

Two-loop corrections and techniques for Higgs production and decay with massless gauge bosons

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- I. Introduction & Motivation
- II. Calculation & Techniques
- III. Results & Discussion
- IV. Summary & Conclusion

In collaboration with: S. Actis, G. Passarino, S. Uccirati

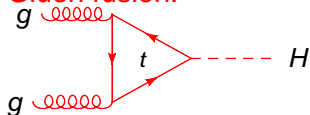
Nucl. Phys. **B811**, 182, 2009, Phys. Lett. **B670**, 12, 2008,
Phys. Lett. **B669**, 62, 2008, Phys. Lett. **B655**, 298, 2007.

Introduction & Motivation

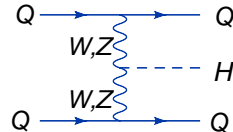
Higgs production

- Main production mechanisms:

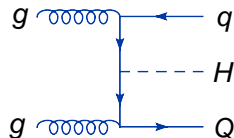
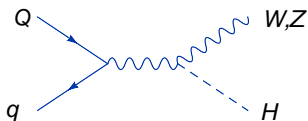
Gluon fusion:



Vector boson fusion:

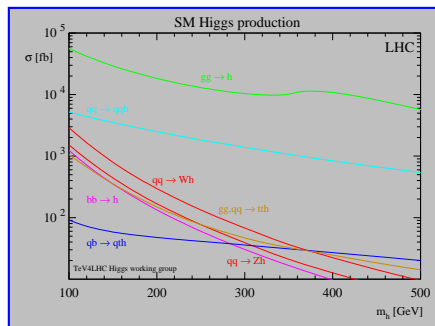


Associated production:



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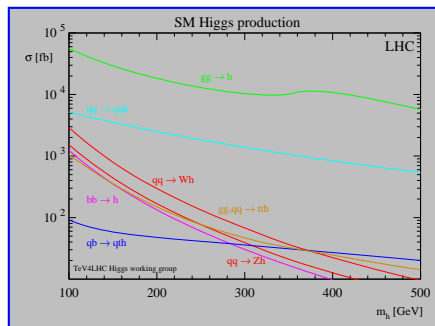
Higgs production



- Dominant production mechanism: Gluon fusion

Introduction & Motivation

Higgs production



- Dominant production mechanism:
Gluon fusion

- NLO QCD

- Heavy top-quark limit:

Dawson; Djouadi, Spira, Zerwas

- Entire Higgs-mass range:

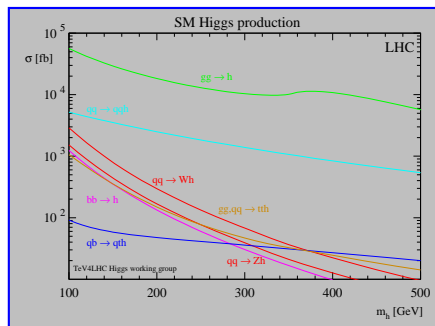
Djouadi, Graudenz, Spira, Zerwas; Harlander, Kant; Anastasiou, Beerli, Bucherer, Daleo, Kunszt; Aglietti, Bonciani, Degrassi, Vincini

- NNLO QCD

Harlander; Catani, Florian, Grazzini;
Harlander, Kilgore; Anastasiou, Melnikov;
Ravindran, Smith, Neerven; Anastasiou,
Melnikov, Petriello; Catani, Grazzini

Introduction & Motivation

Higgs production



- Dominant production mechanism: Gluon fusion

- NLO EW

- Exact light-fermion contribution

Aglietti, Bonciani, Degrassi, Vincini

- Contributions involving top-quark in terms of expansions

Degrassi, Maltoni; Djouadi, Gambino, Kniehl

- ↪ full EW corrections: **in this talk**

with complete dependence on heavy particles M_H, M_W, M_Z, M_t

S. Actis, G. Passarino, C.S., S. Uccirati

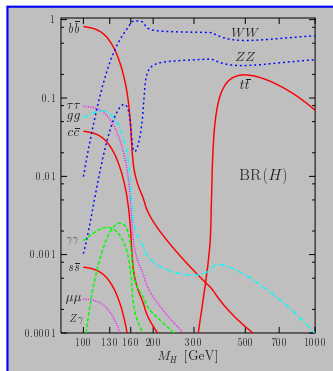
- Mixed QCD-EW corrections

C. Anastasiou, R. Boughezal, F. Petriello

Introduction & Motivation

Higgs decay

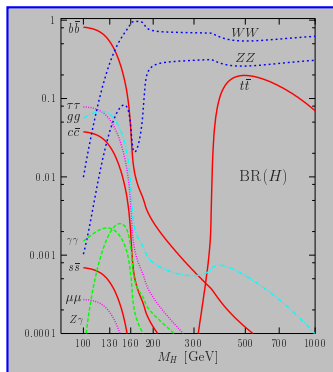
- $H \rightarrow WW, ZZ$ dominant process for heavy Higgs boson
- $H \rightarrow b\bar{b}$ dominant process for light Higgs, but huge QCD background.
- $H \rightarrow \gamma\gamma$ rare process $\text{Br} \sim 10^{-3}$, but experimentally clean



Djouadi, Graudenz, Spira, Zerwas

Introduction & Motivation

Higgs decay

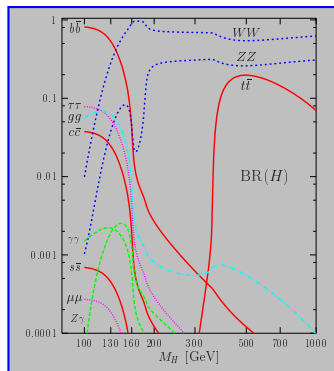


Djouadi, Graudenz, Spira, Zerwas

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- $H \rightarrow \gamma\gamma$ rare process $\text{Br} \sim 10^{-3}$, but experimentally clean
- NLO QCD
 - Expansions below/above threshold
Zheng, Wu; Djouadi, Spira, van der Bji, Zerwas;
Dawson, Kauffman
Djouadi, Spira, Zerwas; Melnikov, Yakovlev;
Inoue, Najima, Oka, Saito
 - Analytical results
Fleischer, Tarasov, Tarasov; Harlander, Kant; Anastasiou,
Beerli, Bucherer, Daleo, Kunstz; Aglietti, Bonciani,
Degrassi, Vicini

Introduction & Motivation

Higgs decay



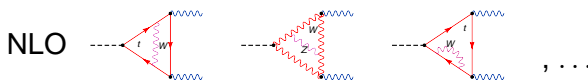
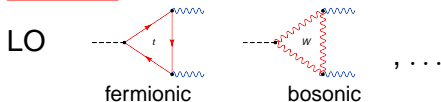
Djouadi, Graudenz, Spira, Zerwas

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- $H \rightarrow \gamma\gamma$ rare process $\text{Br} \sim 10^{-3}$, but experimentally clean
- NLO EW
 - corrections of $\mathcal{O}(G_f m_t^2)$
Liao, Li; Fugel, Kniehl, Steinhauser
 - corrections of $\mathcal{O}(G_f m_h^2)$
Korner, Melnikov, Yakovlev
 - exact light-fermion contribution
Aglietti, Bonciani, Degrassi, Vincini
 - Contributions involving top-quark and weak bosons exp. in $M_h^2/(4M_w^2)$
Degrassi, Maltoni
- ↪ full EW corrections: **in this talk**
Actis, Passarino, C.S., Uccirati

Calculation & Techniques

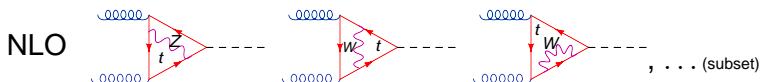
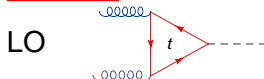
...some diagrams contributing to the EW 2-loop corrections

■ $H \rightarrow \gamma\gamma$:



also unphysical Higgs-Kibble scalar fields and Fadeev-Popov ghost fields

■ $gg \rightarrow H$:



Calculation & Techniques

The amplitude of $H(P) \rightarrow \gamma(p_1) + \gamma(p_2)$, (analog $gg \rightarrow H$)

- Decomposition of amplitude with respect to tensor structures:

$$\mathcal{A}^{\mu\nu}(H \rightarrow \gamma\gamma) = \frac{g^3 s_\theta^2}{16\pi^2} \left[F_D \delta^{\mu\nu} + \sum_{i,j=1}^2 F_P^{(ij)} p_i^\mu p_j^\nu + F_\epsilon \epsilon(\mu, \nu, p_1, p_2) \right]$$

- Require interference of 2-loop with 1-loop amplitude,
At 1-loop $F_\epsilon = 0$, need only F_D and F_P :

$$\mathcal{A}^{\mu\nu}(H \rightarrow \gamma\gamma) = \frac{g^3 s_\theta^2}{16\pi^2} (F_D \delta^{\mu\nu} + F_P^{(21)} p_2^\mu p_1^\nu).$$

- Introduce projectors for extraction of $F_D = P_D^{\mu\nu} \mathcal{A}_{\mu\nu}$, $F_P^{(21)} = P_P^{\mu\nu} \mathcal{A}_{\mu\nu}$
- Ward identity(WI): $p_1^\nu \mathcal{A}_{\mu\nu} p_2^\nu = 0$, $F_D + p_1 \cdot p_2 F_P = 0$
 - Valid order by order in perturbation theory
 - Only one form factor required (e.g. F_D) for computation of width
 - Can be used as a check of the calculation

Calculation & Techniques

2-loop contributions are computed numerically:

- Diagrams: *GraphShot*

S. Actis, A. Ferroglia, G. Passarino, M. Passera, C.S., S. Uccirati

Form3 based package for automatic generation and manipulation of 1- and 2-loop Feynman diagrams:

insert Feynman-rules, perform traces, remove reducible scalar products, symmetrize integrals, reduction, counter terms, extracts pole-part of loop diagrams, renormalization,...

- \rightsquigarrow UV-finite amplitude,
 integrals classified into different topologies,
 subdivided in scalar, vector and tensor type integrals
 \rightsquigarrow mapped on form factors

- Form factors are evaluated numerically in parametric space

- Before num. integration:

Cancel collinear sing. + Study threshold behavior

(For a moment consider $H \rightarrow \gamma\gamma$ without loss of generality)

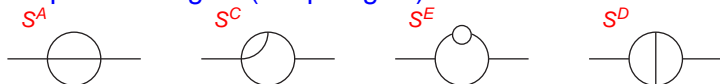
Calculation & Techniques

Classification different topologies

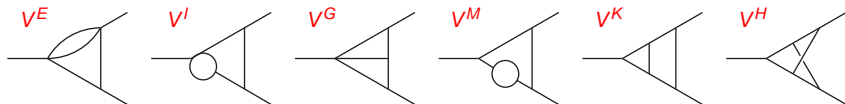
- 1-loop functions: well known A_0 -, B_0 -, C_0 -functions
- 2-loop tadpoles (2 topologies, related by IBP)



- 2-loop self-energies (4 topologies)



- 2-loop vertices (6 topologies)

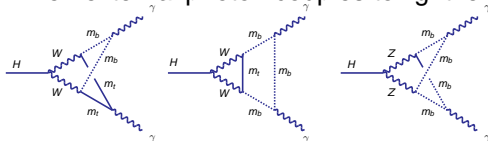


- We end with integrals up to rank 2

Calculation & Techniques

Collinear singularities

- Appear when external photon couples to light fermions, e.g.:



- Collinear singularities are regularized by small fermion mass
 \rightsquigarrow singularities become manifest in $\log(m)$
- Verification of their cancellation:
 Requires universal representation of the coefficient
- Introduce integrals of one-loop functions:

$$= \ln \frac{m^2}{s} \int_0^1 dz \text{ [diagram] } + \text{coll. finite.}$$

→ use their well-known properties to make cancellation explicit

Calculation & Techniques

Threshold singularities, Behavior at WW -threshold ($H \rightarrow \gamma\gamma$)

Amplitude develops threshold singularities in $\beta_W = \sqrt{1 - 4M_W^2/s}$, when $s = M_h^2 \sim 4M_W^2$, possible sources:

I) Square-root singularities:

$$\begin{aligned}
 & \text{Loop diagram} = - \text{Tree diagram} \times \text{Loop diagram} + (\text{reg. part at } \beta = 0) \\
 & \text{Tree diagram} = B_0(-s, M_W, M_W) = -\frac{1}{\beta} \ln \frac{\beta+1}{\beta-1} \quad \leftarrow \text{singular for } M_h = 2M_W
 \end{aligned}$$

II) Finite W -mass renormalization (derivative of C-function)

III) Higgs wave function factor (derivative of B-function)

IV) Logarithmic singularities:

$$H \rightarrow \gamma\gamma \text{ via } W \text{ loop} \sim \log(\beta)$$

$$H \rightarrow \gamma\gamma \text{ via } t \text{ loop}$$

same config. with top loop finite,
log multiplied by β_t^2 ,
(spin structure)

Calculation & Techniques

Complex poles Denner, Dittmaier, Roth, Wackerroth, Wieders

Cure problems with crossing of thresholds through complex poles:

- “Minimal” introduction of complex poles:

Decompose Amplitude:

$$\mathcal{A} = \mathcal{A}_{1,W}^{div} / \beta_W + \mathcal{A}_{1,Z}^{div} / \beta_Z + \mathcal{A}_2^{div} \log(-\beta_W^2 - i0) + \mathcal{A}^{fin}$$

- $\mathcal{A}_1^{div}, \mathcal{A}_2^{div}$ form a gauge invariant subset
- Replace M_V^2 with $s_V = \mu_V(\mu_V - i\gamma_V)$ in both $\mathcal{A}_1^{div}, \mathcal{A}_2^{div}$ and threshold factor β_V where $\mu_V^2 = M_V^2 - \Gamma_V^2, \gamma_V = \Gamma_V \left(1 - \frac{\Gamma_V^2}{2M_V^2}\right)$
- ⇒ Regularize threshold singularities
- ⇒ Square-root singularities originating from V_M and finite mass renormalization cancel ($H \rightarrow \gamma\gamma$)
- ⇒ Square-root sing. from wave function and $\log \beta$ survive

- “Complete” introduction of complex poles:

Replace M_V^2 in all divergent and finite terms

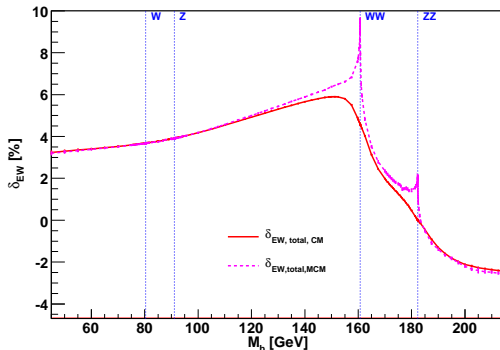
- ↪ Remove artificially large cusps coming from enhancements in finite contributions

Result:

The partonic process $gg \rightarrow H$

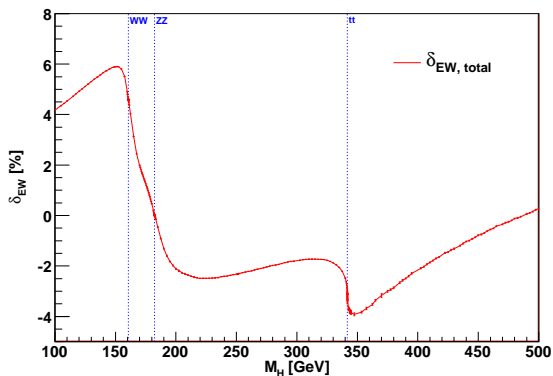
IPS: $M_W = 80.398 \text{ GeV}$, $M_Z = 91.1876 \text{ GeV}$, $m_t = 170.9 \text{ GeV}$,
 $G_F = 1.16637 \cdot 10^{-5} \text{ GeV}^{-2}$, $\Gamma_Z = 2.4952 \text{ GeV}$, $\Gamma_W = 2.093 \text{ GeV}$

- LO parton cross section
 $\sigma_{LO} = \sigma_0 M_H^2 \delta(s - M_H^2)$
- electroweak correction parametrized by
 $\sigma = \sigma_0 (1 + \delta_{EW})$
- **MCM:** minimal complex masses
 cures threshold singularity
- **CM:** complex masses
 avoids incongruent effects
 around thresholds
- Result obtained with real masses divergent at thr.,
 good approx. below/above



Result:

The partonic process $gg \rightarrow H$



Maximum around WW -threshold, minimum around tt -threshold
 → The percentage correction varies between +6% and -4%

Result:

The hadronic process $pp \rightarrow H + X$

- Hadronic process:

$$\sigma(s, M_H^2) = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f_{i/h_1}(x_1, \mu_F^2) f_{j/h_2}(x_2, \mu_F^2) \times \int_0^1 dz \delta(z - \frac{M_H^2}{s x_1 x_2}) z \sigma^{(0)} G_{ij}(z; \alpha_S(\mu_R^2), M_H^2/\mu_R^2; M_H^2/\mu_F^2),$$

Parton densities: $f_{i/h_1}, f_{j/h_2}, i, j \in \{g, q, \bar{q}\}$

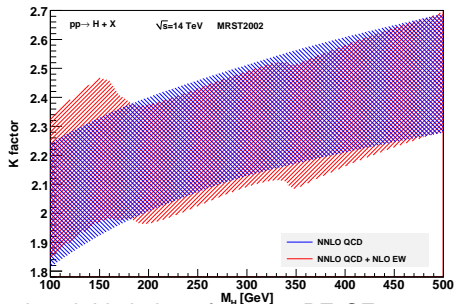
QCD corrections: $G_{ij} = \alpha_S^2(\mu_R) \delta_{ig} \delta_{jg} \delta(1-z) + \alpha_S^3(\mu_R) G_{ij}^{(1)} + \dots$

- Complete factorization(CF): $\sigma^{(0)} G_{ij} \rightarrow \sigma^{(0)} (1 + \delta_{EW}) G_{ij}$
amounts to an overall re-scaling of QCD result,
consider electroweak correction point like
- Partial factorization(PF): $\sigma^{(0)} G_{ij} \rightarrow \sigma^{(0)} [G_{ij} + \alpha_S^2(\mu_R^2) \delta_{EW} G_{ij}^{(0)}]$
equivalent to add electroweak corrections to QCD ones

Result:

The hadronic process $pp \rightarrow H + X$

- Use Fortran program HiggsNNLO by [M. Grazzini](#)
- K-factor: Ratio cross section with higher orders over LO result



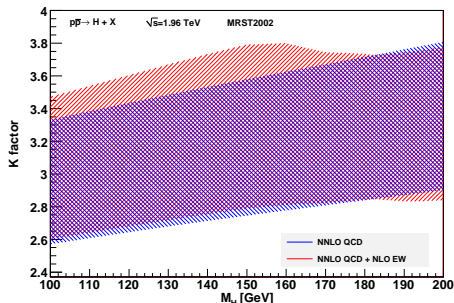
- Uncertainty band: Variation of μ_R, μ_F, PF, CF
- Stronger dependence on H -mass, follows shape of partonic process
- $M_h < 180\text{GeV}$ CF(PF) fixes upper(lower) bound, $M_h > 180\text{GeV}$ inversely
- Central value for cross section is shifted by 2-5% ($M_H = 120\text{ GeV}$) (CF,PF)

Update with MSTW2008 PDF \rightarrow [de Florian](#), [Grazzini](#); [Anastasiou](#), [Boughezal](#), [Petriello](#)

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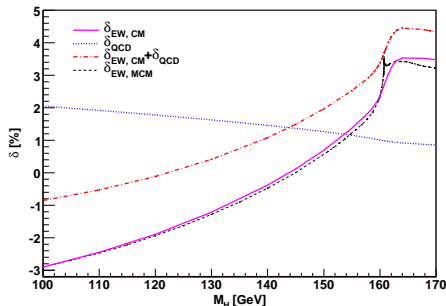


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Result:

The decay $H \rightarrow \gamma\gamma$, correction in percent for the partial width



- QCD corrections > 0 , ranging from $+1.8\%$ (120 GeV) to $+0.9\%$ (170 GeV)
- EW effects range from -1.9% (120 GeV) to 3.5% (170 GeV)
- EW effects directly below threshold small -0.47% (140 GeV) (cancellations)

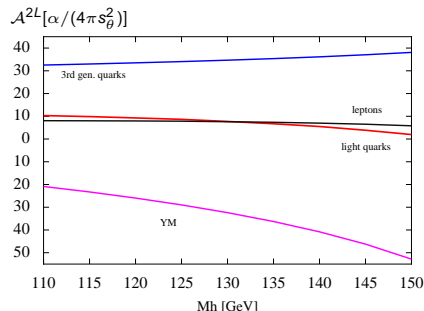
- Cancellation between δ_{QCD} and δ_{EW} below threshold -0.1% (120 GeV)
- Around the threshold both positive leading to a sizable **total correction** of about **+4%**

Results:

Anatomy of EW corrections to $\Gamma(H \rightarrow \gamma\gamma)$ for $110 \text{ GeV} < M_H < 150 \text{ GeV}$

Decompose 2-loop amplitude: $\mathcal{A}^{2L} = \mathcal{A}_{lep}^{2L} + \mathcal{A}_{lq}^{2L} + \mathcal{A}_{YM}^{2L} + \mathcal{A}_{3gen}^{2L}$

M_H [GeV]	δ_{EW} [%]
115	-2.22
120	-1.93
125	-1.63
130	-1.28
135	-0.90
140	-0.47



- Agreement with lep and lq Aglietti, Bonciani, Degrassi, Vicini, 3rd gen. quarks and YM Degrassi, Maltoni (expansion), corrected renormalization
- Analog for partonic $gg \rightarrow H$, decomposition in lq and 3rd gen. (contribution of 3rd gen. small below threshold)

Summary & Conclusion

- Gluon fusion ($gg \rightarrow H$) is the dominant production process for the Higgs boson at LHC
- The decay $H \rightarrow \gamma\gamma$ provides a clean environment for its discovery
- The two-loop electroweak corrections with complete dependence of all heavy particles (M_H, M_W, M_Z, M_t) have been computed:
- For $gg \rightarrow H$ the partonic cross section varies between

$$\delta \sim -4\% - 6\% \quad (M_H = 100 - 500 \text{ GeV})$$
 The had. cross section is shifted by $+2\% - 5\%$ ($M_H = 120 \text{ GeV}$)
- For $H \rightarrow \gamma\gamma$ the relative corrections to the decay width are

$$\delta \sim -1\% - 4\% \quad (M_H = 100 - 170 \text{ GeV})$$