

# Top quark production at Tevatron and LHC

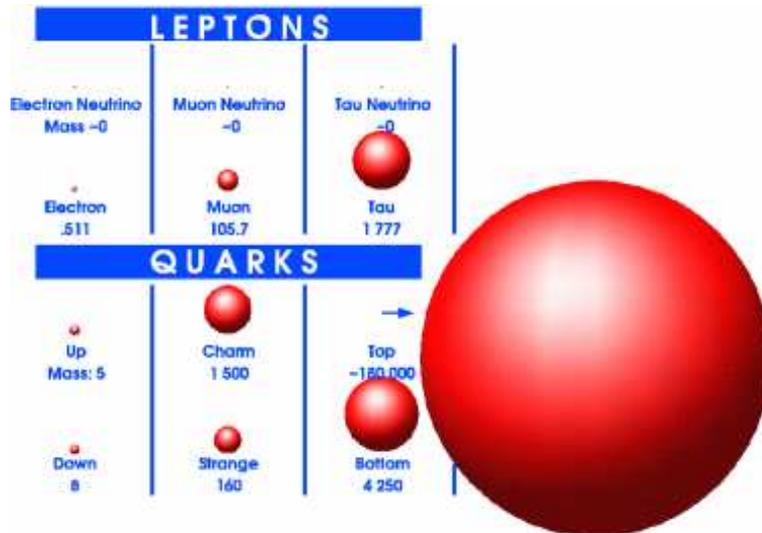
**Sven-Olaf Moch**

Sven-Olaf.Moch@desy.de

DESY, Zeuthen

# Plan

- Some new results on the heaviest elementary particle



- Report based on recent work done in collaboration with
  - P. Uwer on [arXiv:0804.1476](https://arxiv.org/abs/0804.1476) and on [arXiv:0807.2794](https://arxiv.org/abs/0807.2794)
  - Y. Kiyo, J.H. Kühn, M. Steinhauser and P. Uwer on [arXiv:0812.0919](https://arxiv.org/abs/0812.0919)
  - U. Langenfeld on [arXiv:0901.0802](https://arxiv.org/abs/0901.0802)
  - U. Langenfeld and P. Uwer to appear

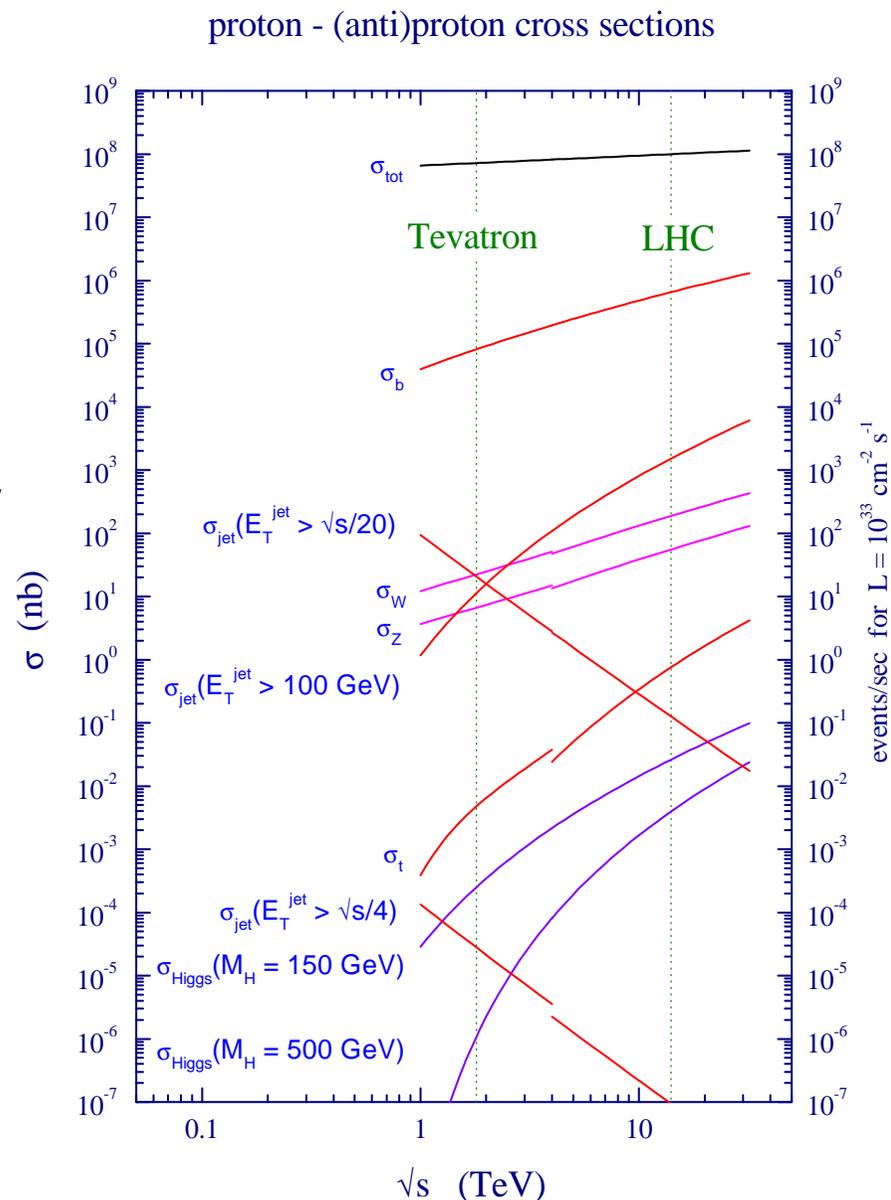
# Proton colliders

- Tevatron: energy frontier at  $\sqrt{S} = 1.96\text{TeV}$   
top quark discovery
- LHC: in commissioning phase  
Higgs boson search at highest energies:  $\sqrt{S} = 14\text{TeV}$



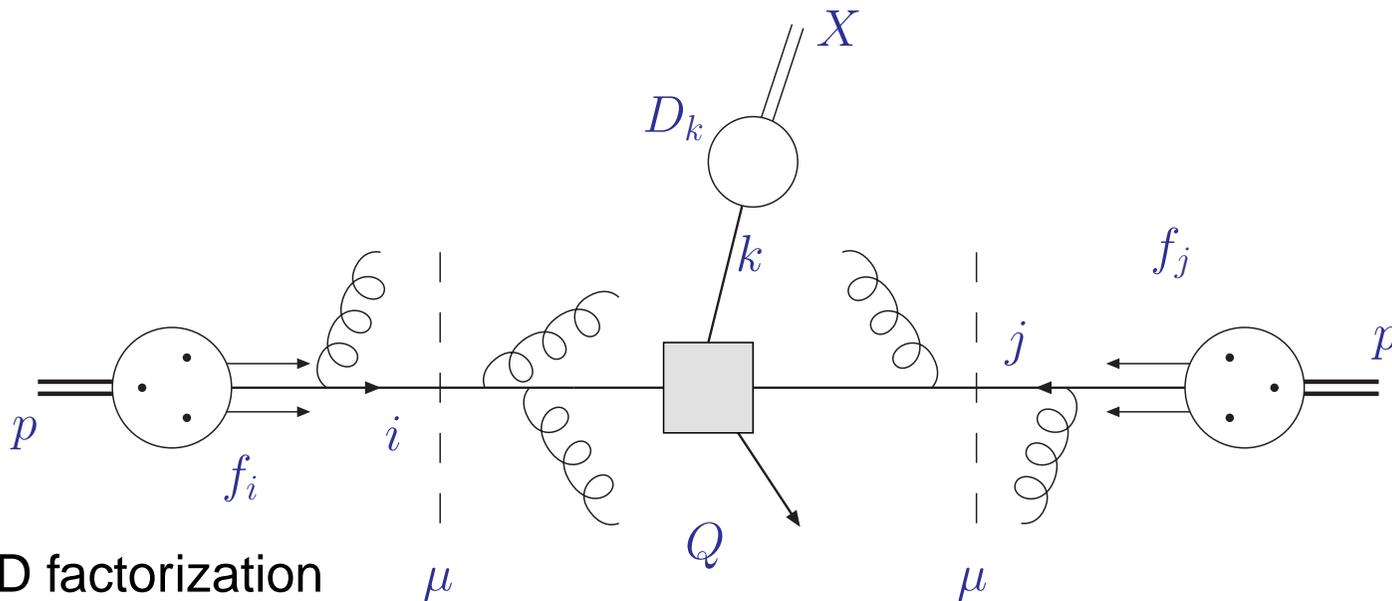
# Top quarks at proton colliders

- Top quark discovery at Tevatron
  - a lot of integrated luminosity for analyses
  - $t\bar{t}$ -pairs and single top
- LHC will accumulate very high statistics for  $t\bar{t}$ -pairs
  - low luminosity run:  $8 \cdot 10^6$  events/year (high luminosity run: 10 times more)
  - mass measurement  
 $\Delta m_t = \mathcal{O}(1)\text{GeV}$   
 (constraints on Standard Model Higgs mass  $m_h$ )
  - top-spin correlations
  - search for anomalous couplings
- Tops make up large part of background in Higgs or new physics searches



# Perturbative QCD at colliders

- Hard hadron-hadron scattering
  - constituent partons from each incoming hadron interact at short



- QCD factorization
  - separate sensitivity to dynamics from different scales

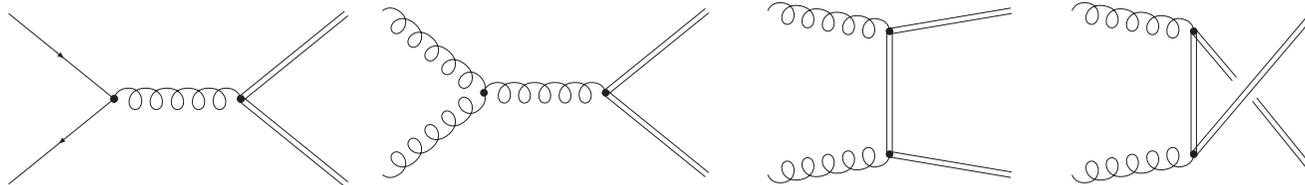
$$\sigma_{pp \rightarrow X} = \sum_{ijk} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij \rightarrow k}(\alpha_s(\mu^2), Q^2, \mu^2) \otimes D_{k \rightarrow X}(\mu^2)$$

- factorization scale  $\mu$ , subprocess cross section  $\hat{\sigma}_{ij \rightarrow k}$  for parton types  $i, j$  and hadronic final state  $X$

# Top quark production

- Leading order Feynman diagrams

$$q + \bar{q} \longrightarrow Q + \bar{Q}$$
$$g + g \longrightarrow Q + \bar{Q}$$



- NLO in QCD [Nason, Dawson, Ellis '88](#); [Beenakker, Smith, van Neerven '89](#); [Mangano, Nason, Ridolfi '92](#); [Bernreuther, Brandenburg, Si, Uwer '04](#); [Mitov, Czakon '08](#); ...
  - accurate to  $\mathcal{O}(15\%)$  at LHC
- Much activity towards higher orders in QCD
  - one-loop squared terms (NLO  $\times$  NLO) [Anastasiou, Mert Aybat '08](#); [Kniehl, Merebashvili, Körner, Rogal '08](#)
  - analytic two-loop fermionic corrections for  $q\bar{q} \rightarrow t\bar{t}$  [Bonciani, Ferroglia, Gehrmann, Maitre, Studerus '08](#)
  - numerical result for two-loop virtual  $q\bar{q} \rightarrow t\bar{t}$  [Czakon '08](#)

# Strategy

- First steps towards higher orders in QCD: explore limits

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## Small mass limit

- Study of massive QCD amplitudes in limit  $m \rightarrow 0$ 
  - look at soft and collinear limits
  - exploit relation of massive to massless amplitudes
- Two-loop virtual corrections to  $q\bar{q} \rightarrow t\bar{t}$  and  $gg \rightarrow t\bar{t}$  in small-mass limit  $m^2 \ll s, t, u$  S.M., Czakon, Mitov '07

## Threshold resummation

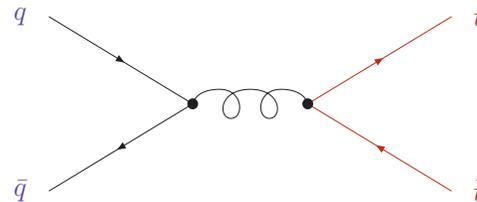
- Partonic threshold  $s \simeq 4m^2$ 
  - Sudakov-type logarithms  $\ln \beta$  with velocity of heavy quark  $\beta = \sqrt{1 - 4m^2/s}$
  - long history of resummation Kidonakis, Sterman '97; Bonciani, Catani, Mangano, Nason '98; Kidonakis, Laenen, S.M., Vogt '01; ...



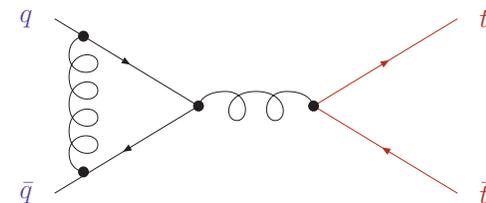
# Sudakov logarithms

- Intuitive aspects of higher order corrections

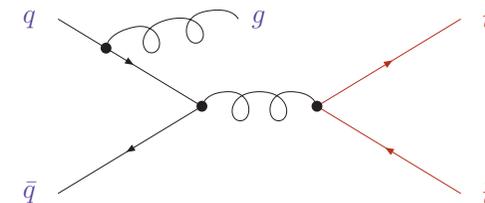
- lowest order, elastic



- first order correction  
*virtual* < 0 (elastic)



- first order correction  
*Brems* > 0 (inelastic)



- at threshold for  $t\bar{t}$ -creation

- strong Sudakov-suppression inelastic tendency

$$\sigma \sim \exp[-\alpha_s \ln^2(1 - 4m_t^2/s)]$$

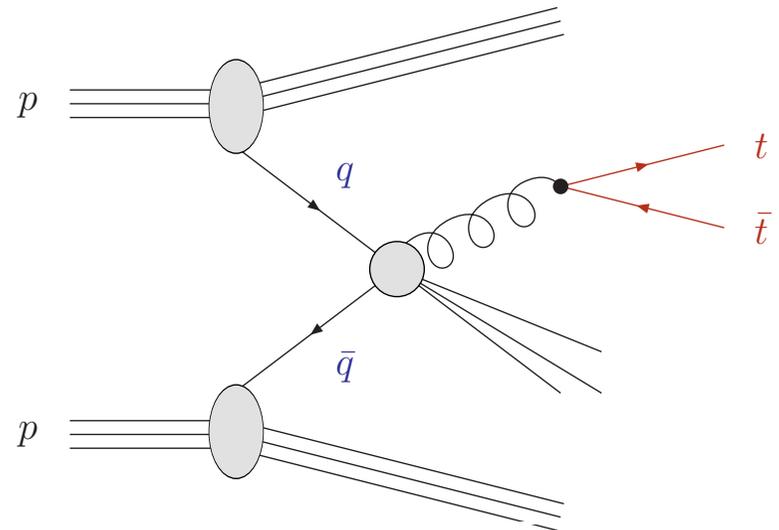
- universal factor for parton splittings (leading log accuracy)  
modelling of MC parton showers

- Hadronic reaction  $p\bar{p}$ :

- recall master equation

$$\sigma_{pp \rightarrow t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij \rightarrow t\bar{t}}$$

- initial partons: also Sudakov-suppressed



- Parton cross section  $\hat{\sigma}_{ij \rightarrow t\bar{t}}$

- Sudakov-enhancement after mass factorization

$$\hat{\sigma}_{ij \rightarrow t\bar{t}} = \frac{\sigma_{pp \rightarrow t\bar{t}}}{f_i \otimes f_j} = \frac{e^{-\alpha_s \ln^2(\dots)}}{\left(e^{-\alpha_s \ln^2(\dots)}\right)^2} = e^{+\alpha_s \ln^2(\dots)}$$

- large double logarithms

# Threshold resummation

- Threshold at  $s \simeq 4m_t^2$ 
  - parton cross section exhibit Sudakov-type logarithms  $\ln(\beta)$  with velocity of heavy quark  $\beta = \sqrt{1 - 4m_t^2/s}$  at  $n^{\text{th}}$ -order
- All order resummation of large logarithms  $\alpha_s^n \ln^{2n}(\beta) \longleftrightarrow \alpha_s^n \ln^{2n}(N)$ 
  - resummation in Mellin space (renormalization group equation)
- Resummed cross section in Mellin space

$$\frac{\hat{\sigma}_{ij,I}^N(m^2)}{\hat{\sigma}_{ij,I}^{(0),N}(m^2)} = g_{ij,I}^0(m^2) \cdot \exp\left(G_{ij,I}^{N+1}(m^2)\right) + \mathcal{O}(N^{-1} \ln^n N)$$

- exponent in singlet-octet color basis decomposition  $I = 1, 8$

$$G_{q\bar{q}/gg,I}^N = G_{\text{DY}/\text{Higgs}}^N + \delta_{I,8} G_{Q\bar{Q}}^N$$

- Renormalization group equations for functions  $G_{\text{DY}/\text{Higgs}}^N$  and  $G_{Q\bar{Q}}^N$ 
  - well-known exponentiation from factorization in soft/collinear limit

# The radiative factors

- Production of color singlet final state from parton-parton scattering described by  $G_{\text{DY}/\text{Higgs}}^N$

$$G_{\text{DY}/\text{Higgs}}^N =$$

$$\int_0^1 dz \frac{z^{N-1} - 1}{1-z} \int_{\mu_f^2}^{4m^2(1-z)^2} \frac{dq^2}{q^2} 2 A_i(\alpha_s(q^2)) + D_i(\alpha_s(4m^2[1-z]^2))$$

- well known anomalous dimensions  $A_i$  (collinear gluon emission) and  $D_i$  (process dependent gluon emission at large angles)  
Vogt '00; Catani, Grazzini, de Florian, Nason '03; S.M., Vermaseren, Vogt '05

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- $G_{Q\bar{Q}}^N$  accounts for gluon emission from octet final state

$$G_{Q\bar{Q}}^N = \int_0^1 dz \frac{z^{N-1} - 1}{1-z} D_{Q\bar{Q}}(\alpha_s([1-z]^2 4m^2))$$

- anomalous dimension  $D_{Q\bar{Q}}$  (cf. pole of form factor for massive quarks)

$$D_{Q\bar{Q}}^{(1)} = -A_g^{(1)}, \quad D_{Q\bar{Q}}^{(2)} = -A_g^{(2)}$$

$D_{Q\bar{Q}}^{(2)}$  consistent with Mitov, Sterman, Sung '09; Becher, Neubert '09

# Accuracy under control

- Control over logarithms  $\ln(N)$  with  $\lambda = \beta_0 \alpha_s \ln(N)$  to  $N^k$  LL accuracy

$$G_{ij, I}^N = \ln N \cdot g_{ij}^1(\lambda) + g_{ij, I}^2(\lambda) + \alpha_s g_{ij, I}^3(\lambda) + \dots$$

- $g^1(\lambda)$ : LL

Laenen, Smith, v.Neerven '92; Berger, Contopanagos '95; Catani, Mangano, Nason, Trentadue '96

- $g^2(\lambda)$ : NLL

Bonciani, Catani, Mangano, Nason '98; Kidonakis, Laenen, S.M., Vogt '01

- $g^3(\lambda)$ : NNLL

S.M., Uwer '08

- Resummed  $G^N$  predicts fixed orders in perturbation theory
  - generating functional for towers of large logarithms

# New results

- NNLO cross section for heavy-quark hadro-production near threshold (all powers of  $\ln \beta$  and Coulomb corrections) S.M., Uwer '08; Langenfeld, S.M., Uwer to appear
  - e.g.  $gg$ -fusion for  $n_f = 5$  light flavors at  $\mu = m_t$

$$\hat{\sigma}_{gg \rightarrow t\bar{t}}^{(1)} = \hat{\sigma}_{gg \rightarrow t\bar{t}}^{(0)} \left\{ 96 \ln^2 \beta - 9.5165 \ln \beta + 35.322 + 5.1698 \frac{1}{\beta} \right\}$$

$$\hat{\sigma}_{gg \rightarrow t\bar{t}}^{(2)} = \hat{\sigma}_{gg \rightarrow t\bar{t}}^{(0)} \left\{ 4608 \ln^4 \beta - 1894.9 \ln^3 \beta + \left( -912.35 + 496.30 \frac{1}{\beta} \right) \ln^2 \beta + \left( 3031.1 + 321.14 \frac{1}{\beta} \right) \ln \beta + 68.547 \frac{1}{\beta^2} - 196.93 \frac{1}{\beta} + C_{gg}^{(2)} \right\}$$

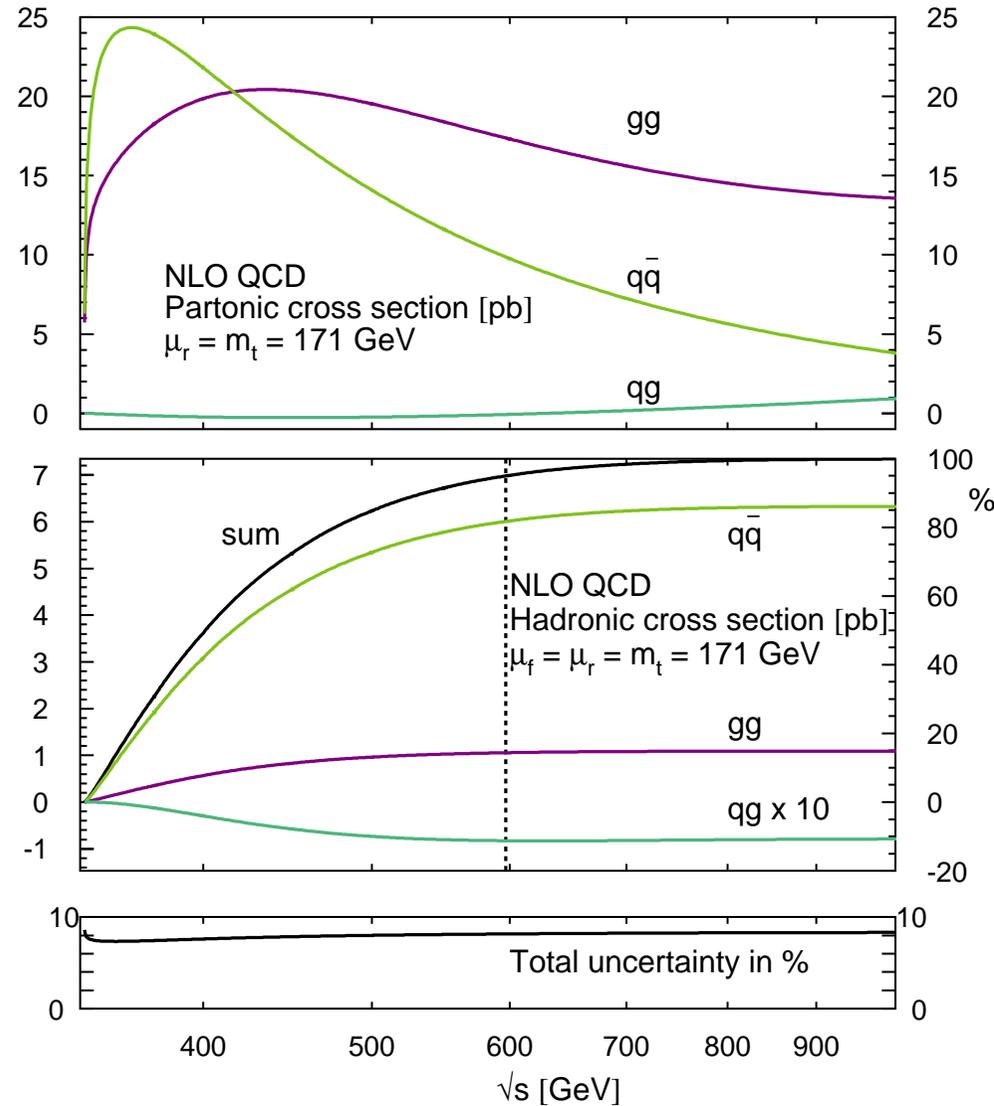
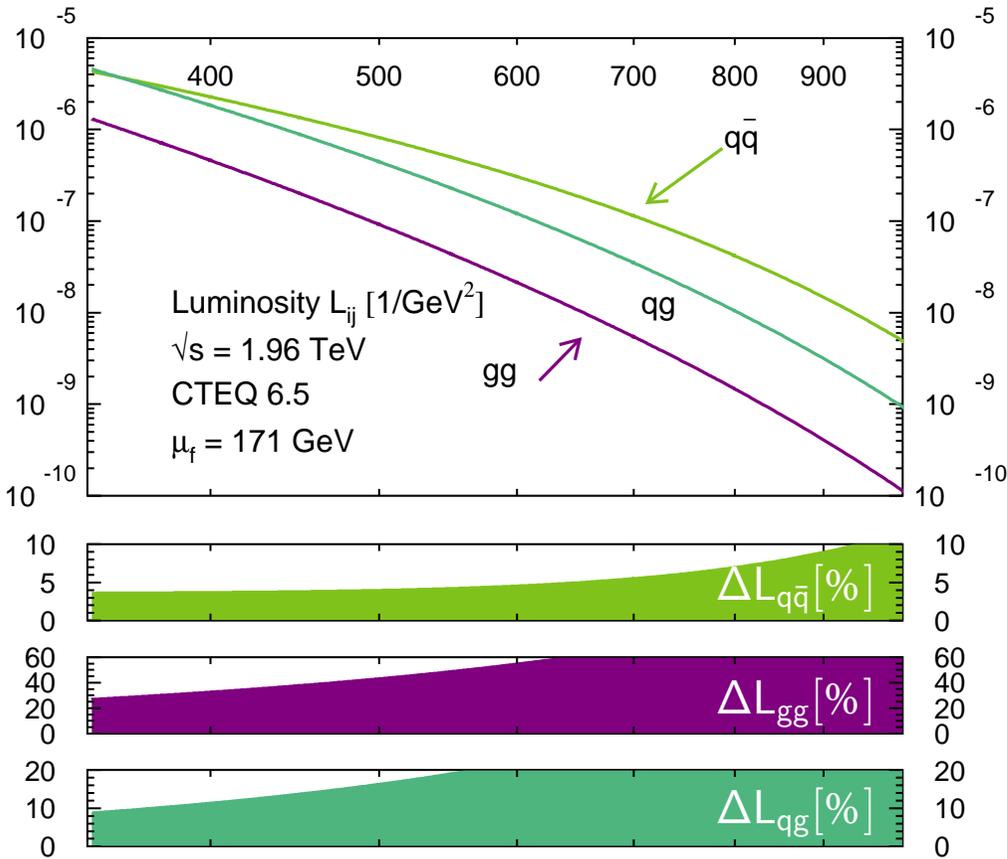
- Add all scale dependent terms
  - $\ln(\mu/m_t)$ -terms exactly known from renormalization group methods

## Upshot

- Best approximation to complete NNLO
- Similar results for new massive colored particles (4th generation quarks, squarks, gluinos, ...)  
S.M., Uwer '08; S.M., Langenfeld '08

# Total cross section at Tevatron

$$\sigma_{pp \rightarrow t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij \rightarrow t\bar{t}}$$

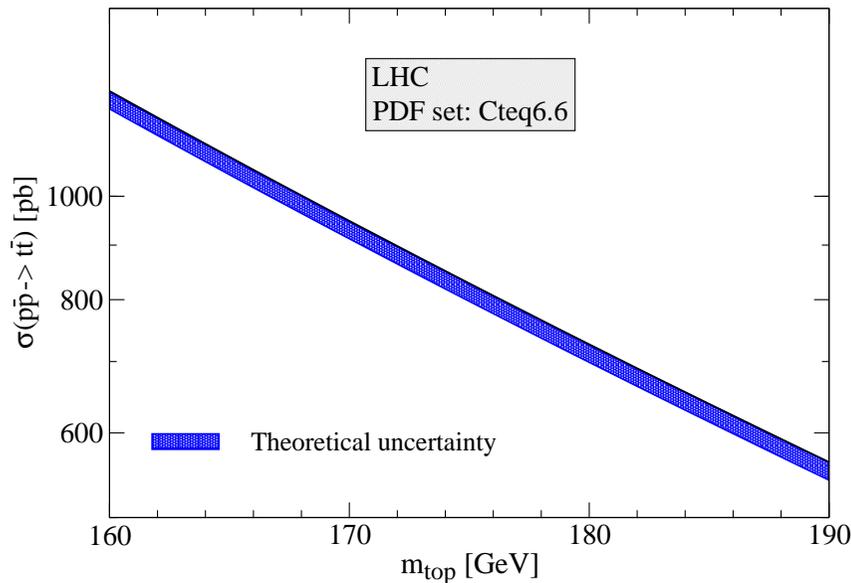




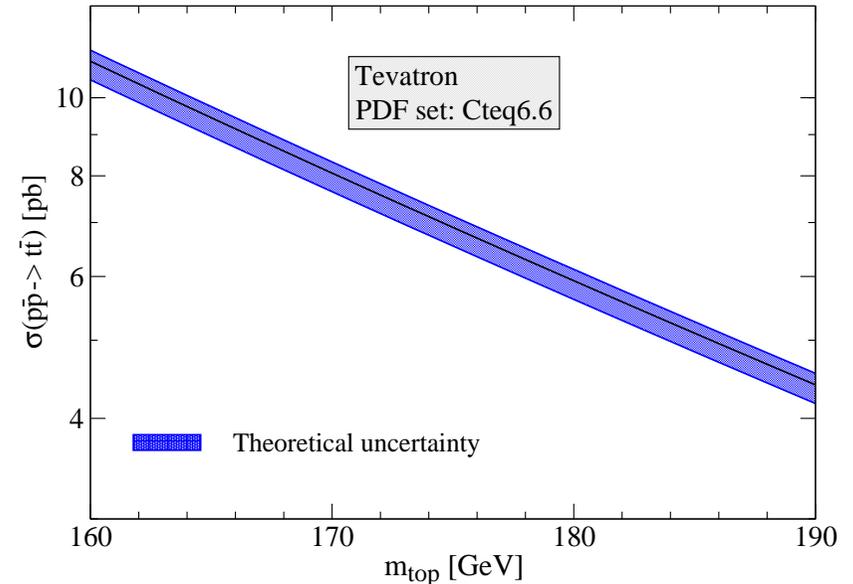


# Mass dependence

- Parametrize mass dependence with a fit around  $x = (m_t/\text{GeV} - 173)$   
$$\sigma(\mu) = a + bx + cx^2 + dx^3 + ex^4 + fx^5 + gx^6$$
- fit precise to per mille accuracy in range  $150 \text{ GeV} \leq m_t \leq 220 \text{ GeV}$
- various scale and PDF choices:  $\mu = m_t/2, m_t, 2m_t$ , CTEQ6.6, MSTW2008, ...



**LHC**



**Tevatron**

# Theory improvements

- Variation of renormalization and factorization scale  $\mu_R \neq \mu_F$
- Improved matching at NLO
  - consistent color-singlet and color-octet contributions at NLO Petrelli, Cacciari, Greco, Maltoni, Mangano '97; Hagiwara, Sumino, Yokoya '08
- Fits to exact NLO calculation Mitov, Czakon '08
- $gq$ -channel included at two loops (large  $gq$ -parton luminosity at LHC)
  - leading term near threshold  $\sim \beta^3 \ln^3 \beta$  (power suppressed  $\beta^2$ )

# Scale dependence

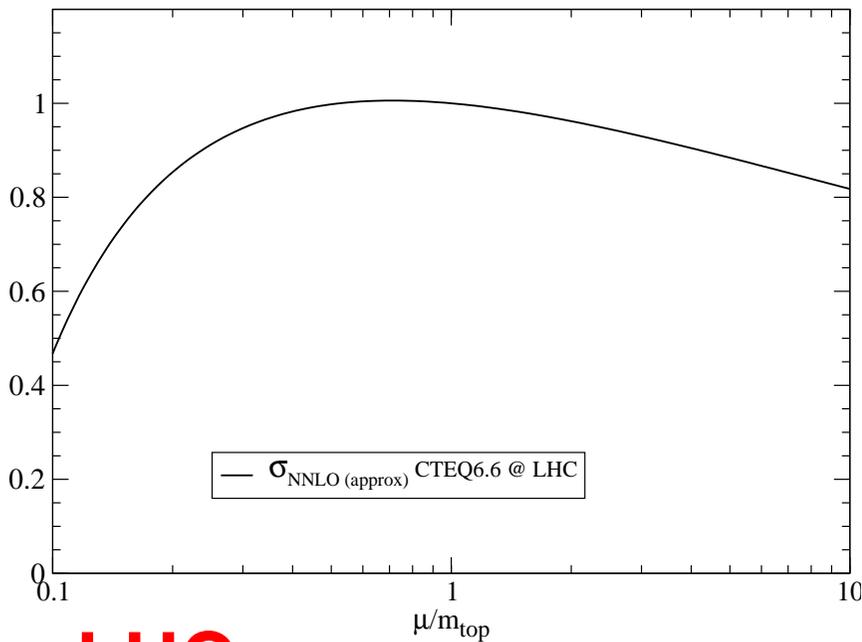
- Renormalization group methods predict all terms  $L = \ln(\mu^2/m_t^2)$

$$\begin{aligned} \sigma_{t\bar{t}} &= \sigma^{(0)} + \alpha_s(\mu) \left\{ \sigma^{(1)} + L \sigma_L^{(1)}(\sigma^{(0)}, \beta_0, P_0) \right\} \\ &+ \alpha_s^2(\mu) \left\{ \sigma^{(2)} + L \sigma_L^{(2)}(\sigma^{(0)}, \sigma^{(1)}, \beta_0, \beta_1, P_0, P_1) + L^2 \sigma_{L^2}^{(2)}(\sigma^{(0)}, \beta_0, P_0) \right\} \end{aligned}$$

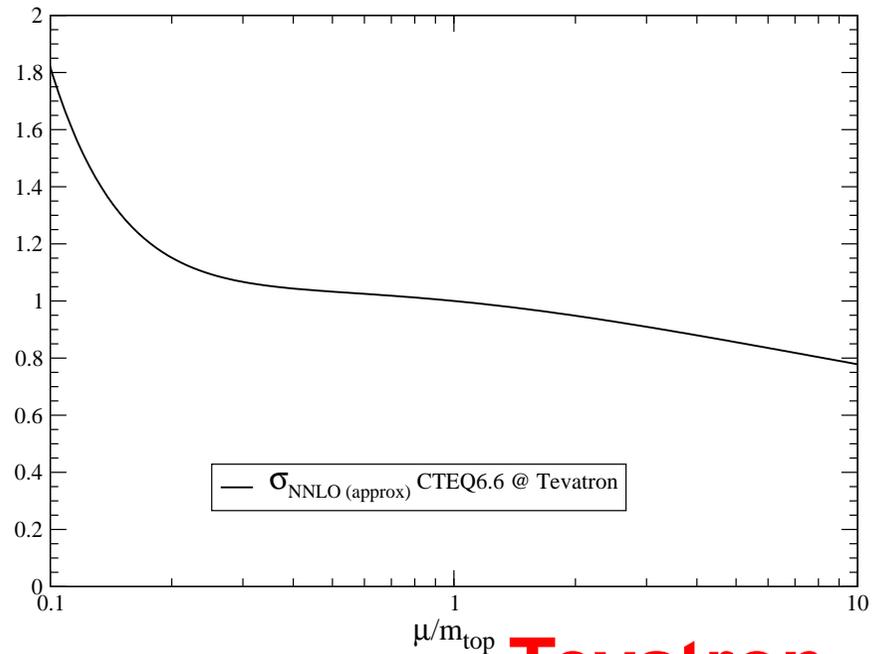
- relax  $\mu = \mu_R = \mu_F$  in study of theoretical uncertainty
- allow for independent variation  $\mu_R \neq \mu_F$

# Scale dependence (I)

- Theoretical uncertainty from variation of scales  $\mu = \mu_R = \mu_F$ 
  - plot with PDF set CTEQ6.6 (but largely independent on PDFs)
  - very stable predictions in range  $\mu \in [m_t/2, 2m_t]$ 
    - $-3\% \leq \Delta\sigma \leq +0.5\%$  at LHC
    - $-4\% \leq \Delta\sigma \leq +3\%$  at Tevatron



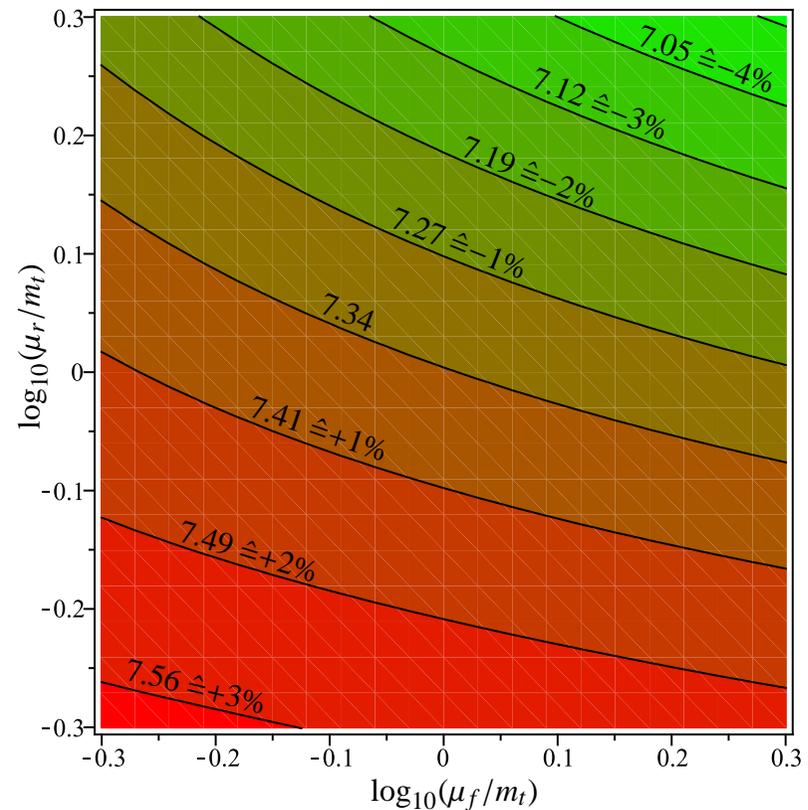
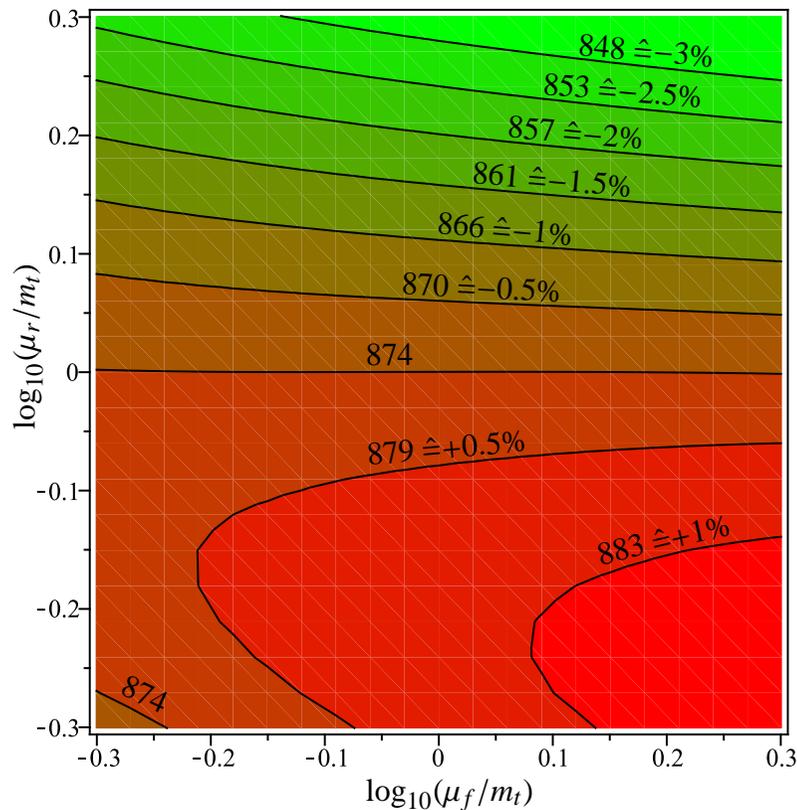
**LHC**



**Tevatron**

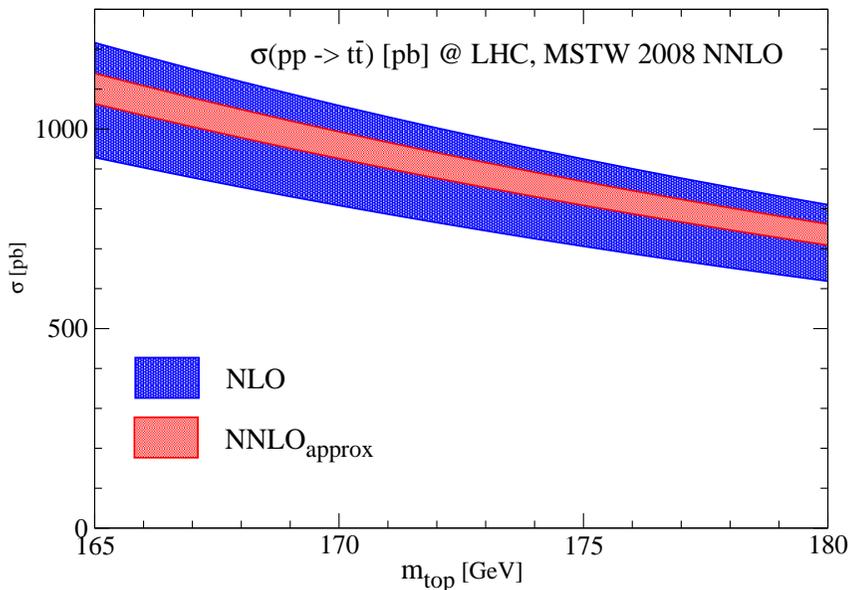
## Scale dependence (II)

- Contour lines of total cross section for  $\mu_R \neq \mu_F$ 
  - independent variation of renormalization and factorization scale
  - range corresponds to  $\mu_R, \mu_F \in [m_t/2, 2m_t]$
  - plot with PDF set CTEQ6.6 (but largely independent on PDFs)

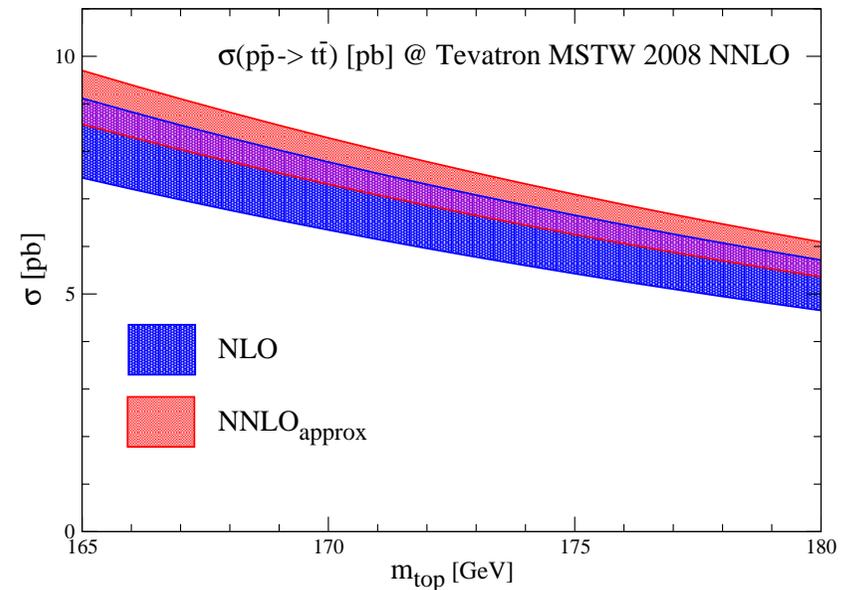


# Total theory uncertainty

- NLO (with MSTW2008 PDF set)
  - scale uncertainty  $\mathcal{O}(10\%) \oplus$  PDF uncertainty  $\mathcal{O}(5\%)$
- NNLO<sub>approx</sub> (with MSTW2008 PDF set)
  - scale uncertainty  $\mathcal{O}(3\%) \oplus$  PDF uncertainty  $\mathcal{O}(2\%)$
- Theory at NNLO matches anticipated experimental precision  $\mathcal{O}(10\%)$



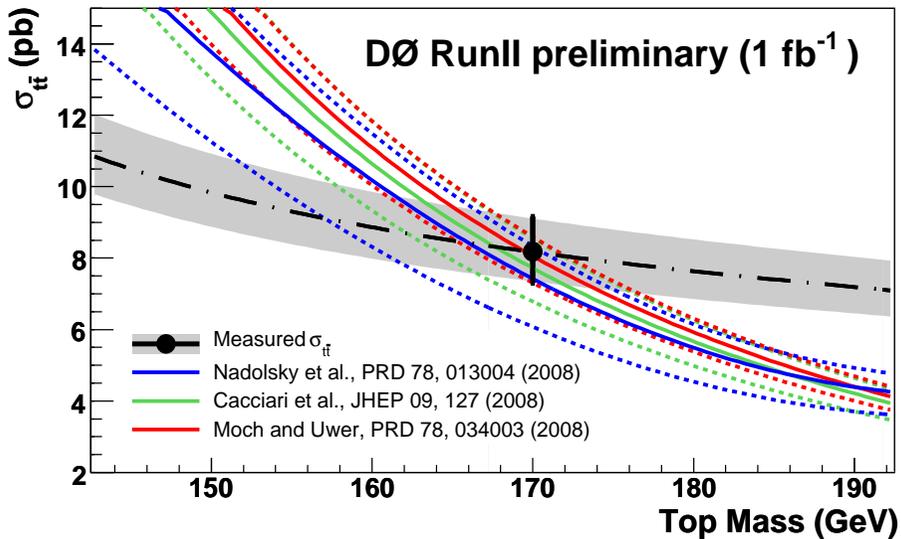
**LHC**



**Tevatron**

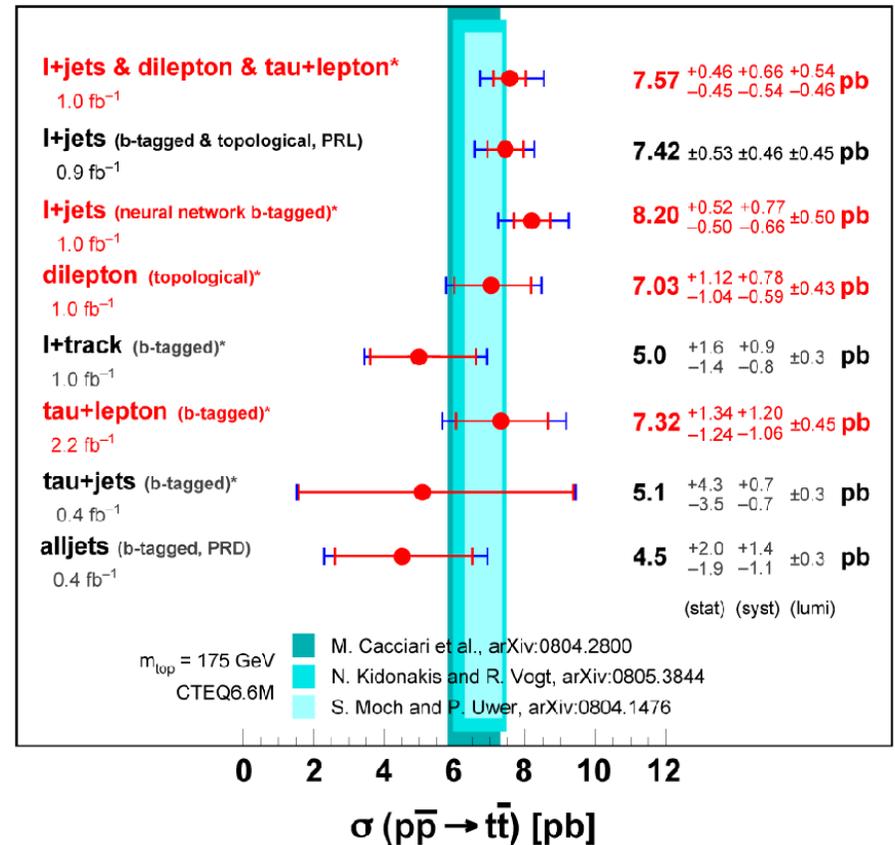
# Tevatron analyses

- Total cross section and different channels of Tevatron analyses (theory uncertainty band from scale variation)
- NNLO allows for precision determinations of  $m_t$  from total cross section (slope  $d\sigma/dm_t$ )



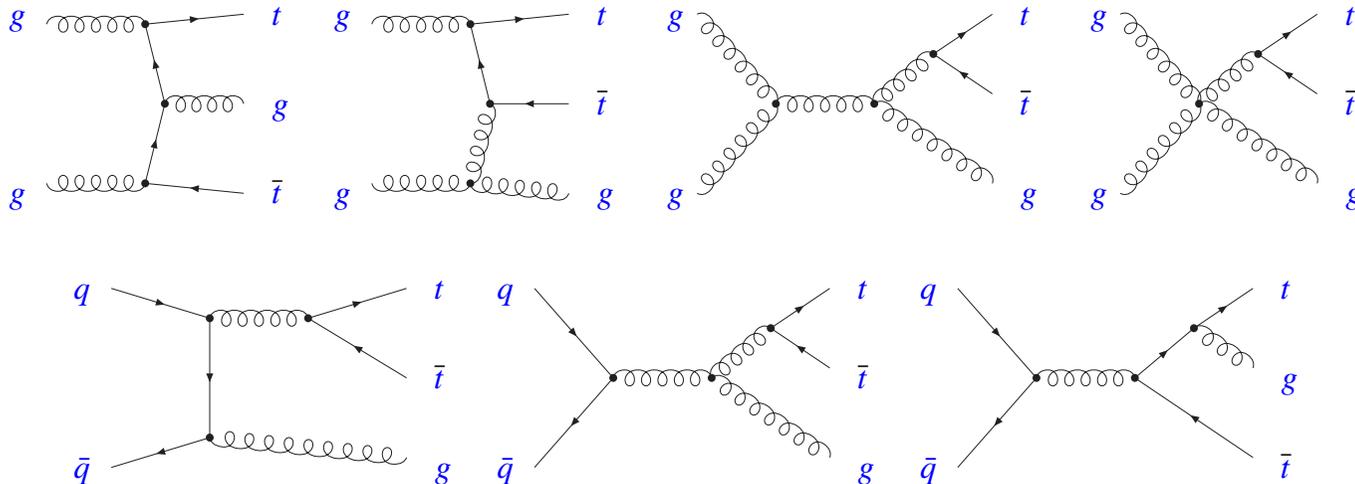
## DØ Run II \* = preliminary

April 2009



## $t\bar{t} + \text{jet}$ production (I)

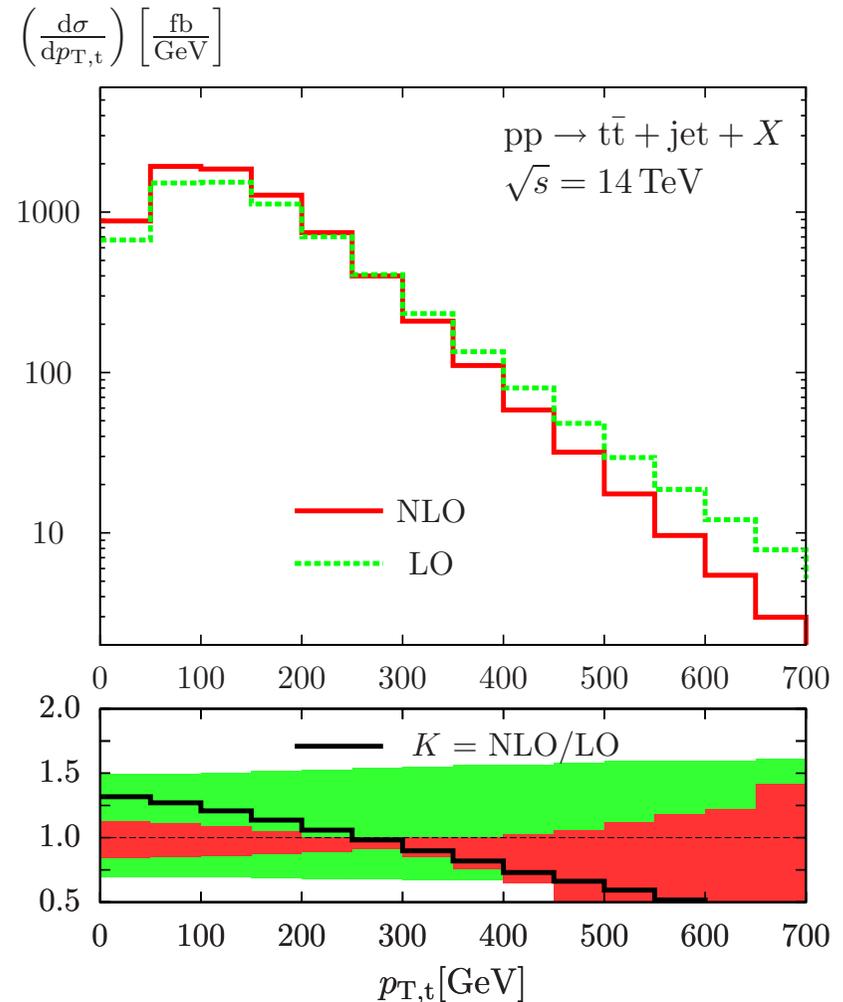
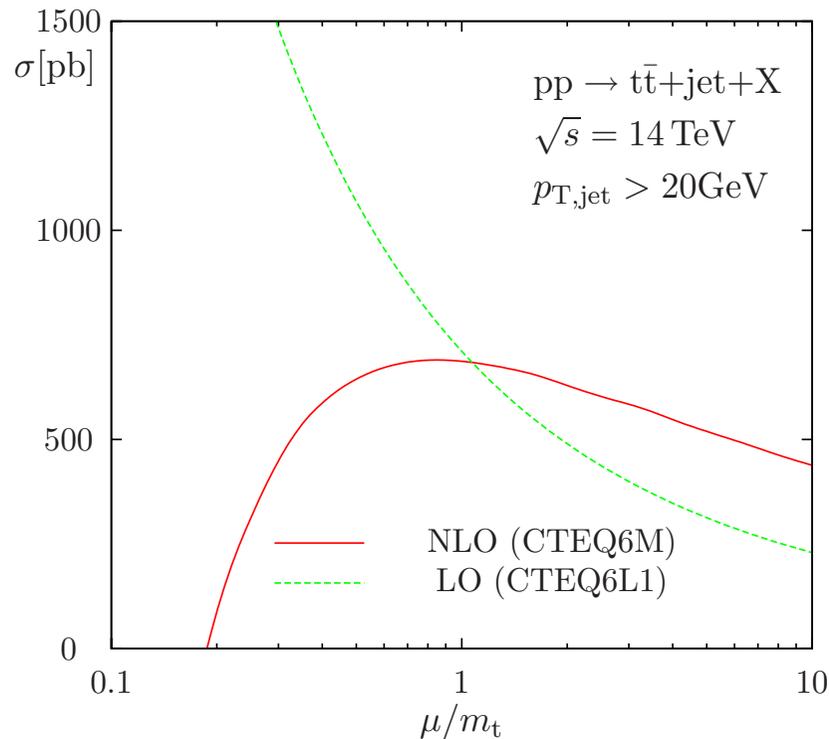
- LHC: large rates for production of  $t\bar{t}$ -pairs with additional jets
  - sample Feynman diagrams for  $t\bar{t} + \text{jet}$  production at LO



- NLO corrections to  $t\bar{t} + \text{jet}$  production are part of NNLO corrections for inclusive  $t\bar{t}$  production
  - at scale  $\mu_R = \mu_F = m_t$  corrections are almost zero
  - threshold resummation captures dominant contributions



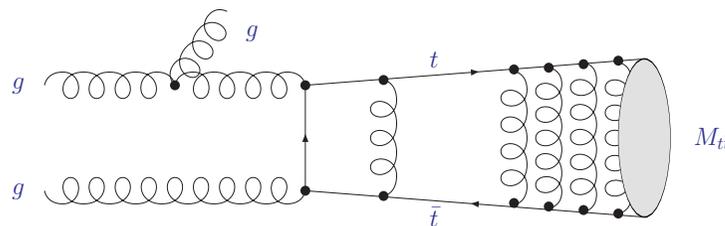
## $t\bar{t}$ + jet production (II)



- Impressive state-of-the-art NLO QCD result [Dittmaier, Uwer, Weinzierl '07-'08](#)
  - much improved scale dependence of total rate
  - transverse-momentum distributions of top-quark  $p_{T,t}$  along with K-factor and scale variation  $m_t/2 \leq \mu \leq 2m_t$

# Coulomb corrections

- Heavy quark production very close to threshold
  - resummation of Coulomb corrections  $\sim 1/\beta$  to all orders (non-relativistic QCD)
  - NRQCD factorization Bodwin, Braaten, Lepage '95
- Much work (theory and phenomenology) for ILC
  - fixed center-of-mass energy  $S$  allows threshold scan at  $\sqrt{S} \sim 2m_t$
  - dominant color-singlet production  $\rightarrow t\bar{t} \left( {}^3S_1^{[1]} \right)$
- Effects on top-mass measurement at LHC Hagiwara, Sumino, Yokoya '08
- Detailed study in NRQCD assembling existing knowledge at NLO/NLL Kiyoyama, Kühn, S.M., Steinhauser, Uwer '08
  - complete NLO NRQCD result Petrelli, Cacciari, Greco, Maltoni, Mangano '97 (corrections by Hagiwara, Sumino, Yokoya '08)
  - NLL resummation Cacciari '99

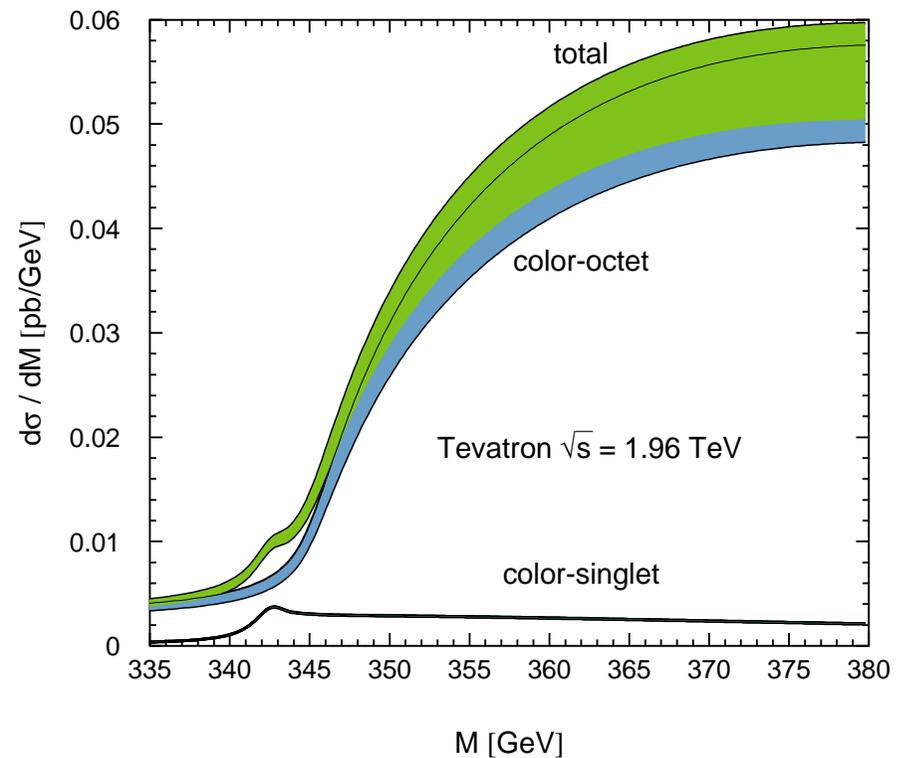
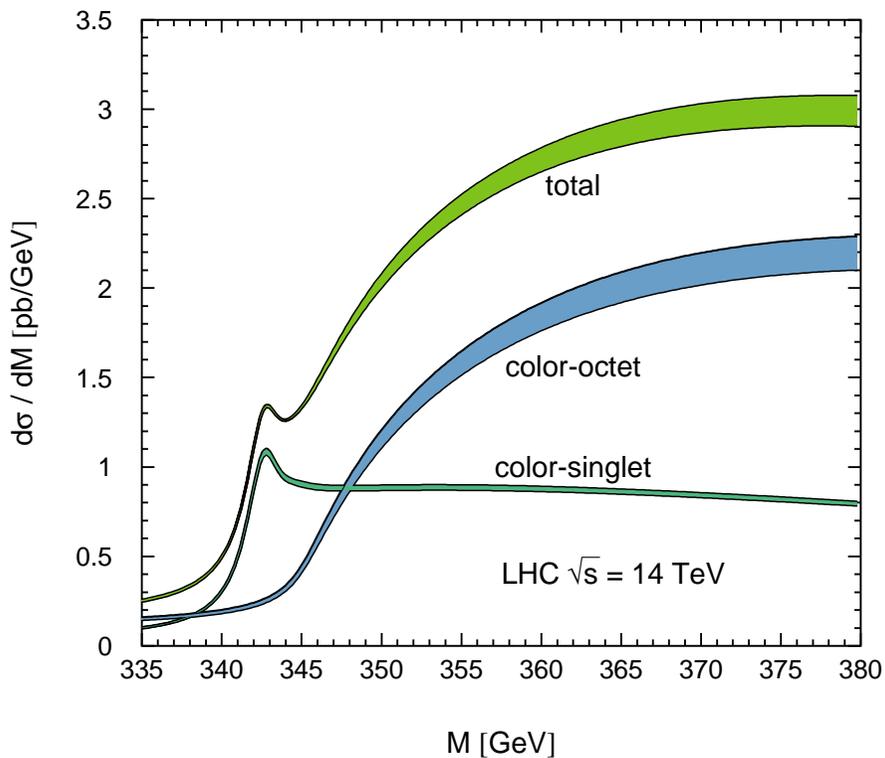


# Coulomb corrections

- Recall master equation 
$$\sigma_{pp \rightarrow t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij \rightarrow t\bar{t}}$$
- Convolution with PDFs  $f_i \otimes f_j$ 
  - top-quark pairs produced as color-singlets and color-octets  
 $\rightarrow t\bar{t} \left( {}^{2s+1}S_J^{[1,8]} \right)$
  - threshold at  $M_{t\bar{t}} \sim 2m_t$  with  $M_{t\bar{t}} = (p_t + p_{\bar{t}})^2$
- NRQCD factorization of partonic cross section into  
$$\hat{\sigma}_{ij \rightarrow t\bar{t}} = F_{ij \rightarrow T} \otimes G(M_{t\bar{t}})$$
  - free  $t\bar{t}$  production rate  $F$
  - evolution factor into “boundstate” (Green’s function)  $G$
- Differential kinematics 
$$\frac{d\hat{\sigma}_{ij \rightarrow t\bar{t}}}{dM_{t\bar{t}}^2} = F_{ij \rightarrow T} \times \Im G^{[1,8]}(M_{t\bar{t}})$$
  - factorization of soft-collinear dynamics (real emission radiation)
  - matching at NLO and NLL resummation

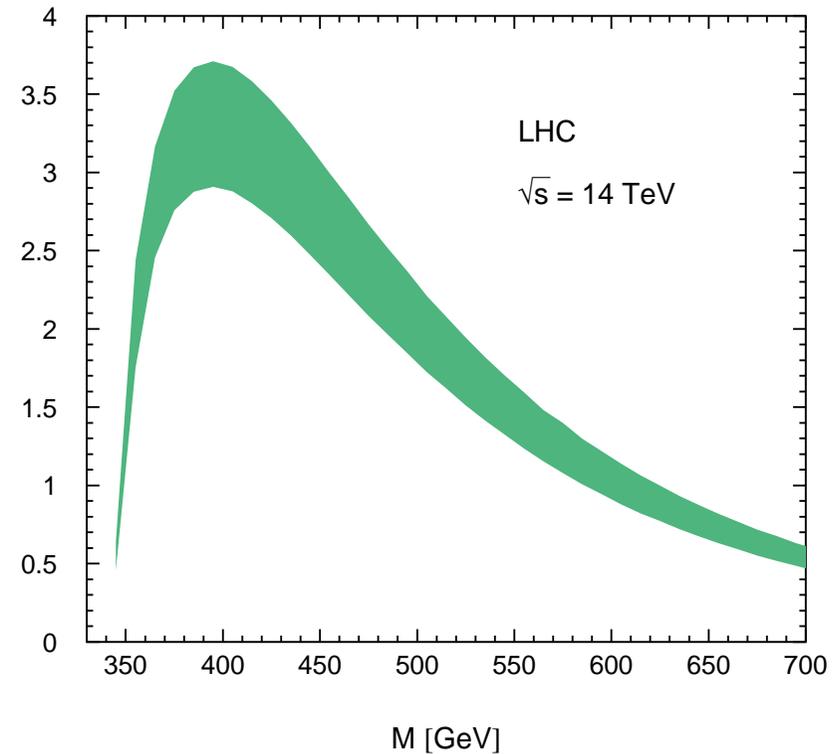
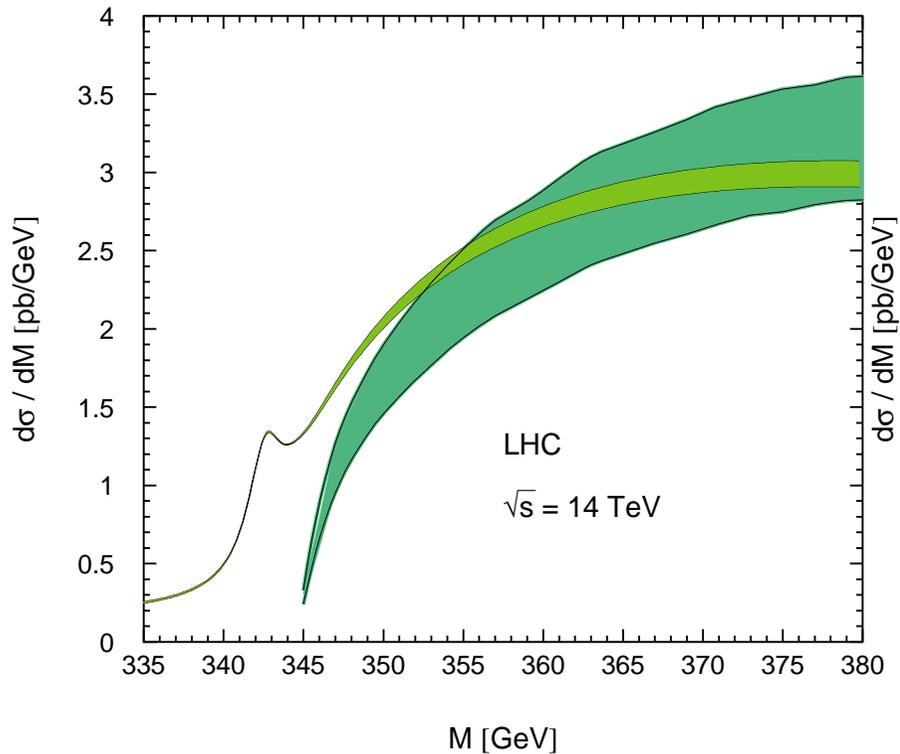
# Invariant mass distribution

- $d\sigma/dM_{t\bar{t}}$  at LHC driven by large gluon luminosity
  - $gg \rightarrow t\bar{t} \left( {}^1S_0^{[1]} \right)$  dominates
- $d\sigma/dM_{t\bar{t}}$  at Tevatron with small bound state effects
  - $q\bar{q}$ -channel large with only color-octet configurations only



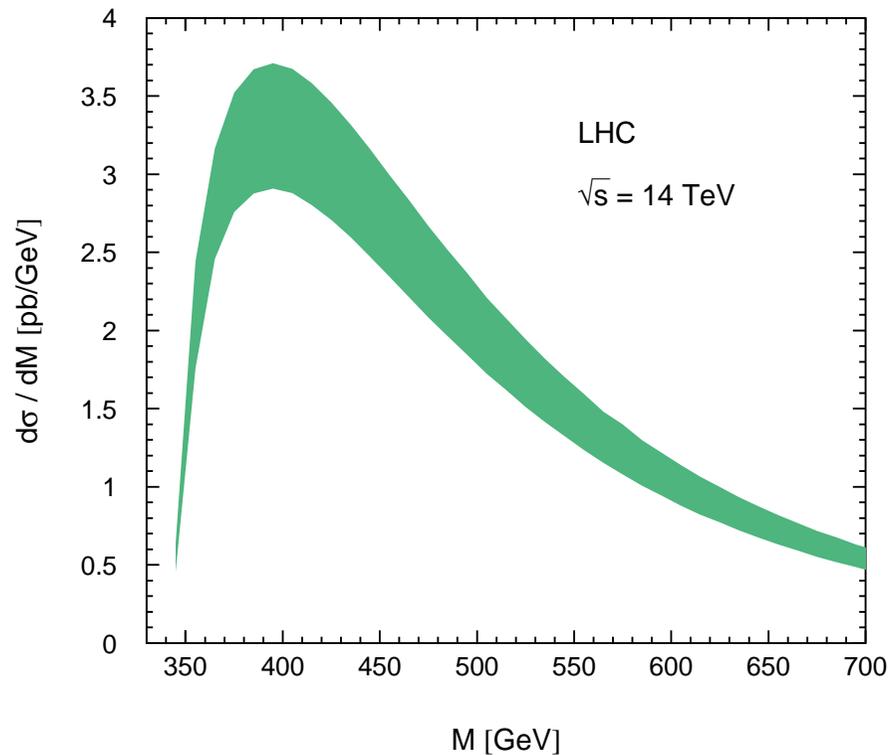
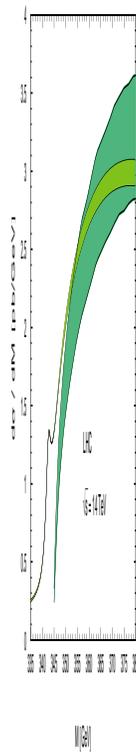
# Matching to fixed order

- $d\sigma/dM_{t\bar{t}}$  with at LHC
  - compare NLL resummed result in NRQCD (plain vanilla) NLO  
Mangano, Nason, Ridolfi '92
  - consistency check OK

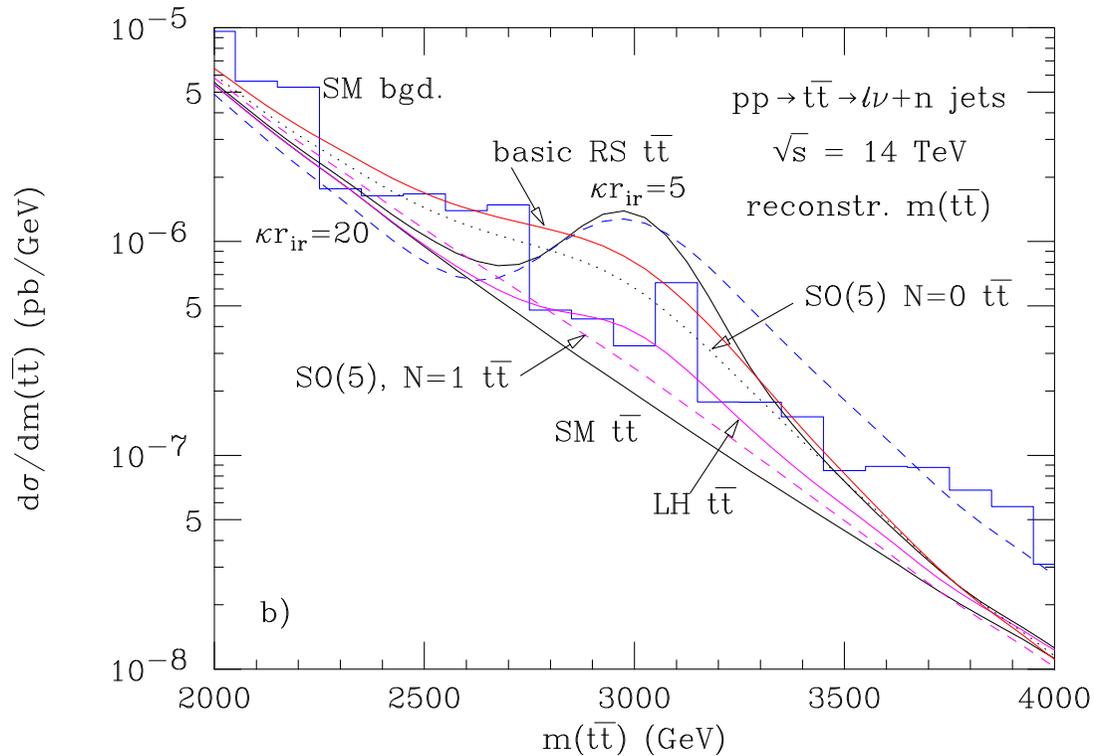


# Invariant mass distribution

- Resolution of bound state effects in  $d\sigma/dM_{t\bar{t}}$  at LHC difficult (requires rather fine binning)
  - uncertainty of total cross section  $\Delta\sigma \simeq \mathcal{O}(10)$  pb from
  - extrapolation of  $M_{t\bar{t}}$ -distribution affected by  $gg \rightarrow t\bar{t} \left( {}^1S_0^{[1]} \right)$



# High- $M_{t\bar{t}}$ tail of invariant mass distribution



- New physics searches in high-end tail of invariant mass  $M_{t\bar{t}}$ -spectrum at LHC
  - Kaluza-Klein resonances (e.g.  $s$ -channel graviton exchange)  
 Frederix, Maltoni '07; Baur, Orr '08

# Summary

## Collider phenomenology

- Top quark theory
  - improved understanding of theory and application of new concepts
  - resummation important for Tevatron and LHC phenomenology
- Total cross section
  - NNLO<sub>approx</sub> prediction with exact scale dependence  $\mu_R \neq \mu_F$  ( $\ln(\mu_R/m)$ ,  $\ln(\mu_F/m)$ -terms)
  - investigation of theoretical uncertainty
    - $\Delta\sigma \sim \mathcal{O}(\pm 3)\%$  at LHC
    - $\Delta\sigma \sim \mathcal{O}(\pm 4)\%$  at Tevatron

## Higher orders

- QCD results for hard scattering at **new level of precision**