

Heavy Quarks

*... lessons we've learned
& future applications*



Fred Olness
SMU

Conspirators:

I Schienbein, S. Berge, P. Nadolsky,

J.-Y. Yu, J. Owens, J. Morfin, W. Tung, C. Keppel, ...

Loopfest VIII

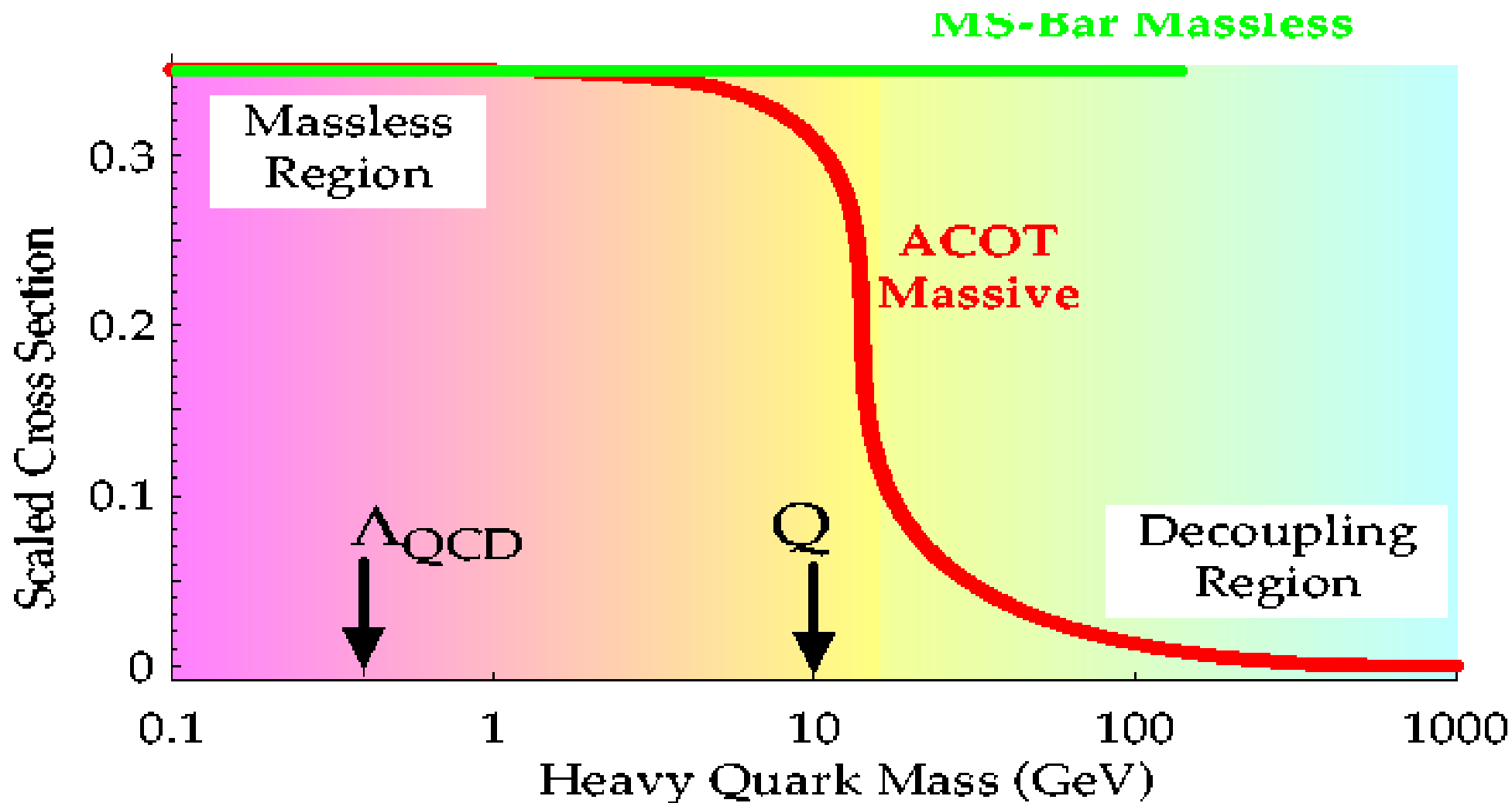
9 May 2009

$m = 0$: Massless case.

Mass plays no dynamic role
Well understood.

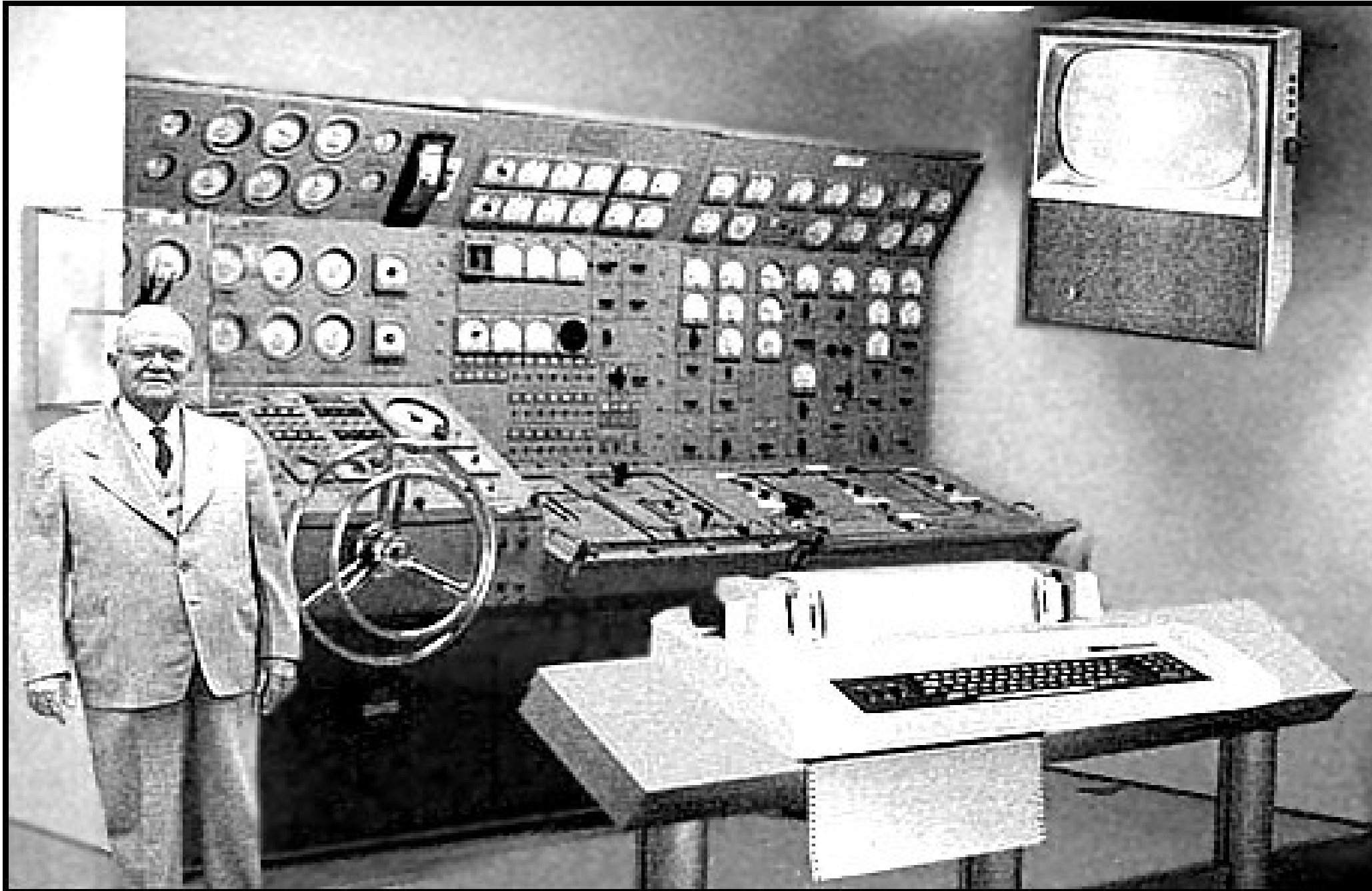
$m = \infty$: Infinite case.

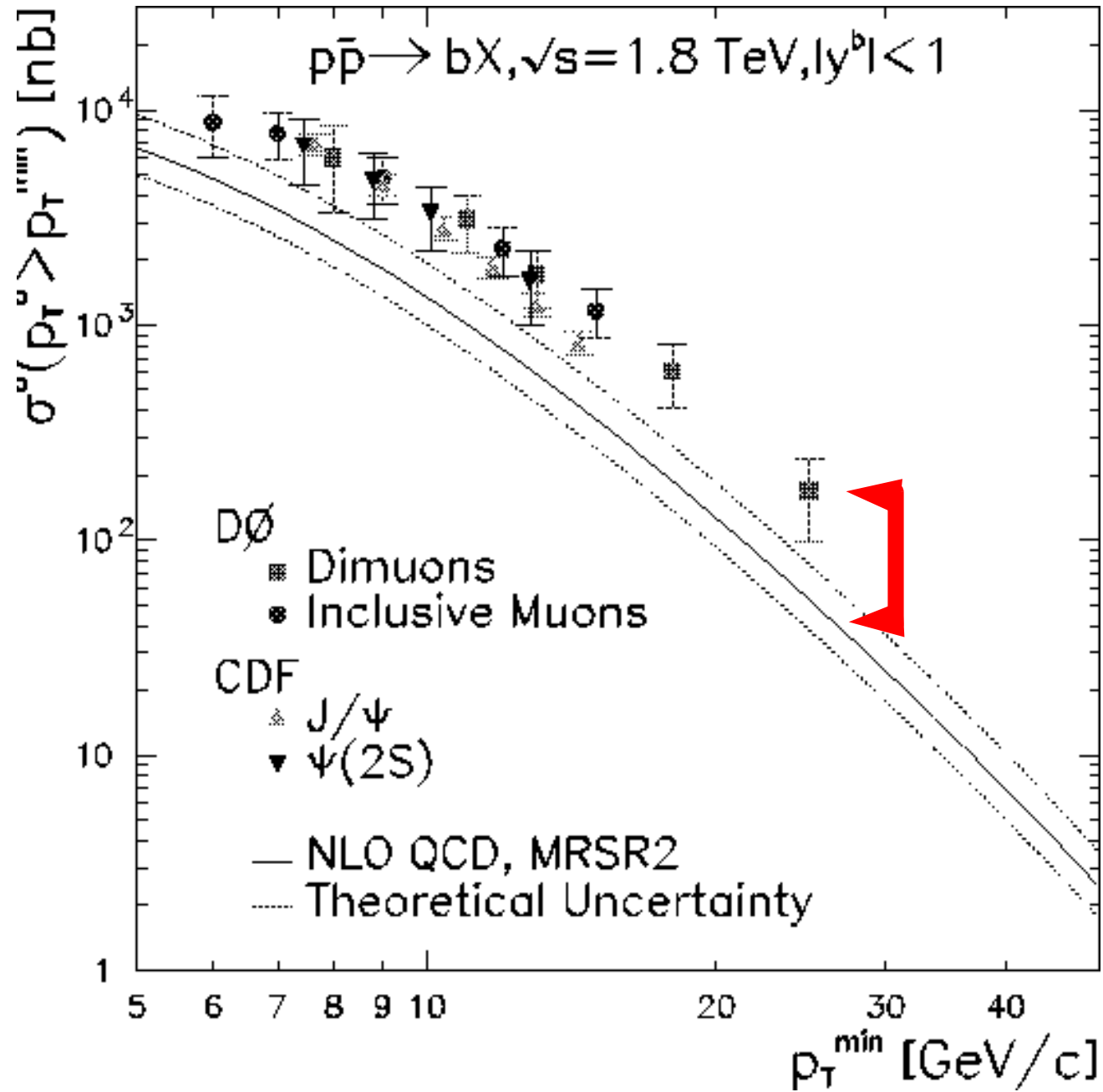
Mass Decouples.
We can forget about this object



E.g., e^+e^- total cross section to 3-loops

HISTORY

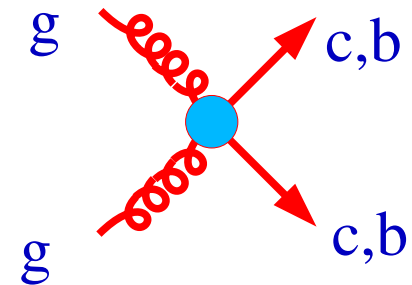


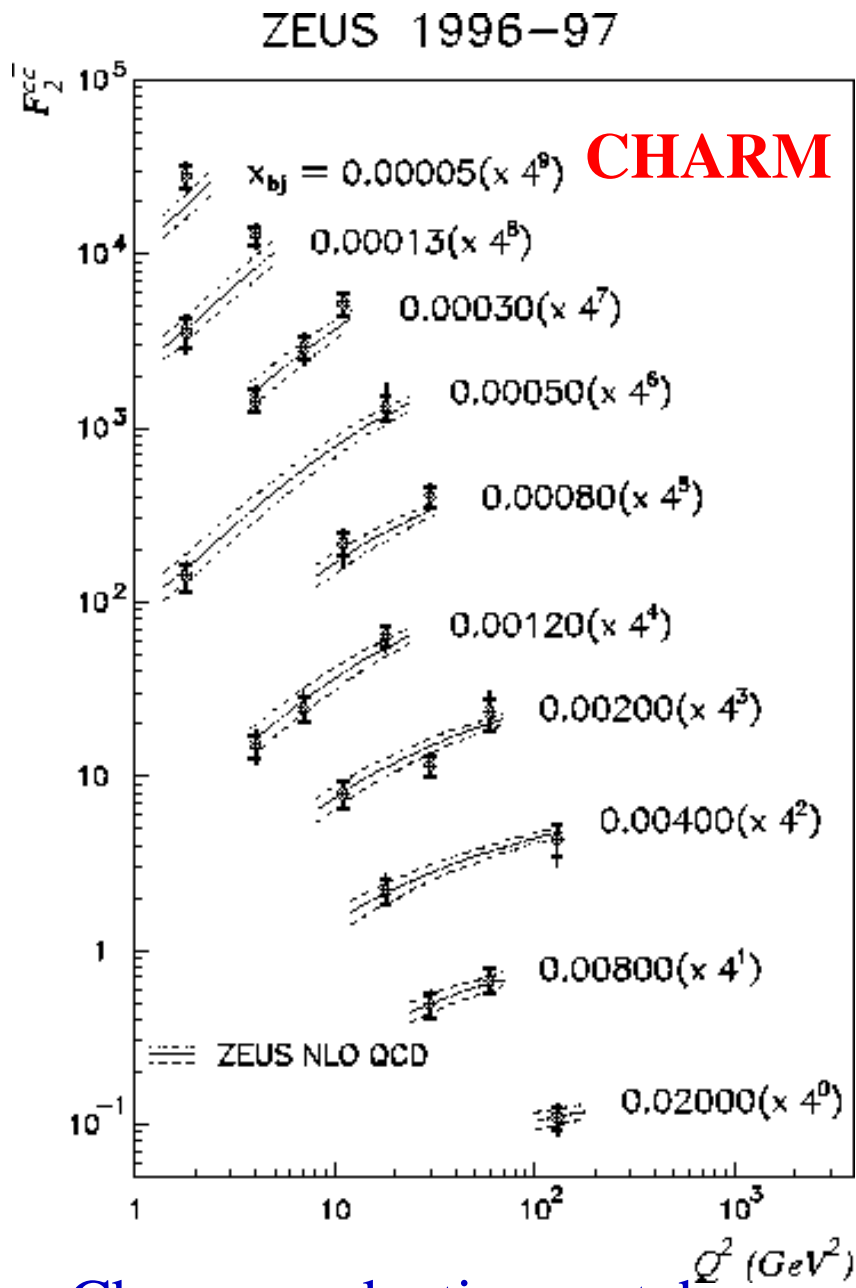


Comparison of Run I data with NLO Theory

Data is high by factor of 2x or 3x

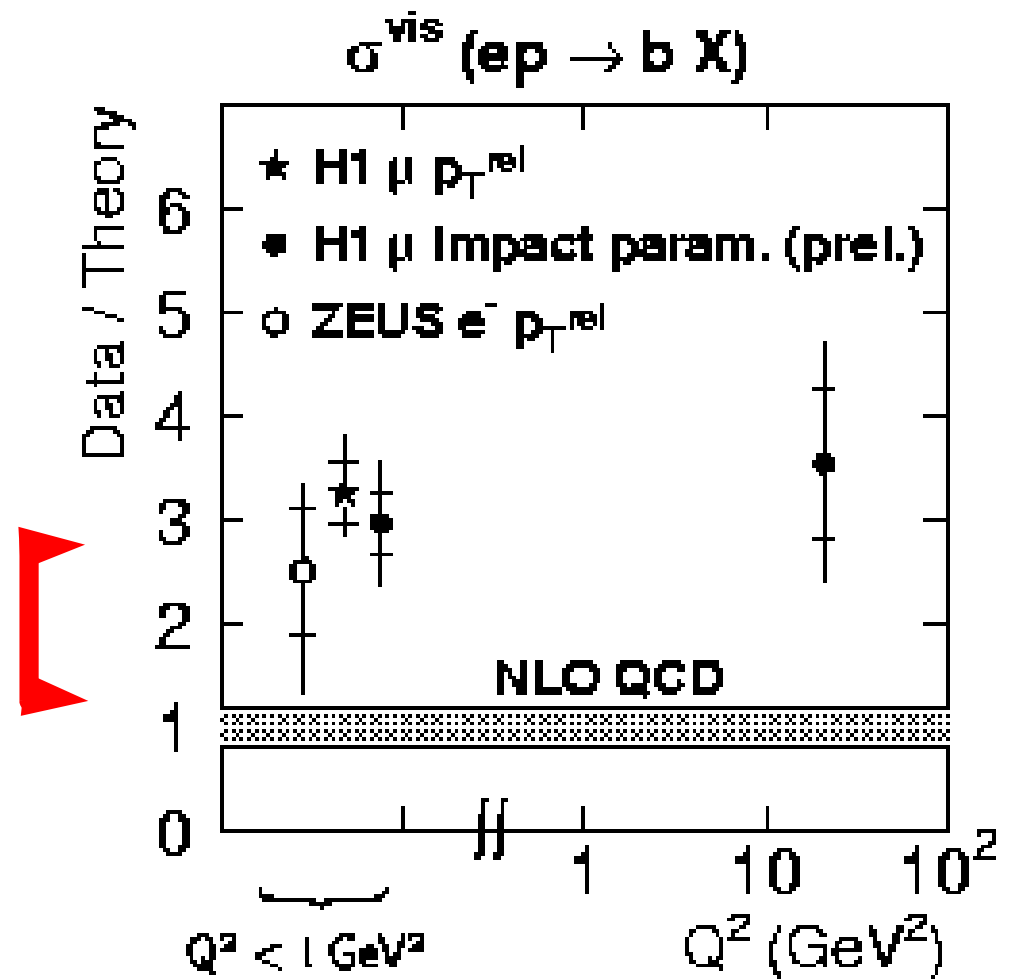
... even given μ variation



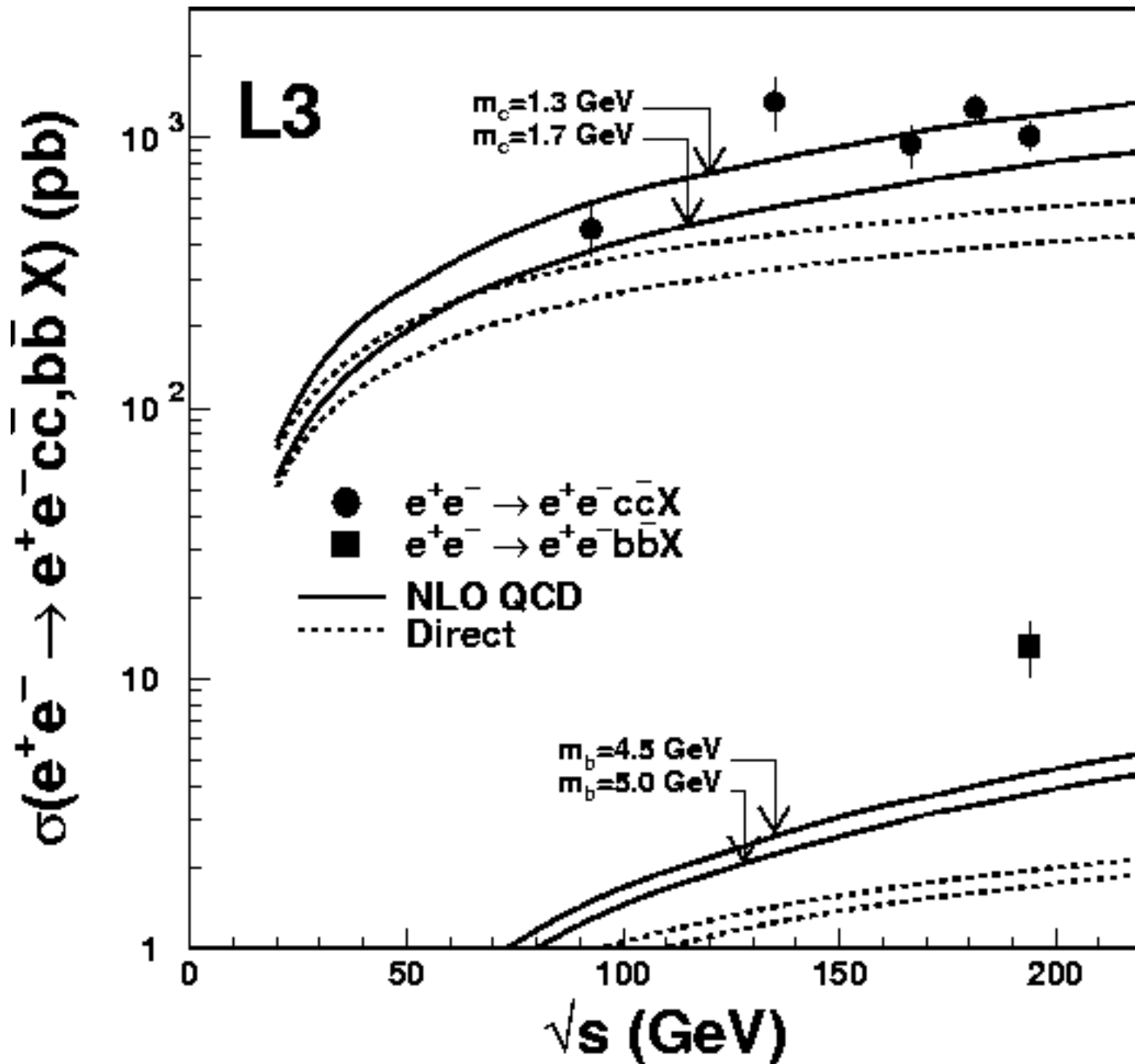


Charm production matches well with NLO calculation

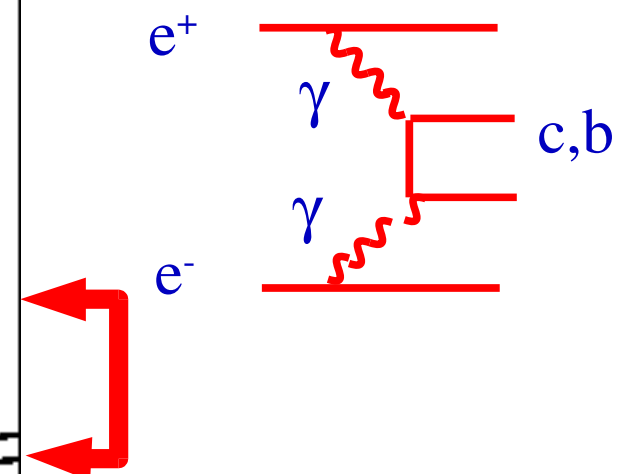
BEAUTY



Data is high by factor of 2x or 3x



Comparison of LEP data with NLO Theory

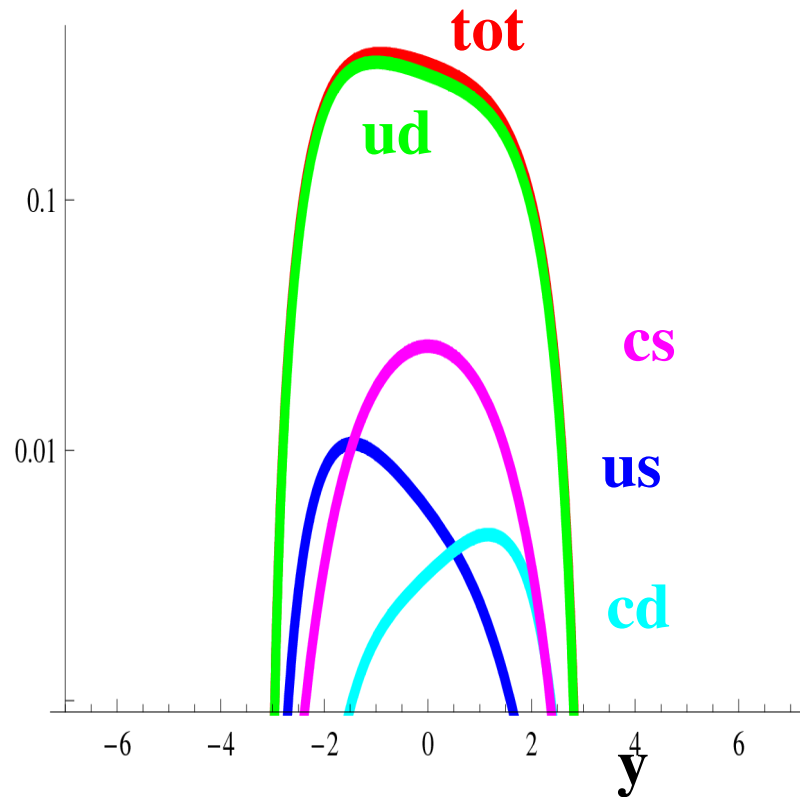
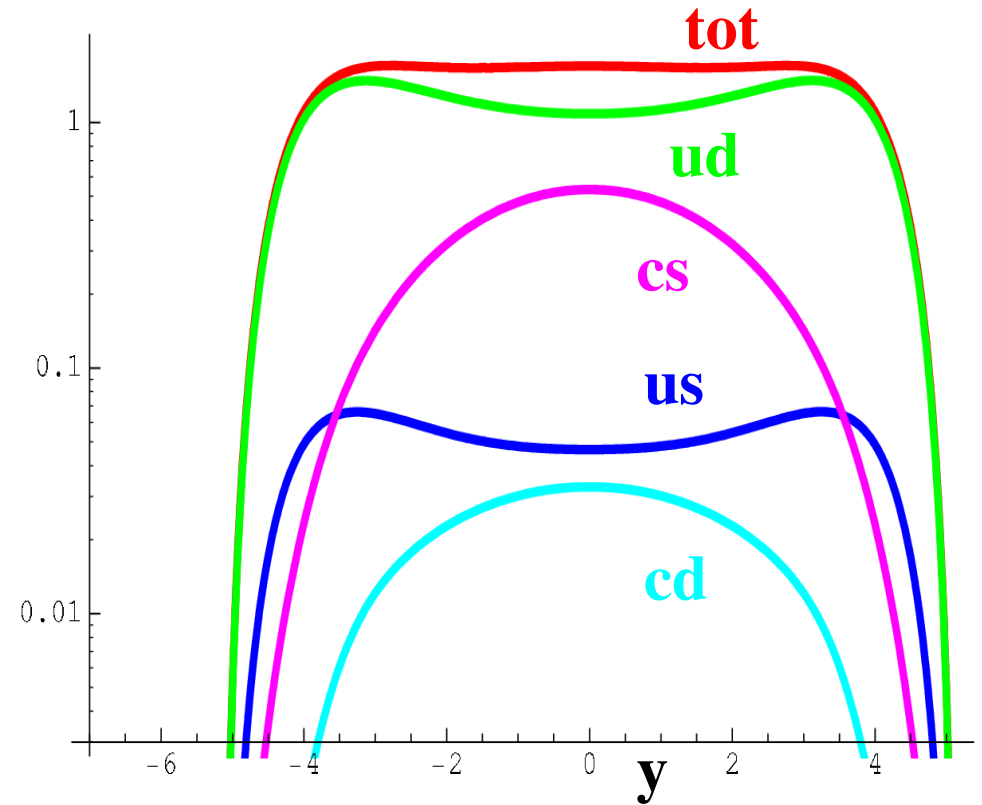


Charm is reasonable

Bottom data is high by factor of 2x or 3x

FUTURE



$d\sigma/dy(W^+)$ at Tevatron $d\sigma/dy(W^+)$ at LHC

- Larger fraction of heavy quarks
- W/Z are “Benchmark” Cross Sections
... will be measured in early run

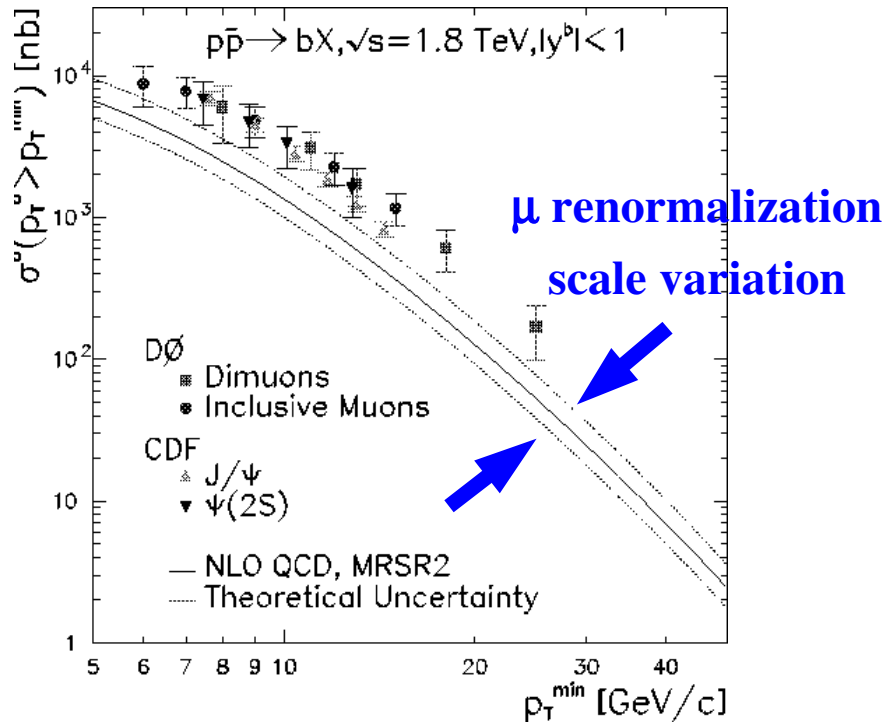
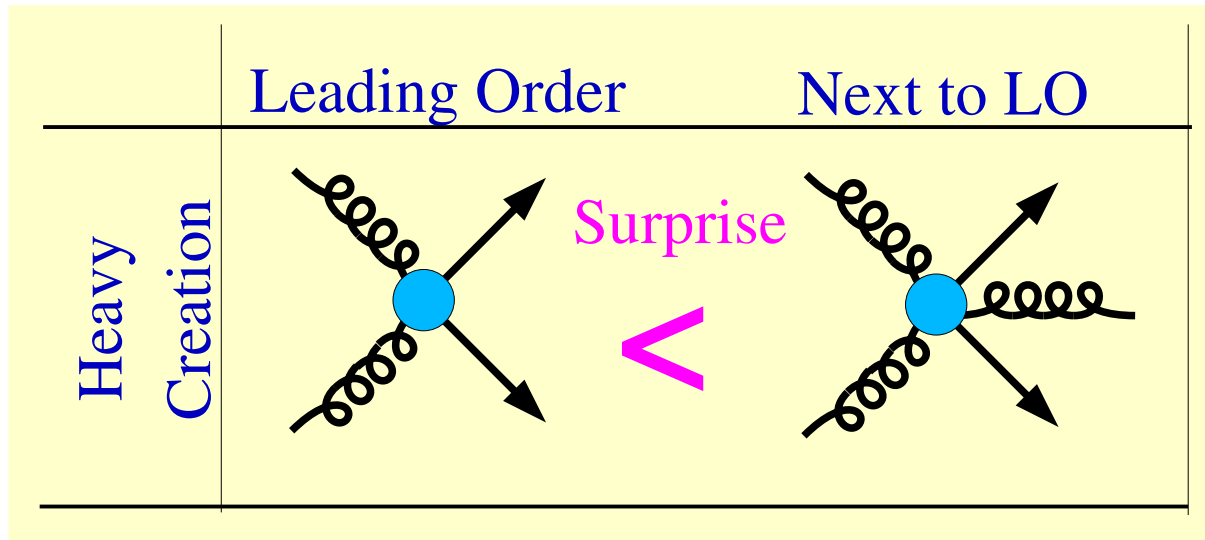
HEAVY is a relative term

Calculating

Heavy

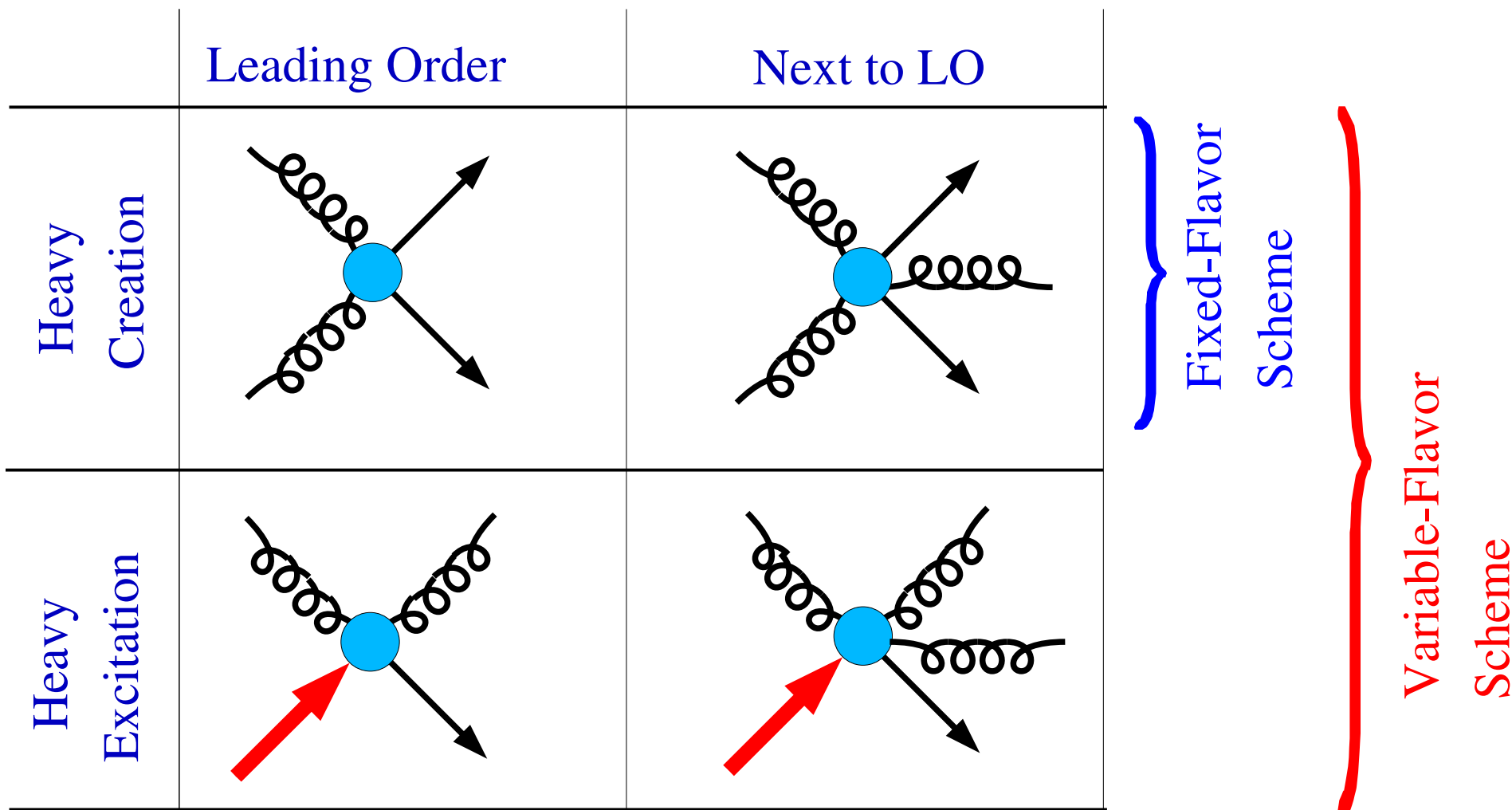
Quarks

Part I



Surprise:
NLO / LO Big

But, ...
theory still
below data



heavy quark is integral to the proton

Calculating Heavy Quarks

*Heavy Quarks in the
CTEQ PDFs*

VFN: Variable Flavor Number:

Introduce new partonic components

when the scale μ exceeds the heavy quark mass.

E.g., charm and bottom are included in the proton at high enough scales

Massless $\overline{\text{MS}}$ Evolution:

Evolution kernels in DGLAP are Mass-Independent: $\partial f \sim P \otimes f$.

ACOT (*Aivazis, Collins, Olness, Tung*) A general framework for

including the heavy quark components. *Phys.Rev.D50:3102-3118,1994.*

S-ACOT (*Simplified-ACOT*) ACOT with the simplification that initial-

state heavy quark masses can be set to zero. *Phys.Rev.D62:096007,2000.*

χ -Prescription: ACOT- χ & S-ACOT- χ :

As above with a generalized slow-rescaling $x \rightarrow x(1 + (m_1 + m_2)^2/Q^2)$

Problem:

Heavy Quark introduces new scale:
... life gets interesting.

$$\log\left(\frac{Q^2}{\mu^2}\right) \quad \log\left(\frac{M_H^2}{\mu^2}\right)$$

Solution:

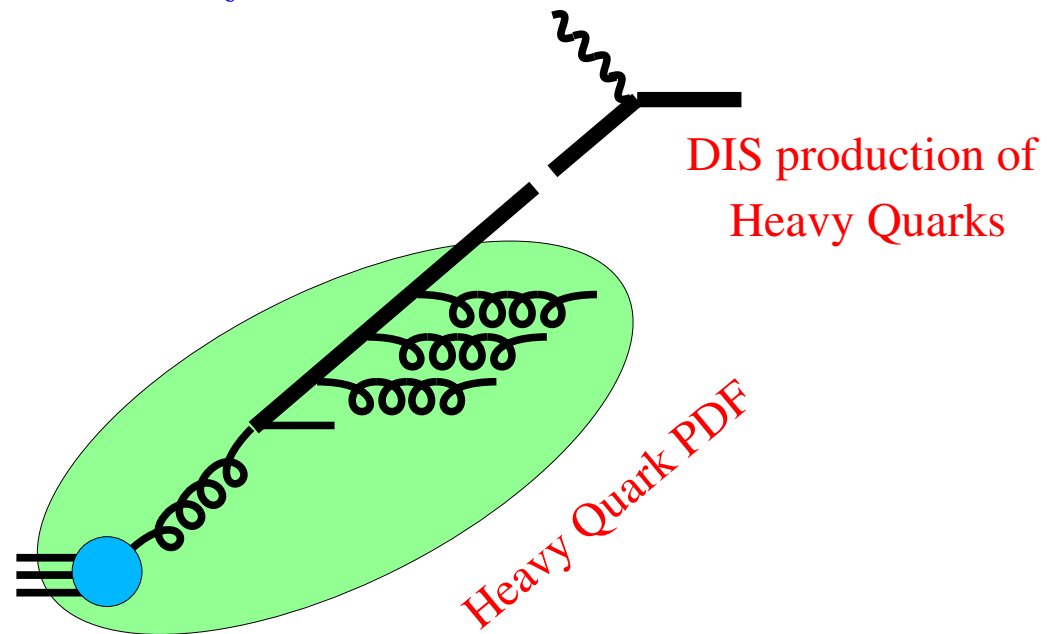
Resum $\text{Log}(M_H)$ in the Heavy Quark PDF's:

... i.e., as in the ACOT renormalization scheme

ACOT, PRD 50, 3102

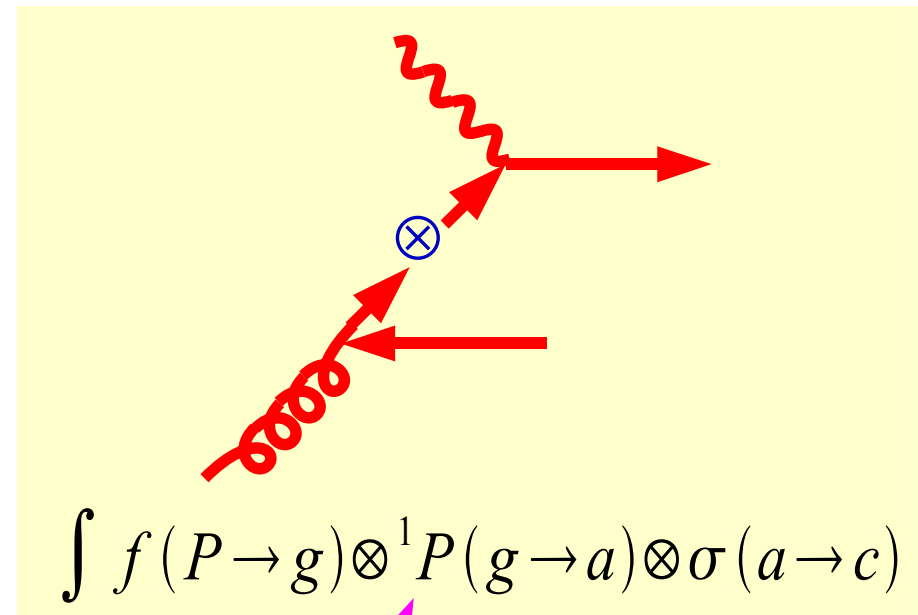
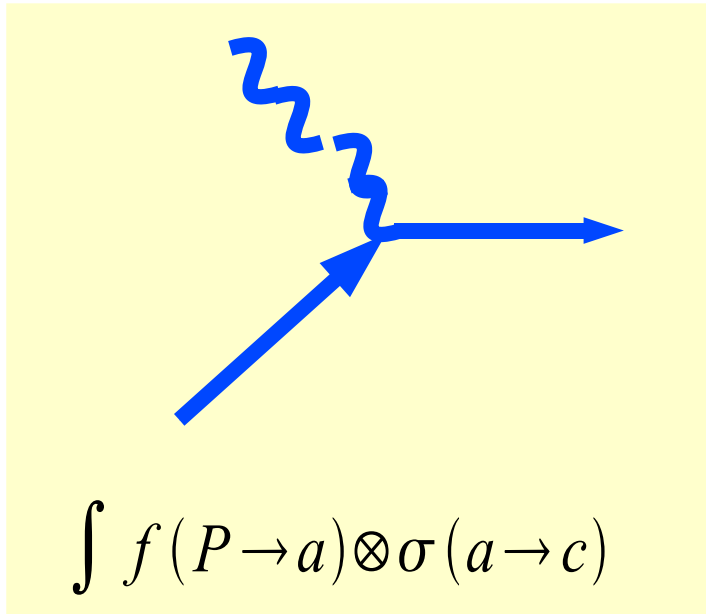
DGLAP equation

Resums iterative splittings
 inside the proton



Result:

We can describe the full kinematic range from low to high
implemented in the CTEQ6HQ PDF's with finite M_Q



near threshold \approx $f(P \rightarrow g) \otimes {}^1P(g \rightarrow a)$
($M_H \sim Q$)

1P splittings must match

Near threshold ($M_H \sim Q$), mass effects cancel between HE and SUB

Above threshold ($M_H \ll Q$), mass effects can be ignored

Choice of DGLAP Kernels is a **Scheme Choice!!!**

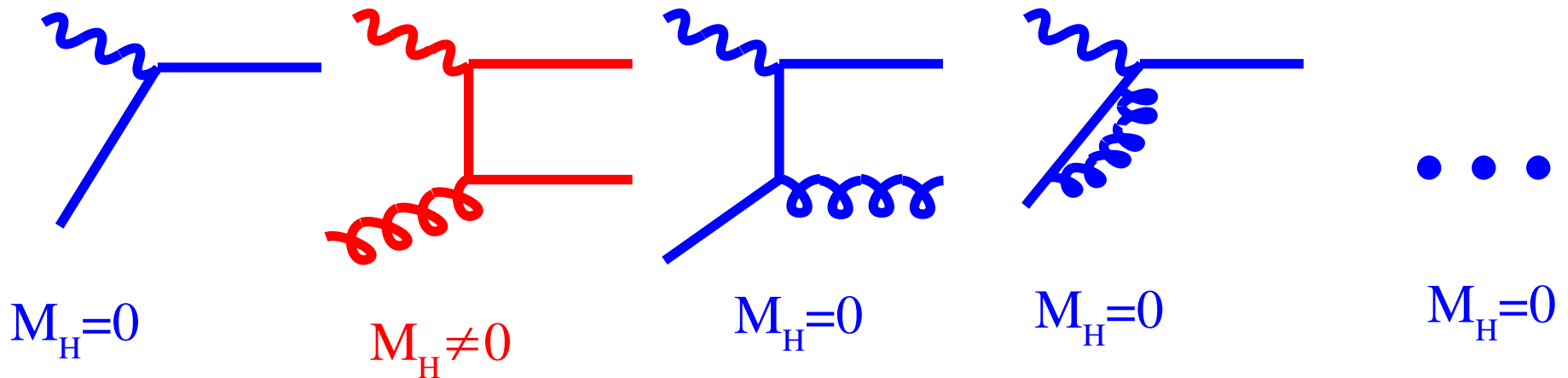
This is NOT an approximation!!!

Development: Factorization proof extended to Heavy Quark case. *Collins (1998)*

Observation: Simplified-ACOT Scheme:

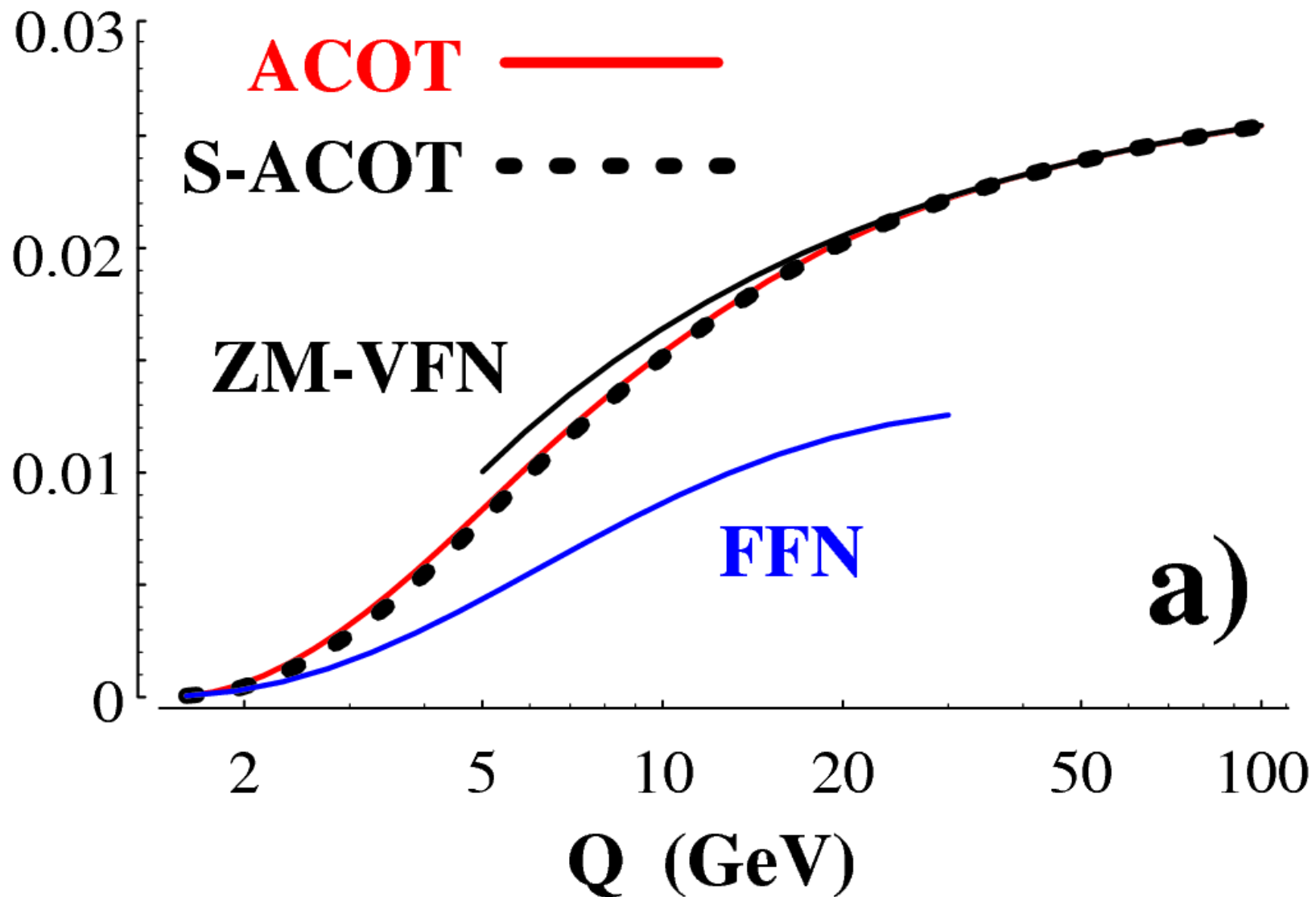
Set $M_H=0$ on
incoming HQ lines

Kramer, Olness, Soper (2000)



Result: 1) Comparable Numerics & 2) Simpler Calculations

Simplified result numerically comparable to full result



... what about the analytic result???

Which would you prefer to calculate?

ACOT

$$\begin{aligned}
 \hat{f}_1^Q(\hat{s}_1) &= \frac{8}{\Delta'^2} \left\{ -\Delta^2(S_+ \Sigma_{+-} - 2m_1 m_2 S_-) L_{\mathcal{E}'} + 2m_1 m_2 S_- \left(\frac{1}{\hat{s}_1} [\Delta'^2 + 4m_2^2 \Sigma_{+-}] \right. \right. \\
 &+ 2\Sigma_{+-} - \Sigma_{++} + \frac{\Sigma_{++} + \hat{s}_1}{2} + \frac{\hat{s}_1 + m_2^2}{\Delta' \hat{s}_1} \left[\Delta'^2 + 2\Sigma_{+-} \Sigma_{++} + (m_2^2 + Q^2) \hat{s}_1 \right] L_{\mathcal{E}'} \\
 &+ S_+ \left(\frac{-m_2^2 \Sigma_{+-}}{(\hat{s}_1 + m_2^2) \hat{s}_1} (\Delta^2 + 4m_2^2 \Sigma_{+-}) - \frac{1}{4(\hat{s}_1 + m_2^2)} \left[3\Sigma_{+-}^2 + 4m_2^2 (10\Sigma_{++} \Sigma_{+-} \right. \right. \\
 &- \Sigma_{+-} \Sigma_{+-} - m_2^2 \Sigma_{+-}) + \hat{s}_1 (-7\Sigma_{+-} \Sigma_{++} + 18\Delta^2 - 4m_2^2 (7Q^2 - 4m_2^2 + 7m_2^2)) \\
 &+ 3\hat{s}_1^2 (\Sigma_{+-} - 2m_2^2) - \hat{s}_1^2 \left. \right] + \frac{\hat{s}_1 + m_2^2}{2\Delta'} \left[\frac{-2}{\hat{s}_1} \Sigma_{+-} (\Delta^2 + 2\Sigma_{+-} \Sigma_{++}) \right. \\
 &\left. \left. + (4m_2^2 m_2^2 - 7\Sigma_{+-} \Sigma_{++}) - 4\Sigma_{+-} \hat{s}_1 - \hat{s}_1^2 \right] L_{\mathcal{E}'} \right\} \\
 \hat{f}_2^Q(\hat{s}_1) &= \frac{16}{\Delta'^4} \left\{ -2\Delta^4 S_+ L_{\mathcal{E}'} + 2m_1 m_2 S_- \left(\frac{\hat{s}_1 + m_2^2}{\Delta'} (\Delta'^2 - 6m_2^2 Q^2) L_{\mathcal{E}'} \right. \right. \\
 &- \frac{\Delta'^2 (\hat{s}_1 + \Sigma_{++})}{2(\hat{s}_1 + m_2^2)} + (2\Delta^2 - 3Q^2 (\hat{s}_1 + \Sigma_{++})) \left. \right) + S_+ \left(-2(\Delta^2 - 6m_2^2 Q^2) (\hat{s}_1 + m_2^2) \right. \\
 &- 2(m_2^2 + m_2^2) \hat{s}_1^2 - 8m_2^2 \Sigma_{+-} + \Delta^2 (2\Sigma_{++} - m_2^2) + 2\hat{s}_1 (2\Delta^2 + (m_2^2 - 5m_2^2) \Sigma_{+-}) \\
 &+ \frac{(\Delta'^2 - 6Q^2 (m_2^2 + \hat{s}_1)) \Sigma_{++} (\hat{s}_1 + \Sigma_{++})}{2(\hat{s}_1 + m_2^2)} - \frac{2\Delta^2}{\hat{s}_1} (\Delta^2 + 2(2m_2^2 + \hat{s}_1) \Sigma_{+-}) \\
 &+ \frac{(\hat{s}_1 + m_2^2)}{\Delta'} \left[\frac{-2}{\hat{s}_1} \Delta^2 (\Delta^2 + 2\Sigma_{+-} \Sigma_{++}) - 2\hat{s}_1 (\Delta^2 - 6m_2^2 Q^2) \right. \\
 &\left. \left. - (\Delta'^2 - 18m_2^2 Q^2) \Sigma_{++} - 2\Delta^2 (\Sigma_{++} + 2\Sigma_{+-}) \right] L_{\mathcal{E}'} \right\} \\
 \hat{f}_3^Q(\hat{s}_1) &= \frac{16}{\Delta'^2} \left\{ -2\Delta^2 R_+ L_{\mathcal{E}'} + 2m_1 m_2 R_- \left(1 - \frac{\Sigma_{+-}}{\hat{s}_1} + \frac{(\hat{s}_1 + m_2^2) (\hat{s}_1 + \Sigma_{+-})}{\Delta' \hat{s}_1} L_{\mathcal{E}'} \right) \right. \\
 &+ R_+ \left(\Sigma_{+-} - 3\Sigma_{+-} - \frac{2}{\hat{s}_1} (\Delta^2 + 2m_2^2 \Sigma_{+-}) - \frac{(\hat{s}_1 - \Sigma_{+-}) (\hat{s}_1 + \Sigma_{+-})}{2(\hat{s}_1 + m_2^2)} \right. \\
 &\left. \left. + \frac{\hat{s}_1 + m_2^2}{\Delta' \hat{s}_1} [-\hat{s}_1^2 + 4(m_2^2 \Sigma_{+-} - \Delta^2) - 3\hat{s}_1 \Sigma_{+-}] L_{\mathcal{E}'} \right) \right\}
 \end{aligned}$$

with

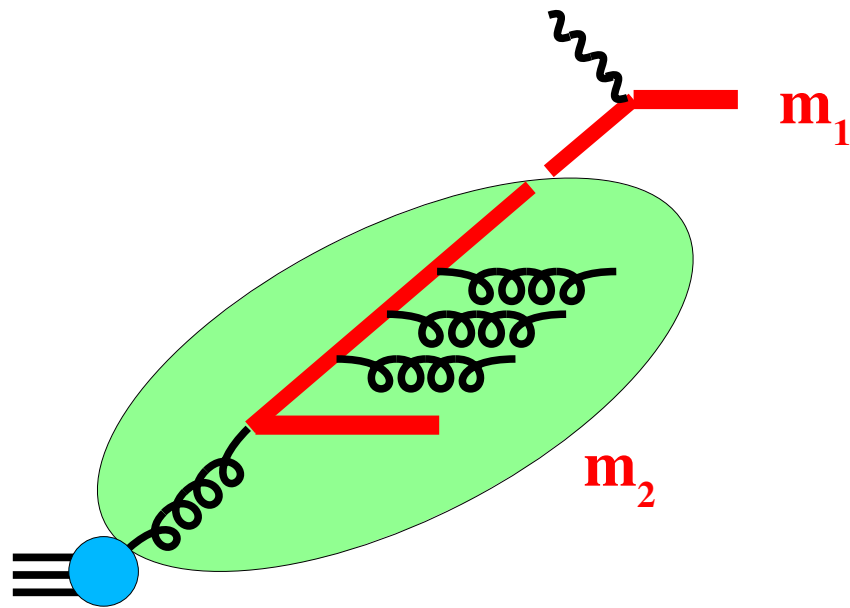
$$L_{\mathcal{E}'} \equiv \ln \left(\frac{\Sigma_{++} + \hat{s}_1 - \Delta'}{\Sigma_{++} + \hat{s}_1 + \Delta'} \right)$$

and

$$I_{\mathcal{E}'} = \left(\frac{\hat{s}_1 + 2m_2^2}{\hat{s}_1^2} + \frac{\hat{s}_1 + m_2^2}{\Delta' \hat{s}_1^2} \Sigma_{++} L_{\mathcal{E}'} \right).$$

S-ACOT

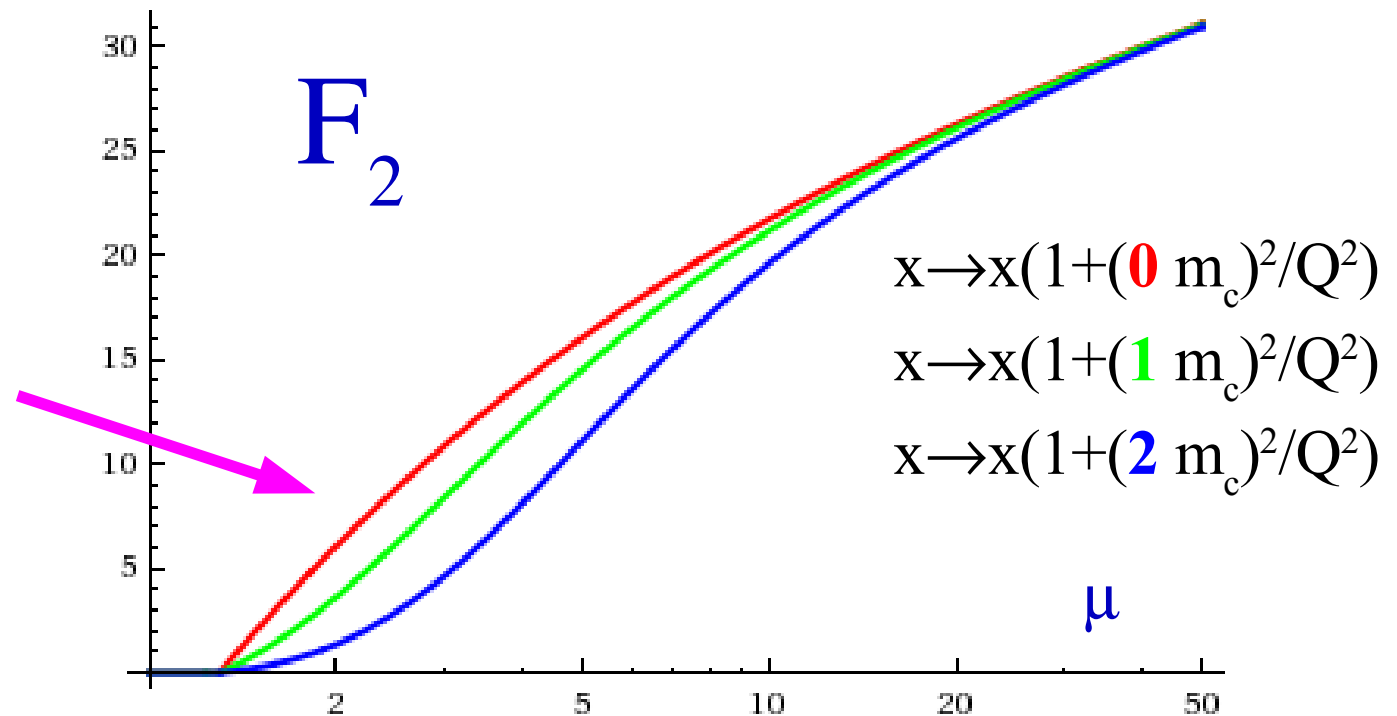
$$\begin{aligned}
 C_2^{(V, \mathcal{H})} &= C_2(F) \frac{x}{2} \left[\frac{1+x^2}{1-x} \left(\ln \frac{1-x}{x} - \frac{3}{4} \right) + \frac{1}{4} (9+5x) \right], \\
 C_1^{(V, \mathcal{H})} &= \frac{1}{2x} C_2^{(V, \mathcal{H})} - C_2(F) \frac{1}{2} x, \\
 C_3^{(V, \mathcal{H})} &= \frac{1}{x} C_2^{(V, \mathcal{H})} - C_2(F) (1+x),
 \end{aligned}$$



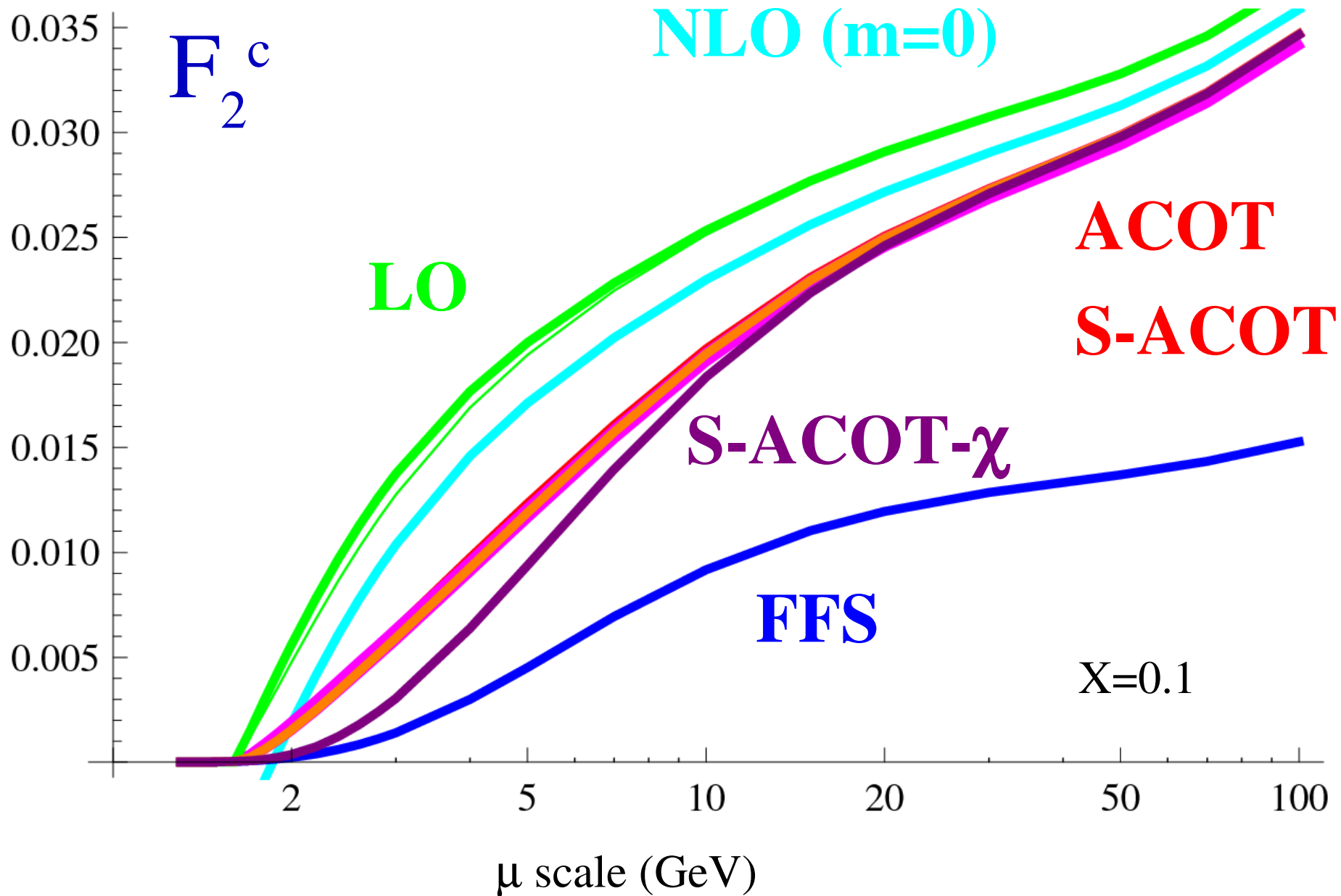
χ -prescription helps with threshold kinematics

$$\chi = x \left[1 + \frac{(m_1 + m_2)^2}{Q^2} \right]$$

Kinematic suppression at threshold

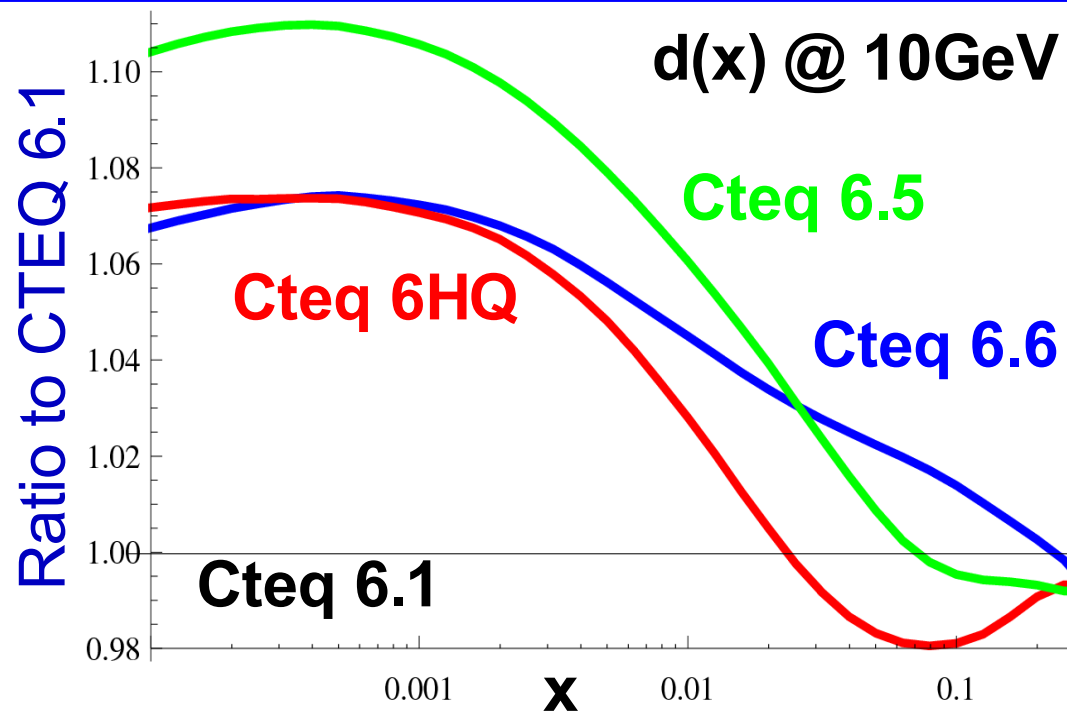
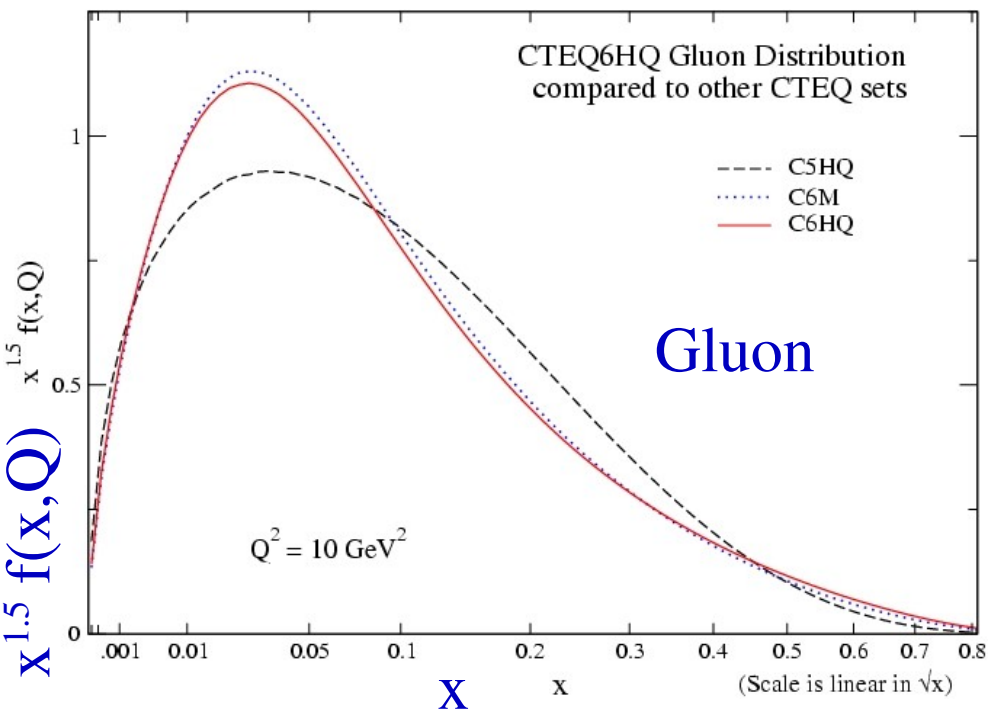


Results



A man with one watch know what time it is. A man with two is never sure.

Impact



χ^2/DOF

- Large shift C5M \Rightarrow C6M
 - (New DIS & Jet data)
- Charm PDF tied to gluon ($g \rightarrow cc$)
- Small visual difference but ...
 - Shift due to both scheme and uncertainty

Set	# pts	6HQ	6M	6M \otimes GM	6HQ \otimes ZM
ZEUS	104	0.91	0.98	2.84	3.72
H1	484	1.02	1.04	1.50	1.22
TOTAL	1925	1.04	1.06	1.26	1.30

Mixed Schemes



Data set	# pts	CTEQ6HQ	CTEQ6M	C6M ⊗ GM	C6HQ ⊗ ZM
Bcdms_p	339	370 (1.09)	370 (1.09)	370 (1.11)	373 (1.10)
Bcdms_d	251	269 (1.07)	279 (1.11)	274 (1.07)	281 (1.12)
Zeus	104	94 (0.91)	102 (0.98)	258 (2.84)	387 (3.72)
H1a	126	124 (0.99)	130 (1.03)	135 (1.11)	123 (0.98)
H1b	129	103 (0.80)	111 (0.86)	119 (0.84)	104 (0.80)
H1c	229	266 (1.16)	261 (1.14)	474 (2.11)	364 (1.59)
Nmc_p	201	304 (1.51)	299 (1.49)	273 (1.35)	366 (1.82)
Nmc_d/p	123	112 (0.91)	111 (0.91)	111 (0.90)	114 (0.92)
Ccft_F2	69	90 (1.30)	120 (1.74)	116 (1.82)	107 (1.55)
Ccft_F3	86	35 (0.41)	37 (0.43)	36 (0.40)	36 (0.42)
E605	119	102 (0.86)	103 (0.86)	101 (0.86)	102 (0.86)
Cdf_wasy	11	9 (0.78)	9 (0.83)	9 (0.83)	9 (0.78)
E866	15	5 (0.34)	6 (0.43)	6 (0.43)	5 (0.34)
DD_jet	90	71 (0.79)	49 (0.55)	49 (0.55)	71 (0.79)
Cdf_jet	33	55 (1.66)	50 (1.51)	50 (1.51)	55 (1.66)
All	1925	2008 (1.04)	2037 (1.06)	2431 (1.26)	2496 (1.30)

A 3 σ effect



HERA experiments sensitive to Mixed scheme



Encouraging that C6M and C6HQ are comparable

Encouraging that Mixed schemes yield large χ^2

Will affect PDFs in region of low x and low Q

CTEQ6HQ

CTEQ6M

MIXED

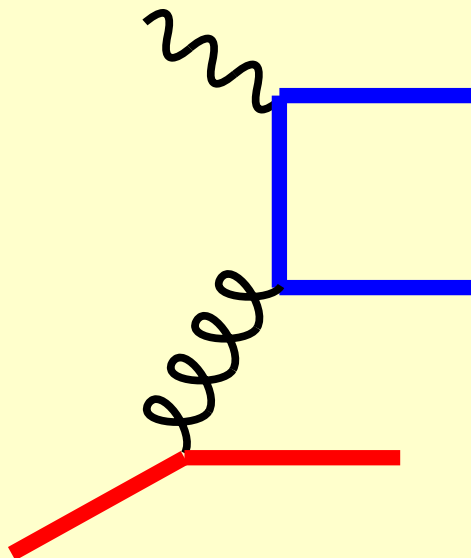
MIXED

NNILO

Multiple heavy quark lines:

S-ACOT more complex

... compute full massive result



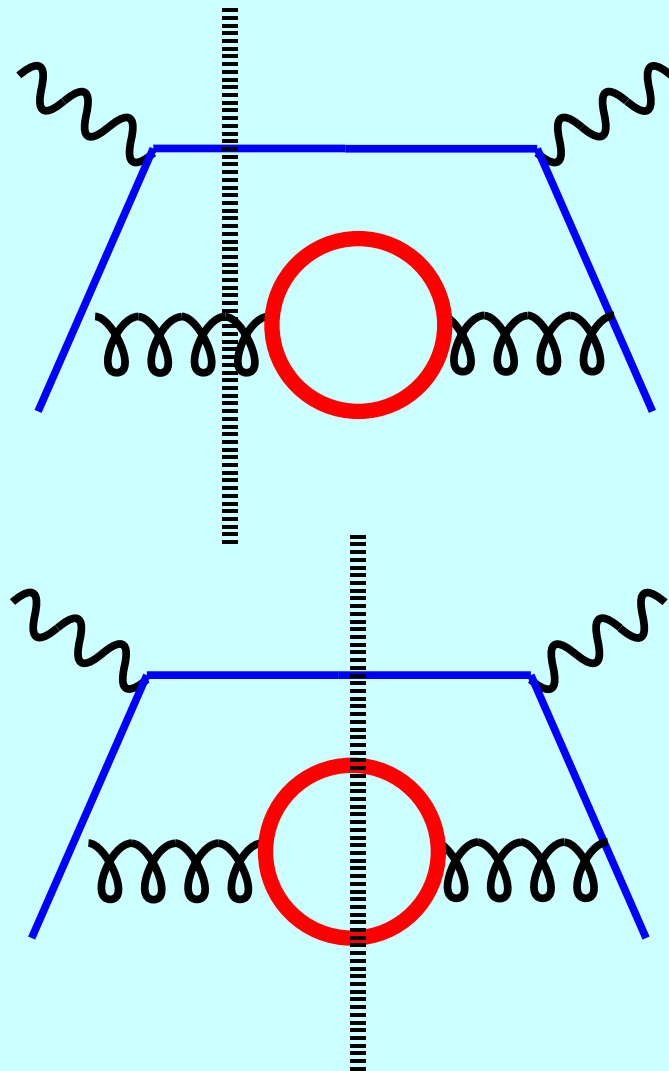
.... recent work, for example

M. Czakon, A. Mitov, S. Moch, (2008)

Bierenbaum, Blumlein, Klein (2009)

Ambiguities in F_2^C

- *Ill defined,*
- *additional cutoff required*

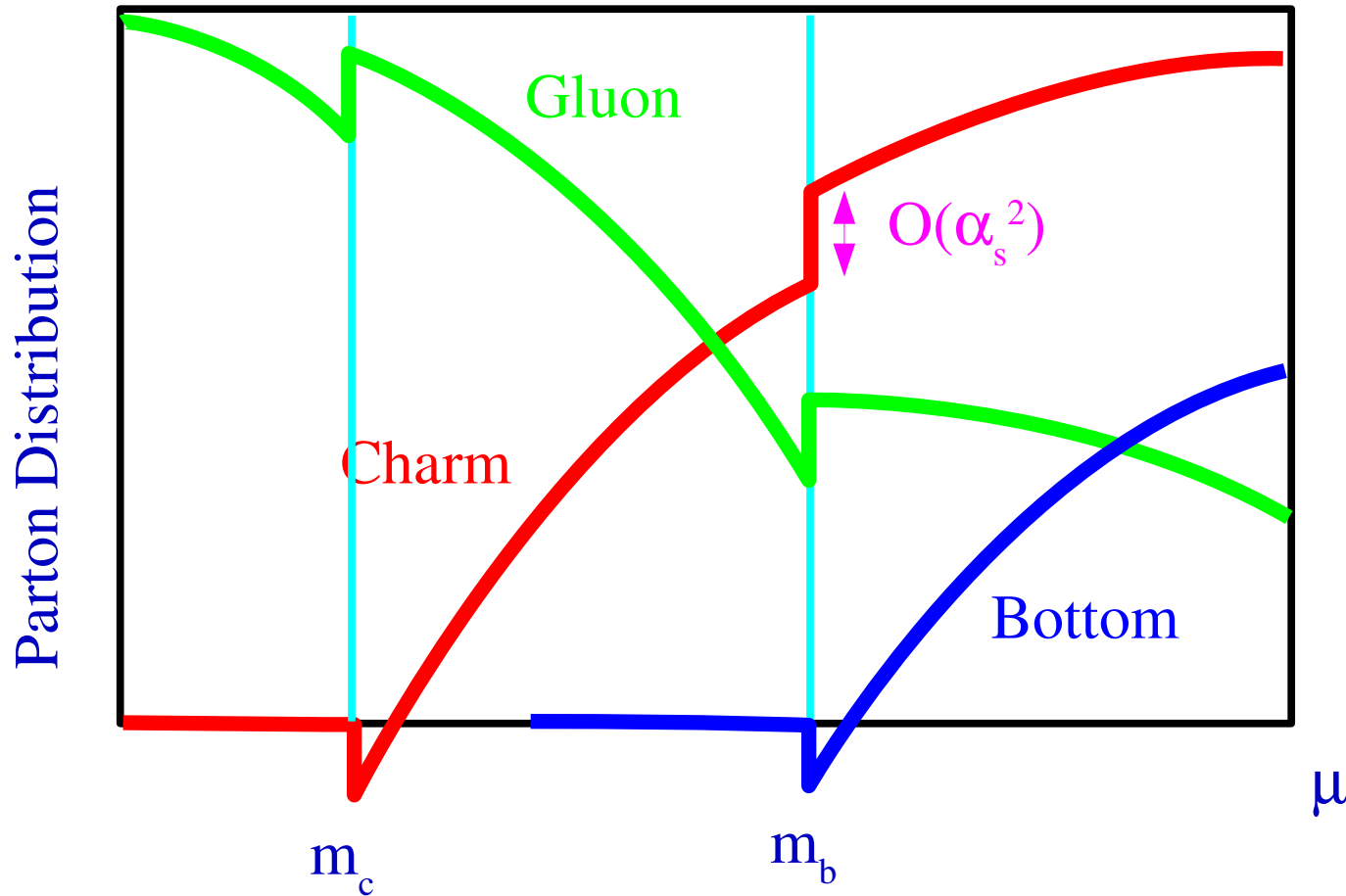


Buza, Matiounine, Smith, van Neerven (1997)

Bierenbaum, Blumlein, Klein (2009)

NNLO

*A proposal for NNLO
PDF implementation*



Not continuous at $O(\alpha_s^2)$

$$f_k^{n_f+1}(\mu^2, m_H^2) = A_{kj}(\mu^2/m_H^2) \otimes f_j^{n_f}(\mu^2),$$

relate N and N+1 PDF's

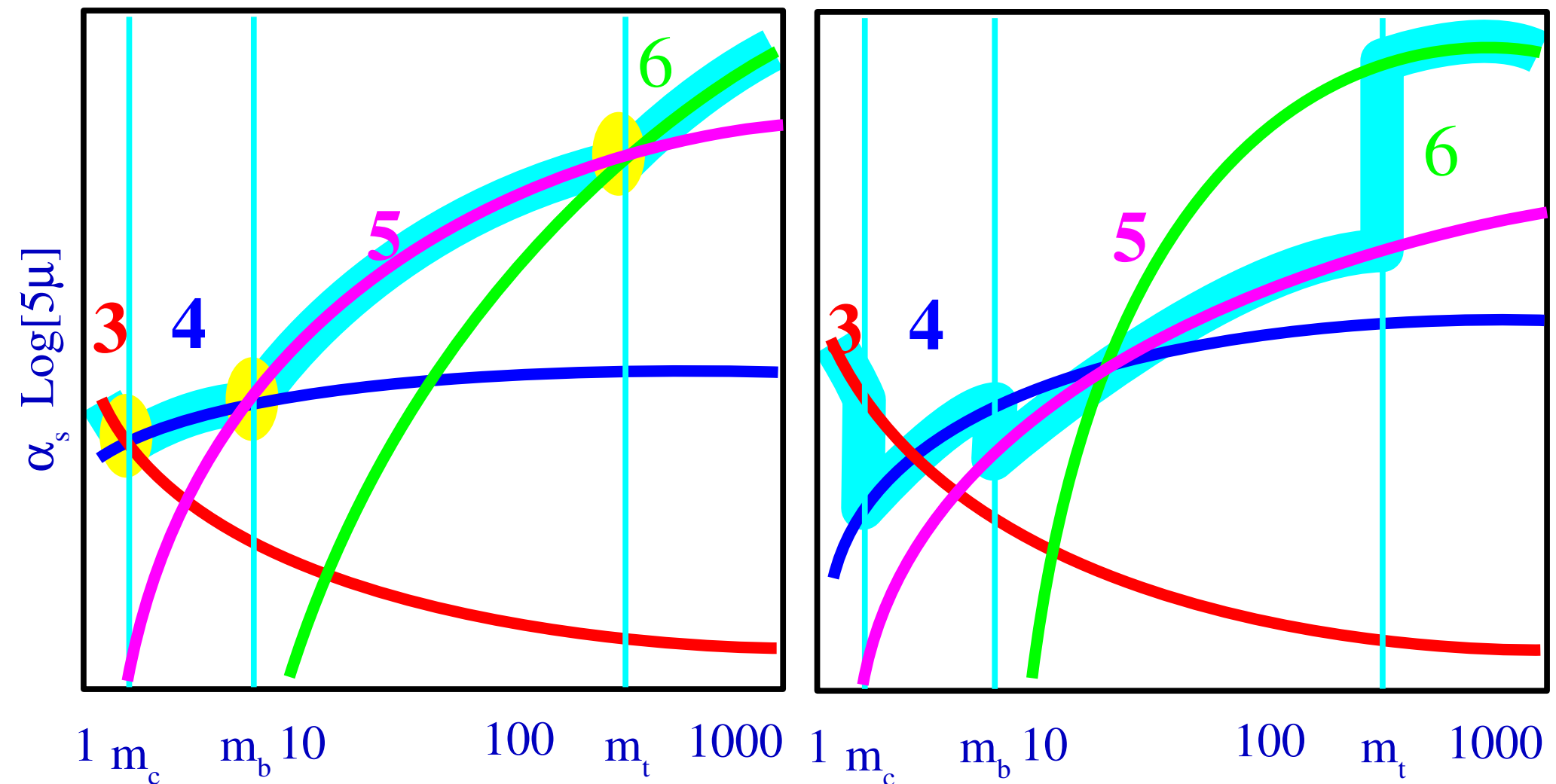
implied relation of C's

$$\begin{aligned} F(x, Q^2) &= C_k^{FFNS}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2) \\ &= C_j^{VFNS}(Q^2/m_H^2) \otimes f_j^{n_f+1}(Q^2) \equiv C_j^{VFNS}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2) \end{aligned}$$

Note:
FFNS ~ N
VFNS ~ N+1

At 1-loop and 2-loops,
continuous at thresholds

At $O(\alpha_s^3)$, not even
continuous at thresholds

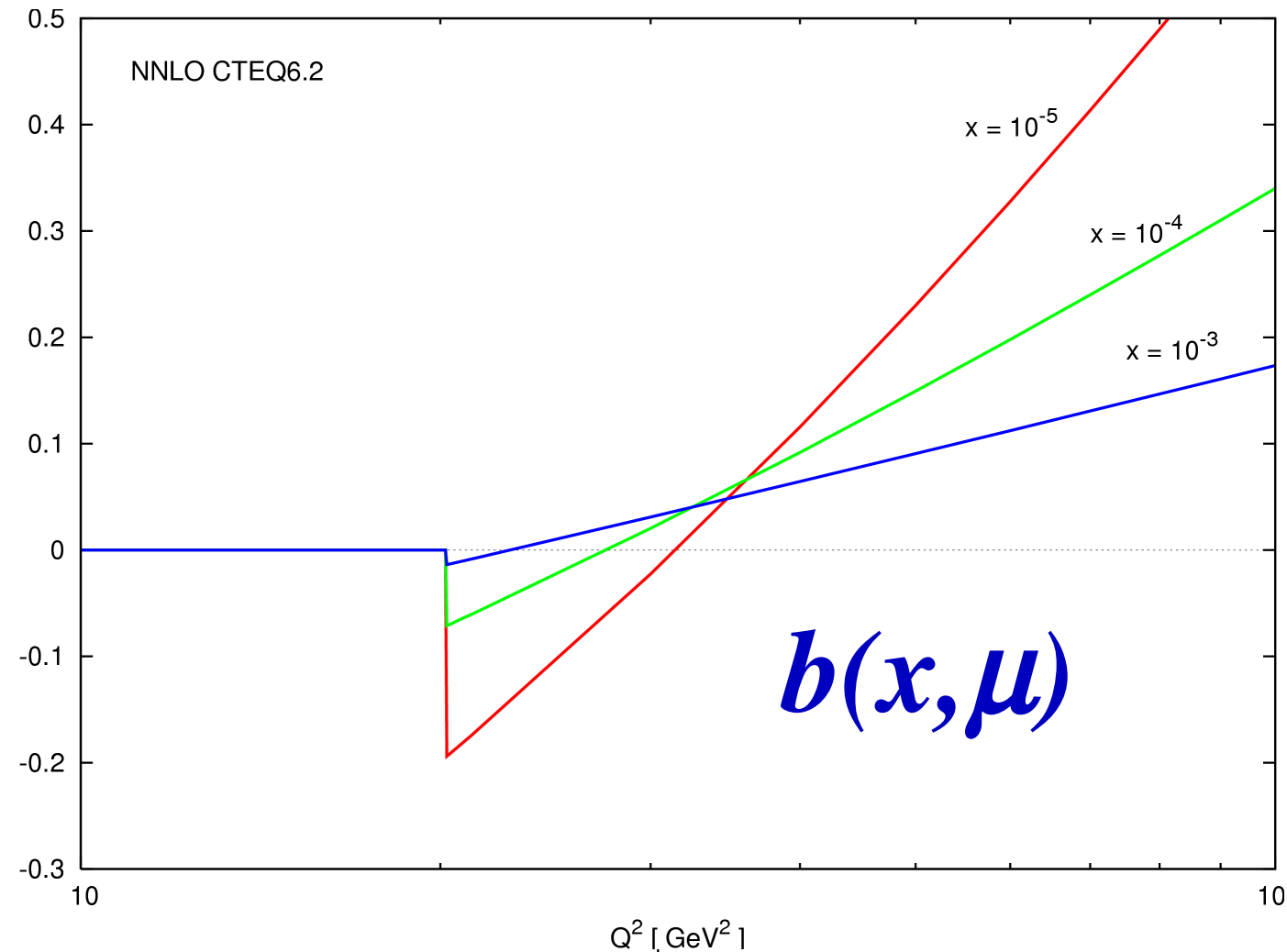
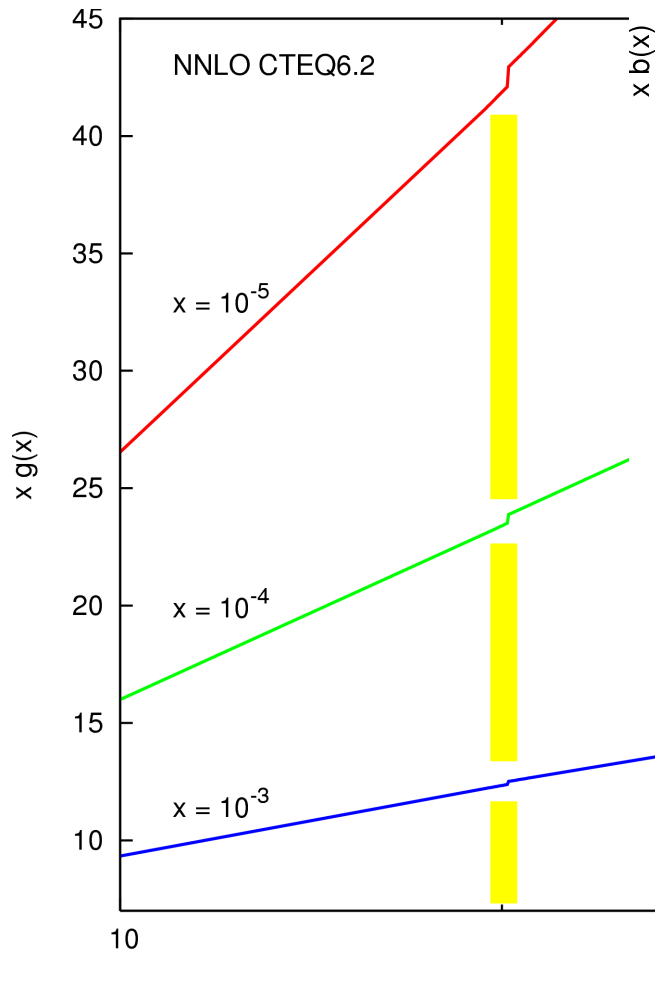


$$\alpha_{(n_f)}(M) = \alpha_{(n_f-1)}(M) - \frac{11}{72\pi^2} \alpha_{(n_f-1)}^3(M) + \mathcal{O}(\alpha_{(n_f-1)}^4)$$

NNLO

Buza, Matiounine, Smith,
van Neerven (1997)

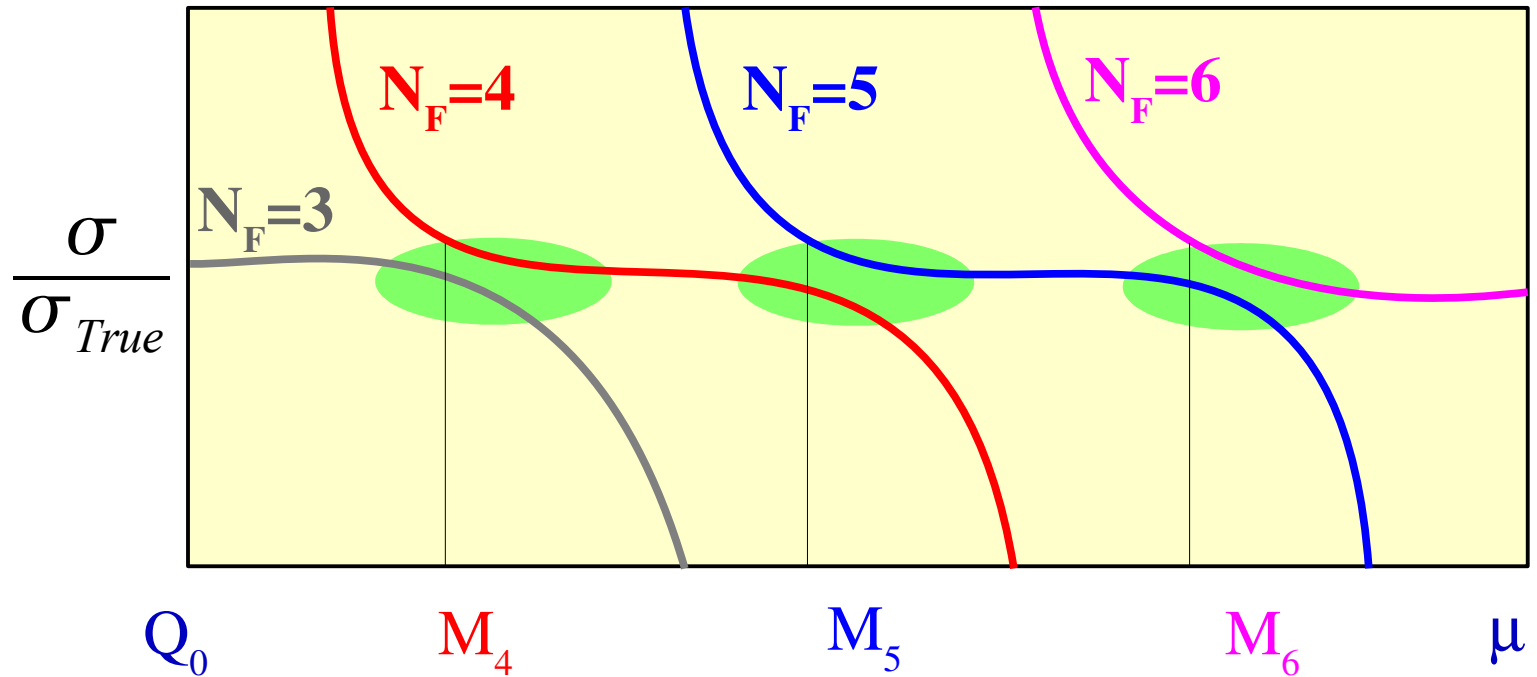
Bierenbaum, Blumlein, Klein (2009)



$g(x, \mu)$

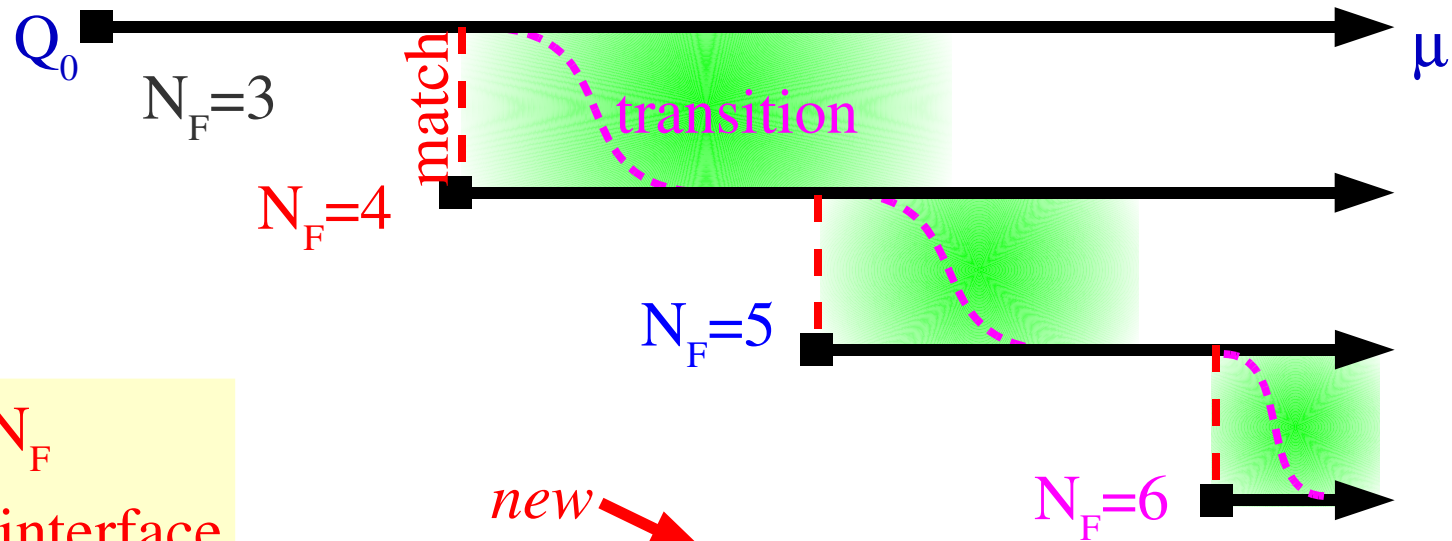
Field, Olness, Smith (2006)

Match at $\mu=m$
Transition at ...



Olness & Schienbein,
arXiv:0812.3371 [hep-ph]

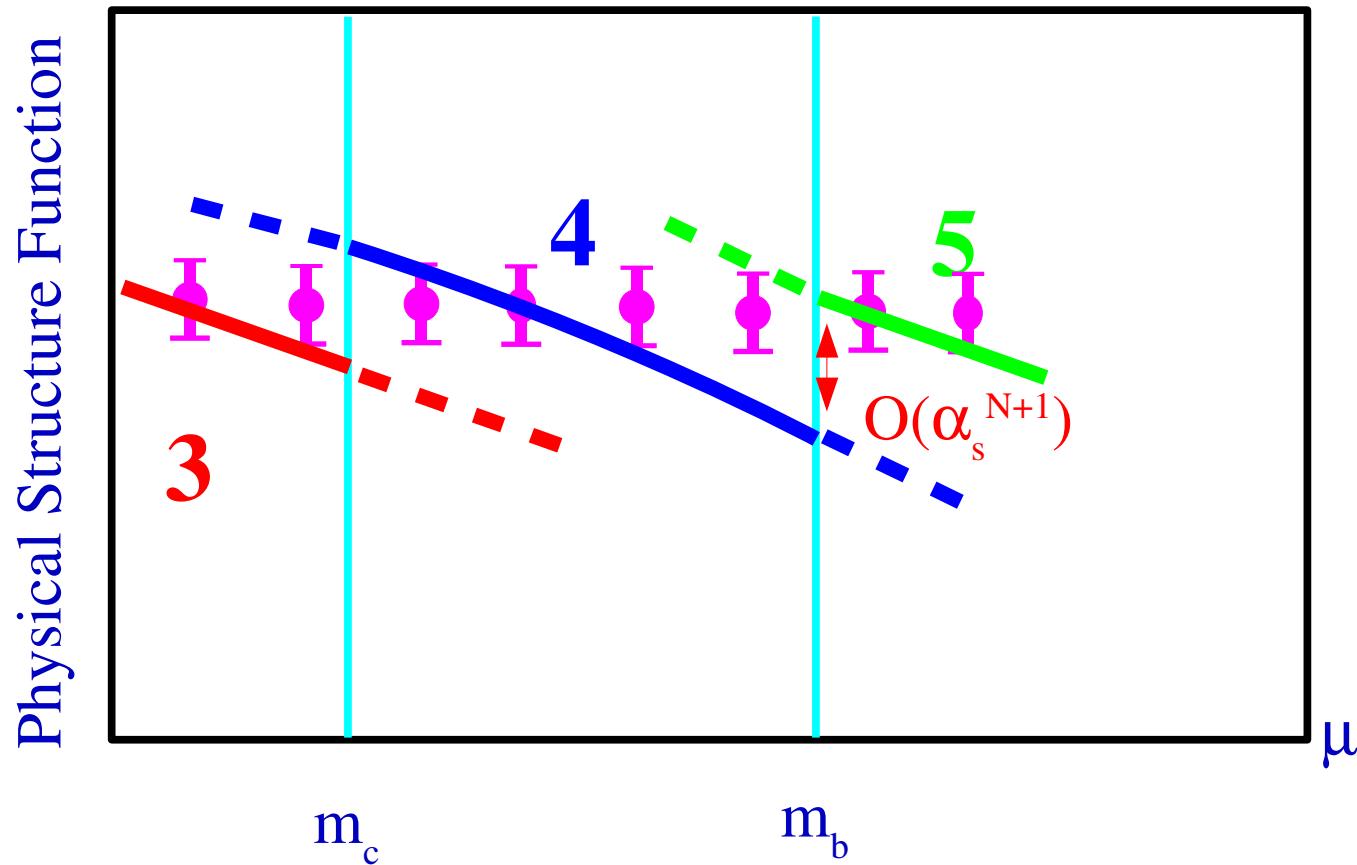
For $\mu \sim m$,
N and N+1
Schemes Co-exist



- Freedom to specify N_F
- Requires N_F in PDF interface
- Simplified Numerics

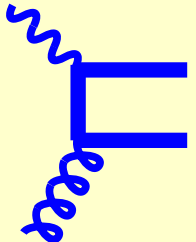
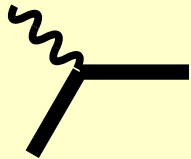
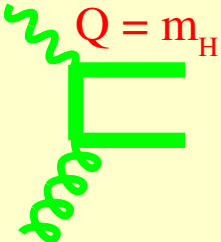
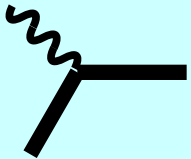
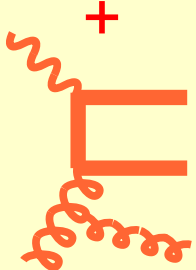
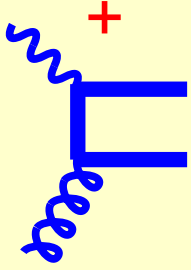
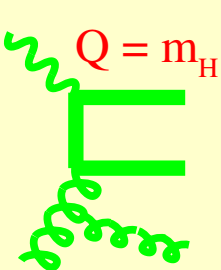
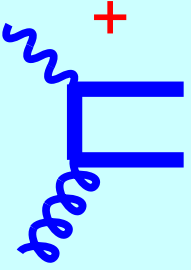
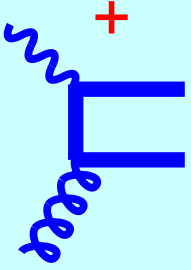
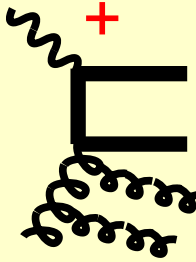
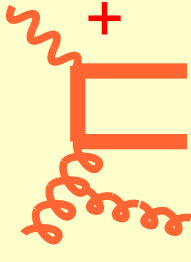
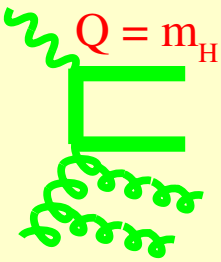
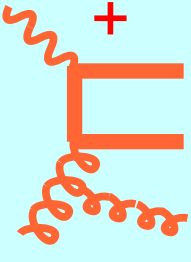
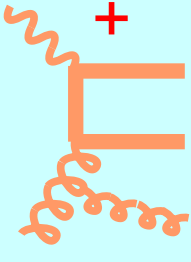
new →

$$f_{a/p}(x, Q, N_F)$$



- * Difference represents the theoretical uncertainty
- * Gaps will decrease with higher orders (*they must as physical quantities*)
 (*note: gaps of PDF's and α_s do not--these are unphysical quantities*)
- * If data prefers one scheme \Rightarrow optimal perturbative organization
- * Gaps between schemes reflects limit of theory uncertainty

Scheme
choices and
global fits

	TR type schemes			ACOT type schemes			
	$Q < m_H$	$Q > m_H$	constant term	$Q < m_H$	$Q > m_H$	constant term	
LO				LO	\emptyset		\emptyset
NLO				NLO			\emptyset
NNLO				NNLO			\emptyset

*Are we ready
for the LHC*

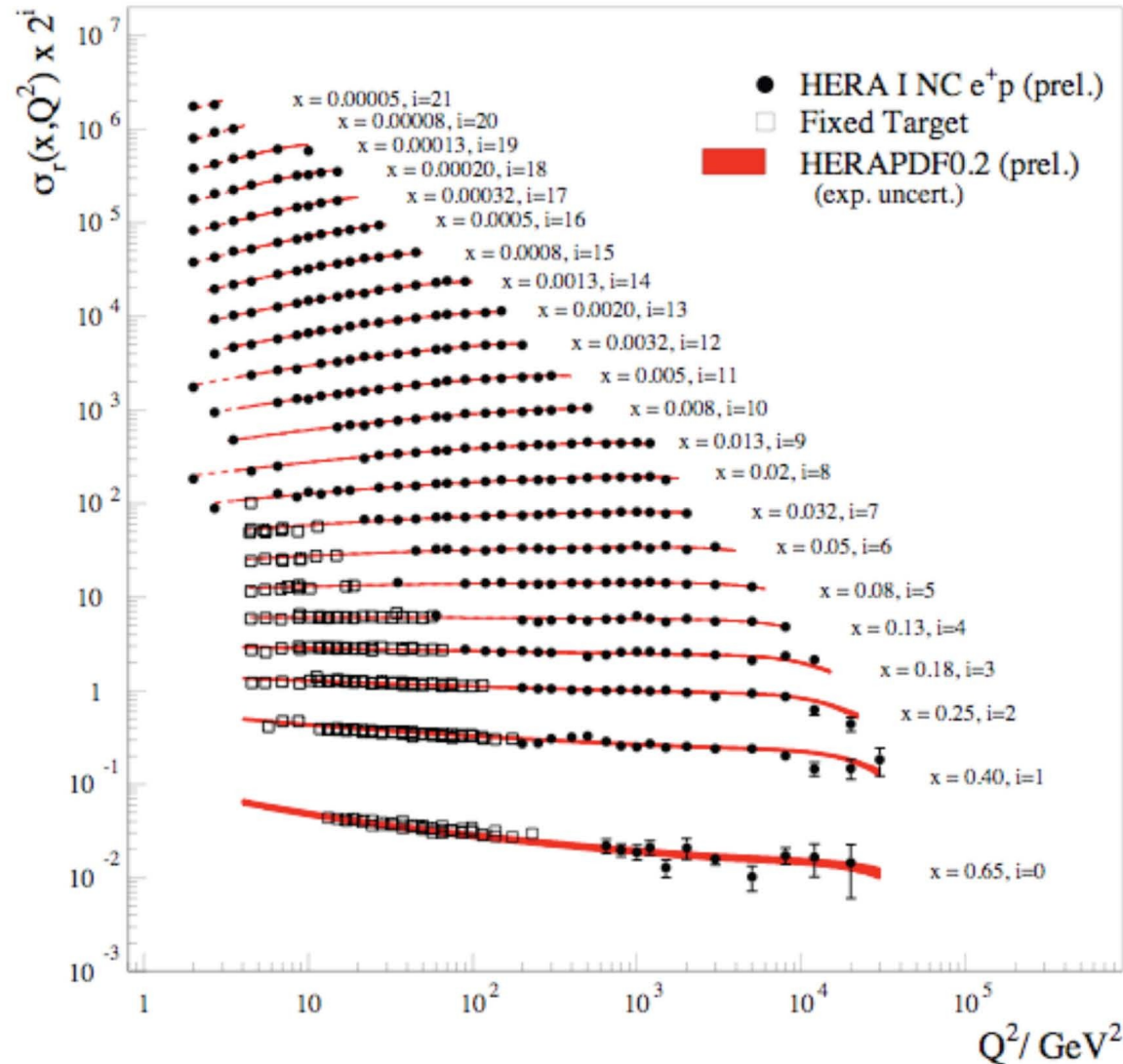
... yes ... no



Fit Results

- ($\chi^2 / \text{dof} = 576/592$)
- Plots show the extended kinematic range of the HERA data as compared to the fixed target measurements
- Plots include experimental uncertainties on both data and fit

H1 and ZEUS Combined PDF Fit



April 2009

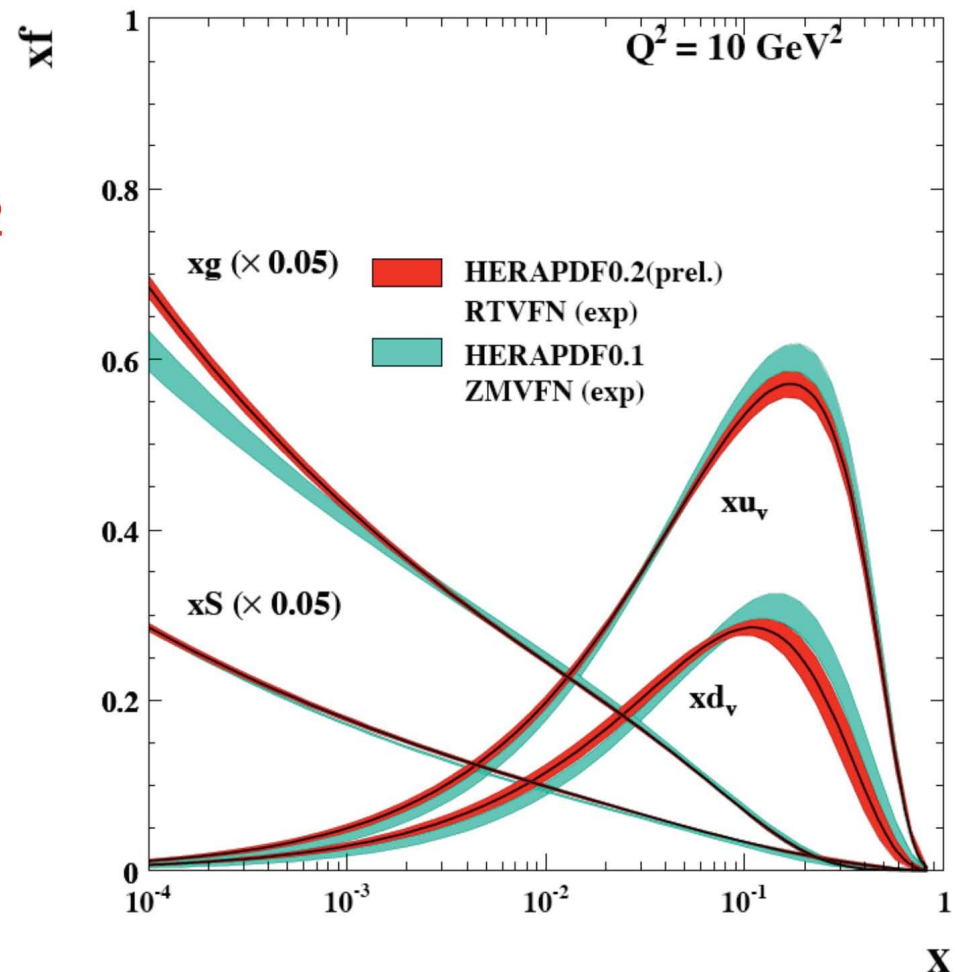
HERA Structure Functions Working Group

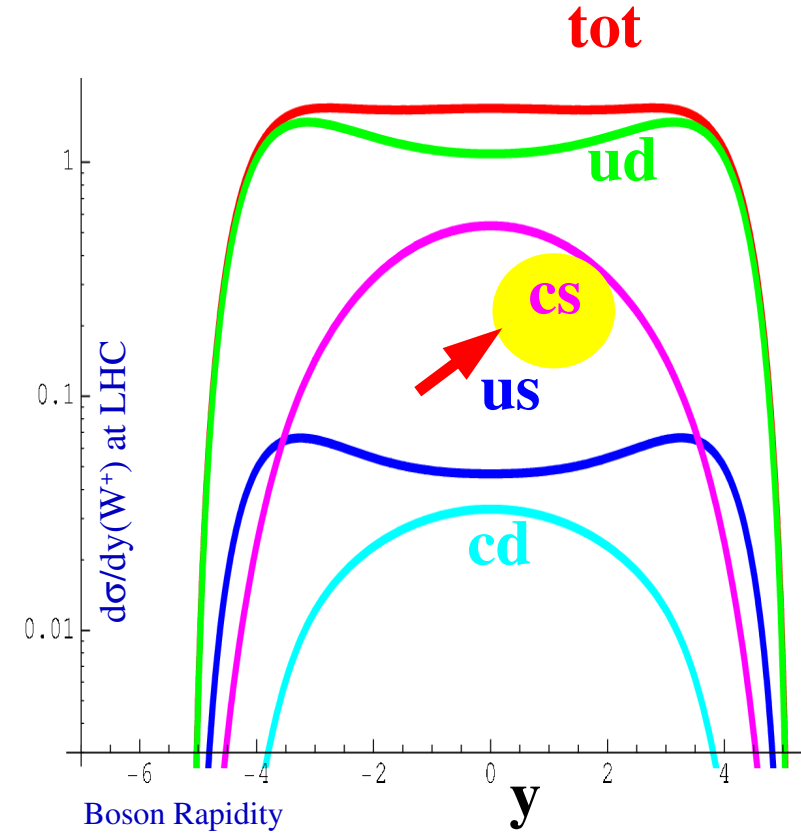
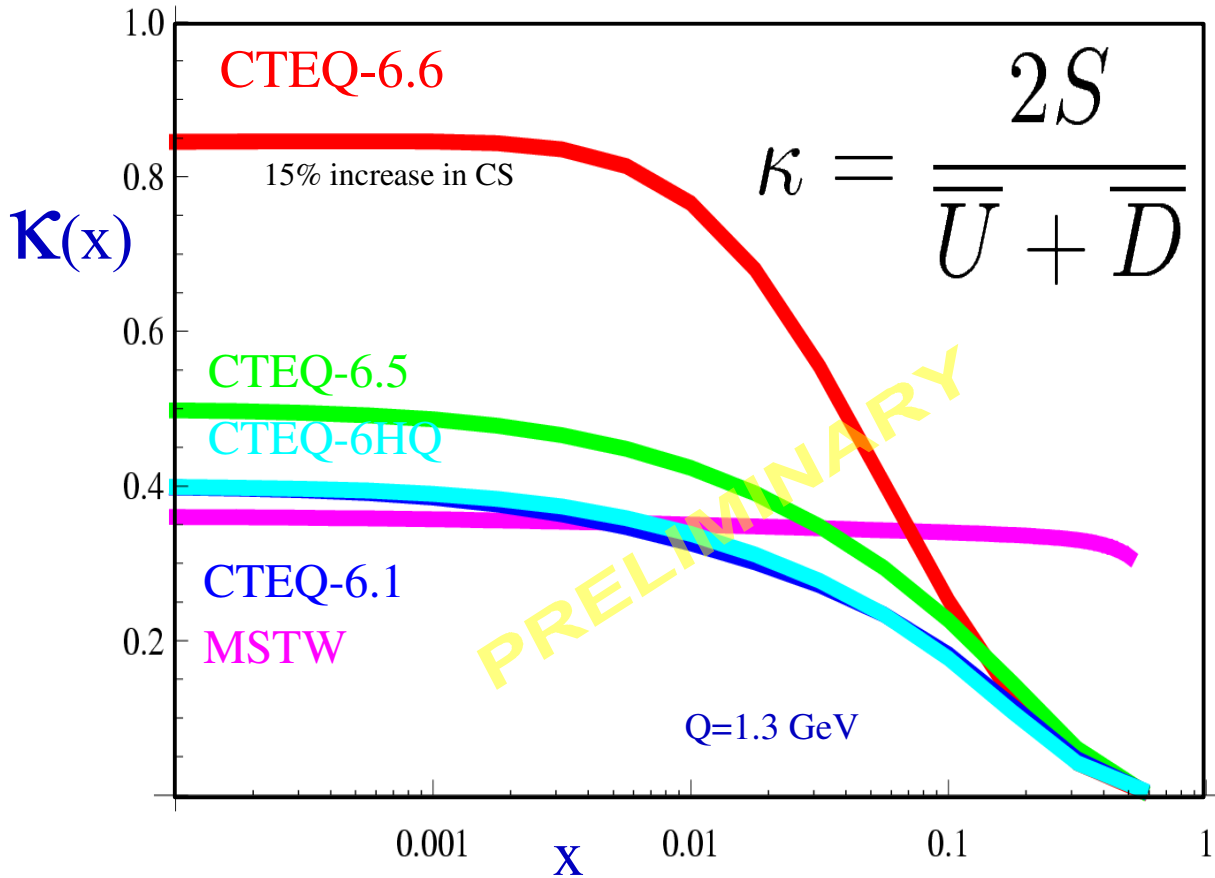
HERAPDF0.1 vs HERAPDF0.2

- For consistency, when comparing the HERA PDF sets only the **experimental errors** are used:
 - ⇒ The model uncertainties of the two PDF sets are not identical
 - ⇒ HERAPDF0.1 did not consider the uncertainty due to PDF parametrisation

- **Observations:**

- ⇒ Errors are smaller for **HERAPDF0.2**
 - ⇒ d_{val} is softer
 - ⇒ Gluon is steeper:
 - This is expected due to the heavy flavour treatment
- HERAPDF0.1** - massless quarks (ZM-VFNS)
- HERAPDF0.2** - massive quarks (TR-VFNS)





⇒ Different Y Distribution

... but HQ Uncertainties will feed into LHC “Benchmark” processes

Conclusions

“I have no data yet.

It is a capital mistake to theorize before one has data. Insensibly one begins to twist facts to suit theories, instead of theories to suit facts.“

The Adventures of Sherlock Holmes

Sir Arthur Conan Doyle

A Scandal in Bohemia

... data keeps one honest ...

* **Historically:** Calculation of Heavy Quarks challenging

* **Significant Progress:**

New theoretical tools enable reliable calculations

Heavy Quarks are included in DGLAP evolution

Heavy Quarks masses are included in Global Analysis

* **Current Challenges**

Complex at NNLO

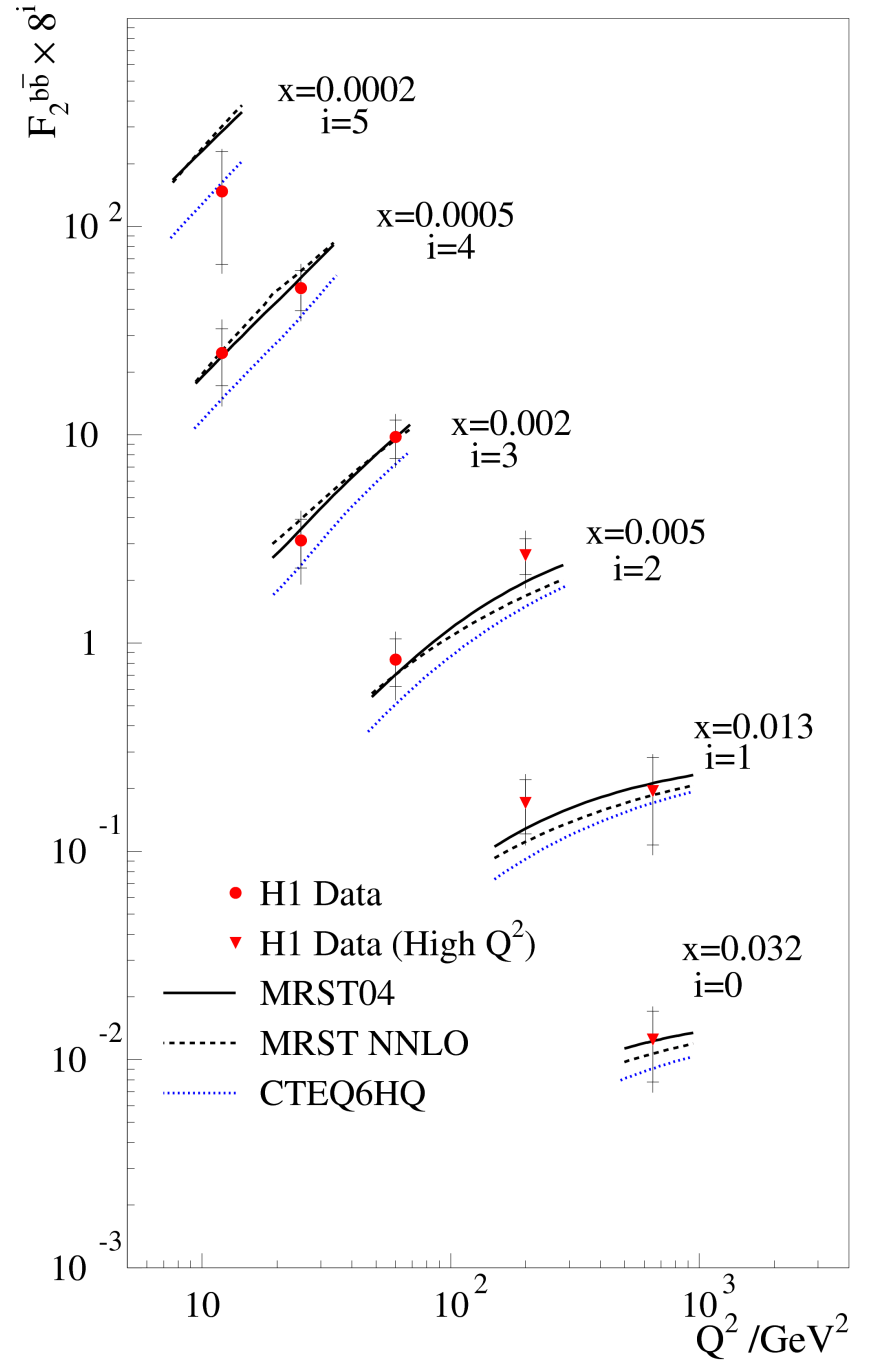
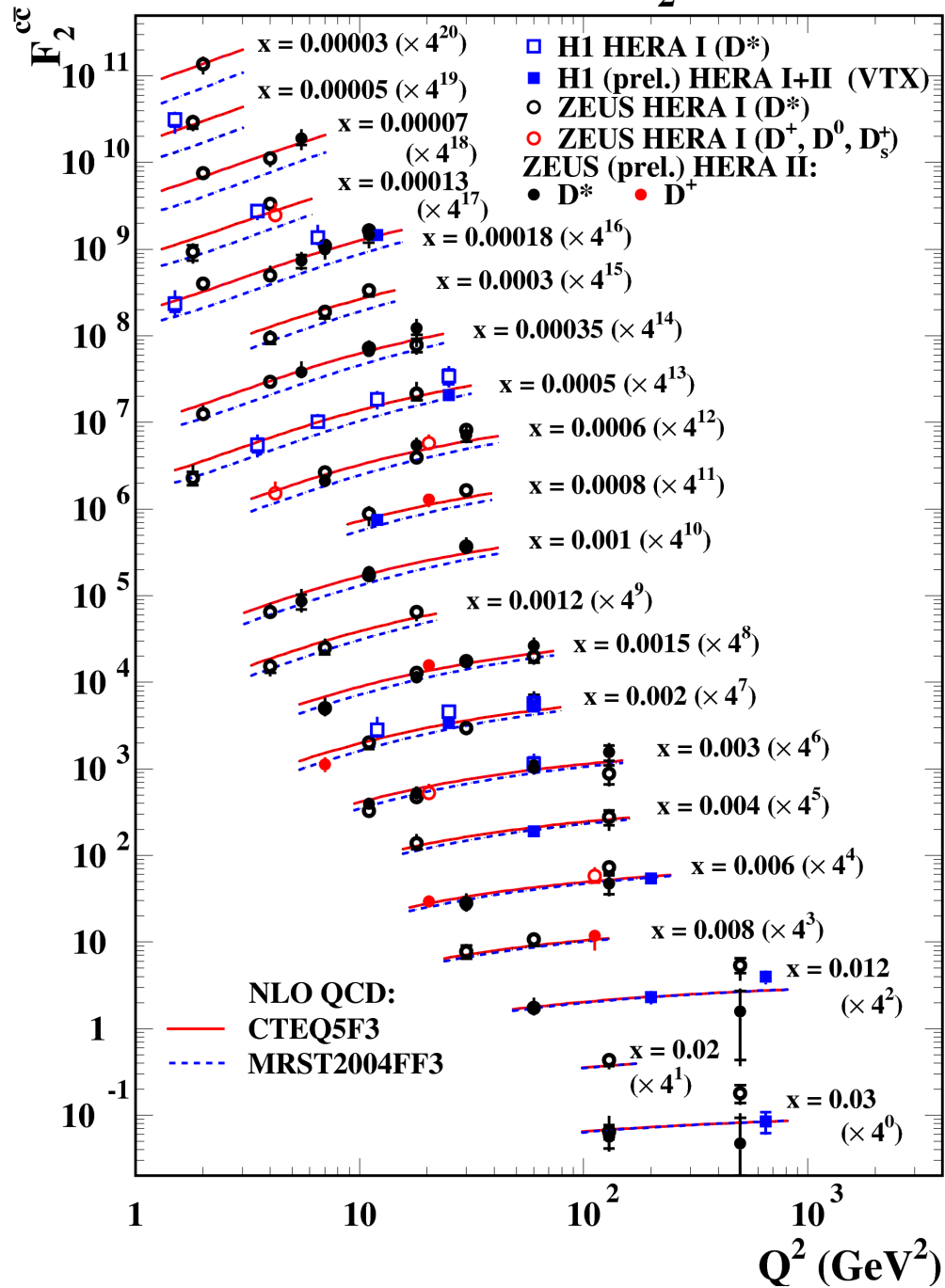
Heavy Quarks play a more prevalent role at the LHC

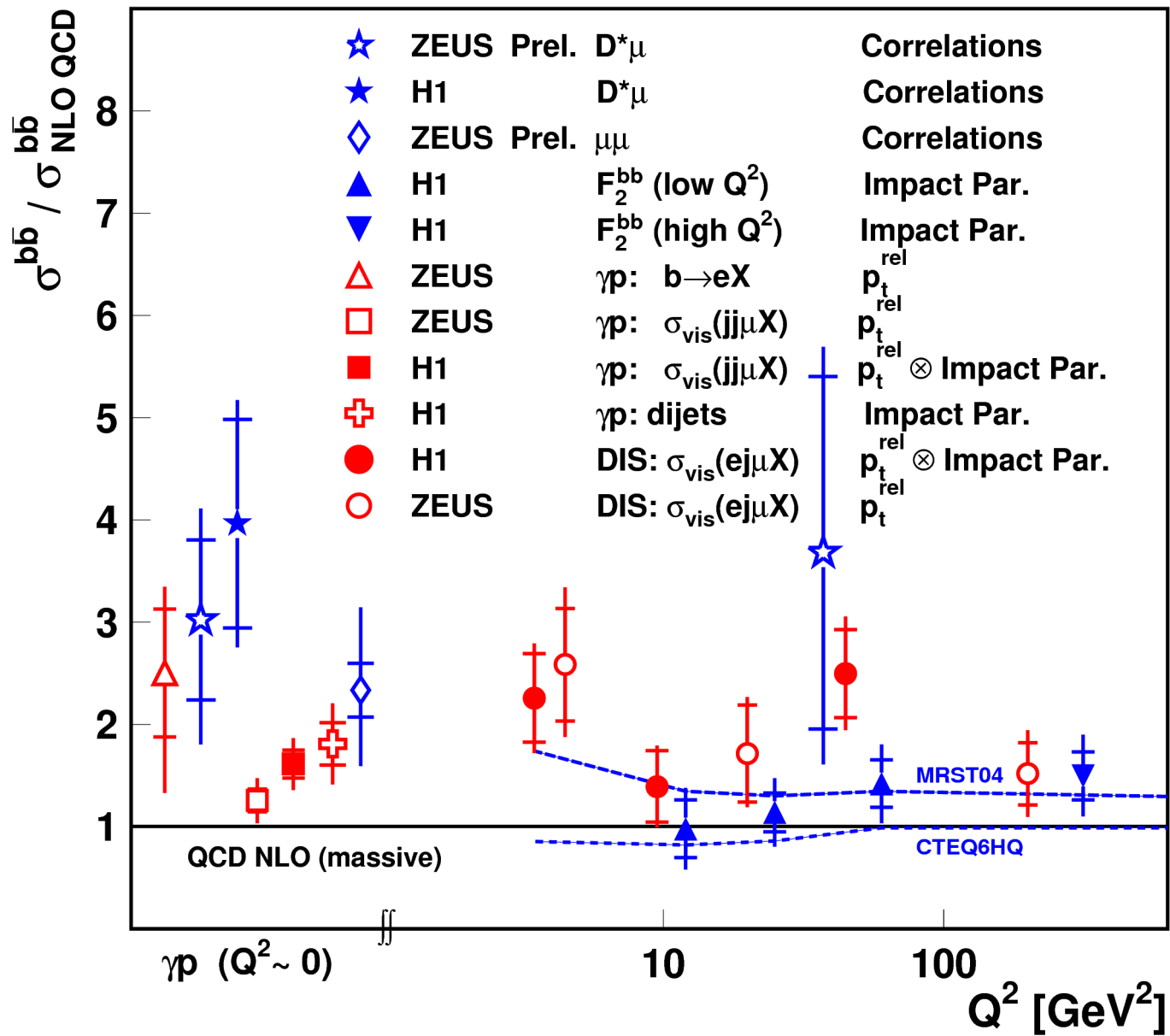
Need to accommodate these effects and uncertainties
to make best use of HERA & Tevatron data.
Also, be prepared to cross-check using early LHC data

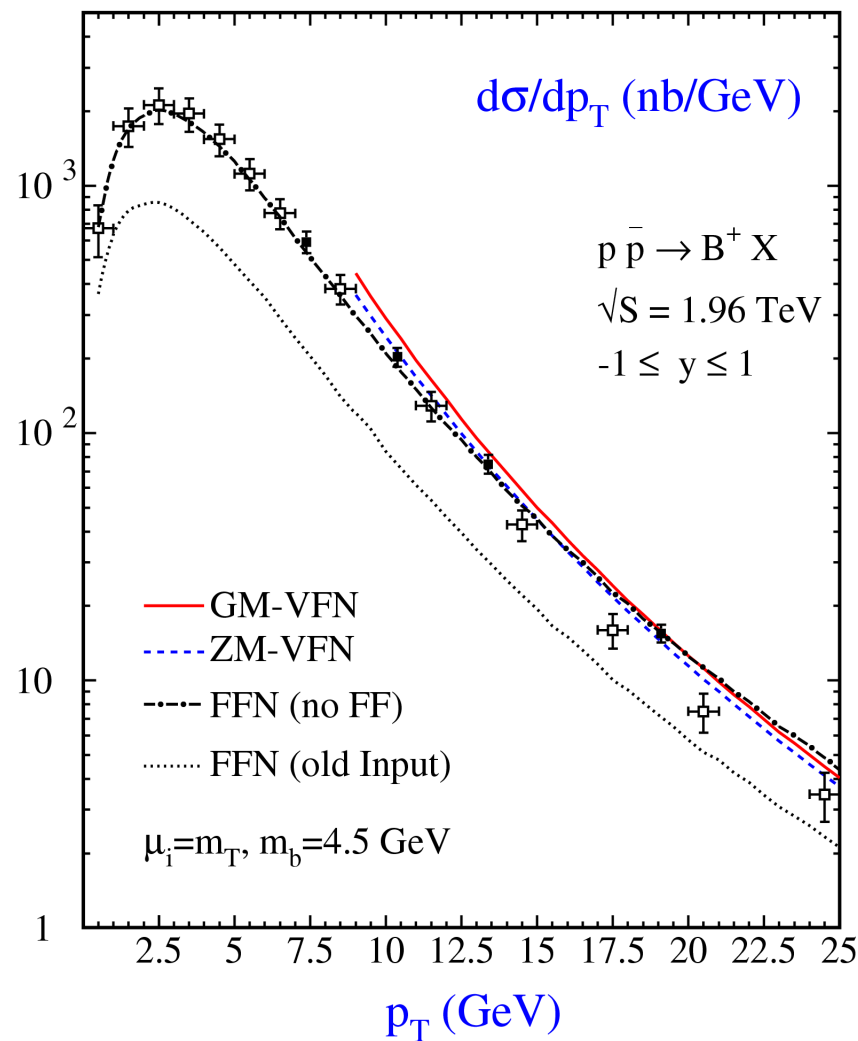
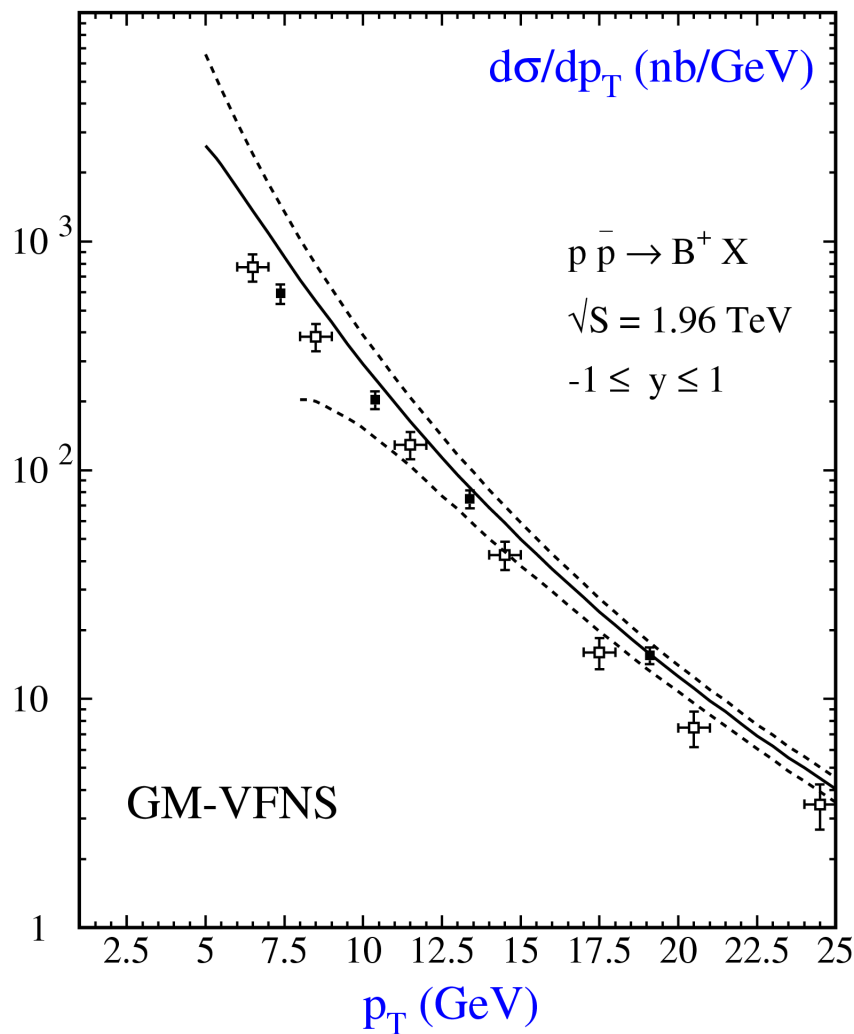
Backup

F Charm & Bottom

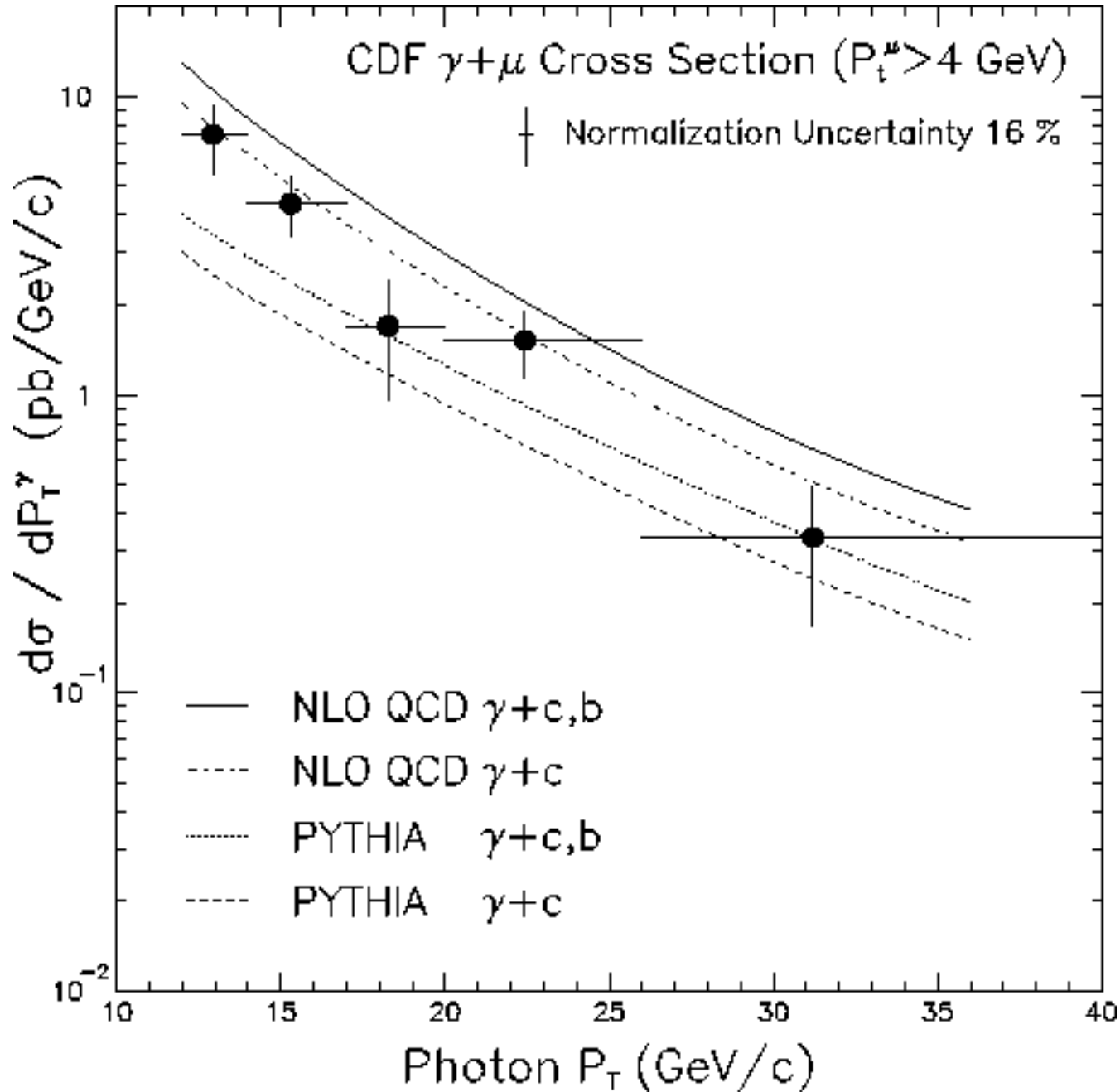
HERA $F_2^{c\bar{c}}$





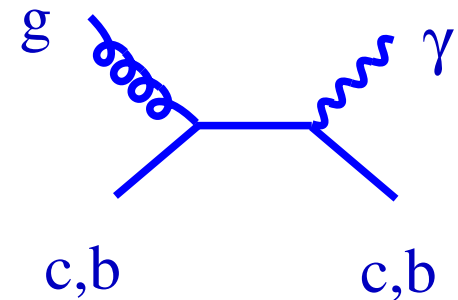


- GM-VFN prediction yields agreement w/ Run II data
- Contrast FFN w/ old input

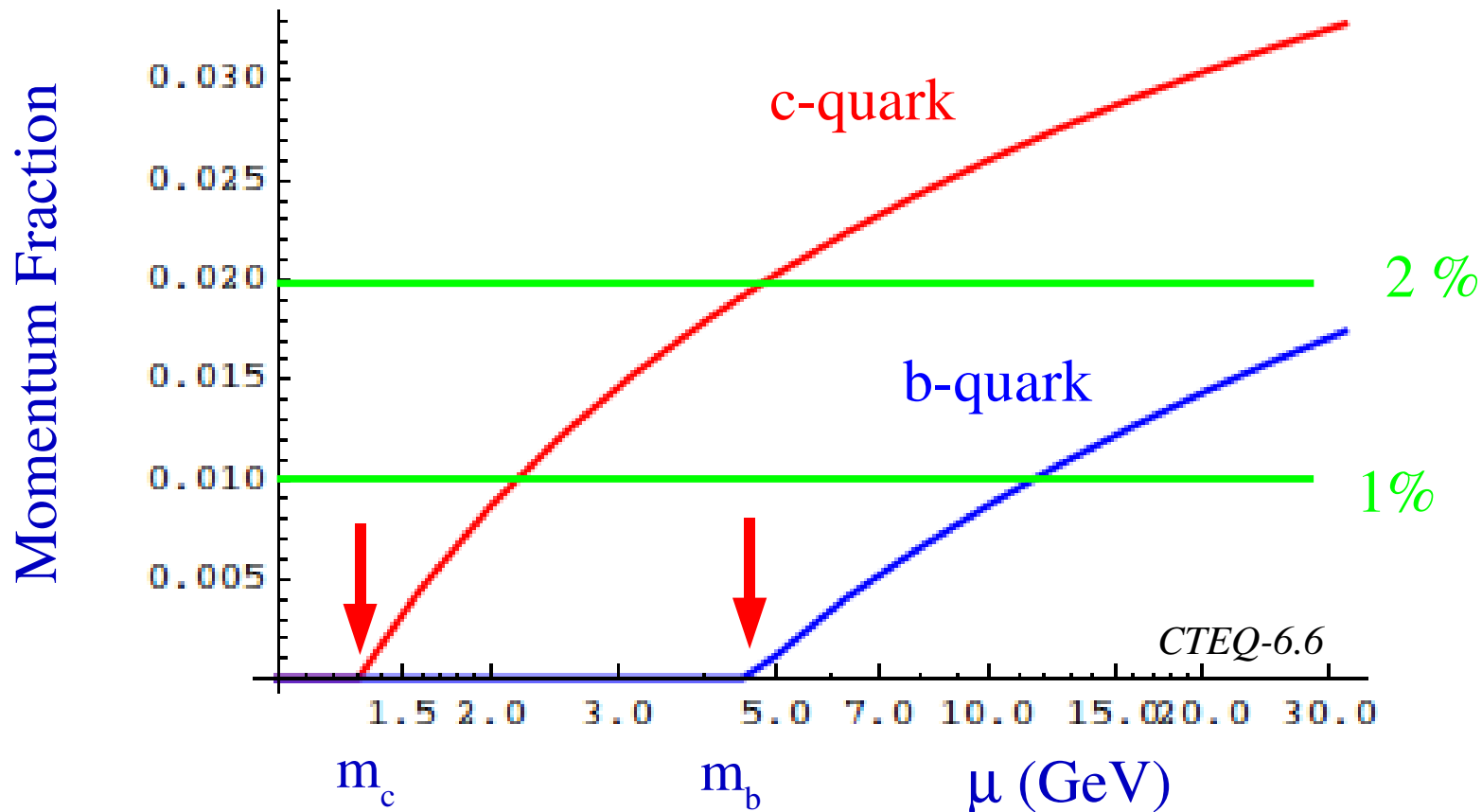


Comparison of Run I
Data with NLO Theory

Reasonable agreement
within theory
uncertainty



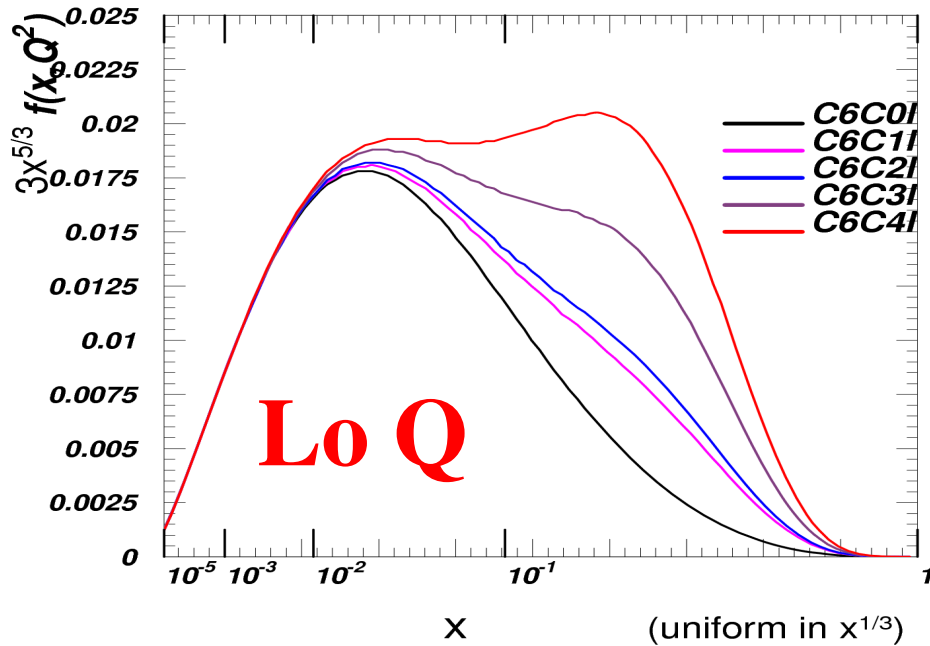
Intrinsic Heavy Quarks



- * Most sensitive near threshold
- * What happens if we allow the evolution to determine charm?

Zero:	No intrinsic charm
Positive:	Intrinsic charm
Negative:	Inconsistent

Charm PDF, $Q^2 = 10 \text{ GeV}^2$



Charm PDF, $Q = 85 \text{ GeV}$

