ON-SHELL METHODS FOR ONE-LOOP AMPLITUDES

Darren Forde (SLAC)

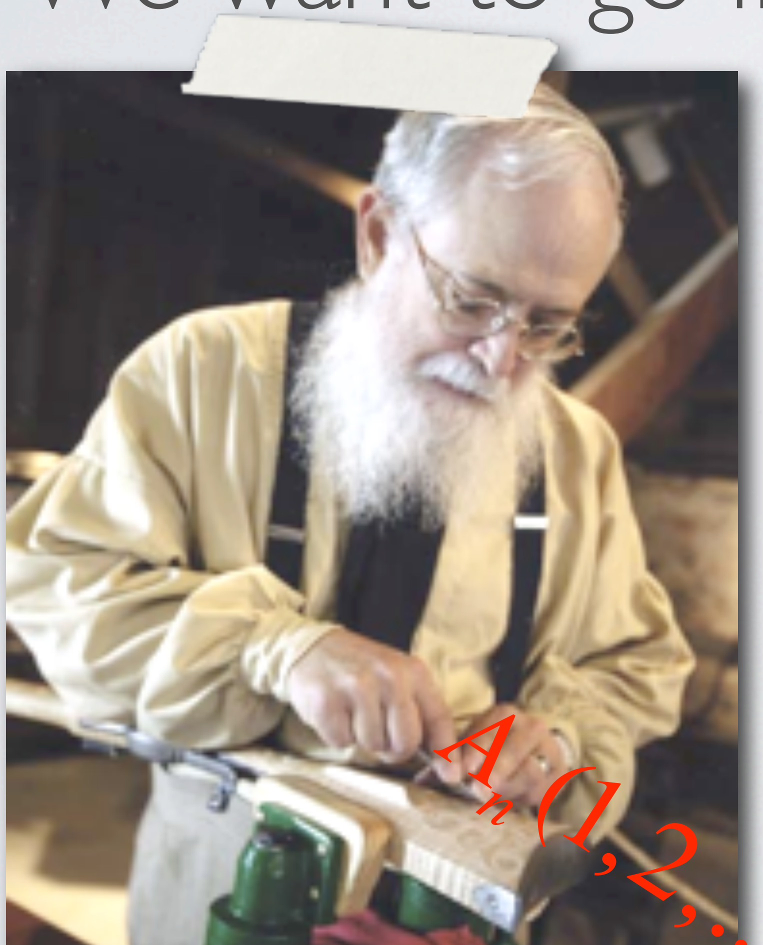
In collaboration with C. Berger, Z. Bern, L. Dixon, F. Febres Cordero, T. Gleisberg, D. Maitre, H. Ita & D. Kosower.

OVERVIEW

- We want one-loop amplitudes to produce NLO corrections for LHC processes.
- Automate the computation of these terms, `BlackHat`.
  - On-shell recursion relations.
  - Generalised unitarity techniques in 4 dimensions.
  - Rational extraction - Uses generalised unitarity techniques in *D*-dimensions.
- Full W+3 jets at NLO including the sub-leading terms.

AUTOMATION

We want to go from



$A_n(1,2,\ldots,n)$

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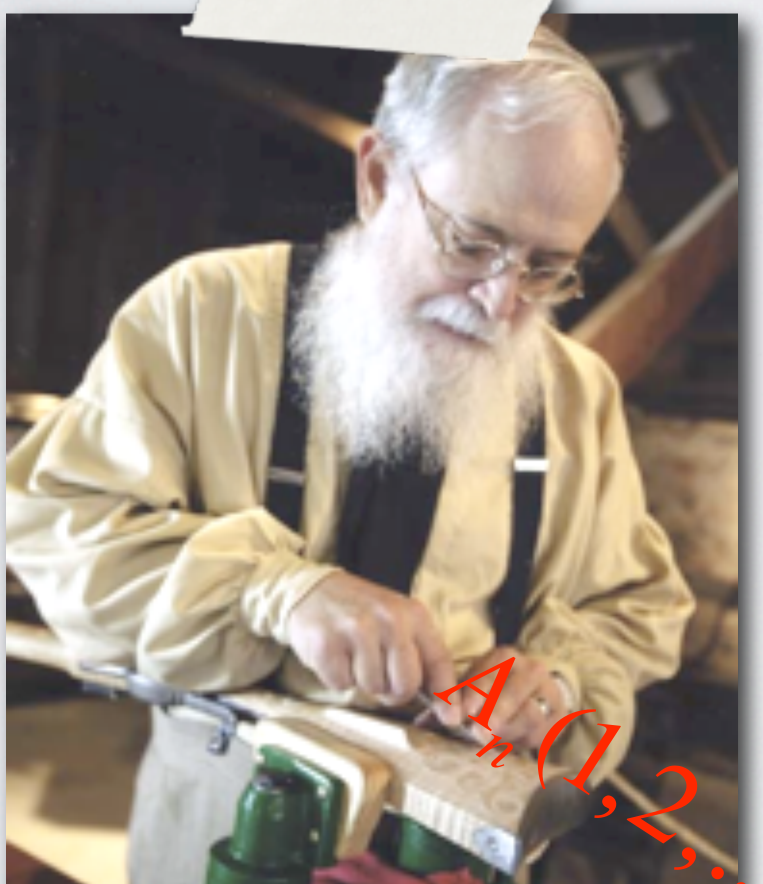
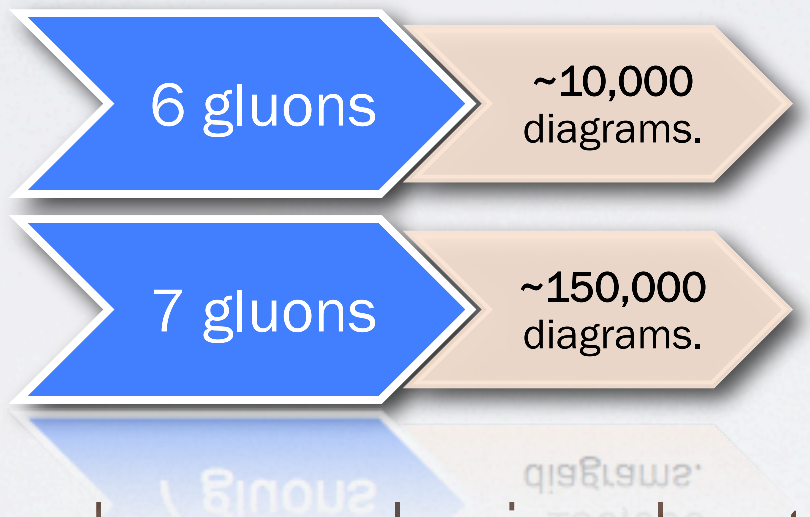
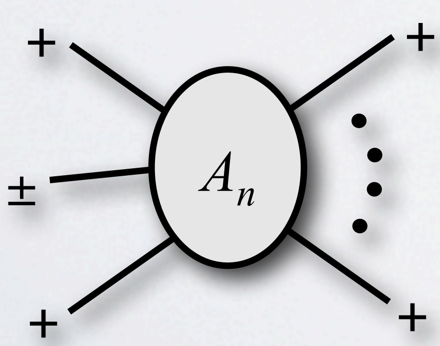


NEW TECHNIQUES

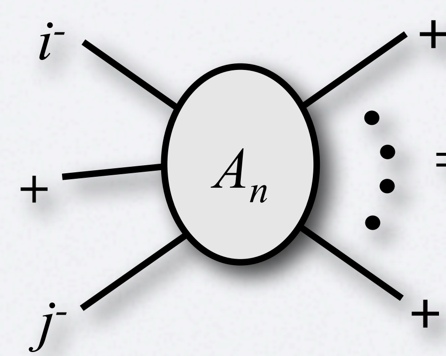
- Feynman diagrams have a **factorial** growth in the number of terms, particularly bad for large numbers of gluons.



- Calculated Amplitudes much simpler than expected. e.g.



$$=0$$



$$= \frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle (n-1)n\rangle\langle n1\rangle}$$

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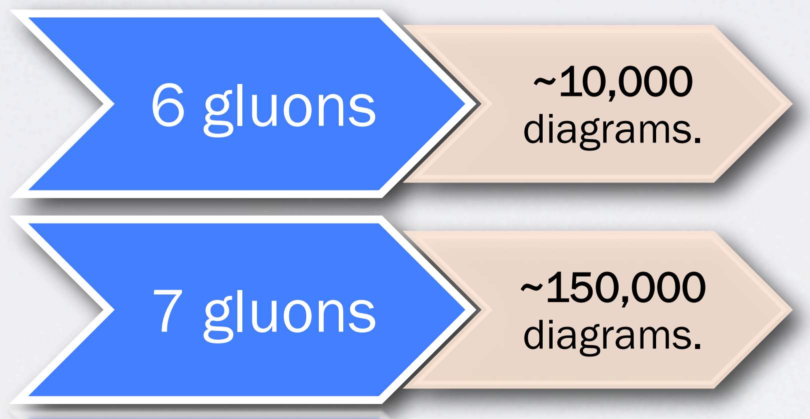
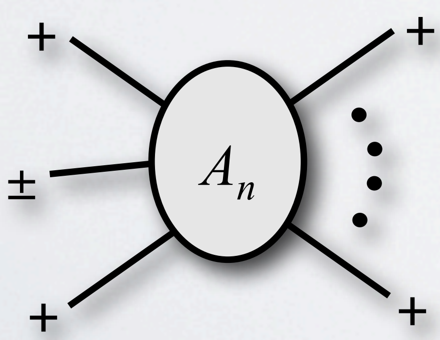


Want to use on-shell quantities. Avoid large cancellations due to gauge dependance.

- Calculated Amplitudes much simpler than expected. e.g.



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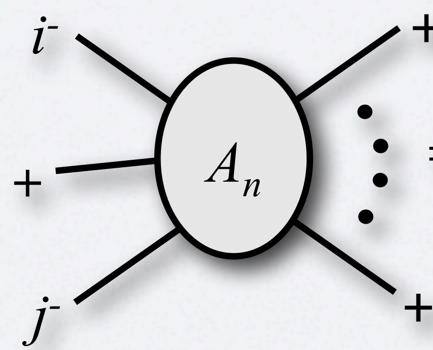


$$= \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle (n-1) n \rangle \langle n1 \rangle}$$

WHAT HAS BEEN DONE?

- Many important 5 processes have been computed using Feynman diagram approaches, including pp$\rightarrow$ vector bosons, quarks, Higgs, etc. (Jäger, Oleari, Zeppenfeld, Bozzi, Ciccolini, Denner, Dittmaier, Campbell, Ellis, Zanderighi, Ciccolini, Figy, Hankele, Zeppenfeld, Beenakker, Krämer, Plümper, Spira, Zerwas, Dawson, Jackson, Reina, Wackeroth, Lazopoulos, Petriello, Melnikov, McElmurry, Campanario, Prestel, Kallweit, Uwer, Febres Cordero, Weinzierl, Bredenstein, Pozzorini).

- Limited 6 point results. (e.g. Bredenstein, Denner, Dittmaier, Pozzorini).

- Usually require new techniques.

WHAT HAS BEEN DONE?

Les Houches "wish list", (2007)

Process ($V \in \{Z, W, \gamma\}$)	Comments
4. $pp \to t\bar{t}\, b\bar{b}$	relevant for $t\bar{t}H$
5. $pp \to t\bar{t}$+2jets	relevant for $t\bar{t}H$
6. $pp \to VV\, b\bar{b}$,	relevant for VBF $\to H \to VV$, $t\bar{t}H$
7. $pp \to VV$+2jets	relevant for VBF $\to H \to VV$ VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld.
8. $pp \to V$+3jets	various new physics signatures
NLO calculations added to list in 2007	
9. $pp \to b\bar{b}b\bar{b}$	Higgs and new physics signatures

- Many i… d using Feynm… tor bosons, quarks, … Denner, Dittmaier, Campbell … nakker, Krämer, Plümper, … oulos, Petriello, Melnikov, … ordero, Weinzierl, Bredenste…
- Limite… Pozzorini).
- Usually require new techniques.

W case computed by `BlackHat+SHERPA`. (Pieces computed by (Ellis, Menlikov, Zanderighi))

AUTOMATED TOOLS

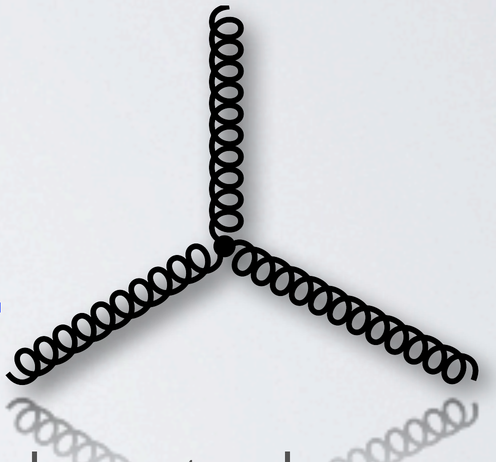
- Let the computer(s) do the hard work!
- New generation of automated tools based on new methods.
  - `BlackHat` - W+3 jet NLO computation (with SHERPA). (Berger, Bern, Dixon, DF, Febres Cordero, Gleisberg, Maitre, Ita, Kosower)
  - **Rocket** - Partial W+3 jet NLO computation. (Ellis, Melnikov,Zanderighi), (Ellis, Giele, Melnikov, Kunszt, Zanderighi)
  - **Cuttools** - pp→VVV at NLO. A number of "wish-list" amplitudes. (van Hameren, Papadopoulos, Pittau), (Ossola, Papadopoulos, Pittau)
  - Other amplitude level codes (Giele, Winter), (Lazopoulos), (Schulze)

THE COMPLEX PLANE

- A **key feature** of new developments has been the use of **complex momenta**.

- Benefits

  - Define a non-zero on-shell **three-point function**.



  - Build **all** amplitudes from just this term (in general not clear from the Lagrangian).

  - Take better advantage of **analytic properties** of amplitudes.

AMPLITUDES & POLES

- An amplitude is a function of its external momenta (& helicity)

$$A_n(k_1^{h_1}, k_2^{h_2}, \ldots, k_i^{h_i}, \ldots, k_j^{h_j}, \ldots, k_n^{h_n})$$

- Shift the momenta of two external legs so they become complex. (Britto, Cachazo, Feng, Witten)
  - Keeps both legs on-shell.
  - Conserves Momentum.
- Turns physical poles of the amplitude into poles in z.

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$$k_i^\mu \to k_i^\mu(z) = k_i^\mu - \frac{z}{2}\left\langle i^- \middle| \gamma^\mu \middle| j^- \right\rangle, \; k_j^\mu \to k_j^\mu(z) = k_j^\mu + \frac{z}{2}\left\langle i^- \middle| \gamma^\mu \middle| j^- \right\rangle$$

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Only possible with complex momenta

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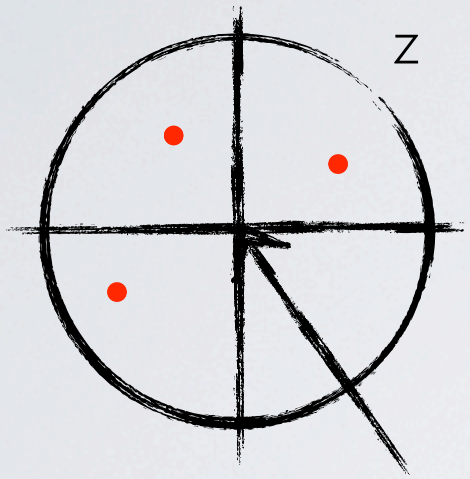
A SIMPLE IDEA



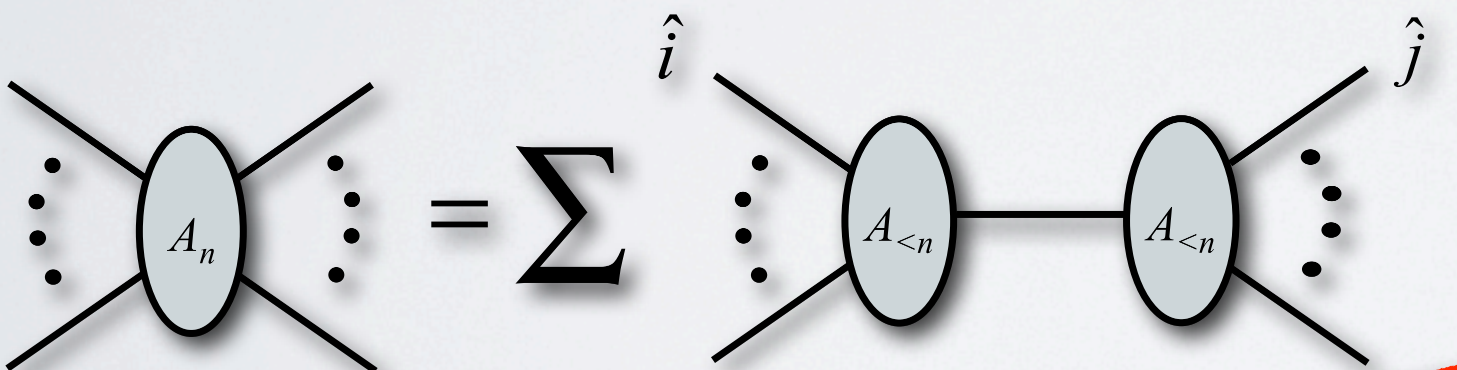
$A(0)$ the amplitude we want,
with real momentum

Contour Integral

$$\frac{1}{2i\pi}\oint_C dz\,\frac{A_n(z)}{z} = 0$$

Cauchy's Theorem

$$A_n(0) = -\sum_{\text{poles}} \mathrm{Res}_z\,\frac{A_n(z)}{z}$$



Relate to factorisation

On-shell recursion

$$A_n \overset{P^2\to 0}{=} \sum_{i\in L,\, j\in R} A_L(\ldots,i,\ldots,P)\frac{1}{P^2}A_R(\ldots,j,\ldots,P)$$

ONE-LOOP AMPLITUDES

- Split one-loop structure into rational and cut parts.

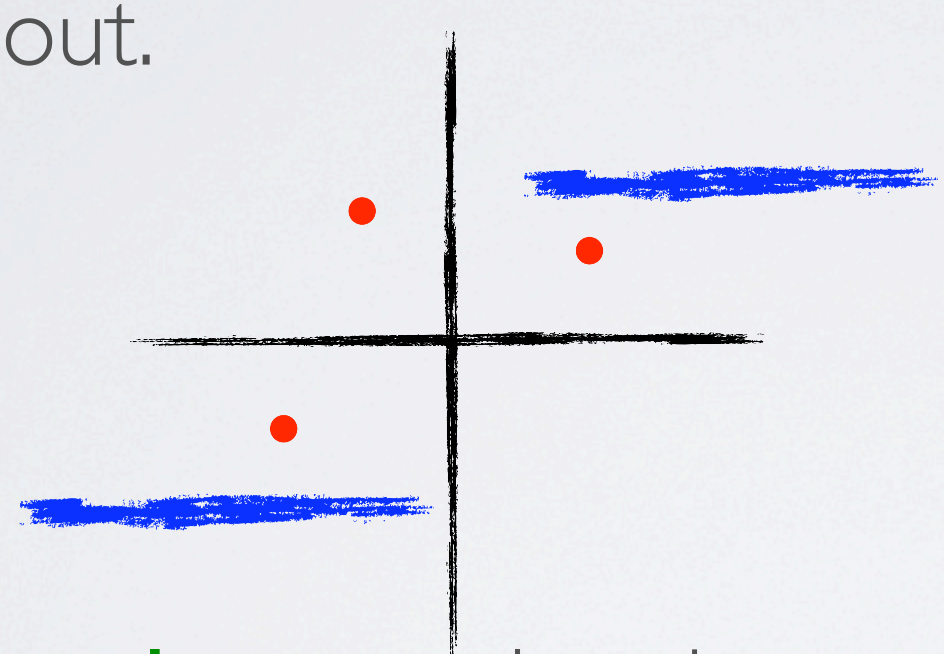
Rational terms + Log's, Polylog's, etc. = Loop amplitude

$$\frac{\langle 1\,3\rangle^2 (s_{12}+s_{23})}{[1\,2]\,[2\,3]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle} \qquad \mathrm{L}_{2a}\left(\frac{-s_{34}}{-s_{51}}\right)$$

- Cut terms contain branch cuts.
- Rational terms contain only poles, split into two kinds (Bern, Dixon, Kosower)
    - Factorising poles, appear in the complete result.
    - Spurious poles (cancel with the cut terms).

POLES & RATIONAL TERMS

- **Branch cuts** give the cut terms, compute separately and subtract out.



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  - Compute by extracting residue of spurious pole from cut.
- **Recursive poles** from complex factorisation. (Berger, Bern, Dixon, DF, Kosower), (Berger, Bern, Dixon, DF, Febres Cordero, Ita, Maitre, Kosower)

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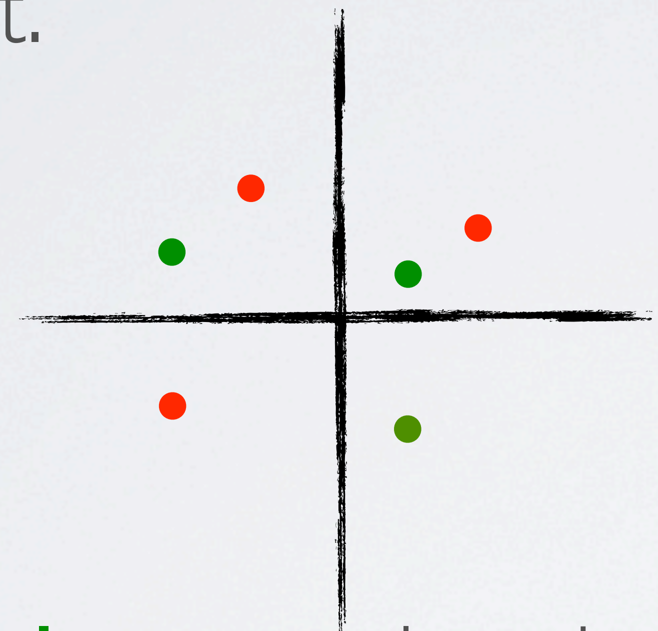

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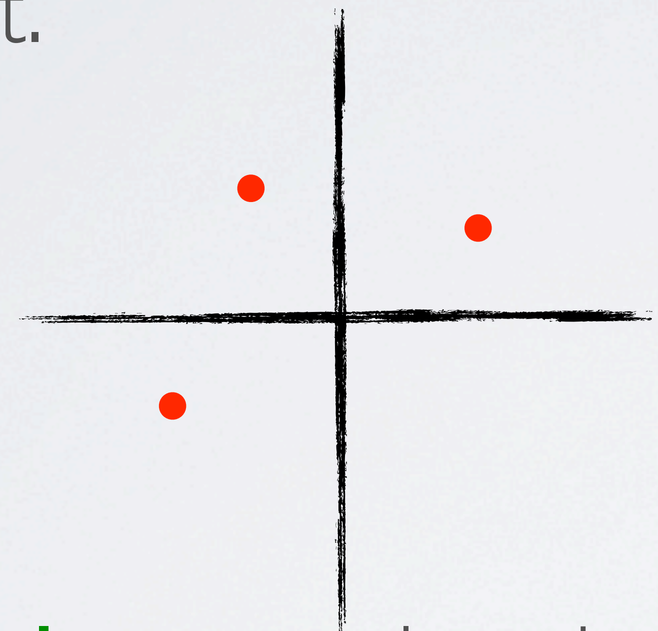

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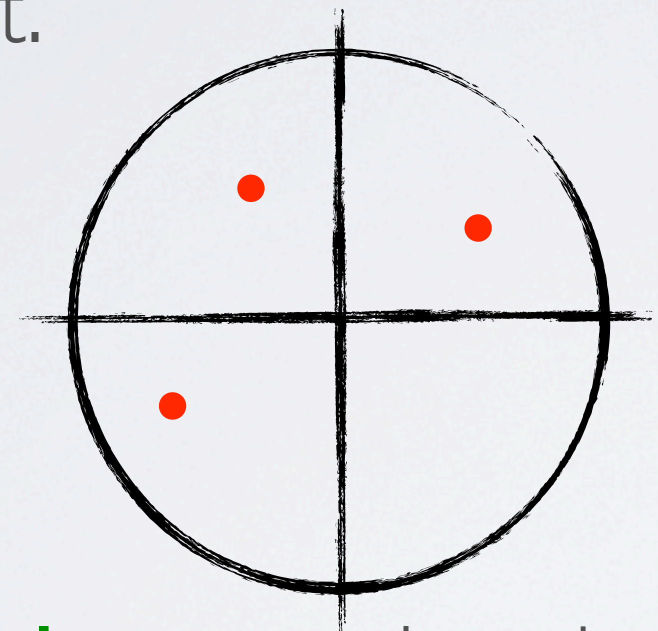

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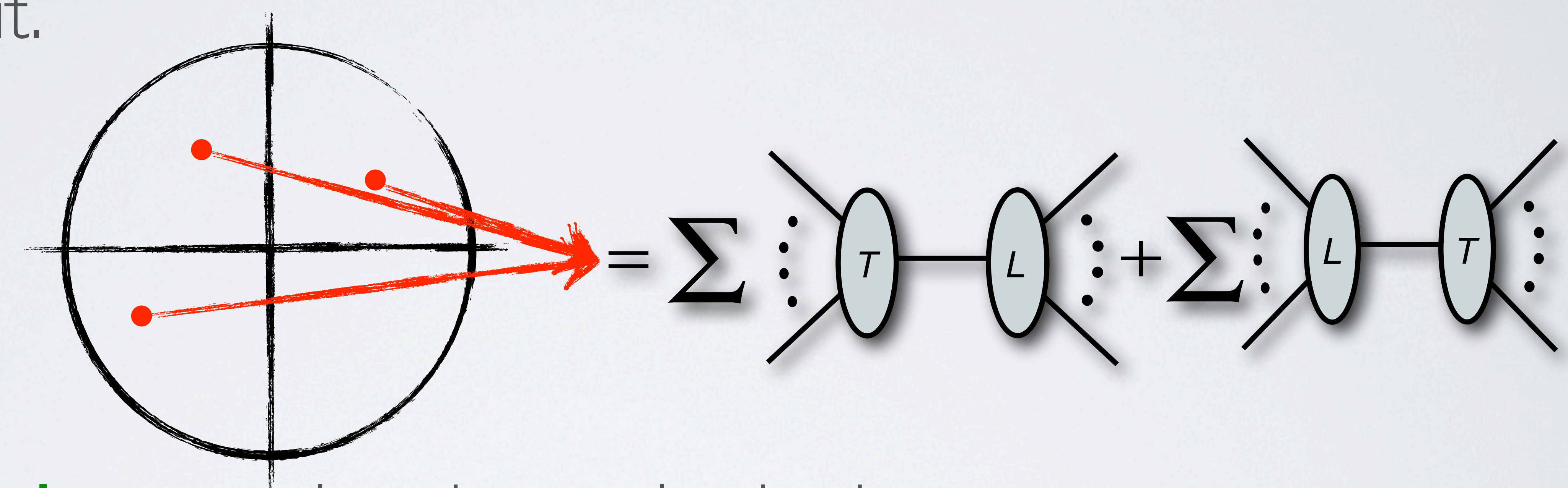

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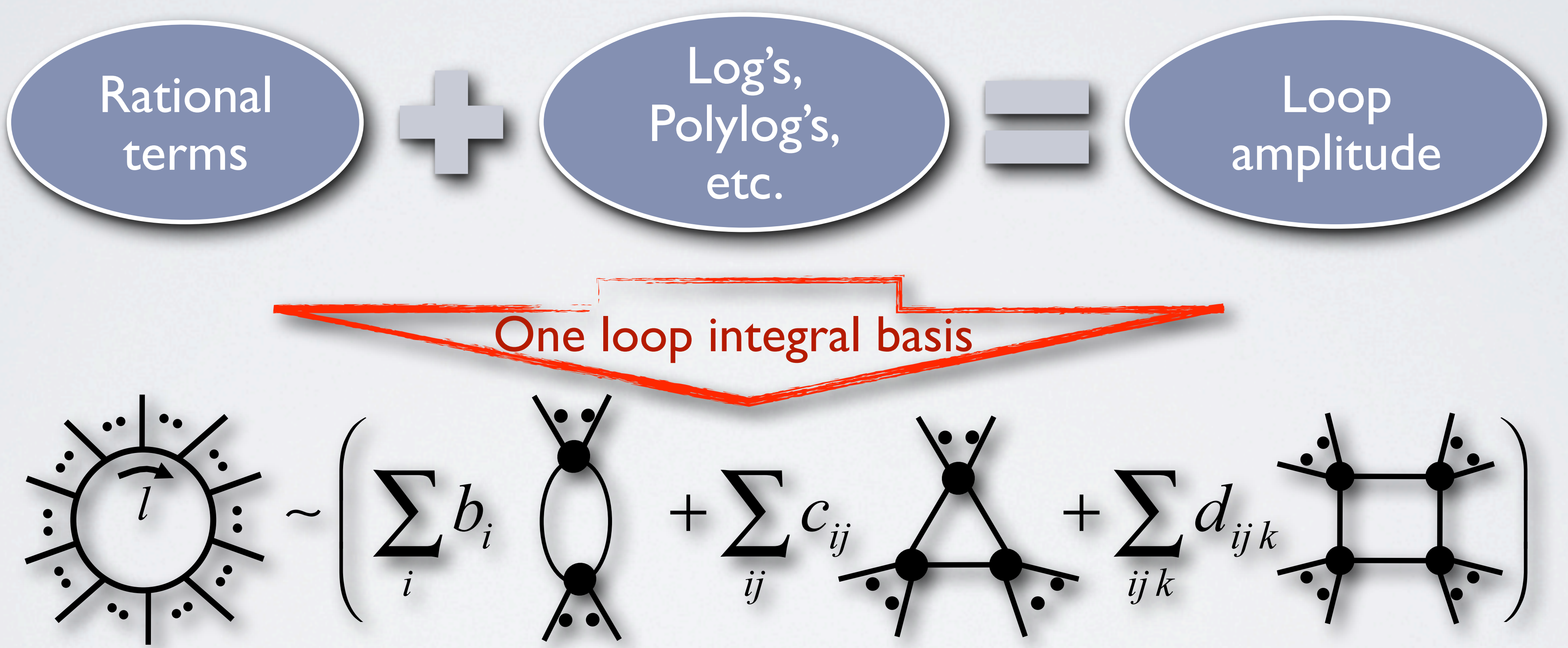
CUTS & UNITARITY

- Use unitarity to compute the cut terms



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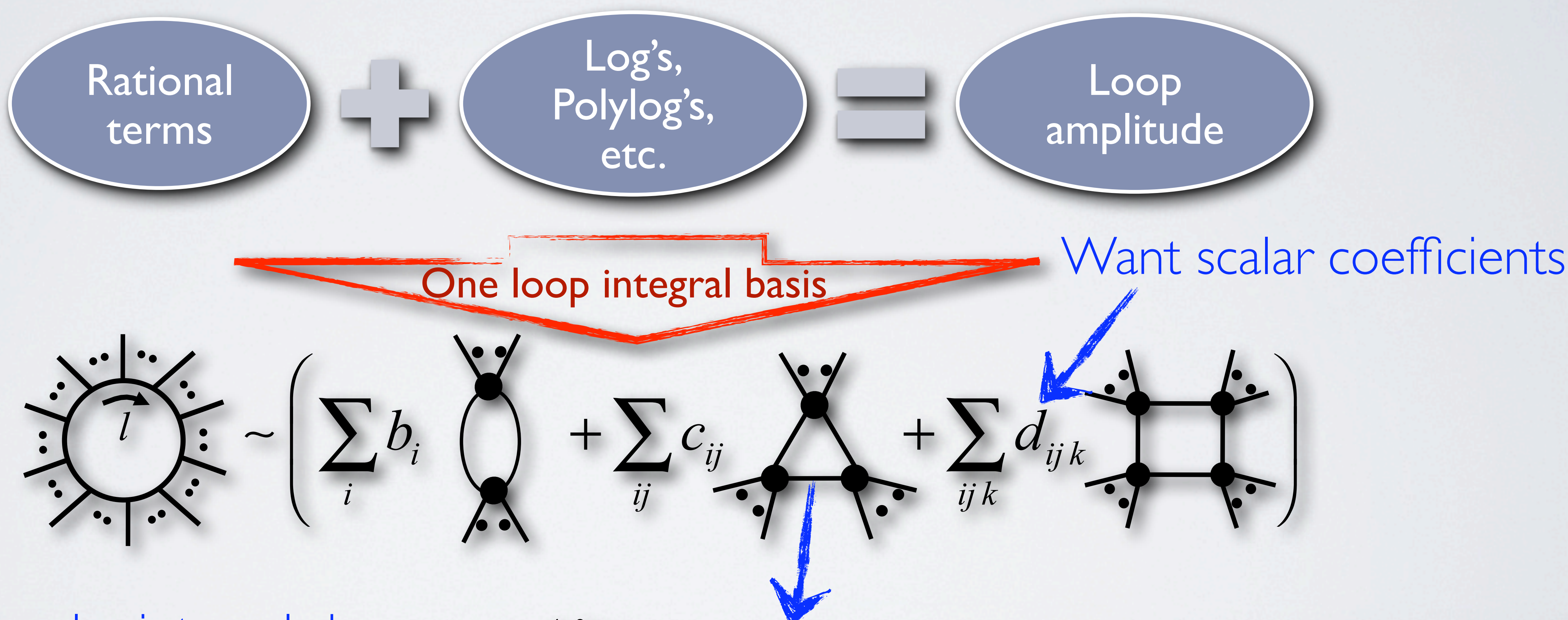
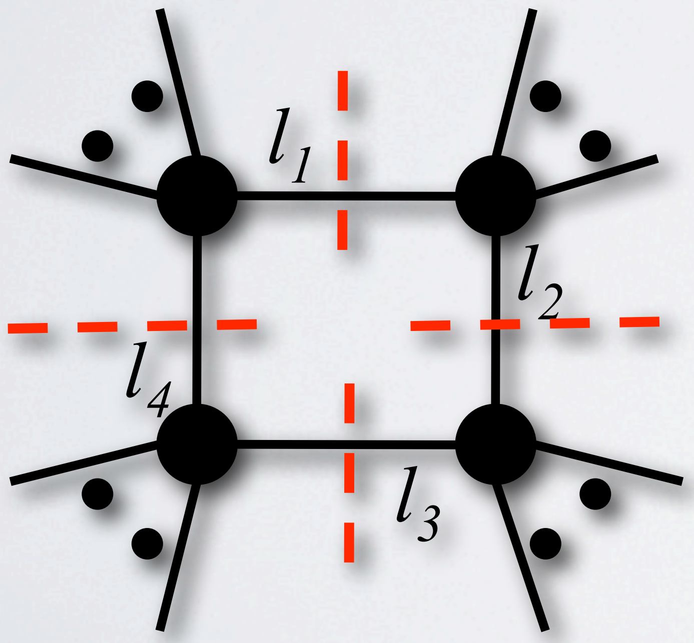


One loop scalar integrals known
(Ellis, Zanderighi), (Denner, Nierste, Scharf)
(van Oldenborgh, Vermaseren) + many others

$$\int \frac{d^{4-2\varepsilon} l}{(2\pi)^{4-\varepsilon}} \frac{1}{l^2 (l-p_1)^2 (l-p_2)^2} \sim a\,\mathrm{Li}_2\left(s_i, s_j\right) + b\,\mathrm{Log}\left(s_i\right) + \ldots$$

BOX COEFFICIENTS

- Generalised unitarity, cut the loop more than two times.
- Quadruple cuts freezes the box integral. (Britto, Cachazo, Feng)



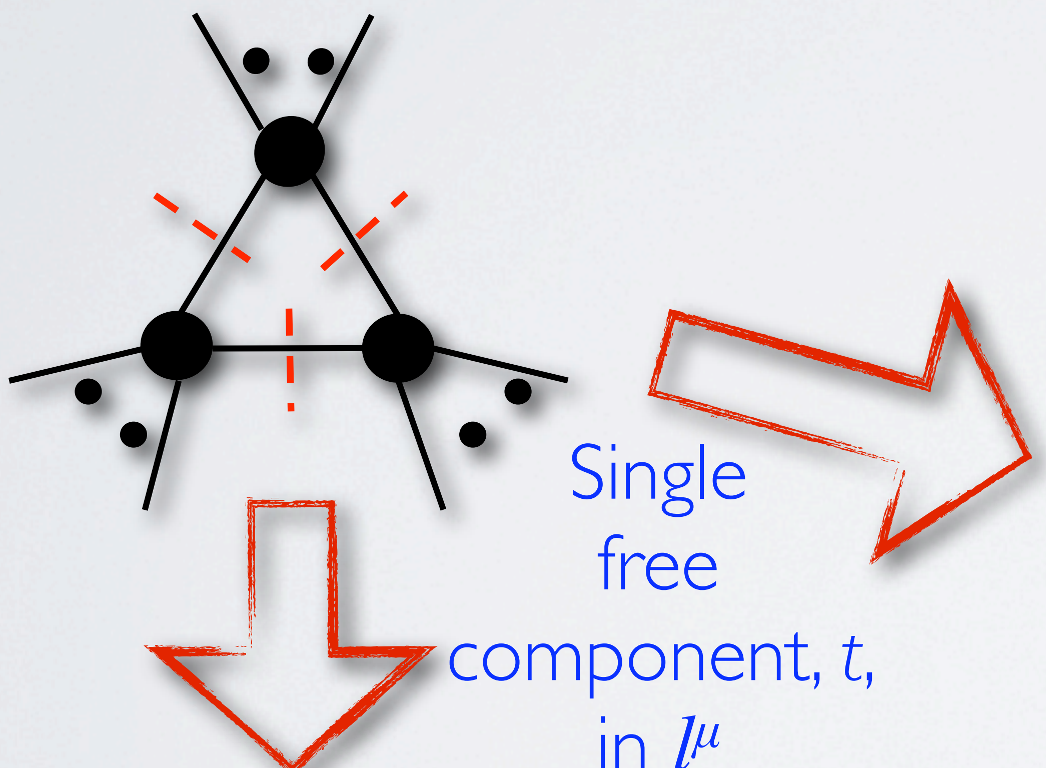
No free components in l^μ, fixed by
4 constraints in 4 dimensions.

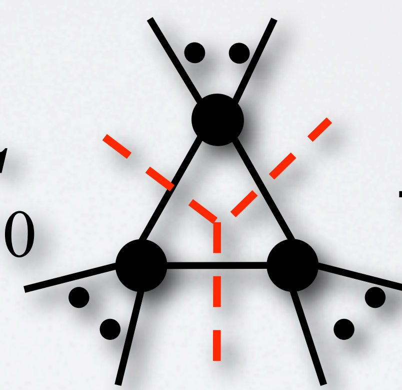
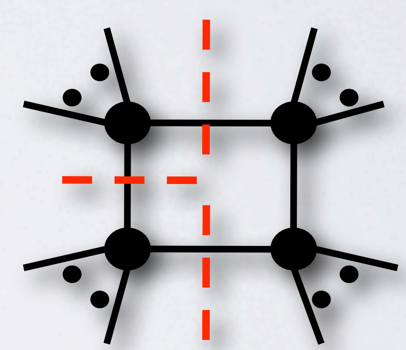
$$d = \frac{1}{2} \sum_{a=1,2} A_1(l_a) A_2(l_a) A_3(l_a) A_4(l_a)$$

Generally requires complex momenta

DIRECT EXTRACTION

- Triple cut isolates a single triangle coefficient. (DF)



C_0  $+\sum_i d_i$ 

$$\int d^4 l\,\delta(l_1^2)\delta(l_2^2)\delta(l_3^2)$$

$$\times A_1(l)A_2(l)A_3(l)$$

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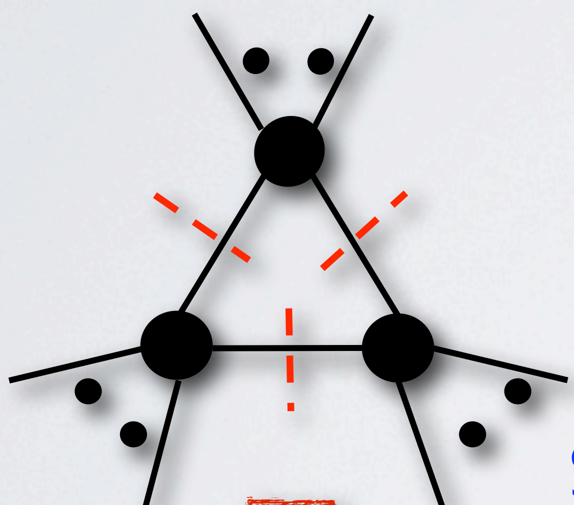
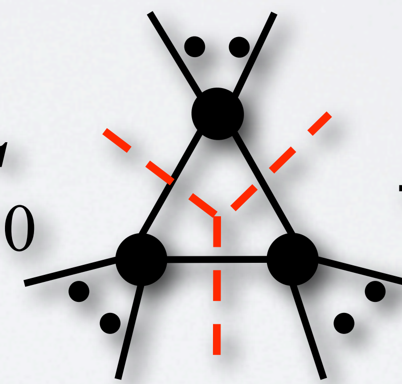
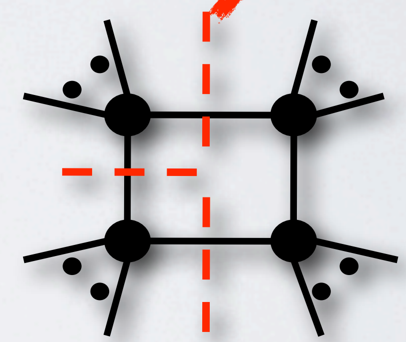


Single free component, t, in l^μ

$$\int d^4 l\,\delta(l_1^2)\delta(l_2^2)\delta(l_3^2) \times A_1(l)A_2(l)A_3(l)$$

C_0  $+\sum_i d_i$ 

$$\frac{d_i}{\zeta(t-t_i)}$$

$$\int dt\,\frac{\tilde{C}_{-3}}{t^3}+\frac{\tilde{C}_{-2}}{t^2}+\frac{\tilde{C}_{-1}}{t}+\tilde{C}_0+t\tilde{C}_1+t^2\tilde{C}_2+t^3\tilde{C}_3$$

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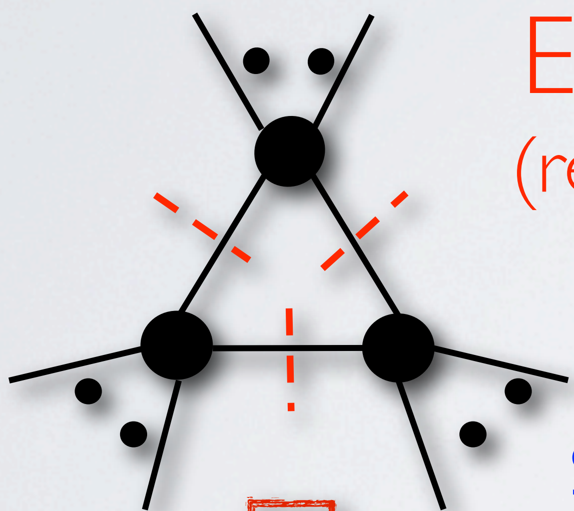
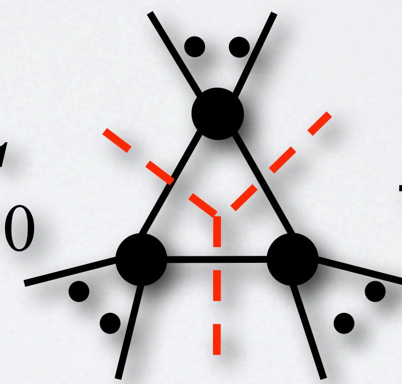


Extract coefficient from large t limit

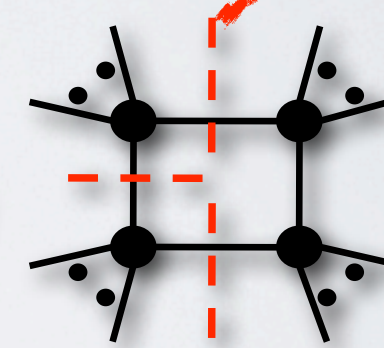
(require param of l^μ where integrals over t vanish)

Single free component, t, in l^μ

$$C_0$$



$$+\sum_i d_i$$



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$$\int d^4 l\, \delta(l_1^2)\delta(l_2^2)\delta(l_3^2) \times A_1(l)A_2(l)A_3(l)$$

$$\int dt \frac{\tilde{C}_{-3}}{t^3}+\frac{\tilde{C}_{-2}}{t^2}+\frac{\tilde{C}_{-1}}{t}+\tilde{C}_0+t\tilde{C}_1+t^2\tilde{C}_2+t^3\tilde{C}_3$$

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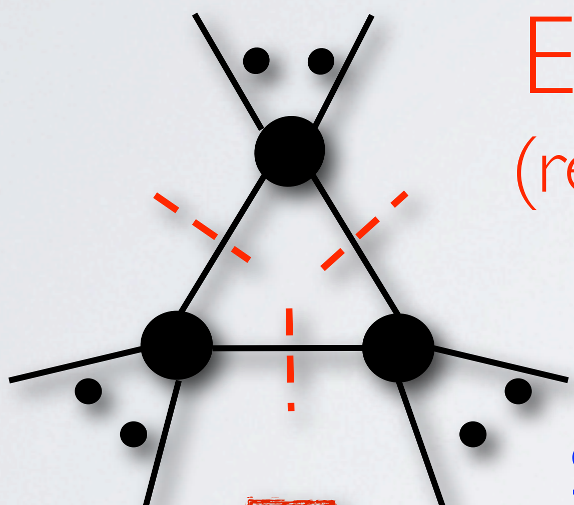

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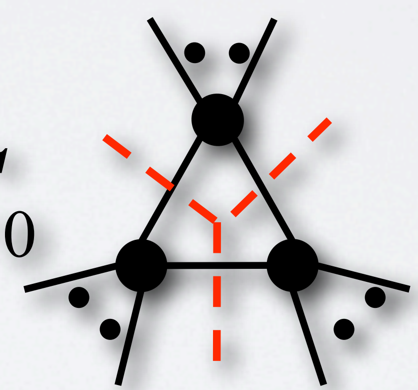
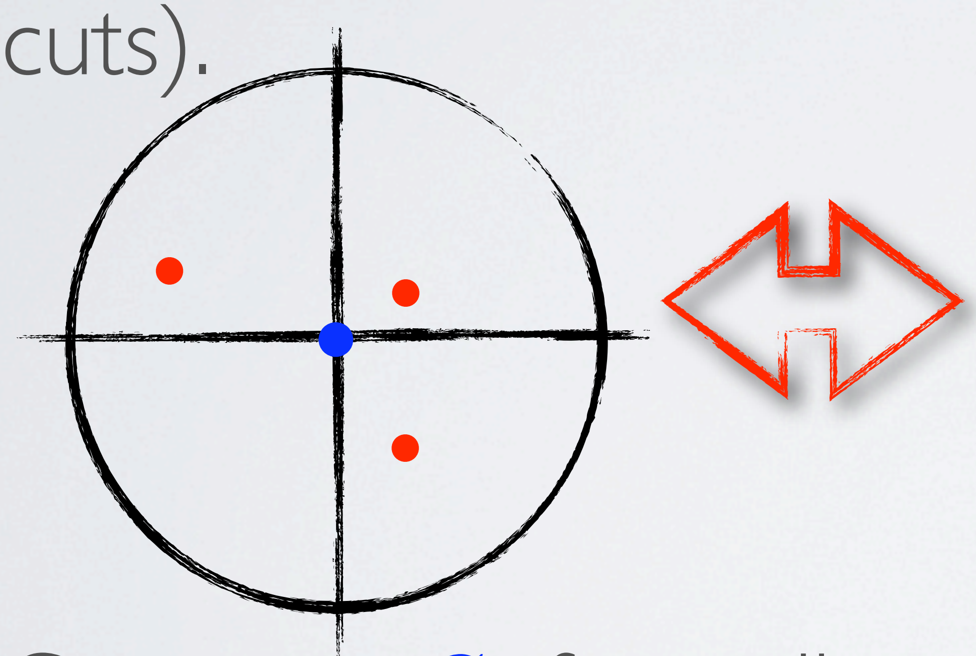


$$\int d^4 l \, \delta(l_1^2)\delta(l_2^2)\delta(l_3^2) \times A_1(l)A_2(l)A_3(l)$$

$$= \tilde{C}_0 \int dt$$

SUBTRACTING POLES

- Numerically taking large t limit is difficult.
- Subtract box poles from triple cut, (computed from quadruple cuts).



$$\oint T_3(t) = \oint \frac{C_{-3}}{t^3} + \frac{C_{-2}}{t^2} + \frac{C_{-1}}{t} + C_0 + tC_1 + t^2 C_2 + t^3 C_3 + \sum_{i,\sigma=\pm} \frac{d_i^\sigma}{\xi_i^\sigma (t - t_i^\sigma)}$$

- Compute C_0 from discrete Fourier projection.
- Alternative approach, solve for all coefficients. (Ossola, Papadopoulos, Pittau), (Ellis, Giele, Kunszt)

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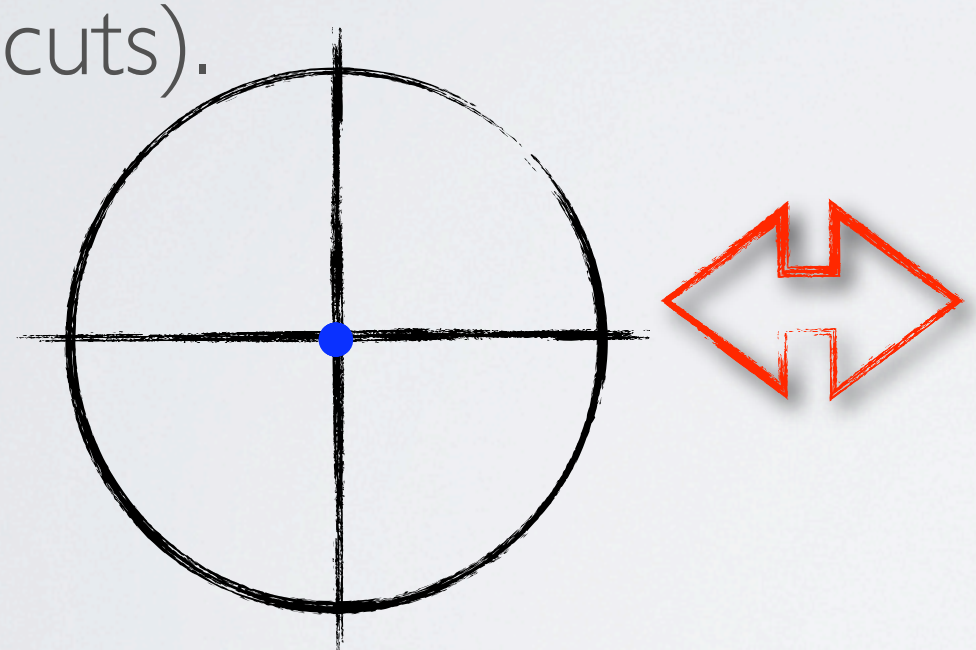

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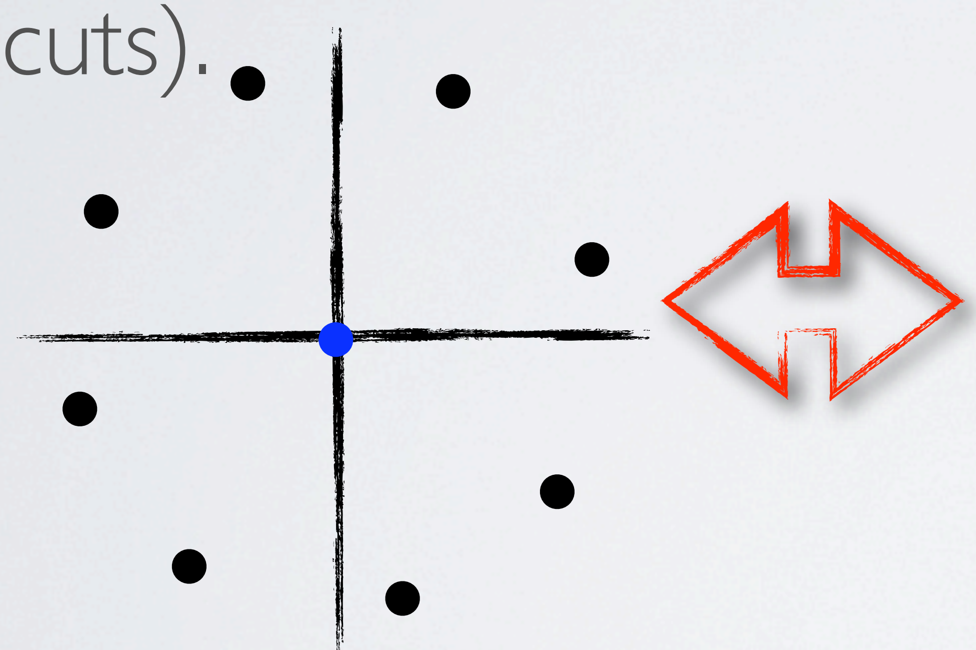

$$\oint T_3(t) = \sum_{i=1,..,7} T_3(t_i) = C_0$$

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REVISITING THE RATIONAL TERMS

- Cuts in 4 dimensions miss the rational terms.
- Perform cuts in *D*-Dimensions, $D=4-2\varepsilon$.
  - Introduces branch cuts for rational terms.
  - Can compute the rational terms from just trees.
- Two approaches,
  - Work "masslessly" in more than 4 dimensions. (Giele, Kunszt, Melnikov), (Giele, Winter)
  - Work in 4 dimensions with a *D*-Dimensional "mass". (Bern, Morgan), (Badger), (Ossola, Papadopoulos, Pittau), (Draggiotis, Garzelli, Papadopoulos, Pittau), (Anastasiou, Britto, Feng, Kunszt, Mastrolia)

D-DIMENSIONAL UNITARITY

- Relate higher dimensional approach to massive approach by decomposing *D*-Dimensional loop momenta,

$$l^{\nu} = \bar{l}^{\nu} + \mu \tilde{l}^{\nu}$$

- Need only massive part after splitting up the *D*-Dimensional loop contributions (in FDH scheme)

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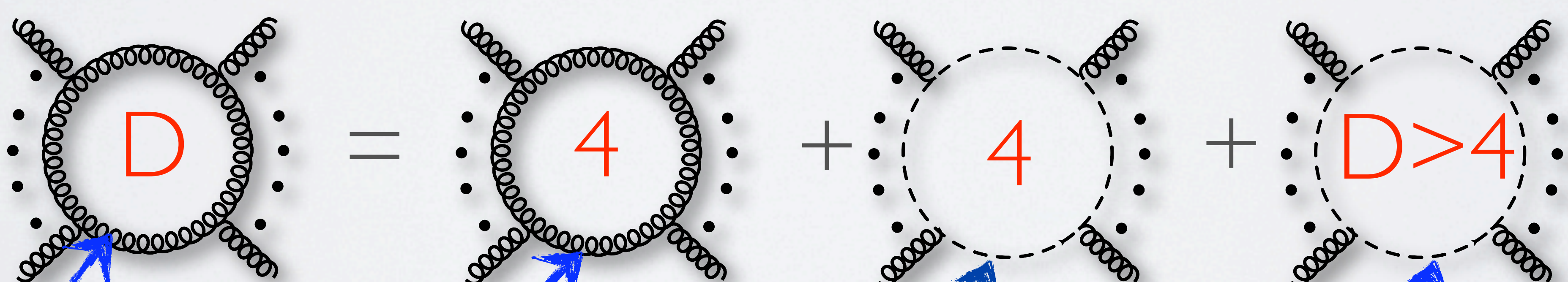
Massive (μ^2) 4 dim momenta   orthogonal *D*>4 dimensional momenta

- Need only massive part after splitting up the *D*-Dimensional loop contributions (in FDH scheme) e.g. for a gluon loop



Massless D-dim gluon   Massless gluon   Massive scalar   Scalar in D>4 dims

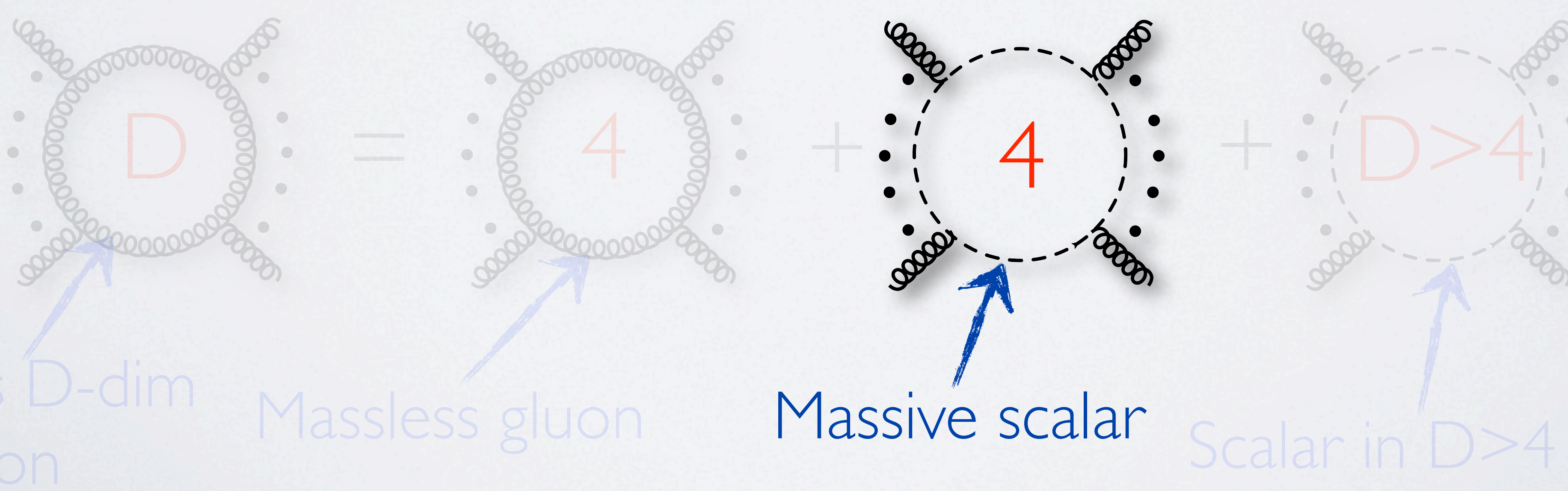
D-DIMENSIONAL UNITARITY

- Relate higher dimensional approach to massive approach by decomposing *D*-Dimensional loop momenta,

Massless *D* dim momenta → $l^\nu = \bar{l}^\nu + \mu \tilde{l}^\nu$

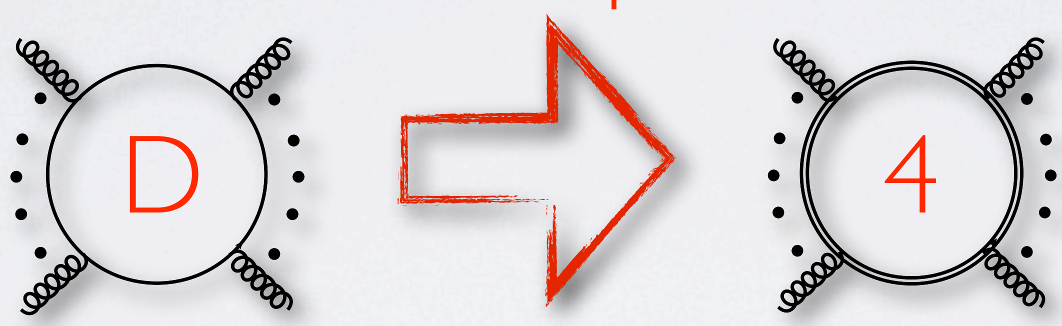
Massive (μ^2) 4 dim momenta   orthogonal *D*>4 dimensional momenta

- Need only massive part after splitting up the *D*-Dimensional loop contributions (in FDH scheme) e.g. for a gluon loop

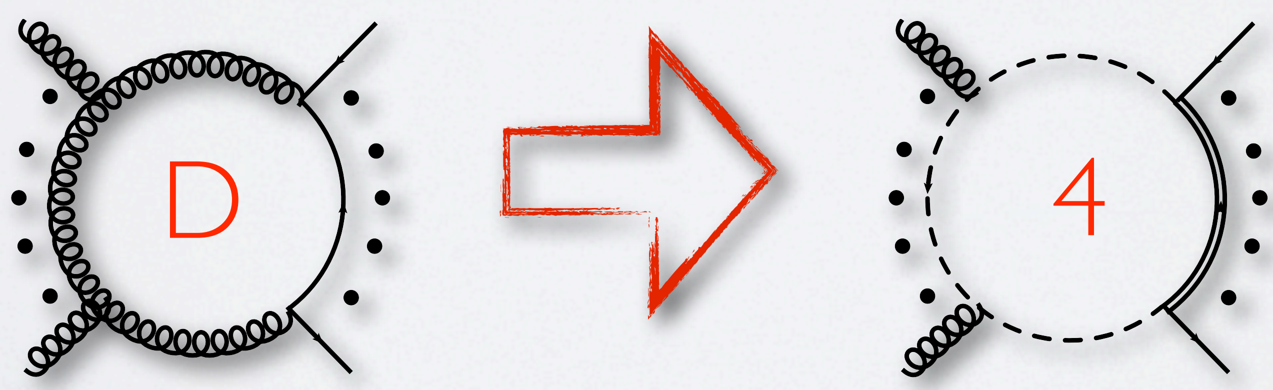


ISOLATING THE RATIONAL TERMS

- Similar rule for quarks, replace a massless D-Dimensional quark with a massive 4 dimensional quark.

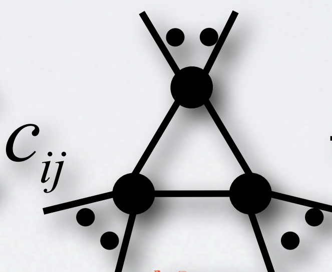
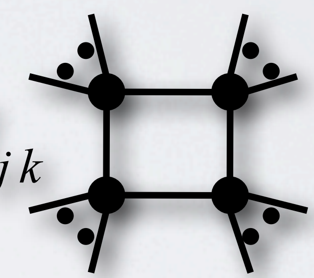


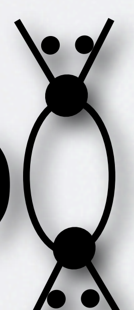
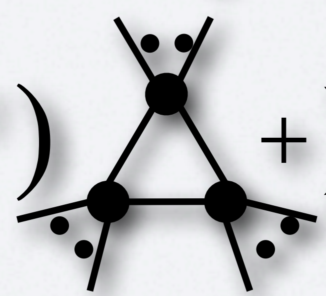
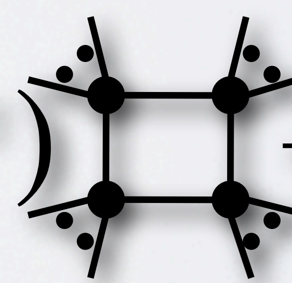
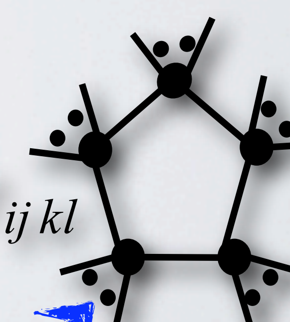
- Mixed gluon/quark loops are replaced by mixed massive scalar/ quark loops



NEW INTEGRAL BASIS

- In *D*-Dimensions the coefficients of the basis integrals pick up a *D*-Dimensional mass, μ, dependance.

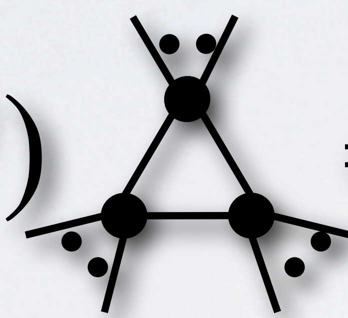
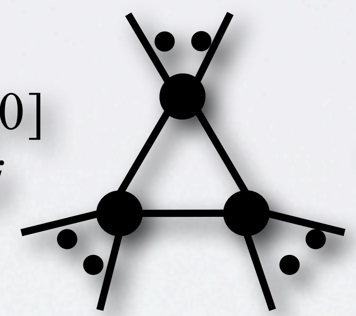
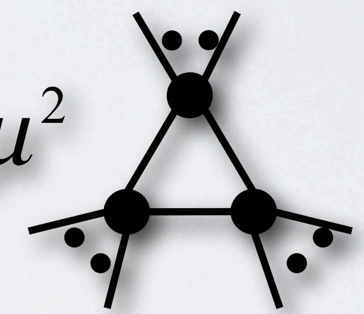
4 Dims $R_n + \sum_i b_i$  $+ \sum_{ij} c_{ij}$  $+ \sum_{ijk} d_{ijk}$ 

D Dims $\sum_i b_i(\mu^2)$  $+ \sum_{ij} c_{ij}(\mu^2)$  $+ \sum_{ijk} d_{ijk}(\mu^2)$  $+ \sum_{ijkl} e_{ijkl}$ 

Can now have pentagon contributions

RATIONAL TERMS FROM COEFFICIENTS

- Rational terms from mass dependant parts of the coefficients, e.g. triangle

$c_{ij}(\mu^2)$  $= c_{ij}^{[0]}$  $+ c_{ij}^{[1]}\mu^2$ 

- Only even powers of μ^2, max power related to max tensor power of l^μ, e.g. 4 for a box, 2 for a triangle.

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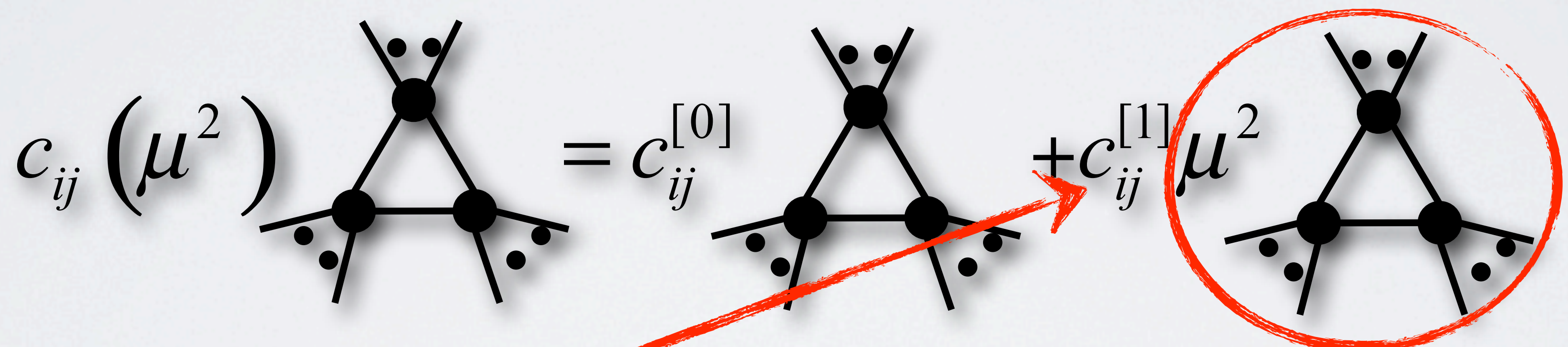


Extract the coefficient of this integral in the same way as the cut terms

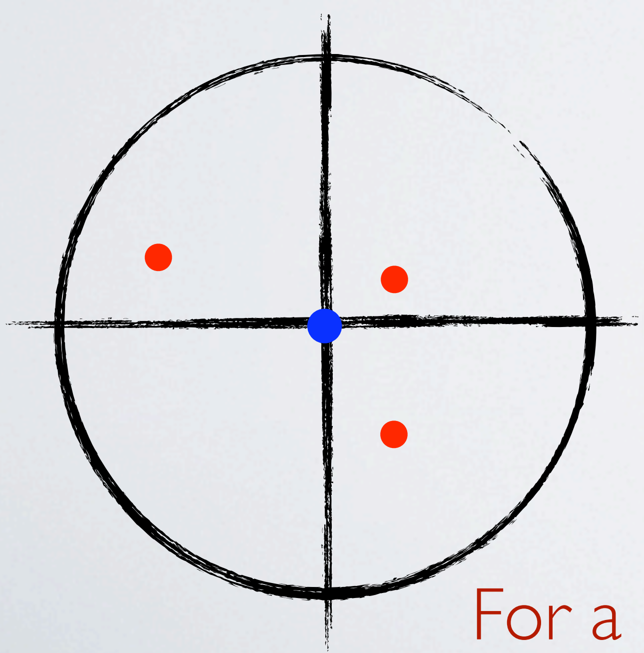
Integral gives finite contribution when $\varepsilon \to 0$, e.g. $I_3^{4-2\varepsilon}\left[\mu^2\right] \xrightarrow{\varepsilon\to 0} -\frac{1}{2}$

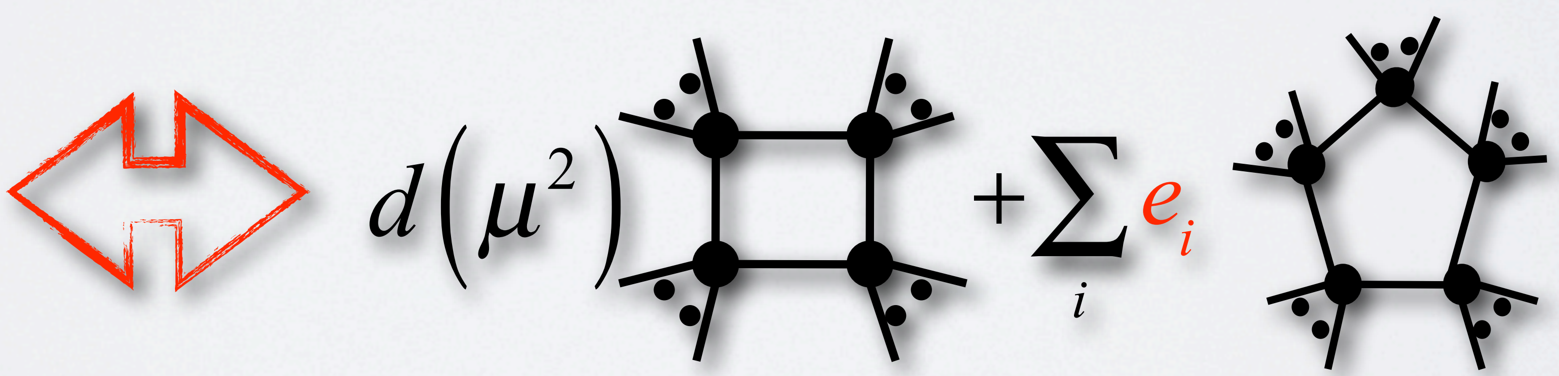
- Only even powers of μ^2, max power related to max tensor power of l^μ, e.g. 4 for a box, 2 for a triangle.

RATIONAL EXTRACTION

- Could use large parameter behaviour to extract coefficients, **numerically unstable**. (Badger)

- Subtract poles and perform discrete Fourier projection in μ^2 (in addition to the 4 dimensional cut parameters, e.g. t)





For a box subtract the pentagons, for the triangle subtract the boxes etc.

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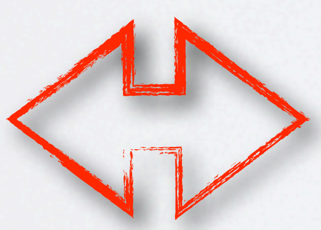
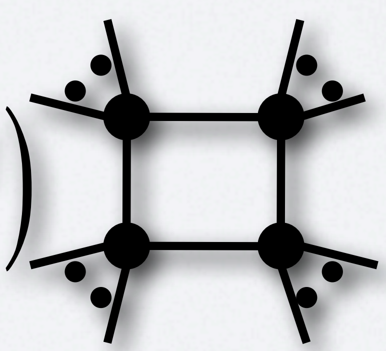


$d\left(\mu^2\right)$



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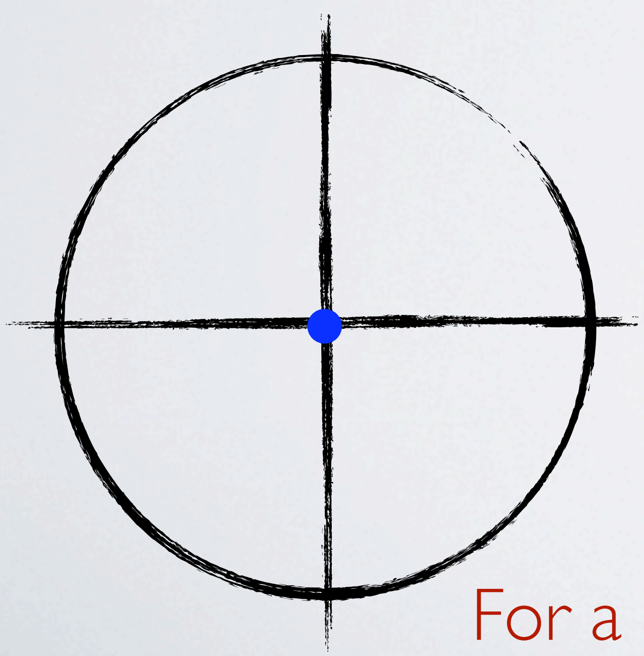
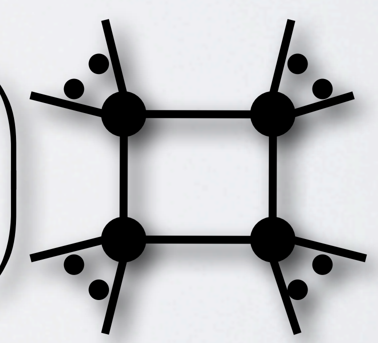


$$\Leftrightarrow \left(d_0 + d_2\mu^2 + d_4\left(\mu^2\right)^2\right)$$



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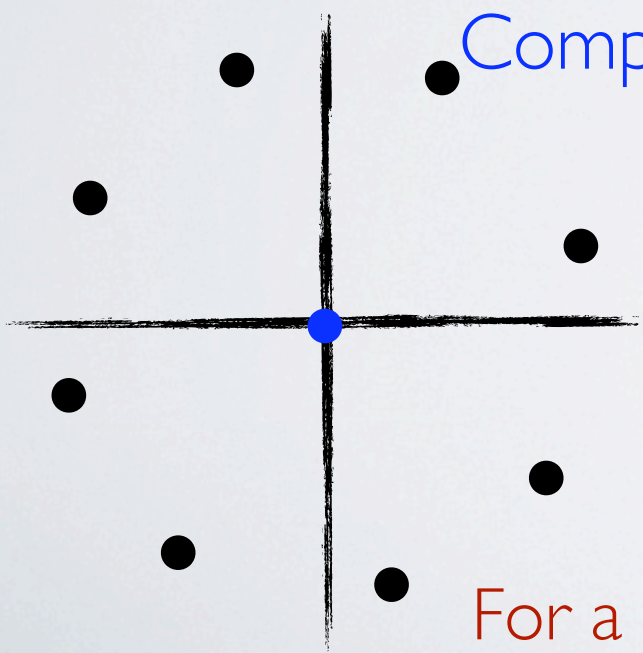
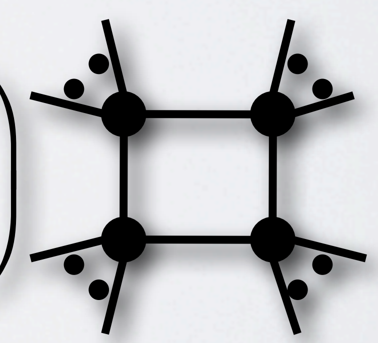
Compute this coefficient on circle around μ=0 on the complex plane



$$\left(d_0 + d_2\mu^2 + d_4\left(\mu^2\right)^2\right)$$



For a box subtract the pentagons, for the triangle subtract the boxes etc.

NUMERICAL ACCURACY

- Want to guarantee the accuracy of our numerical results.
- A number of checks. As an example, a powerful test for the rational terms uses the vanishing of higher tensor coefficients.
  - e.g. in the bubble, test how close to zero the coefficient of $\mu^2 y$ is. Gives a "free" test of the accuracy.
  - Re-compute just the coefficient that fails.
- A number of other tests, such as IR poles etc.

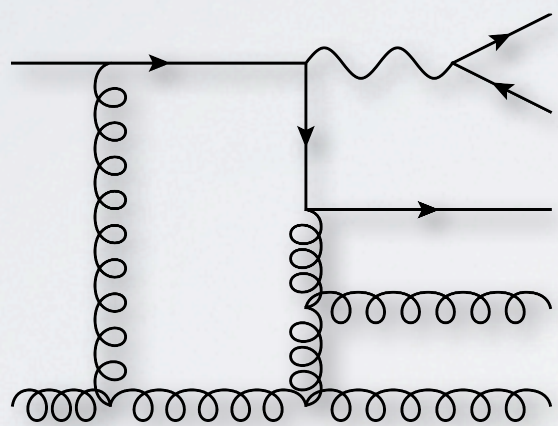
RATIONAL TERMS & BLACKHAT

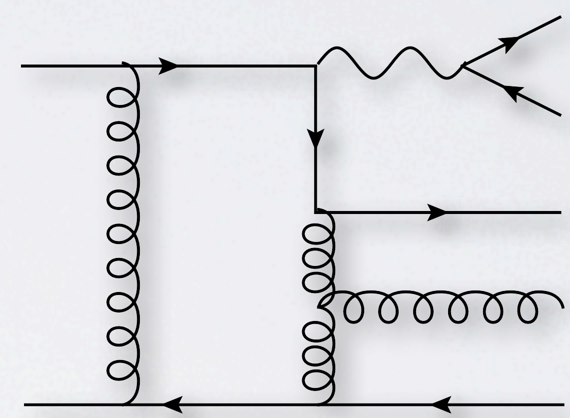
- Both on-shell recursion and rational extraction approach for rational terms now implemented in `BlackHat`.
  - Fermion & vector particles, with any number of legs.
- Use the best approach for a particular contribution.
  - *Rational extraction* approach just relies on knowing trees (useful when we don't want to think!)
  - *On-shell recursion* need to know a bit more the amplitude, can be made faster.
- Used to compute sub-leading contributions in new complete W+3 jets result.

W+3 JETS

- Leading Colour gives majority of the contribution,

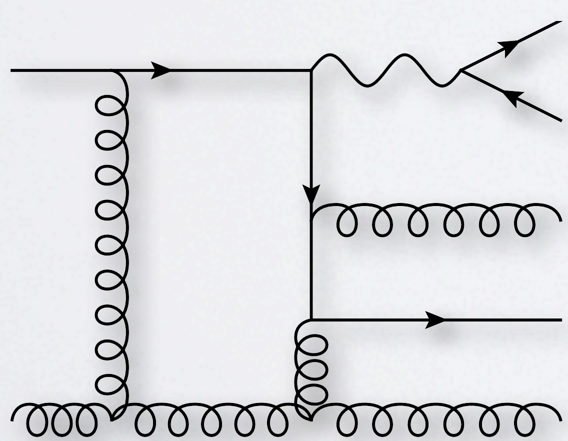
On-shell recursion

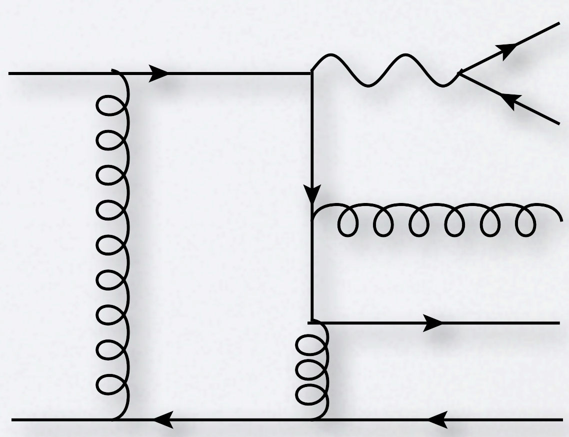


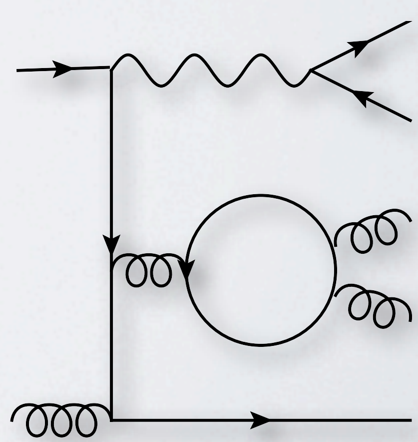


- Additional contributions for sub-leading colour, these include

Rational Extraction







TOTAL CROSS SECTIONS

- For the total cross section at the Tevatron,

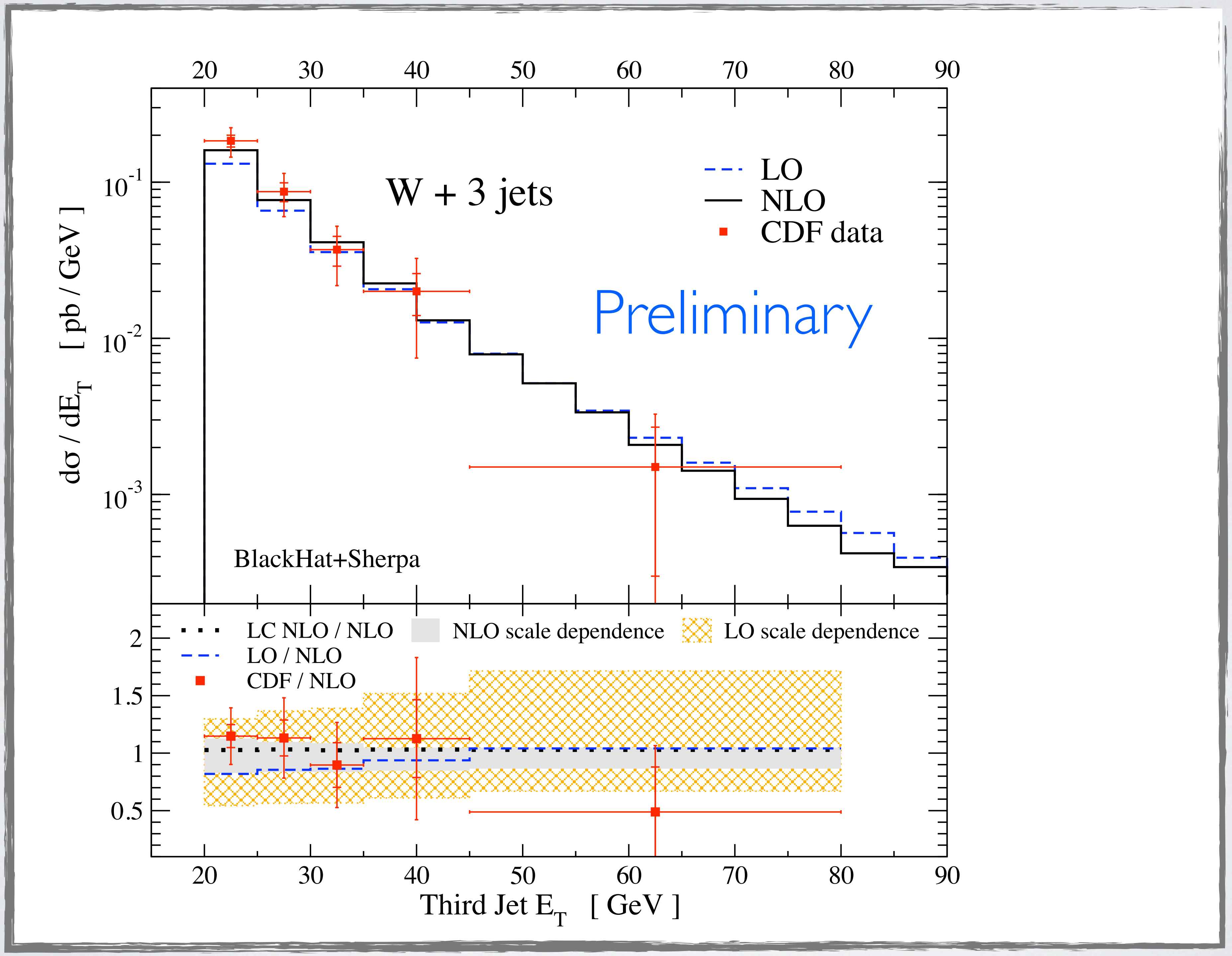
number of jets	CDF	LC NLO	NLO
1	53.5 ± 5.6	$58.3^{+4.6}_{-4.6}$	$57.8^{+4.4}_{-4.0}$
2	6.8 ± 1.1	$7.81^{+0.54}_{-0.91}$	$7.62^{+0.62}_{-0.86}$
3	0.84 ± 0.24	$0.908^{+0.044}_{-0.142}$	$0.882(5)^{+0.057}_{-0.138}$

Preliminary

- The Leading colour (LC) approximation is very good. Only an ~3% contribution from the Sub-leading colour.

- Per phase-space point sub-leading is much more demanding, but sample ~1/20 fewer points for same error as LC.

EFFECT ON DISTRIBUTIONS



CLIFFHANGER

- To be continued.... (See H. Ita's and F. Febres Cordero's talks tomorrow)

CONCLUSIONS

- Implemented *rational extraction* approach for computing rational terms alongside on-shell recursion within BlackHat.
- Computed the full W+3 jets contributions, including Sub-leading colour at both the Tevatron and the LHC.
- Small contribution from sub-leading terms to final result, leading colour approximation works well.