

Four legs, no loops: Flavor physics in a model with a warped extra dimension*

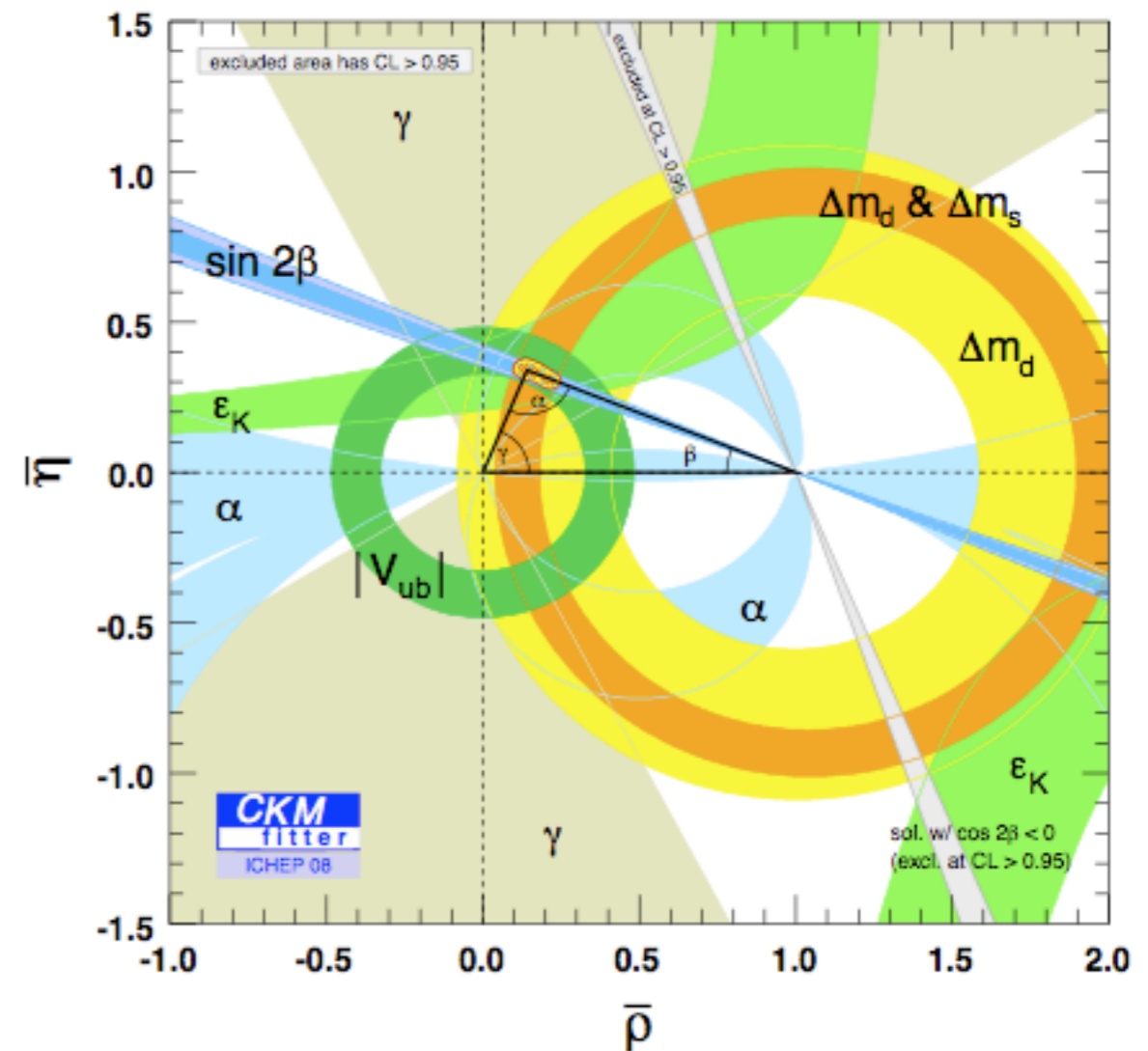
Ulrich Haisch
Johannes Gutenberg Universität Mainz

LoopFest VIII
University of Wisconsin at Madison
8th of May 2009

Main lesson from quark flavor physics

Fact: Standard Model (SM) of particle physics, *i.e.*, theory of electroweak and strong interaction (QCD) is very successful in describing quark flavor mixing

This is quite clear from looking at consistency of various constraints appearing in Cabibbo-Kobayashi-Maskawa (CKM) fit ...



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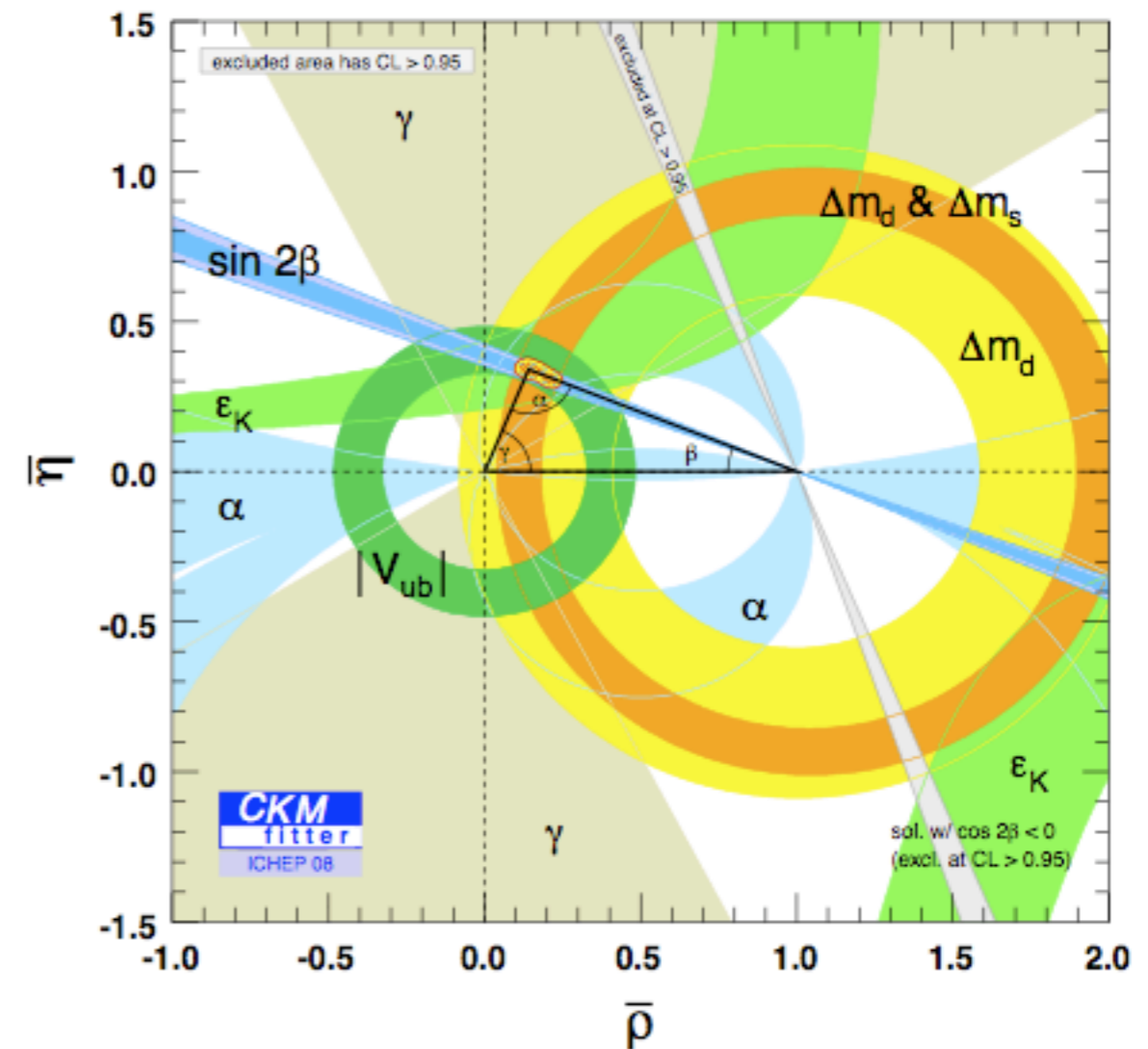
N. Cabibbo

M. Kobayashi

T. Maskawa

Nobel Prize in Physics 2008 awarded to Kobayashi and Maskawa:

“for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature”*



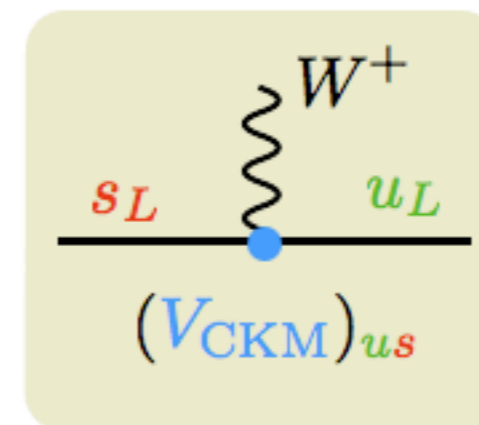
*http://nobelprize.org/nobel_prizes/physics/laureates/2008/

Main lesson from quark flavor physics

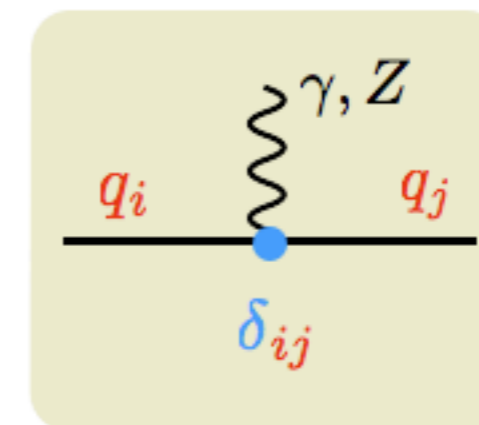
Fact: Standard Model (SM) of particle physics, *i.e.*, theory of electroweak and strong interaction (QCD) is very successful in describing quark flavor mixing

... and from absence of excessive Flavor-Changing Neutral Currents (FCNCs) such as $D-\bar{D}$ mixing, $K_L \rightarrow \mu^+ \mu^-$, $B \rightarrow X_s \gamma$, *etc.* that are forbidden at tree level in SM

Upshot: effects of Beyond SM (BSM) physics in quark flavor mixing can only appear as small corrections to leading CKM mechanism



$V_{CKM} = CKM$
matrix



$\delta =$ diagonal
matrix

This is too nice and too simple to be true ...

- Since SM does not include gravity it is merely low-energy limit of more fundamental theory valid up to energy scale $\Lambda_{UV} < M_{GUT}, M_{Pl}$
- Yet if one wants to describe phenomena at energies $E \ll \Lambda_{UV}$ one is allowed to perform expansion in powers of $E/\Lambda_{UV} \ll 1$
- Effective Field Theories (EFTs) provide systematic way to perform such expansion: powers of E/Λ_{UV} correspond to dimension of operators

Schematically:

$$\mathcal{L}_{BSM} \xrightarrow{E/\Lambda_{UV} \ll 1} \mathcal{L}_{EFT} = \mathcal{L}_{SM} + \underbrace{\frac{\mathcal{L}^{(5)}}{\Lambda_{UV}}}_{\sim E/\Lambda_{UV}} + \underbrace{\frac{\mathcal{L}^{(6)}}{\Lambda_{UV}^2}}_{\sim (E/\Lambda_{UV})^2} + \dots$$

and indeed there is a problem of flavor ...

$$\mathcal{L}_{\text{EFT}} = \underbrace{\Lambda_{\text{UV}}^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2}_{\text{electroweak symmetry breaking}} + \mathcal{L}_{\text{SM}}^{\text{gauge}} + \mathcal{L}_{\text{SM}}^{\text{Yukawa}} + \underbrace{\frac{\mathcal{L}^{(5)}}{\Lambda_{\text{UV}}} + \frac{\mathcal{L}^{(6)}}{\Lambda_{\text{UV}}^2}}_{\text{generic flavor structure}} + \dots$$

Higgs mass
large FCNCs

$$\sim \frac{g_T^2}{16\pi^2} \Lambda_{\text{UV}}^2$$

no fine-tuning \Downarrow

$$\sim \frac{g_X^2}{\Lambda_{\text{UV}}^2}$$

bounds on flavor mixing \Downarrow

$$\Lambda_{\text{Higgs}} \lesssim 1 \text{ TeV} \quad \rightarrow \quad \text{increasing energy scale} \quad \rightarrow \quad \Lambda_{\text{flavor}} \gtrsim 10^3 \text{ TeV}$$

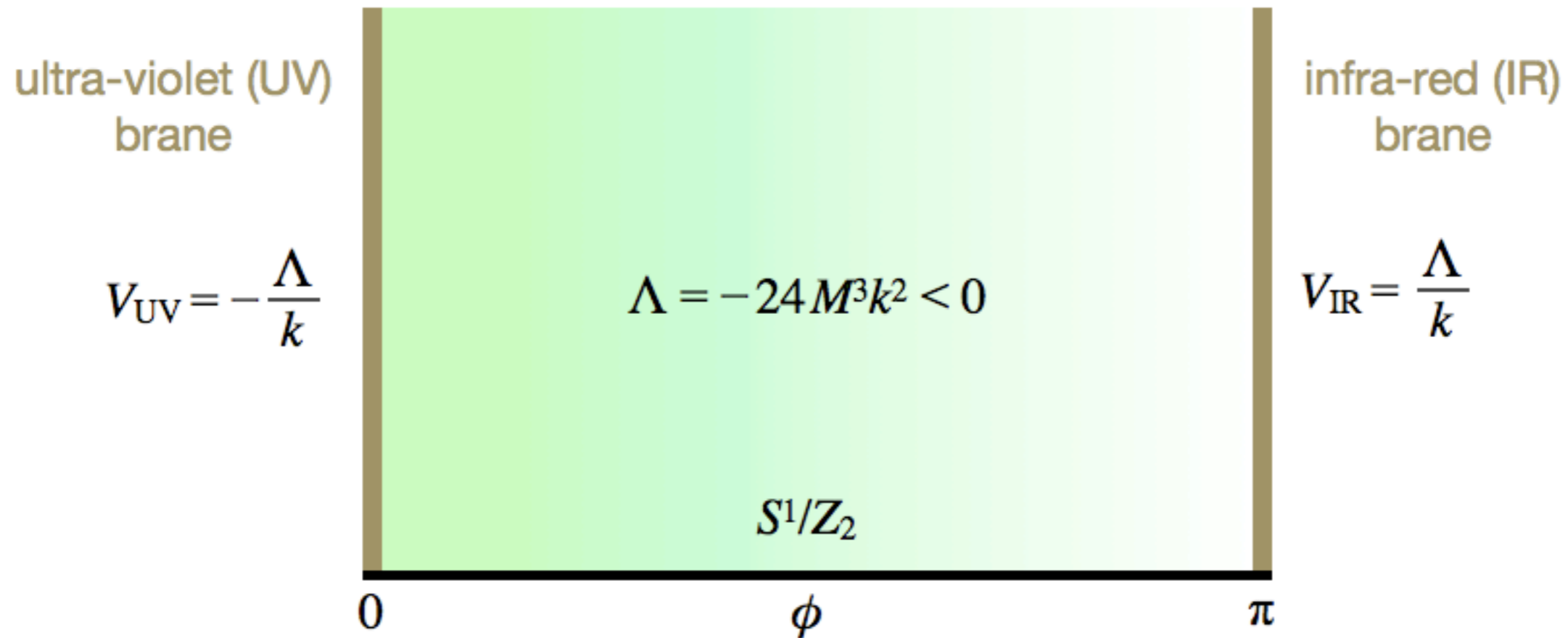
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$$\mathcal{L}_{\text{EFT}} = \underbrace{\Lambda_{\text{UV}}^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2}_{\text{no fine-tuning} \downarrow} + \mathcal{L}_{\text{SM}}^{\text{gauge}} + \mathcal{L}_{\text{SM}}^{\text{Yukawa}} + \underbrace{\frac{\mathcal{L}^{(5)}}{\Lambda_{\text{UV}}} + \frac{\mathcal{L}^{(6)}}{\Lambda_{\text{UV}}^2}}_{\text{bounds on flavor mixing} \downarrow} + \dots$$

$$\Lambda_{\text{Higgs}} \lesssim 1 \text{ TeV} \quad \xrightarrow{\text{increasing energy scale}} \quad \Lambda_{\text{flavor}} \gtrsim 10^3 \text{ TeV}$$

- Solution to flavor problem and explanation for $\Lambda_{\text{Higgs}} \ll \Lambda_{\text{flavor}}$:
 - $\Lambda_{\text{UV}} \gg 1 \text{ TeV}$: new particles too heavy to be discovered at LHC
 - $\Lambda_{\text{UV}} \approx 1 \text{ TeV}$: quark flavor mixing protected by flavor symmetry

Randall-Sundrum (RS) model: Geometry*

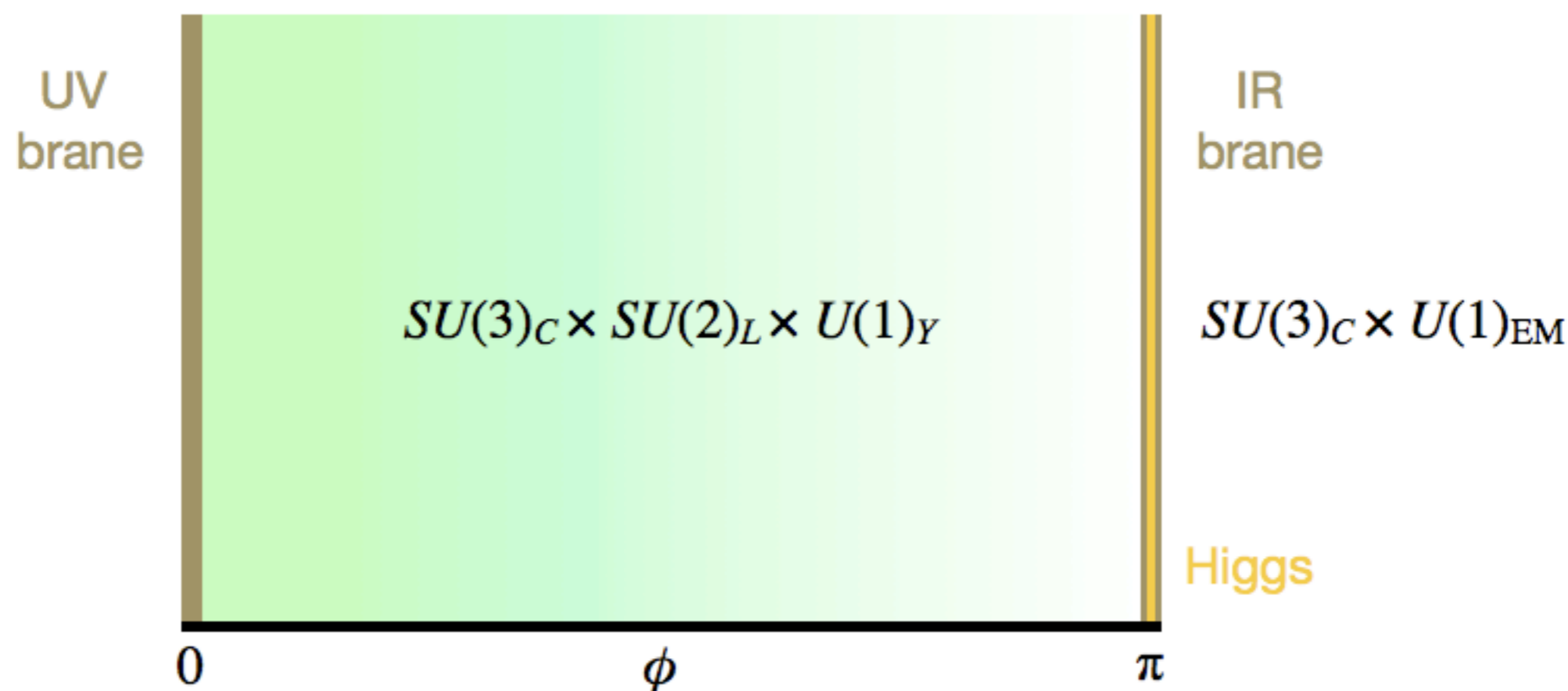


Slice of AdS_5 with curvature k :

$$ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu - r^2 d\phi^2, \quad \sigma = kr|\phi|$$

$$\epsilon = \frac{\Lambda_{\text{IR}}}{\Lambda_{\text{UV}}} \equiv e^{-kr\pi} \approx \frac{m_W}{M_{\text{Pl}}} \approx 10^{-16}, \quad L \equiv -\ln \epsilon \approx 37, \quad M_{\text{KK}} \equiv k\epsilon = \text{few TeV}$$

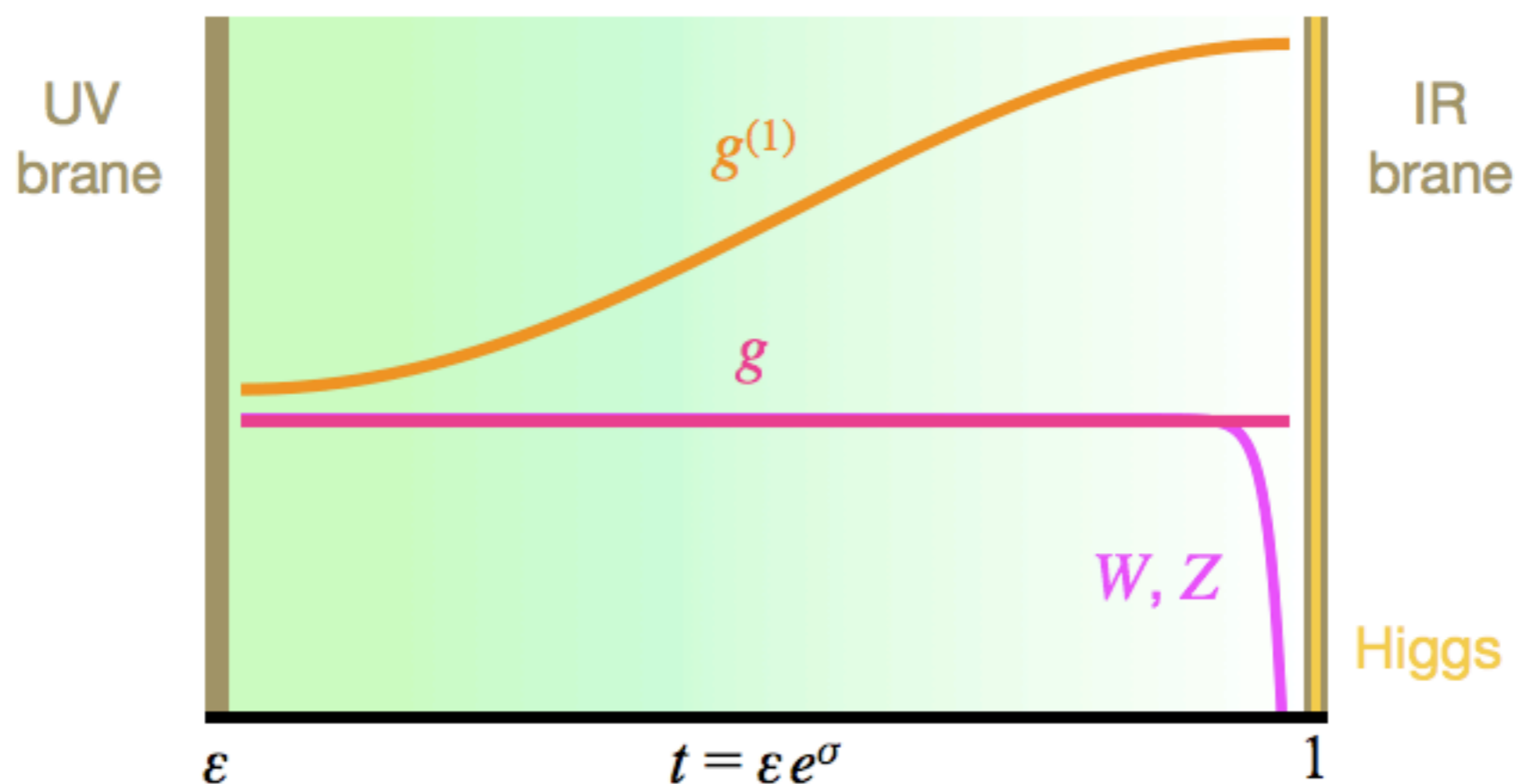
RS model: Symmetry breaking



Pattern of symmetry breaking:

- ▶ bulk gauge group $SU(2)_L \times U(1)_Y$ broken by IR brane-localized Higgs to $U(1)_{EM}$
- ▶ after electroweak symmetry breaking, heavy gauge bosons and their Kaluza-Klein (KK) excitations get masses $m_0, m_1 \approx 2.45 M_{KK}, m_2 \approx (2.45 + \pi) M_{KK}, \dots$

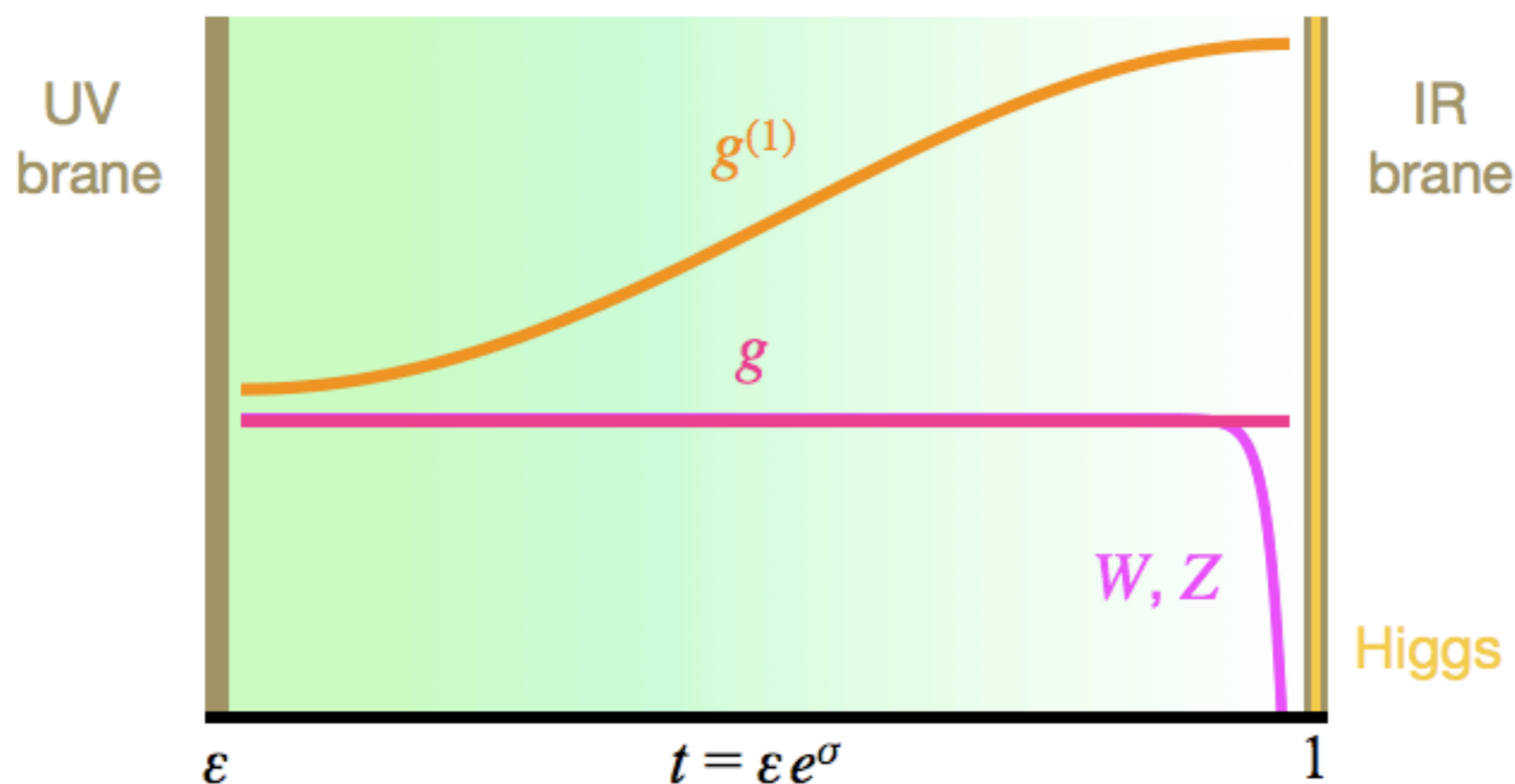
RS model: Gauge bosons*



Profiles of gauge fields:

$$\chi_{g,\gamma}(\phi) = \frac{1}{\sqrt{2\pi}}, \quad \chi_{W,Z}(\phi) \approx \frac{1}{\sqrt{2\pi}} \left[1 + \frac{m_{W,Z}^2}{M_{\text{KK}}^2} \left(1 - \frac{1}{L} + t^2 (1 - 2L - 2 \ln t) \right) \right]$$

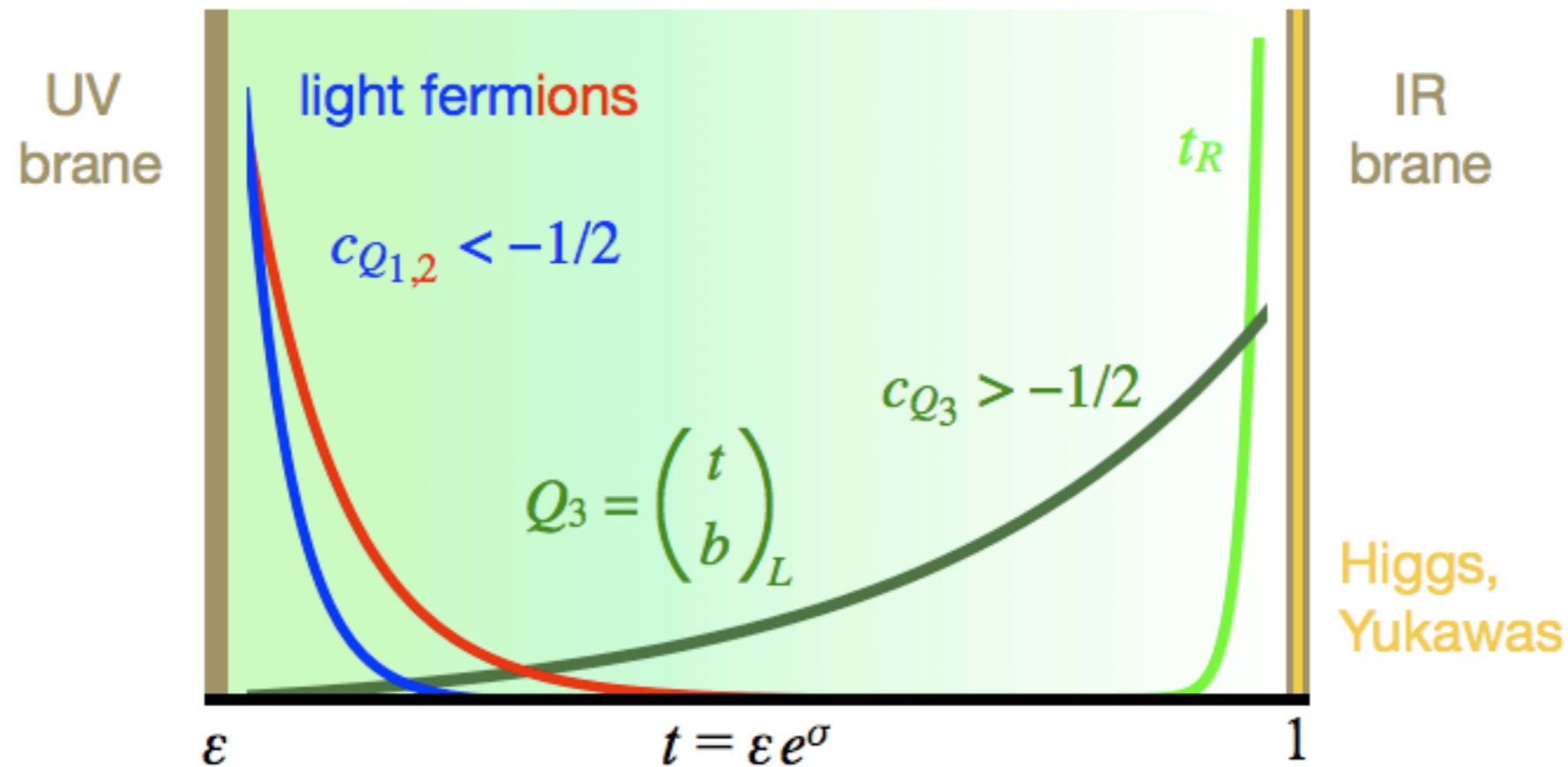
RS model: Gauge bosons*



Profiles of gauge fields:

- ▶ while profiles of photon and gluon are flat, wave functions of heavy gauge bosons and profiles of KK modes peaked at IR brane where Higgs lives
- ▶ non-trivial profiles entering overlap integrals alter interactions compared to SM

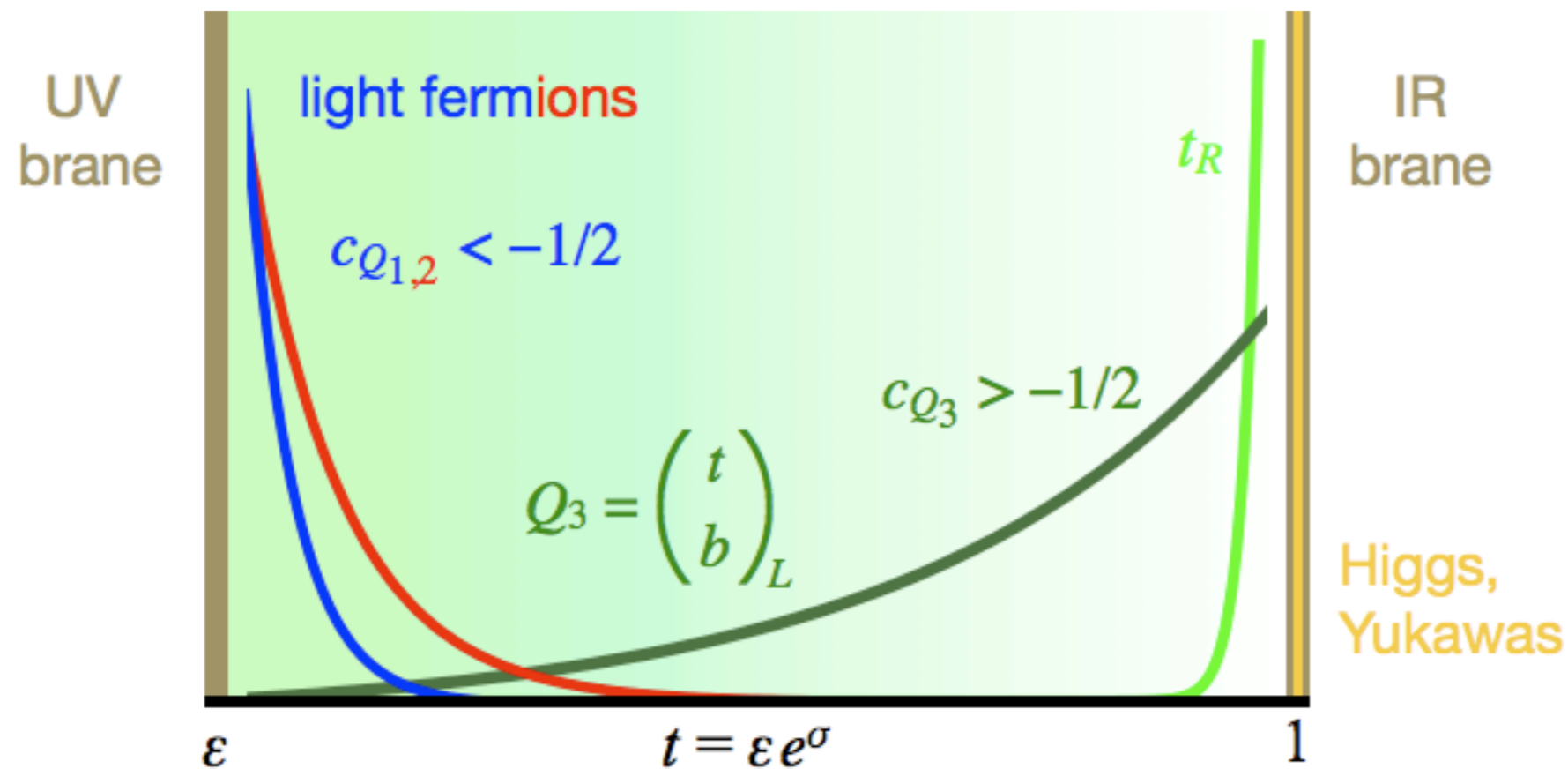
RS model: Fermions*



Profiles of fermion fields:

$$C_n^{(A)}(\phi) \approx \sqrt{\frac{L\epsilon}{\pi}} F_{c_A} t^{c_A}, \quad S_n^{(A)}(\phi) \approx \pm \text{sgn}(\phi) \sqrt{\frac{L\epsilon}{\pi}} \frac{m_n}{M_{\text{KK}}} \left(\frac{t^{-c_A}}{F_{c_A}} + \frac{t^{1+c_A} - t^{-c_A}}{1 - 2c_A} F_{c_A} \right)$$

RS model: Fermions*



Profiles of fermion fields:

- ▶ localization of fermion profiles in extra dimension controlled by bulk mass parameters $c_{Q,q} = \pm M_{Q,q}/k$
- ▶ top quark lives in IR to generate its large mass, while light fermions live in UV

RS model: Yukawa interactions*

$$S_{\text{Yukawa}} = - \int d^4x r \int_{-\pi}^{\pi} d\phi \frac{\sqrt{2}v e^{-3\sigma}}{kr} \delta(|\phi| - \pi) [\bar{u}_L \mathbf{Y}_u u_R^c + \bar{d}_L \mathbf{Y}_d d_R^c + \text{h.c.}]$$

KK decomposition:

$$q_L(x, t) \propto \text{diag}[F_{c_{Q_i}} t^{c_{Q_i}}] \mathbf{U}_q q_L^{(0)}(x), \quad q_R^c(x, t) \propto \text{diag}[F_{c_{q_i}} t^{c_{q_i}}] \mathbf{W}_q q_R^{(0)}(x)$$

effective Yukawa couplings:

$$\mathbf{Y}_q^{\text{eff}} = \text{diag}[F_{c_{Q_i}}] \mathbf{Y}_q \text{diag}[F_{c_{q_i}}] = \mathbf{U}_q \boldsymbol{\lambda}_q \mathbf{W}_q^\dagger, \quad \boldsymbol{\lambda}_q = \frac{\sqrt{2}}{v} \text{diag}[m_{q_i}]$$

- Hierarchy in $\mathbf{Y}_q^{\text{eff}}$ can be generated naturally from structureless matrices \mathbf{Y}_q with $\mathcal{O}(1)$ complex elements, by localizing quarks at different points in 5th dimension since this leads to exponential different values for $F_{c_{A_i}} \sim e^{L(c_{A_i}+1/2)}$

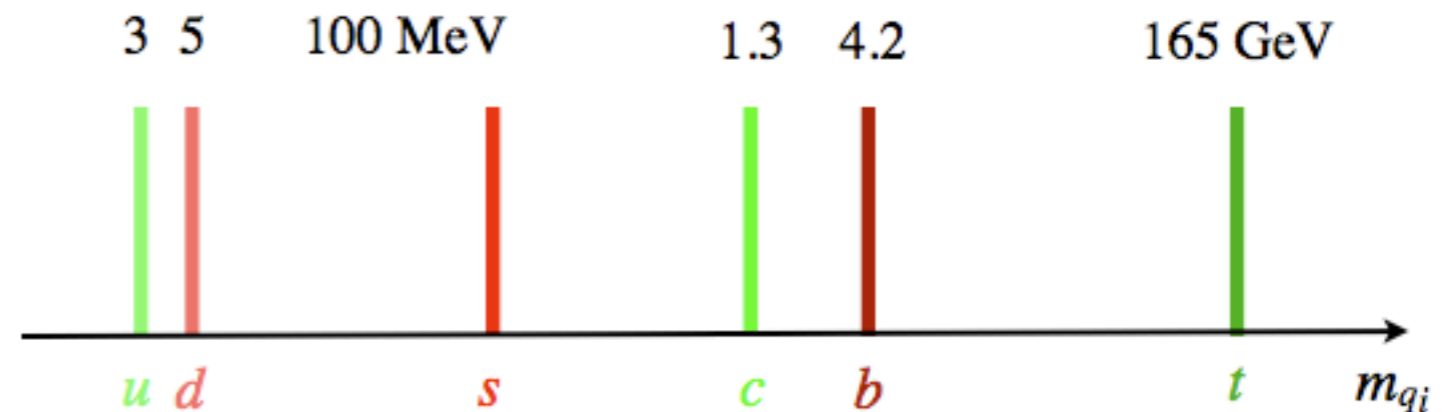
RS model: Masses and mixings*

$$m_{q_i} = \mathcal{O}(1) \frac{v}{\sqrt{2}} F_{c_{Q_i}} F_{c_{q_j}},$$

$$\lambda = \mathcal{O}(1) \frac{F_{c_{Q_1}}}{F_{c_{Q_2}}},$$

$$A = \mathcal{O}(1) \frac{F_{c_{Q_2}}^3}{F_{c_{Q_1}}^2 F_{c_{Q_3}}^2},$$

$$\bar{\rho} - i\bar{\eta} = \mathcal{O}(1)$$



$$V_{\text{CKM}} \approx \begin{pmatrix} 1 & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta}) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\lambda \approx 0.23, \quad A \approx 0.81, \quad \bar{\rho} \approx 0.14, \quad \bar{\eta} \approx 0.34$$

- CKM angles fix hierarchy of $F_{c_{Q_1}}/F_{c_{Q_2}} \sim \lambda$, $F_{c_{Q_2}}/F_{c_{Q_3}} \sim \lambda^2$, $F_{c_{Q_1}}/F_{c_{Q_3}} \sim \lambda^3$, while quark masses determine $F_{c_{u_1}}/F_{c_{u_2}} \sim m_u/m_t 1/\lambda^3$, $F_{c_{u_2}}/F_{c_{u_3}} \sim m_c/m_t 1/\lambda^2$, ...

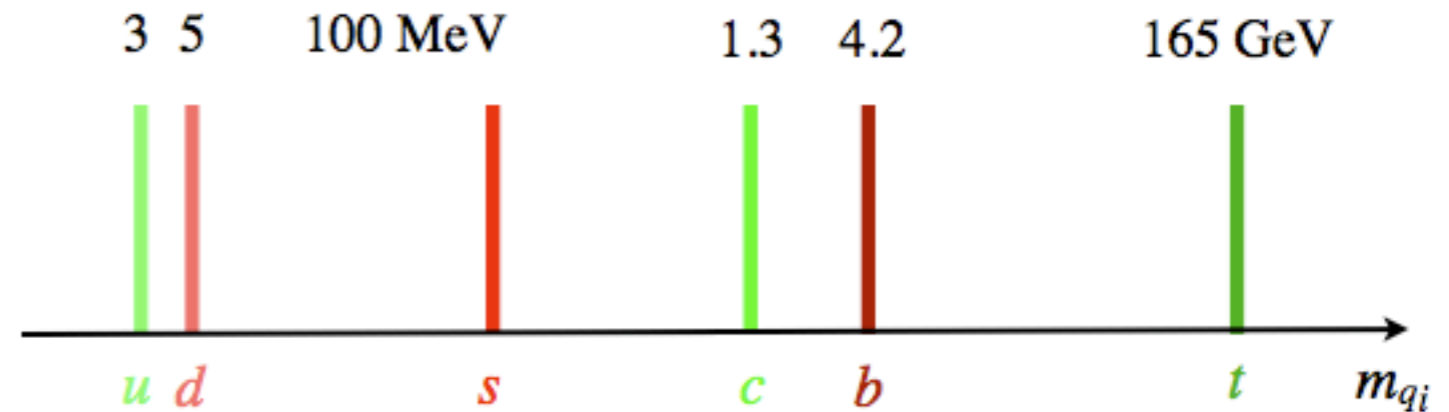
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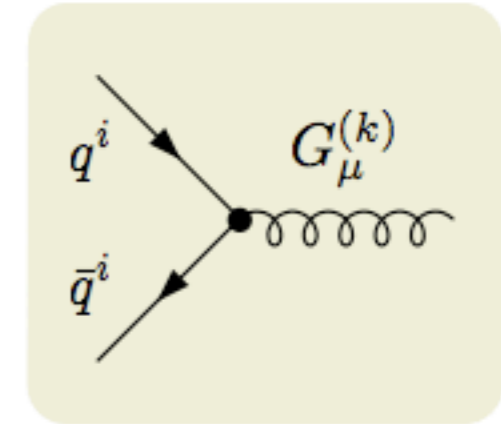
$$\lambda \approx 0.23, \quad A \approx 0.81, \quad \bar{\rho} \approx 0.14, \quad \bar{\eta} \approx 0.34$$

- To first order, $\bar{\rho} - i\bar{\eta}$ is independent of fermion profiles $F_{c_{A_i}}$ at IR brane. Thus precise amount of CP violation remains unexplained in RS framework

RS-GIM mechanism*

- Quark-quark-gluon vertex in flavor eigenbasis:

$$\bar{q}^i G_\mu^{(k)} q^i \sim -ig_s \gamma_\mu \sqrt{L} F_{c_{q^i}}^2, \quad F_{c_{q^i}} \sim e^{L(c_{q^i} + 1/2)}$$



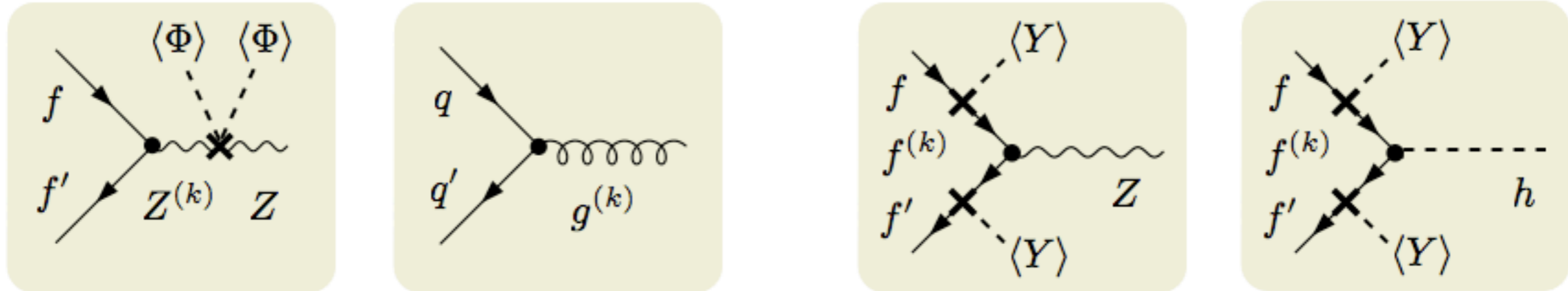
- Quark-quark-gluon vertex in mass eigenbasis:

$$\bar{q}_{L,i}^{(0)} G_\mu^{(k)} q_{L,j}^{(0)} \sim -ig_s \gamma_\mu \sqrt{L} F_{c_{Q_i}} F_{c_{Q_j}}, \quad \bar{q}_{R,i}^{(0)} G_\mu^{(k)} q_{R,j}^{(0)} \sim -ig_s \gamma_\mu \sqrt{L} F_{c_{q_i}} F_{c_{q_j}}$$

Important features:

- ▶ unitary rotations $(U_q)_{ij} \sim F_{c_{Q_i}}/F_{c_{Q_j}}$ and $(W_q)_{ij} \sim F_{c_{q_i}}/F_{c_{q_j}}$ turn flavor-diagonal, but non-universal tree-level couplings into flavor-violating ones
- ▶ since FCNCs are proportional to $F_{c_{A_i}} F_{c_{A_j}}$, exponential suppression of fermion profiles $F_{c_{A_i}}$ at IR brane, guarantees flavor protection called RS-GIM

Sources of flavour violation*

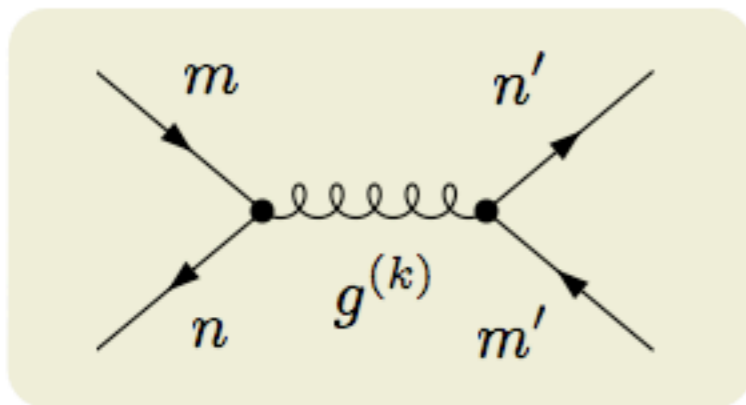


Flavour violation arises from:

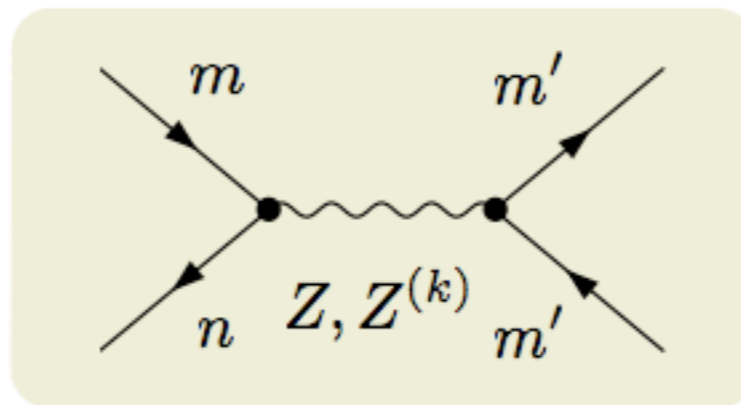
- ▶ modification of W , Z boson profiles due to electroweak symmetry breaking on IR brane
- ▶ non-trivial overlap integrals of KK gauge-boson profiles with SM fermion wave functions
- ▶ non-orthonormality of fermion profiles interpreted as mixing of $SU(2)_L$ singlet and doublets

Anatomy of tree-level FCNC processes

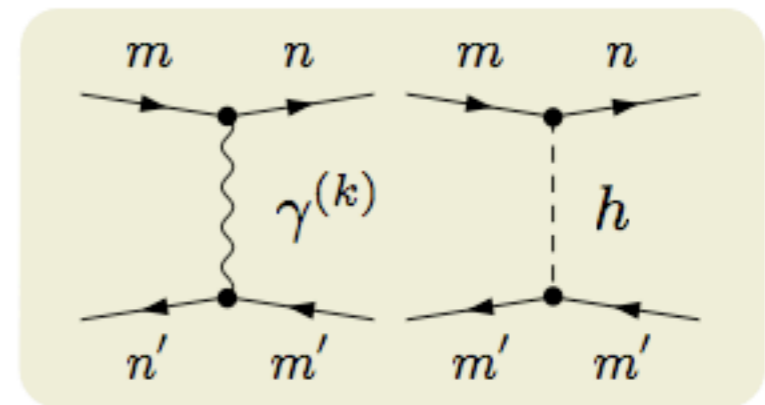
- Three types of generic contributions to dimension six operators:



dominant contribution to
 $\Delta F = 2$ processes



dominant contribution to
 $\Delta F = 1$ processes

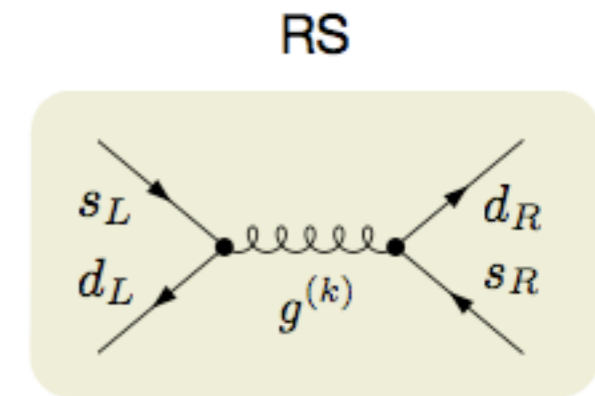
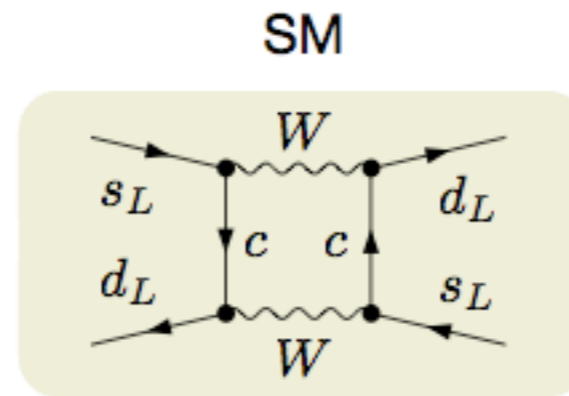


small contributions to
 $\Delta F = 1, 2$ processes

- Like in SM, dimension five dipole-type operators contributing to $B \rightarrow X_s \gamma$ or $\mu \rightarrow e \gamma$ arise first at one-loop level

Meson mixing: Effective Hamiltonian

$$\mathcal{H}_{\text{eff}}^{\Delta S=2} = \sum_{i=1}^5 C_i Q_i + \sum_{i=1}^3 \tilde{C}_i \tilde{Q}_i$$



$$Q_1 = (\bar{d}_L^a \gamma_\mu s_L^a) (\bar{d}_L^b \gamma^\mu s_L^b),$$

$$Q_2 = (\bar{d}_R^a s_L^a) (\bar{d}_R^b s_L^b),$$

$$Q_3 = (\bar{d}_R^a s_L^b) (\bar{d}_R^b s_L^a),$$

$$Q_4 = (\bar{d}_R^a s_L^a) (\bar{d}_L^b s_R^b),$$

$$Q_5 = (\bar{d}_R^a s_L^b) (\bar{d}_L^b s_R^a),$$

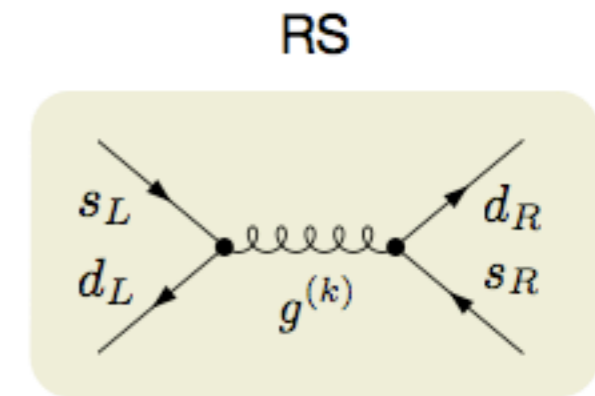
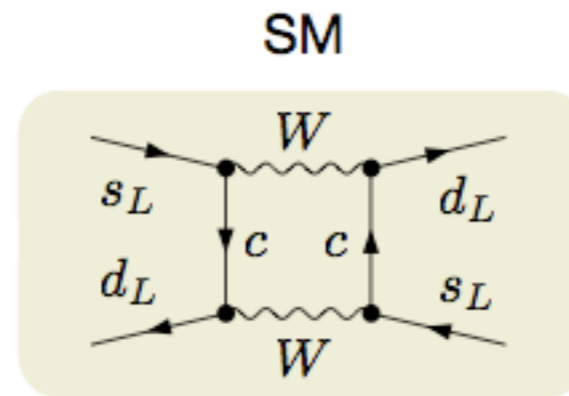
$$\tilde{Q}_{1,2,3} : L \leftrightarrow R$$

- Contribution from Wilson coefficient of Q_4 to CP-violating quantity ϵ_K strongly enhanced through renormalization group evolution and chiral factor $(m_K/m_s)^2$ in matrix element:

$$|\epsilon_K|_{\text{RS}} \propto \text{Im} \left[C_{1,K}^{\text{RS}} + 115 \left(C_{4,K}^{\text{RS}} + \frac{C_{5,K}^{\text{RS}}}{3} \right) \right]$$

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$$Q_2 = (\bar{d}_R^a s_L^a)(\bar{d}_R^b s_L^b),$$

$$Q_3 = (\bar{d}_R^a s_L^b)(\bar{d}_R^b s_L^a),$$

$$Q_4 = (\bar{d}_R^a s_L^a)(\bar{d}_L^b s_R^b),$$

$$Q_5 = (\bar{d}_R^a s_L^b)(\bar{d}_L^b s_R^a),$$

$$\tilde{Q}_{1,2,3} : L \leftrightarrow R$$

$$C_{1,K}^{\text{RS}} \sim \frac{4\pi L}{M_{\text{KK}}^2} \left[\frac{\alpha_s}{3} + 1.12\alpha \right] F_{c_{Q_1}}^2 F_{c_{Q_2}}^2,$$

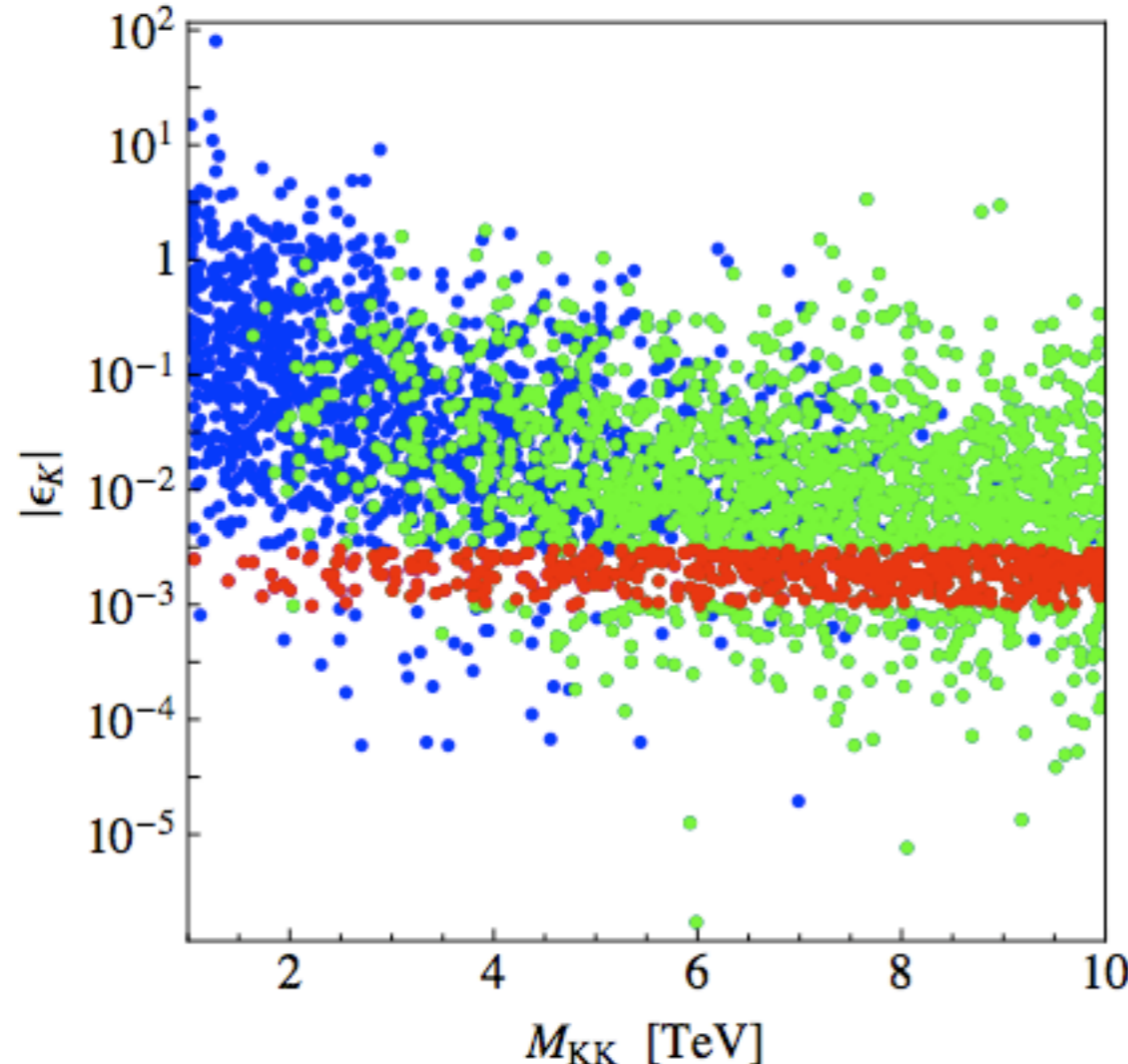
$$C_{4,K}^{\text{RS}} \sim \frac{4\pi L}{M_{\text{KK}}^2} [-2\alpha_s] F_{c_{Q_1}} F_{c_{Q_2}} F_{c_{d_1}} F_{c_{d_2}}$$

$$\sim \frac{4\pi L}{M_{\text{KK}}^2} [-2\alpha_s] \frac{2m_d m_s}{v^2 |Y_q|^2}$$

- Size of $C_{4,K}^{\text{RS}}$ can be reduced (enhanced) by making L ($|Y_q|$) smaller (larger)

Meson mixing: Neutral kaons*

- Generically $|\epsilon_K|/|\epsilon_K|_{\text{exp}} = \mathcal{O}(100)$ in RS model with $|\epsilon_K|_{\text{exp}} = (2.23 \pm 0.01) \cdot 10^{-3}$.
But $|\epsilon_K| \approx |\epsilon_K|_{\text{exp}}$ possible even for $M_{\text{KK}} = 1$ TeV after fine-tuning

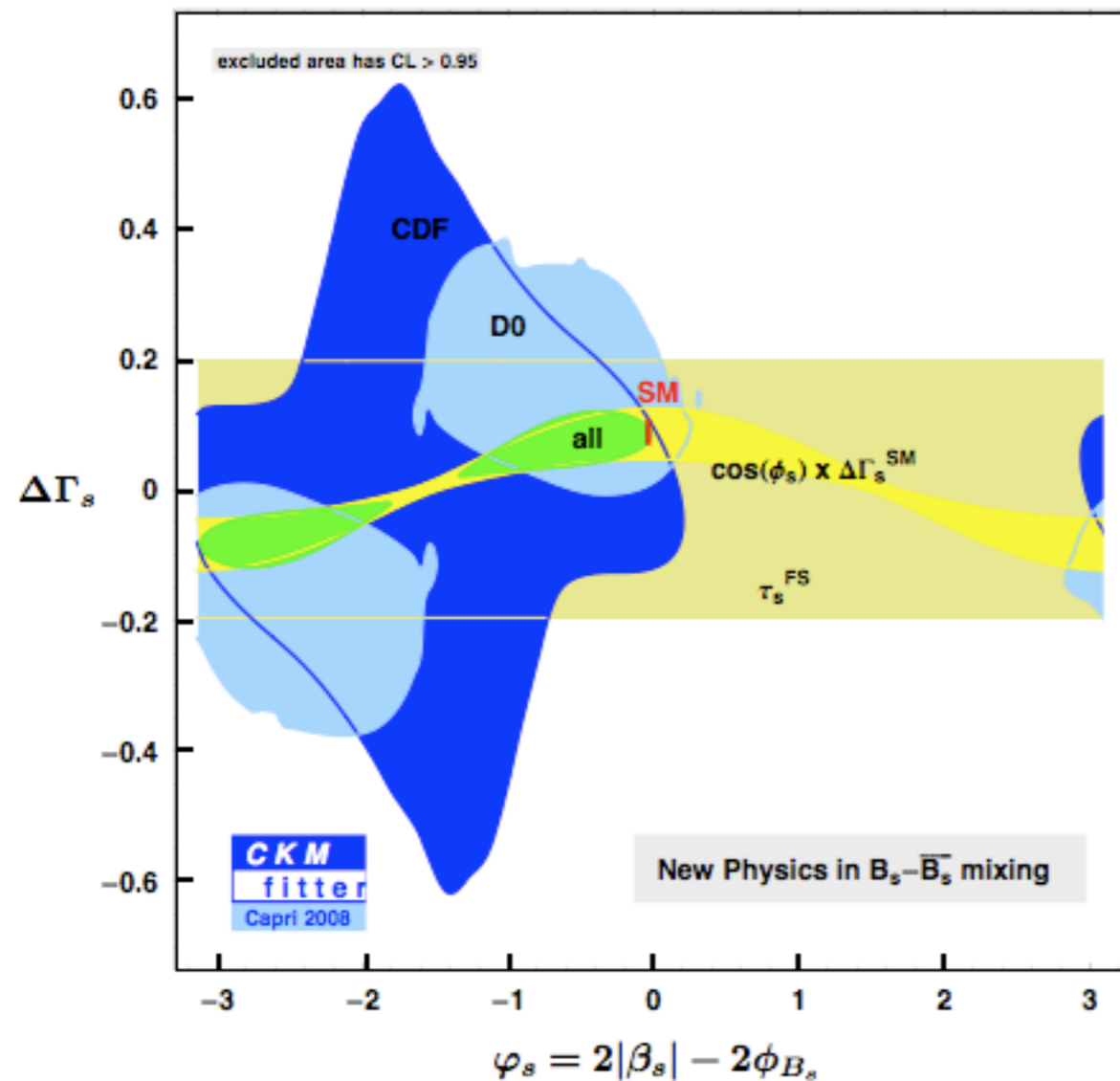


3000 randomly chosen RS points with $|Y_q| < 3$ reproducing quark masses and CKM parameters with $\chi^2/\text{dof} < 11.5/10$ corresponding to 68% CL

- satisfying 95% CL limit
 $|\epsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$
- without $Z \rightarrow b\bar{b}$ constraint
- with $Z \rightarrow b\bar{b}$ constraint at 95% CL

BSM physics in B_s mixing*

- Tantalizing hints for new physics phase in $B_s - \bar{B}_s$ mixing from flavor-tagged analysis of mixing-induced CP violation in $B_s \rightarrow J/\psi\phi$ by CDF and DØ



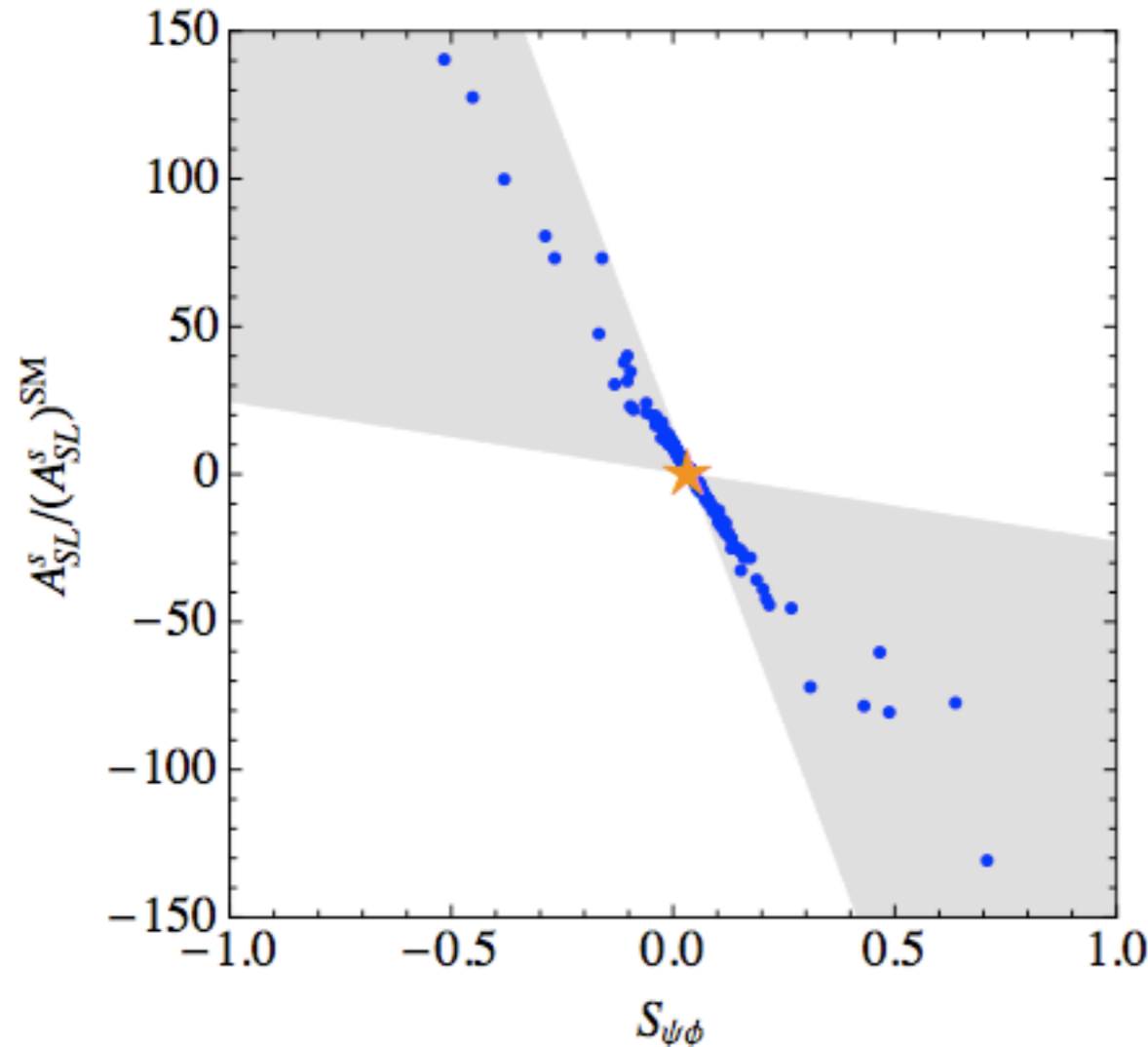
CKMfitter combination:

- ▶ CDF data only 2.1σ
- ▶ DØ data only 1.9σ
- ▶ CDF and DØ data 2.7σ
- ▶ full BSM physics fit 2.5σ

Discrepancy of $\varphi_s = 2|\beta_s| - 2\phi_{B_s}$ with respect to SM value $\varphi_s \approx 2^\circ$ at around 2σ level. Issue will be clarified at LHCb

Meson mixing: Neutral B_s mesons*

- In RS model significant corrections to semileptonic CP asymmetry A_{SL}^s and $S_{\psi\phi} = \sin(2|\beta_s| - 2\phi_{B_s})$ consistent with $|\varepsilon_K|$ can arise



$$A_{SL}^s = \frac{\Gamma(\bar{B}_s \rightarrow l^+ X) - \Gamma(B_s \rightarrow l^- X)}{\Gamma(\bar{B}_s \rightarrow l^+ X) + \Gamma(B_s \rightarrow l^- X)}$$

$$= \text{Im} \left(\frac{\Gamma_{12}^s}{M_{12}^s} \right)$$

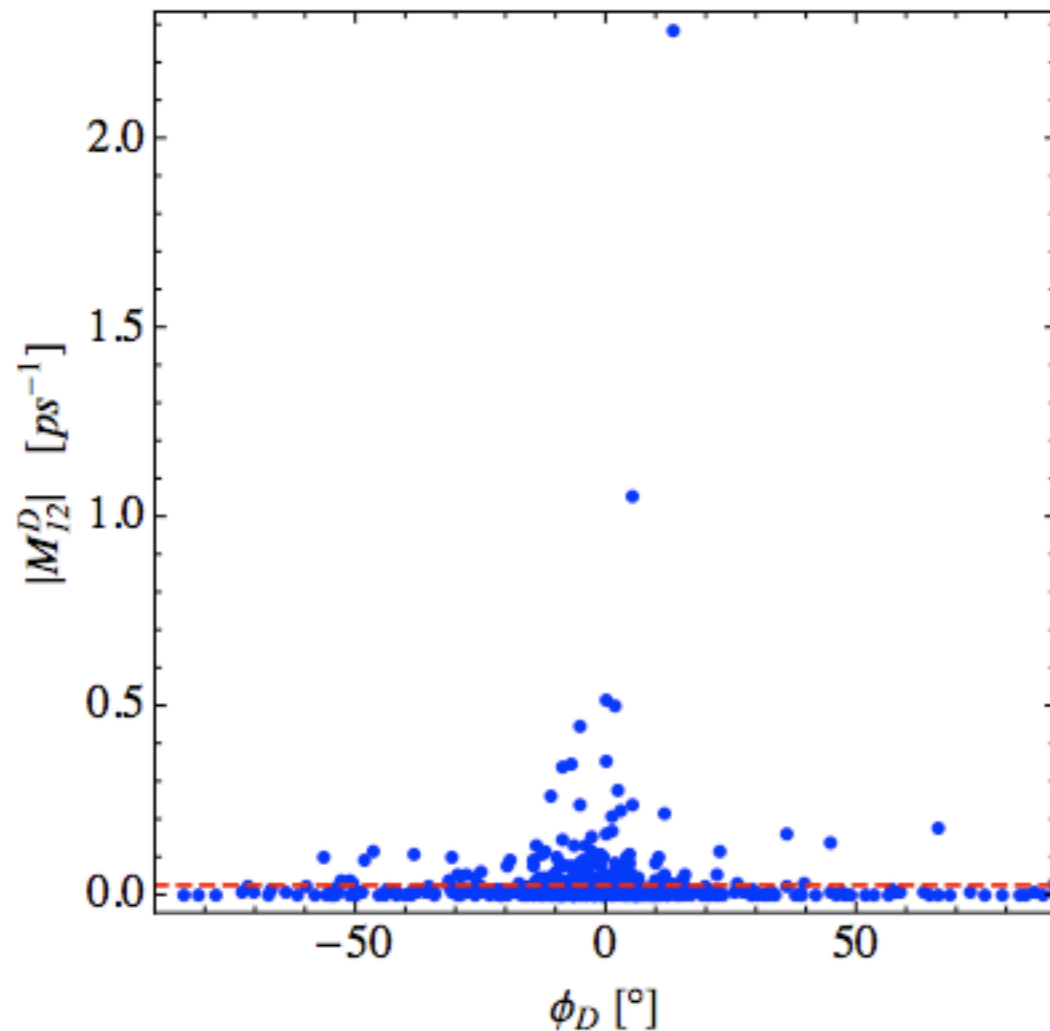
★ SM: $A_{SL}^s \approx 2 \cdot 10^{-5}$, $S_{\psi\phi} \approx 0.04$

■ model-independent prediction

● consistent with quark masses, CKM parameters, and 95% CL limit $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

Meson mixing: Neutral D mesons*

- Very large effects possible in $D - \bar{D}$ mixing, including large CP violation. Prediction might be testable at LHCb

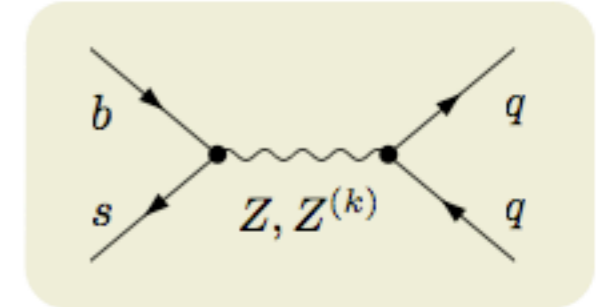
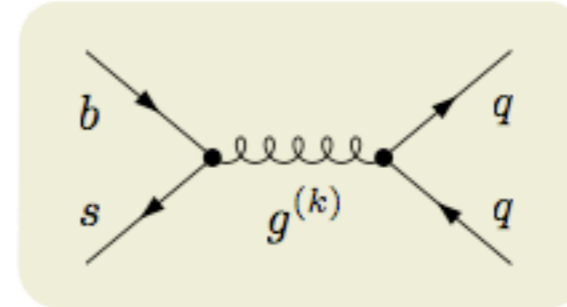


$$\begin{aligned}(M_{12}^D)^* &= \langle \bar{D} | \mathcal{H}_{\text{eff,RS}}^{\Delta C=2} | D \rangle \\ &= |M_{12}^D| e^{2i\phi_D}\end{aligned}$$

- - maximal allowed SM effect with no significant CP phase
- consistent with quark masses, CKM parameters, and 95% CL limit $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

Rare decays: Effective Hamiltonian*

$$\mathcal{H}_{\text{eff,RS}}^{b \rightarrow sq\bar{q}} = \sum_{i=3}^{10} \left(C_i^{\text{RS}} Q_i + \tilde{C}_i^{\text{RS}} \tilde{Q}_i \right)$$



$$Q_3 = 4 (\bar{s}_L^a \gamma^\mu b_L^a) \sum_q (\bar{q}_L^b \gamma_\mu q_L^b),$$

$$\vdots$$

$$Q_6 = 4 (\bar{s}_L^a \gamma^\mu b_L^b) \sum_q (\bar{q}_R^b \gamma_\mu q_R^a),$$

$$Q_7 = 6 (\bar{s}_L^a \gamma^\mu b_L^a) \sum_q Q_q (\bar{q}_R^b \gamma_\mu q_R^b),$$

$$\vdots$$

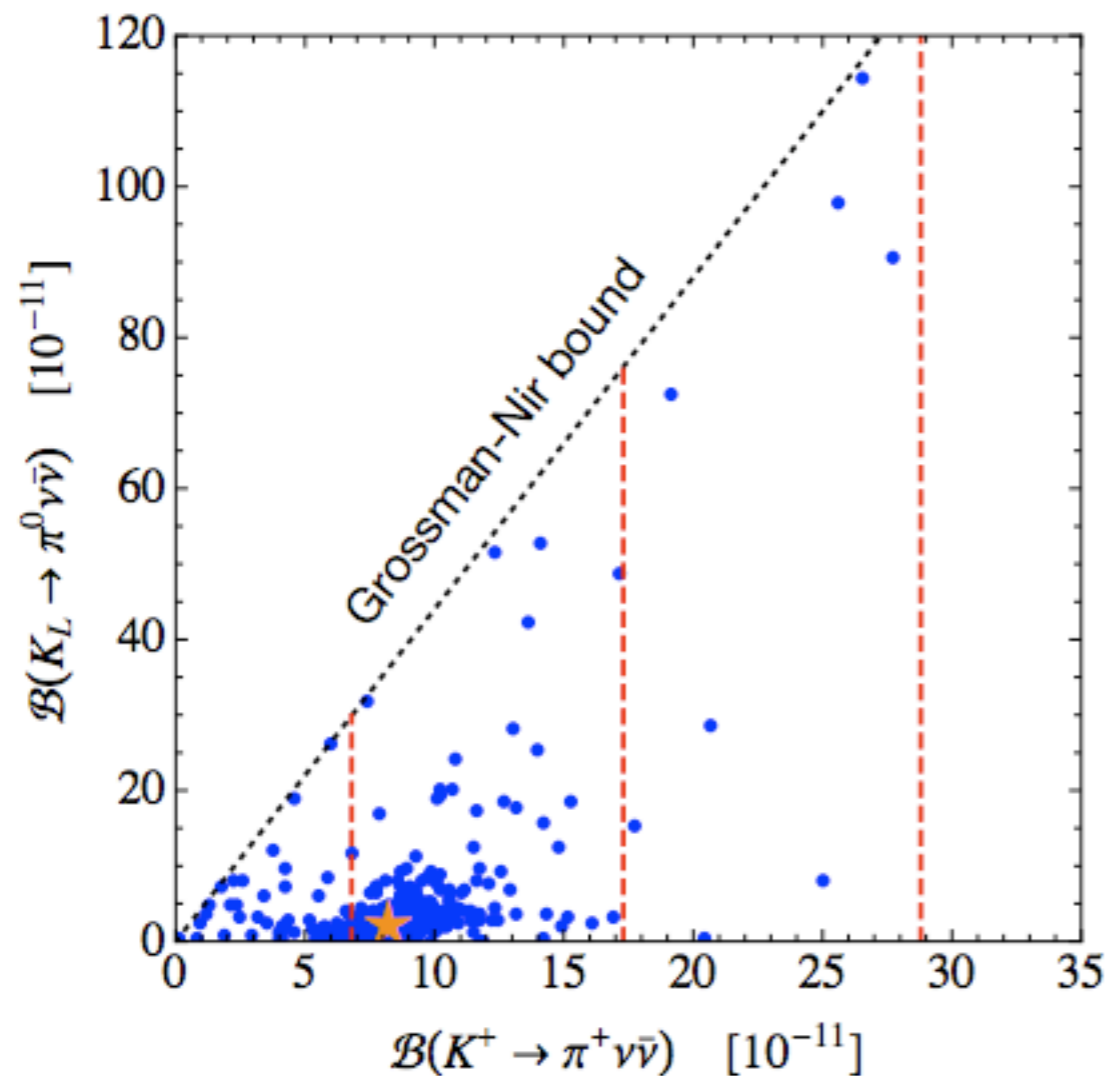
$$Q_{10} = 6 (\bar{s}_L^a \gamma^\mu b_L^b) \sum_q Q_q (\bar{q}_L^b \gamma_\mu q_L^a),$$

$$\tilde{Q}_{3-10} : L \leftrightarrow R$$

- KK gluons give dominant contribution to QCD penguins Q_{3-6} . Electroweak penguins Q_{7-10} arise almost entirely from exchange of Z and its KK modes

Rare K decays: Golden modes*

- Spectacular corrections in very clean $K \rightarrow \pi \nu \bar{\nu}$ decays. Even Grossman-Nir bound, $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 4.4 \mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, can be saturated



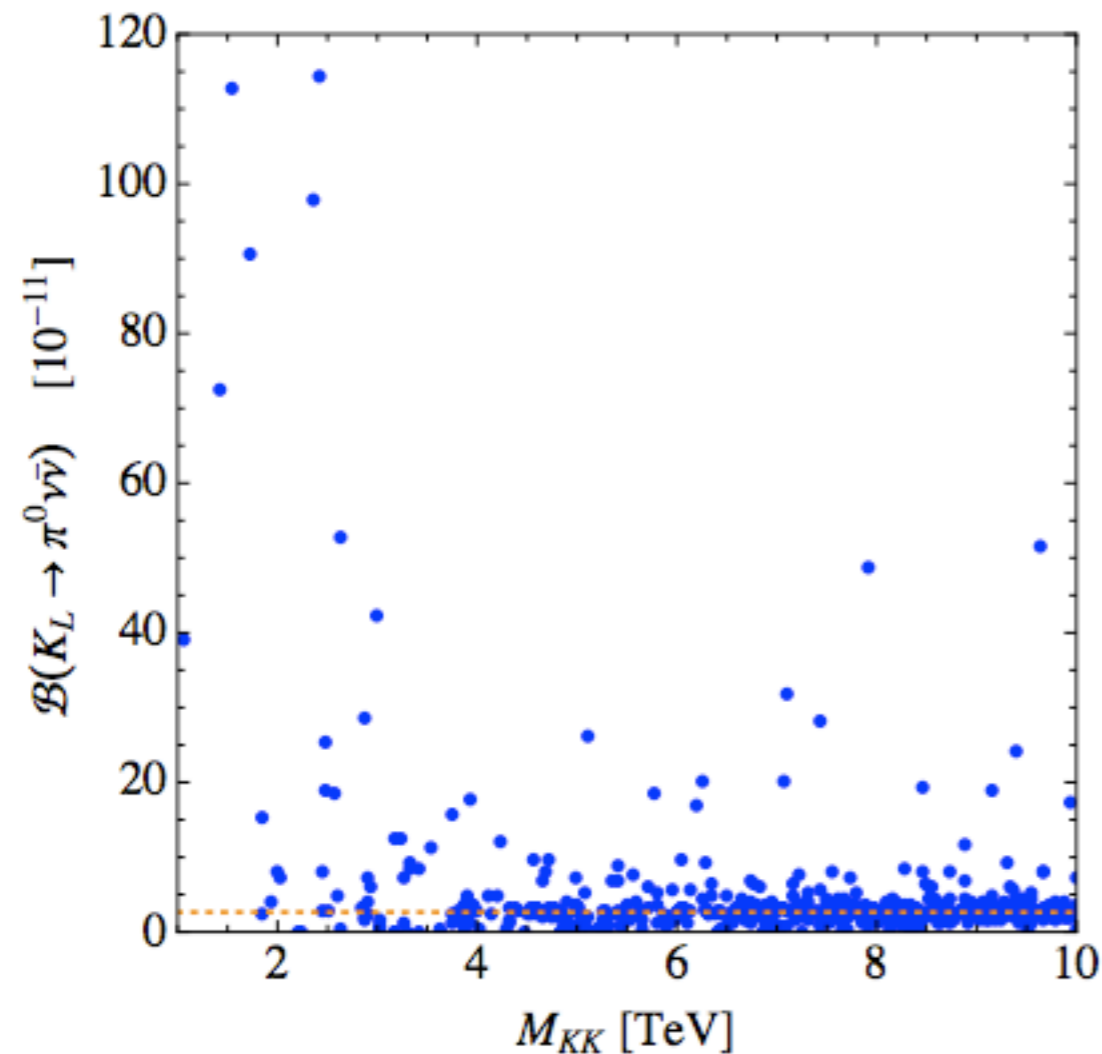
★ SM: $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \approx 8.3 \cdot 10^{-11}$,
 $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \approx 2.7 \cdot 10^{-11}$

-- central value and 68% CL limit
 $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (17.3_{-10.5}^{+11.5}) \cdot 10^{-11}$
from E949

• consistent with quark masses,
CKM parameters, and 95% CL
limit $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

Rare K decays: Golden modes*

- Sensitivity to KK scale extends far beyond LHC reach. $K \rightarrow \pi\nu\bar{\nu}$ modes offer unique window to BSM physics at and beyond terascale



$$m_{Z^{(1)}} \approx 2.50 M_{KK} ,$$

$$m_{Z^{(2)}} \approx 5.59 M_{KK} ,$$

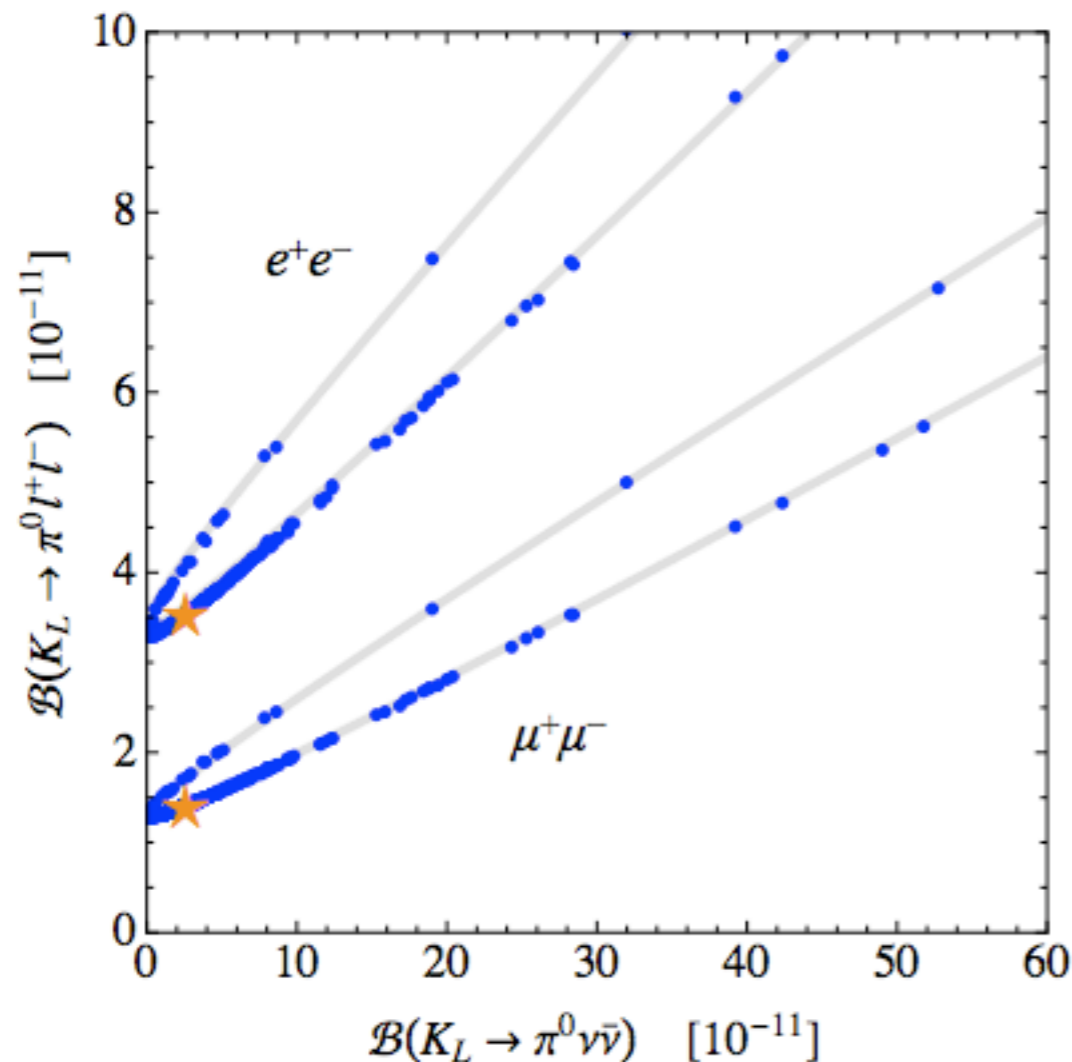
\vdots

..... SM: $\mathcal{B}(K_L \rightarrow \pi^0 \nu\bar{\nu}) \approx 2.7 \cdot 10^{-11}$

- consistent with quark masses, CKM parameters, and 95% CL limit $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

Rare K decays: Silver modes*

- Deviations from SM expectations in $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 l^+ l^-$ follow specific pattern, arising from smallness of vector and scalar contributions



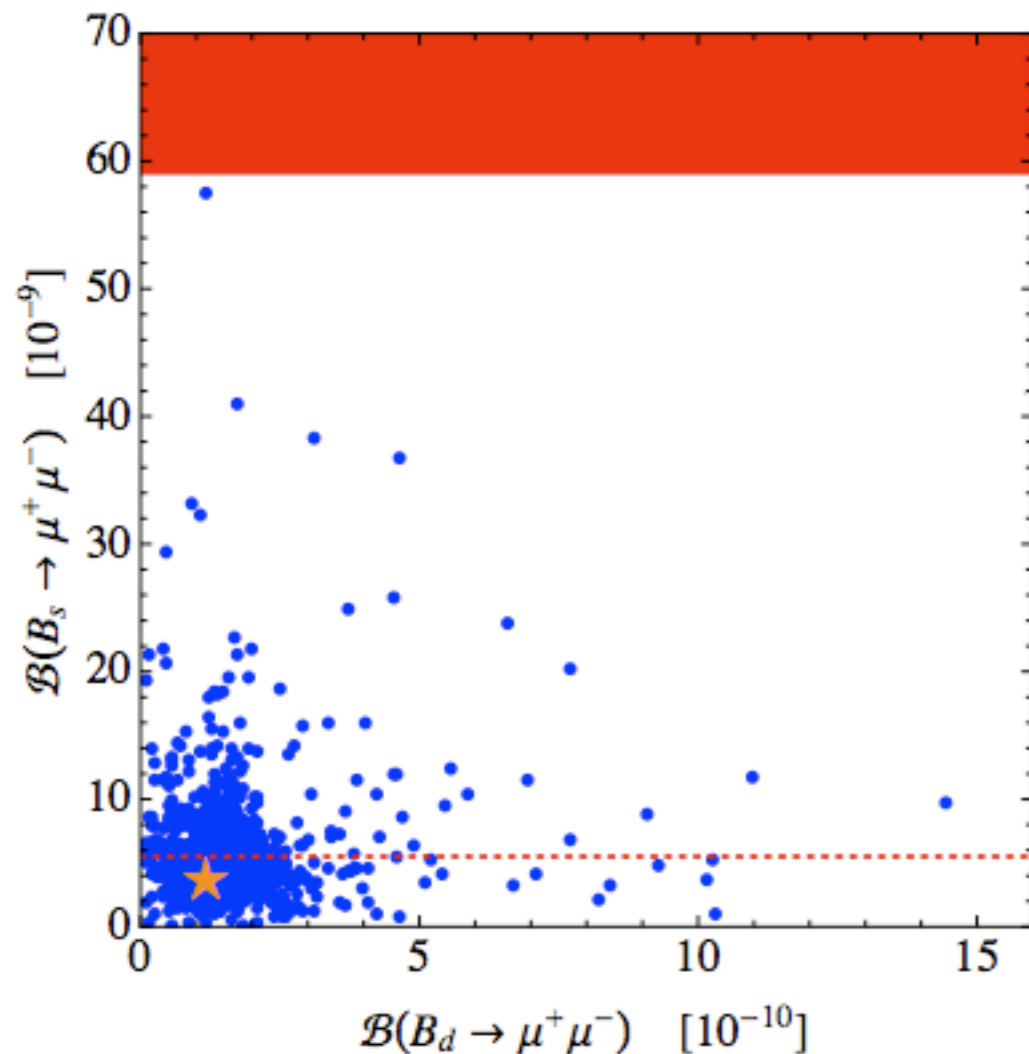
- ★ SM: $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \approx 2.7 \cdot 10^{-11}$,
 $\mathcal{B}(K_L \rightarrow \pi^0 e^+ e^-) \approx 3.6 \cdot 10^{-11}$,
 $\mathcal{B}(K_L \rightarrow \pi^0 \mu^+ \mu^-) \approx 1.4 \cdot 10^{-11}$
 for constructive interference

— model-independent prediction

- consistent with quark masses, CKM parameters, and 95% CL limit $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

Rare B decays: Purely leptonic modes*

- Factor ten enhancements possible in rare $B_{d,s} \rightarrow \mu^+ \mu^-$ modes without violation of $Z \rightarrow b\bar{b}$ constraints. Effects largely uncorrelated with $|\varepsilon_K|$



★ SM: $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-) \approx 1.2 \cdot 10^{-10}$,
 $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) \approx 3.9 \cdot 10^{-9}$

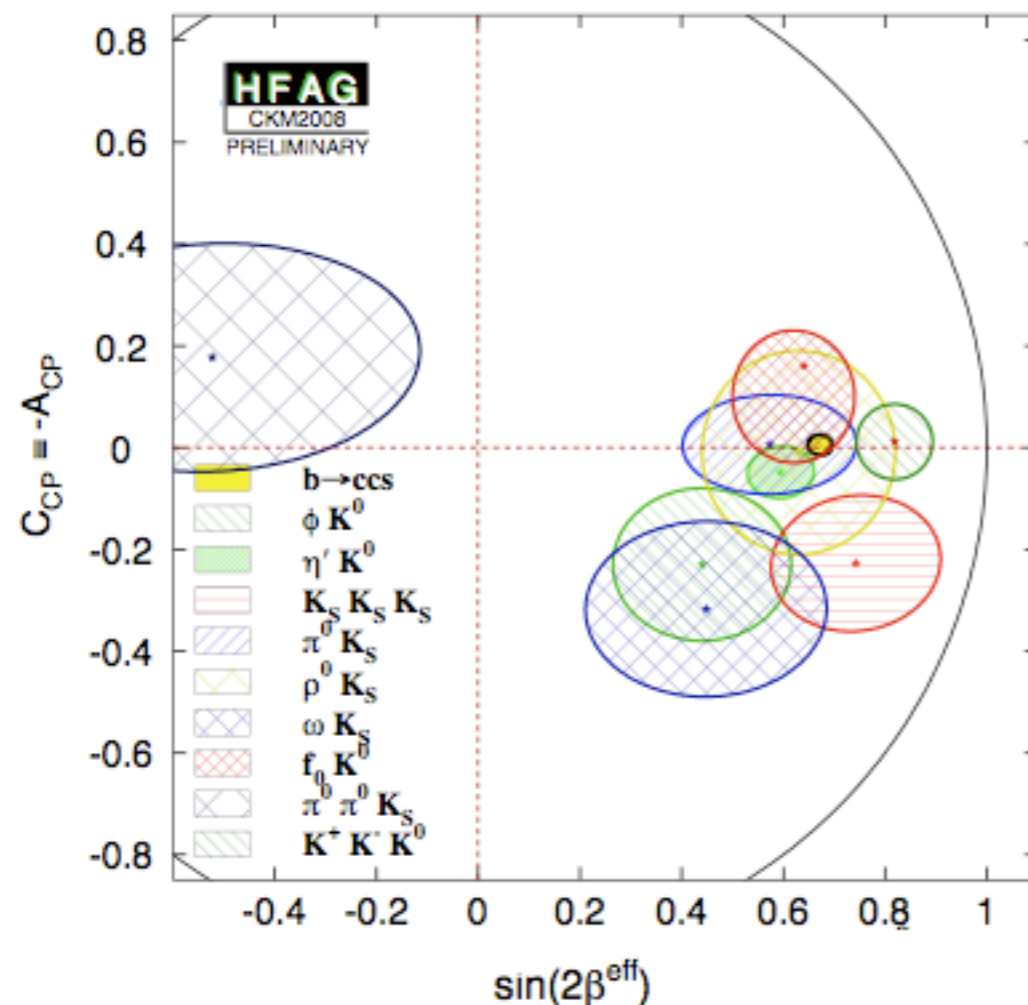
..... minimum of $5.5 \cdot 10^{-9}$ for 5σ
discovery by LHCb, 2 fb^{-1}

■ 95% CL upper limit from CDF
 $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) < 5.8 \cdot 10^{-8}$

● consistent with quark masses,
CKM parameters, and 95% CL
limit of $Z \rightarrow b\bar{b}$

Non-leptonic B and K decays*

- Electroweak penguin effects in rare hadronic decays such as $B \rightarrow K\pi$ or $B \rightarrow \phi K$ are naturally of order one compared to SM and can introduce new large CP-violating phases. Similar effects can occur in $K \rightarrow \pi\pi$

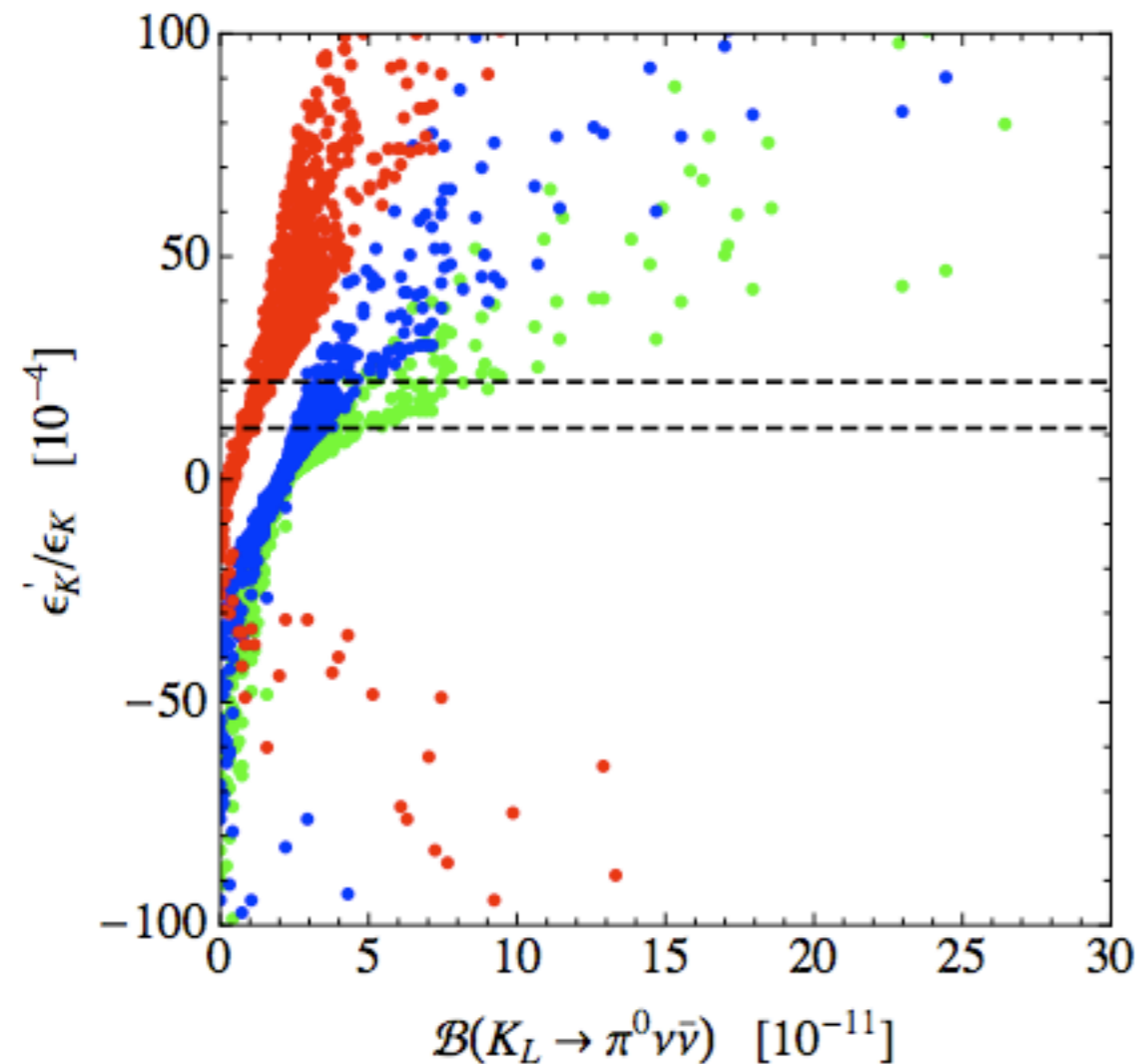


Potentially relevant for:

- ▶ explaining large CP asymmetries in $B \rightarrow K\pi$ and determining of $\sin(2\beta^{\text{eff}})$ from penguin-dominated modes
- ▶ studying of correlations between ratio $\varepsilon'_K/\varepsilon_K$ measuring direct and indirect CP violation in $K \rightarrow \pi\pi$ and large effects in rare K decays

Correlations between $\varepsilon'_K/\varepsilon_K$ and rare K decays*

- Even in view of large theoretical uncertainties, data on $\varepsilon'_K/\varepsilon_K$ imply non-trivial constraints on possible BSM effects in rare K decay



-- experimental 95% CL limit
 $\varepsilon'_K/\varepsilon_K \in [11.5, 21.9] \cdot 10^{-4}$

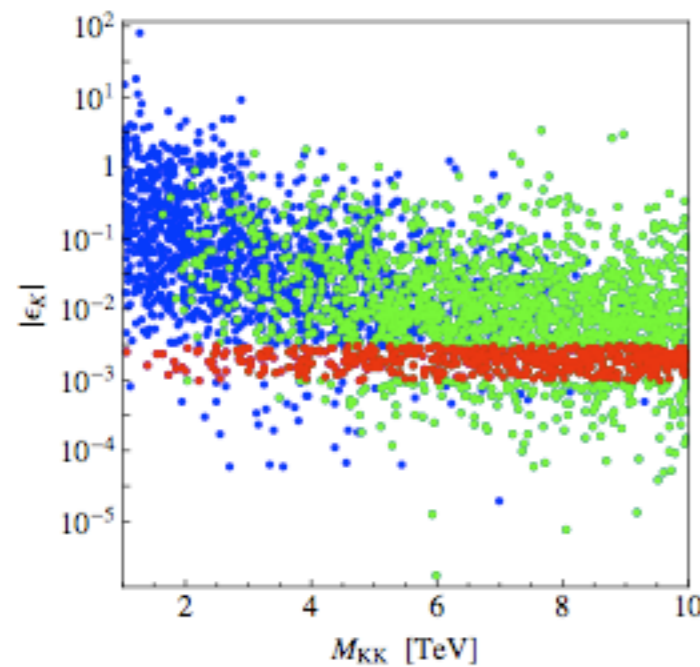
- upper
- central
- lower value

consistent with quark masses,
CKM parameters, and 95% CL
limit $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

Conclusions

- LHC is there, but LHC discoveries alone unlikely to allow for full understanding of new phenomena observed
- Flavor physics can play key role in this respect, since it offers unique window to BSM physics at and beyond terascale
- Warped extra dimensions offer compelling geometrical explanation of gauge and fermion hierarchy problem, mysteries left unexplained in SM
- Flavor-changing tree-level transitions of K and B_s mesons particularly interesting as their sensitivity to KK scale extends beyond LHC reach
- Developed techniques form basis for loop calculations of electroweak precision observables including S and T as well as rare decays such as $B \rightarrow X_s \gamma$ or $\mu \rightarrow e \gamma$

Meson mixing: Ideas to reduce fine-tuning in $|\varepsilon_K|^*$



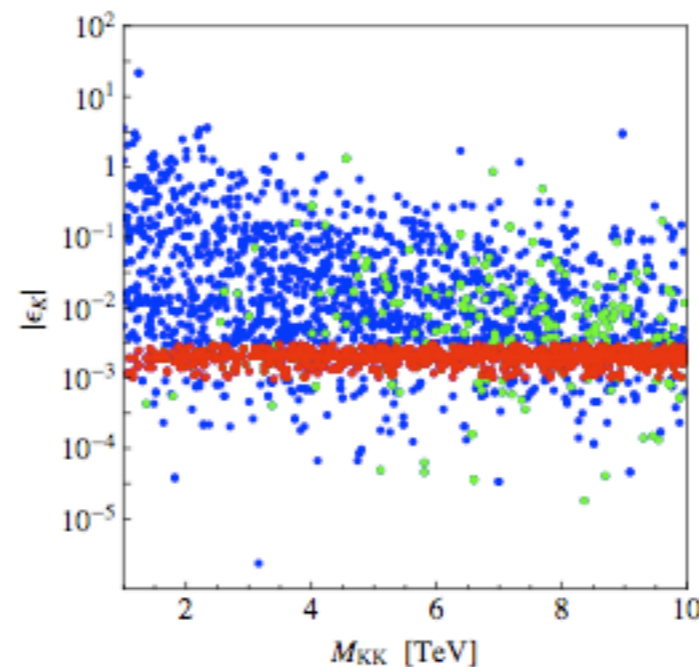
S1: Standard

$$|Y_q| < 3$$

● 16%

● 59%

13% pass



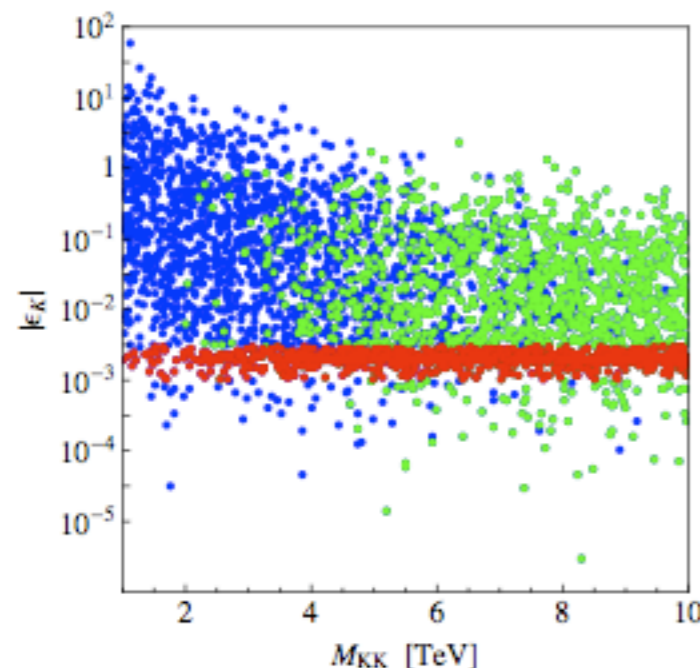
S2: Big Yukawas

$$|Y_q| < 12$$

● 44%

● 26%

19% pass



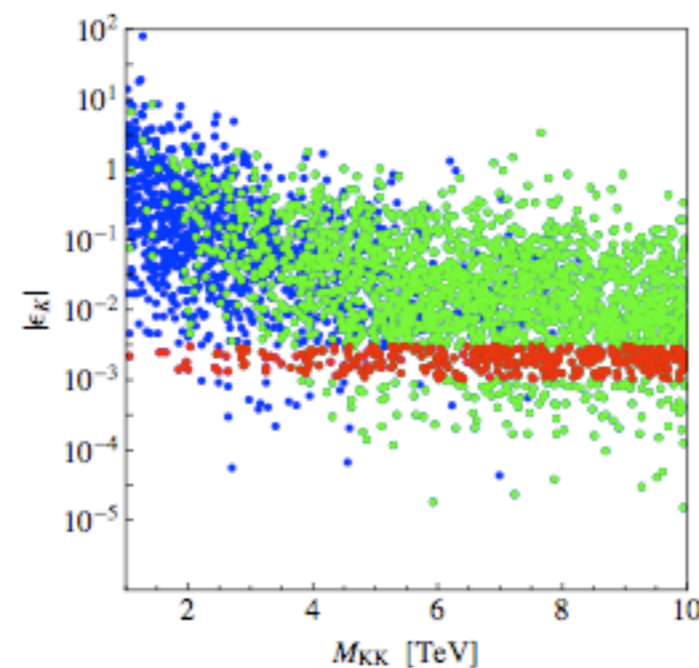
S3: Alignement

$$C_{d_1} = C_{d_2} = C_{d_3}$$

● 48%

● 24%

16% pass



S4: Little RS

$$L = 7$$

● 11%

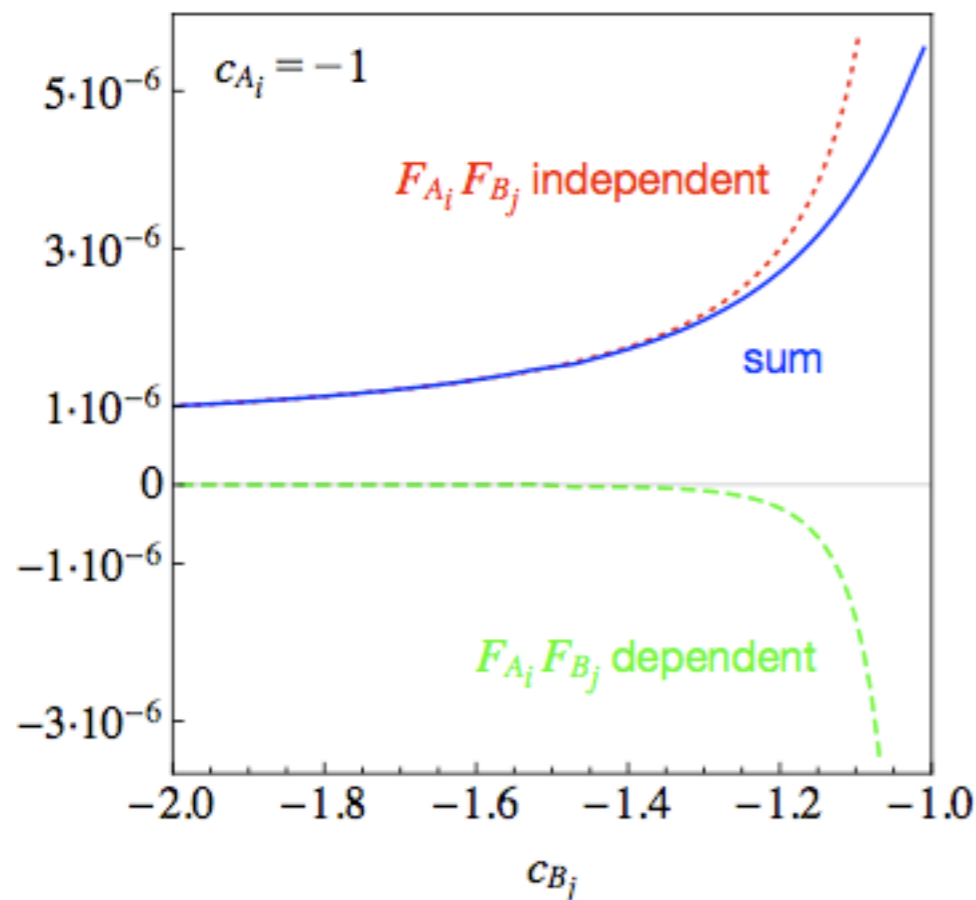
● 68%

9% pass

*Davoudiasl *et al.*, arXiv:0802.0203; Santiago, arXiv:0806.1230; Bauer *et al.*, arXiv:0811.3678, arXiv:09xx.xxxx

$|\varepsilon_K|$ in little RS models*

- Since many amplitudes in RS model are enhanced by logarithm of warp factor L harmful effects can naively be suppressed by volume truncation



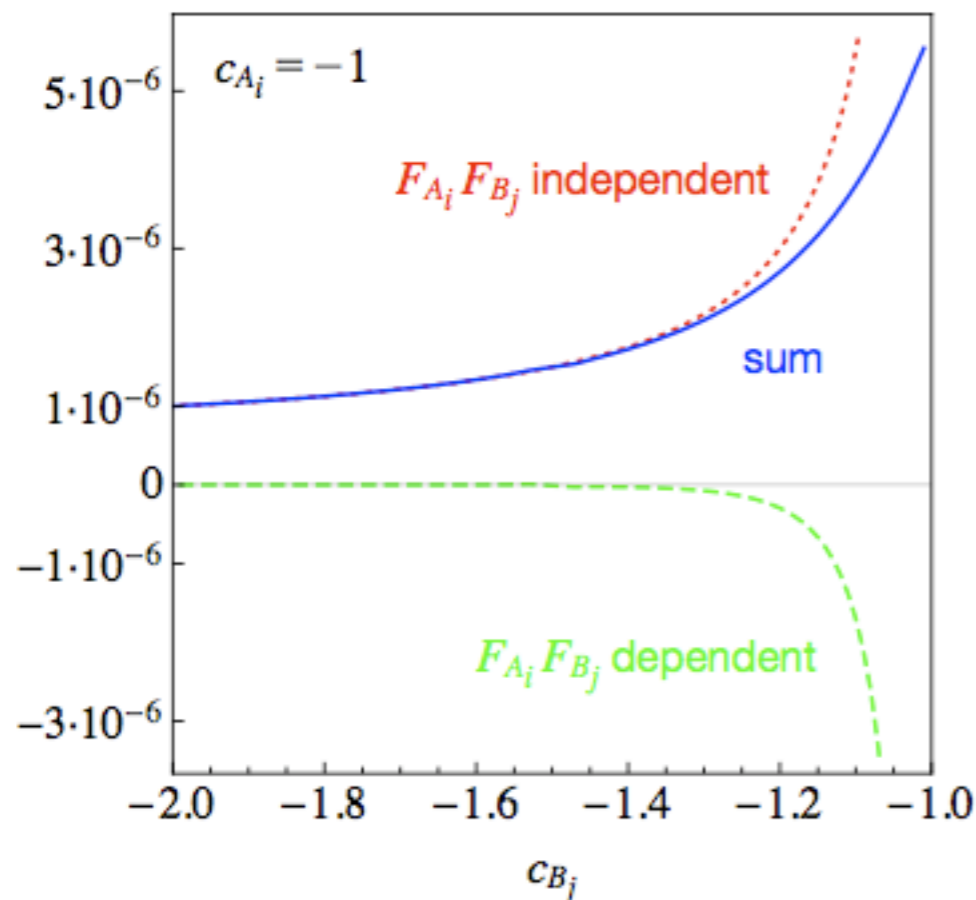
Typical bulk parameters for $L = 7$:

$$\begin{aligned}
 c_{Q_1} &= -1.06, & c_{Q_2} &= -0.77, & c_{Q_3} &= -0.61, \\
 c_{u_1} &= -1.92, & c_{u_2} &= -0.96, & c_{u_3} &= +0.34, \\
 c_{d_1} &= -1.75, & c_{d_2} &= -1.53, & c_{d_3} &= -0.93
 \end{aligned}$$

- For $c_{A_i} + c_{B_j} < -2$ weight factor t_{ζ}^2 appearing in overlap integrals of $\tilde{\Delta}_A \otimes \tilde{\Delta}_B$ not sufficient to suppress light quark profiles in UV. This partially evades RS-GIM suppression

$|\varepsilon_K|$ in little RS models*

- Since many amplitudes in RS model are enhanced by logarithm of warp factor L harmful effects can naively be suppressed by volume truncation



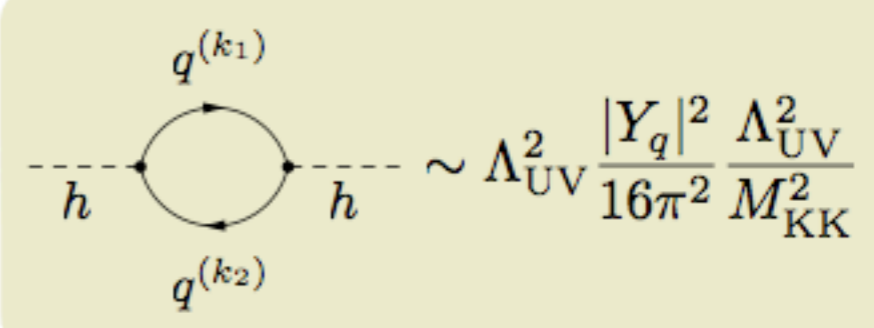
Typical bulk parameters for $L = 7$:

$$\begin{aligned} c_{Q_1} &= -1.06, & c_{Q_2} &= -0.77, & c_{Q_3} &= -0.61, \\ c_{u_1} &= -1.92, & c_{u_2} &= -0.96, & c_{u_3} &= +0.34, \\ c_{d_1} &= -1.75, & c_{d_2} &= -1.53, & c_{d_3} &= -0.93 \end{aligned}$$

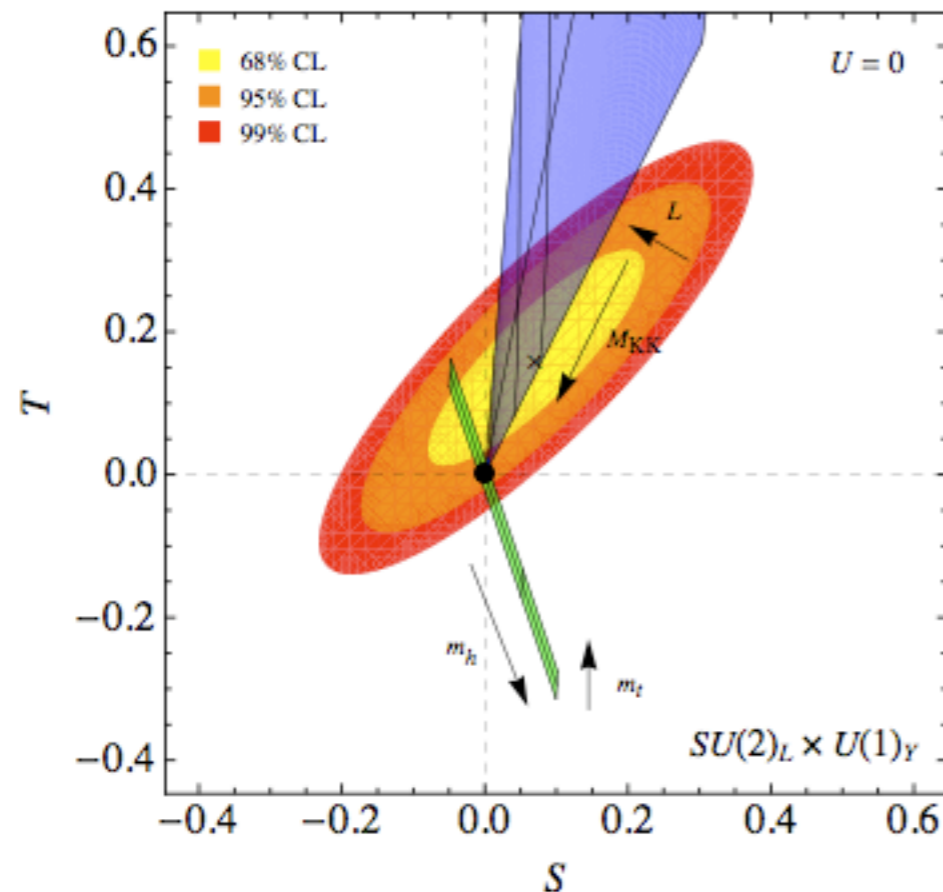
- Condition $c_{Q_2} + c_{d_2} > -2$ implies $L > 8.2$ corresponding to $\Lambda_{UV} > \text{few } 10^3 \text{ TeV}$. UV dominance in $|\varepsilon_K|$ is thus natural feature of little RS models

S and T parameters in minimal RS model*

- In warped models with brane-localized Higgs sector, m_h naturally of order M_{KK} . Heavy Higgs allows for $M_{\text{KK}} > 2.6$ TeV at 99% CL consistent with S and T



$$\sim \Lambda_{\text{UV}}^2 \frac{|Y_q|^2}{16\pi^2} \frac{\Lambda_{\text{UV}}^2}{M_{\text{KK}}^2}$$

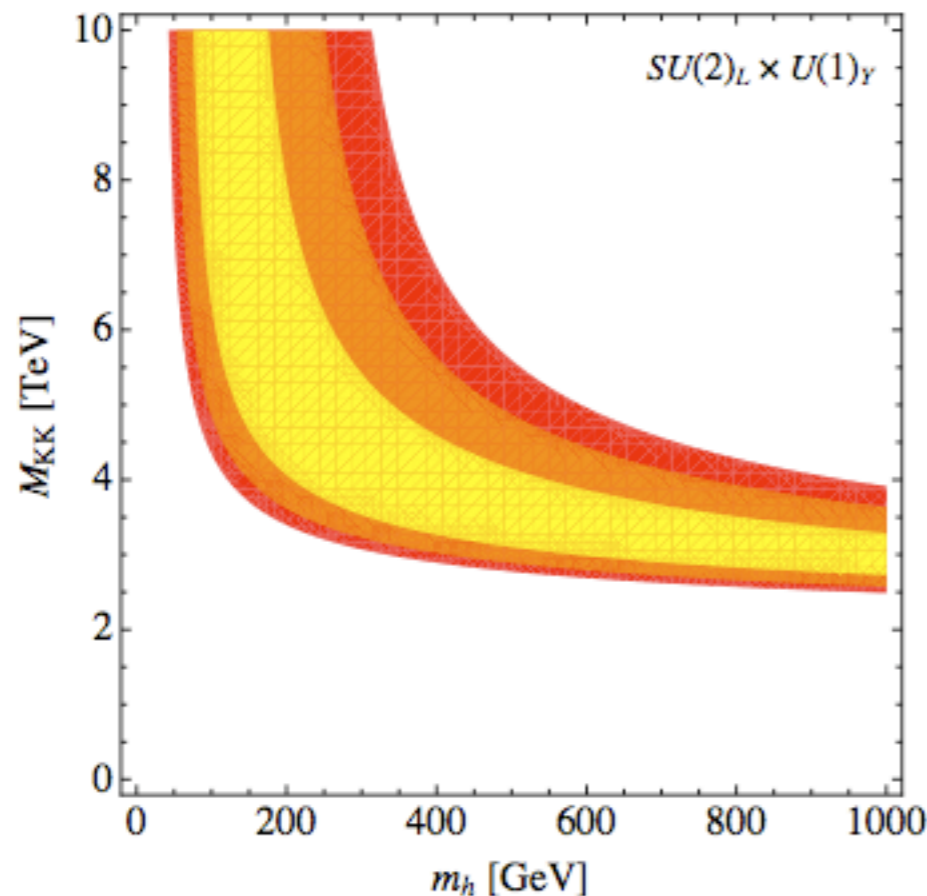
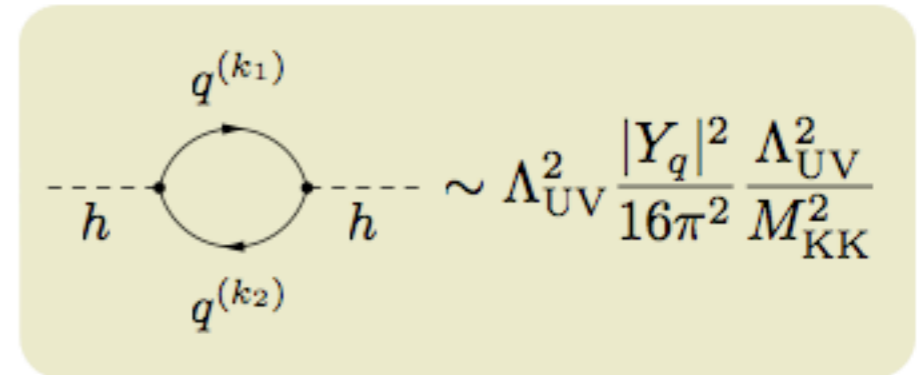


$$\Delta S = \frac{1}{6\pi} \ln \frac{m_h}{m_h^{\text{ref}}}, \quad \Delta T = -\frac{3}{8\pi c_w^2} \ln \frac{m_h}{m_h^{\text{ref}}}$$

- minimal RS prediction for $M_{\text{KK}} \in [1, 10]$ TeV and $L \in [5, 37]$
- SM reference point for $m_h \in [60, 1000]$ GeV and $m_t = (172.6 \pm 1.4)$ GeV
- SM reference point for $m_h = 150$ GeV

S and T parameters in minimal RS model*

- In warped models with brane-localized Higgs sector, m_h naturally of order M_{KK} . Heavy Higgs allows for $M_{\text{KK}} > 2.6$ TeV at 95% CL consistent with S and T



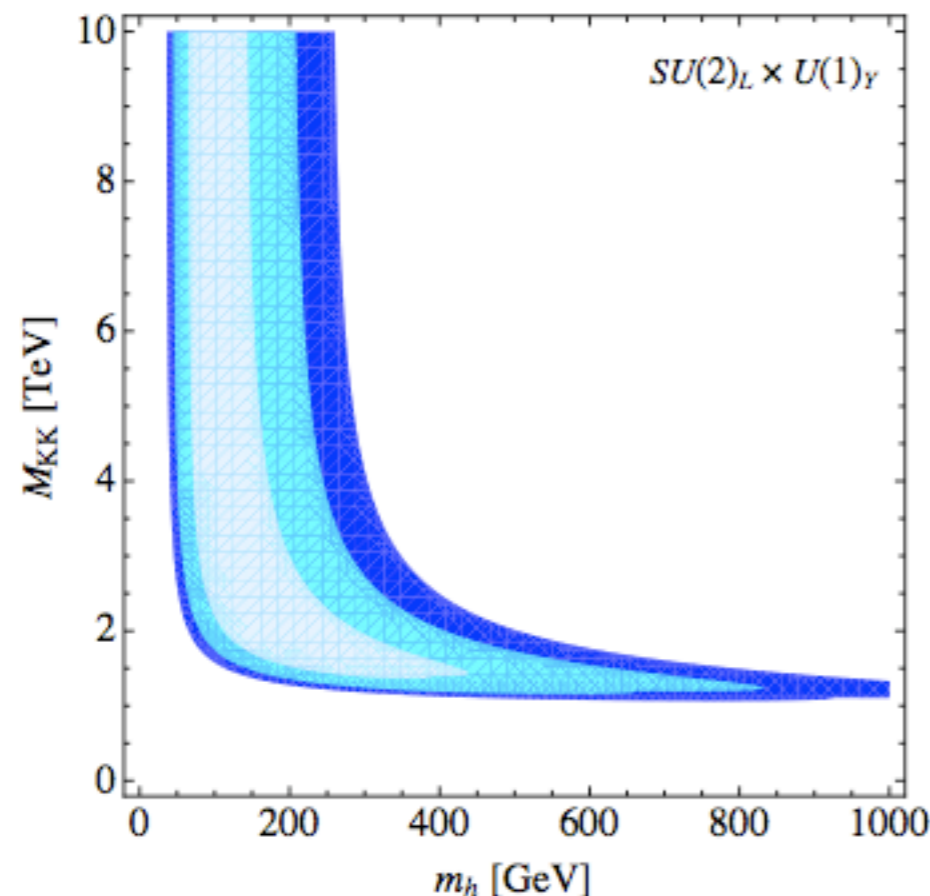
$$\Delta S = \frac{1}{6\pi} \ln \frac{m_h}{m_h^{\text{ref}}}, \quad \Delta T = -\frac{3}{8\pi c_w^2} \ln \frac{m_h}{m_h^{\text{ref}}}$$

- 68% CL
- 95% CL regions from S and T in minimal
- 99% CL RS model for $L = \ln(10^{16}) \approx 37$

*Carena *et al.*, hep-ph/0305188; Casagrande *et al.*, arXiv:0807.4537

S and T parameters in little RS model*

- Another way to protect T from vast corrections consists in giving up on solution to full gauge hierarchy problem by working in volume-truncated RS background. For $L = \ln(10^3) \approx 7$, allowed KK scale is lowered to $M_{\text{KK}} > 1.5 \text{ TeV}$ at 99% CL for $m_h = 150 \text{ GeV}$



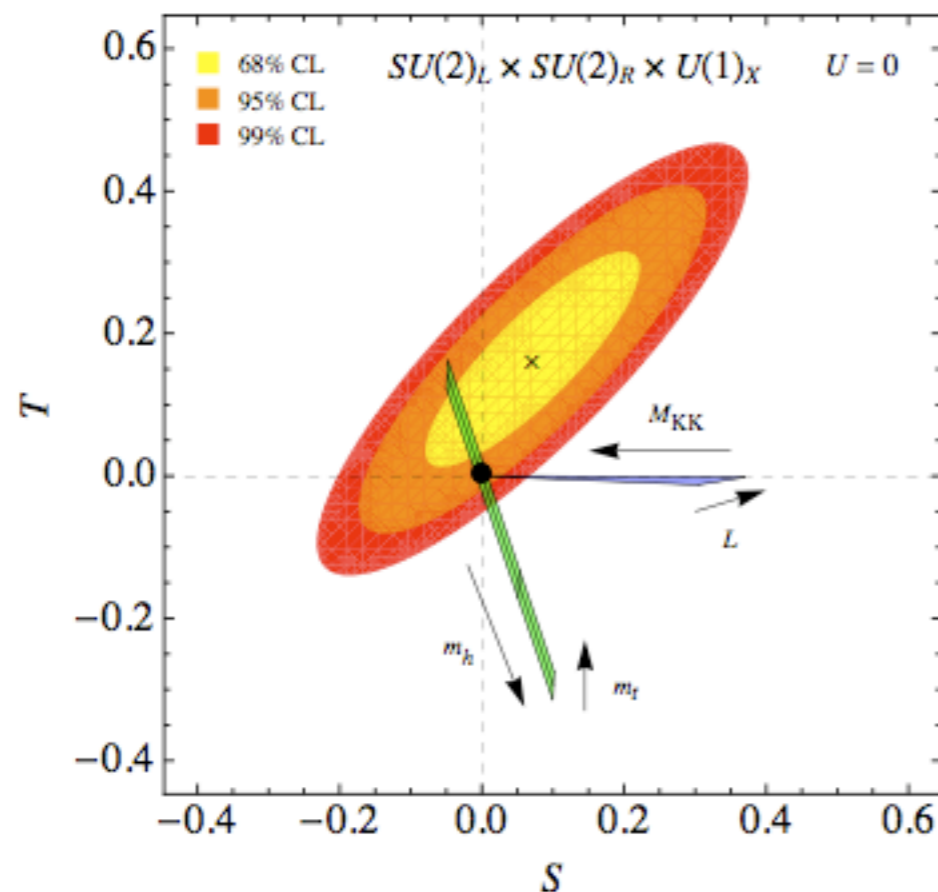
$$S = \frac{2\pi v^2}{M_{\text{KK}}^2} \left(1 - \frac{1}{L} \right),$$

$$T = \frac{\pi v^2}{2c_w^2 M_{\text{KK}}^2} \left(L - \frac{1}{2L} \right)$$

- 68% CL
 - 95% CL
 - 99% CL
- regions from S and T in little RS model for $L = \ln(10^3) \approx 7$

S and T parameters in extended RS model*

- Most elegant cure for excessive contributions to T parameter is custodial $SU(2)_R$ symmetry. Lower bound of KK scale follows then from constraint on S . For $m_h = 150$ GeV one finds $M_{\text{KK}} > 2.4$ TeV at 99% CL. Yet presence of heavy Higgs boson could spoil global electroweak fit

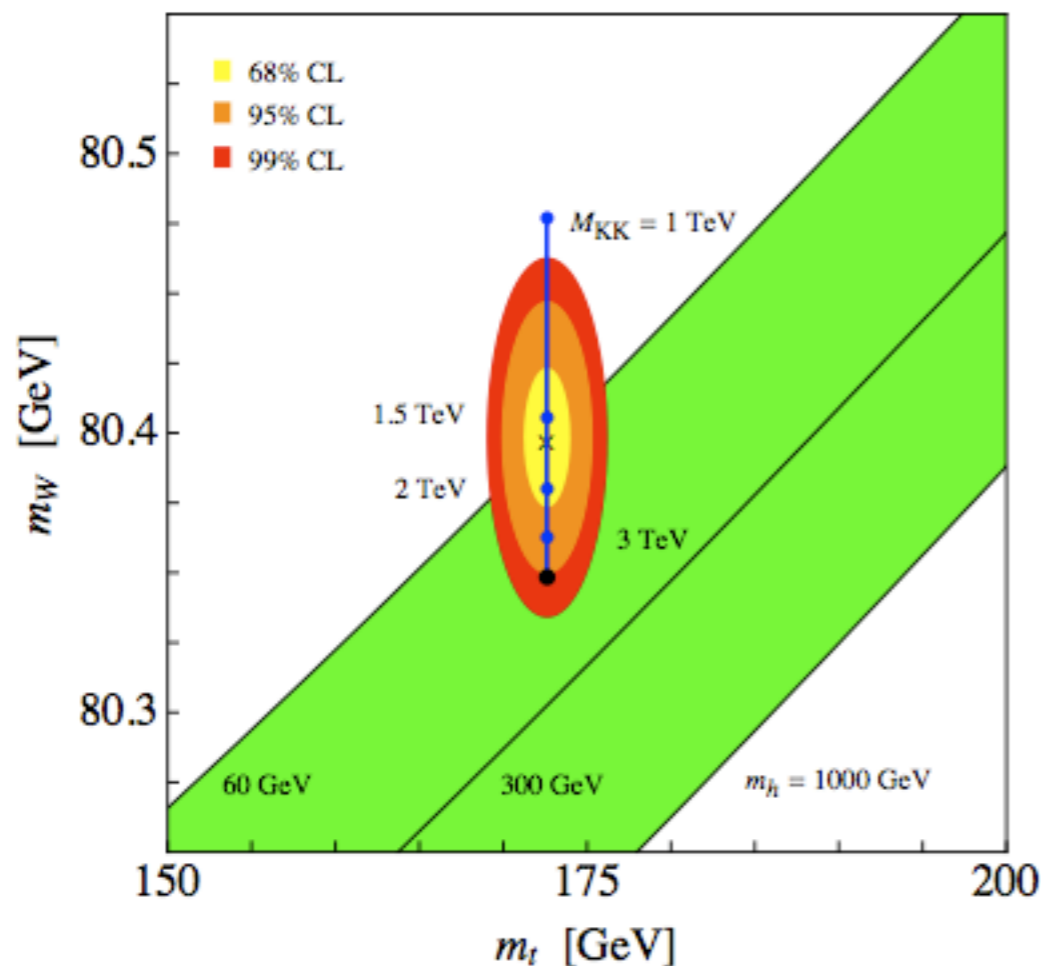
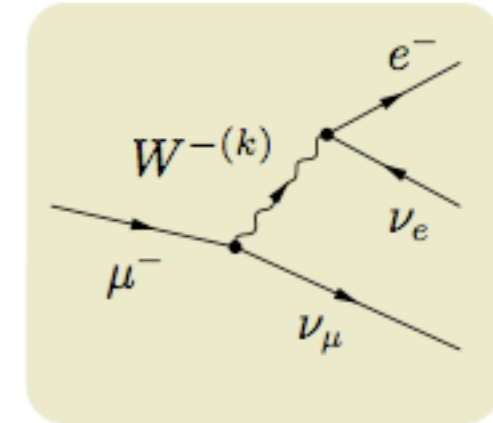


$$S = \frac{2\pi v^2}{M_{\text{KK}}^2} \left(1 - \frac{1}{L}\right), \quad T = -\frac{\pi v^2}{4c_w^2 M_{\text{KK}}^2} \frac{1}{L}$$

- prediction in extended RS model for $M_{\text{KK}} \in [1, 10]$ TeV and $L \in [5, 37]$
- SM reference point for $m_h \in [60, 1000]$ GeV and $m_t = (172.6 \pm 1.4)$ GeV
- SM reference point for $m_h = 150$ GeV

Mass of W boson*

- RS model allows to explain 50 MeV difference between direct and indirect extractions of W -boson mass $m_W \approx 80.40$ GeV and $(m_W)_{\text{ind}} \approx 80.35$ GeV

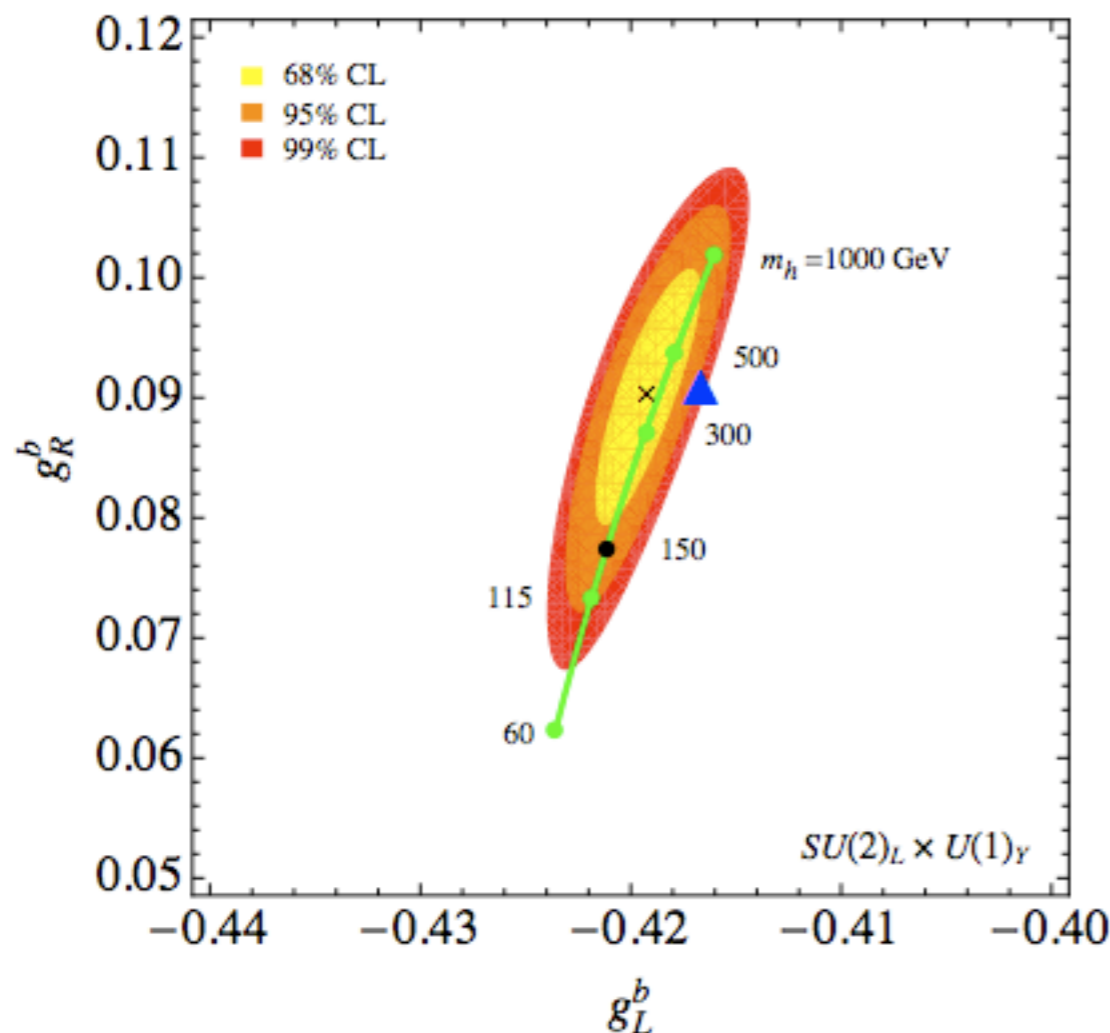


$$(m_W)_{\text{ind}} \approx m_W \left[1 - \frac{m_W^2}{4M_{\text{KK}}^2} \left(1 - \frac{1}{2L} \right) \right]$$

- $(m_W)_{\text{ind}}$ in SM for $m_h \in [60, 1000]$ GeV
- $(m_W)_{\text{ind}}$ in SM for $m_h = 150$ GeV
- $(m_W)_{\text{ind}}$ in RS model for $M_{\text{KK}} \in [1, 3]$ TeV

$Z \rightarrow b\bar{b}$ in minimal RS model*

- Heavy Higgs boson improves quality of fit to pseudo observables R_b^0 , A_b , and $A_{\text{FB}}^{0,b}$. Minimal RS model thus offer indirect explanation of 2.1σ anomaly in $A_{\text{FB}}^{0,b}$ since in this setup Higgs-boson mass is expected to large

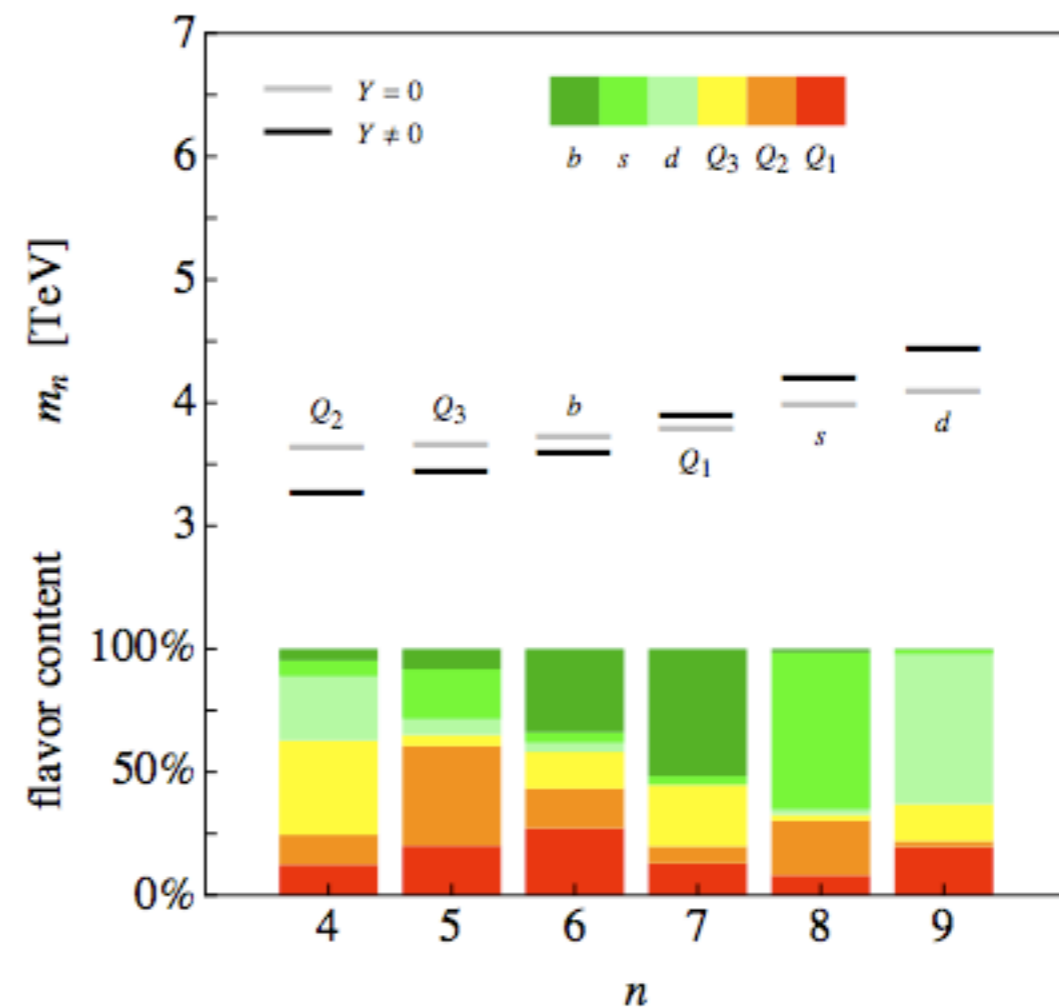
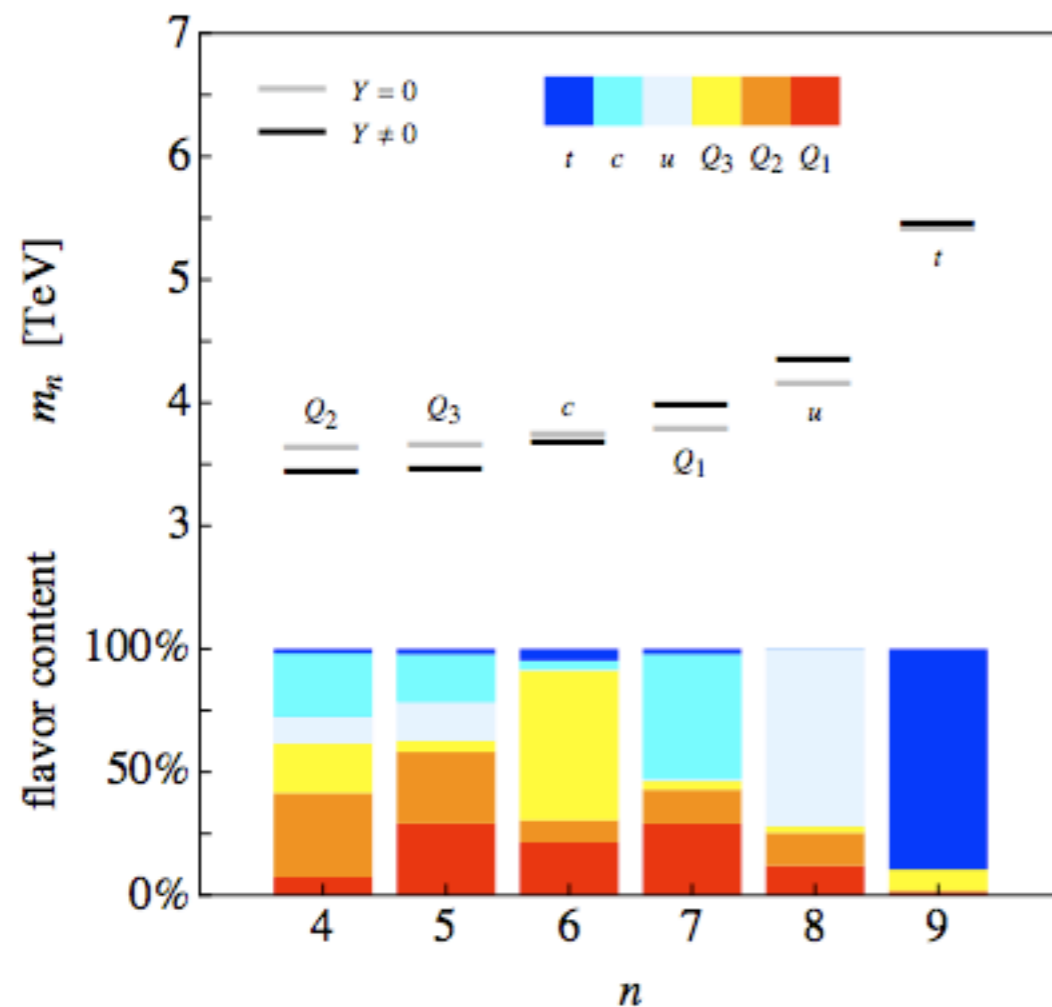


$$\Delta A_{\text{FB}}^{0,b} = -2.7 \cdot 10^{-3} \ln \frac{m_h}{m_h^{\text{ref}}}$$

- ▲ minimal RS prediction for reference point with $M_{\text{KK}} = 1.5 \text{ TeV}$ and $m_h = 400 \text{ GeV}$
- SM prediction for $m_h \in [60, 1000] \text{ GeV}$
- SM prediction for $m_h = 150 \text{ GeV}$

Mass and mixing of KK fermions*

- Since mass splittings of undisturbed KK states typical of order 100 GeV order, Yukawa couplings introduce large mixings among KK modes of same level. Mixings give rise to FCNCs when inserted into loop diagrams



Non-unitarity of CKM matrix*

- Typical RS prediction:

$$1 - (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2) = -0.00048,$$

$$1 + \frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} + \frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}} = -0.0068 + 0.0209 i$$

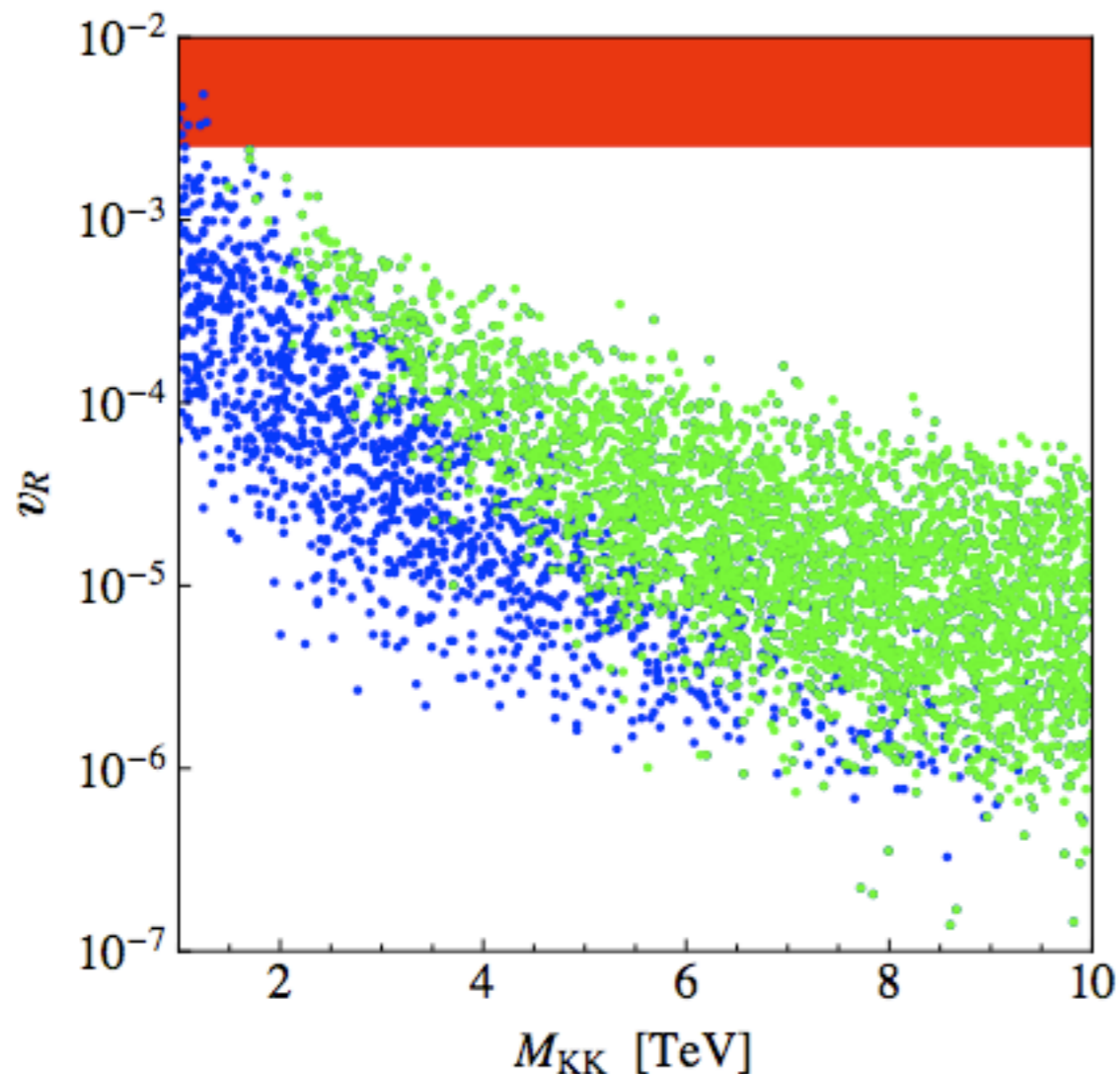
- Effects of similar magnitude as current uncertainties of global CKM fit:

$$1 - (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2) = 0.00022 \pm 0.00051_{V_{ud}} \pm 0.00041_{V_{us}},$$

$$\bar{\rho} = 0.147 \pm 0.029, \quad \bar{\eta} = 0.343 \pm 0.016$$

Right-handed charged current couplings*

- Induced right-handed charged current couplings are too small to lead to observable effects. Most pronounced effects occur in Wtb coupling v_R

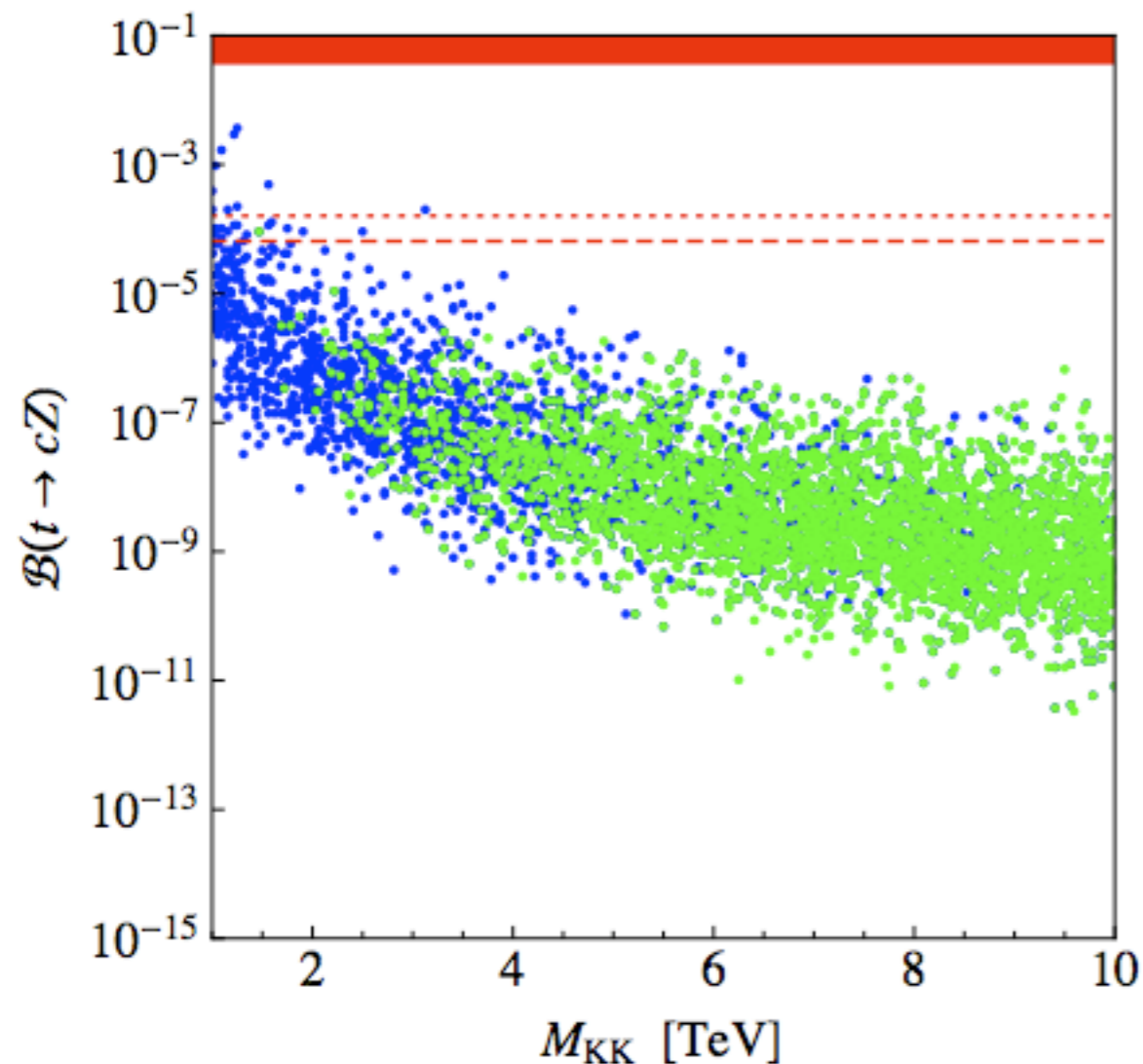


3000 randomly chosen RS points with $|Y_q| < 3$ reproducing quark masses and CKM parameters with $\chi^2/\text{dof} < 11.5/10$ corresponding to 68% CL

- $v_R \in [-0.0007, 0.0025]$ at 95% CL exclusion bound from $B \rightarrow X_s \gamma$
- without $Z \rightarrow b\bar{b}$ constraint
- with $Z \rightarrow b\bar{b}$ constraint at 95% CL

Rare FCNC top decays*

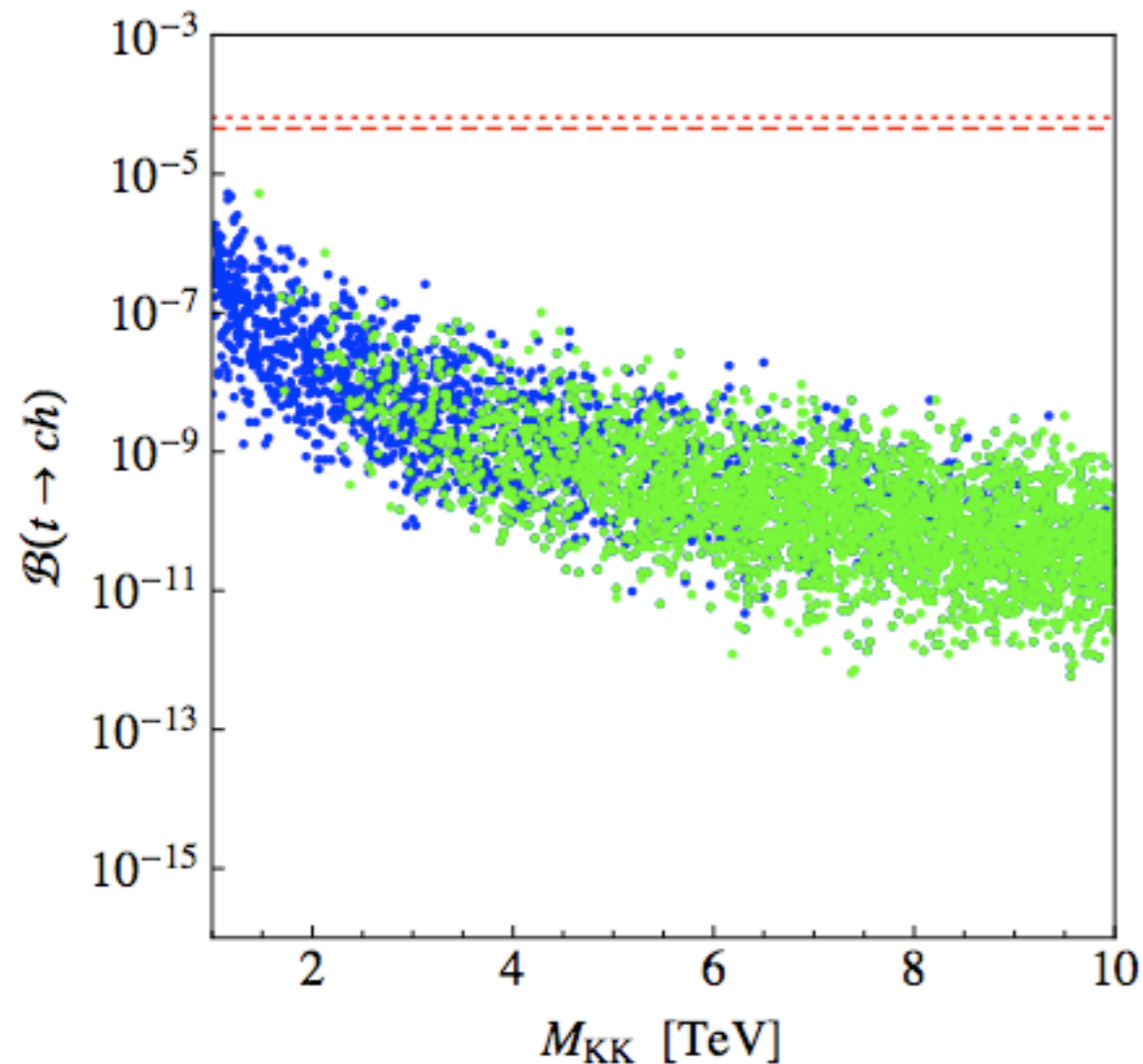
- Predictions of branching ratios for $t \rightarrow cZ$ and $t \rightarrow ch$ in minimal RS model typically below LHC sensitivity



- minimum of $1.6 \cdot 10^{-4}$ for 5σ discovery by ATLAS, 100 fb^{-1}
- - - 95% CL limit of $6.5 \cdot 10^{-5}$ from ATLAS, 100 fb^{-1}
- ■ 95% CL upper bound from CDF $B(t \rightarrow u(c)Z) < 3.7\%$
- ● without $Z \rightarrow b\bar{b}$ constraint
- ● with $Z \rightarrow b\bar{b}$ constraint at 95% CL

Rare FCNC top decays*

- Predictions of branching ratios for $t \rightarrow cZ$ and $t \rightarrow ch$ in minimal RS model typically below LHC sensitivity



..... minimum of $6.5 \cdot 10^{-4}$ for 3σ evidence by LHC

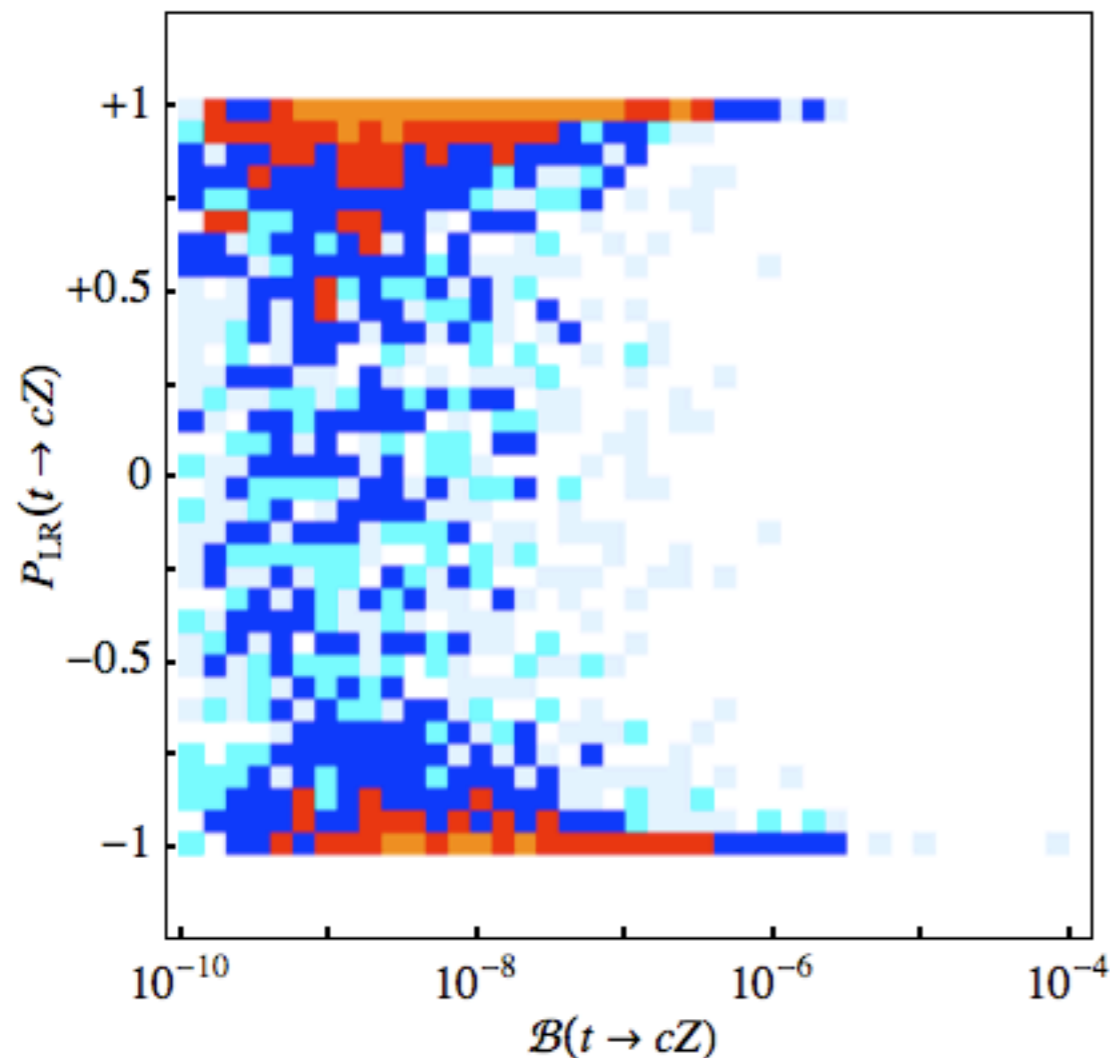
- - 95% CL limit from LHC
 $B(t \rightarrow ch) < 4.5 \cdot 10^{-5}$

• without $Z \rightarrow b\bar{b}$ constraint

• with $Z \rightarrow b\bar{b}$ constraint at 95% CL

Rare FCNC top decays*

- RS model does not lead to firm prediction for chirality of Ztc interactions, although $Z \rightarrow b\bar{b}$ constraint restricts more strongly left-handed coupling



$$P_{LR}(t \rightarrow cZ) = \frac{\Gamma_L(t \rightarrow cZ) - \Gamma_R(t \rightarrow cZ)}{\Gamma_L(t \rightarrow cZ) + \Gamma_R(t \rightarrow cZ)}$$

orange < 150

red < 50

blue < 20

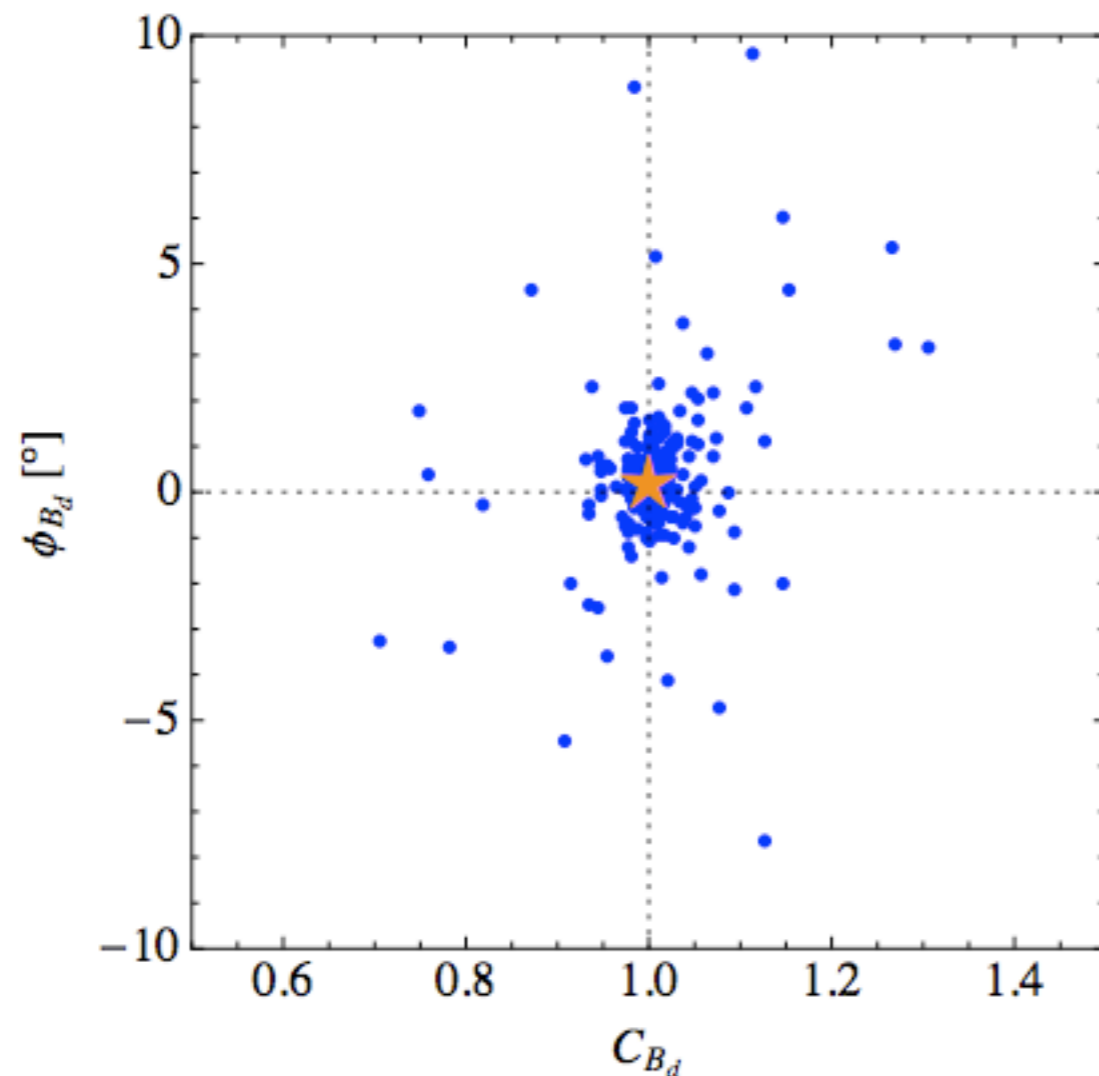
cyan < 7

light blue < 3

events in bin after imposing
 $Z \rightarrow b\bar{b}$ constraint at 95% CL

Meson mixing: Neutral B_d mesons*

- Even after imposing $|\varepsilon_K|$ constraint, sizable effects in magnitude and phase of B_d meson mixing amplitude possible

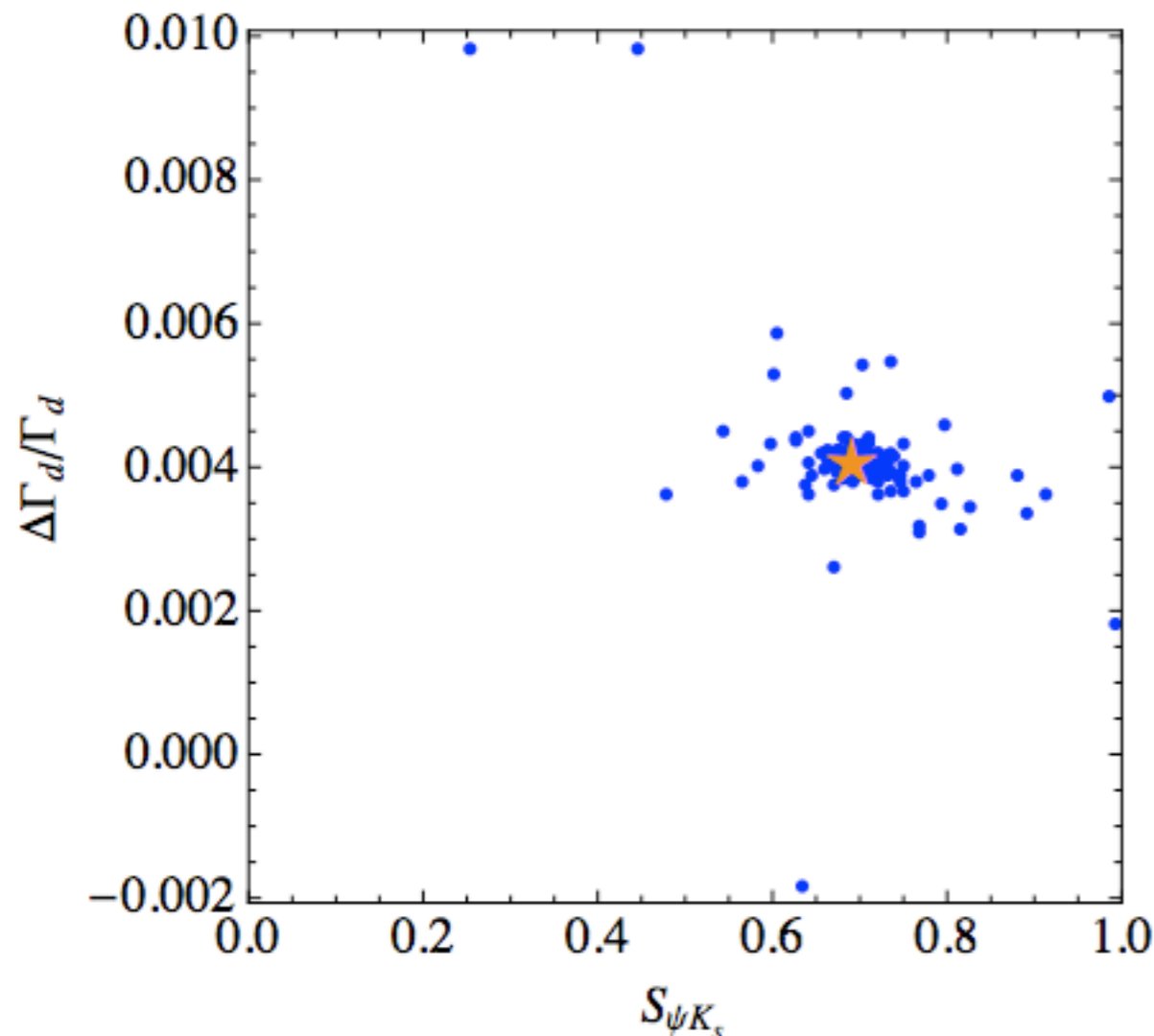


$$C_{B_d} e^{2i\phi_{B_d}} = \frac{\langle B_d | \mathcal{H}_{\text{eff,full}}^{\Delta B=2} | \bar{B}_d \rangle}{\langle B_d | \mathcal{H}_{\text{eff,SM}}^{\Delta B=2} | \bar{B}_d \rangle}$$

- ★ SM: $C_{B_d} = 1, \phi_{B_d} = 0^\circ$
- consistent with quark masses, CKM parameters, and 95% CL limit $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

Meson mixing: Neutral B_d mesons*

- Constraint from $|\varepsilon_K|$ does not exclude order one effects in width difference $\Delta\Gamma_d/\Gamma_d$ of B_d system



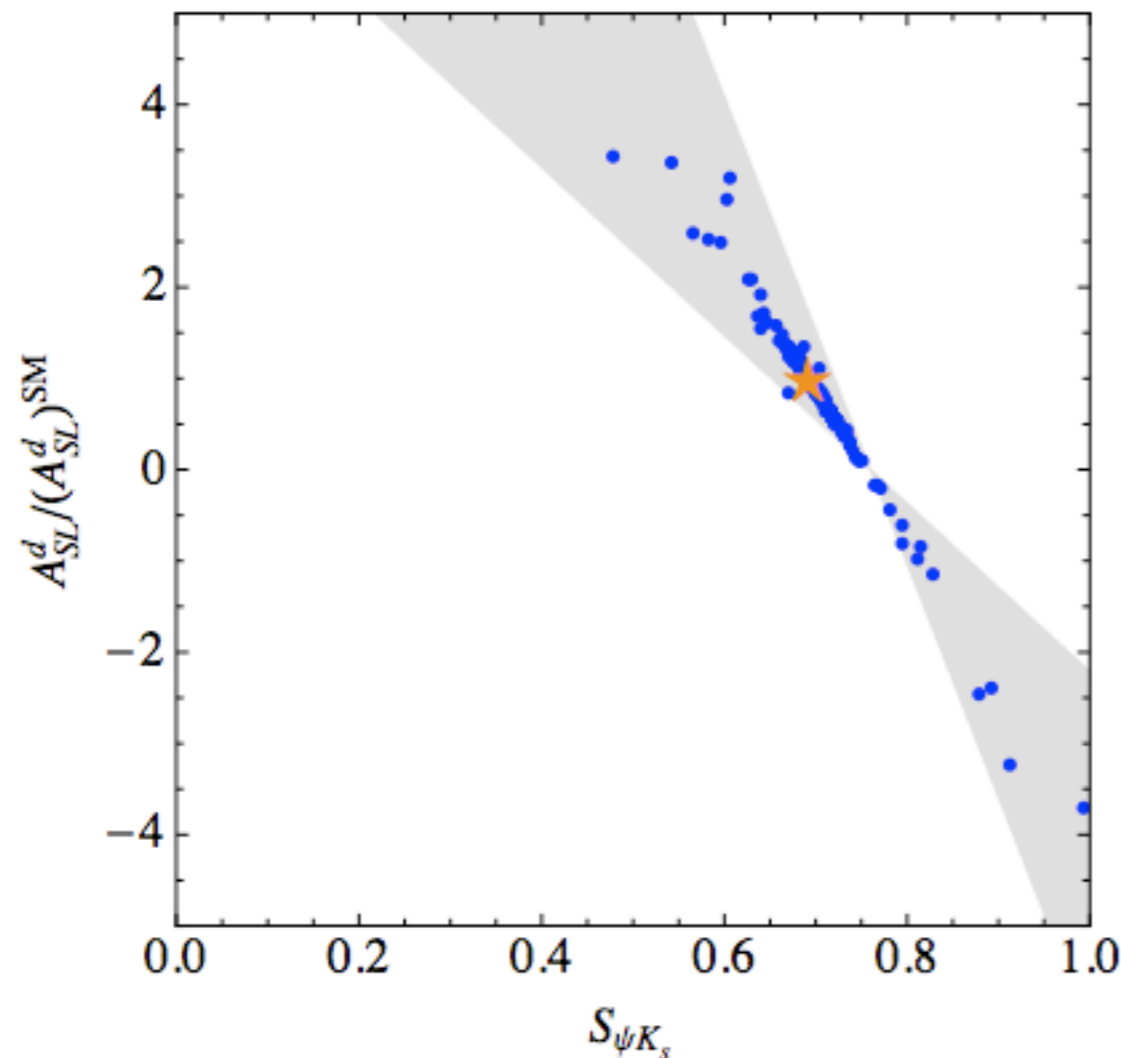
$$\begin{aligned}\Delta\Gamma_d &= \Gamma_L^d - \Gamma_S^d \\ &= 2 |\Gamma_{12}^d| \cos(2\beta + 2\phi_{B_d})\end{aligned}$$

★ SM: $\Delta\Gamma_d/\Gamma_d \approx 0.004$, $S_{\psi K_S} \approx 0.69$

- consistent with quark masses, CKM parameters, and 95% CL limit $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

Meson mixing: Neutral B_d mesons*

- In RS model, significant corrections to semileptonic CP asymmetry A_{SL}^d and $S_{\psi K_S} = \sin(2\beta + 2\phi_{B_d})$ consistent with $|\varepsilon_K|$ can arise



$$A_{SL}^d = \frac{\Gamma(\bar{B}_d \rightarrow l^+ X) - \Gamma(B_d \rightarrow l^- X)}{\Gamma(\bar{B}_d \rightarrow l^+ X) + \Gamma(B_d \rightarrow l^- X)}$$

$$= \text{Im} \left(\frac{\Gamma_{12}^d}{M_{12}^d} \right)$$

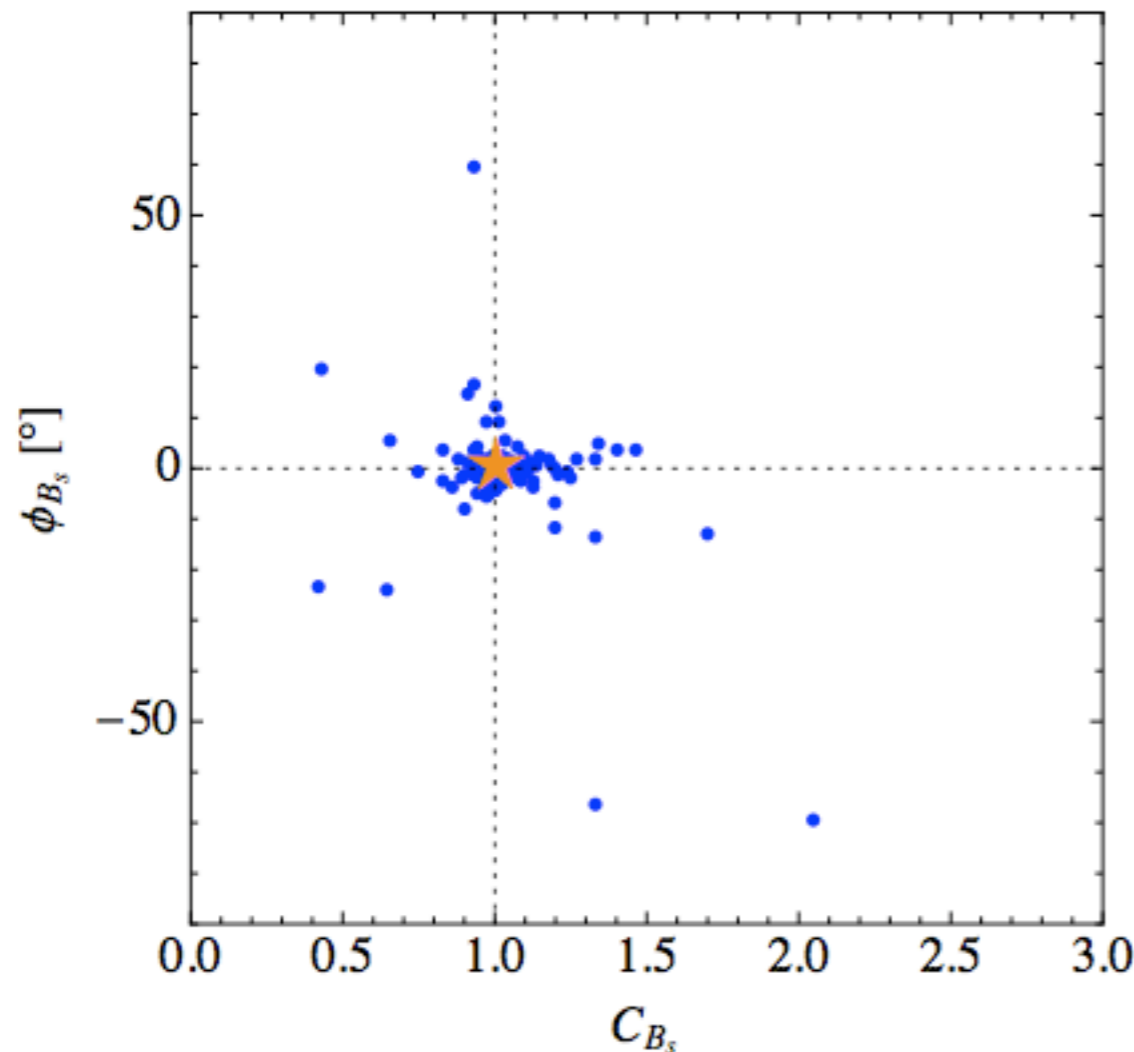
★ SM: $A_{SL}^d \approx -5 \cdot 10^{-4}$, $S_{\psi K_S} \approx 0.69$

■ model-independent prediction

● consistent with quark masses, CKM parameters, and 95% CL limit $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

Meson mixing: Neutral B_s mesons*

- Even after imposing $|\varepsilon_K|$ constraint, sizable effects in magnitude and phase of B_s meson mixing amplitude possible

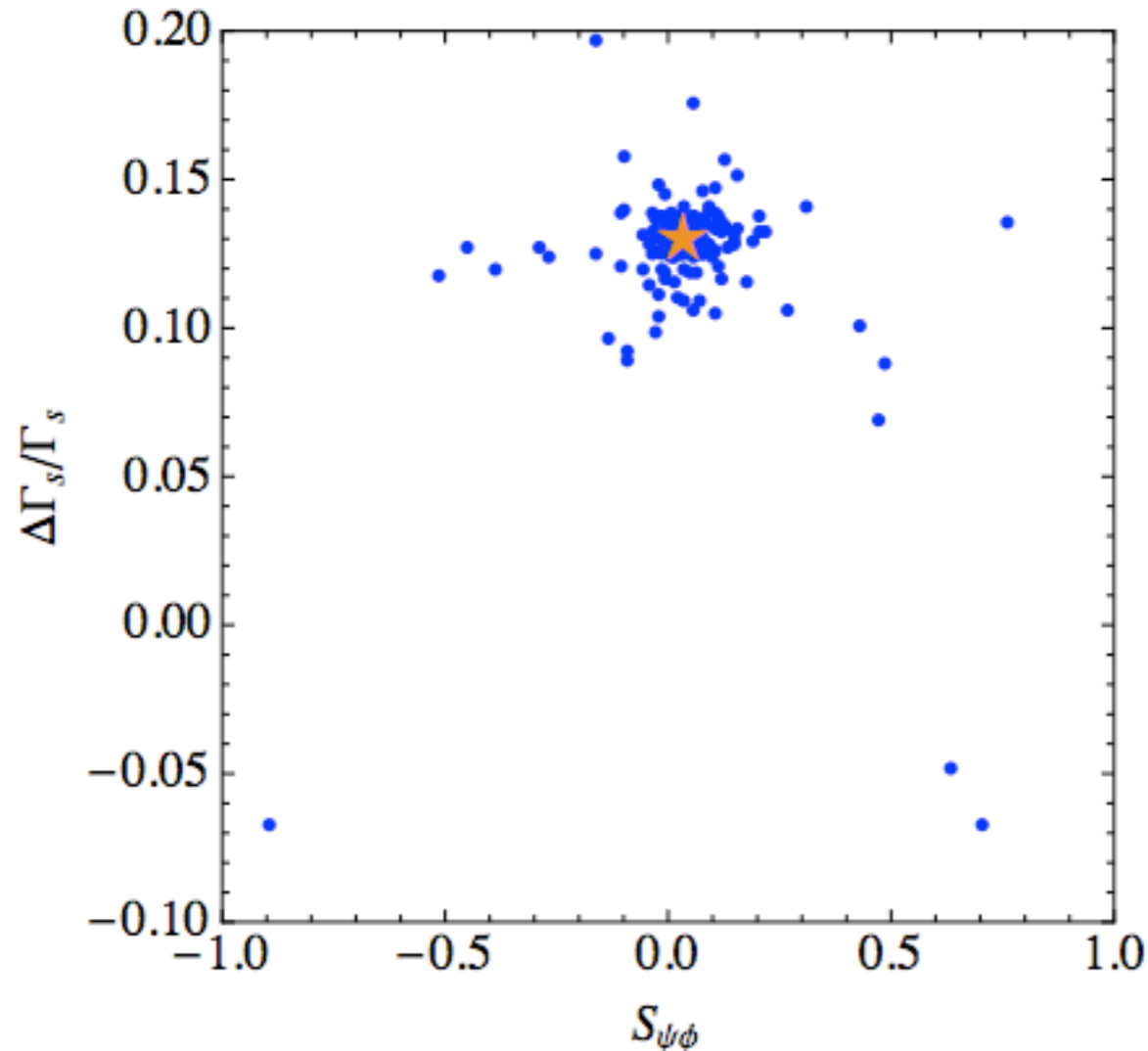


$$C_{B_s} e^{2i\phi_{B_s}} = \frac{\langle B_s | \mathcal{H}_{\text{eff,full}}^{\Delta B=2} | \bar{B}_s \rangle}{\langle B_s | \mathcal{H}_{\text{eff,SM}}^{\Delta B=2} | \bar{B}_s \rangle}$$

- ★ SM: $C_{B_s} = 1, \phi_{B_s} = 0^\circ$
- consistent with quark masses, CKM parameters, and 95% CL limit $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

Meson mixing: Neutral B_s mesons*

- Constraint from $|\varepsilon_K|$ does not exclude order one effects in width difference $\Delta\Gamma_s/\Gamma_s$ of B_s system

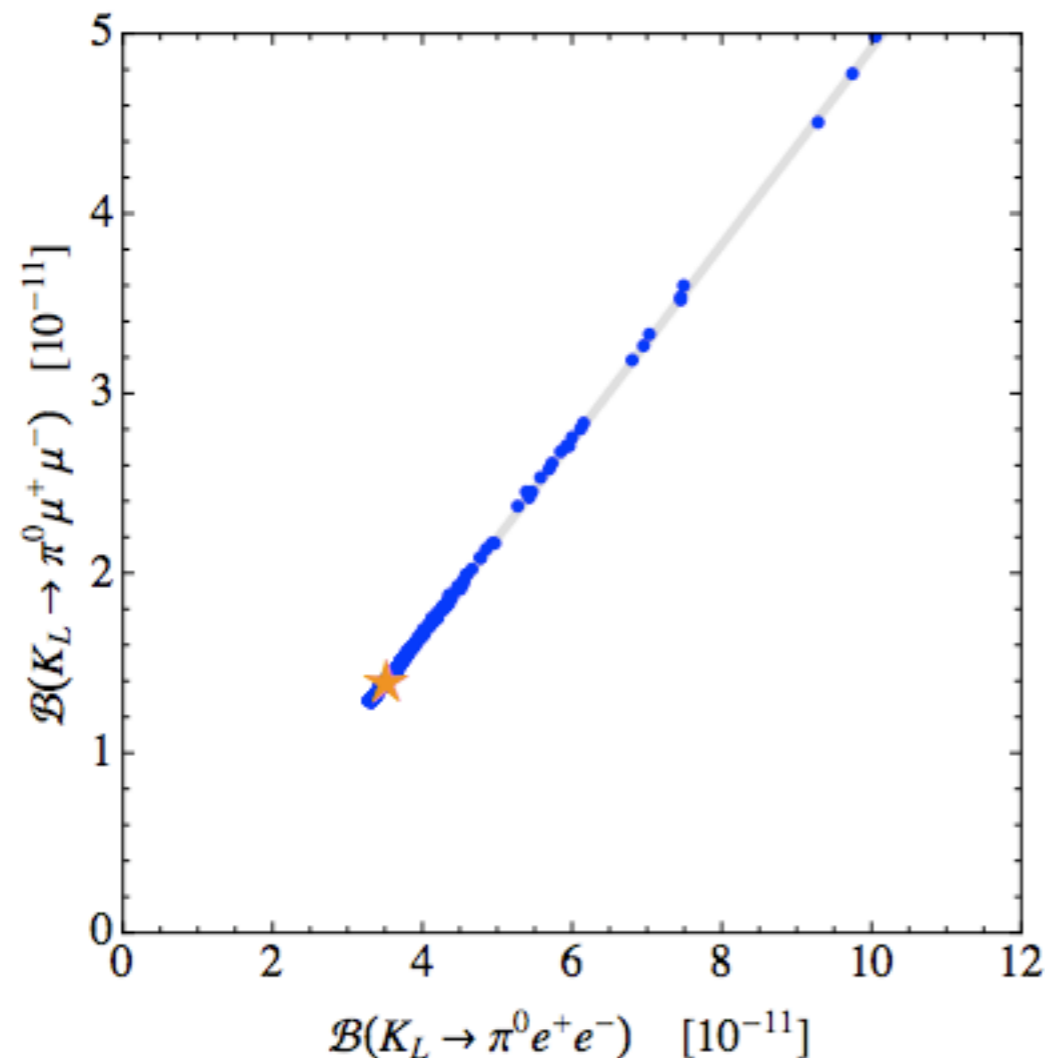


$$\begin{aligned}\Delta\Gamma_s &= \Gamma_L^s - \Gamma_S^s \\ &= 2 |\Gamma_{12}^s| \cos(2|\beta_s| - 2\phi_{B_s})\end{aligned}$$

- ★ SM: $\Delta\Gamma_s/\Gamma_s \approx 0.13$, $S_{\psi\phi} \approx 0.04$
- consistent with quark masses, CKM parameters, and 95% CL limit $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

Rare K decays: Silver modes*

- Order one enhancements possible in $K_L \rightarrow \pi^0 l^+ l^-$ modes. Effects in $e^+ e^-$ and $\mu^+ \mu^-$ channel are strongly correlated due to axial-vector dominance



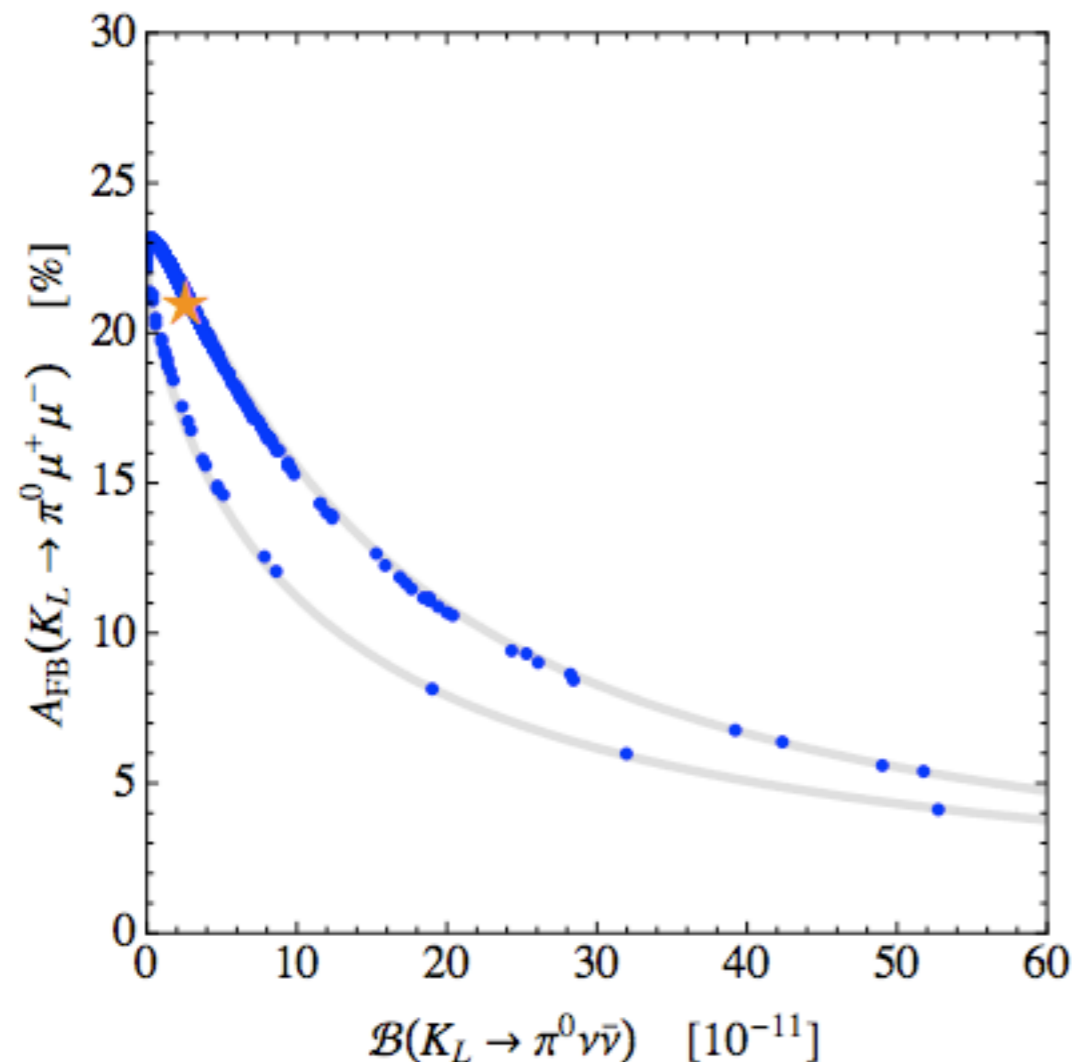
★ SM: $B(K_L \rightarrow \pi^0 e^+ e^-) \approx 3.6 \cdot 10^{-11}$,
 $B(K_L \rightarrow \pi^0 \mu^+ \mu^-) \approx 1.4 \cdot 10^{-11}$
for constructive interference

— model-independent prediction

• consistent with quark masses,
CKM parameters, and 95% CL
limit $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

Rare K decays: Silver modes*

- Deviations from SM expectations in $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 l^+ l^-$ follow specific pattern, arising from smallness of vector and scalar contributions



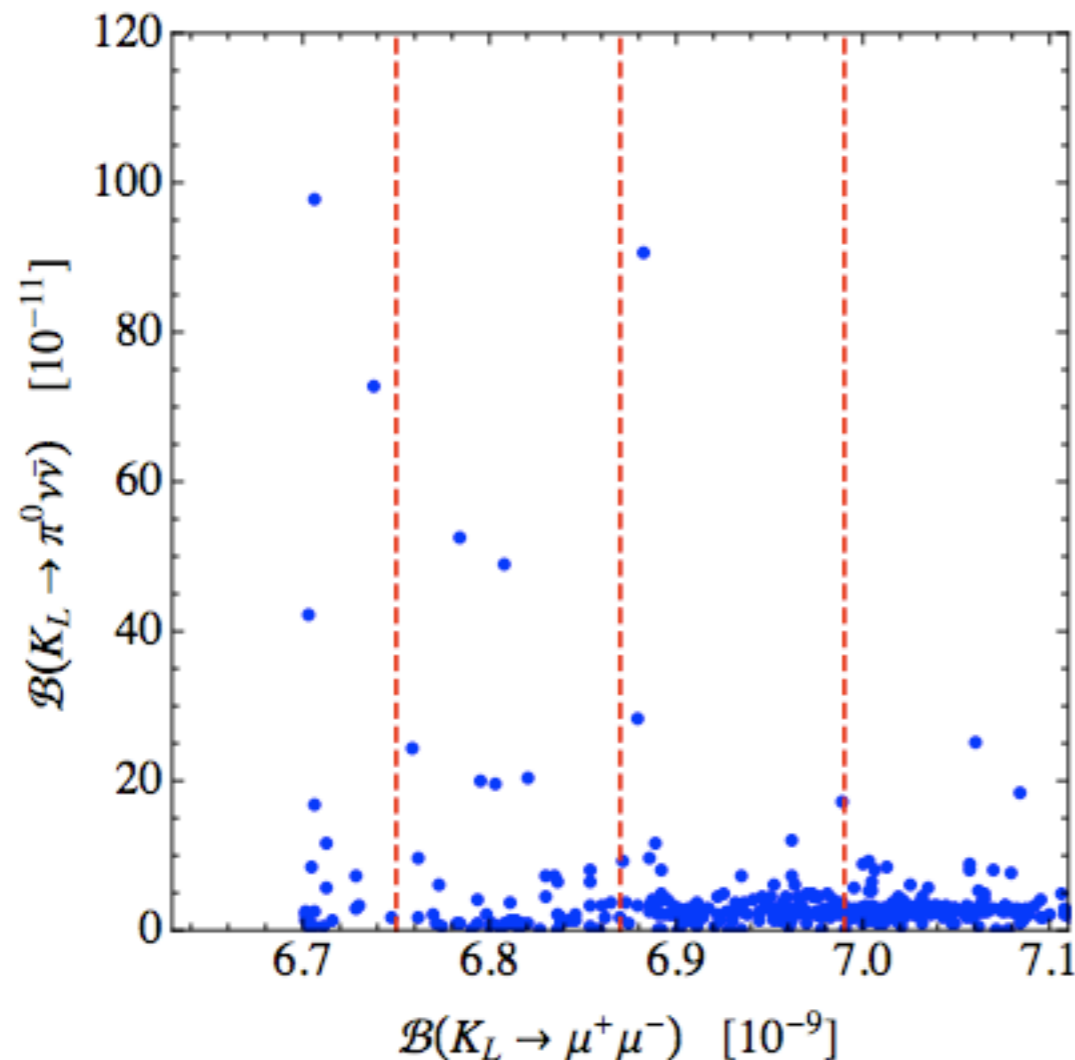
★ SM: $B(K_L \rightarrow \pi^0 \nu \bar{\nu}) \approx 2.7 \cdot 10^{-11}$,
 $A_{\text{FB}}(K_L \rightarrow \pi^0 \mu^+ \mu^-) \approx 21\%$
for constructive interference

— model-independent prediction

● consistent with quark masses,
CKM parameters, and 95% CL
limit $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

Rare K decays: Bronze mode*

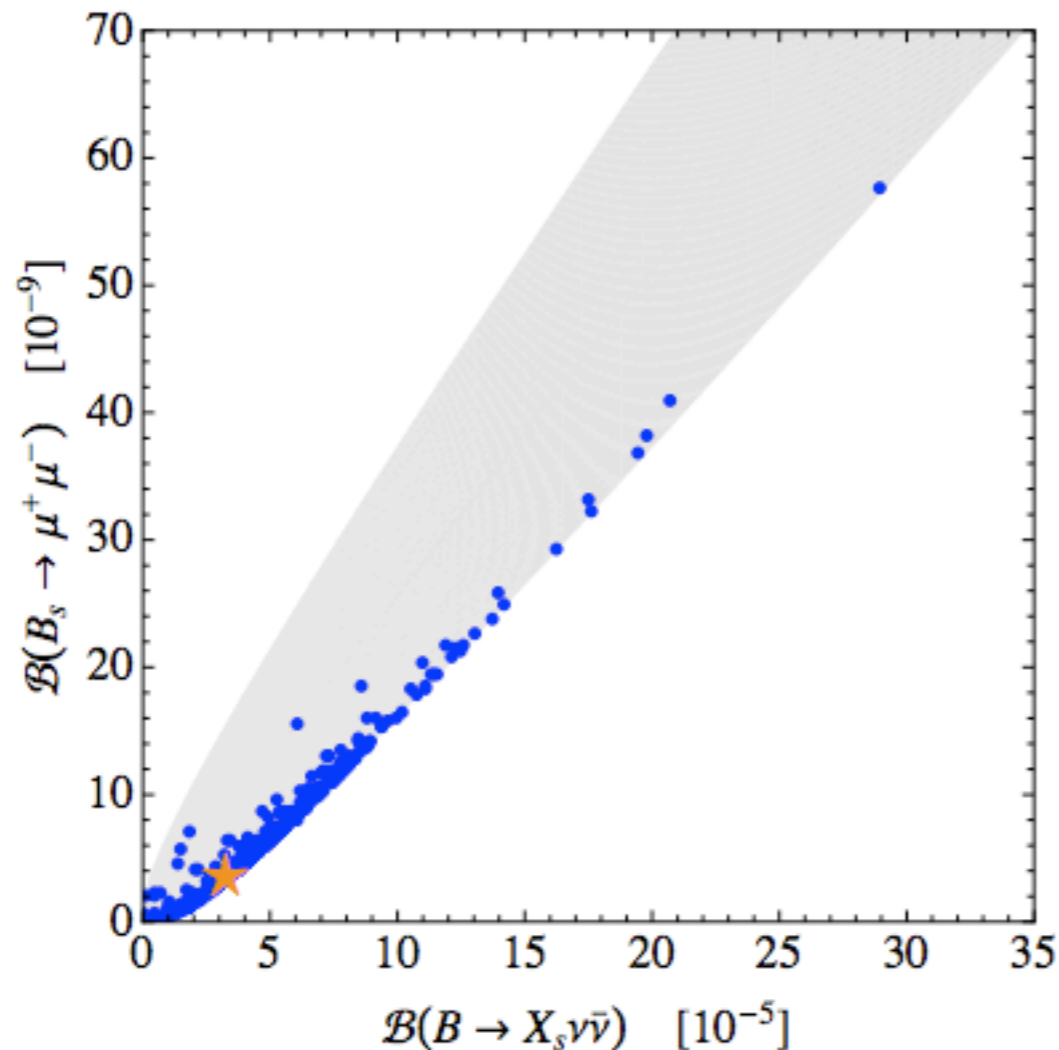
- Better theoretical understanding of precisely measured $K_L \rightarrow \mu^+ \mu^-$ mode could allow to constrain possible enhancement of $K_L \rightarrow \pi^0 \nu \bar{\nu}$



- - PDG central value and 3σ range
 $\mathcal{B}(K_L \rightarrow \mu^+ \mu^-) = (6.87 \pm 0.12) \cdot 10^{-9}$
- consistent with quark masses, CKM parameters, and 95% CL limit $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

Rare B decays: Purely leptonic modes*

- Enhancements in $B_{d,s} \rightarrow \mu^+ \mu^-$ strongly correlated with ones in very rare decays $B \rightarrow X_{d,s} \nu \bar{\nu}$. Pattern again result of axial-vector dominance



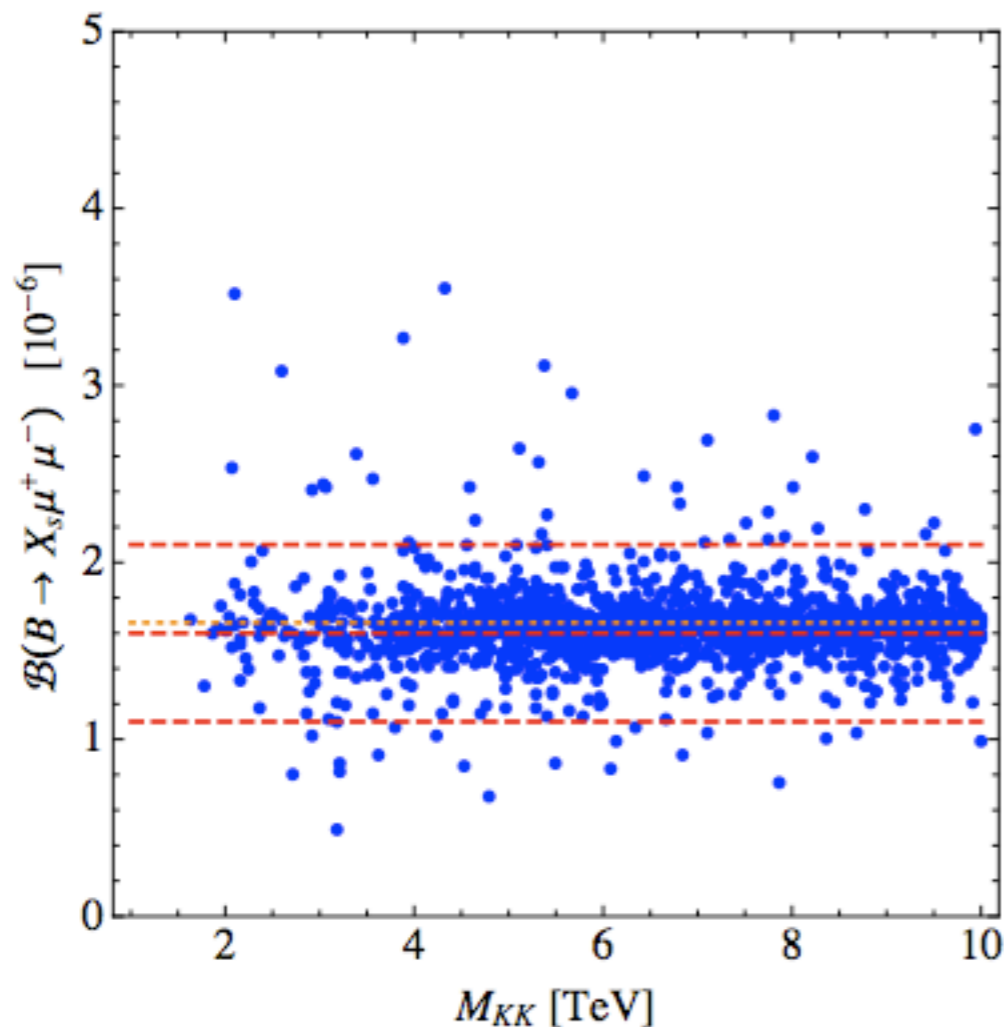
★ SM: $B(B_s \rightarrow \mu^+ \mu^-) \approx 3.9 \cdot 10^{-9}$,
 $B(B \rightarrow X_s \nu \bar{\nu}) \approx 3.5 \cdot 10^{-5}$

■ model-independent prediction

● consistent with quark masses,
CKM parameters, and 95% CL
limit of $Z \rightarrow b \bar{b}$

Rare B decays: Inclusive semileptonic modes*

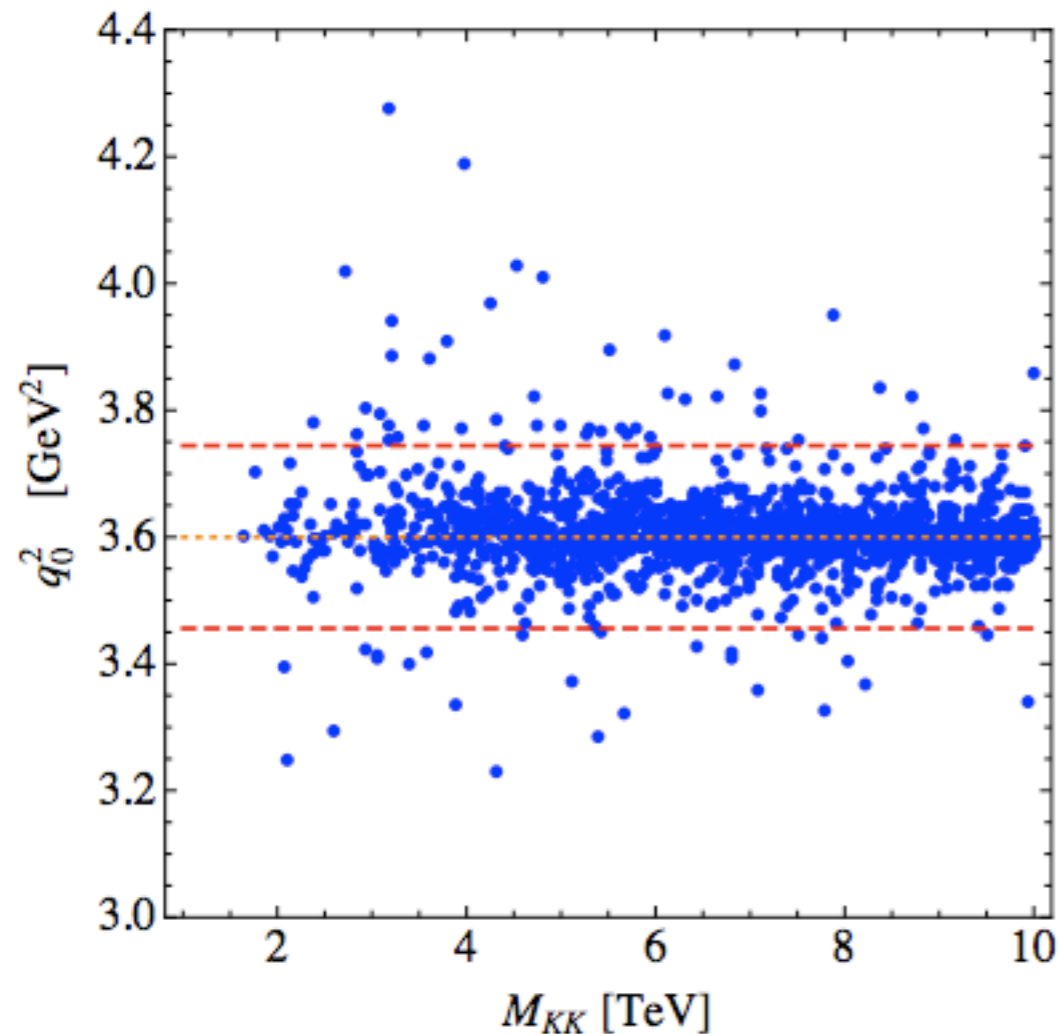
- Once $Z \rightarrow b\bar{b}$ constraint is satisfied, values for $B \rightarrow X_s \mu^+ \mu^-$ branching ratio arising from Z and $Z^{(k)}$ exchange are typically within experimental limits



- SM: $\mathcal{B}(B \rightarrow X_s \mu^+ \mu^-) \approx 1.7 \cdot 10^{-6}$
for $q^2 \in [1, 6] \text{ GeV}^2$
- - - central value and 68% CL limit
 $\mathcal{B}(B \rightarrow X_s \mu^+ \mu^-) = (1.6 \pm 0.5) \cdot 10^{-6}$
from BaBar and Belle
- • consistent with quark masses,
CKM parameters, and 95% CL
limit of $Z \rightarrow b\bar{b}$

Rare B decays: Inclusive semileptonic modes*

- Deviations of zero of forward-backward asymmetry, q_0^2 , in $B \rightarrow X_s \mu^+ \mu^-$ from SM prediction might be observable at high-luminosity flavor factory



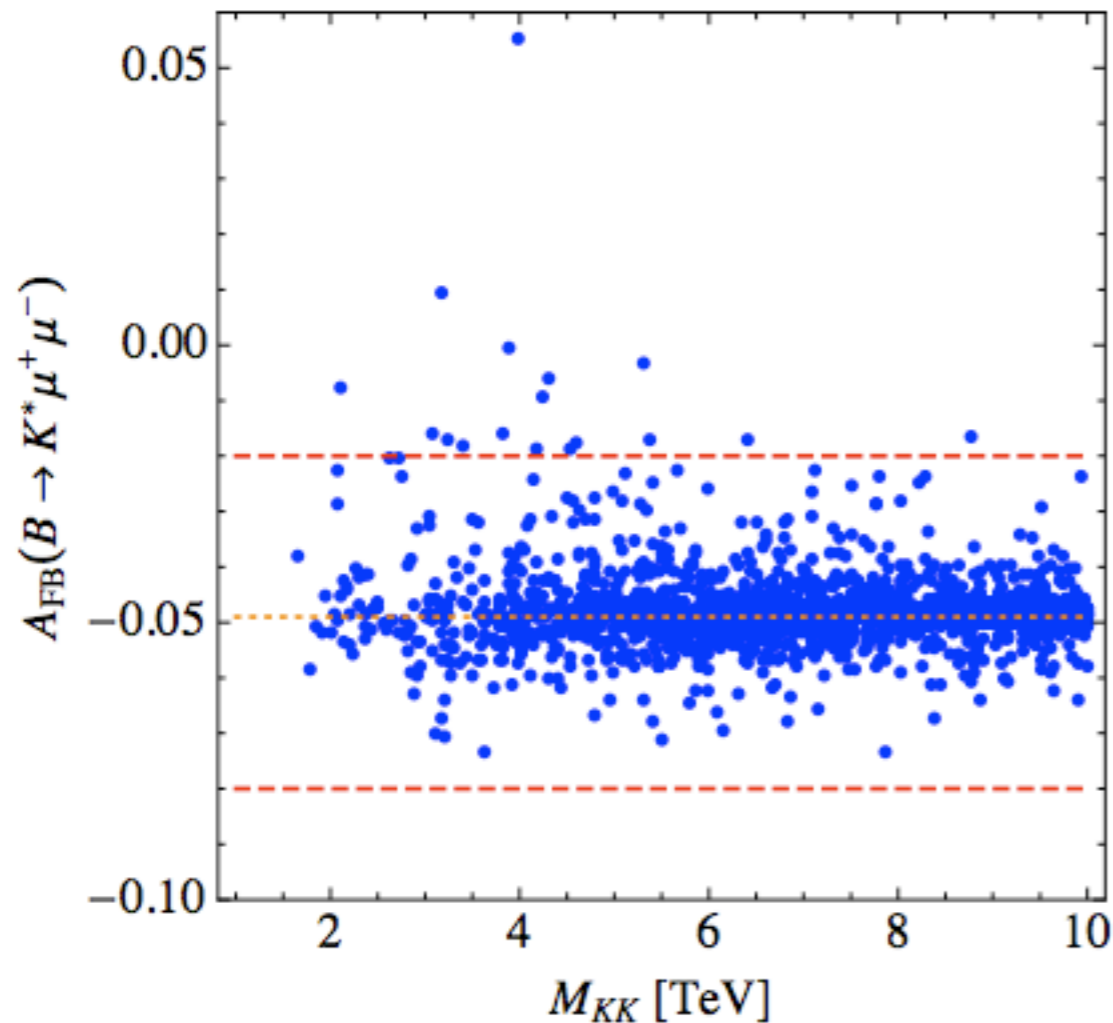
..... SM: $q_0^2 \approx 3.6 \text{ GeV}^2$

- - - expected sensitivity at SuperB
factory, 75 ab^{-1}

• consistent with quark masses,
CKM parameters, and 95% CL
limit of $Z \rightarrow b\bar{b}$

Rare B decays: Exclusive semileptonic modes*

- Corrections to $A_{\text{FB}}(B \rightarrow K^* \mu^+ \mu^-)$ on average below LHCb sensitivity. Other angular distributions such as $A_{\text{T}}^{(3)}(B \rightarrow K^* \mu^+ \mu^-)$ might offer better prospects



- SM: $A_{\text{FB}}(B \rightarrow K^* \mu^+ \mu^-) \approx -0.05$ for $q^2 \in [1, 6] \text{ GeV}^2$
- - - expected sensitivity of LHCb, 2 fb^{-1}
- • consistent with quark masses, CKM parameters, and 95% CL limit of $Z \rightarrow b\bar{b}$

Physical parameters in quark sector*

Flavor is violated by:

▶ bulk parameters c_Q, c_u, c_d - 3×3 hermitian matrices	3×6 real parameters 3×3 complex phases
▶ Yukawa couplings Y_u, Y_d - 3×3 complex matrices	2×9 real parameters 2×9 complex phases
<hr/>	
	36 real parameters 27 complex phases
▶ global $U(3)^3$ flavor symmetry can be used to remove	9 real parameters $18 - 1_B = 17$ complex phases

Physical parameters: $6_m + 12_\alpha + 9_c = 27$ moduli and $1_{\text{CKM}} + 9_\phi = 10$ phases

Warped-space Froggatt-Nielsen mechanism*

Bulk fermions in RS:

$$(Y_q^{\text{eff,RS}})_{ij} \propto (Y_q)_{ij} e^{-kr\pi(c_{Q_i} - c_{q_j})}$$

- ▶ self-similarity along ϕ
- ▶ bulk parameter c_{Q_i, q_i}
- ▶ IR brane at $\phi = \pi$
- ▶ warp factor $e^{-2kr\pi}$

Froggatt-Nielsen (FN) symmetry:

$$(Y_q^{\text{eff,FN}})_{ij} \propto (Y_q)_{ij} e^{(a_i - b_{q_j})}$$

- ▶ $U(1)_F$ symmetry
- ▶ $U(1)_F$ charges $Q_F = a_i, b_{q_i}$
- ▶ $\langle \phi \rangle \neq 0$ of scalar ϕ , $Q_F = 1$
- ▶ $\varepsilon = \langle h \rangle / \langle \phi \rangle \ll 1$

- Models with warped spatial extra dimension provide compelling geometrical interpretation of flavor symmetry

Reparametrization invariance*

- Expressions for quark masses and mixing matrices are invariant under two reparametrizations RPI-1 and RPI-2

RPI-1:

$$F_{c_Q} \rightarrow e^{-\xi} F_{c_Q},$$

$$F_{c_q} \rightarrow e^{+\xi} F_{c_q},$$

$$\left[c_Q \rightarrow c_Q - \frac{\xi}{L} \right],$$

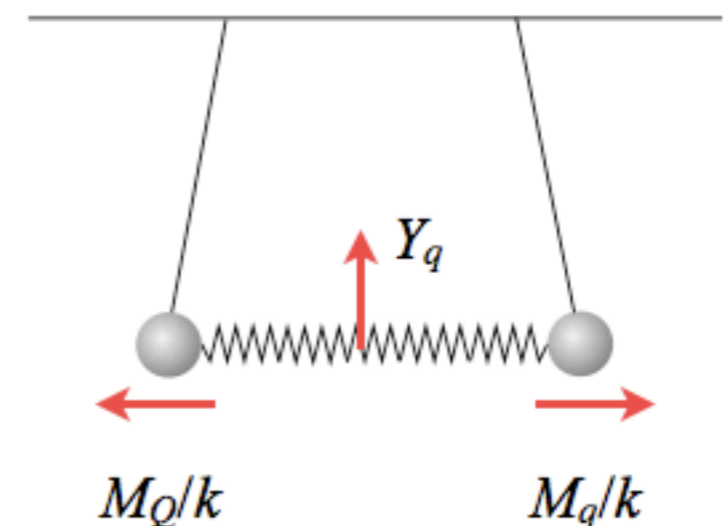
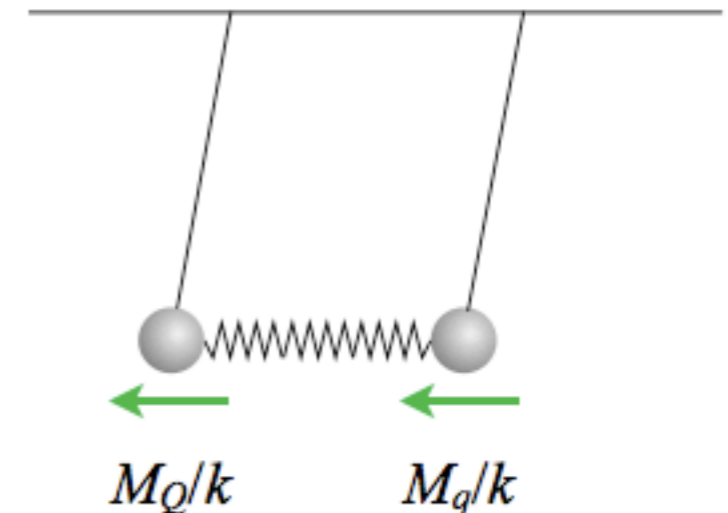
$$\left[c_q \rightarrow c_q + \frac{\xi}{L} \right]$$

RPI-2:

$$F_{c_A} \rightarrow \zeta F_{c_A},$$

$$Y_q \rightarrow \frac{1}{\zeta^2} Y_q$$

$$\left[c_A \rightarrow c_A - \frac{\ln \zeta}{L} \right],$$



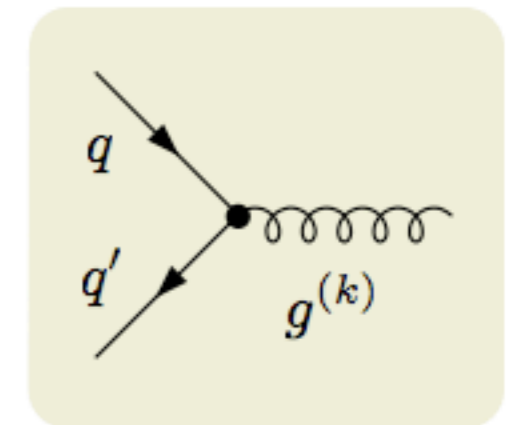
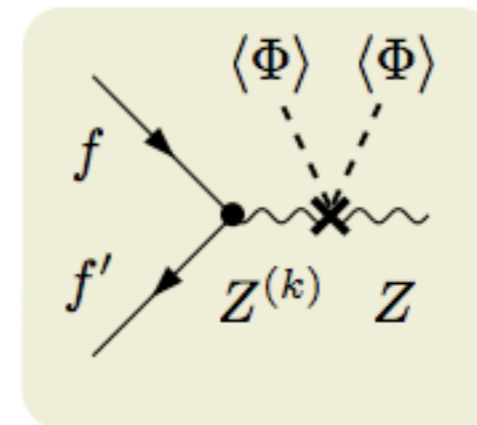
Mixing matrices: Gauge and KK boson effects

$$(\Delta_Q)_{ij} \rightarrow \left(U_q^\dagger \text{diag} \left[\frac{F_{c_{Q_i}}^2}{3 + 2c_{Q_i}} \right] U_q \right)_{ij}, \quad (\Delta_q)_{ij}, (\Delta'_q)_{ij}: Q_i \rightarrow q_i, U_q \rightarrow W_q,$$

$$(\Delta'_Q)_{ij} \rightarrow \left(U_q^\dagger \text{diag} \left[\frac{5 + 2c_{Q_i}}{2(3 + 2c_{Q_i})^2} F_{c_{Q_i}}^2 \right] U_q \right)_{ij}, \quad V_{\text{CKM}} \rightarrow U_u^\dagger U_d$$

Effects due to gauge-boson profiles*:

- ▶ parameterized by four mixing matrices Δ_A, Δ'_A build out of $F_{c_{A_i}}$ and left- and right-handed rotations U_q and W_q
- ▶ Δ_A contributions are enhanced with respect to Δ'_A corrections by logarithm L of warp factor



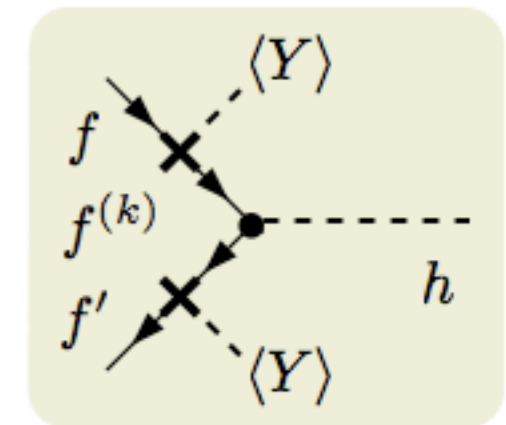
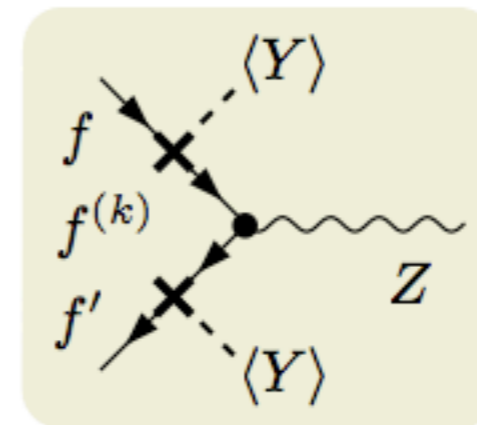
Mixing matrices: Fermion mixing

$$(\delta_Q)_{ij} \rightarrow \left(\mathbf{x}_q \mathbf{W}_q^\dagger \text{diag} \left[\frac{1}{1 - 2c_{q_i}} \left(\frac{1}{F_{c_{q_i}}^2} - 1 + \frac{F_{c_{q_i}}^2}{3 + 2c_{q_i}} \right) \right] \mathbf{W}_q \mathbf{x}_q \right)_{ij},$$

$$(\delta_q)_{ij} : c_{q_i} \rightarrow c_{Q_i}, \quad \mathbf{W}_q \rightarrow \mathbf{U}_q, \quad \mathbf{x}_q \equiv \frac{\mathbf{m}_q}{M_{\text{KK}}} = \frac{\text{diag}(m_{q_1}, m_{q_2}, m_{q_3})}{M_{\text{KK}}}$$

Effects due to fermion mixing*:

- ▶ mixing matrices δ_A are parametrically of same order as Δ_A, Δ'_A as they are not suppressed by v^2/M_{KK}^2 in Feynman rules
- ▶ fermion mixing is only source of flavor-breaking in Higgs-boson couplings that are proportional to $\mathbf{m}_q/v \delta_q + \delta_Q \mathbf{m}_q/v$



Mixing matrices: Scaling relations

$$(U_q)_{ij} \sim (V_{\text{CKM}})_{ij} \sim \begin{cases} \frac{F_{c_{Q_i}}}{F_{c_{Q_j}}}, & i \leq j, \\ \frac{F_{c_{Q_j}}}{F_{c_{Q_i}}}, & i > j, \end{cases} \quad (W_q)_{ij} \sim \begin{cases} \frac{F_{c_{q_i}}}{F_{c_{q_j}}}, & i \leq j, \\ \frac{F_{c_{q_j}}}{F_{c_{q_i}}}, & i > j, \end{cases}$$

$$(\Delta_Q^{(l)})_{ij} \sim F_{c_{Q_i}} F_{c_{Q_j}}, \quad (\delta_Q)_{ij} \sim \frac{m_{q_i} m_{q_j}}{M_{\text{KK}}^2} \frac{1}{F_{c_{q_i}} F_{c_{q_j}}} \sim \frac{v^2 Y_q^2}{M_{\text{KK}}^2} F_{c_{q_i}} F_{c_{q_j}},$$

$$(\Delta_q^{(l)})_{ij} \sim F_{c_{q_i}} F_{c_{q_j}}, \quad (\delta_q)_{ij} \sim \frac{m_{q_i} m_{q_j}}{M_{\text{KK}}^2} \frac{1}{F_{c_{Q_i}} F_{c_{Q_j}}} \sim \frac{v^2 Y_q^2}{M_{\text{KK}}^2} F_{c_{Q_i}} F_{c_{Q_j}}$$

- $F_{c_{A_i}} F_{c_{A_j}}$ factors present in expressions for Δ_A , Δ'_A , and δ_A mixing matrices makes RS-GIM suppression explicit

Mixing matrices: Transformation properties

RPI-1:

$$\begin{aligned}\Delta_Q &\rightarrow e^{-2\xi} \Delta_Q, & \Delta_q &\rightarrow e^{+2\xi} \Delta_q, \\ \delta_Q &\rightarrow e^{+2\xi} \delta_Q, & \delta_q &\rightarrow e^{-2\xi} \delta_q,\end{aligned}$$

RPI-2:

$$\begin{aligned}\Delta_Q &\rightarrow \zeta^2 \Delta_Q, & \Delta_q &\rightarrow \zeta^2 \Delta_q, \\ \delta_Q &\rightarrow \frac{1}{\zeta^2} \delta_Q, & \delta_q &\rightarrow \frac{1}{\zeta^2} \delta_q,\end{aligned}$$

Reparametrization transformations imply*:

- ▶ relative importance of left- and right-handed couplings, $\Delta_Q, \delta_Q \leftrightarrow \Delta_q, \delta_q$, as well as contributions due to non-trivial gauge-boson profiles and fermion mixing, $\Delta_{Q,q} \leftrightarrow \delta_{Q,q}$, can be reshuffled
- ▶ but it is not possible to make all contributions simultaneously small