NNLO Higgs production via gluon fusion with finite top mass

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(work done in collaboration with Robert Harlander)

Outline

Introduction

The effective theory approach

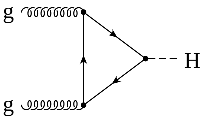
Expansion in the full theory

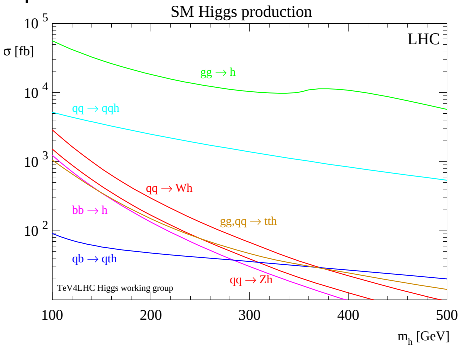
Results

Conclusion

$gg \to H$ in the SM

- production is via a top loop





- LO cross section known

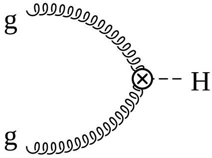
$$\sigma_{LO} = \frac{G_F \alpha_s^2(\mu^2)}{128\sqrt{2}\pi} \tau^2 \, \delta(1-x) |1 + (1-\tau) f(\tau)|^2$$

$$\tau = \frac{4m_t^2}{m_H^2}$$

[Georgi, Glashow, Machacek, Nanopoulos]

The Heavy Top Effective Theory

- If $\frac{m_H}{2m_{top}} \ll 1$ work in effective theory
- Top 'integrated out' of the theory
- ...but leaves its legacy in the form of altered couplings and new vertices



$$\mathcal{L}_{eff} = -\frac{H}{4v} C_1 G_{\mu\nu} G^{\mu\nu}$$

$$C_1 = -\frac{1}{3}\frac{\alpha_s}{\pi}\left\{1 + \frac{11}{4}\frac{\alpha_s}{\pi} + \cdots\right\}$$

Major benefit: reduces number of loops by one

Quantum Corrections

- QCD corrections huge - $\mathcal{O}(100\%)$
  NLO (effective theory) [Dawson'91]
  NLO (HIGLU) [Spira,Djouadi,Graudenz,Zerwas'95]
  NNLO (effective theory) [Harlander,Kilgore'02]
  [Anastasiou,Melnikov'02]
  [Ravindran,Smith,van Neerven'03]
- Electroweak
  [Actis, Passarino, Sturm, Uccirati '08]
- Mixed QCD-Electoweak
  [Anastasiou, Boughezal, Petriello '08]
- NNLO+NNLL - $\mathcal{O}(\%)$
  [Catani, de Florian, Grazzini, Nason '03]
- N^3LO threshold enhanced corrections
  [Moch, Vogt '05], [Laenen, Magnea '05], [Ravindran '05]
  [Kidonakis '05], [Idilbi, Ju, Yuan '05]
- "π^2-resummation"
  [Ahrens, Becher, Neubert, Yang '08]

Differential Quantities

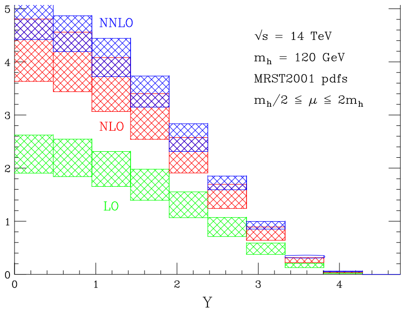
- Fully differential cross sections available through NNLO in the effective theory

  [Anastasiou, Petriello, Melnikov]
  [Grazzini]

- gg $\longrightarrow$ H + 2 jet cross section studied with full m_t dependence

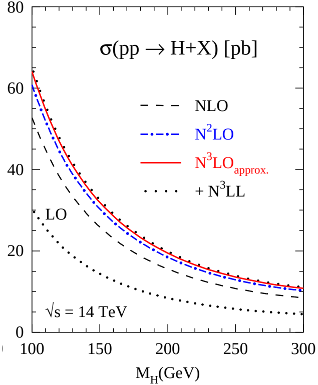
  [Del Duca, Kilgore, Oleari, Schmidt, Zeppenfeld]

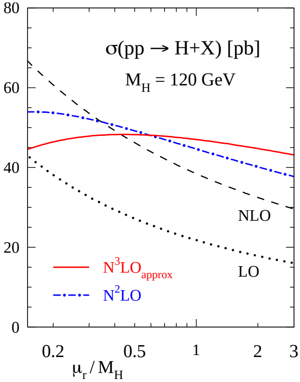
- Expect effective theory to be accurate provided m_H, p_T remain below m_t



Anastasiou, Petriello, Melnikov

Overview of QCD Corrections



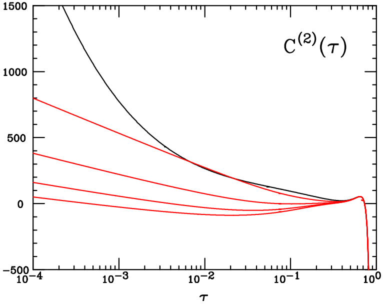


Moch, Vogt

- QCD effects well under control
- Residual scale uncertainty $\sim 5\%$
- See also updated analyses
  [Anastasiou, Boughezal, Petriello ’08]
  [de Florian, Grazzini ’09]

How accurate is the effective theory at NNLO?

Top Mass Effects - high energy region



- Infinite top mass approx breaks down when $\tau = m_H^2/\hat{s}$ is small
- Observe spurious behaviour as $\tau \to 0$
- These contributions can be separately evaluated

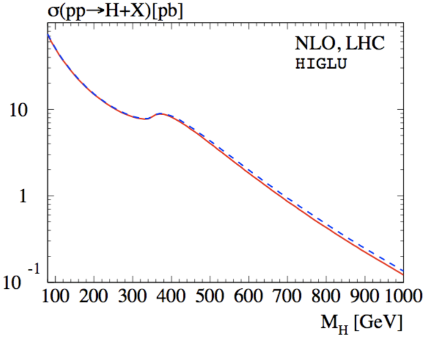
Marzani,Ball,Del Duca,Forte,Vicini [arXiv:0801.2544]

$$C^{(2)}(\tau,\infty) \xrightarrow{\tau\to 0} c_1 \ln\tau + c_2 \ln^2\tau + c_3 \ln^3\tau$$

$$C^{(2)}(\tau,m_t) \xrightarrow{\tau\to 0} d_1 \ln\tau$$

- d_1 evaluated numerically for various m_H values
- Can lead to $\sim 5\%$ corrections

Top Mass Effects at NLO



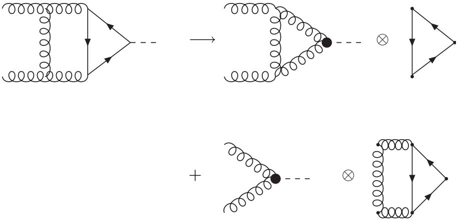
- Define $K = \frac{\sigma_{LO}^{full}}{\sigma_{LO}^{\infty}}$
- Capture mass dependence at NLO with $K \times \sigma_{NLO}^{\infty}$
- Works remarkably well

  [Krämer, Laenen, Spira '96]

- Also used at NNLO

Asymptotic Expansion

- Full NNLO calculation with top mass not currently feasible
- We perform an asymptotic expansion in $\frac{1}{m_t}$



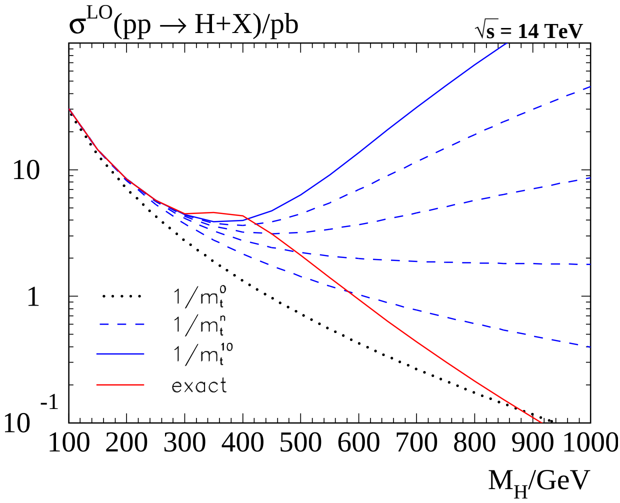
$$\sigma = \sum_n \left(\frac{m_H^2}{4m_t^2} \right)^n \sigma_n$$

- First term σ_0 is the effective theory result
- First non-leading $1/m_t$ term at NLO known

  [Dawson,Kauffman '93]
- Tools exist to automatize the calculation

  QGRAF, EXP, FORM, MATAD, MINCER

Expansion of σ_{LO}



- Converges up to $m_H \sim 2m_t$ threshold

NNLO ingredients

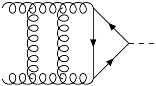
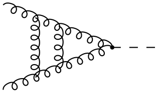
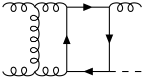
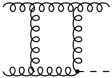
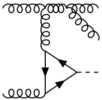
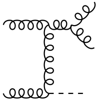
We have three contributions:

- double virtual
- single real emission
- double real emission

$$\sigma = \int M^{(3)}_{gg\to H}\left[M^{(1)}_{gg\to H}\right]^* + |M^{(2)}_{gg\to H}|^2$$
$$+ \iint M^{(2)}_{gg\to Hg}\left[M^{(1)}_{gg\to Hg}\right]^*$$
$$+ \iiint |M^{(1)}_{gg\to Hgg}|^2$$

- Each integrated over relevant phase space volume

Comparison

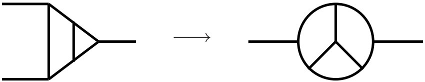
Full Theory	diagrams	Effective Theory	diagrams
	623		120
	327		87
	114		39

- All diagrams generated with QGRAF, expanded with EXP, processed with FORM

Loop Amplitudes

- After expansion, double virtual part consists of 2-loop 3-point diagrams
- Use Baikov-Smirnov method to map onto known 3-loop 2-point diagrams

  [hep-ph/0001192]



- First used to evaluate the virtual contribution to the Higgs cross section in the effective theory

  Harlander [hep-ph/0007289]

- Can treat arbitrary propagator powers

Phase Space Integration (1)

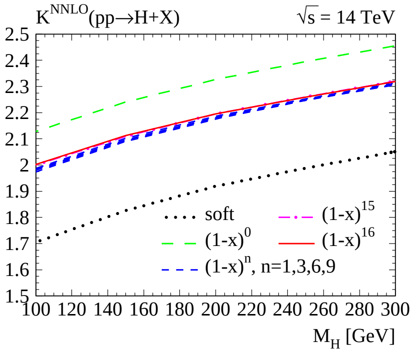
- One particle final states are easy $\longrightarrow \delta(1 - \frac{m_H^2}{\hat{s}})$
- Two particle final states are somewhat less easy
  - Amplitudes come with ${}_2F_1$ hypergeometrics
  - Direct integration yields extended hypergeometrics $({}_3F_2,\ {}_4F_3)$

$$\begin{aligned} \int_0^1 & dv\ v^{\alpha}(1-v)^{\beta}\ {}_2F_1(1,-\epsilon,1-\epsilon,z\ v) \\ = & B(\alpha,\beta)\ {}_3F_2\left[\{1,-\epsilon,1+\alpha\},\{1-\epsilon,\alpha+\beta+2\},z\right] \end{aligned}$$

- Use HypExp package to expand these in ϵ
  [Huber,Maitre]

Phase Space Integration (2)

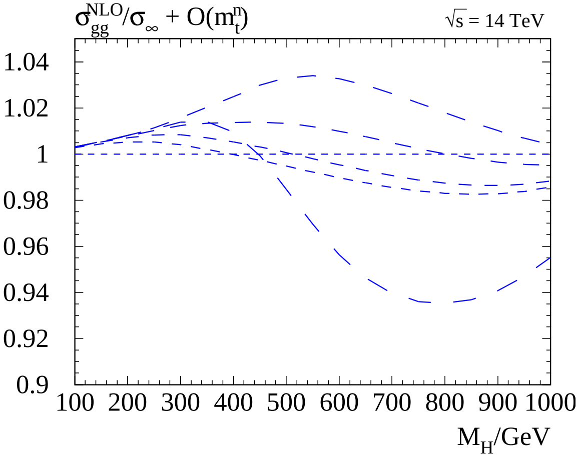
- Three particle final states very difficult
- Expand amplitude and phase space in powers of $(1 - \frac{m_H^2}{\hat{s}})$
- Series converges quickly



[Harlander,Kilgore]

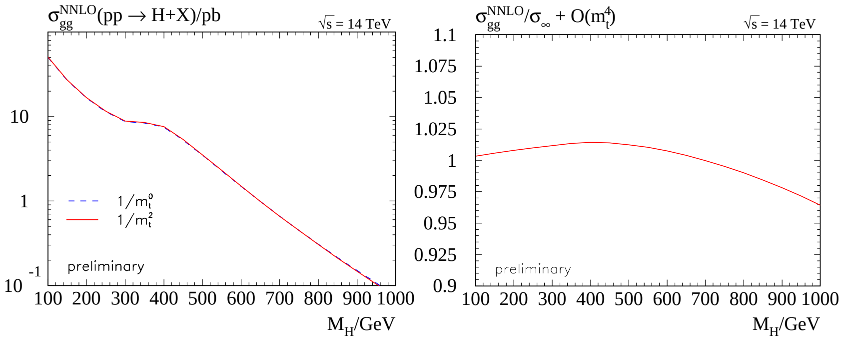
- In order to cancel poles must also expand the single real contribution

Preliminary Results - NLO



- First five terms in $1/m_t$ expansion shown
- Converges well below threshold

Preliminary Results - NNLO



- Use weight factor $\frac{\sigma_{LO}^{full}}{\sigma_{LO}^{exp}}$
- Very small effect - well below scale and PDF uncertainty - $\mathcal{O}(10\%)$

Summary and Outlook

- Long standing problem: how accurate is the large m_t limit at NNLO?
- We have shown that top mass effects are $\mathcal{O}(1\%)$, at least below threshold in the gg channel
- Use of effective theory is justified

Next Steps:

- Add other channels
- Consider effects on exclusive quantities