Precision Event Shapes with Effective Field Theory

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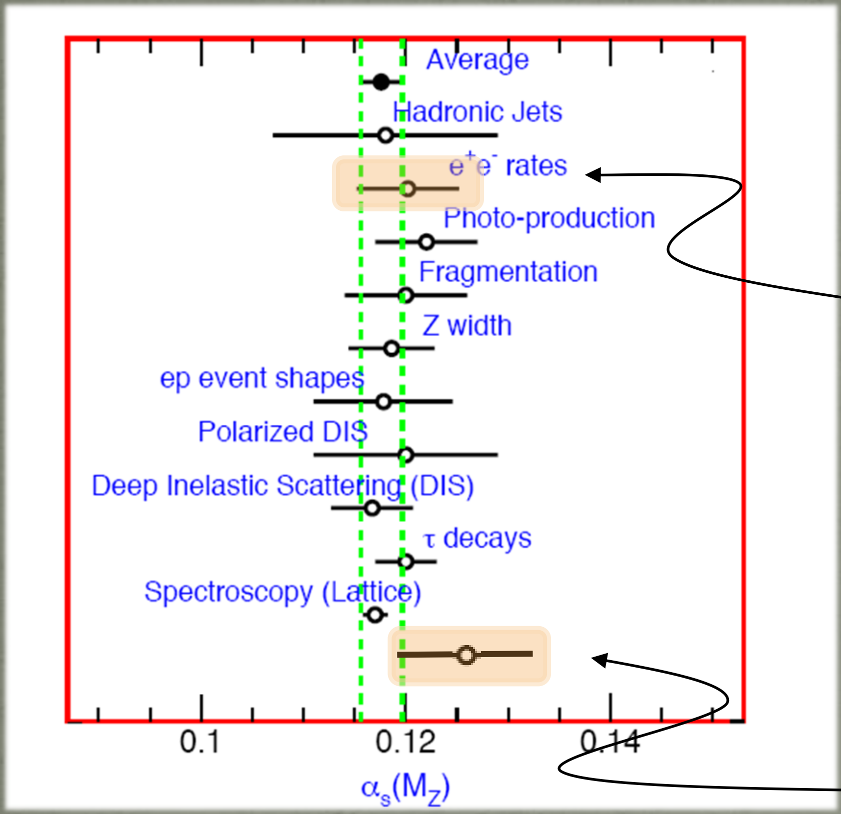
Harvard University

The Strong Coupling Constant

Source: PDG



World average

PDG (2007)

$$\alpha_s(M_Z) = 0.1176 \pm 0.002$$

LEP QCD working group avg.

$$\alpha_s(M_Z) = 0.1202 \pm 0.0003\,(\text{stat}) \pm 0.0049\,(\text{syst})$$

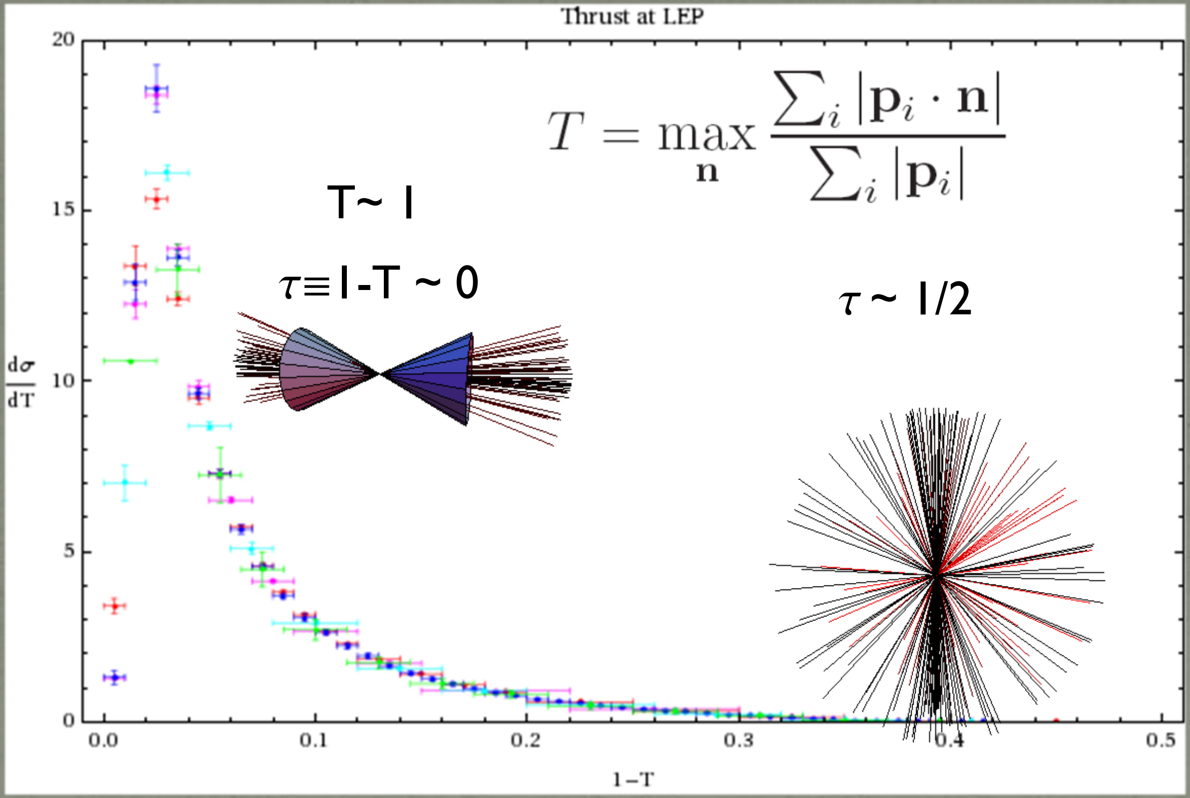
"totally dominated by theoretical uncertainties"

Thrust only, LEP 91.2 GeV data

$$\alpha_s(M_Z) = 0.1264 \pm 0.0008\,(\text{exp.}) \pm 0.0066\,(\text{theory})$$

ALEPH Collaboration, **Eur.Phys.J.C35:457-486,2004**

Thrust



Thrust at LEP is some of the best data in the world – 1 million clean events

ALEPH Collaboration, **Eur.Phys.J.C35:457-486,2004**

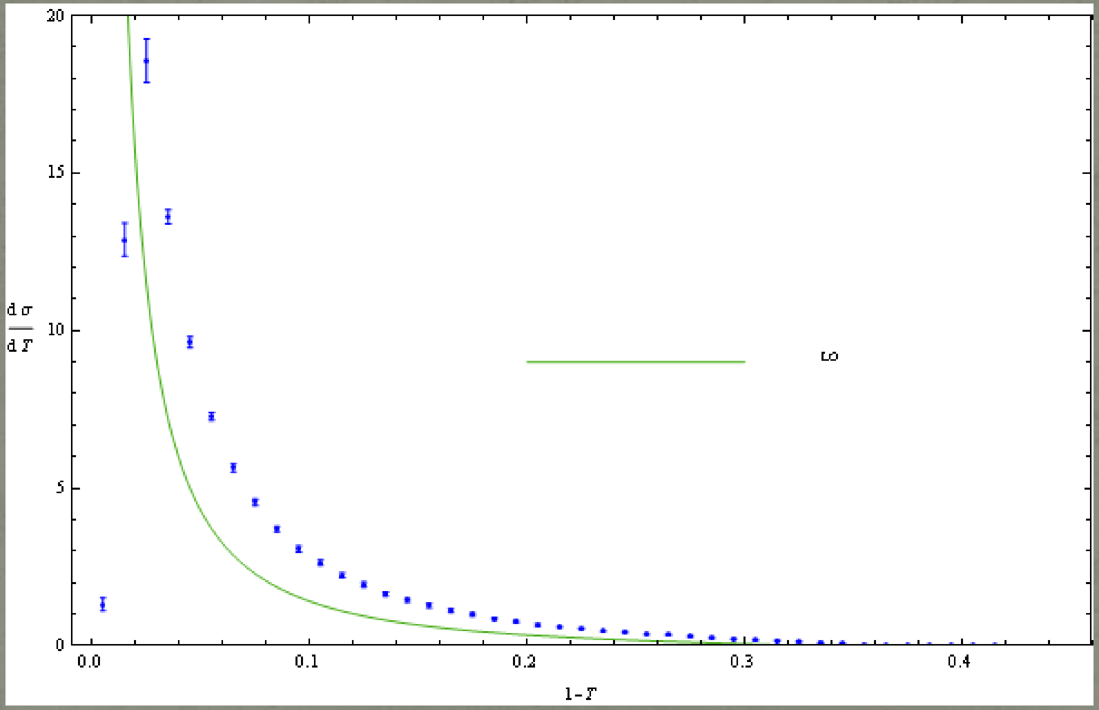
$$\alpha_s(M_Z) = 0.1264 \pm 0.0008\,(\text{exp.}) \pm 0.0066\,(\text{theory})$$

Why hasn't this data led to the world's best test of QCD?

Thrust at Leading Order (LO) in α_s

- Thrust can be calculated in perturbation theory
- At leading order, it is a textbook field theory exercise

$$\left[\frac{1}{\sigma_0}\frac{d\sigma}{d\tau}\right]_{\mathrm{LO}} = \delta(\tau) + \frac{C_F}{2\pi}\alpha_s\left[\frac{-4\log\tau - 3}{\tau} - 8 + 2\log\tau + 23\tau - \frac{44}{3}\tau^2 + \cdots\right]$$

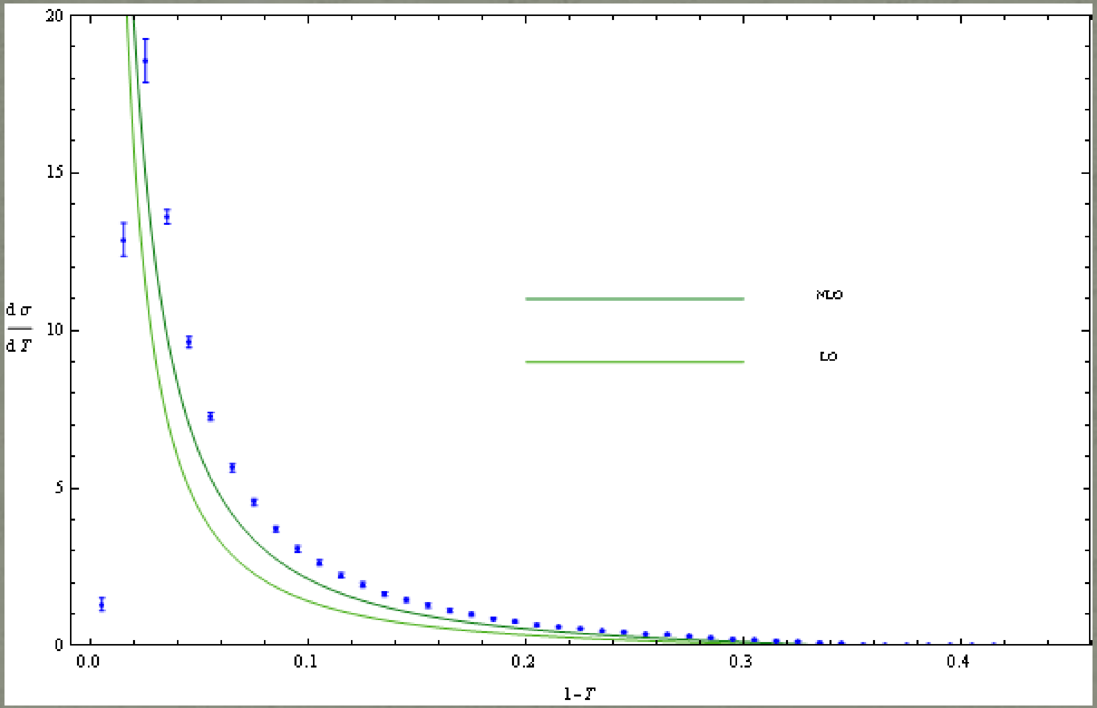


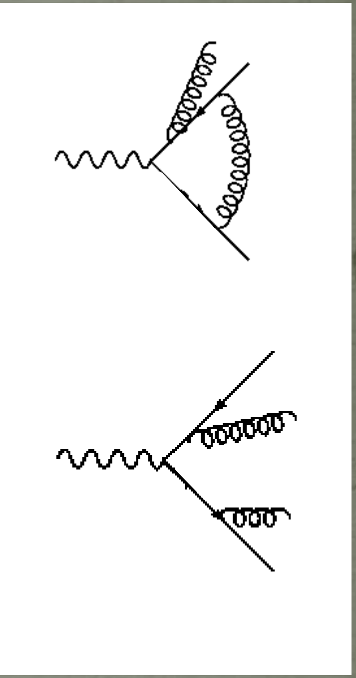
Not a very good fit to data!

Next-to-Leading Order (NLO)

- At next-to-leading order, it is an extremely difficult calculation
- Involves complicated integrals with overlapping divergences
- Answer only known numerically

Ellis, Ross, Terrano (**Nucl.Phys.B178:421,1981**)



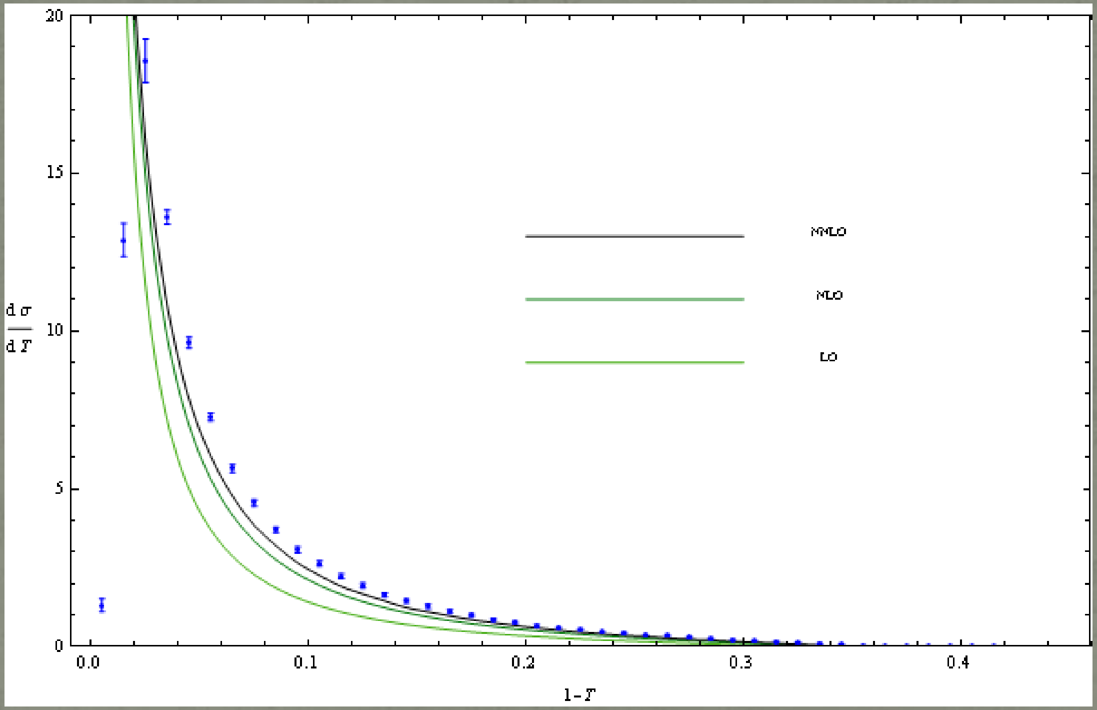


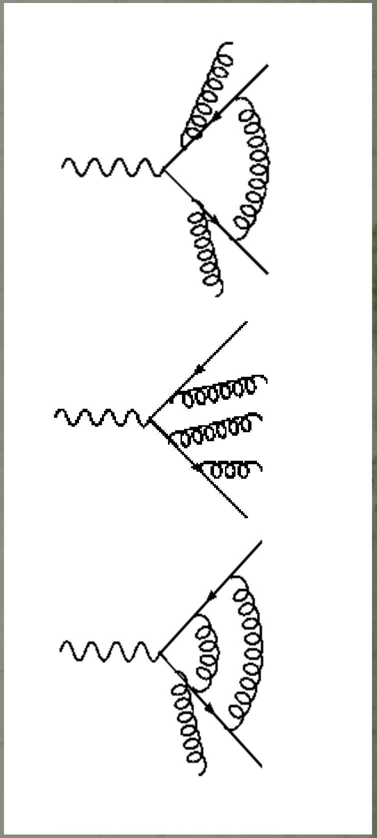
Still not a great fit to data!

Next-to-Next-to-Leading Order (NNLO)

- Involves nearly impossible loop calculations with multiple overlapping divergences
- Impressive culmination of many years of effort
- Answer only known numerically – using a supercomputer

Gehrmann-de Ridder, Gehrmann, Glover and Heinrich JHEP 0711:058, 2007





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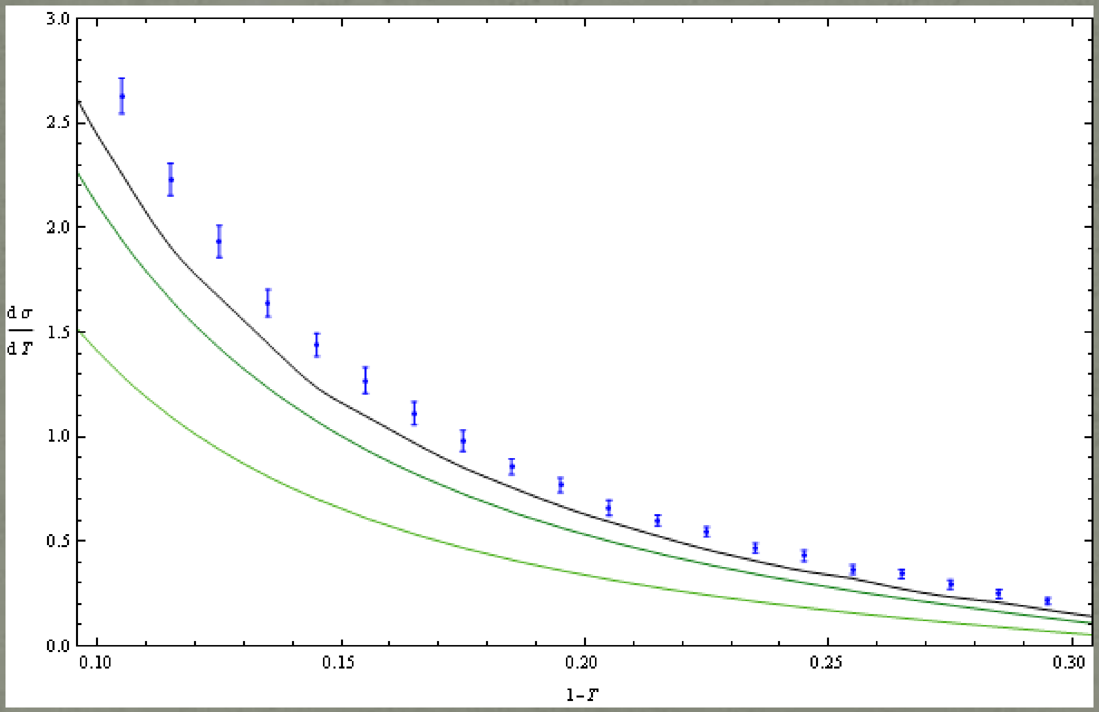
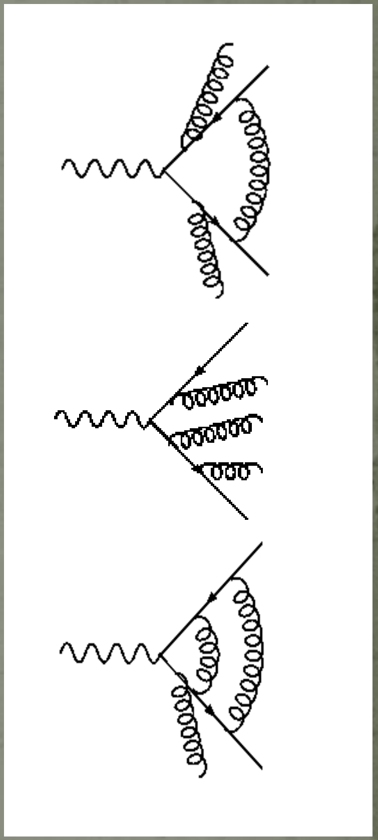
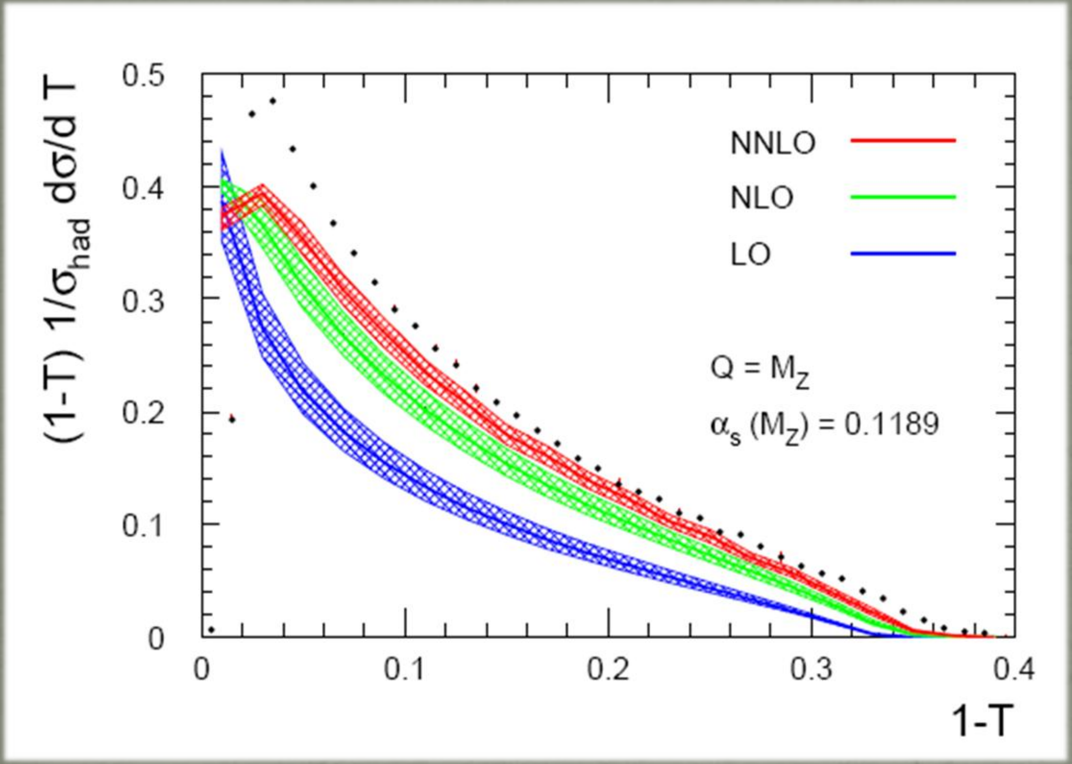




Fit for α_s at NNLO

Dissetori et al. (arXiv:0712.0327)



Fit to LEP data:

$$\alpha_s(M_Z) = 0.1274 \pm 0.0047$$

(Thrust only)

compare to world average

$$\alpha_s(M_Z) = 0.1176 \pm 0.002$$

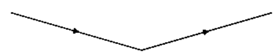
off by ~ 10% ~2σ!

What's missing?

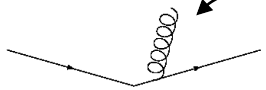
Recall the Leading Order thrust is blows up at small thrust:

$$\left[\frac{1}{\sigma_0}\frac{d\sigma}{d\tau}\right]_{\rm LO} = \delta(\tau) + \frac{C_F}{2\pi}\alpha_s\left[\frac{-4\log\tau - 3}{\tau} - 8 + 2\log\tau + 23\tau - \frac{44}{3}\tau^2 + \cdots\right]$$

Where do the divergences come from?



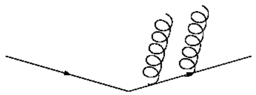
$= \delta(t)$



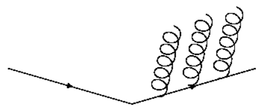
$= \alpha_s\frac{\log\tau}{\tau}$

Dominant contribution comes from soft and collinear radiation

More radiation makes it worse



$= \alpha_s^2\frac{\log^3\tau}{\tau}$



$= \alpha_s^3\frac{\log^5\tau}{\tau}$

Even if α_s is small, if ($\alpha_s\log^2 t$) is large, and the whole series is important

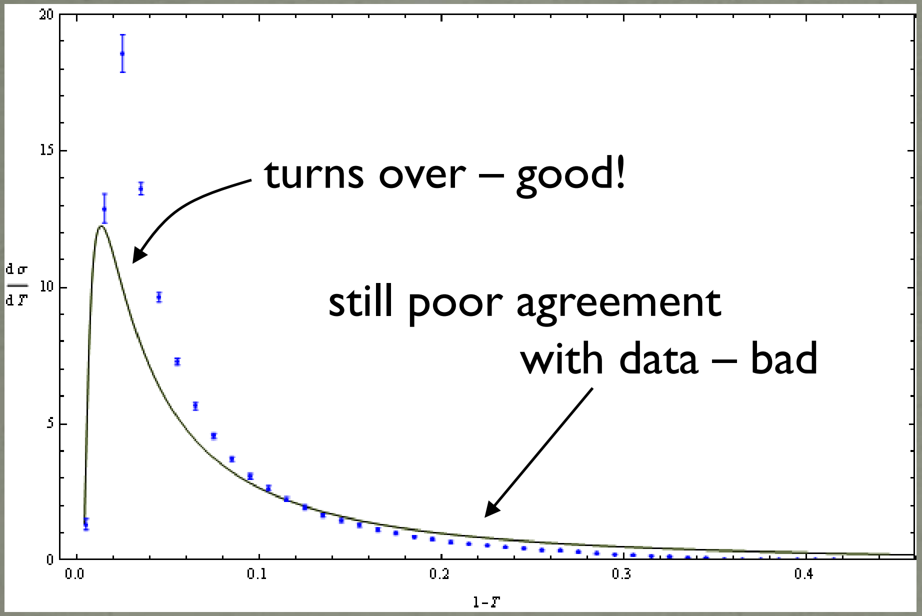
$$\frac{d\sigma}{d\tau} = \alpha_s\frac{\log\tau}{\tau}\left[1 - \alpha_s\log^2\tau + \frac{1}{2}\left(\alpha_s\log^2\tau\right)^2 + \cdots\right]$$

Sum up radition

We can sum the series:

$$\frac{d\sigma}{d\tau} = \alpha_s \frac{\log \tau}{\tau} \exp\left[-\alpha_s \log^2 \tau\right]$$

•This is equivalent to integrating the radiation semi-classically



The semi-classical resummation of thrust was done first in 1993

Catani et al. (**Nucl.Phys.B407:3-42,1993)**

NLL resummation

•Semi-classical approach gets stuck at this order

•Little progress since then – often believed that colliders are too messy to calculate anything more accurately

Traditional Resummation

- Soft Logs
  - summed in Eikonal Approximation

  $S(k) \approx e^{ikx}\; W(x)W(0)$

- Collinear logs
  - summed in semi-classical Jet Functions

  $J(m)$ = Probability for finding Jet of mass m

- Hard logs
  - Variation of α_s at Hard Scale

- Factorization Theorem
  - Heuristic, based on phase space decomposition

Effective Field Theory

- Soft Logs
  - Eikonal approximation derived from SCET Lagrangian

  $S(k) \approx e^{ikx}\; W(x)W(0)$

- Collinear logs
  - matrix elements of collinear fields
  - derived from SCET Lagrangian

- Hard logs
  - Matching calculation QCD$\rightarrow$ SCET

- Factorization Theorem
  - Also Heuristic, based on power counting

Advantages of SCET

- Resummation done through renormalization group
  - From operator anomalous dimensions, not radiation probabilities
- Systematically improvable
  - Anomalous dimensions are easier to calculate than loops in full QCD
  - Power corrections (eg. m_b corrections from HQET)
  - Factorize off universal non-perturbative shape functions
- Physical scales manifest
  - Hard Scale Q, Jet Scale p, Soft Scale p^2/Q
  - Distinguishes $\log\frac{Q^2}{p^2}$ from $\log\frac{p^2}{(p^2/Q)^2}$ → More honest estimate of theoreteical uncertainties
- Resummation done in momentum space
  - Avoids integrating over Landau pole during Mellin transform

Fleming, Hoang, Mantry, Stewart **(hep-ph/0703207)**

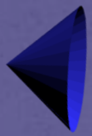
MDS, PRD:77.14026 (2008)

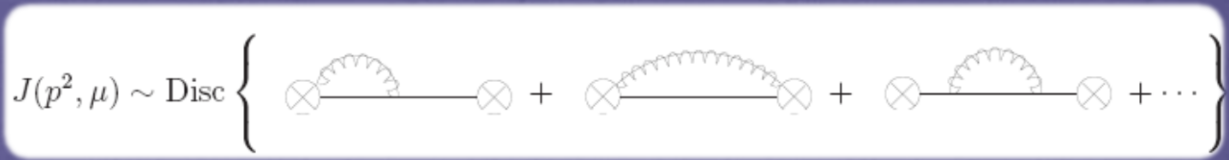
Factorization for thrust:

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{d\tau} = |C_H(Q)|^2 \int dp^2\, dq^2 J(p^2)\, J(q^2)\, S_T\!\left(\tau Q - \frac{p^2+q^2}{Q}\right)$$

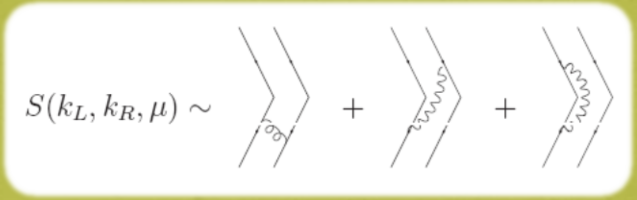
Hard Function:

Jet Function:





Soft Function:



4^{th} order resummation

For example, jet function at 1-loop:

$$J(p^2) = \delta(p^2) + C_F\left(\frac{\alpha_s}{4\pi}\right)\left\{\left(\frac{7}{4} - \frac{\pi^2}{4}\right)\delta(p^2) + \left[\frac{4\log\frac{p^2}{Q^2} - 3}{p^2}\right]_\star\right\} + \cdots$$

Bauer and Manohar, PRD:70.034024 (2004)
Bosch, Lange, Neubert, Paz, NPB 699 335 (2004)

We have 4-loop β-function
4-loop cusp anomalous dimensions (Pade)
3-loop anomalous dimensions
2-loop hard and jet finite parts

Becher, Neubert, Pecjak JHEP 0701:076,2007

- Soft function finite part known analytically at 1-loop
- 2-loop soft function can be computed numerically

MDS, PRD: 77.14026 (2008)

MDS, T. Becher, arXiv:0803.0342

Next-to-next-to-next-to-leading log resummation (NNNLL)

(without effective field theory, only NLL available)

vs Fixed Order

Effective Field Theory is an approximation

- it gets the large part rights to all order in α_s
- but it *only* gets large parts right, not finite remainders

Expand the effective field theory thrust distribution in α_s:

MDS, PRD:77.14026 (2008)

$$\left[\frac{1}{\sigma_0}\frac{d\sigma}{d\tau}\right]_{\mathrm{SCET}} = \frac{1}{\tau} e^{4S(Q,Q\tau)+6A(Q,Q\tau)+\cdots}\left[1+\alpha_s(Q\sqrt{\tau})(c_1+c_2\partial_\eta+c_3\partial_\eta^2)+\cdots\right]\frac{e^{-2\gamma_E\eta}}{\Gamma[2\eta]}$$

$$= \delta(\tau) + \frac{C_F}{2\pi}\alpha_s\left[\frac{-4\log\tau-3}{\tau}\right]+\cdots$$

This should approach the fixed order as $\tau \to 0$

$$\left[\frac{1}{\sigma_0}\frac{d\sigma}{d\tau}\right]_{\mathrm{LO}} = \delta(\tau) + \frac{C_F}{2\pi}\alpha_s\left[\frac{-4\log\tau-3}{\tau} - 8 + 2\log\tau + 23\tau - \frac{44}{3}\tau^2 + \cdots\right]$$

✓ It successfully reproduces the singular behavior of the leading fixed-order result

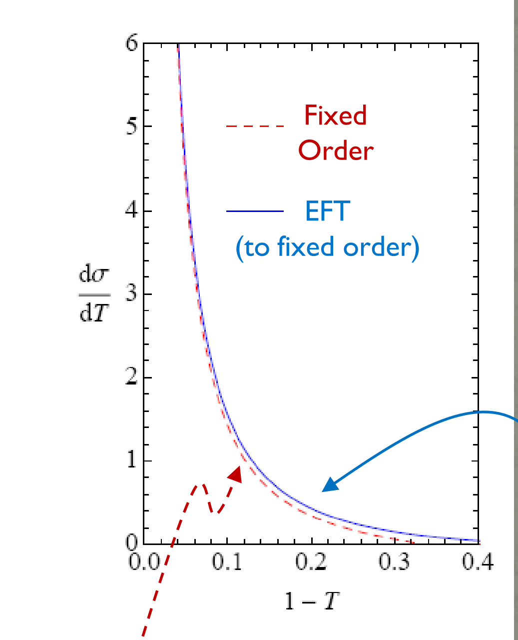
Compare to Fixed Order

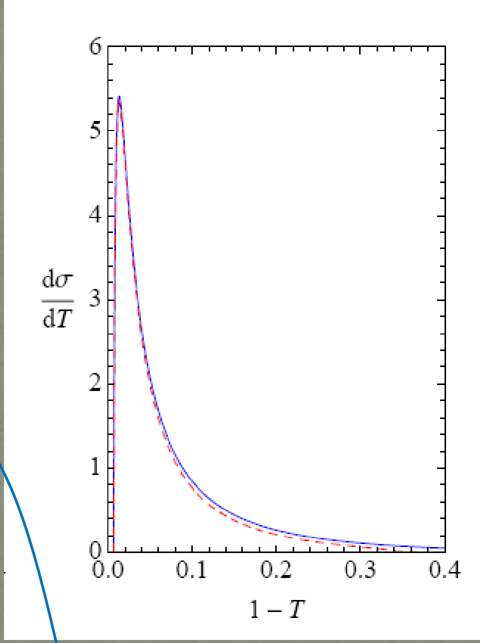
- Effective Field Theory result known exactly
- Beyond leading order, fixed-order results known only numerically

MDS, T. Becher, arXiv:0803.0342

leading order

second order





$$\frac{-4\log\tau - 3}{\tau} - 8 + 2\log\tau + 23\tau - \frac{44}{3}\tau^2$$

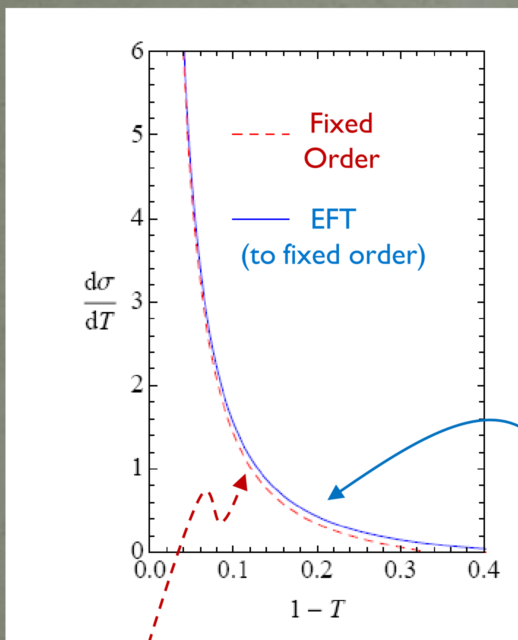
$$\frac{-4\log\tau - 3}{\tau}$$

Compare to Fixed Order

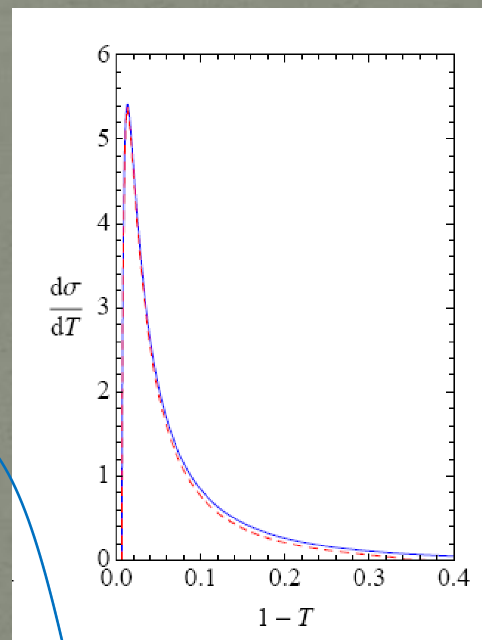
- Effective Field Theory result known exactly
- Beyond leading order, fixed-order results known only numerically

MDS, T. Becher, arXiv:0803.0342

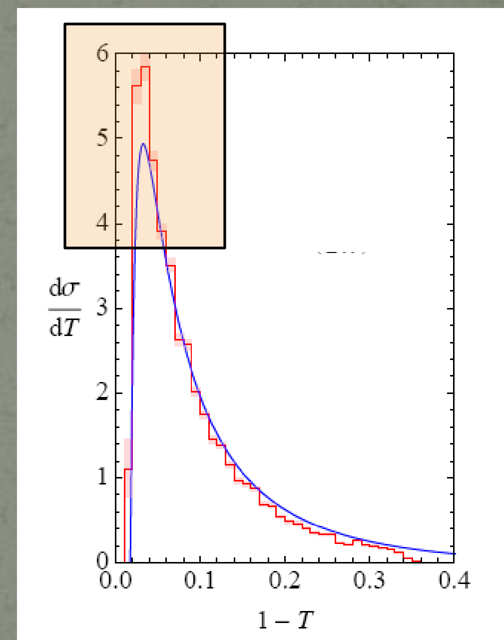
leading order



second order



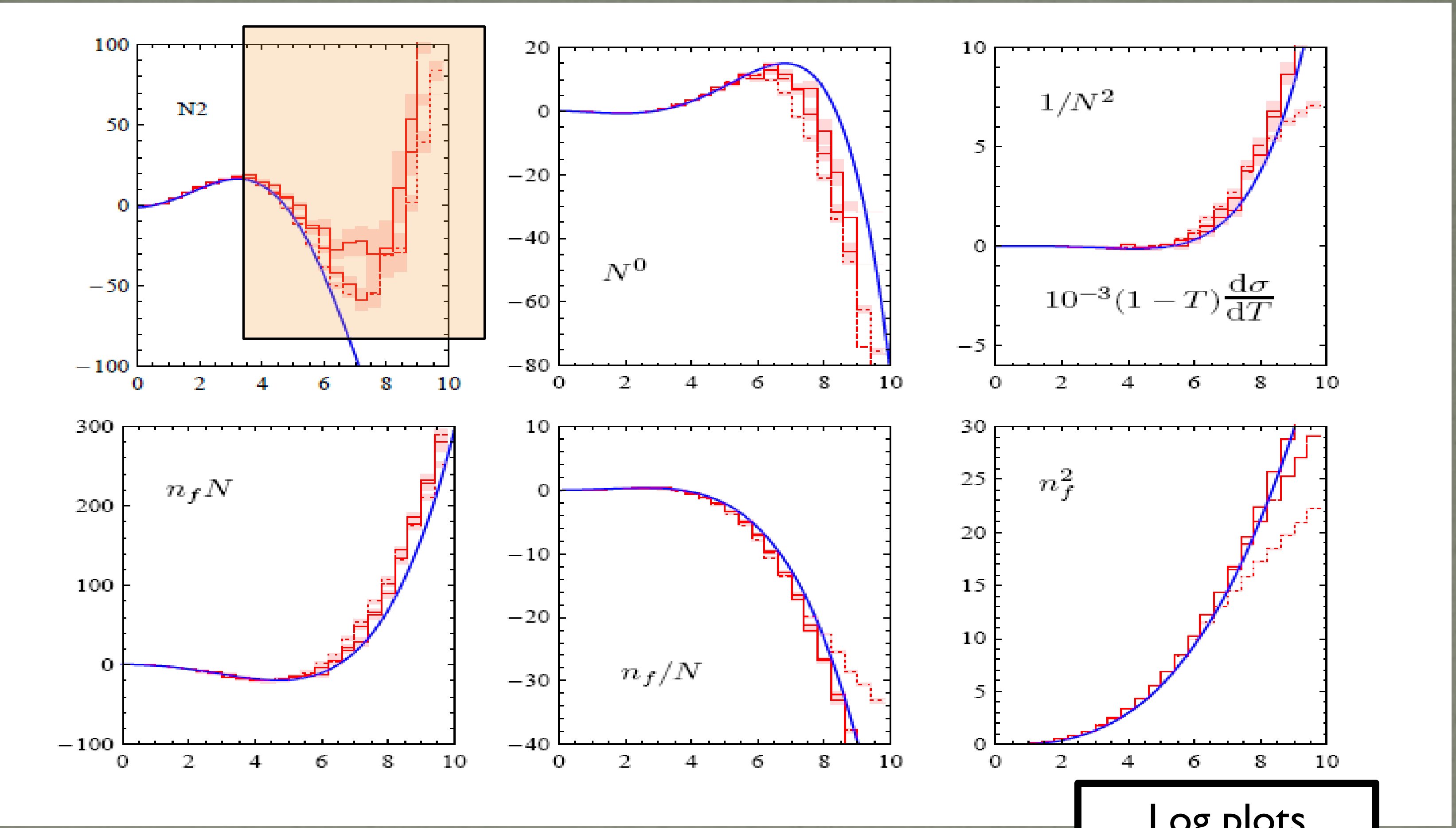
third order



$$\frac{-4\log\tau-3}{\tau}-8+2\log\tau+23\tau-\frac{44}{3}\tau^2$$

$$\frac{-4\log\tau-3}{\tau}$$

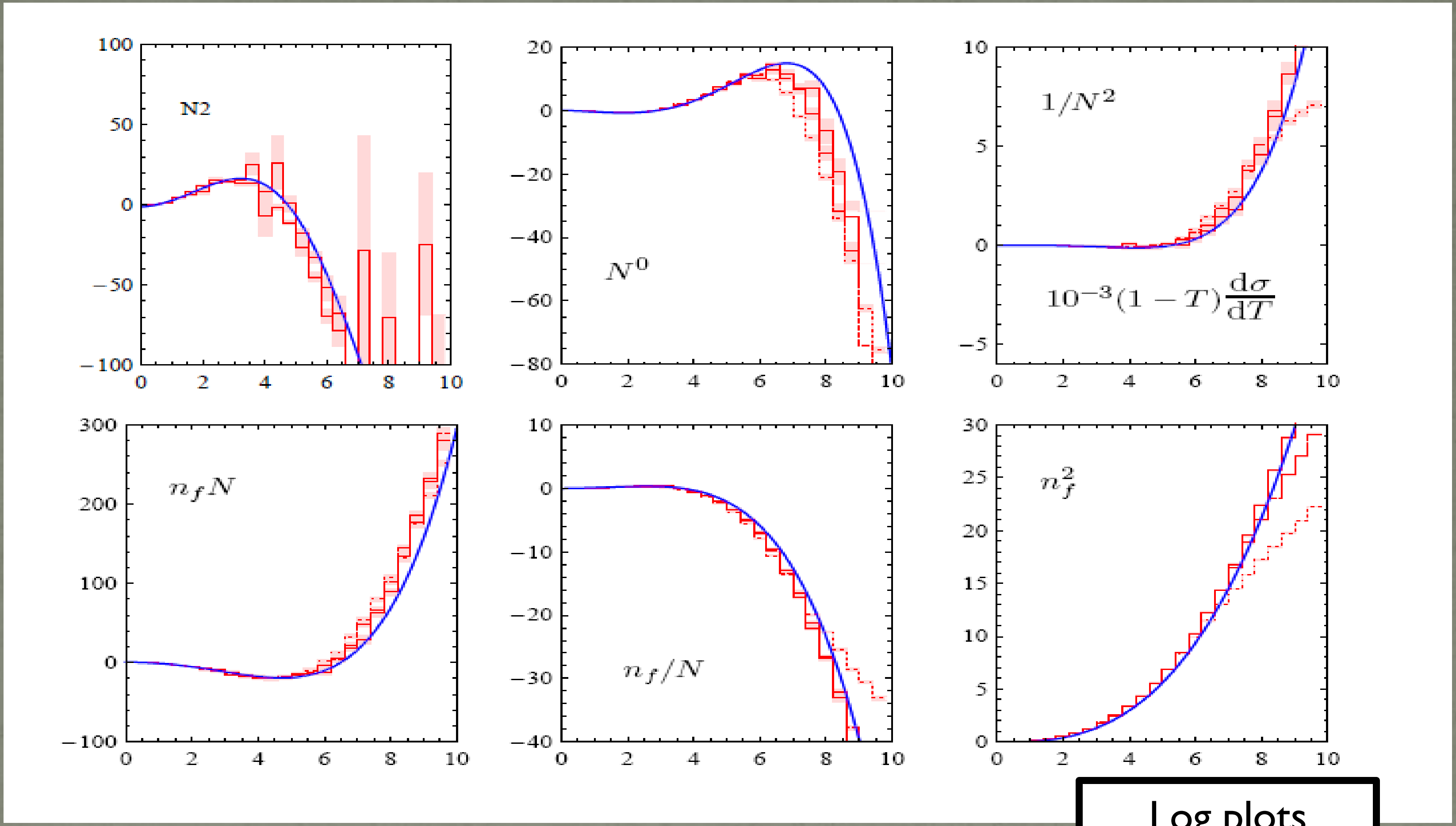
NNLO color structures



Log plots

Data generously provided by Gerhmann et al.

NNLO color structures



Log plots

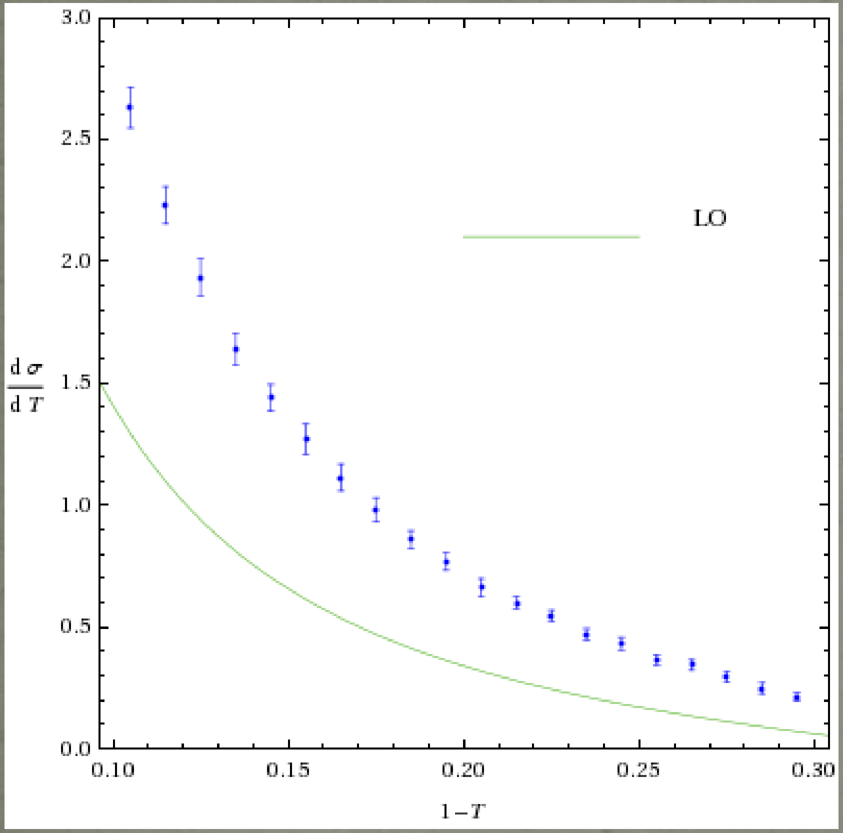
Corrected histograms of Gerhmann et al.

Consistent with observation of incomplete subtraction by S. Weinzierl

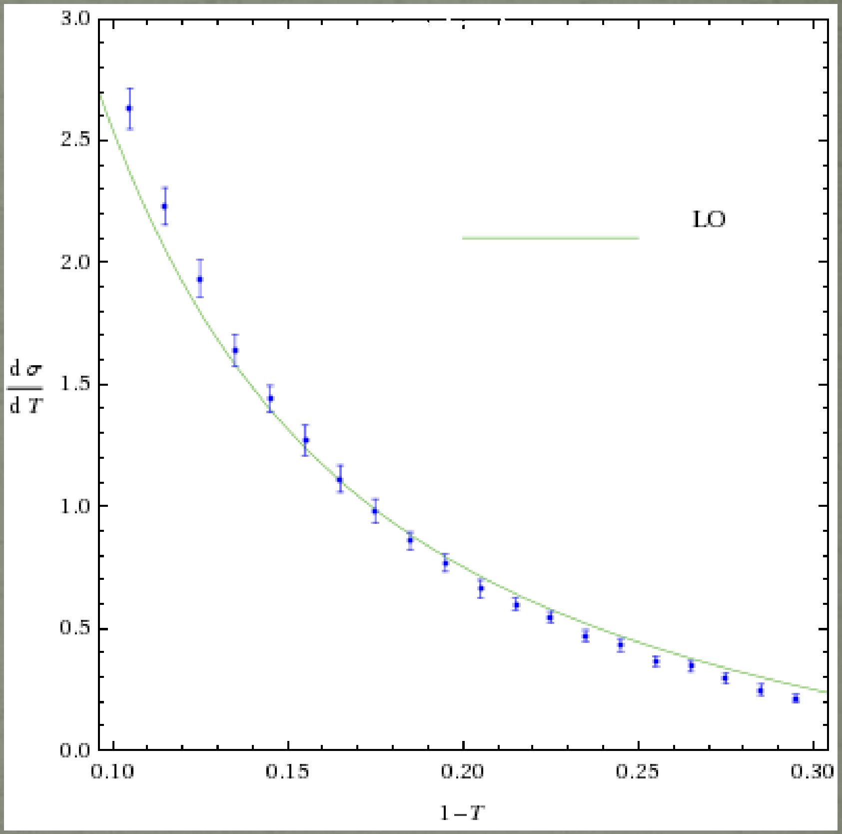
S. Weinzierl PRL 101:162001, 2008

Convergence

Fixed Order



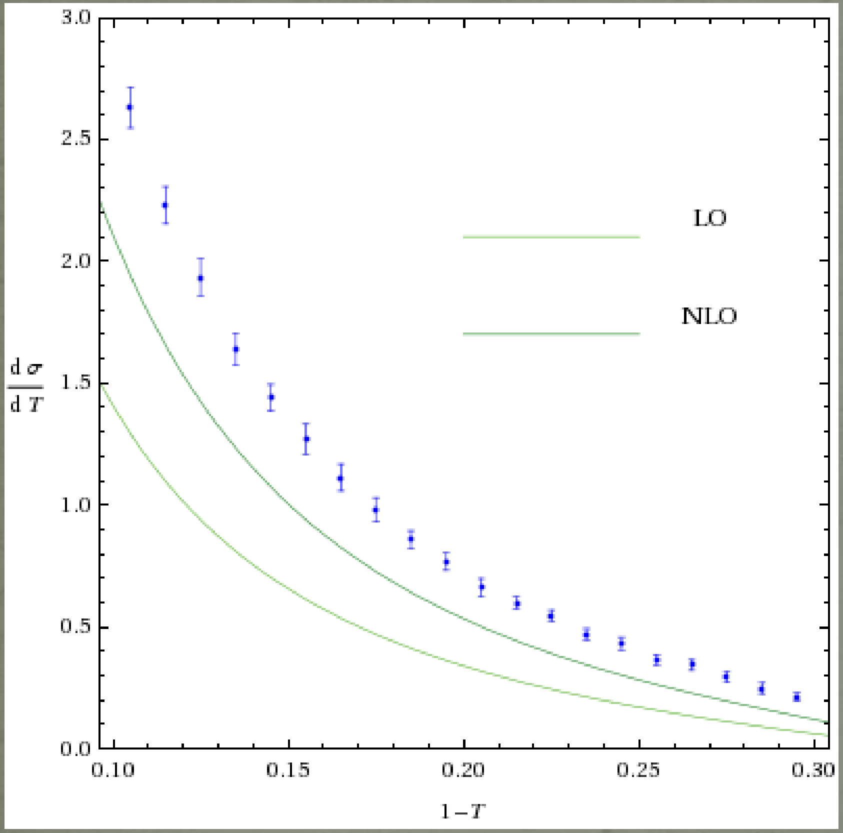
Effective Field Theory
(matched to Fixed Order)



At fixed $\alpha_s(M_Z)$ = 0.1168

Convergence

Fixed Order



Effective Field Theory
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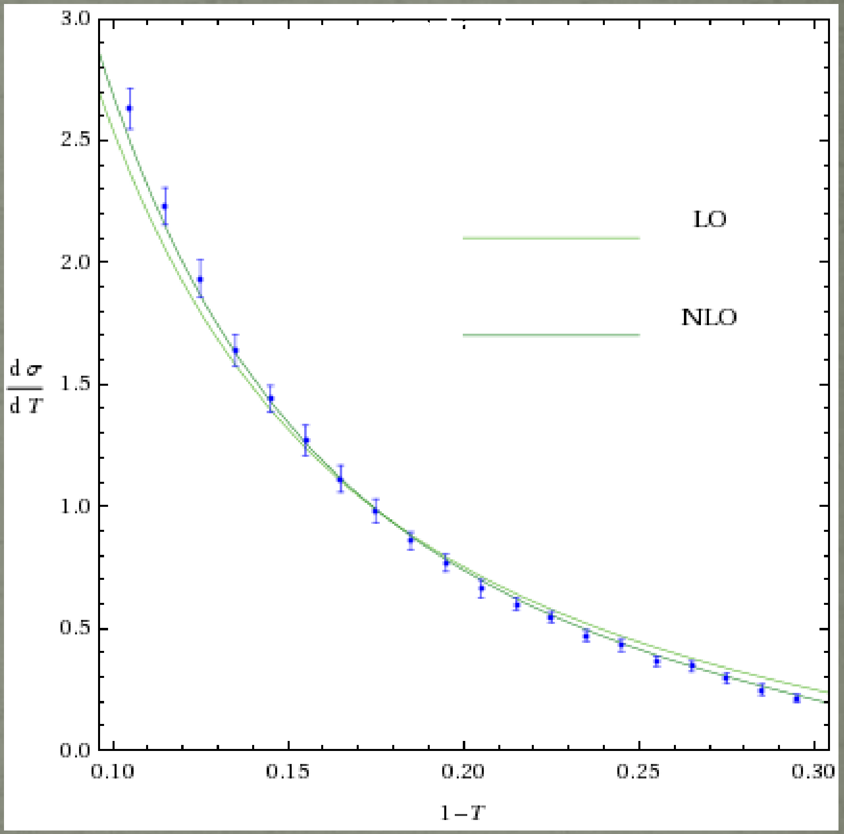
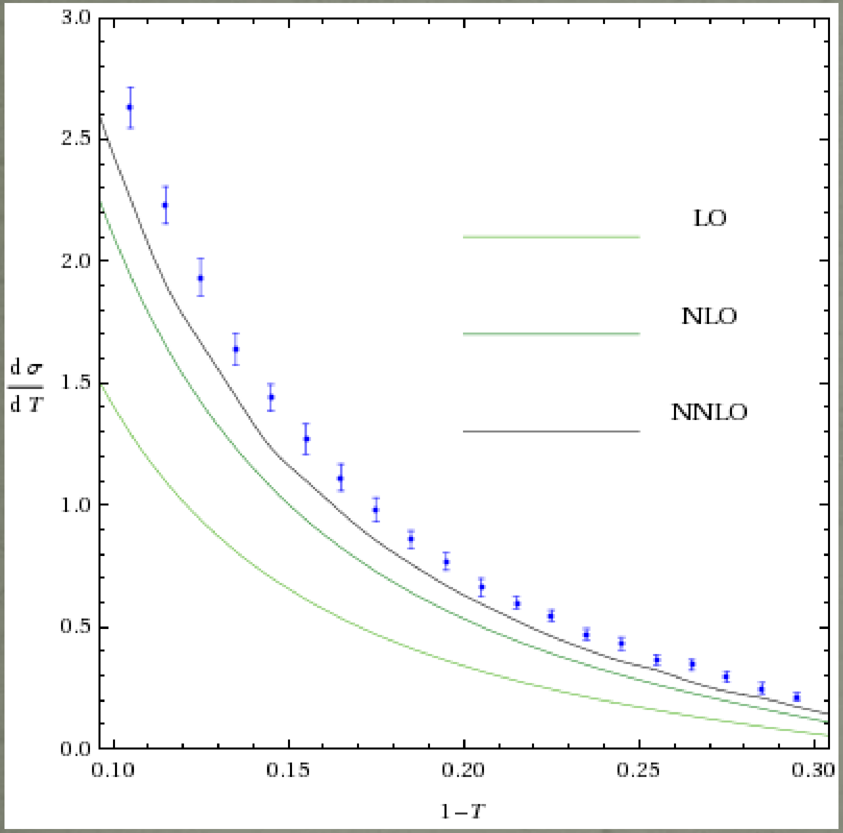


At fixed $\alpha_s(M_Z) = 0.1168$

Convergence

Fixed Order



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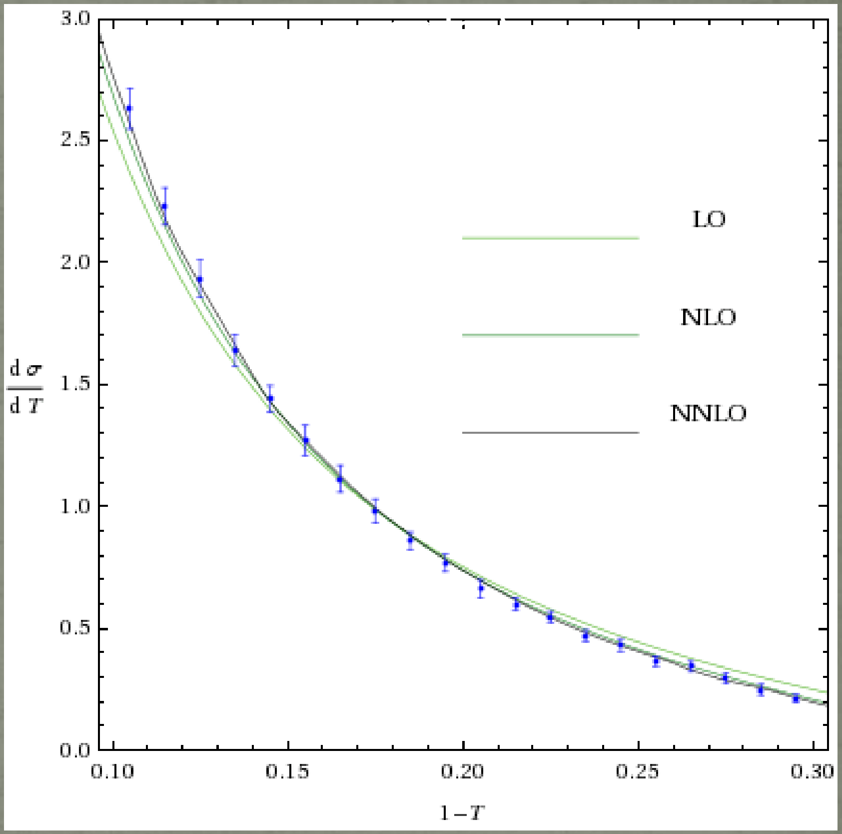
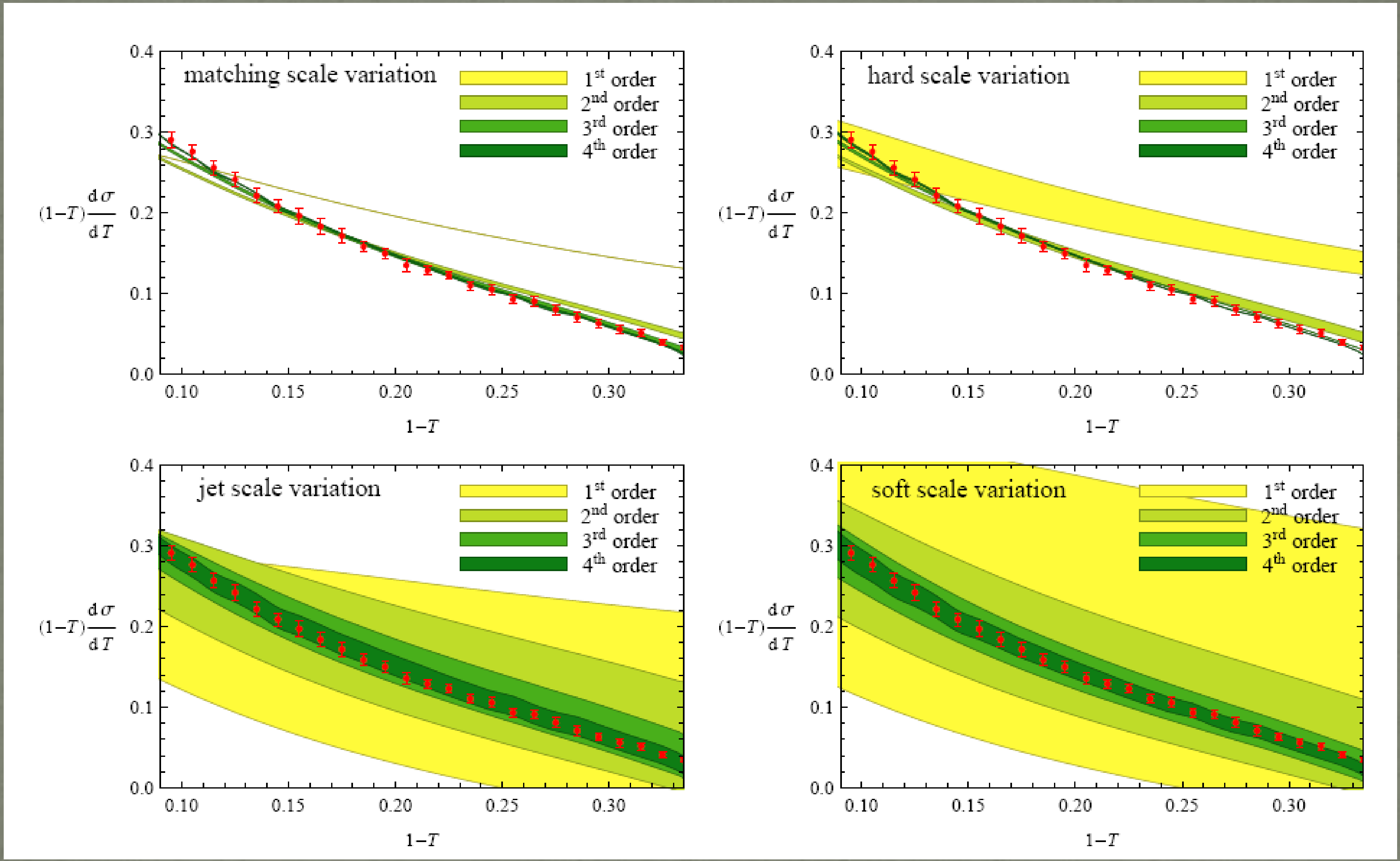
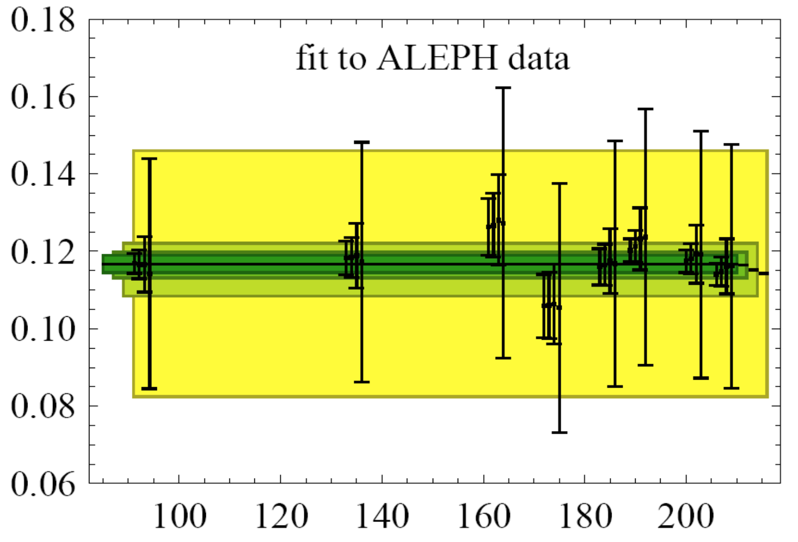


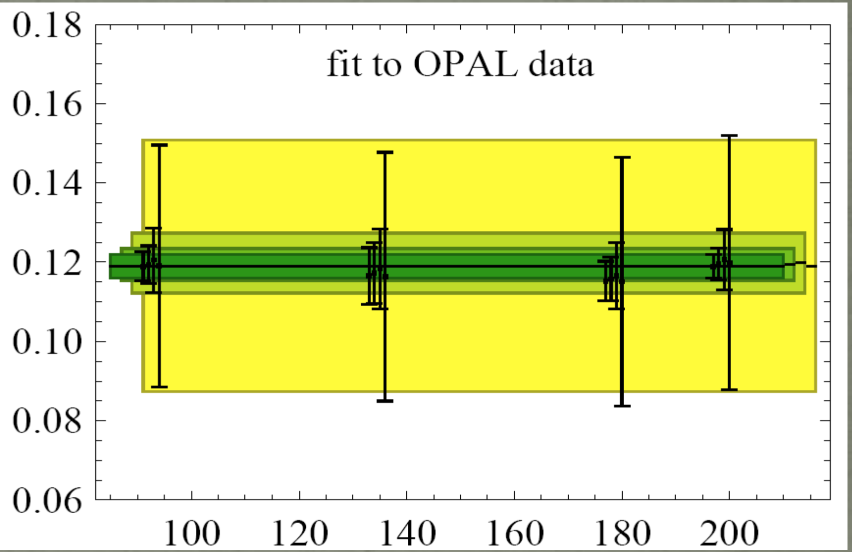
At fixed $\alpha_s(M_Z) = 0.1168$

Perturbative Uncertainties



LEP I and LEP II





LEP1: $\alpha_s(M_Z)$ = 0.1177 ± 0.0001 (stat)
± 0.0008 (sys)
± 0.0014 (had)
± 0.0013 (pert)

LEP1/LEP2: $\alpha_s(M_Z)$ = 0.1168 ±0.0022

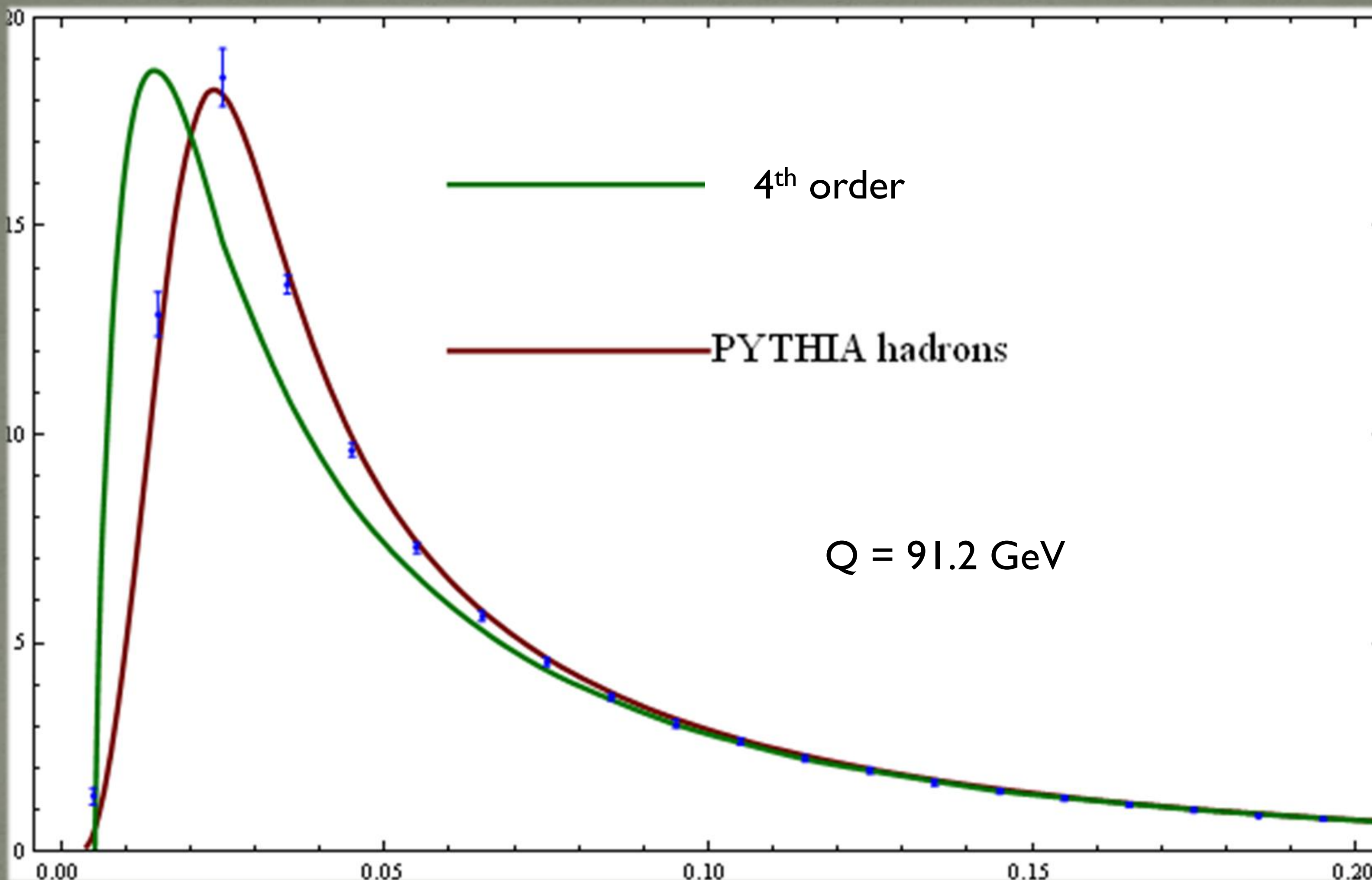
LEP1: $\alpha_s(M_Z)$ = 0.1179 ± 0.0001 (stat)
± 0.0011 (sys)
± 0.0031 (had)
± 0.0014 (pert)

LEP1/LEP2: $\alpha_s(M_Z)$ = 0.1189 ±0.0030

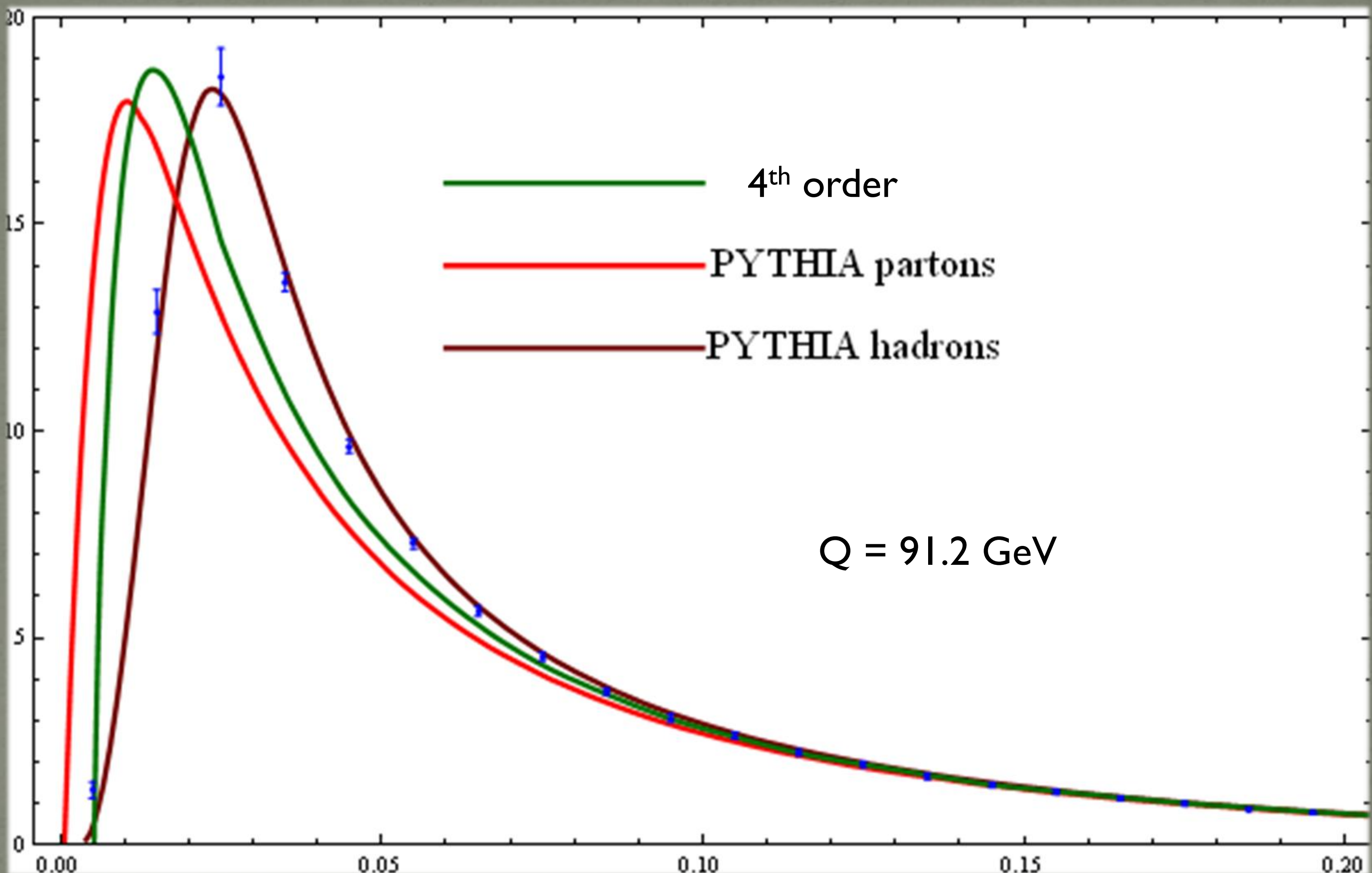
$\alpha_s(M_Z)$ = 0.1172 ±0.0022

$\alpha_s(M_Z)$ = 0.1176 ±0.0020 (World Average)

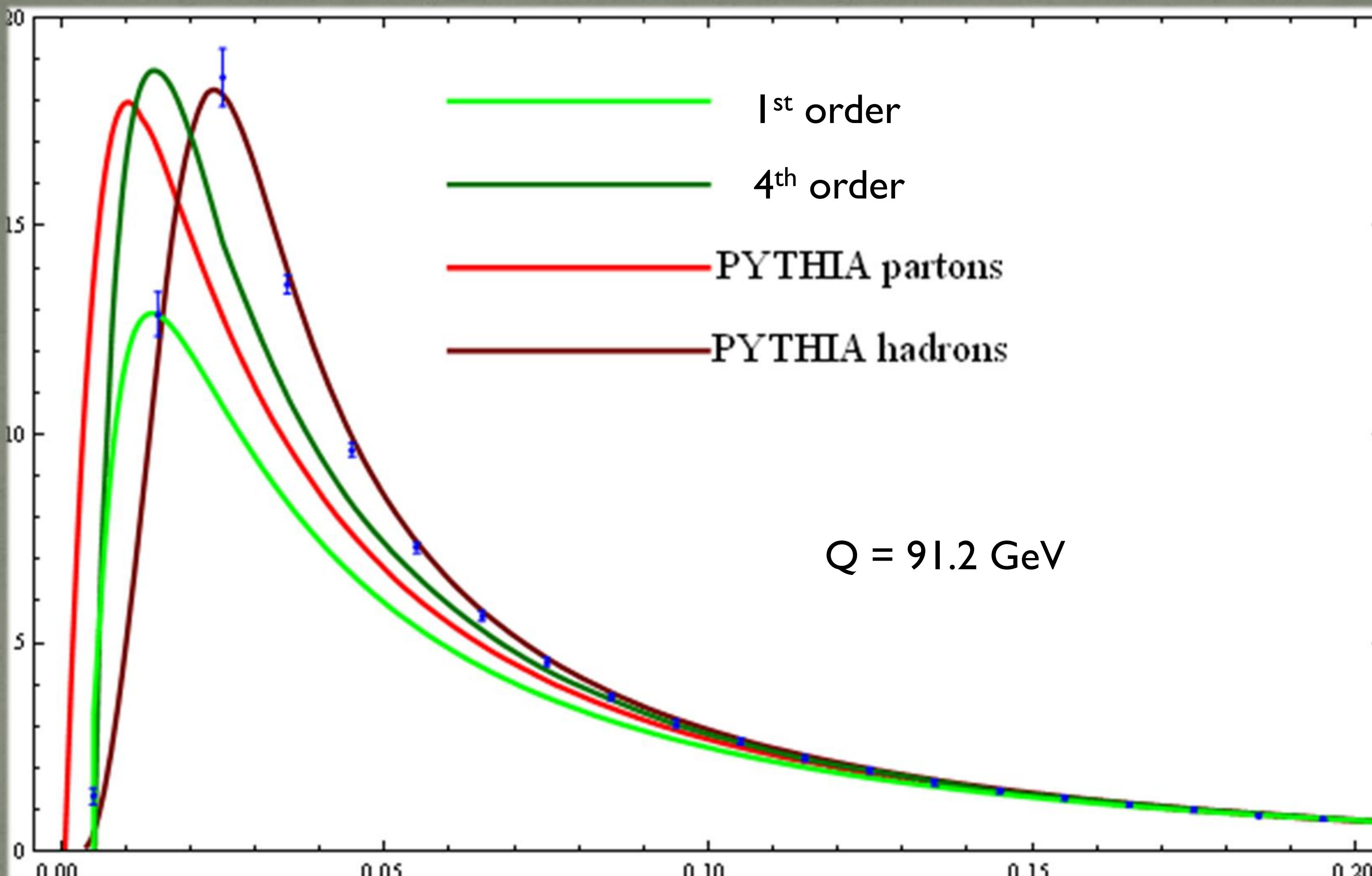
SCET vs Pythia



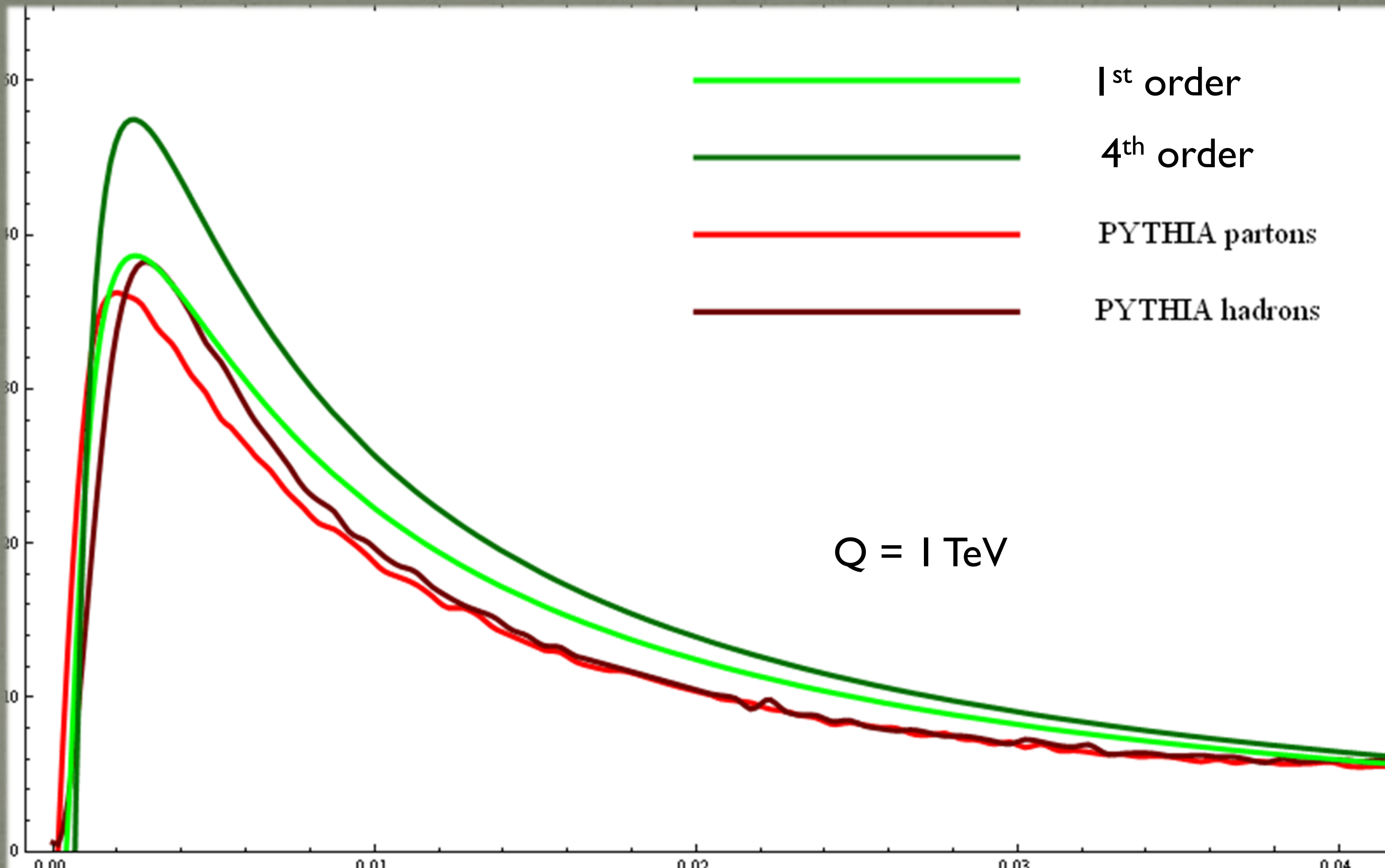
SCET vs Pythia



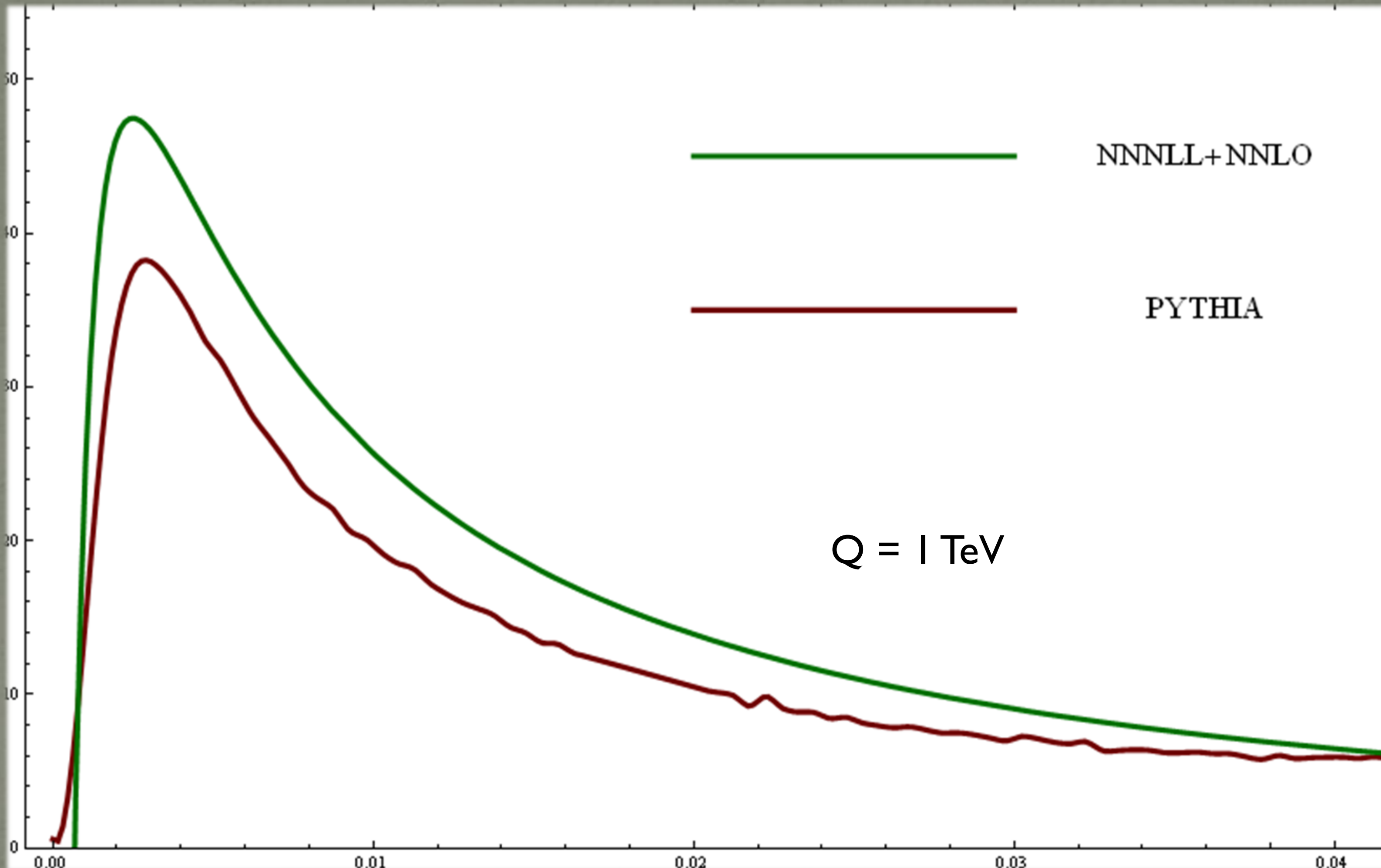
SCET vs Pythia



SCET vs Pythia



SCET vs Pythia

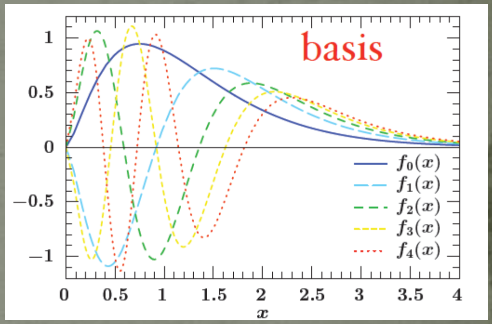


Power Corrections

Work in progress by Abbate, Fickinger, Hoang, Mateu and Stewart

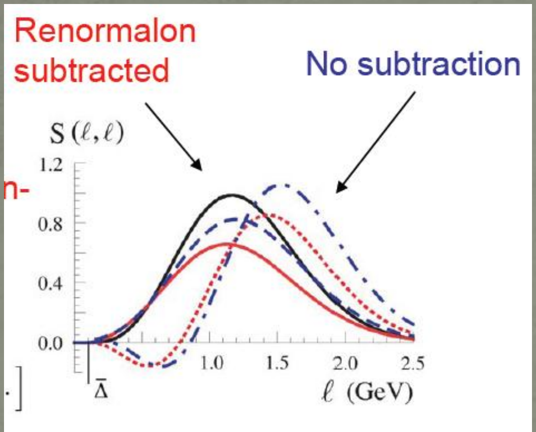
- Convolute perturbative soft function with non-perturbative shape function

$$S(k,\mu) \to \int \mathrm{d}k' S(k-k',\mu) S_{\mathrm{NP}}(k',\mu)$$

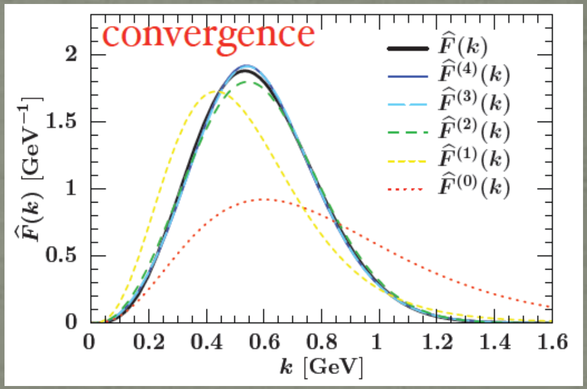


- Universal
- Orthonomal basis

- Remove renormalon ambiguity



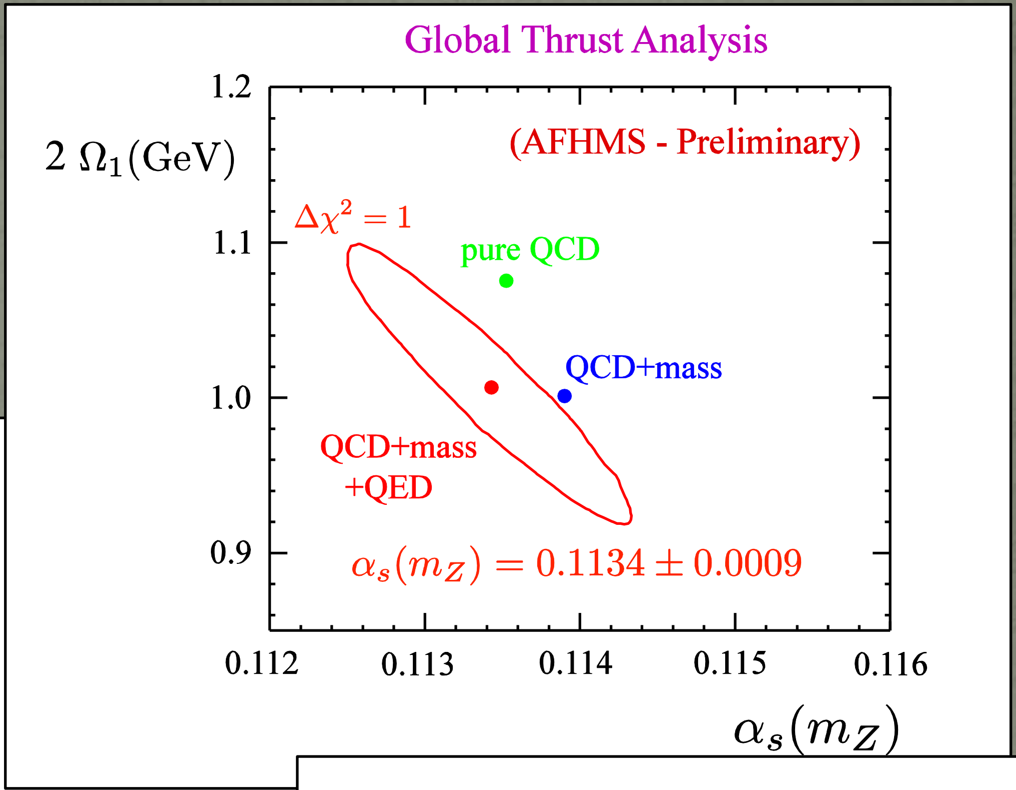
Hoang and Stewart Phys.Lett.B660:483-493,2008

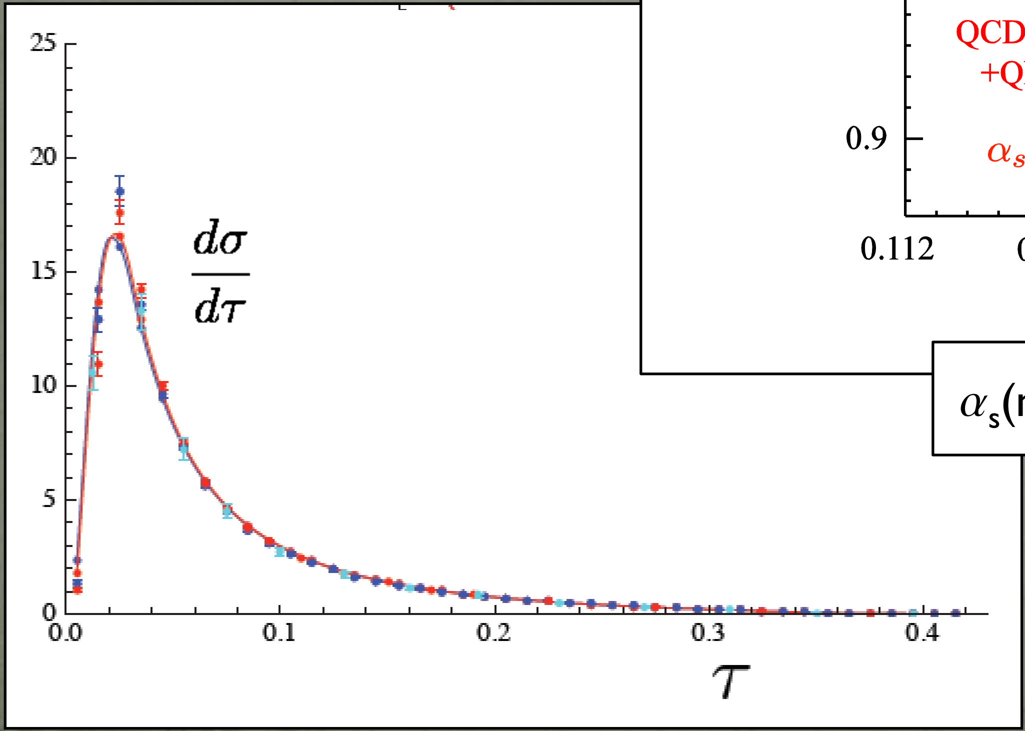


Ligeti, Stewart and Tackmann, Phys.Rev.D78:114014,2008

- m_b (1-2%) and QED effects (2%)

Results with Power Corrections





$\alpha_s(m_Z)$= 0.1134 ± 0.0009 ± 0.0010

Sys + Stat + Had uncertainty

Perturbative uncertainty

Conclusions

- Soft-Collinear Effective Theory is a powerful tool for collider physics
  - Combines resummation with fixed order calculations
  - Systematically includes power corrections
  - Allows for resummation well beyond NLL (NNNLL for thrust)

- Measurement of as from LEP has been theory limited
  - Systematics of SCET remove limitation

$\alpha_s(m_Z)$= 0.1134 ± 0.0013

$\alpha_s(m_Z)$= 0.1183 ± 0.0008 (lattice)

$\alpha_s(m_Z)$= 0.1213 ± 0.0006 (tau decays)

- Next Stop: LHC