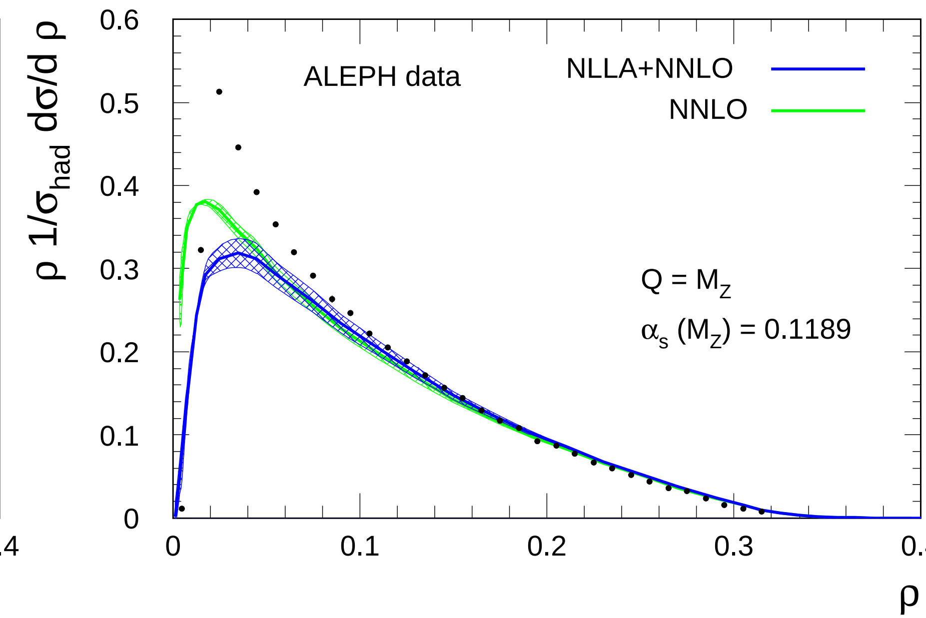


- substantial improvement towards lower $y_{\rm cut}$
- two-jet rate now NNNLO

Event shapes at NNLO+NLLA

NNLO+NLLA thrust and heavy mass





- (NNLO +NLLA) compared to (NNLO) prediction
  - slightly better description towards the 2-jet limit
  - In the 3-jet region, two predictions in agreement
  - further improvement needed: by including hadronization corrections