Threshold resummation for pair-production of coloured heavy particles at hadron colliders

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- Introduction and motivation
- Coulomb and threshold-logarithm resummation
- Squark-antisquark cross section at NLL
- Conclusions

$$p_i(r_i) + p_j(r_j) \rightarrow H(R)H'(R') + X \qquad H, H' \equiv t, \ \tilde{q}, \ \tilde{g}, \ \dots$$

Partonic cross section for pair-production of heavy particles at hadron colliders contains terms kinematically enhanced in the partonic threshold region $\hat{s} \sim 4\bar{m}^2$, $\bar{m} \equiv (m_H + m_{H'})/2$.

- Coulomb singularities: ~ αⁿ_s/βⁿ, β = √1 − 4m̄²/ŝ ⇔ Coulomb interactions of slowly-moving particles
- Threshold logarithms: $\sim \alpha_s^n \ln^{2n} \beta^2 \Leftrightarrow$ soft-gluon exchange

Small coupling but effectively "non-perturbative" dynamics \Rightarrow Must be resummed to all orders when the partonic threshold region dominates the total hadronic cross section!

- Absolute normalisation of the total cross section
- Generally observed to reduce factorisation-scale dependence

Resummation in Mellin-moment space

In recent years threshold resummation has been applied to several different processes:

- $t\bar{t}$ [Catani et al. '96; Bonciani et al '98; Kidonakis et al '01; Moch, Uwer '08; Czakon, Mitov '08]
- Squarks, gluinos [Kulesza, Motyka '08; Langenfeld, Moch '09]
- Colour-octet scalars [Idilbi, Kim, Mehen, '09]

The theoretical basis for resummation is the <u>factorisation</u> of hard and soft contributions in the threshold region (more generally for $Q^2 \sim \hat{s}$, even if $Q^2 \neq 4\bar{m}^2$)

$$\hat{\sigma} = H \otimes S$$

• Resummation traditionally performed in Mellin-moment space:

$$H \otimes S \Rightarrow H(N)S(N) \quad \alpha_s^n \ln^{2n} \beta \Rightarrow \alpha_s^n \ln^{2n} N$$

- Threshold logs exponentiated by solving evolution equations for H(N) and S(N).
- Requires numerical inversion of the Mellin transform and prescription to deal with Landau poles in the integrand

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Resummation in momentum space

In this talk: apply formalism proposed by [Neubert and Becher '06] to resummation of the total cross section for $p_i p_j \rightarrow HH' + X$.

- Based on effective-theory description of the process (<u>SCET</u>+NRQCD)
- Threshold resummation performed directly in momentum space

Extra factorisation of the cross section near the true partonic threshold $\hat{s} \sim (m_H + m_{H'})^2$



- <u>Hard function C</u>: depends on the precise nature of the physics model
- Process-independent soft function W: analogous to S in usual factorisation formula
- <u>Potential function J</u> encoding Coulomb interactions

J and W depend only on SU(3) representation of p_i, p_j, H, H'

The formalism

Near threshold $(\beta \ll 1)$ partonic cross section receives contributions from hard $(k^2 \sim \bar{m}^2)$ and long-distance $(k^2 \leq \bar{m}^2 \beta^2)$ dynamical modes:

- collinear: $k_{-} \sim m, k_{+} \sim m\beta^2, k_{\perp} \sim m\beta$

Long-distance modes

- **potential** : $k_0 \sim m\beta^2$, $|\vec{k}| \sim m\beta$ (\Leftrightarrow Coulomb singularities)
- soft: $k_0 \sim |ec{k}| \sim m eta^2$

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Long-distance modes

- **potential** : $k_0 \sim m\beta^2$, $|\vec{k}| \sim m\beta$ (\Leftrightarrow Coulomb singularities) - **soft**: $k_0 \sim |\vec{k}| \sim m\beta^2$

The full theory is matched on an effective Lagrangian from which hard modes are removed.

$$\mathcal{L}_{\text{full}} \rightarrow \mathcal{L}_{\text{SCET}} + \mathcal{L}_{\text{PNRQCE}}$$

• \mathcal{L}_{SCET} : describes interactions of collinear (ξ_c , A_c) and soft (A_s) modes

$$\mathcal{L}_c \quad = \quad \bar{\xi}_c \left(i n \cdot D + i \not\!\!D_{\perp c} \frac{1}{i \overline{n} \cdot D_c} i \not\!\!D_{\perp c} \right) \frac{\overline{\eta}}{2} \, \xi_c - \frac{1}{2} \, \mathrm{tr} \left(F_c^{\mu\nu} F_{\mu\nu}^c \right)$$

• \mathcal{L}_{PNRQCD} : contains interactions of potential (ψ, ψ', A_p) and soft (A_s) modes

$$\mathcal{L}_{\text{PNRQCD}} = \psi^{\dagger} \left(i D_s^0 + \frac{\vec{\partial}^2}{2m_H} + \frac{i \Gamma_H}{2} \right) \psi + \psi'^{\dagger} \left(i D_s^0 + \frac{\vec{\partial}^2}{2m_{H'}} + \frac{i \Gamma_{H'}}{2} \right) \psi' + \int d^3 \vec{r} \left[\psi^{\dagger} \mathbf{T}^{(R)a} \psi \right] (x + \vec{r}) \left(\frac{\alpha_s}{r} \right) \left[\psi'^{\dagger} \mathbf{T}^{(R')a} \psi' \right] (x)$$

Structure of EFT amplitudes and soft-gluon decoupling

$$\mathcal{A}_{\{a\}}(pp' \to HH'X) = \sum_{\ell,i} c_{\{b\}}^{(i)} C_{pp'}^{(\ell,i)}(4\bar{m}^2,\mu) \langle H_{a_3}H'_{a_4}X|\mathcal{O}_{pp',\{b\}}^{(\ell)}(0)|p_{a_1}p'_{a_2}\rangle$$

- Tower of <u>hard coefficients</u> encoding the short-distance structure of the pair-production process
- $\mathcal{O}_{pp', \{b\}}^{(0)} \sim \phi_{\bar{c}, b_2} \phi_{c, b_1} \psi'_{b_4} \psi_{b_3}$, with $\phi_c = \{W_c \xi_c, \mathcal{A}_c\}$.
- Matrix element evaluated using the EFT Lagrangian ⇒ <u>soft gluons</u> interacting with everything and <u>Coulomb interactions</u> between the two heavy particles

Structure of EFT amplitudes and soft-gluon decoupling

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At leading order in PNRQCD soft gluons can be removed from the effective Lagrangian via a field redefinition involving <u>soft Wilson lines</u>:

$$\begin{split} \phi_{c}(x) &= S_{n}^{(r_{i})}(x_{-})\phi_{c}^{(0)}(x) \\ \psi(x) &= S_{v}^{(R)}(x_{0})\psi_{b}^{(0)}(x) \\ S_{n}^{(r_{i})\dagger}(in \cdot D)S_{n}^{(r_{i})} &\Rightarrow in \cdot D_{c} \\ S_{v}^{(R)\dagger}(iD_{s}^{0})S_{v}^{(R)} &\Rightarrow i\partial^{0} \end{split}$$

After soft-gluon decoupling:

$$\hat{\sigma}_{pp'}(\hat{s},\mu) = \frac{1}{2\hat{s}N_{pp'}} \sum_{i,i',\ell,\ell'} \sum_{R_{\alpha}} C_{pp'}^{(\ell,i)} C_{pp'}^{(\ell',i')*} \int d\omega J_{R_{\alpha}}^{(\ell,\ell')}(E-\frac{\omega}{2}) W_{ii'}^{R_{\alpha}}(\omega,\mu)$$

Colour structure of the factorisation formula

- Hard coefficients $C_{pp'}^{(l,i)}$ is decomposed on a basis of colour operators $o_{\{c\}}^{(i)}$
- Potential function $J_{R_{\alpha}}^{(\ell,\ell')}$ is projected over <u>irreducible representations</u> of the final state: $R \otimes R' = \sum_{\alpha} R_{\alpha}$
- $W_{ii'}^{R_{\alpha}}$ is given by a <u>set of matrices</u> acting on the vector space spanned by $o_{\{c\}}^{(i)}$

$$P_{\{k\}}^{R_{\alpha}} \left\{ \begin{array}{l} \psi_{[a]} \\ \psi_{[a]} \\$$

Different soft function $W_{ii'}^{R_{\alpha}}$ for each irreducible representation R_{α} !

All-order colour structure of $W_{ii'}^{R_{\alpha}}$ for $3 \otimes \bar{3}$

Consider pair production of particle-antiparticle in the fundamental representation (ex. $t\bar{t}, \tilde{q}\bar{\tilde{q}}$)

$$\mathbf{3}\otimes\bar{\mathbf{3}}=\mathbf{1}\oplus\mathbf{8}$$

Projectors on the irreducible representations: $P_{\{k\}}^{S} = \frac{1}{N_{C}} \delta_{k_{1}k_{2}} \delta_{k_{3}k_{4}}$ $P_{\{k\}}^{8} = 2T_{k_{2}k_{1}}^{A}T_{k_{3}k_{4}}^{A}$

• Quark-antiquark channel: $q\bar{q} \rightarrow H^{(3)}H^{(\bar{3})}$

$$c_{\{a\}}^{(1)} = \frac{1}{N_C} \delta_{a_1 a_2} \delta_{a_3 a_4} \qquad c_{\{a\}}^{(2)} = \frac{2}{\sqrt{N_C^2 - 1}} T^A_{a_2 a_1} T^A_{a_3 a_4}$$

$$\begin{split} &W_{ii'}^{S}(z,\mu) = \operatorname{diag}\left(W_{DY},0\right) \\ &W_{ii'}^{8}(z,\mu) = \operatorname{diag}\left(0,\frac{1}{(N_{C}^{2}-1)}\langle 0|\operatorname{Tr}[\overline{T}[S_{n}^{\dagger}T^{a}S_{\nu,ac}^{(8)}S_{\overline{n}}](z)\operatorname{T}[S_{\overline{n}}^{\dagger}S_{\nu,cb}^{(8),\dagger}T^{b}S_{n}](0)]|0\rangle\right) \end{split}$$

-
$$W^{S/8}_{i \neq i'}(z,\mu) \propto \operatorname{Tr}[T^A] = 0$$

- $W_{DY} = \frac{1}{N_C} \langle 0 | \mathrm{Tr}\overline{\mathrm{T}}[S_n^{\dagger} S_{\overline{n}}](z) \mathrm{T}[S_{\overline{n}}^{\dagger} S_n](0) | 0 \rangle.$
- Conventional soft function $W_{ii'} = \sum_{\alpha} W_{ii'}^{R_{\alpha}} = \text{diag}(W_{DY}, W_{22}^8)$

All-order colour structure of $W^{R_{\alpha}}_{ii'}$ for $3 \otimes \overline{3}$

• Gluon-gluon channel: $gg \to H^{(3)}H^{(\overline{3})}$

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All-order colour structure of $W_{ii'}^{R_{\alpha}}$ for $3 \otimes \overline{3}$

• Gluon-gluon channel: $gg \to H^{(3)}H^{(\overline{3})}$

- At threshold soft function for $r_i + r_j \rightarrow R + R'$ is reduced to sum of soft functions for $r_i + r_j \rightarrow R_{\alpha}$, where $R \otimes R' = \sum_{\alpha} R_{\alpha}$
- Conventional soft function $W_{ii'} = \sum_{\alpha} W_{ii'}^{R_{\alpha}}$ for $t\bar{t}/\tilde{q}\bar{\tilde{q}}$ at threshold is diagonal to all order in α_s in the same colour basis that diagonalises the one-loop soft function (Extends recent results for the two-loop massive soft anomalous dimension in the threshold limit [Mitov et al. '09; Neubert et al. '09])

Resummation of threshold logarithms

Factorisation-scale independence of the total cross section translates into evolution equations for hard matching coefficient and soft function

$$\frac{d}{d\ln\mu}C^{i}_{pp'}(4\bar{m}^{2},\mu) = \left[\frac{1}{2}\left(\Gamma^{r}+\Gamma^{r'}\right)\ln\left(-\frac{4\bar{m}^{2}}{\mu^{2}}\right)+\gamma^{C}\right]_{ij}C^{j}_{pp'}(4\bar{m}^{2},\mu)$$

$$\frac{d}{d\ln\mu}\mathbf{W}^{R_{\alpha}}(\omega,\mu) = -\int_{0}^{\omega}d\omega\left(\frac{1}{\omega-\omega'}\right)_{[\mu]}\left[\Gamma_{R_{\alpha}}\mathbf{W}^{R_{\alpha}}(\omega',\mu)+\mathbf{W}^{R_{\alpha}}(\omega',\mu)\Gamma^{\dagger}_{R_{\alpha}}\right]$$

$$-\gamma^{S}_{R_{\alpha}}\mathbf{W}^{R_{\alpha}}(\omega,\mu)-\mathbf{W}^{R}(\omega,\mu)\gamma^{S\dagger}_{R_{\alpha}}$$

$$\sum_{\alpha}\Gamma_{R_{\alpha}}=\Gamma^{r}+\Gamma^{r'}\sum_{\alpha}\gamma^{S}_{R_{\alpha}}=\gamma^{S}$$

• Γ^r and $\Gamma_{R_{\alpha}}$ control resummation of double logs, γ_{ij}^C , $\gamma^{S,R_{\alpha}}$ resum single logs.

• In general mixing of different colour structures. With suitable choice of colour basis reduce to set of decoupled equations for $W_{ii}^{R_{\alpha}}$

One-loop soft function and anomalous dimensions



One-loop soft function for arbitrary initial and final-state particles:

$$W_{ij}^{R_{\alpha},(1)}(L) = \frac{\alpha_s}{4\pi} \left[\frac{\Gamma_{R_{\alpha},ij}^{r,(0)} + \Gamma_{R_{\alpha},ij}^{r',(0)}}{4} \left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon}L + L^2 + \frac{\pi^2}{6} \right) - \gamma_{R_{\alpha},ij}^{\mathcal{S},(0)} \left(\frac{1}{\epsilon} + L + 2 \right) \right]$$
$$L = 2\ln\left(\frac{iz_0\mu e^{\gamma_E}}{2}\right)$$

For R_{α} relevant for $t\bar{t}/\tilde{q}\bar{\tilde{q}}$ one obtains:

$q\bar{q}$ channel		gg channel	
$\Gamma_{S}^{3,(0)} = \text{diag}(4C_{F}, 0)$ $\Gamma_{8}^{3,(0)} = \text{diag}(0, 4C_{F})$	$ \begin{aligned} \gamma_{S}^{S,(0)} &= \text{diag}(0,0) \\ \gamma_{S}^{S,(0)} &= \text{diag}(0,-2N_{C}) \end{aligned} $	$ \Gamma_{S}^{8,(0)} = \operatorname{diag}(4N_{C}, 0, 0) \Gamma_{8}^{8,(0)} = \operatorname{diag}(0, 4N_{C}, 4N_{C},) $	$\begin{split} \gamma_{8}^{S,(0)} &= \text{diag}(0,0,0) \\ \gamma_{8}^{S,(0)} &= \text{diag}(0,-2N_{C},-2N_{C}) \end{split}$

 $\sum_{\alpha} \Gamma_{R_{\alpha}}^{r,(0)}, \sum_{\alpha} \gamma_{R_{\alpha}}^{s,(0)}$ gives the usual one-loop cusp and soft anomalous dimensions!

Pietro Falgari (IPPP Durham)

Resummed soft function and matching coefficient

Threshold logarithms are resummed by renormalising $C_{pp'}^i$ and $\mathbf{W}^{R_{\alpha}}$ at a hard scale μ_h and a soft scale μ_s respectively, and evolving them to the common scale μ .

$$\begin{split} C_{pp'}^{i,\mathrm{res}}(4\bar{m}^{2},\mu) &= \exp[2S(\mu_{h},\mu) - a_{i}^{C}(\mu_{h},\mu)] \left(-\frac{4\bar{m}^{2}}{\mu_{h}^{2}}\right)^{-a_{\Gamma}(\mu_{h},\mu)} C_{pp'}^{i}(4\bar{m}^{2},\mu_{h}) \\ W_{ii}^{R_{\alpha},\mathrm{res}}(\omega,\mu) &= \exp[-4S_{i}^{R_{\alpha}}(\mu_{s},\mu) + 2a_{i}^{S,R_{\alpha}}(\mu_{s},\mu)]\tilde{s}_{ii}^{R_{\alpha}}(\partial_{\eta},\mu_{s})\frac{1}{\omega} \left(\frac{\omega}{\mu_{s}}\right)^{2\eta} \theta(\omega) \frac{e^{-2\gamma_{E}\eta}}{\Gamma(2\eta)} \\ S(\nu,\mu) &= -\int_{\alpha_{s}(\nu)}^{\alpha_{s}(\mu)} d\alpha_{s} \frac{\Gamma'(\alpha_{s}) + \Gamma'(\alpha_{s})}{2\beta(\alpha_{s})} \int_{\alpha_{s}(\nu)}^{\alpha_{s}} \frac{d\alpha_{s}'}{\beta(\alpha_{s}')} \\ a_{\Gamma}(\nu,\mu) &= -\int_{\alpha_{s}(\nu)}^{\alpha_{s}(\mu)} d\alpha_{s} \frac{\Gamma^{(0)r}(\alpha_{s}) + \Gamma'(\alpha_{s})}{2\beta(\alpha_{s})} \\ a_{i}^{X}(\nu,\mu) &= -\int_{\alpha_{s}(\nu)}^{\alpha_{s}(\mu)} d\alpha_{s} \frac{\gamma_{i}^{X}(\alpha_{s})}{\beta(\alpha_{s})} \end{split}$$

 μ_h and μ_s must be chosen such that fixed-order perturbative expansions of $C^i_{pp'}(4\bar{m}^2,\mu_h)$ and $W^{R_{\alpha}}_{ii}(\omega,\mu_s)$ are well behaved.

Resummed Coulomb corrections

Well known from quarkonia physics! $J_{R_{\alpha}}^{(\ell,\ell')}$ related to zero-distance Green function of Schrödinger operator $-\vec{\nabla}^2/(2m_{\text{red}}) - \alpha_s(-C_{R_{\alpha}})/r$:

$$J_{R_{\alpha}}^{(\ell,\ell')}(E) \leftrightarrow \operatorname{Im} G_{\mathrm{C},\mathrm{R}_{\alpha}}^{(0)}(0,0;E) \qquad E = \sqrt{s} - 2\bar{m}$$

In the MS

$$G_{C,R_{\alpha}}^{(0)}(0,0;E) = -\frac{(2m_{\rm red})^2}{4\pi} \left\{ \sqrt{-\frac{E}{2m_{\rm red}}} + \alpha_s (-C_{R_{\alpha}}) \left[\frac{1}{2} \ln \left(-\frac{8 m_{\rm red} E}{\mu^2} \right) -\frac{1}{2} + \gamma_E + \psi \left(1 - \frac{\alpha_s (-C_{R_{\alpha}})}{2\sqrt{-E/(2m_{\rm red})}} \right) \right] \right\}$$

with $C_{R_{\alpha}} = (-C_F, 1/(2N_C))$ for $R_{\alpha} = (S, 8)$.

Finite width effects can be implemented by the replacement $E \rightarrow E + i\Gamma_H$ (see also **Stefano Actis' talk** tomorrow)

Squark-antisquark production at the LHC ($\sqrt{s} = 14 \text{ TeV}$)

In the rest of this talk:

$$PP
ightarrow ilde{q} ar{ ilde{q}} + X$$

Perform NLL resummation of soft-gluon corrections + Coulomb singularities:

- Two-loop cusp anomalous dimension Γ and QCD β -function
- One-loop soft anomalous dimension γ^s
- Tree-level fixed-order matching coefficient $C_{pp'}^i$ and soft functions $W_{ii'}^{R_{\alpha}}$

 μ set by default to $m_{\tilde{q}} \Rightarrow$ no need to resum the hard matching coefficient $C_{pp'}^i$

The effective-theory resummed cross section is matched onto the full NLO result [Zerwas et al., '96]

$$\hat{\sigma}_{pp'}^{\text{match}}(\hat{s},\mu_f) = \left[\hat{\sigma}_{pp'}^{\text{NLL}}(\hat{s},\mu_f) - \hat{\sigma}_{pp'}^{\text{NLL}}(\hat{s},\mu_f)|_{\text{NLO}}\right] + \hat{\sigma}_{pp'}^{\text{NLO}}(\hat{s},\mu_f)$$

Full NLO result computed using fitted scaling functions provided by [Langenfeld, Moch, '09]

NLL corrections to $\tilde{q}\tilde{\tilde{q}}$ production [PRELIMINARY]

$$K_{\rm NLL} - 1 = \frac{\sigma^{\rm match}}{\sigma^{\rm NLO}} - 1$$



NLL corrections to $\tilde{q}\bar{\tilde{q}}$ production [PRELIMINARY]

$$K_{\rm NLL} - 1 = \frac{\sigma^{\rm match}}{\sigma^{\rm NLO}} - 1$$



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Soft-scale dependence

Resummation introduces extra scale $\mu_s \Rightarrow$ How does the resummed result depend on μ_s ?



In the traditional Mellin-space resummation this is not visible (soft scale implicitly fixed to $\mu_s = 2m_{\tilde{q}}/N$)

Factorisation-scale dependence

Resummation of threshold logarithms sensibly reduces the factorisation-scale dependence of the cross section

 $m_{\tilde{a}} = 500 \,\mathrm{GeV}$ $m_{\tilde{a}} = 1 \,\mathrm{TeV}$ σ [pb] σ [pb] ³⁵ F 0.7 r LO 10 NLO 30 F NLO NLL 0.6 NLL. 25 0.5 20 0.4 15 0.3 10 0.2 0.1 5 μ 0 0.0 0.1 0.2 0.5 1.0 2.0 5.0 ma 0.1 0.2 0.5 1.0 2.05.0 ma

Green band obtained by varying $\bar{\mu}_s/2 < \mu_s < 2\bar{\mu}_s$

- Momentum-space resummation based on effective-theory framework works well and is in good agreement with analogous results in moment space
- Formalism allows for <u>all-order generalisation</u> of known one-loop and two-loop results $(\mathbf{W}^{R_{\alpha}} \text{ diagonal to all-order in } \alpha_s)$
- For squark-antisquark production resummation effects beyond NLO amount to 3 10% in the range 0.2 2 TeV
- Even for small squark masses resummation dramatically improves factorisation-scale dependence of the cross section

Outlook

• Formalism can be applied to <u>arbitrary final states</u> (squark-squark, squark-gluino, gluino-gluino, etc...)

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