Two-loop soft anomalous dimensions for heavy quark production

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- **Soft anomalous dimensions**
- **Two-loop eikonal calculations**
- **Top quark production**

Soft gluon corrections

Incomplete cancellations of infrared divergences between virtual diagrams and real diagrams with soft (low-energy) gluons

terms of the form $\left[\frac{\ln^k(1-z)}{1-z}\right]_+$

Resum (exponentiate) these soft corrections

At NLL (NNLL) accuracy requires one-loop (two-loop) calculations in the eikonal approximation

Near threshold soft corrections are dominant and can provide good approximations to the complete cross section

Examples: top pair and single top production

jet, direct photon, or W production at high p_T

Resummed cross section

Resummation follows from factorization properties of the cross section

- performed in moment space

H: **hard-scattering function**

S: **soft-gluon function**

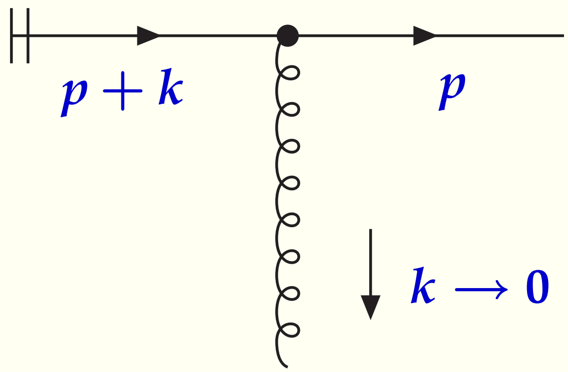
$$\hat{\sigma}^{res}(N) = \exp\left[\sum_i E_i(N)\right] \exp\left[\sum_i 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu}\, \gamma_{i/i}\left(N, \alpha_s(\mu)\right)\right]$$
$$\times \exp\left[\sum_i 2 d_{\alpha_s} \int_{\mu_R}^{\sqrt{s}} \frac{d\mu}{\mu}\, \beta\left(\alpha_s(\mu)\right)\right] H\left(\alpha_s(\mu_R)\right)$$
$$\times \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}} \frac{d\mu}{\mu}\, \Gamma_S^\dagger\left(\alpha_s(\mu)\right)\right] S\left(\alpha_s\left(\frac{\sqrt{s}}{\tilde{N}}\right)\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}} \frac{d\mu}{\mu}\, \Gamma_S\left(\alpha_s(\mu)\right)\right]$$

where

Γ_S **is the soft anomalous dimension - a matrix in color space**

and a function of kinematical invariants s, t, u

Eikonal approximation



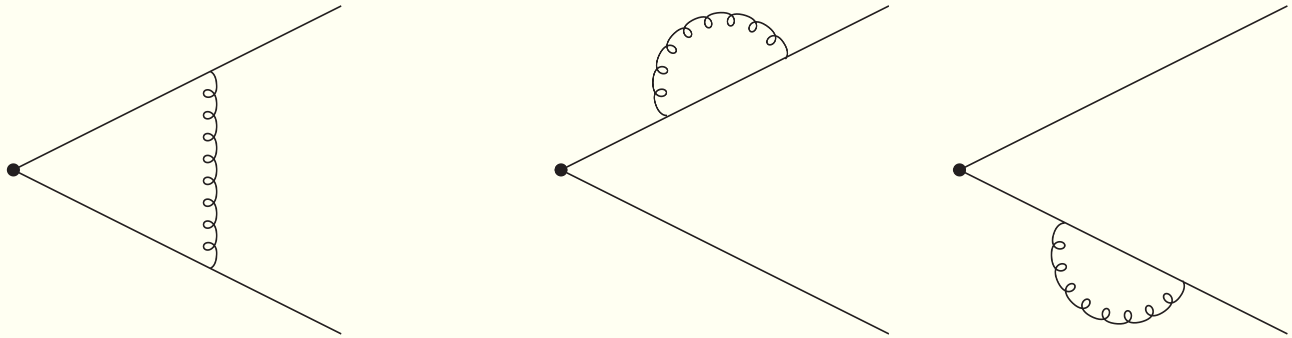
$$\bar{u}(p)\,(-ig_s T_F^c)\,\gamma^\mu \frac{i(\not p + \not k + m)}{(p+k)^2 - m^2 + i\epsilon} \longrightarrow \bar{u}(p)\, g_s T_F^c\, \gamma^\mu \frac{\not p + m}{2p\cdot k + i\epsilon} = \bar{u}(p)\, g_s T_F^c \frac{v^\mu}{v\cdot k + i\epsilon}$$

with $p \propto v$, T_F^c generators of SU(3)

Perform calculation for massive quarks in momentum space and Feynman gauge

Complete two-loop results for $e^+e^- \longrightarrow t\bar{t}$

One-loop diagrams



(a) (b)

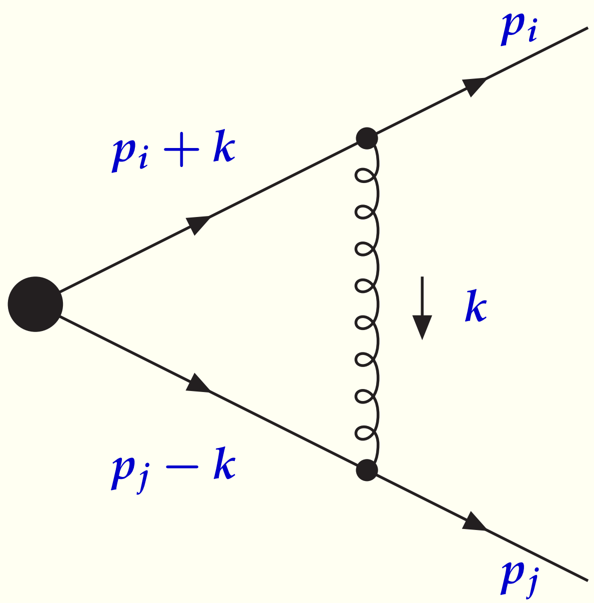
The one-loop soft anomalous dimension, $\Gamma_S^{(1)}$, can be read off the coefficient of the ultraviolet (UV) pole of the one-loop diagrams

$$\Gamma_S = (\alpha_s/\pi)\Gamma_S^{(1)} + (\alpha_s/\pi)^2\Gamma_S^{(2)} + \cdots$$

$$\Gamma_S^{(1)} = C_F\left[-\frac{(1+\beta^2)}{2\beta}\ln\left(\frac{1-\beta}{1+\beta}\right) - 1\right]$$

with $\beta = \sqrt{1-\frac{4m^2}{s}}$

Example: one-loop vertex correction



$$I_{1a} = g_s^2 \int \frac{d^n k}{(2\pi)^n} \frac{(-i) g^{\mu\nu}}{k^2} \frac{v_i^\mu}{v_i \cdot k} \frac{(-v_j^\nu)}{(-v_j \cdot k)}$$

Using Feynman parametrization, this can be rewritten as

$$I_{1a} = -2i g_s^2 \frac{v_i \cdot v_j}{(2\pi)^n} \int_0^1 dx \int_0^{1-x} dy \int \frac{d^n k}{\left[x k^2 + y v_i \cdot k + (1-x-y) v_j \cdot k\right]^3}$$

which, after the integration over k, gives

$$I_{1a} = g_s^2\, v_i \cdot v_j\, 2^{6-2n}\, \pi^{-n/2}\, \Gamma\left(3-\frac{n}{2}\right) \int_0^1 dx\, x^{3-n} \times \int_0^{1-x} dy \left[-y^2 v_i^2 - (1-x-y)^2 v_j^2 - 2y\, v_i \cdot v_j (1-x-y)\right]^{n/2-3}$$

After several manipulations, and with $n = 4 - \epsilon$ and $\beta = \sqrt{1-4m^2/s}$,

$$I_{1a} = \frac{\alpha_s}{\pi} (-1)^{-1-\epsilon/2}\, 2^{5\epsilon/2}\, \pi^{\epsilon/2}\, \Gamma\left(1+\frac{\epsilon}{2}\right) (1+\beta^2) \int_0^1 dx\, x^{-1+\epsilon} (1-x)^{-1-\epsilon} \times \left\{ \int_0^1 dz \left[4z\beta^2(1-z)+1-\beta^2\right]^{-1} - \frac{\epsilon}{2} \int_0^1 dz \frac{\ln\left[4z\beta^2(1-z)+1-\beta^2\right]}{4z\beta^2(1-z)+1-\beta^2} + \mathcal{O}\left(\epsilon^2\right) \right\}$$

integral over x contains both UV and IR singularities - isolate UV singularities

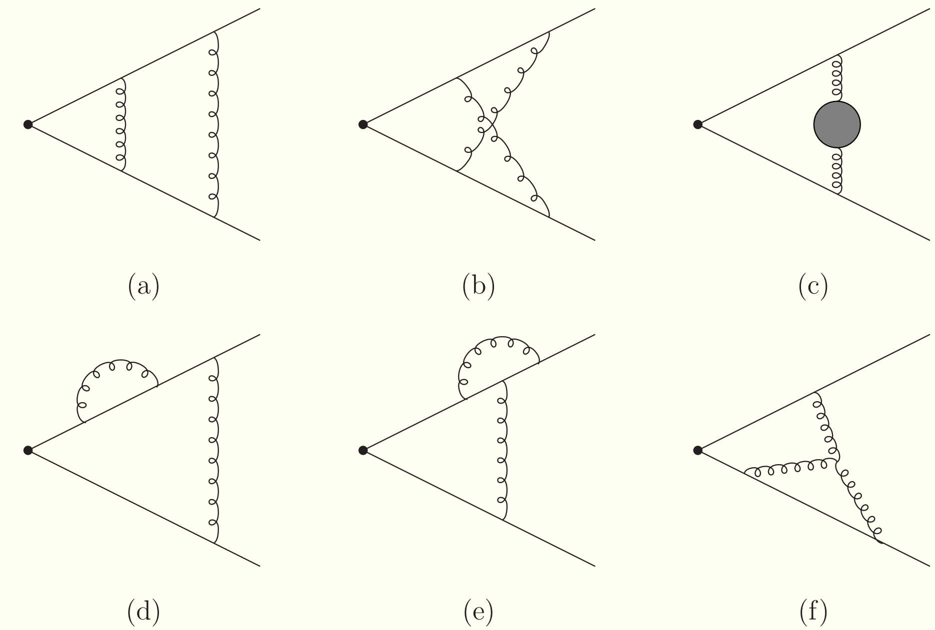
$$\int_0^1 dx\, x^{-1+\epsilon} (1-x)^{-1-\epsilon} = \frac{1}{\epsilon} + \mathrm{IR}$$

Then

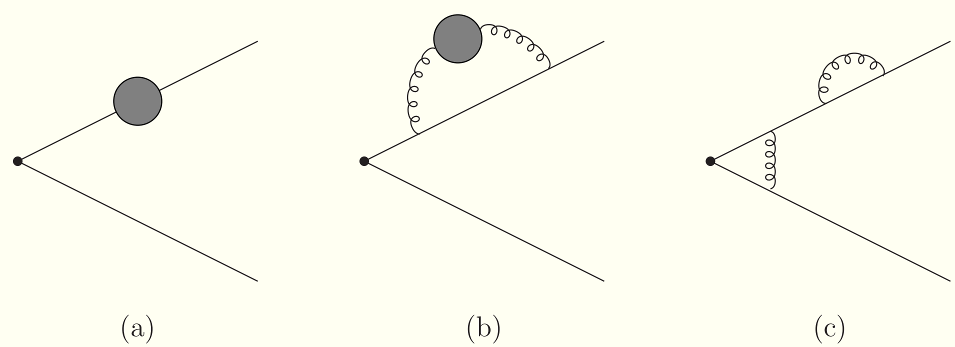
$$I_{1a}^{UV} = \frac{\alpha_s}{\pi} \frac{(1+\beta^2)}{2\beta} \frac{1}{\epsilon} \ln\left(\frac{1-\beta}{1+\beta}\right)$$

Two-loop diagrams

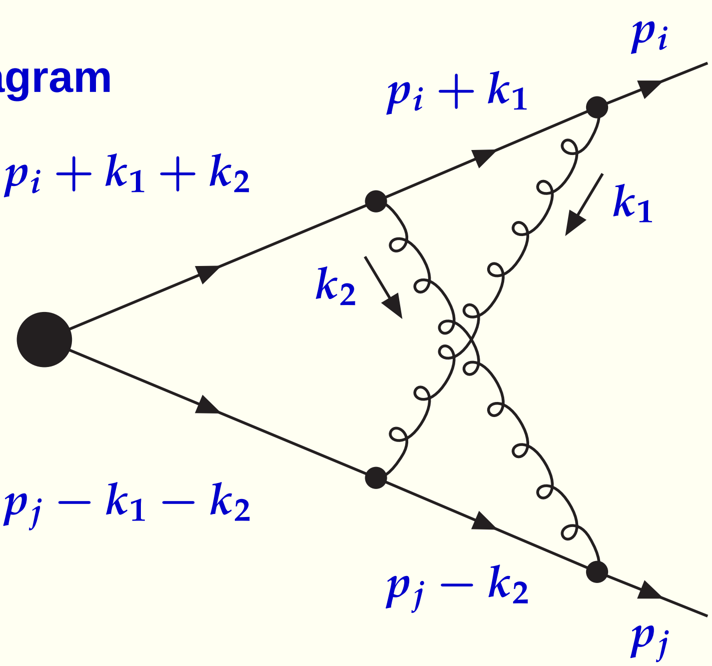
Vertex correction graphs



Heavy-quark self-energy graphs



Example: two-loop crossed diagram



$$I_{2b} = g_s^4 \int \frac{d^n k_1}{(2\pi)^n} \frac{d^n k_2}{(2\pi)^n} \frac{(-i) g^{\mu\nu}}{k_1^2} \frac{(-i) g^{\rho\sigma}}{k_2^2} \frac{v_i^\mu}{v_i \cdot k_1} \frac{v_i^\rho}{v_i \cdot (k_1 + k_2)} \frac{(-v_j^\nu)}{-v_j \cdot (k_1 + k_2)} \frac{(-v_j^\sigma)}{-v_j \cdot k_2}$$

Perform k_2 integral first

$$I_{2b} = -i \frac{\alpha_s^2}{\pi^2} 2^{-4+\epsilon} \pi^{-2+3\epsilon/2} \Gamma\left(1 - \frac{\epsilon}{2}\right) \Gamma(1+\epsilon)(1+\beta^2)^2 \int_0^1 dz \int_0^1 \frac{dy\,(1-y)^{-\epsilon}}{\left[2\beta^2 (1-y)^2 z^2 - 2\beta^2 (1-y) z - \frac{(1-\beta^2)}{2}\right]^{1-\epsilon/2}}$$

$$\times \int \frac{d^n k_1}{k_1^2 \, v_i \cdot k_1 \left[\left((v_i - v_j) z + v_j\right) \cdot k_1\right]^{1+\epsilon}}$$

Then proceed with the k_1 integral, and isolate UV and IR poles. After many steps

$$I_{2b}^{UV} = \frac{\alpha_s^2}{\pi^2} \frac{(1+\beta^2)^2}{8\beta^2} \frac{1}{\epsilon} \left\{ -\ln\left(\frac{1-\beta}{1+\beta}\right) \left[\mathrm{Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right) + \zeta_2\right] - \frac{1}{3} \ln^3\left(\frac{1-\beta}{1+\beta}\right) + \mathrm{Li}_3\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right) - \zeta_3 \right\}$$

Include counterterms for all graphs and multiply with corresponding color factors

Determine two-loop soft anomalous dimension from UV poles of the sum of the graphs

$$\Gamma_S^{(2)} = \left\{\frac{K}{2}+\frac{C_A}{2}\left[\ln\left(\frac{1-\beta}{1+\beta}\right)+\frac{\zeta_2}{2}\right]+\frac{(1+\beta^2)}{4\beta}C_A\left[\mathrm{Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)+\zeta_2\right]\right\}\Gamma_S^{(1)}$$
$$+C_F C_A\left\{\frac{1}{2}+\frac{1}{2}\ln\left(\frac{1-\beta}{1+\beta}\right)-\frac{(1+\beta^2)^2}{8\beta^2}\left[\frac{1}{3}\ln^3\left(\frac{1-\beta}{1+\beta}\right)-\mathrm{Li}_3\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)+\zeta_3\right]\right.$$
$$\left.-\frac{(1+\beta^2)}{2\beta}\left[\ln\left(\frac{1-\beta}{1+\beta}\right)\ln\left(\frac{(1+\beta)^2}{4\beta}\right)-\mathrm{Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)\right]-2\,F(\beta)\right\}$$

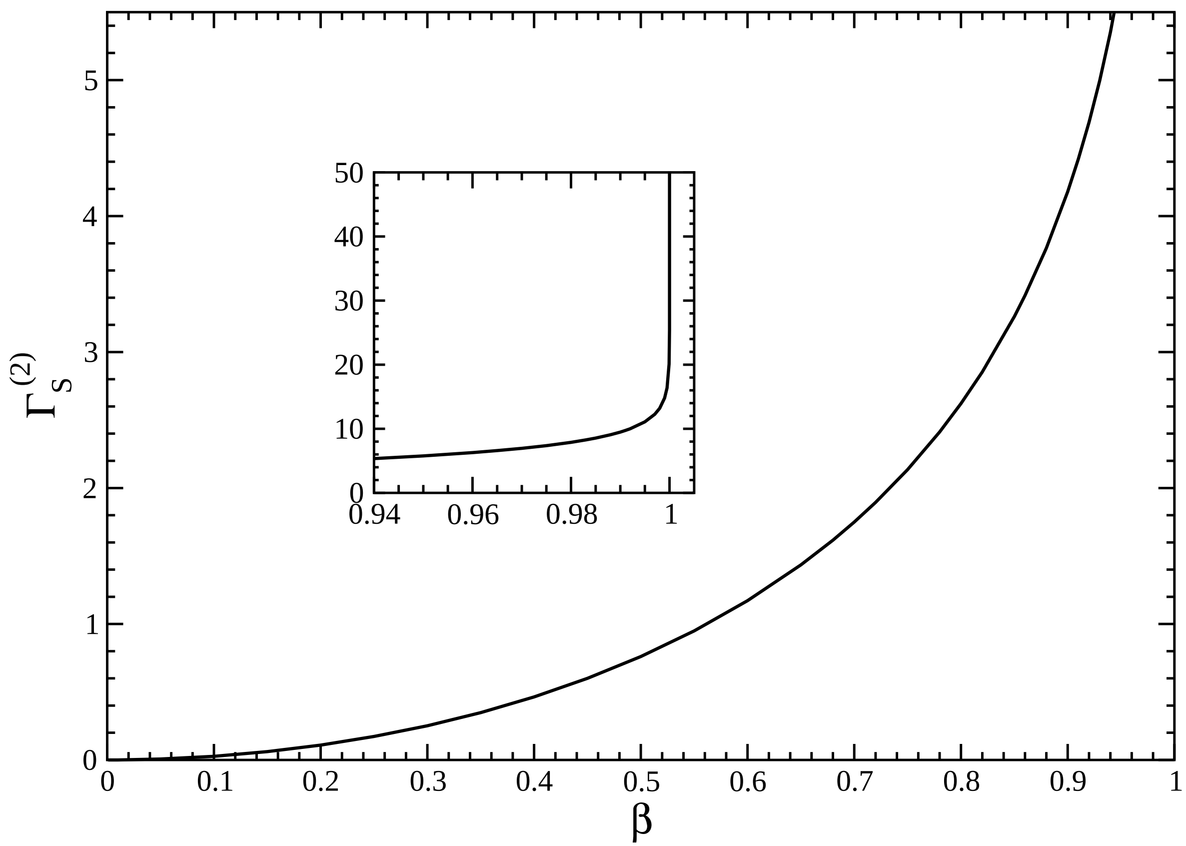
where $K=C_A(67/18-\zeta_2)-5n_f/9$

and

$$F(\beta)=\frac{(1+\beta^2)}{4\beta}\int_0^1\frac{dx\,\ln x}{x(1-x^2)\sqrt{(1-\beta^2)^2+4\beta^2x^2}}\left\{2\beta x\right.$$
$$\left.-\sqrt{(1-\beta^2)^2+4\beta^2x^2}\left[-\ln(1-\beta^2)+\ln\left(2\beta x+\sqrt{(1-\beta^2)^2+4\beta^2x^2}\right)\right]\right\}$$

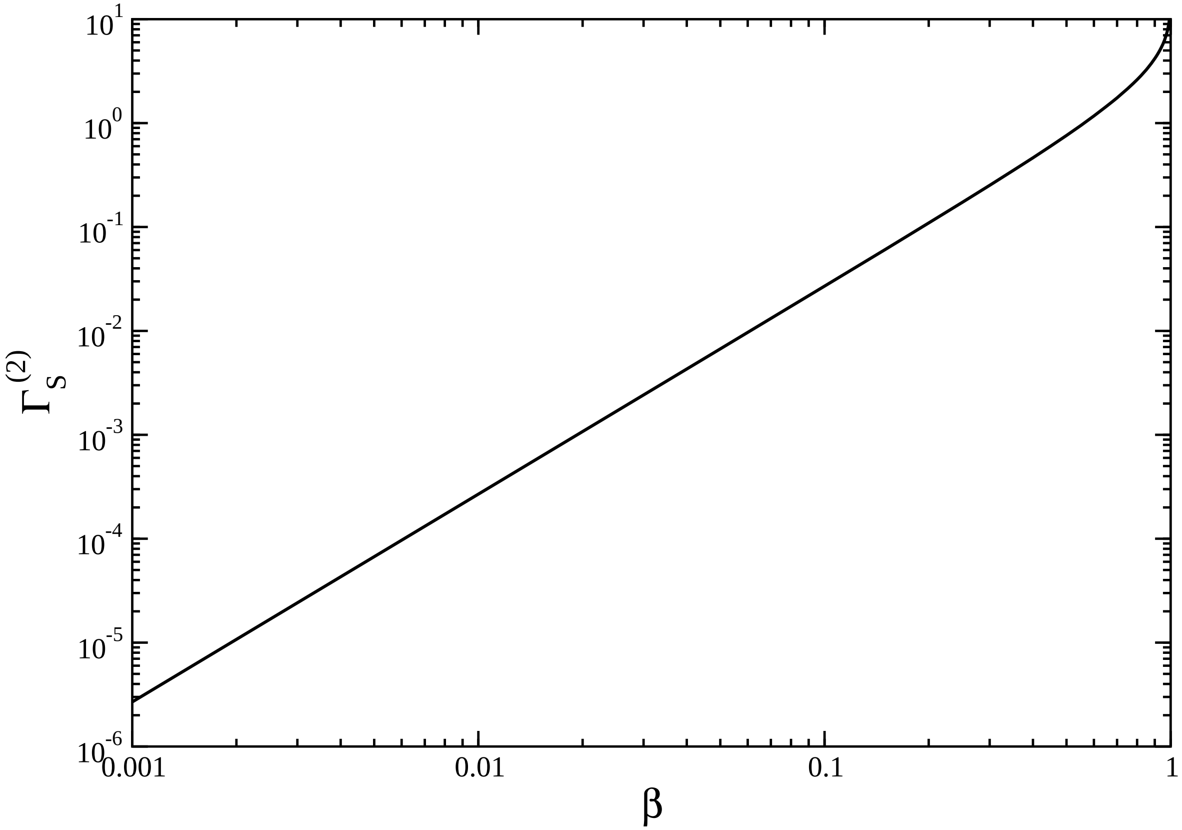
The color structure of $\Gamma_S^{(2)}$ involves only the factors $C_F C_A$ and $C_F n_f$

Two-loop soft anomalous dimension $\Gamma_S^{(2)}$ for $e^+e^- \rightarrow t\bar{t}$



$\Gamma_S^{(2)}$ vanishes at $\beta = 0$, the threshold limit, and diverges at $\beta = 1$, the massless limit.

Logarithmic plot of $\Gamma_S^{(2)}$ for $e^+e^- \to t\bar{t}$



Small and large β behavior of $\Gamma_S^{(2)}$

Small β behavior - expand around $\beta = 0$

$$\Gamma_{S\,\exp}^{(2)} = -\frac{2}{27}\beta^2\left[C_F C_A(18\zeta_2 - 47) + 5C_F n_f\right] + \mathcal{O}(\beta^4)$$

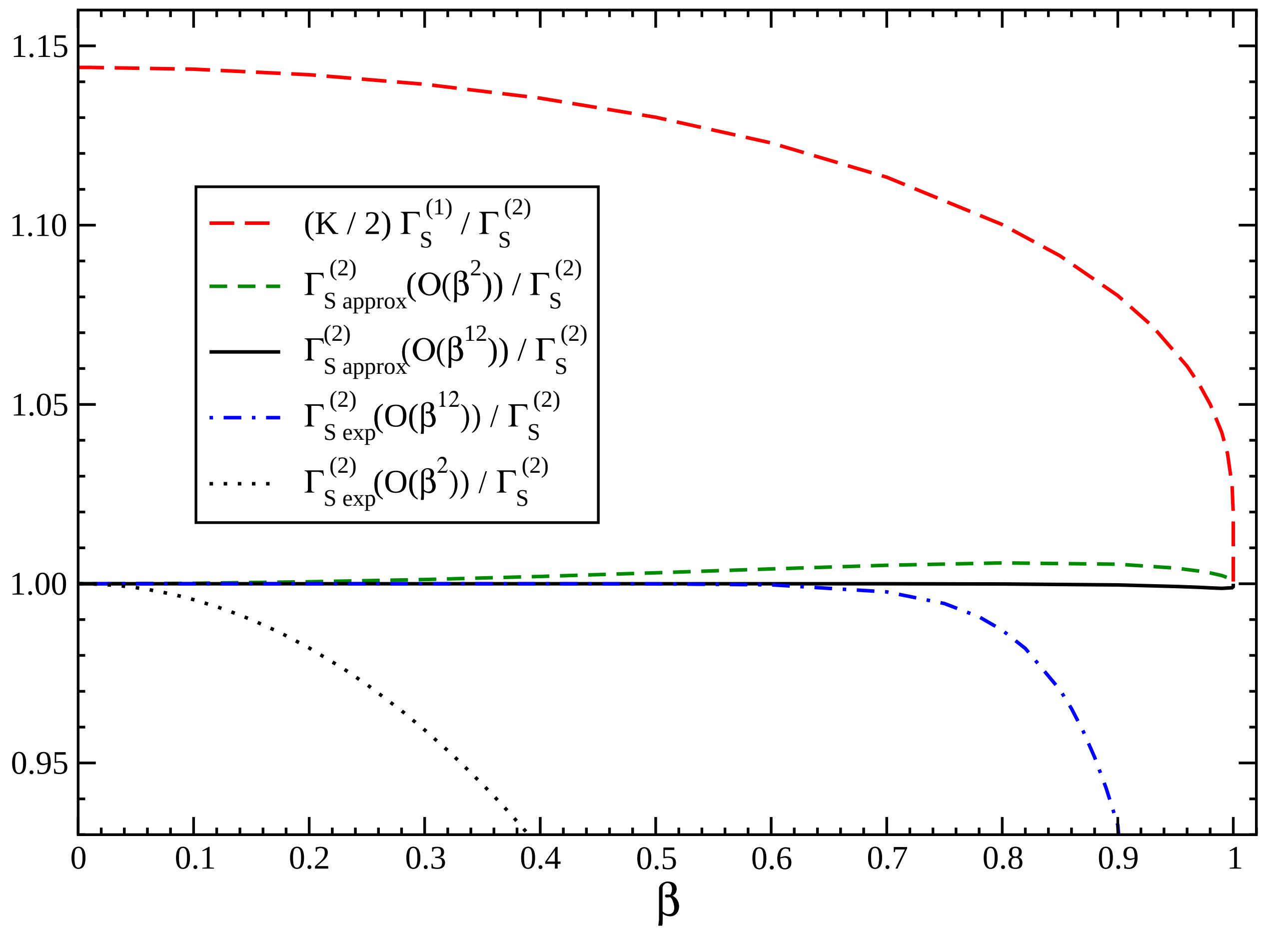
$\Gamma_S^{(2)}$ is an even function of β

Large β behavior - as $\beta \to 1$, $\Gamma_S^{(2)} \to \frac{K}{2}\Gamma_S^{(1)}$

Construct approximation for all β

$$\begin{aligned}\Gamma_{S\,\text{approx}}^{(2)} &= \Gamma_{S\,\exp}^{(2)} + \frac{K}{2}\Gamma_S^{(1)} - \frac{K}{2}\Gamma_{S\,\exp}^{(1)}\\ &= \frac{K}{2}\Gamma_S^{(1)} + C_F C_A\left(1 - \frac{2}{3}\zeta_2\right)\beta^2 + \mathcal{O}\left(\beta^4\right)\end{aligned}$$

Expansions and approximations to $\Gamma_S^{(2)}$ for $e^+e^- \to t\bar{t}$



$\Gamma_{S\,\rm approx}^{(2)}$ is a remarkably good approximation to complete $\Gamma_S^{(2)}$

In general $\Gamma_S^{(2)} \neq \frac{K}{2}\Gamma_S^{(1)}$, i.e. more complicated than massless case, for all heavy quark processes ($e^+e^- \longrightarrow t\bar{t}$ and heavy quark hadroproduction)

For $e^+e^- \longrightarrow t\bar{t}$ soft logarithms of the form $\frac{\ln^{n-1}(\beta^2)}{\beta^2}$

At the Tevatron and the LHC the $t\bar{t}$ cross section receives most contributions in the region around $0.3 < \beta < 0.8$ which peak roughly around $\beta \sim 0.6$.

NNLO approximate cross sections include soft-gluon contributions

Top quark production

Dominant process is pair production $q\bar{q} \longrightarrow t\bar{t}$ and $gg \longrightarrow t\bar{t}$

Very good agreement of theory (with soft-gluon corrections) with Tevatron data

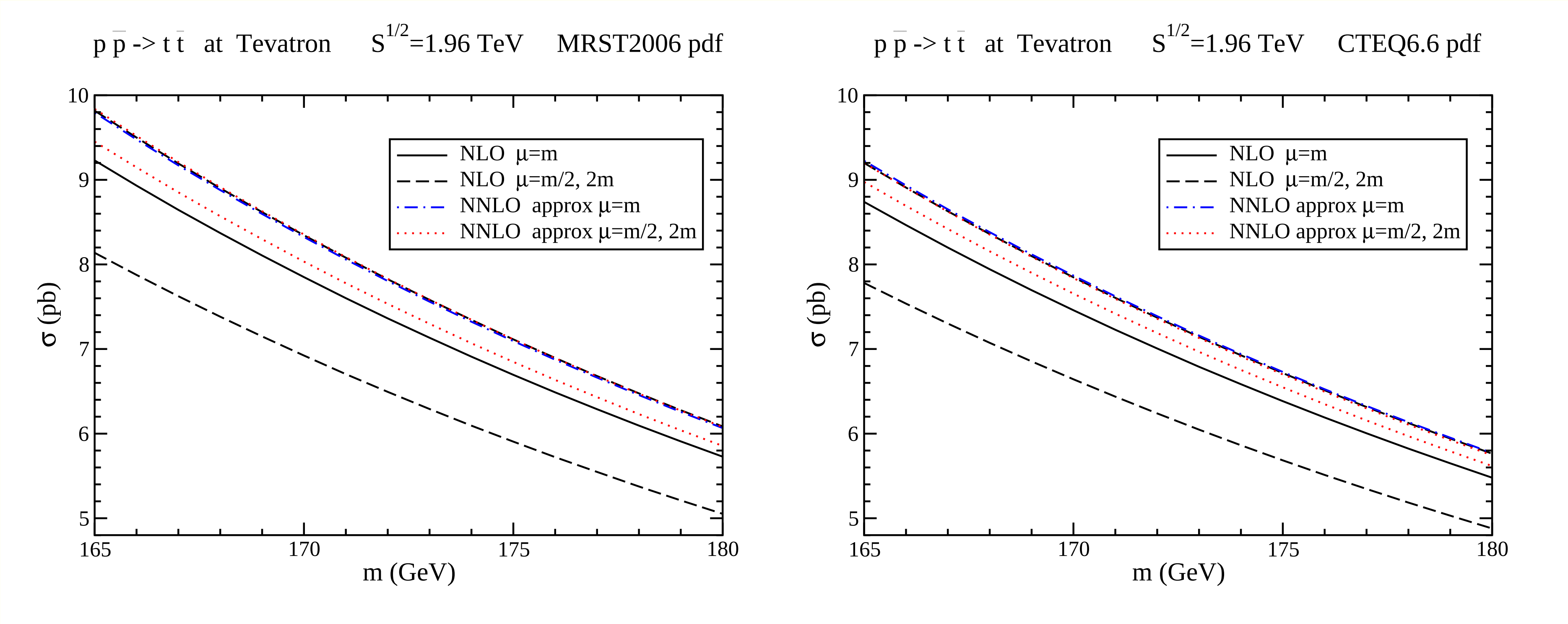
Recent observation for single top production - cross section consistent with theory

Opportunities for study of electroweak properties of the top

Top quark mass known with better precision

Resummed results and expansions at the differential level

Top quark pair cross section at the Tevatron

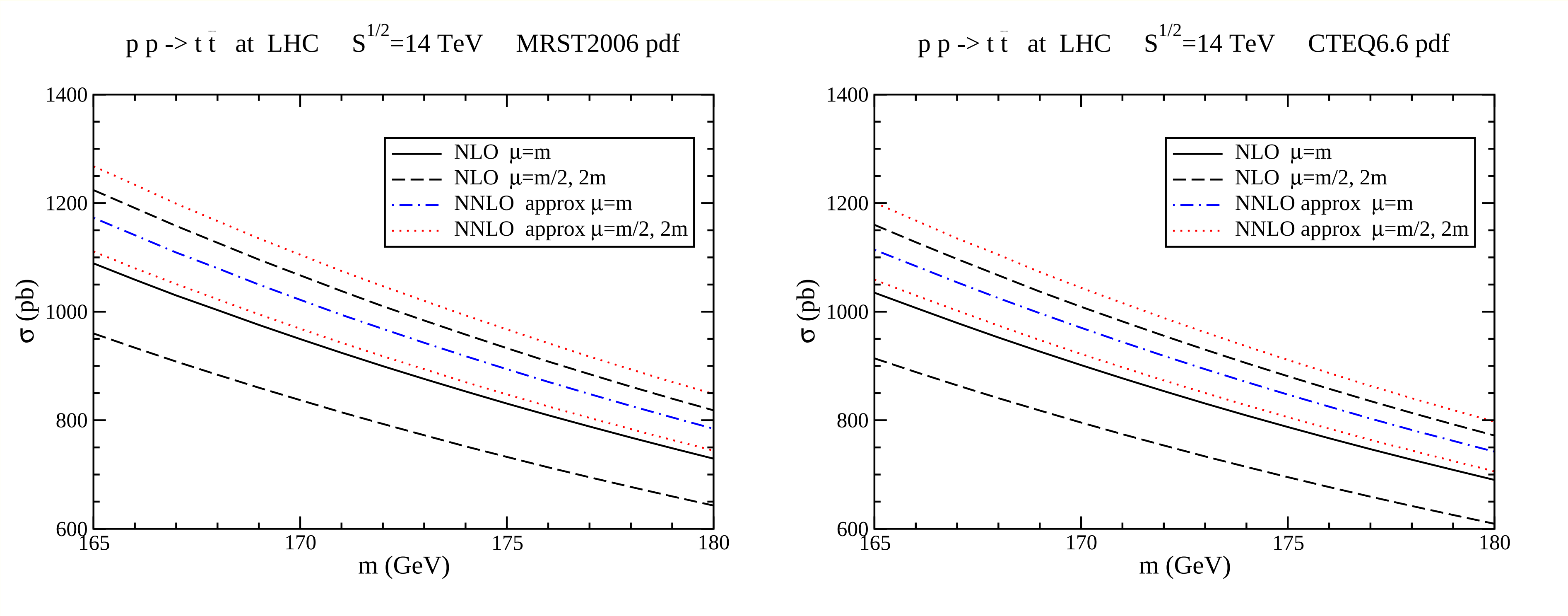


$$\sigma^{\rm NNLOapprox}_{p\bar{p}\to t\bar{t}}(1.96\,{\rm TeV}, m=172\,{\rm GeV}, {\rm MRST}) = 7.80 \pm 0.31 {}^{+0.03}_{-0.27} {}^{+0.23}_{-0.19}\,{\rm pb} = 7.80 {}^{+0.39}_{-0.45}\,{\rm pb}$$

$$\sigma^{\rm NNLOapprox}_{p\bar{p}\to t\bar{t}}(1.96\,{\rm TeV}, m=172\,{\rm GeV}, {\rm CTEQ}) = 7.39 \pm 0.30 {}^{-0.03}_{-0.20} {}^{+0.48}_{-0.37}\,{\rm pb} = 7.39 {}^{+0.57}_{-0.52}\,{\rm pb}$$

Kinematics uncertainty, scale variation, pdf errors **(NK, R. Vogt, PRD 78)**

Top quark pair cross section at the LHC

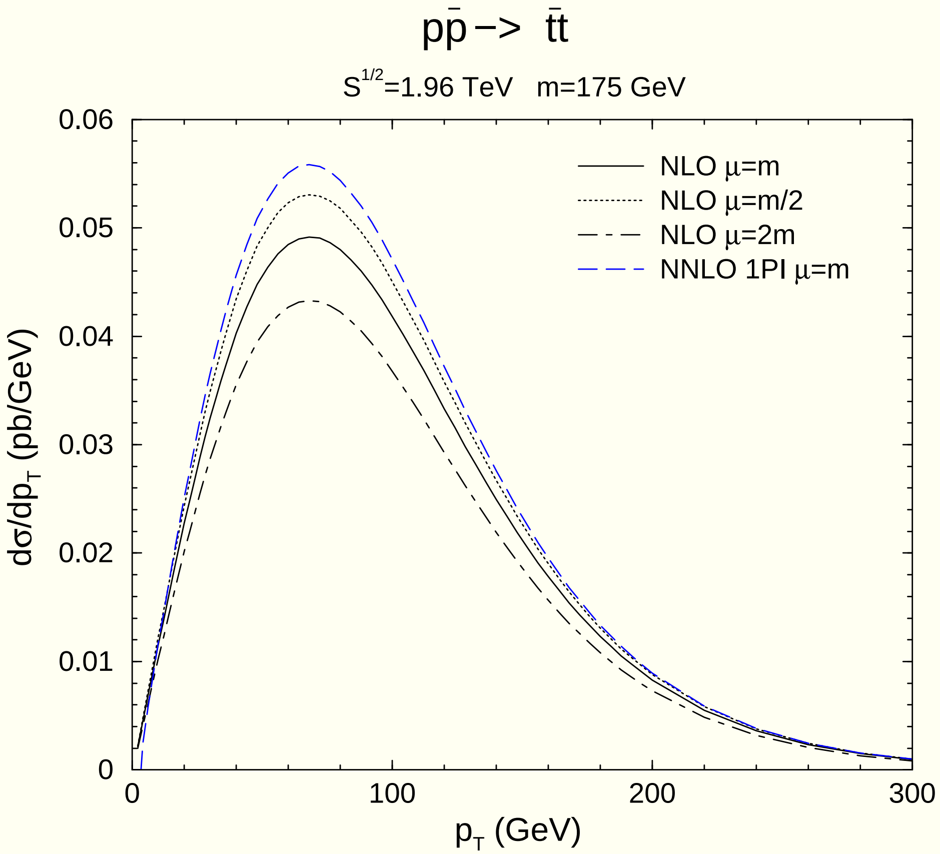


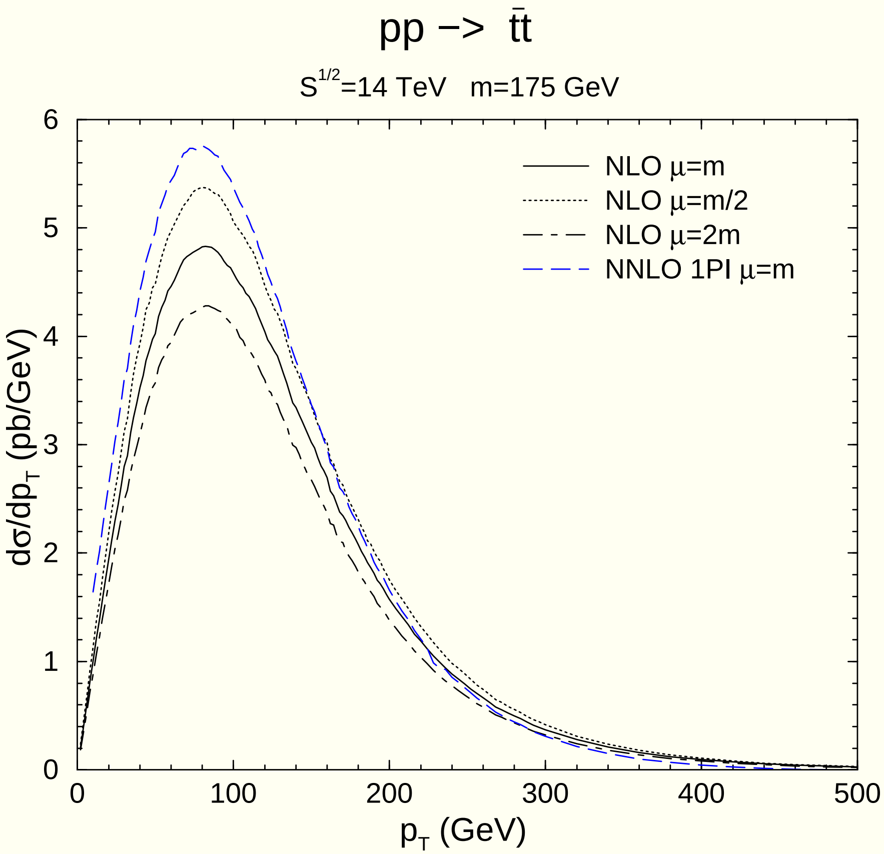
$$\sigma^{\rm NNLOapprox}_{pp\to t\bar{t}}(14\,{\rm TeV}, m=172\,{\rm GeV}, {\rm MRST}) \quad = \quad 968 \pm 4 \, {}^{+79}_{-50} \, {}^{+12}_{-13} \, {\rm pb} = 968 \, {}^{+80}_{-52} \, {\rm pb}$$

$$\sigma^{\rm NNLOapprox}_{pp\to t\bar{t}}(14\,{\rm TeV}, m=172\,{\rm GeV}, {\rm CTEQ}) \quad = \quad 919 \pm 4 \, {}^{+70}_{-45} \, {}^{+29}_{-31} \, {\rm pb} = 919 \, {}^{+76}_{-55} \, {\rm pb}$$

Kinematics uncertainty, scale variation, pdf errors **(NK, R. Vogt, PRD 78)**

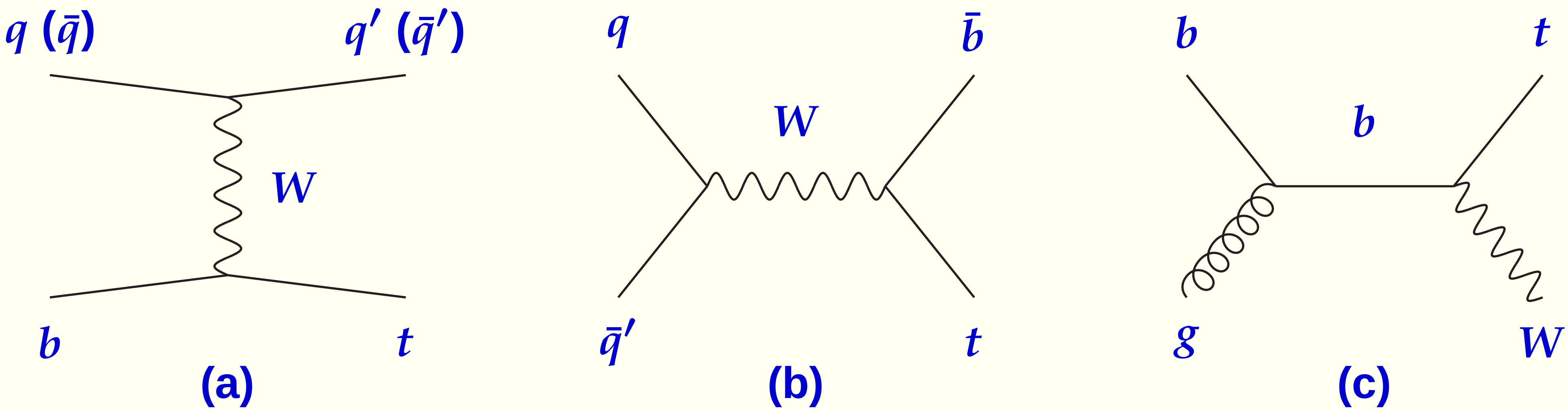
Top quark p_T distribution at Tevatron and LHC





Single top quark production

Partonic processes at LO

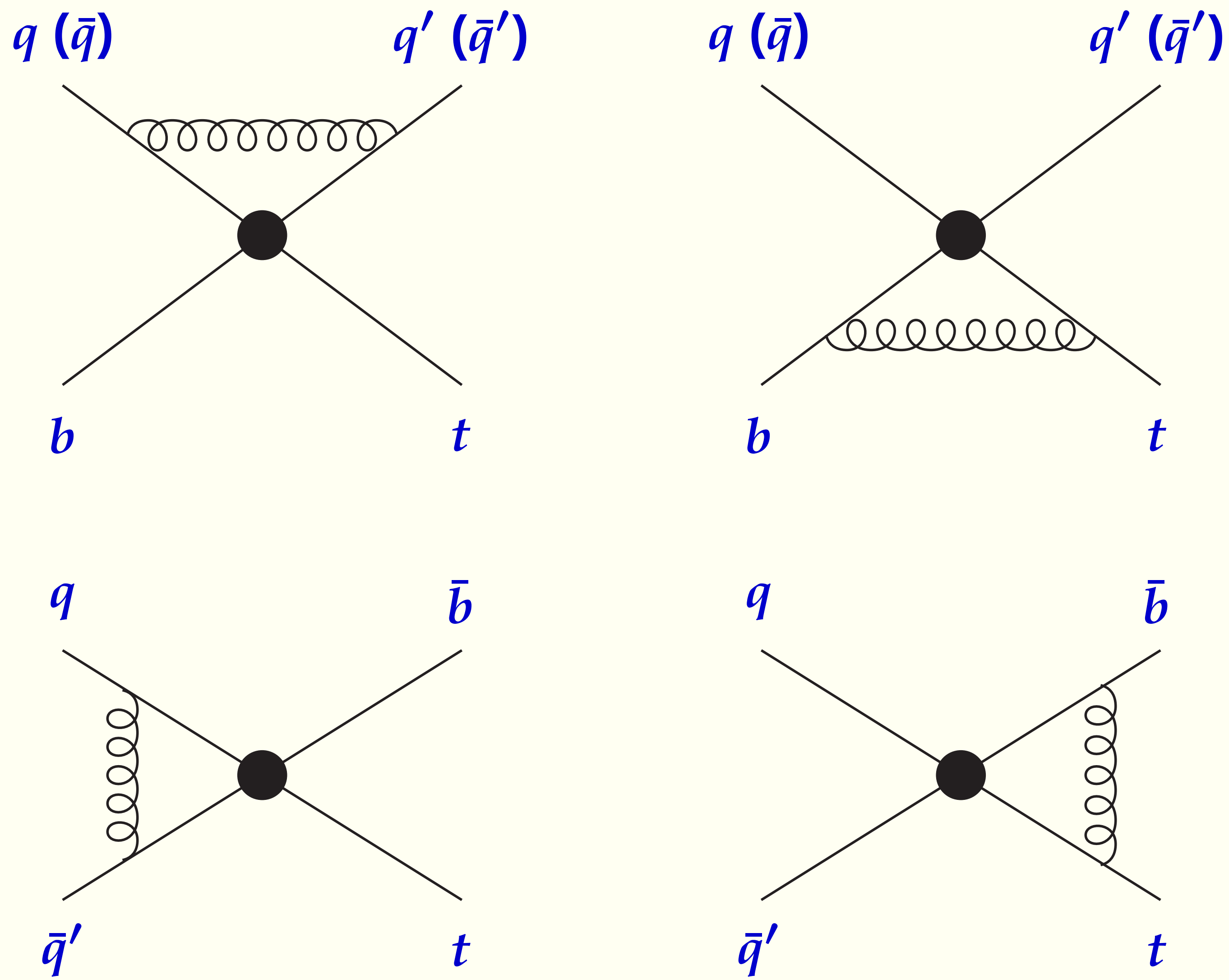


(a) t **channel:** $qb \longrightarrow q't$ **and** $\bar{q}b \longrightarrow \bar{q}'t$ **(**$ub \longrightarrow dt$ **and** $\bar{d}b \longrightarrow \bar{u}t$**, etc.)**

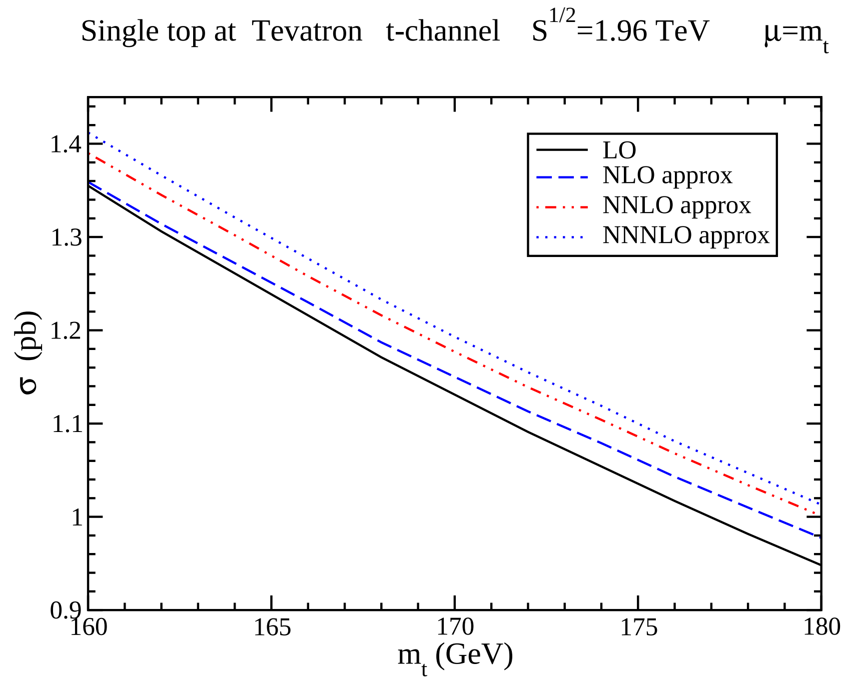
(b) s **channel:** $q\bar{q}' \longrightarrow \bar{b}t$ **(**$u\bar{d} \longrightarrow \bar{b}t$**, etc)**

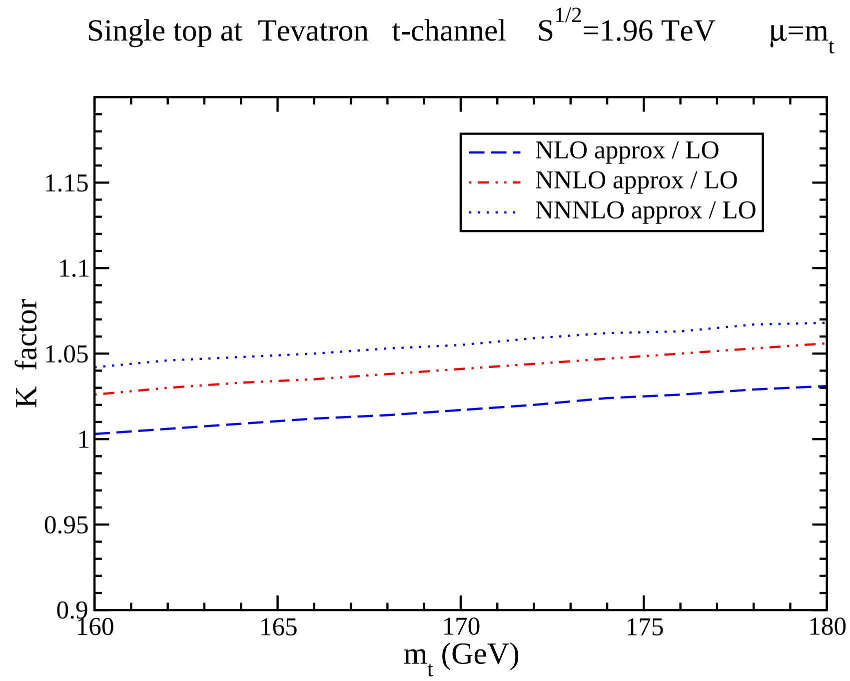
(c) associated tW **production:** $bg \longrightarrow tW^-$

One-loop eikonal vertex corrections to the soft function in the t and s channels



Single top production at the Tevatron - t channel





Matched cross section (exact NLO + soft gluon corrections through NNNLO)

$$\sigma^{t-\text{channel}}(m_t = 170\,\text{GeV}) = 1.17^{+0.02}_{-0.01} \pm 0.06\ \text{pb} = 1.17 \pm 0.06\ \text{pb}$$

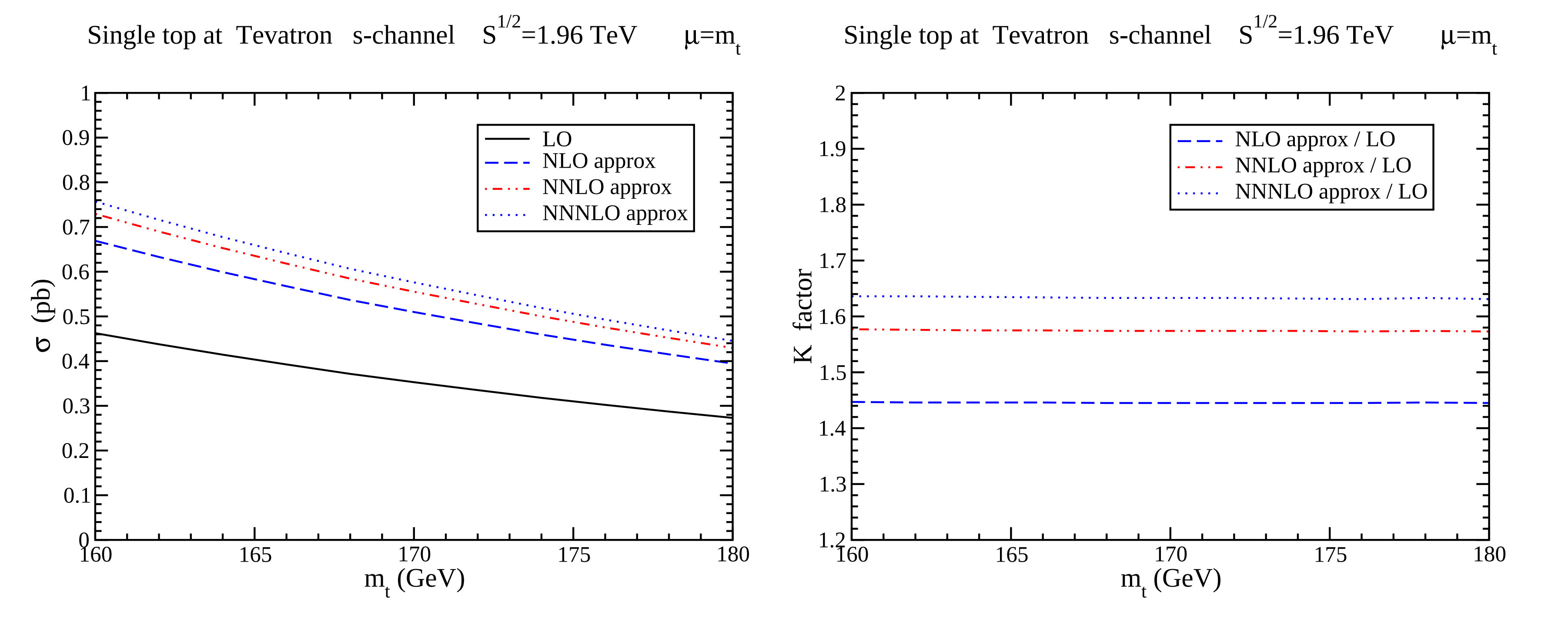
↑ scale ↑ pdf

$$\sigma^{t-\text{channel}}(m_t = 175\,\text{GeV}) = 1.08^{+0.02}_{-0.01} \pm 0.06\ \text{pb} = 1.08 \pm 0.06\ \text{pb}$$

↑ scale ↑ pdf

Cross section for anti-top production is identical

Single top production at the Tevatron - s channel



Matched cross section (exact NLO + soft gluon corrections through NNNLO)

$$\sigma^{s-\mathrm{channel}}(m_t = 170\,\mathrm{GeV}) = 0.56 \pm 0.02 \pm 0.01 \text{ pb} = 0.56 \pm 0.03 \text{ pb}$$

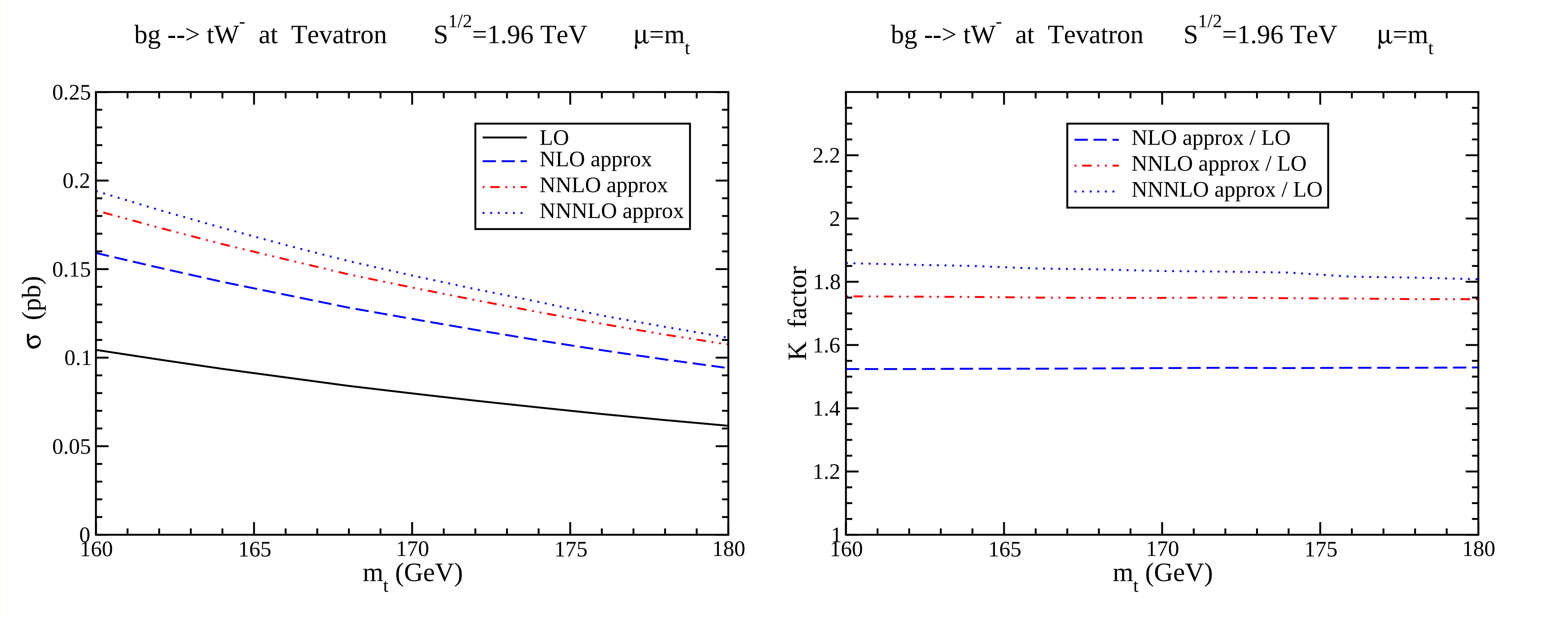
↑ scale ↑ pdf

$$\sigma^{s-\mathrm{channel}}(m_t = 175\,\mathrm{GeV}) = 0.49 \pm 0.02 \pm 0.01 \text{ pb} = 0.49 \pm 0.02 \text{ pb}$$

↑ scale ↑ pdf

Cross section for anti-top production is identical

Single top production at the Tevatron - tW channel



Approximate NNNLO cross section

$$\sigma^{tW}(m_t = 170\,\mathrm{GeV}) = 0.15 \pm \underset{\text{scale}}{0.02} \pm \underset{\text{pdf}}{0.03}\ \mathrm{pb} = 0.15 \pm 0.03\ \mathrm{pb}$$

$$\sigma^{tW}(m_t = 175\,\mathrm{GeV}) = 0.13 \pm \underset{\text{scale}}{0.02} \pm \underset{\text{pdf}}{0.02}\ \mathrm{pb} = 0.13 \pm 0.03\ \mathrm{pb}$$

Cross section for anti-top production is identical

Summary

- **Soft-gluon corrections and resummation**

- **Two-loop calculations in eikonal approximation**

- **Massive quarks involve further complications**

- $\Gamma_S^{(2)}$ **calculated for** $e^+e^- \rightarrow t\bar{t}$

- **Top pair and single top cross section at Tevatron and LHC**