# Resummation and top-pair production

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Work with:

Michael Czakon
arXiv:0812.0353
arXiv:0811.4119
in progress …

George Sterman and Ilmo Sung
arXiv:0903.3241
in progress …

# Current status

Top-pair cross-section, 20 years later:
The state of the art is still NLO QCD corrections ☺

Nason, Dawson, Ellis (1988-90)
Beenakker, Kuijf, van Neerven, Smith (1989)
Beenakker, van Neerven, Meng, Schuler, Smith (91)
Mangano, Nason, Ridolfi (1992)

Bernreuther et al. (2004)
M. Czakon, A.M. (2008)

- The only improvement over 20 years: now we know it analytically.
- Such slow progress is for a good reason:
  top is very hard to calculate (more later).
- Theoretical uncertainties are not as small as we would like them to be:

  NLO corrections 50%
  NLO uncertainty 10% (more details to follow).

How can we get the uncertainties down to few percent?

# What to do?

- Clearly, the best way is to just calculate the NNLO corrections.

This is very complicated! The complexity is ~ 3-loop massive box !!

The best strategy is known, and people are working hard on this:

M. Czakon and A. M.

- Second source: soft gluon (threshold) resummation.

The only source of new information in top production in the last > 10 years

Developed (NLL): Sterman et al mid-90's
Bonciani, Catani, Mangano, Nason '98
Applied (NLL): Kidonakis, Laenen, Moch, Vogt;
Cacciari et al, Moch Uwer, Czakon AM

In the following: myths and facts about the resummation ☺

# The Usual Wisdom for Doing Soft-Resummation

- The soft terms are dominant ones in the partonic cross-section. We can predict them – no need for calculations. So we are done.

- The partonic flux is largest close to threshold, i.e. the pdf's sample the threshold region and not the rest. So we are $(\text{done})^2$

All this is a very nice story.

Sometimes it is true.

Let us have a look.

It will turn out that, as usual, everything is in the details ☺

# The partonic cross-section

The observed cross-section is an integral over the product of:

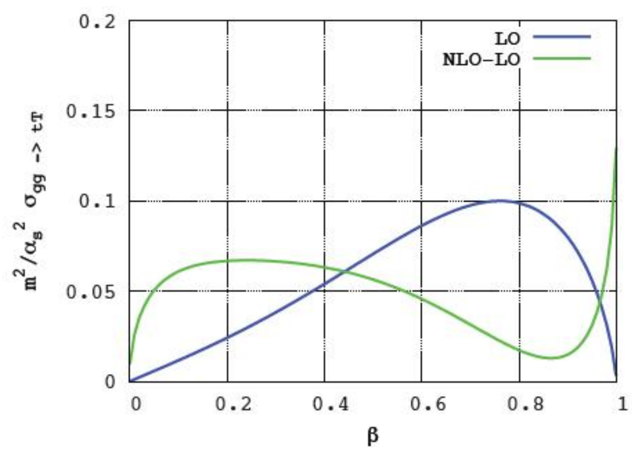
- Partonic cross-section,
- Partonic flux.

$$\sigma(s_{\rm had}) = \sum_{ij} \int_0^{\beta_{\rm max}} d\beta \, \Phi(\beta) \, \hat{\sigma}_{\rm part}(\beta)$$

$$\rho = \frac{4m_t^2}{s}$$

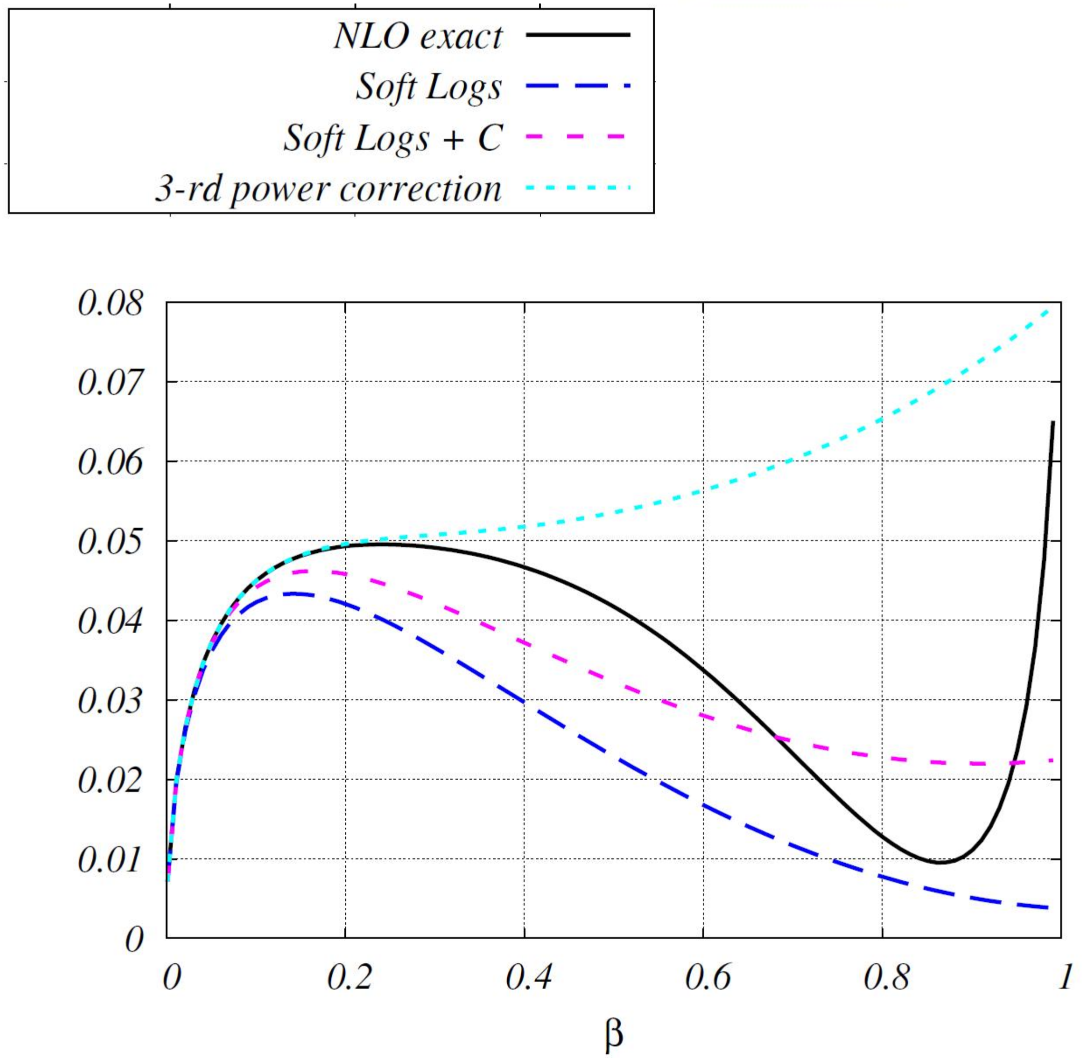
$$\beta = \sqrt{1-\rho}$$

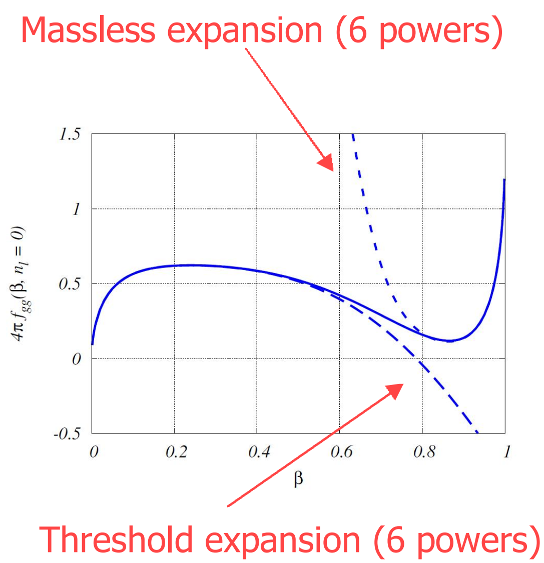
$$\eta = \frac{s}{4m_t^2} - 1$$




Large NLO corrections!

# The partonic cross-section: power expansion



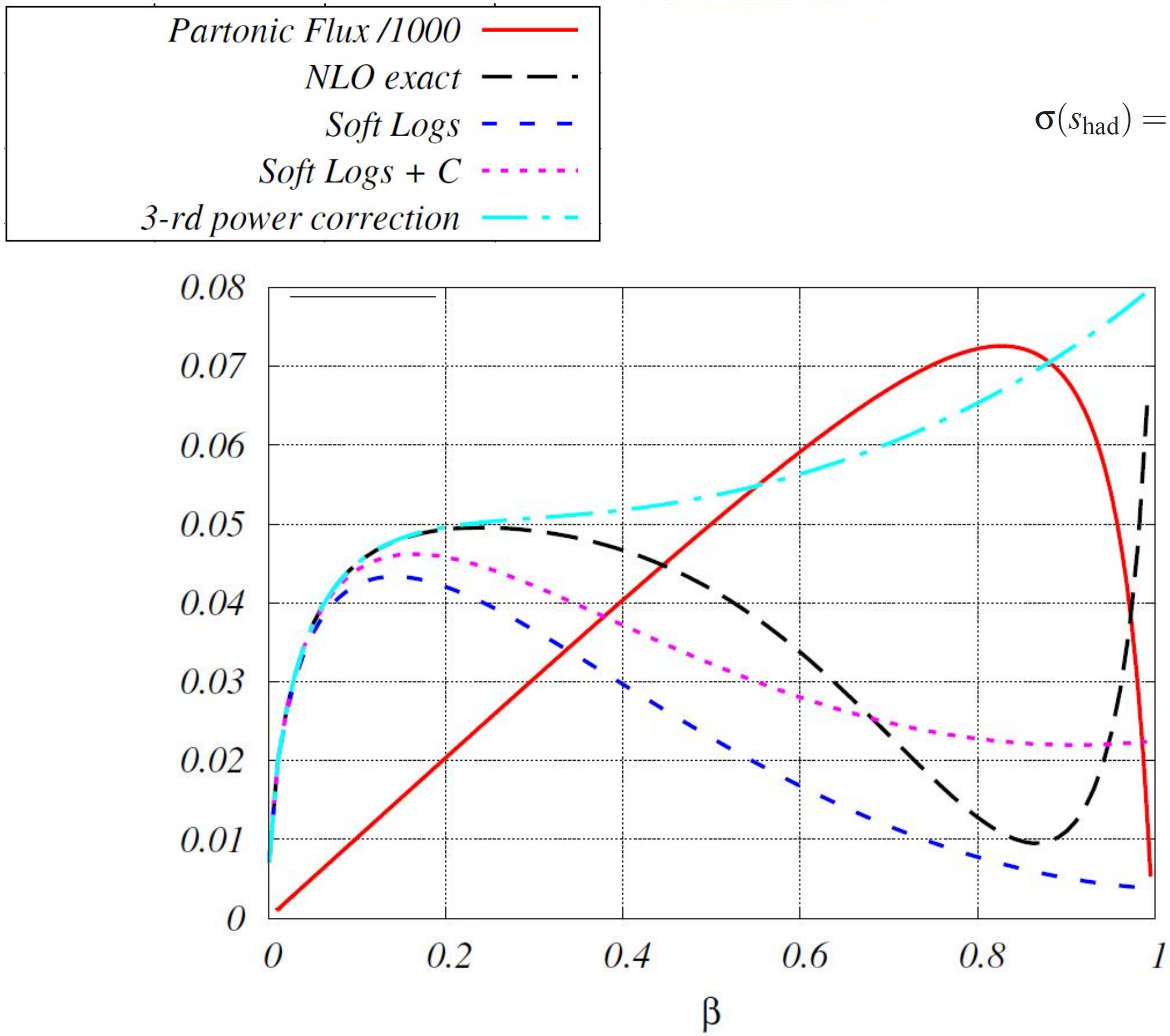




Czakon, AM '08

The sub-leading powers are large (and divergent)!

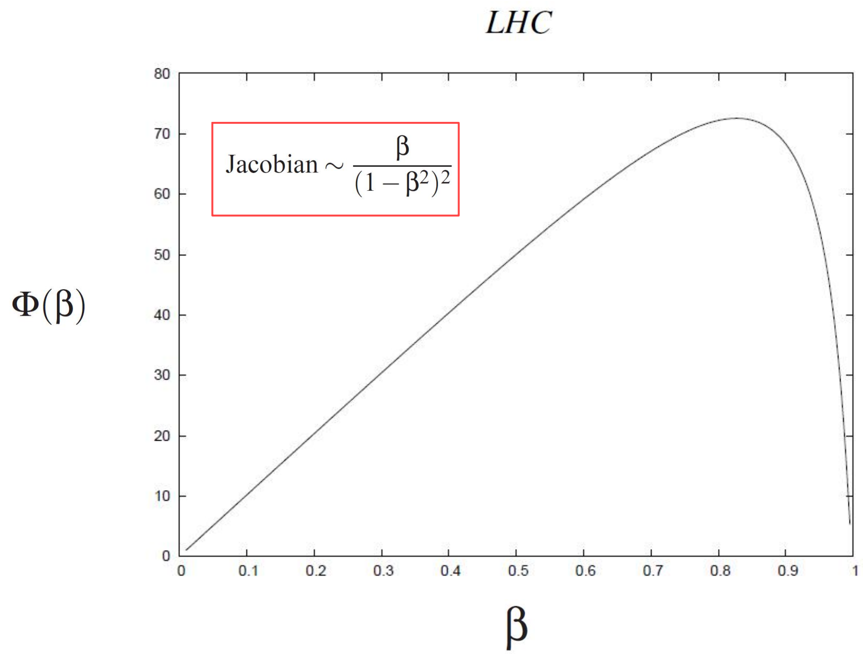
# Does the flux change things?

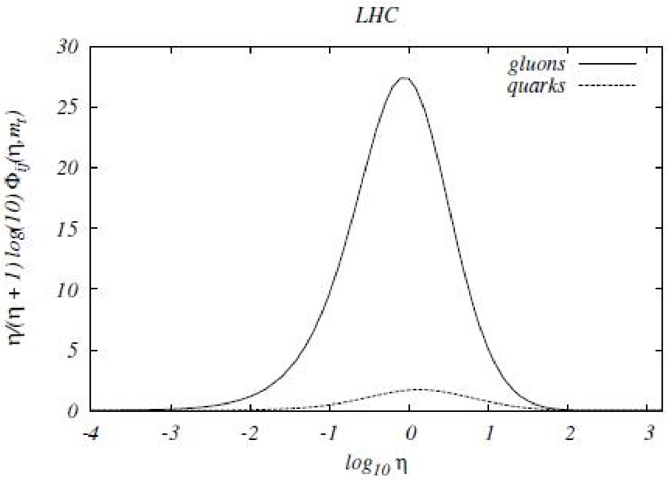
$$\sigma(s_{\text{had}}) = \sum_{ij} \int_0^{\beta_{\max}} d\beta \, \Phi(\beta) \, \hat{\sigma}_{\text{part}}(\beta)$$

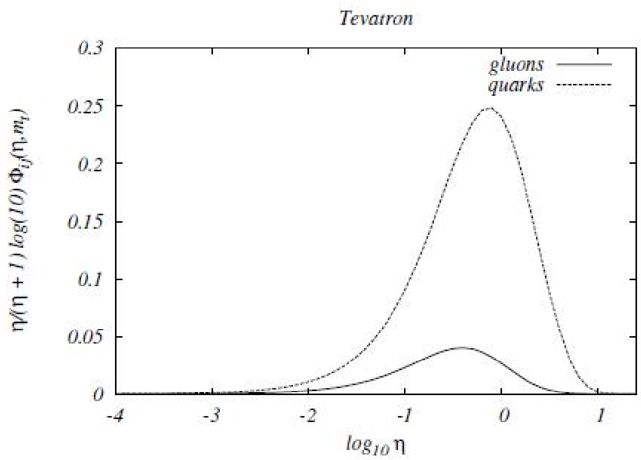



The flux does not predominantly sample the threshold region!

# The partonic flux



# What do we learn from the known NLO results?

- The soft approximation is not a good approximation to FO
- The partonic flux does not help much either ☺
- Yet, at hadronic level, the soft approximation is only few % away from the exact NLO.

This requires the hard matching constant!
Constant itself not predicted by the resummation;
comes from FO calculation.

Similar observation in Higgs
talk by de Florian

Let's say we try to use it for guessing NNLO;

how does the soft resummation work?

# How is the threshold resummation done?

The resummation of soft gluons is driven mostly by kinematics:

Sterman ‘87
Catani, Trentadue ‘89

- Only soft emissions possible due to phase space suppression (hence kinematics)
- That’s all there is for almost all “standard” processes: Higgs, Drell-Yan, DIS, $e^+e^-$

Key: the number of hard colored partons < 4

In top pair production (hadron colliders) new feature arises:

Color correlations due to soft exchanges (n>=4)

Non-trivial color algebra in this case.

# How is the threshold resummation done?

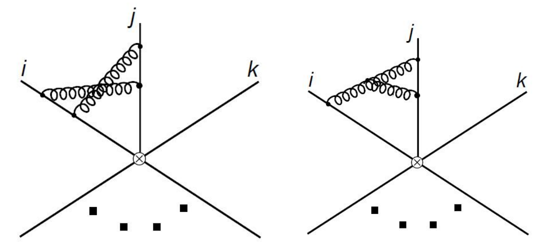
For a general n-colored parton amplitude we have:

$$M_I = J \cdot S_{IJ} \cdot H_J$$

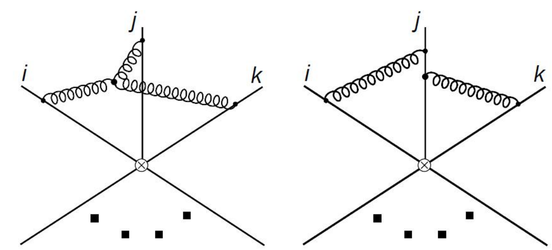
Soft function:

- contains all soft color correlations,
- matrix (in color space) for > 3 partons

Two-particle correlations:
(easy ☺ - massive formfactor)




Three-particle correlations:
(very hard – massive box)




They are never small no matter how soft the gluons!

# How is the threshold resummation done?

The soft function (and its anomalous dimension) control the single poles of n-loop amplitudes; appear in resummations at hadron colliders.

Talks by Becher, Kidonakis, Neubert

In the massive case it is know explicitly at 1 loop

Kidonakis, Sterman '97
Catani, Dittmaier, Trocsanyi '01

First 2-loop results appeared very recently:

A.M., Sterman, Sung '09
Becher, Neubert '09
N. Kidonakis '09

The results are not totally explicit (yet),
but few very important properties were established:

- The matrix diagonalizes at 2-loops close to partonic threshold,
- Its structure is very different from the massless case.

# How is the threshold resummation done?

How is all that related to top resummation?

The relation $M_I = J \cdot S_{IJ} \cdot H_J$

Implies that, close to threshold, one has:

Hard matching coefficients

$$M_I \sim e^{\Gamma_{IJ}} \cdot H_J = e^{\Gamma_1}\, H_1 + e^{\Gamma_8}\, H_8$$

Singlet-octed eigenvalues

Implication: for top-pair we need two independent Sudakov exponents.

$$\sigma_{ij}^{\mathrm{TOT}}(N) = \sigma_{ij,\mathbf{1}}(N) + \sigma_{ij,\mathbf{8}}(N)$$

$$\sigma_{ij,\mathbf{I}}(N) = \sigma_{ij,\mathbf{I}}^{\mathrm{Born}}(N)\ \sigma_{ij,\mathbf{I}}^{\mathrm{H}}\ \Delta_{ij,\mathbf{I}}(N)$$

i,j=gg,qq

N-Mellin moment: $f(N) = \int_0^1 \rho^{N-1} f(\rho) d\rho.$

Sudakov exponent

# Get the cross-section

How we put all this to work?

- Match fixed order and resummed results:

$$\sigma_{RESUM} = \sigma_{NLO} + \sigma_{SUDAKOV} - \sigma_{OVERLAP}$$

Known at NLO, not at NNLO

- $\sigma_{NLO}$ is known exactly,
- $\sigma_{SUDAKOV}$ : anomalous dimensions and matching coefficients needed.

Known at NLO M. Czakon, A.M. '08

# Get the cross-section

The NLO hard matching coefficients were calculated only very recently:

$$\sigma^{H}_{q\bar{q},\mathbf{1}} = \mathcal{O}(\alpha_s^2),$$

$$\sigma^{H}_{q\bar{q},\mathbf{8}} = 1+\frac{\alpha_s}{\pi}\left[C_F\left(-8+\frac{2\pi^2}{3}+3\log 2\right)+C_A\left(\frac{59}{9}-\frac{\pi^2}{4}-3\log 2\right)\right.$$
$$\left. + \; n_l\left(-\frac{5}{9}+\frac{2\log 2}{3}\right)-\frac{8}{9}+\log\left(\frac{\mu^2}{m^2}\right)\left(-\frac{3}{2}C_F+\frac{11}{6}C_A-\frac{n_l+1}{3}\right)\right]+\mathcal{O}(\alpha_s^2)$$

M. Czakon, A.M. '08

$$\sigma^{H}_{gg,\mathbf{1}} = 1+\frac{\alpha_s}{\pi}\left[C_F\left(-5+\frac{\pi^2}{4}\right)+C_A\left(1+\frac{5\pi^2}{12}\right)-\frac{1}{3}\log\left(\frac{\mu^2}{m^2}\right)\right]+\mathcal{O}(\alpha_s^2)$$

$$\sigma^{H}_{gg,\mathbf{8}} = 1+\frac{\alpha_s}{\pi}\left[C_F\left(-5+\frac{\pi^2}{4}\right)+C_A\left(3+\frac{7\pi^2}{24}\right)-\frac{1}{3}\log\left(\frac{\mu^2}{m^2}\right)\right]+\mathcal{O}(\alpha_s^2)$$

Surprisingly simple expressions!

Agree with results from quarkonium production:

Hagiwara, Sumino, Yokoya '08
Kuhn, Mirkes '93
Petrelli, Cacciari, Greco, Maltoni, Mangano '97

# Numerical findings

At that point we have everything we need at NLO.

By putting it together we find unexpected numerical effects:

Threshold expansion $\beta \to 0$ :
(i.e. $4m^2 \to s$)

$$f_{gg}^{(1)}(\beta) = \frac{1}{4\pi^2} f_{gg}^{(0)}(\beta) \left( \left( C_F - \frac{(N^2-4)\,C_A}{2(N^2-2)} \right) \frac{\pi^2}{2\beta} + 2C_A \log^2(8\beta^2) - \frac{(9N^2-20)C_A}{N^2-2} \log(8\beta^2) \right.$$
$$+ C_A \left( \frac{21N^2-50}{N^2-2} - \frac{(17N^2-40)\,\pi^2}{24\,(N^2-2)} + \frac{(N^2-4)\log 2}{N^2-2} - 2\log^2 2 \right)$$
$$\left. + C_F \left( -5 + \frac{\pi^2}{4} \right) + o(\beta) \right). \quad (27)$$

Extraction of the constant in the threshold limit:

$$C_A \left( \frac{21N^2-50}{N^2-2} - \frac{(17N^2-40)\,\pi^2}{24\,(N^2-2)} + \frac{(N^2-4)\log 2}{N^2-2} - 2\log^2 2 \right) + C_F \left( -5 + \frac{\pi^2}{4} \right)$$
$$= \frac{1111}{21} - \frac{283\pi^2}{168} + \frac{15\log 2}{7} - 6\log^2 2 \simeq 34.88\,,$$

$$\frac{768\pi}{7} a_0^{gg} \simeq 37.25\,.$$

Used in earlier literature

Interested lesson in numerics: the earlier numeric cross-section is better than 1% but its derivative (close to threshold) is 7% off.

# Numerical findings

From resummation, the following 2 loop logs can be predicted:

$$\sigma_{gg}(\beta) = \sigma_{gg}^{\text{Born}}(\beta) + \frac{\alpha_s}{4\pi}\sigma_{gg}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \sigma_{gg}^{(2)} + O(\alpha_s^3)$$

$$\sigma_{gg}^{(2)} = \sigma_{gg}^{\text{Born}}(\beta)\left(4608\log^4\beta + 1894.9\log^3\beta - 3.4811\log^2\beta + O(\log\beta)\right)$$

Moch Uwer '08

It turns out the coefficient of $\ln^2(\beta)$ is of the form: $-14306.9505 + 384C_3$

where: $C_3 = 37.23$ Old numeric value

$C_3 = 34.88$ New analytic value

Therefore the coefficient of $\ln^2(\beta)$ becomes -912.35

Note: the reason is pure numerics!

i.e. a change by a factor of 260 !

# Numerical Findings

$$\sigma_{gg}^{\mathrm{H\,(BCMN)}} = 1 + \frac{\alpha_s}{\pi}\, 14.39 + \mathcal{O}(\alpha_s^2),$$
$$\sigma_{gg}^{\mathrm{H\,(BCMN)}}\Big|_{C_3\ \mathrm{exact}} = 1 + \frac{\alpha_s}{\pi}\, 12.04 + \mathcal{O}(\alpha_s^2),$$
$$\sigma_{gg,\mathbf{1}}^{\mathrm{H}} = 1 + \frac{\alpha_s}{\pi}\, 9.16 + \mathcal{O}(\alpha_s^2),$$
$$\sigma_{gg,\mathbf{8}}^{\mathrm{H}} = 1 + \frac{\alpha_s}{\pi}\, 13.19 + \mathcal{O}(\alpha_s^2),$$

$C_3$ numerics: -5%,
color singlet channel: -12%,
color octet channel: -3%,

Their implications :

- ✓ Formally these effects are beyond NLL; yet significant numerically
- ✓ Must be taken into account beyond NLL !

# Top quark pair: "the numbers"

The central values (LHC):

- FO NLO / FO LO: 50%
- NLL / FO NLO: 4%
- New NLO effects / FO NLO: 1-1.5% Czakon, AM
- Beyond NLL effects / FO NLO: 0.8% Moch, Uwer

Current theory error estimate (NLO/NLL): ~ 10%

Uncertainty =/= just scale variation !!!

Important: No genuine NNLO term is known (could easily give 5%) !

# Top quark pair: state of the art

Comparison of central values for:

- $m_{top}$=172.4 GeV
- $\mu$=m
- correct exact hard matching coefficients
- Coulombic effects not elaborated upon.

$\alpha_s(M_Z)$ :

CTEQ 6.6: 0.118
MRST 2006 nnlo: 0.119
MSTW 2008 nnlo: 0.117
MSTW 2008 nlo: 0.120

**MRST 2006 nnlo**

NLO = 890 pb
NLO+NLL = 918 pb

**CTEQ 6.6**

NLO = 844 pb
NLO+NLL = 871 pb

**MSTW 2008 nnlo**

NLO = 857 pb
NLO+NLL = 885 pb

**MSTW 2008 nlo**

NLO = 906 pb
NLO+NLL = 935 pb

5.4 %

1.6 %

7.3 %

Czakon, AM in progress

# Conclusions

NLO/NLL exact and complete only now. M. Czakon, A.M. (2008)
This is the most complete (and consistent) description of top-pair we have at the moment.

- Soft gluon approximation is not really good at parton level
- Partonic flux does not predominantly sample the threshold region (LHC)

- Yet, when integrated to hadron level SG approx produces good results!

Question: The uncertainty/how to = ? (it is close to 10%, not 3%)

- Now we have good understanding of the "A-constant":

Bonciani, Catani, Mangano, Nason '98

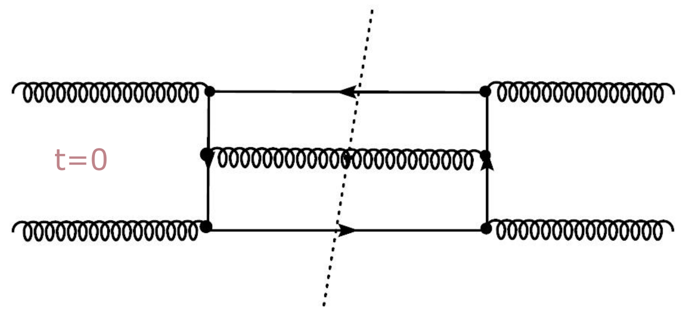
Sub-leading powers are non-negligible; in fact are equally important!

- The new set MSTW 2008 NNLO is (much) closer to CTEQ6.6 (for top-pair)

- Two-loop anomalous dimensions will appear. That is the best that can be done before the NNLO calculation is ready!

# Top quark: How complicated is the NLO?

Here are few sample diagrams at NLO:

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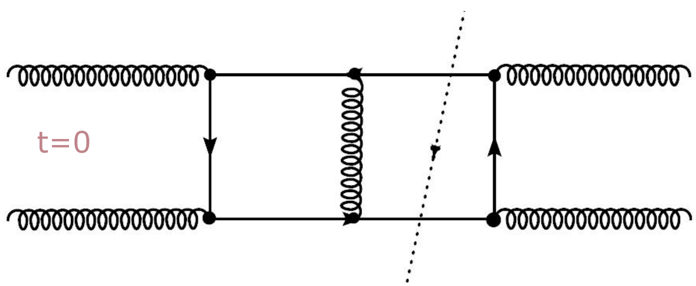


+ crossed

- Note: these are 2 loop (cut) boxes with masses.
  Not studied before.

# Main details of the exact NLO calculation

- For 20 years $\sigma_{TOT}$ was known as a numerically derived fit
- Newly calculated analytical results (new techniques):
  - The whole problem is mapped into 37 masters (real+virtual)
  - We find that the cross-section develops new unphysical singularities!
  - Appearance of elliptic functions,
  - We confirm the high numerical accuracy of the earlier FO results (< 1%)

Czakon, AM '08